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The welfare effect of disclosure through media: a zero-sum case

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The welfare effect of disclosure through media: a zero-sum case

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Abstract

We extend the beauty contest framework to allow the disclosure of the authority to be received with an additional noise, the realization of which varies among agents. In this setup, we find that there could be a situation in which an anti-transparency policy maximizes welfare however precise the signal the authority can obtain.

Keywords: Public information; Interpretation errors; Welfare
JEL classification: D82

1 Introduction

In a beauty contest framework developed by Morris and Shin (2002, henceforth MS), public information, which is interpreted as a disclosure of economic forecast by the authority, may be harmful to social welfare; that is, anti-transparency may be optimal. However, the robustness of their result has been questioned. Angeletos and Pavan (2004) and Hellwig (2005) show that MS’s result depends on the form of the payoff function.1 Svensson (2006) claims that even in MS’s model, public information increases welfare under plausible parameter values.

In this note, we extend MS’s model to allow the disclosure to be received by agents with an additional noise, the realization of which varies among agents.2

In other words, we introduce the noise with which the message sent from the authority is received by market participants (receiver errors, or interpretation errors). There are several reasons for the existence of such interpretation errors. First, in many cases market participants receive announcements by the

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1Angeletos and Pavan (2007) analyze the welfare effect of public information in the more general form of payoff function.

2Myatt and Wallace (2008) analyze the Lucas island model in a similar information structure, while we analyze the beauty contest model of stock market.
authority through the media, and the messages that are stressed vary among media. Second, even if all media release the exact message of the authority, if the authority does not have sufficient credibility, market participants would not completely believe it and would interpret the message in various ways. In this setup, we show that, anti-transparency may maximize welfare however precise the information the authority can obtain about economic fundamentals. This note is organized as follows. Section 2 describes the model. Section 3 analyzes the welfare effect of greater precision of the public announcement. Section 4 seeks the point at which anti-transparency maximizes welfare. Section 5 concludes.

2 The setup and the equilibrium

The model is an extended version of MS, where the message from the authority is received by agents with interpretation errors.

In the economy, there is a continuum of agents, indexed by the unit interval \([0,1]\). Agent \(i\) chooses an action \(a_i \in \mathbb{R}\) to maximize his or her payoff. We write \(\theta \in \mathbb{R}\) for the state of the economy and \(a\) for the action profile of all agents. Agent \(i\)’s payoff function is given by

\[
u_i(a, \theta) \equiv -(1 - r)(a_i - \theta)^2 - r(L_i - \bar{L}), \tag{1}\]

where \(r\) is a constant with \(0 < r < 1\),

\[
L_i \equiv \int_0^1 (a_j - a_i)^2 dj, \quad \text{and} \quad \bar{L} \equiv \int_0^1 L_j dj. \tag{2}\]

We assume that agents face uncertainty concerning \(\theta\), but they have access to private signals and the disclosure of the authority through media. Agent \(i\) observes a private signal \(x_i = \theta + \varepsilon_i\) as in MS. The authority receives a signal \(y = \theta + \eta\) and releases it to the public. However, we assume that the authority does not have the technology to inform the public about the exact value of the signal, so that agent \(i\) receives the signal with a noise \(\zeta_i\), which we call agent \(i\)’s interpretation error. Therefore the message from the authority received by agent \(i\) is

\[
y_i = \theta + \zeta_i, \tag{3}\]

where \(\zeta_i \equiv \eta + \xi_i\). We assume that

\[
\varepsilon_i \sim N(0, 1/\beta), \tag{4}
\eta \sim N(0, 1/\alpha), \tag{5}
\xi_i \sim N(0, 1/\gamma); \tag{6}\]

hence,

\[
\zeta_i \sim N(0, 1/\delta), \tag{7}\]

with \(\delta^{-1} = \alpha^{-1} + \gamma^{-1}\). Thus, our model nests MS in the sense that it is the same as MS when \(\gamma^{-1} = 0\).
Under these assumptions, agent $i$’s expected value of $\theta$ is

$$E_i(\theta) = \frac{\beta x_i + \delta y_i}{\beta + \delta},$$

(8)

where $E_i(\cdot) \equiv E(\cdot | x_i, y_i)$ denotes the expectation operator on agent $i$’s information set.

