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A New Keynesian Model with Endogenous Frictions

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Abstract

We develop a New Keynesian model that incorporates rigidities in the ability of households and firms to adjust their utility-efficient / profit-efficient resource allocation in response to shocks. These rigidities reflect the fact that households and firms enter into commitments for several periods of time regarding the allocation of resources limiting their ability to flexibly respond to unforeseen shocks. We show that these rigidities can adversely impact the productivity of firms and households' utility and result in the appearance of higher statistical moments in the demand and supply curves which are not exogenously constant but system-endogenous. As a result, we will derive the appearance of an inflation bias which exists even in the case of an efficient natural output and which cannot be removed by a rule-based monetary policy. Further, we show that monetary policy faces an additional trade-off in managing the friction losses due to inflation uncertainty and output uncertainty. (JEL D81, E10, E52).

I Introduction

In this paper we elaborate the impact of rigidities in the ability of economic subjects to adjust their resource allocation to unforeseen income and inflation shocks on households and firms as well as welfare. Further we derive the implications of these rigidities for the conduct of monetary policy.

To be precise, we argue that in addition to price and wage rigidities widely taken into account in research, economic subjects also face rigidities regarding their consumption and resource allocation, i.e. households commit part

of their income for consumption over several periods of time. Likewise, firms commit resources for several periods in advance and hence are limited in their ability to adjust their resources to shocks. We show that uncertainty regarding future income and inflation creates friction losses when consumption on the demand side and resources on the supply side are not fully flexible, but can only be changed within a certain time frame due to the fact that both households and firms commit resources for several time periods.

As a consequence, we will show that the model equations for demand and supply will not only incorporate expectations but also higher statistical moments of endogenous variables representing the afore-mentioned friction losses caused by rigidities in the form of the limited ability of economic subjects to adjust their resource plans to shocks. Further, we will show that these statistical moments show the property that we call endogenous uncertainty, i.e. they are not exogenously prescribed but defined within the economic system. In particular, they depend on the conduct of monetary policy.

The key results that we derive in our model are

- The natural interest rate and the natural output level are dependent on the second order statistical moments representing friction losses and are hence influenced by the conduct of monetary policy through the determination of the underlying probability distribution of endogenous variables.
- On the demand side we see that frictions reduce the impact of future income expectations on present demand, since households are aware of the risk of future friction losses and hence take them into account by saving more as a buffer against potential future losses.
- On the supply side frictions caused by firms' inflexibility to adjust to shocks reduce the average productivity and hence the natural output level, because firms are on average inefficiently invested in labor. These friction losses in production are passed on to consumers in the form of higher markups. Further we see that firms facing uncertain future inflation levels have a tendency to "over-price" their products, resulting in an additional inflation bias which exists even in the case where the natural output is efficient and which cannot be removed by a rule-based monetary policy.

- Monetary policy faces a new trade-off in managing friction losses due to output variability and inflation variability. Thus, the choice of a monetary policy regime does not only influence demand and supply directly by the corresponding policy function, but also indirectly through the statistical moments of the probability distribution of endogenous variables that result from the choice of the respective policy.

The global impact of these frictions on the supply side reducing productivity and on the demand side absorbing part of a households' purchasing power can be summarized in the following stylized equations:

$$\text{Gross household income} = \text{Output} - \text{frictions in production} \quad (1)$$

$$\begin{aligned} \text{Disposable household income} &= \text{Gross household income} \\ &- \text{frictions in consumption} \end{aligned} \quad (2)$$

It is important to mention that the analysis of both aspects – uncertainty and rigidities – within the New Keynesian model framework has been an important focus of recent research. In particular the inclusion of costs of re-allocation is also considered by other authors, e.g. Smets and Wouters (2003) and (2007) and Christiano et al. (2005) develop a model where adjustment to the utilization of the capital stock of households incurs costs of re-allocation and where the empirically observed persistence of consumption is introduced into the model framework by external habit formation. Our description of costs of re-allocation is different in two aspects: First of all, the persistence of consumption is not introduced by external habit formation but is explained by the costs economic subjects face when they have to alter long-term consumption plans and hence provides an alternative explanation for households' consumption persistence. Secondly, we include costs of re-allocation on the supply side as well, where they will appear in the form of overhead costs due to labor surplus when demand is below expectations or additional short-term hiring costs whenever demand for goods exceeds previous expectations. Hiring costs have been integrated into a New Keynesian model framework before, in particular by Blanchard and Gali 2008, who assumed that period t labor demand N_t is given by a fixed factor of previous employment $(1 - \delta)N_{t-1}$ plus new hires causing hiring costs depending on the tightness of the job market. Our model differs in the sense that the driver of hiring costs are surprise fluctuations in the demand for a firm's good and the limited ability of firms to adjust their labor force in a timely manner,

whereas in the model of Blanchard and Gali (2008) the costs are determined by the absolute amount of hiring and the tightness of the job market.

As regards the role of uncertainty within New Keynesian models, an important focus of research is the so-called robust control theory (cf Hansen and Sargent, 2001; Dennis, 2007 and Svensson and Williams, 2007), which analyzes a monetary authority under model uncertainty, i.e. one assumes that the "real" model for the economy which provides the observer with economic data differs from the model used by central banks. The intention of the robust control theory is to derive monetary decision rules, that work in the set of models close to the known model.

Another important concept is the well-known certainty equivalence theorem, which basically states that the behavior of the economic system and the way monetary policy should be conducted is determined by expectations of endogenous variables. Uribe and Schmitt-Grohe (2003) have proved that in a very general equilibrium model setup, higher order approximations lead to model equations containing higher statistical moments of shock variables, but only as constant terms, i.e. the reaction coefficients of monetary policy remain unimpacted. This can be interpreted as a further justification of the certainty equivalence theorem. A similar result is found by other authors, e.g. the recognition of signal extraction in the model setup as outlined in Swanson (2000) leads to a modification of the certainty equivalence result expressed as terms which contain constant higher statistical moments.

It is important to mention that in the context of the robust control theory and the certainty equivalence theorem probability distributions of shock variables are exogenously constant which implies constant statistical moments of the distribution functions, in contrast to system-endogenously defined probability distributions that we will discover in this paper.

Compared with the above-mentioned areas of research, our paper elaborates a different aspect of uncertainty: We analyze the role of uncertainty in the decision-making process of households and firms facing rigidities in their resource allocation due to commitment, whereas the above-mentioned robust control theory focuses on the role of uncertainty from the policy makers' point of view. In brief, our model approach uses the following line or argument:

- Households and firms face rigidities regarding their allocation of resources over time, i.e. part of their resources is pre-committed for several periods and hence cannot be adjusted flexibly when shocks occur or the adjustment after shocks implies frictions in the form of additional costs (e.g. contractual penalty fees) to re-arrange the committed resource plans.
- These rigidities appear in the optimization problem of households and firms and will result in second (and possibly higher) order statistical moments in the model equations for demand and supply, representing friction losses due to inefficient resource allocation.
- These higher statistical moments are not exogenously constant but are system endogenous. In particular, they depend on the type of shocks to the system and the conduct of monetary policy, i.e. the monetary authority can actively influence the underlying probability distributions and hence the corresponding statistical moments and thereby actively manage the trade-off between friction losses caused by output variability and inflation variability.

The derivation of the log-linearized model equations for demand and supply is performed in two steps:

1. At first we take the expectation of the equations for demand and supply over those endogenous variables that cause friction losses for economic subjects. This reflects the fact that economic subjects are aware of these potential losses and take them into account in their future resource planning in the present period.
2. Afterwards, we perform a standard log-linearization of the system.

Performing step one before step two ensures that friction losses appear in the form of second order moments in the model.

For both demand and supply we will show that rigidities in the flexibility of adjusting resources after shocks can be introduced into the model in two different ways: Firstly, by directly imposing a rule of inflexibility regarding shocks for several periods of time. Secondly, by imposing costs of re-allocation, i.e. economic subjects adjusting a pre-committed resource path

face additional costs, which for instance occur when contractual commitments have to be adjusted. We will show that the second concept is more general and hence will be used to derive our model equations.

The structure of this paper is as follows: In section two we integrate the above-mentioned inflexibility of households for adjusting their resource path after shocks into their optimizing behavior and derive the corresponding New Keynesian IS curve. Section three develops a model for profit-maximizing firms facing rigidities in their ability to adjust their labor force in response to demand shocks for their goods in the short-term to derive an inflation adjustment curve (IA curve). Section four analyzes the role of monetary policy within a log-linearized model framework before the conclusion in section five.

II Decision making of households

We want to derive a model that shows the impact of rigidities in the ability of households to optimally adjust their resources when shocks occur. To be precise, one can distinguish between two types of goods:

1. C_t is a good for which a household has (contractually) pre-committed a certain consumption path over several periods of time and hence faces rigidities when trying to adjust this consumption path after the occurrence of unforeseen shocks. For instance, mortgages, apartment rents, leasing contracts etc. typically imply a commitment of a household's consumption for several periods.
2. D_t denotes a fully flexible good, i.e. households can increase or reduce their consumption in this good in response to sudden income changes. For instance, food, clothes etc. would be deemed to be flexible goods.

For the sake of simplicity and without loss of generality, we will neglect fully flexible goods D_t in the description of households, because this type of good has become the standard description of consumption within the family of New Keynesian models and it is straightforward to see that adding a flexible good D_t to the model setup below will not change the results we develop in the following.

The period- t utility function of households entailing consumption C_t , real

money holding M_t and labor supply N_t reads:

$$u_t := \frac{C_t^{1-\alpha}}{1-\alpha} + \frac{\tau}{1-\beta} M_t^{1-\beta} - \chi \frac{N_t^{1+\eta}}{1+\eta} \quad (3)$$

with $0 < \alpha < 1$, $0 < \beta < 1$, $\tau > 0$, $\chi > 0$ and

$$C_t = \int C_t^{\frac{\theta-1}{\theta}}(i) di, \quad N_t = \int N_t^{\frac{\theta-1}{\theta}}(i) di$$

The household income consists of the real wage earned $(W_t/P_t)N_t$ based on its labor supply N_t and real profits received from firms Π_t :

$$V_t = \frac{W_t}{P_t} N_t + \Pi_t \quad (4)$$

Further, households can hold bonds B_t and hence the budget constraint reads:

$$V_t = C_t + (1 + \pi_t)M_{t+1} - M_t + \frac{B_{t+1}}{1 + r_t} - B_t \quad (5)$$

Equation (5) basically states that the expected income surplus $V_t - C_t$ not used for consumption in period t can be used to either increase expected real money balances or bond holdings, where $(1 + \pi_t)$ denotes the amount of money an economic subject has to save at the beginning of period t to own one real money unit at the beginning of period $t + 1$, whereas $\frac{1}{1+r_t}$ denotes the price of a bond at time t that pays off one real money unit at time $t + 1$.

A Frictions in consumption

To assess the impact of shocks on households, we make the following assumption: In period T the household calculates its optimal allocation including consumption $C_{t|T}$ for $t \geq T+1$, based on the expected future income $V_{t|T}$ and inflation levels $\pi_{t|T}$. We assume that the household enters into a commitment (e.g. by contractual arrangements) for the consumption path $C_{t|T}$ for several periods of time. However, in the following period $T+1$ we assume a shock to the household's budget (5) occurs, which can either be caused by an income

shock ϵ_{T+1}^V or an inflation shock ϵ_{T+1}^π to the previously expected income and inflation levels, i.e. actual income and actual inflation read:

$$V_{T+1} = V_{T+1|T} + \epsilon_{T+1}^V \quad (6)$$

$$\pi_{T+1} = \pi_{T+1|T} + \epsilon_{T+1}^\pi \quad (7)$$

According to equation (5) this results in an effective budget impact of

$$\epsilon_{T+1}^B = \epsilon_{T+1}^\pi M_{T+2|T} - \epsilon_{T+1}^V \quad (8)$$

which we will include into the budget (5) when solving the household's utility optimization problem.

Consequently, in period $T + 1$ the household re-optimizes its expected future utility based on (3) for all future periods $t \geq T + 1$ by choosing the optimal path $(C_t(\epsilon_{T+1}^B), M_t(\epsilon_{T+1}^B), N_t(\epsilon_{T+1}^B))$ of consumption, money holding and labor supply in response to the shock ϵ_{T+1}^B it has perceived in period $T + 1$.

Furthermore, we argue that there are two different ways of modeling the afore-mentioned inflexibility of households to adjust their consumption plans to a budget shock ϵ_{T+1}^B :

Cost of reallocation:

When a household wants to change its future consumption plan in a period $T + 1$ due to a shock ϵ_{T+1}^B from the previously contractually committed values $C_{t|T}$ to the the adjusted values $C_t(\epsilon_{T+1}^B)$ then additional costs are incurred for a number of periods $t = T + 1..T + K$, which we denote by a cost function

$$f_t(C_t) = f_t(C_t(\epsilon_{T+1}^B) - C_t(0))$$

with $C_t(0) = C_{t|T}$ denoting the pre-committed value and $C_t(\epsilon_{T+1}^B)$ the optimally adjusted value. These costs occur for example if a household wants to change a contractually pre-committed consumption scheme, e.g. when leasing or mortgage contracts are canceled before maturity, or if a household has to change location due to lower than expected income or job searching costs after an adverse income shock (e.g. due to unemployment).

We require the cost function to be increasing in the difference between the optimally adjusted value $C_t(\epsilon_{T+1}^B)$ and the un-shocked value $C_t(0)$:

$$f_t(C_t(0)) = 0, \quad f_t(C_t) \geq 0, \quad \text{sgn} f'_t(C_t) = \text{sgn}[C_t(\epsilon_{T+1}^B) - C_t(0)] \quad (9)$$

The optimization problem of the household expressed as Lagrange-function reads:

$$\begin{aligned}
L := & \sum_{t=T+1}^{\infty} \beta^t \left(u(C_t(\epsilon), M_t(\epsilon), N_t(\epsilon)) + \lambda_t \left[\frac{W_t}{P_t} N_t(\epsilon) + \Pi_t + \delta_t |_{T+1} \epsilon \right. \right. \\
& - f_t(C_t(\epsilon) - C_t(0)) - (C_t(\epsilon) + (1 + \pi_t) M_{t+1}(\epsilon) - M_t(\epsilon) \\
& \left. \left. + \frac{B_{t+1}(\epsilon)}{1 + r_t} - B_t(\epsilon)) \right] \right) \Big|_{\epsilon = \epsilon_{T+1}^B} \quad (10)
\end{aligned}$$

with

$$\delta_{ij} = \begin{cases} 1, & i = j \\ 0, & i \neq j \end{cases} \quad (11)$$

The actual budget in (10) is composed of the budget (5) plus the budget shock (8) and the cost of reallocation. To promote a model, we use a cost function of the following form:

$$f_t(C_t(\epsilon)) = C_t(0) \left(1 - e^{-\delta(C_t(\epsilon) - C_t(0))^2} \right) \quad (12)$$

Function (12) defines the cost of adjusting consumption after a budget shock as a percentage of the value of the committed consumption $C_t(0)$, where the percentage term increases monotonically in the deviation of the adjusted consumption $C_t(\epsilon_{T+1}^B)$ from committed consumption $C_t(0)$ from zero to the saturation level of 1. The factor $\delta > 0$ indicates the sensitivity of adjustment costs to the afore-mentioned deviations. Further, (12) fulfills condition (9):

$$f'_t(C_t(\epsilon)) = -2(C_t(\epsilon) - C_t(0)) \delta e^{-\delta(C_t(\epsilon) - C_t(0))^2} \quad (13)$$

The optimality conditions derived from (10) with respect to consumption and money holdings are

$$u_M(M_{t+1}(\epsilon_{T+1}^B)) = ((1 + \pi_t)(1 + r_t) - 1) \frac{u_C(C_{t+1}(\epsilon_{T+1}^B))}{1 + f'_t(C_{t+1}(\epsilon_{T+1}^B))} \quad (14)$$

$$\frac{u_C(C_t(\epsilon_{T+1}^B))}{1 + f'_t(C_t(\epsilon_{T+1}^B))} = \beta(1 + r_t) \frac{u_C(C_{t+1}(\epsilon_{T+1}^B))}{1 + f'_t(C_{t+1}(\epsilon_{T+1}^B))} \quad (15)$$

with $u_C(C_t) = C_t^{-\alpha}$ and $u_M(M_t) = \tau M_t^{-\beta}$. Further, the optimality condition for the labor market reads (with f'_t given in (13)):

$$\chi \frac{N_t^\eta}{C_t^{-\alpha}} (1 + f'_t(C_t)) = \frac{W_t}{P_t} \quad (16)$$