The best response function of agent $i$ is given by the first-order condition:

$$a_i = (1 - r)E_i(\theta) + rE_i(\bar{a}).$$

(9)

where $\bar{a} \equiv \int_0^1 a_i \, di$ represents the average of their actions. As in MS, we can construct the following linear equilibrium strategy:

$$a_i = \mu x_i + (1 - \mu) y_i,$$

(10)

and hence

$$\bar{a} = \mu \theta + (1 - \mu) y.$$

(11)

From (8), (9), and (11),

$$a_i = (1 - r)E_i(\theta) + rE_i(\bar{a})$$

$$= [1 - r(1 - \mu)]E_i(\theta) + r(1 - \mu)E_i(y)$$

$$= [1 - r(1 - \mu)]\frac{\beta x_i + \delta y_i}{\beta + \delta} + r(1 - \mu)y_i$$

$$= \left\{ [1 - r(1 - \mu)]\frac{\beta}{\beta + \delta} \right\} x_i + \left\{ [1 - r(1 - \mu)]\frac{\delta}{\beta + \delta} + r(1 - \mu) \right\} y_i$$

(12)

Comparing (10) with (12), we obtain

$$\mu = \frac{\beta(1 - r)}{\delta + \beta(1 - r)}.$$

(13)

3 The welfare effect of greater precision of signals

The social welfare function is defined as

$$W(a, \theta) \equiv \frac{1}{1 - r} \int_0^1 u_i(a, \theta) \, di$$

$$= - \int_0^1 (a_i - \theta)^2 \, di.$$  

(14)

From (10) and (13),

$$a_i - \theta = \frac{\beta(1 - r)}{\delta + \beta(1 - r)} (x_i - \theta) + \frac{\delta}{\delta + \beta(1 - r)} (y_i - \theta).$$

(15)

Substituting (15) into (14) and calculating the expected value of the social welfare conditional on $\theta$, we obtain $ex \ ante$ welfare,

$$E[W(a, \theta)|\theta] = - \frac{\delta + \beta(1 - r)^2}{\delta + \beta(1 - r)^2},$$

(16)
Using (16) and the fact that $\delta^{-1} = \alpha^{-1} + \gamma^{-1}$, we analyze the welfare effect of greater precision of the private signal and the signal received by the authority, respectively. First,

$$\frac{\partial E(W|\theta)}{\partial \beta} = \frac{(1 - r)(1 + r)\delta + (1 - r)^2\beta}{[\delta + \beta(1 - r)^3]} > 0,$$

(17)

therefore greater precision of private information always increases ex ante welfare, as in MS. For the signal received by the authority, we can find the following result.

**Proposition 1.** Greater precision of the signal received by the authority (an increase of $\alpha$) decreases ex ante welfare if and only if:

$$\beta > -\frac{\gamma^2}{(2r - 1)(1 - r)} + \frac{\gamma}{(2r - 1)(1 - r)} \text{ and } r > \frac{1}{2}.$$  

(18)

**Proof.** It holds that

$$\frac{d\delta}{d\alpha} > 0 \text{ and } \frac{\partial E(W|\theta)}{\partial \delta} = \frac{\delta - (2r - 1)(1 - r)\beta}{[\delta + \beta(1 - r)^3]};$$

(19)

hence,

$$\frac{dE(W|\theta)}{d\alpha} = \frac{\partial E(W|\theta)}{\partial \delta} \cdot \frac{d\delta}{d\alpha} < 0$$

(20)

if and only if

$$\frac{\beta}{\delta} > \frac{1}{(2r - 1)(1 - r)} \text{ and } r > \frac{1}{2}.$$  

(21)

Using the fact $\delta^{-1} = \alpha^{-1} + \gamma^{-1}$, we can see that (21) is equivalent to (18).

Equation (16) is the same as in MS except that $\alpha$ is replaced by $\delta$; hence when $\gamma^{-1} = 0$ our model is the same as in MS and a decrease of $\gamma$ in our model is equivalent to a decrease of $\alpha$ in MS. Figure 1 illustrates the welfare effect of greater precision of the signal received by the authority. When $r > 0.5$, there are ranges of the parameters where greater precision of the signal received by the authority is harmful to welfare, as in MS. Furthermore, the ranges are no smaller than in MS and they become wider as interpretational errors increase (lower $\gamma$). In particular, it is important that the borderline is a hyperbola rather than a straight line in MS. From this fact, we establish the following result.