Deriving the optimality conditions with respect to ϵ_{T+1}^B yields:

$$\begin{aligned} u_{MM}^{t+1} M'_{t+1} &= [(1 + \pi_t)(1 + r_t) - 1] u_{cc}^{t+1} C'_{t+1} \\ \implies M'_{t+1} &= \frac{(1 + \pi_t)(1 + r_t) - 1}{u_{MM}^{t+1}} u_{cc}^{t+1} C'_{t+1} \end{aligned} \quad (17)$$

$$u_{cc}^t C'_t = \beta(1 + r_t) u_{cc}^{t+1} C'_{t+1} \implies C'_{t+1} = \frac{u_{cc}^t}{\beta(1 + r_t) u_{cc}^{t+1}} C'_t \quad (18)$$

$$\begin{aligned} 0 &= \delta_{tT+1} - C'_t(1 + f'(C_t)) - (1 + \pi_t) M'_{t+1} + M'_t \\ &\quad - \frac{B'_{t+1}}{1 + r_t} + B'_t \end{aligned} \quad (19)$$

with $u_{CC}^t := u_{CC}(C_t) = -\alpha C_t^{-\alpha-1}$ and $u_{MM}^t := u_{MM}(M_t) = -\beta M_t^{-\beta-1}$. Plugging (17) and (18) into (19) yields:

$$\begin{aligned} & C'_t \underbrace{\left(\frac{(1 + \pi_{t-1})(1 + r_{t-1}) - 1}{u_{MM}^t} u_{cc}^t - (1 + \pi_t) \frac{(1 + \pi_t)(1 + r_t) - 1}{\beta(1 + r_t) u_{MM}^{t+1}} u_{cc}^t - 1 - f'(C_t) \right)}_{:k_t} \\ & + \delta_{tT+1} - \frac{B'_{t+1}}{1 + r_t} + B'_t = 0 \end{aligned} \quad (20)$$

which can be used to calculate the derivatives of $C_t(\epsilon_{T+1}^B)$, $M_t(\epsilon_{T+1}^B)$, $B_t(\epsilon_{T+1}^B)$ with respect to the budget shock ϵ_{T+1}^B iteratively starting from period $T + 1$ for all future periods, indicating the change of its resource allocation in response to a period- $T + 1$ budget shock.

Inflexible consumption:

The consumption plan $C_{t|T}$ as committed in period T cannot be changed at all for a number of periods $t = T + 1..T + K$, because the household is contractually bound which implies indirect wealth losses because the household cannot select a utility-efficient resource plan (C_t, M_t, N_t) for these periods. Thus consumption C_t is independent of ϵ_{T+1}^B for $t = T + 1..T + K$ and the corresponding Lagrange-function reads:

$$\begin{aligned} L &:= \sum_{t=T+1}^{\infty} \beta^t \left(u(C_t(1_{(t>T+K)}\epsilon), M_t(\epsilon), N_t(\epsilon)) + \lambda_t [V_t + \delta_{tT+1}\epsilon \right. \\ &\quad \left. - (C_t(1_{(t>T+K)}\epsilon)(1 + \pi_t)M_{t+1}(\epsilon) - M_t(\epsilon) + \frac{B_{t+1}(\epsilon)}{1 + r_t} - B_t(\epsilon))] \right) \Bigg|_{\epsilon = \epsilon_{T+1}^B} \end{aligned} \quad (21)$$

The factor $1_{(t>T+K)}$ equals one if $t > T + K$ and zero otherwise and ensures that for the first K periods consumption cannot be adjusted to the shock ϵ_{T+1}^B . In the following we will show that the inflexibility in consumption can be replicated by a cost-function using the following approach:

$$f_t(C_t(\epsilon_{T+1}^B)) = h(\epsilon_{T+1}^B) \frac{(C_t(\epsilon_{T+1}^B) - C_t(0))^2}{2} \quad (22)$$

To replicate the optimality conditions of the Lagrange function (10) the cost function has to be tailored such that the derivative of (10) with respect to C_t equals the derivative of the Lagrange function (21) with respect to C_t , i.e. for $t = T + 1..T + K$ we have the condition (with $u_C^0 := u_C(C_t(0))$):

$$\begin{aligned} u_C(C_t(\epsilon_{T+1}^B)) + \lambda_t \left(1 + h(\epsilon_{T+1}^B)(C_t(\epsilon_{T+1}^B) - C_t(0))\right) &= \lambda_t + u_C^0 = 0 \\ \implies h(\epsilon_{T+1}^B) &= \frac{\frac{u_C(C_t(\epsilon_{T+1}^B))}{u_C^0} - 1}{C_t(\epsilon_{T+1}^B) - C_t(0)} \end{aligned} \quad (23)$$

Consequently, the cost function reads:

$$f_t(C_t(\epsilon_{T+1}^B)) = \frac{\frac{u_C(C_t(\epsilon_{T+1}^B))}{u_C^0} - 1}{2} (C_t(\epsilon_{T+1}^B) - C_t(0))$$

Taking into account the properties of the utility function $u_C > 0$, $u_{CC} < 0$ the cost function shows the desired properties (9). Further, it is interesting to note that $f_t(C_t) = 0$ if $t > T + K$, which means that the cost-function f_t appears in the same periods as the inflexibility.

Since the concept of re-allocation costs is more general, we will use it in the following.

B Household demand

As a next step, we apply the optimality condition (15) to the two periods T and $T + 1$ to construct the IS curve reflecting the impact of adjustment costs on the demand side. The conditions (17) till (19) we derived from the

Lagrange-function (10) show the change of the optimal path of consumption, money and bond holdings as a function of a realized budget shock ϵ_{T+1}^B . However, when economic subjects plan future periods they will take the expectation over all possible outcomes of future shock variables, in contrast to the present period T where endogenous variables are known without uncertainty. To be precise, in period T in which expectations are formed all endogenous variables are known without uncertainty and hence the marginal period utility equals $u_C(C_T)$, whereas in period $T + 1$ the marginal expected utility is calculated as the expected value over the shock ϵ_{T+1}^B , taking into account the cost function f_t . Thus condition (15) becomes when applied to the periods T and $T + 1$:

$$u_C(C_T) = \beta(1 + r_{T+1})E_\epsilon \tilde{u}_C(C_{T+1}(\epsilon_{T+1}^B)) \quad (24)$$

with

$$\tilde{u}_C(C_{T+1}(\epsilon_{T+1}^B)) := u_C(C_{T+1}(\epsilon_{T+1}^B))(1 + f'_{T+1}(\epsilon_{T+1}^B)).$$

From (24) we obtain

$$\begin{aligned} u_C(C_T) &= \beta(1 + r_{T+1})\tilde{u}_C(C_{T+1}(0)) + (Z_{T+1}^T)_C \sigma_B^2 \quad (25) \\ &= \beta(1 + r_{T+1})\tilde{u}_C(C_{T+1}(0)) \left(1 + \underbrace{\frac{(Z_{T+1}^T)_C}{\beta(1 + r_{T+1})\tilde{u}_C(C_{T+1})}}_{=:K(C_{T+1})>0} \sigma_B^2 \right) \end{aligned}$$

where we have used the following approximation of second order:

$$E_T \tilde{u}_C(C_{T+1}(\epsilon_{T+1}^B)) \approx \tilde{u}_C(C_{T+1}(0)) + \underbrace{\frac{1}{2} \frac{d^2}{d\epsilon^2} \tilde{u}_C(C_{T+1}(\epsilon)) \Big|_{\epsilon=0}}_{=:Z_{T+1}} \sigma_B^2$$

with $(Z_T)_C := \frac{\partial Z_T}{\partial C}$ and $Z_T < 0$ indicating the sensitivity of the household's utility regarding the volatility of its budget. The coefficient $Z_T < 0$ can be expressed in terms of derivatives of $C_t(\epsilon_{T+1}^B)$ and $M_t(\epsilon_{T+1}^B)$ derived in (17) till (19) above. The budget uncertainty is represented by the term

$$\sigma_B^2 = E_T(\epsilon_{T+1}^V + M_{T+2|T}\epsilon_{T+1}^\pi)^2 = (\sigma^V)^2 + M_{T+2|T}^2 (\sigma^\pi)^2 \quad (26)$$

which we derived using equation (8) with $(\sigma^V)^2$ and $(\sigma^\pi)^2$ denoting the expected future uncertainty of output and inflation. Further, we assumed

stochastic independence between inflation and income shocks and volatilities σ^V and σ^π constant in time to simplify notation.

Taking the log of equation (25) and using $c_i = \log C_i$ we receive the IS curve (using the approximation $\log(1+x) \sim x$):

$$\begin{aligned} \implies c_T &= \left(1 - \frac{2\delta}{\alpha}\right) c_{T+1} - \alpha k(c_{T+1}) \sigma_B^2 - \alpha \log \beta - \alpha r_T \\ &= \left(1 - \frac{2\delta}{\alpha}\right) c_{T+1} - \alpha (i_T - E_T \pi_{T+1} - \bar{r}_T) \end{aligned} \quad (27)$$

with the following definition:

$$k(c_{T+1}) := K(C_{T+1} = e^{c_{T+1}}) > 0 \quad (28)$$

and $i_T = r_T + E_T \pi_{T+1}$ denoting the nominal interest rate and the natural rate

$$\bar{r}_T = -\log \beta - \frac{\sigma}{\alpha} \tilde{k} (\sigma_V^2 + M_{T+2|T} \sigma_\pi^2) = -\log \beta - \frac{\sigma}{\alpha} \tilde{k} \sigma_B^2 \quad (29)$$

Here we used a constant coefficient \tilde{k} for the uncertainty term σ_B^2 .

On the demand side, we can summarize our findings based on equation (27) as follows:

- Households' utility is impacted adversely by income and inflation uncertainty, because households who committed a part of their income for consumption over several periods of time cannot adjust their consumption plans optimally without facing additional costs when a budget shock occurs and hence are forced to follow a sub-optimal consumption plan for the periods of their commitment. Consequently, households will always prefer a regime of low variability in inflation and income.
- The IS curve (27) shows an interesting feature: The expected impact of possible period- $T+1$ shocks is anticipated by rational economic subjects in period T and taken into account in their consumption plan in the sense that future income is not fully taken into account in the current period. Instead, future demand enters the formula for current demand with a reduced factor of $\left(1 - \frac{2\delta}{\alpha}\right)$, reflecting the fact that households are aware of the risk of losing part of their future income due to frictions

and hence are more careful regarding spending in period T . This can be understood as an "insurance effect" in the behavior of households, i.e. potential future friction losses due to uncertain inflation and income levels imply households to spend less and save more in the current period as an insurance against possible future losses.

- The natural interest rate is not constant but depends on output and inflation volatility, reflecting friction losses for households due to uncertain income and inflation levels. These friction losses are endogenously determined, in particular they depend on the conduct of monetary policy as we show in the following.
- We have seen that the concept of re-allocation costs is more general than assuming inflexible consumption, i.e. inflexibility can always be expressed in terms of re-allocation costs in the budget of a household.

III Value maximization by firms

In the following we derive a model for firms facing rigidities in the hiring process. To be precise, we argue that hiring decisions of firms are based on an estimate of future demand for their goods, implying the risk that when demand deviates from previous expectations, firms are over- oder under-invested in labor, creating additional costs for them. This argument is not only true for countries with direct rigidities due to labor laws which directly limit the ability to hire or lay-off labor flexibly in the short term, but also in a flexible job market as found in the United States if one takes into account labor facilitating costs, i.e. necessary investments in office space, hardware etc. which in most cases cannot be adjusted flexibly in the short term in response to sudden demand fluctuations.

In our model, firms use a production technology for the good $Y_t(j)$ using labor input $N_t(j)$

$$Y_t(j) = A_t N_t(j) \tag{30}$$

with productivity A_t . As indicated in equations (1) and (2) in the introduction it is important to mention that in the following we will distinguish between gross output Y_t according to the technology (30) and net output after subtracting all frictions due to resource mis-allocation in the economy,

which is disposable for households for consumption. In other words, only in the steady state where no frictions occur we will have $C_t = Y_t$. Otherwise, if frictions appear due to expectation errors in the planning process of households and firms, consumption will be below output $C_t < Y_t$ due to frictions.

A Frictions in production

We consider the labor market to be flexible from a wage-setting perspective, but assume rigidities in the hiring process. To be precise, we assume that in each period firms have to hire labor for the next period, based on the forecasted demand for their goods in the next period, i.e. in period t firm i forecasts a demand $Y_{t+1|t}(i)$ for its good in the following period and hires the amount of labor in line with its technology (30) in period t :

$$N_{t+1|t}(j) = \frac{Y_{t+1|t}(j)}{A_{t+1}} \quad (31)$$

In the period $t + 1$ the actual demand for its good Y_{t+1} can deviate from the predicted level $Y_{t+1|t}(j)$ by a random variable $\epsilon_{t+1}^d(j)$ representing the forecasting error regarding demand:

$$Y_{t+1}(j) = Y_{t+1|t}(j) + \epsilon_{t+1}^d(j) \quad (32)$$

which would imply the revised labor input:

$$N_{t+1}(j) = \frac{Y_{t+1}(j)}{A_{t+1}} \quad (33)$$

As on the demand side, there are two ways to model rigidities in the ability of firms to adjust their labor force to sudden fluctuations in the demand for their goods:

Inflexible labor:

Firms cannot adjust their labor force in the short term (i.e. in period $t + 1$). If demand turned out to be lower than expected (i.e. $\epsilon_{t+1}^d(j) < 0$) this means that firms face additional costs of

$$\frac{W_{t+1}}{P_{t+1}} \left(N_{t+1|t}(j) - N_{t+1}(j) \right) = -\frac{W_{t+1}}{P_{t+1}A_{t+1}} \epsilon_{t+1}^d(j) > 0$$

compared with the situation where they could flexibly adjust their labor force. On the other hand, if demand exceeds expectations companies face forgone profits to the amount of

$$\frac{W_{t+1}}{P_{t+1}} (N_{t+1}(j) - N_{t+1|t}(j)) = \frac{W_{t+1}}{P_{t+1}A_{t+1}} \epsilon_{t+1}^d(j) > 0.$$

To sum up, the inflexibility to adjust labor in the short term increases effective real labor costs which hence read:

$$\varphi_{t+1}(i) = \varphi_{t+1}^0 (1 + |\epsilon_{t+1}^d(j)|) = \frac{W_{t+1}}{P_{t+1}A_{t+1}} \left(1 + \frac{|Y_{t+1}(j) - Y_{t+1|t}(i)|}{Y_{t+1|t}(j)} \right) \quad (34)$$

with

$$\varphi_{t+1}^0 = \frac{W_{t+1}}{P_{t+1}A_{t+1}}. \quad (35)$$

Cost of reallocation:

Firms can adjust their work force in period t to the desired level (33) but face additional costs for hiring in the short term in period $t+1$. This concept is obviously more general in the sense that using (34) as a cost function will produce the same result as assuming inflexibility in the short term hiring process. However, analogous to the cost function (12) we have used for demand we use a smooth cost function of the form:

$$\varphi_{t+1}(i) = \varphi_{t+1}^0 (1 - e^{-\gamma(\epsilon_{t+1}^d(j))^2}) = \frac{W_{t+1}}{P_{t+1}A_{t+1}} (1 - e^{-\gamma(Y_{t+1}(j) - Y_{t+1|t}(j))^2}) \quad (36)$$

where γ denotes the sensitivity of re-allocation cost regarding the forecasting error. The case $\gamma = 0$ corresponds to the situation of an economy without frictions.