**Corollary 1.** If $r > 0.5$ and $\beta > \frac{\gamma}{(2r - 1)(1 - r)}$, then, for any $\alpha$, greater precision of the signal received by the authority decreases ex ante welfare.

In MS in which $\gamma^{-1} = 0$, greater precision of the signal received by the authority increases welfare when it is sufficiently precise. In contrast, Corollary 1 implies that, when interpretation errors are large as compared with the precision of the private signal, greater precision of the signal received by the authority decreases welfare, however precise it may be.

3We can easily show that the borderline is tangent to MS’s at origin, and that the two lines coincide when $\gamma^{-1} = 0$. 

[Figure 1]
4 The hurdle rate and anti-transparency policy

Following MS and Svensson (2006), we assume that the authority is restricted to choosing $\alpha$ from some given interval $[0, \bar{\alpha}]$. Given $\beta$ and $\gamma$, we write $V(\alpha; \beta, \gamma)$ for the ex ante welfare when the precision of the signal the authority receives is $\alpha$, that is,

$$V(\alpha; \beta, \gamma) \equiv E(W|\theta) = -\frac{\alpha \gamma}{\alpha + \gamma} + \beta(1 - r)^2 \left[\frac{\alpha \gamma}{\alpha + \gamma} + \beta(1 - r)\right]^2,$$  \hspace{1cm} (22)

and define the hurdle rate $\bar{\alpha}^*$ implicitly as

$$\bar{\alpha}^*(\beta, \gamma) = \min \alpha \text{ s.t. } V(\alpha; \beta, \gamma) \geq V(0; \beta, \gamma).$$ \hspace{1cm} (23)

The hurdle rate means that if $\bar{\alpha} < \bar{\alpha}^*$ then welfare with disclosure is lower than welfare without it. Solving (23), we obtain the following result.

Proposition 2.

$$\bar{\alpha}^*(\beta, \gamma) = \begin{cases} \beta(2r - 1) \gamma & \text{if } \gamma > \beta(2r - 1), \\ \gamma - \beta(2r - 1) & \text{otherwise}. \end{cases}$$ \hspace{1cm} (24)

Figure 2 illustrates (24) for given $\gamma$. We can ensure that when interpretation errors are large, anti-transparency maximizes welfare however precise the signal the authority could obtain about fundamentals. Svensson (2006) claims that anti-transparency would be suboptimal, because in reality, the authority can obtain more precise signals than agents, that is, $\bar{\alpha} > \beta$. Our model implies that even if $\bar{\alpha} > \beta$, anti-transparency may be optimal, because interpretational errors are not necessarily smaller than the precision of private information and that the authority should forecast only the fundamentals precisely, but should be credible enough for its message to be believed by the public.

5 Conclusion and future research directions

In this note, we show that, because of interpretation errors, there could be a situation in which anti-transparency maximizes welfare, however precise the signal the authority could obtain about fundamentals. Our analysis is quite stylized, but it contains some interesting directions for future research. First, a similar analysis under a different payoff function would be interesting. In their ongoing research, Arato and Nakamura (2009) find that the payoff function as in Angeletos and Pavan (2004) would let $\alpha$ and $\gamma$ have a richer welfare effect, because welfare depends not only on the heterogeneity of the agents’ actions but also on the volatility of their average action. Second, the extension toward a dynamic setup would bring about richer policy implications. As mentioned in Section 1, $\gamma$ could be interpreted as the authority’s credibility. The authority that makes more efforts toward information acquisition and transparency would become more credible. If so, the dynamic optimal information acquisition policy might be different from the static one.
References


\[
\beta = -\frac{\alpha}{(2r-1)(1-r)} \quad \text{(MS)}
\]

\[
\frac{dE(W|\theta)}{d\alpha} < 0
\]

\[
\beta = -\frac{\gamma^2/(2r-1)(1-r)}{\alpha + \gamma} + \frac{\gamma}{(2r-1)(1-r)}
\]

\[
\frac{dE(W|\theta)}{d\alpha} > 0
\]

Figure 1: Welfare effect of greater precision of public signal

Figure 2: The hurdle rate