B Supply of firms

The key step is to define the profit maximizing behavior of firms, where we will refer to the Calvo model of inflexible prices in the following, i.e. we assume that in each period a fixed percentage $1 - \omega$ of randomly chosen firms can adjust their prices, the remaining firms are bound to the prices of the previous period. When firms are able to adjust prices, they maximize

the present value of future profits, where a future period i is discounted with $\Delta_{i,t+i}$ and weighted with the probability ω^i of not being able to adjust the price set today within the next i periods. We use the demand for the composite good $Y_{t+i}(j)$ with price $P_{t+i}(j)$ of the Dixit-Stiglitz aggregate Y_{t+i}

$$Y_{t+i}(j) = Y_{t+i} \left(\frac{P_t(j)}{P_{t+i}} \right)^{-\Theta} = (Y_{t+i|t} + \epsilon_{t+i}^d) \left(\frac{P_t(j)}{P_{t+i}} \right)^{-\Theta}$$

where we have assumed that the individual demand shocks are generated by one global shock in demand, i.e. $\epsilon_{t+i}^d(j) \equiv \epsilon_{t+i}^d \forall j$ to simplify notation. The expected future profit reads (cf Walsh (2003)):

$$\begin{aligned} \Pi &= E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_t(j)}{P_{t+i}} \right) Y_{t+i}(j) - \varphi_{t+i}(j) Y_{t+i}(j) \right] \\ &= E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_t(j)}{P_{t+i}} \right)^{1-\Theta} - \varphi_{t+i} \left(\frac{P_t(j)}{P_{t+i}} \right)^{-\Theta} \right] Y_{t+i} \end{aligned}$$

where we have approximated $\varphi_{t+i}(j)$ in first order by the total cost function¹:

$$\varphi_{t+i} = \int \varphi_{t+i}(j) dj = \frac{W_t}{P_t A_t} \left(1 - e^{-\gamma(Y_{t+i} - Y_{t+i|t})^2} \right) \quad (37)$$

As mentioned in the introduction, we perform two steps to derive our model equations – at first we take the expectation over the endogenous variables causing friction losses, then we perform a log-linearization of the model. Taking the expectation regarding ϵ_{t+i}^d causing friction losses we obtain:

$$\Pi = E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_t(j)}{P_{t+i}} \right)^{1-\Theta} - \varphi_{t+i}^0 (1 + \gamma \sigma_d^2) \left(\frac{P_t(j)}{P_{t+i}} \right)^{-\Theta} \right] Y_{t+i}$$

where we have assumed ϵ_{t+i}^d to be $N(0, \sigma_d^2)$ leading to the following second order approximation:

$$E_t \varphi_{t+i}(\epsilon_{t+i}^d) = \gamma \sigma_d^2 \quad (38)$$

Deriving the present value of future profits with regards to the price $P_t(j)$ to determine the optimal price P_t^* chosen by all firms adjusting prices in the

¹The approximation is exact in the equilibrium $Y_t(j) \equiv Y$, which is the basis for the derivation of the log-linearized model below.

present period yields with $\tilde{\varphi}_{t+i} := \varphi_{t+i}^0(1 + \gamma\sigma_d^2)$:

$$\begin{aligned}
& E_t \sum_{i=0}^{\infty} \omega^i \Delta_{i,t+i} \left[\left(\frac{P_t^*}{P_{t+i}} \right)^{-\Theta} \frac{1-\Theta}{P_{t+i}} + \tilde{\varphi}_{t+i} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\Theta-1} \frac{\Theta}{P_{t+i}} \right] Y_{t+i} = 0 \\
\implies & E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{-\alpha} \left[\left(\frac{P_t^*}{P_{t+i}} \right) (1-\Theta) + \tilde{\varphi}_{t+i} \left(\frac{P_t^*}{P_{t+i}} \right)^{-\Theta} \frac{\Theta}{P_t^*} \right] Y_{t+i} = 0 \\
\implies & P_t^* E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{-\alpha} (1-\Theta) \left(\frac{1}{P_{t+i}} \right)^{1-\Theta} Y_{t+i} \\
& = -E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{-\alpha} Y_{t+i} \tilde{\varphi}_{t+i} \left(\frac{1}{P_{t+i}} \right)^{-\Theta} \Theta \\
\implies & Q_t := \frac{P_t^*}{P_t} = \frac{\Theta}{\Theta-1} \frac{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{C_{t+i}}{Y_t} \right)^{-\alpha} \tilde{\varphi}_{t+i} \left(\frac{P_{t+i}}{P_t} \right)^{\Theta} Y_{t+i}}{E_t \sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{-\alpha} \left(\frac{P_{t+i}}{P_t} \right)^{\Theta-1} Y_{t+i}} \quad (39)
\end{aligned}$$

Here we used the definition of the discount factor $\Delta_{i,t+i} = \beta^i \left(\frac{Y_{t+i}}{Y_t} \right)^{-\alpha}$. If we apply the expectation operator taking into account that $E_t Y_{t+i} = Y_{t+i|t}$ we obtain:

$$Q_t = \frac{\Theta}{\Theta-1} \frac{\sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i|t}}{Y_t} \right)^{-\alpha} \varphi_{t+i}^0 (1 + \gamma\sigma_d^2) \left(\frac{P_{t+i}}{P_t} \right)^{\Theta} Y_{t+i|t}}{\sum_{i=0}^{\infty} \omega^i \beta^i \left(\frac{Y_{t+i|t}}{Y_t} \right)^{-\alpha} \left(\frac{P_{t+i}}{P_t} \right)^{\Theta-1} Y_{t+i|t}} \quad (40)$$

It is interesting to note that in the state of fully flexible prices (i.e. $\omega \rightarrow 0$) Q_t simplifies to:

$$1 = \frac{\Theta}{\Theta-1} (1 + \gamma\sigma_d^2) \varphi_t^0 = \mu \varphi_t^0 \quad \text{with } \mu := (1 + \gamma\sigma_d^2) \frac{\Theta}{\Theta-1} \quad (41)$$

showing that the markup μ over real labor costs increases in frictions caused due to unpredictability of demand represented by the factor $(1 + \gamma\sigma_d^2)$, i.e. companies are passing on the costs of frictions caused by unforeseen demand fluctuations to clients. Further, noting that μ is the markup over real labor costs we conclude from equation (16)

$$\frac{W_t}{P_t} = \frac{A_t}{\mu} = \frac{\chi N_t^\eta}{C_t^{-\alpha}} (1 + f'_t(C_t)) \quad (42)$$

The second step is to derive a log-linearized model, i.e. we will approximate all future endogenous variables around the steady-state, treating them as

random variables under the expectation operator E_t . For the price level we make use of

$$\begin{aligned} P_t^{1-\Theta} &= (1-\omega)(P_t^*)^{1-\Theta} + \omega P_{t-1}^{1-\Theta} \\ \implies 1 &= (1-\omega)Q_t^{1-\Theta} + \omega \left(\frac{P_{t-1}}{P_t}\right)^{1-\Theta} \end{aligned} \quad (43)$$

Expanding (43) around the steady-state $Q_t = P_t^*/P_t = 1$ up to second order terms yields (with \hat{q} denoting percentage changes of Q and the inflation rate $\pi_t = P_t/P_{t-1}$):

$$\hat{q}_t = \frac{\omega}{1-\omega}\pi_t + \frac{\omega(\Theta - 2(1-\omega))}{2(1-\omega)^2}\pi_t^2 \quad (44)$$

Moreover, to derive a log-linear inflation adjustment curve we expand output Y_t , the price level P_t and the marginal costs φ_{t+i} around the steady-state characterized by $Q_t = \mu\varphi = 1$, where we include second order terms for price level and inflation neglected in the original derivation cited above. Consequently, equation (39) can be approximated as:

$$\begin{aligned} &\left(\frac{C^{1-\alpha}}{1-\omega\beta}\right)(1 + \hat{q}_t - \hat{\mu}_t) + Y^{1-\alpha} \sum_{i=0}^{\infty} \omega^i \beta^i [(1-\alpha)E_t \hat{y}_{t+i} \\ &+ (\Theta - 1)(E_t \hat{p}_{t+i} - \hat{p}_t)] \\ &= \mu \frac{Y^{1-\alpha}}{1-\omega\beta} \varphi + \mu\varphi Y^{1-\alpha} \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+i} + (1-\alpha)E_t \hat{y}_{t+i} + \Theta(E_t \hat{p}_{t+i} - \hat{p}_t)] \\ \implies &\frac{1}{1-\omega\beta}(\hat{q}_t - \hat{\mu}_t) = \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i} - \hat{p}_t] \\ \implies &\hat{q}_t - \hat{\mu}_t + \hat{p}_t = (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i [E_t \hat{\varphi}_{t+i} + E_t \hat{p}_{t+i}] \\ \implies &\hat{q}_t - \hat{\mu}_t + \hat{p}_t = (1-\omega\beta)(\hat{\varphi}_t + \hat{p}_t) + \omega\beta(E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1}) \\ \implies &\hat{q}_t - \hat{\mu}_t = (1-\omega\beta)\hat{\varphi}_t + \omega\beta(E_t \hat{q}_{t+1} + E_t \hat{p}_{t+1} - \hat{p}_t) \\ &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta(E_t \hat{q}_{t+1} + E_t \pi_{t+1}) \end{aligned}$$

Using (44) to substitute \hat{q} yields:

$$\begin{aligned} \frac{\omega}{1-\omega}\pi_t &= (1-\omega\beta)\hat{\varphi}_t + \omega\beta \left[\left(1 + \frac{\omega}{1-\omega}\right) E_t \pi_{t+1} \right] \\ &+ \frac{\omega(\Theta - 2(1-\omega))}{2(1-\omega)^2} (\omega\beta E_t \pi_{t+1}^2 - \pi_t^2) + \hat{\mu}_t \end{aligned}$$

Hence we obtain

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \hat{\varphi}_t + \frac{\Theta - 2(1 - \omega)}{2(1 - \omega)} (\omega \beta (\sigma_\pi^2 - (E_t \pi_{t+1})^2) - \pi_t^2) + u_t \quad (45)$$

with $\kappa := \frac{(1-\omega)(1-\omega\beta)}{\omega}$ and where we used $E_t \pi_{t+1}^2 = \sigma_\pi^2 + (E_t \pi_{t+1})^2$ assuming a constant volatility σ_π^2 . The shock term $u_t = \frac{1-\omega}{\omega} \hat{\mu}_t$ represents inflation shocks due to a change in the markup of companies and hence a change of the degree of monopolistic competition. Equation (45) represents a quadratic equation in π_t and π_{t+1} . Since we are only interested in the link between present inflation π_t and expectations regarding future inflation $E_t \pi_{t+1}$ and its uncertainty σ_π^2 , we drop the quadratic inflation terms – the qualitative behavior of the system is unchanged by this simplification.

Further, since the labor market is assumingly flexible from a wage-setting point of view, we can express the term $\kappa \hat{\varphi}_t$ in terms of percentage changes of output around the steady-state, as we show in the following in two steps:

Definition of steady-state:

The steady-state is the equilibrium state where all goods are produced and consumed in equal quantities and no friction losses occur and hence household income equals output and consumption equals household income, i.e.

$$C_t = V_t = Y_t = Y_{t|t-1} \quad (46)$$

$$C_t(j) \equiv C_t, \quad Y_t(j) \equiv Y_t \quad (47)$$

However, outside of the steady-state we can write the households' gross income consisting of wages earned and corporate profits Π_t received as output minus frictions on the production side as indicated in equation (1) in the introduction :

$$\begin{aligned} V_t &= \frac{W_t}{P_t} N_t + \Pi_t = \frac{W_t}{P_t} N_t + P_t Y_t - \varphi_t Y_t \\ &= Y_t P_t - \frac{W_t}{P_t A_t} \left(1 - e^{-\gamma(Y_t - Y_{t|t-1})^2}\right) \end{aligned} \quad (48)$$

Equation (48) basically states that households' gross income equals output minus productivity losses due to resource misplanning. For small perturbations around the steady state we conclude from equation (48) that in first

order approximation we have with variables with a hat denoting percentage changes around the steady-state:

$$\hat{c}_t = \hat{y}_t \quad (49)$$

Real marginal costs:

As a next step, we calculate percentage changes of real costs for two situations:

- The situation of fully flexible prices, denoted by f : From equation (42) we have

$$\hat{a}_t^f = \eta \hat{n}_t^f + \alpha \hat{c}_t^f \quad (50)$$

Using (49) and $\hat{y}_t^f = \hat{n}_t + \hat{a}_t^f$ derived from (30) we obtain the percentage change of output under full flexibility:

$$\hat{y}_t^f = \left(\frac{1 + \eta}{\alpha + \eta} \right) \hat{a}_t \quad (51)$$

- The situation of non-flexible prices in the sense of Calvo: From equation (35) we conclude the following relationship for percentage changes:

$$\hat{\varphi}_t = \hat{w}_t - \hat{p}_t - \hat{a}_t \quad (52)$$

Using the corresponding relationship derived from labor supply (42) (where again we use relation (49))

$$\hat{w}_t - \hat{p}_t = \eta \hat{n}_t + \alpha \hat{y}_t \quad (53)$$

and the relationship $\hat{y}_t = \hat{n}_t + \hat{a}_t$ following equation (30) we obtain

$$\begin{aligned} \hat{\varphi}_t &= \eta(\hat{y}_t - \hat{a}_t) + \alpha \hat{y}_t - \hat{a}_t \\ &= (\alpha + \eta)\hat{y}_t - (1 + \eta)\hat{a}_t \\ &= (\alpha + \eta)\hat{y}_t - (\alpha + \eta)\hat{y}_t^f \\ &= \varphi x_t \end{aligned}$$

with the output gap $x_t = \hat{y}_t - \hat{y}_t^f$ and $\varphi = (\alpha + \eta)$.

Thus, we obtain the IA curve including uncertainty regarding inflation:

$$\pi_t = \beta E_t \pi_{t+1} + \varphi x_t + \zeta \sigma_\pi^2 + u_t \quad (54)$$

with

$$\zeta = \frac{\Theta - 2(1 - \omega)}{2(1 - \omega)} \omega \beta \quad (55)$$

As regards the order of magnitude of ζ , matching the IA curve to empirical data (for details cf Woodford, 2003) yields $\varphi \approx 0.024$, which means that ω is close to one and hence the coefficient ζ of uncertainty regarding future inflation can be deemed positive. Consequently, present inflation π_t is increased by both higher expected future inflation $E_t \pi_{t+1}$ and uncertainty about future inflation, i.e. firms setting prices today tend to increase prices more if they are not certain about their inflation forecast. The reason can be understood by equation (44) – the optimal price chosen is a convex function of inflation. Consequently, when inflation is a stochastic variable, the expected optimal price will be greater than the price formula (44) evaluated at the expected inflation rate:

$$E\hat{q}(\pi_t) > q(\hat{E}\pi_t)$$

Hence, we conclude that profit-maximizing firms tend to "over-price" their products when the future price level is uncertain.

We can draw the following conclusions for the supply side of the economy:

- Rigidities in the capability of adjusting labor input in the very short term increases average labor costs. This effect is passed on to consumers by increasing the markups accordingly.
- Firms facing uncertain future inflation tend to "over-price" their products due to the concavity of the underlying profit function represented by a second order inflation moment in the inflation adjustment curve and creating an additional inflation bias.
- Due to demand uncertainty, the real wage is perceived asymmetrically by households and firms, i.e. households' labor supply according to equation (16) is based on the real wage W_t/P_t , whereas companies see effective labor costs according to equation (41) which are higher by a factor of $(1 + \gamma\sigma_d^2)$. This means that labor input by firms will be inefficiently low.

IV Monetary policy in a log-linearized model

A Log-linear model with frictions

We analyze a log-linearized system for percentage changes of output around the steady state based on the IS curve (27) and the IA curve (54) after performing two simplifications, i.e. from equation (48) we derive two approximations around the steady-state defined in (46) and (47):

- Equation (49) which we use to reformulate the IS curve (27) in terms of deviations x_t of output around the steady state.
- $\sigma_V^2 = \sigma_x^2$ which we use in the natural interest rate (29) and $\sigma_d^2 = \sigma_x^2$ which we use to approximate demand volatility in (38).

Hence, the log-linear system reads:

$$\text{IS curve } x_t = E_t x_{t+1} - \alpha(i_t - E_t \pi_{t+1} - \bar{r}_t) \quad (56)$$

$$\text{IA curve } \pi_t = \beta E_t \pi_{t+1} + \varphi x_t + \zeta \sigma_\pi^2 + u_t \quad (57)$$

with the natural interest rate $\bar{r}_t = -\log \beta - \frac{\sigma}{\alpha} k(\sigma_x^2 + M_{t+1|t} \sigma_\pi^2)$ and inflation shocks u_t .

A key component for the analysis of monetary policy is the derivation of a global utility loss function, which is used to assess and compare the appropriateness of different monetary policy regimes from a welfare point of view. Woodford (2003) derives the following loss function through a second order approximation of households' utility around the steady state:

$$L_t := \pi_t^2 + \lambda(x_t - x^*)^2 \quad (58)$$

Here $x^* = y_t^e - y_t^f$ denotes the difference between efficient output y_t^e and natural output y_t^f . For the sake of brevity, we refer to Woodford (2003) and only outline the changes in the original derivation of equation (58). In essence, it is straightforward to verify that the derivation of the term π_t^2 representing the second order approximation of utility from consumption around its steady state and the term $(x_t - x^*)^2$ representing the second order approximation of dis-utility from labor around the steady state remain unchanged, except for the term x^* , as we derive in the following. Therefore,

we use equation (30) in condition (42) and obtain for the steady state in which $C_t = Y_t$ and $f'_t(C_t) = 0$:

$$\begin{aligned} \frac{A_t}{\mu} &= \frac{\chi N_t^\eta}{Y_t^{-\alpha}} = \frac{\chi \left(\frac{Y_t}{A_t}\right)^\eta}{Y_t^{-\alpha}} = \frac{\chi Y_t^{\eta+\alpha}}{A_t^\eta} \\ \implies Y_t &= \frac{A_t^{\frac{1+\eta}{\eta+\alpha}}}{(\chi\mu)^{\frac{1}{\eta+\alpha}}} \end{aligned} \quad (59)$$

The natural output y_t^f is the log of Y_t given in equation (59), whereas the efficient output is given by the log of equation (59) by setting $\mu = 1$ to reflect a market under full competition and without any friction losses. Hence we conclude with μ given in equation (41):

$$y_t^e - y_t^f = \log \mu^{\frac{1}{\eta+\alpha}} = \frac{\log \left(\frac{\Theta}{\Theta-1} (1 + \gamma \sigma_x^2) \right)}{\eta + \alpha} \quad (60)$$

$$= \underbrace{\frac{\log \Theta + \log(\Theta - 1)}{\eta + \alpha}}_{=: x_0^*} + \underbrace{\frac{\gamma}{\eta + \alpha} \sigma_x^2}_{=: x_1^*} \quad (61)$$

Thus, we have:

$$x^* = y_t^e - y_t^f = x_0^* + x_1^* \sigma_x^2 \quad (62)$$

where $x_1^* > 0$ shows that higher output uncertainty widens the gap between efficient and natural output level. Thus, we can conclude that we obtain an extended loss function of the form

$$L_t := \pi_t^2 + \lambda(x_t - x_0^* - x_1^* \sigma_x^2)^2 \quad (63)$$

Hence, compared with the standard loss function (58) with constant x^* , equation (63) additionally takes into account friction losses caused by firms that are invested inefficiently – the more the higher the value of σ_x .

B Monetary policy with frictions

In the following, we analyze the impact of endogenous frictions on

- The inflation-output trade-off, in particular the sacrifice ratio;

- The reaction function of monetary policy to demand and supply shocks and
- The role of the well-known inflation bias in monetary policy research.

It is interesting to analyze the steady-state behavior of the IA curve (57), i.e. by setting $\pi^e = \pi_t = E_t\pi_{t+1}$ in equation (57) we obtain

$$\pi^e = \frac{\varphi x}{1 - \beta} + \frac{\zeta}{1 - \beta} \sigma_\pi^2 \quad (64)$$

Regarding the inflation-output trade-off, the impact of endogenous uncertainty is already revealed by the long-term Phillips curve (64), which has the same slope $\frac{\varphi}{1 - \beta}$ as in the standard model but which is shifted upwards by the additional inflation bias proportional to σ_π^2 . Thus, the sacrifice ratio which is typically defined as the slope of the curve (64) denoting the cost of an inflation reduction of one percent in terms of output is unaltered. However, the trade-off is worsened by an additional inflation bias, which can only be removed by additional output costs as indicated in (64). To conclude, the model equation (64) gives a reasoning for accepting a certain base inflation rate, which is the case for most central banks, in particular the Fed and the European Central Bank.

To analyze the impact of endogenous frictions on the conduct of monetary policy, we assess the behavior of the monetary authority by minimizing the loss function (63) with the IA curve (57) as a period constraint.

It is important to mention that in the model we presented inflation shocks u_t occurring in the IA curve (54) with standard deviation σ_u can impact both inflation and output, depending on the conduct of monetary policy, and results in volatile inflation rates with standard deviation σ_π and volatile output with standard deviation σ_x . To be precise, the monetary authority faces the problem to distribute shocks u_t between output x_t and inflation π_t and hence distributing the uncertainty σ_u between σ_π and σ_x and consequently managing the trade-off between friction losses caused by output variability and inflation variability.

Consequently, a straightforward way of prescribing a rule-based policy is to explicitly define the share ν of the shock u_t that has to be absorbed by

output, as proposed in Clarida, Gali and Gertler (1999):

$$x_t = -\nu u_t \implies \sigma_x^2 = \nu^2 \sigma_u^2 \quad (65)$$

Here output is simply counteracting inflation shocks with the factor $\nu > 0$. In practice, the output-rule (65) can be implemented through a nominal interest rate rule by plugging (65) into the IS curve (56):

$$i_t = \pi^e + \bar{r}_t + \frac{\nu}{\alpha} u_t \quad (66)$$

where we assumed output and inflation expectations close to the equilibrium level, i.e. $E_t x_{t+1} \approx 0$ and the steady-state inflation level π^e derived below.

Plugging (65) into the IA curve (54) yields

$$\begin{aligned} \pi_t &= \varphi x_t + \beta E_t \pi_{t+1} + \zeta \sigma_\pi^2 + u_t = \beta_t \pi_{t+1} + \zeta \sigma_\pi^2 + (1 - \varphi \nu) u_t \quad (67) \\ \implies \sigma_\pi^2 &= (1 - \varphi \nu)^2 \sigma_u^2 \quad (68) \end{aligned}$$

The factor ν determines the distribution of the volatility σ_u between σ_x and σ_π and the resulting friction losses due to output uncertainty and inflation uncertainty. Hence, the natural rate \bar{r}_t entering the policy function (66) reads:

$$\begin{aligned} \bar{r}_t &= -\log \beta - \frac{\sigma}{\alpha} k (\sigma_x^2 + M_{t+1|t} \sigma_\pi^2) \\ &= -\log \beta - \frac{\sigma}{\alpha} k (\nu^2 + M_{t+1|t} (1 - \varphi \nu)^2) \sigma_u^2 \quad (69) \end{aligned}$$

which shows that the natural rate and hence the interest rate rule of monetary policy depend on the choice of the trade-off parameter ν . This reflects the fact that the natural rate is no longer constant but is influenced by frictions caused by inflation uncertainty and output uncertainty. Monetary policy can actively manage the trade-off between these two types of frictions as reflected by the rule parameter ν in the natural rate (69). Consequently, higher order terms are not exogenously constant as observed in the analysis of Uribe and Schmitt-Grohe (2003), but are determined by the choice of monetary policy represented by the parameter ν .

Inflation expectations can be found by iterating equation (67) forward in

time (where we assume the shocks u_t not to be auto-correlated and thus $E u_{t+i} = 0$ to simplify notation):

$$\begin{aligned}\pi^e &= E\pi_t = \sum_{i=0}^{\infty} \beta^i E[\varphi x_{t+i} + \zeta \sigma_\pi^2 + u_{t+i}] = \sum_{i=0}^{\infty} \beta^i [(1 - \varphi\nu) E u_{t+i} + \zeta \sigma_\pi^2] \\ &= \frac{\zeta \sigma_\pi^2}{1 - \beta} = \frac{\zeta (1 - \varphi\nu)^2 \sigma_u^2}{1 - \beta}\end{aligned}\quad (70)$$

Thus, the present value of future economic losses reads using the relationship (62):

$$\begin{aligned}L &= E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda(x_{t+i} - x^*)^2] \\ &= E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2 - 2\lambda x^* x_{t+i} + \lambda(x^*)^2] \\ &= E_t \sum_{i=0}^{\infty} \beta^i \lambda [(x^*)^2 + 2\nu x^* E_t u_{t+i}] + E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \lambda x_{t+i}^2] \\ &= \frac{\lambda(x^*)^2}{1 - \beta} + \sum_{i=0}^{\infty} \beta^i E_t \left((\beta E_t \pi_{t+i+1} + \zeta \sigma_\pi^2 + (1 - \varphi\nu) u_{t+i})^2 + \lambda \nu^2 u_{t+i}^2 \right) \\ &= \frac{\lambda(x_0^* + x_1^* \sigma_x^2)^2}{1 - \beta} + \sum_{i=0}^{\infty} \beta^i E_t \left(\left(\frac{\zeta (1 - \varphi\nu)^2 \sigma_u^2}{1 - \beta} + (1 - \varphi\nu) u_{t+i} \right)^2 + \lambda \nu^2 u_{t+i}^2 \right) \\ &= \frac{\lambda(x_0^* + x_1^* \sigma_u^2 \nu^2)^2}{1 - \beta} + \frac{\zeta^2 (1 - \varphi\nu)^4 \sigma_u^4}{(1 - \beta)^3} + \frac{(1 - \varphi\nu)^2 \sigma_u^2}{1 - \beta} + \frac{\lambda \nu^2 \sigma_u^2}{1 - \beta}\end{aligned}\quad (71)$$

Hence, the loss function with uncertainty shows additional terms proportional in terms of x_1^* and ζ compared with the loss function without the incorporation of friction losses. In the following we distinguish between three different monetary policy rules based on the rule (65) and its implementation (66).

Output rule:

The output rule $x_t \equiv 0$ implies a factor $\nu = 0$ in the rule (65) and hence implies the minimal output variability $\sigma_x = 0$ but the maximal inflation variability $\sigma_\pi^2 = \sigma_u^2$. The utility loss simplifies (71) to:

$$L_x = \frac{\lambda(x_0^*)^2}{1 - \beta} + \frac{\sigma_u^2}{1 - \beta} + \frac{\zeta^2 \sigma_u^4}{(1 - \beta)^3}\quad (72)$$

The last term proportional to ζ^2 indicates the additional friction loss due to inflation variability.

Inflation rule:

The other extreme of the rule (65) is total price stabilization $\pi_t \equiv 0$ implying a value of $\nu = 1/\varphi$. In this case inflation variability is minimal $\sigma_\pi = 0$ according to equation (68) but output fluctuates with a volatility of σ_u^2/φ^2 . The utility loss (71) reads:

$$L_\pi = \frac{\lambda(x_0^* + x_1^*\sigma_u^2/\varphi^2)^2}{1 - \beta} + \frac{\lambda\sigma_u^2/\varphi^2}{1 - \beta} \quad (73)$$

where the term proportional to x_1^* shows the adverse impact of friction losses in the productivity of firms on welfare.

Optimal rule:

The optimal policy in the context of the rule (65) and its implementation (66) is apparently a value of ν between the output rule and inflation rule to optimally manage the trade-off between friction losses due to output variability and inflation variability. The loss-minimizing value of ν reads:

$$\nu_{opt} = \frac{\varphi}{\lambda + \varphi^2} - \frac{2\varphi\lambda x_0^*x_1^*}{(\lambda + \varphi^2)^2} - \frac{2(\varphi^2(2\varphi^2 + \lambda) - x_0^*x_1^*\lambda(10\varphi^2 - 2\lambda) - \lambda^2)\varphi\sigma_u^2}{(\lambda + \varphi^2)^3(1 - \beta)^2}\zeta^2$$

with the first term on the right-hand side representing the solution in absence of frictions as found in Clarida, Gali and Gertler (1999). The welfare loss function reads:

$$L_{opt} = \frac{\lambda\sigma_u^2}{(\lambda + \varphi^2)(1 - \beta)} + \frac{\lambda x_0^2}{1 - \beta} + \frac{2\lambda x_0 x_1 \varphi^2 \sigma_u^2}{(\lambda + \varphi^2)^2(1 - \beta)} + \frac{\lambda^4 \sigma_u^4 (\varphi^2 + \lambda + 8\varphi^2 x_0 x_1)}{(\lambda + \varphi^2)^5(1 - \beta)^3}$$

The conduct of monetary policy is influenced by inflation uncertainty and output uncertainty in opposite ways: The optimal value of ν increases in x_1^* , i.e. the sensitivity of the economy regarding output uncertainty and related friction losses, hence making output stabilization more attractive. At the same time, ν decreases with ζ , i.e. the sensitivity regarding inflation uncertainty and related frictions, making inflation stabilization increasingly attractive. Since the effect of inflation uncertainty is quadratic in the inflation sensitivity coefficient ζ but linear in the output sensitivity coefficient x_1^* , at least for low levels of sensitivity we can expect output stabilization to

become more important.

V Conclusion

We have constructed a model with rigidities in the ability of households and firms to flexibly re-optimize their resource allocation after the appearance of shocks. These rigidities reflect the fact that households and firms enter into commitments regarding the allocation of resources limiting their ability to flexibly respond to unforeseen shocks. Mathematically speaking, these frictions can either be formulated as a strict inflexibility of certain endogenous variables for a certain period of time or as penalty costs that economic subjects face when they want to change pre-committed resource plans. The latter concept proved to be more general and formed the basis for our micro-foundation of an IS curve representing demand and an inflation adjustment curve describing firms and the assessment of monetary policy.

The model equations we have derived contain frictions on the demand side and supply side represented by higher order moments in endogenous variables. These statistical moments are not exogenously constant but model-determined and have to be taken into account when assessing the conduct of monetary policy.

The model we presented taking into account the afore-mentioned rigidities in the micro-foundation shows several results not contained in the linear model equations of a New Keynesian model framework: First of all, the natural interest rate and natural output which are typically key components of the policy framework of monetary authorities depend on output and inflation uncertainty and hence are endogenously determined within the economic system. In particular, the way monetary authorities manage the trade-off between output and inflation stabilization and hence the trade-off between frictions due to output and inflation variability determines the natural interest rate and natural output.

Moreover, we have shown that uncertainty regarding inflation and output reduces the productivity of firms, because they will be invested inefficiently in resources. As a consequence, the natural output level is a decreasing function of uncertainty and hence endogenously determined.

Furthermore, uncertainty can explain a positive inflation bias even where $x^* = 0$ (efficient natural output), which prevails under both discretionary and rule-based policies. This inflation bias exists in addition to the inflation bias occurring in the standard model of the new neoclassical synthesis due to the well-known time-inconsistency problem, which can be avoided by a rule-based policy. The reason for this additional inflation bias is the fact that profit-maximizing firms, being aware of uncertain future prices, tend to "over-price" their products compared to the standard Calvo model. This "over-pricing" is an endogenous influence determined within the system by inflation uncertainty. Consequently, the model we presented can explain positive inflation rates even in situations where output is at its natural level and monetary policy is rule-based, which is in accordance with empirical observations, since most modern economies show a positive inflation rate throughout the business cycle (cf Walsh, 2003). Henceforth, our model gives a reasoning for accepting a certain positive base inflation, as most central banks currently do, whereas the sacrifice ratio defined as the cost of an inflation decrease in terms of output is unaltered in our model.

As regards the conduct of monetary policy, we have shown that the behavior of the monetary authority is modified by endogenous uncertainty by more than constant terms only, as it is the case when one assumes exogenous uncertainty. Consequently, the statement proved by Schmitt-Grohe and Uribe (2003) that higher order approximations lead to essentially the same model up to constant terms no longer holds. Instead, higher statistical moments play an important role in the economic system and the conduct of monetary policy, since they are no longer constant but determined by the system itself. The monetary authority faces a new trade-off in minimizing the frictions due to inflation uncertainty and output uncertainty and can determine the economic subjects' expectations regarding higher statistical moments of the distribution of endogenous variables, which enter the model function for sup-

ply since firms are typically described by risk-averse utility or profit functions.

A future focus of research comprises how monetary authorities can take into account its indirect influence on productivity and inflation as indicated by endogenous higher statistical moments in the model equations we presented. This includes the question of transparency, i.e. explaining the influence of higher moments to the public including the corresponding reaction of monetary policy, e.g. for the non-linear inflation bias we have observed in our model.

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