# CCES Discussion Paper Series
## Center for Research on Contemporary Economic Systems

**Graduate School of Economics**
**Hitotsubashi University**

CCES Discussion Paper Series, No.22
November 2009


# Commodity Content in a General Input-Output Model: A Comment


Naoki Yoshihara
(Hitotsubashi University)
Roberto Veneziani
(Queen Mary University of London)



Naka 2-1, Kunitachi, Tokyo 186-8601, Japan
Phone: +81-42-580-9076    Fax: +81-42-580-9102
URL: http://www.econ.hit-u.ac.jp/~cces/index.htm
E-mail: cces@econ.hit-u.ac.jp

# Commodity Content in a General Input-Output Model: A Comment*

Naoki Yoshihara[†] and Roberto Veneziani[‡]

September 2009

**Abstract**

This paper critically analyses the approach to the determination of values, or commodity contents, developed by Fujimoto and Opocher (2009). Even setting aside various problematic definitional issues, the broader implications of the approach for classical theory are unclear. First, the value-theoretic definitions of skill differentials and bads capture at best necessary conditions and it is unlikely that such definitions can be provided by focusing only on the technological data of the economy. Second, the approach has various interesting implications concerning the relation between productivity and exploitation that directly contradict some of the authors' claims.

**JEL Classification Numbers**: D46, B51.

**Keywords**: Value, Bads, Skilled and Unskilled Labour, Productivity and Exploitation.

---

*We are grateful to Arrigo Opocher for his insightful comments on an earlier draft of this comment. The usual disclaimer applies.

[†]The Institute of Economic Research, Hitotsubashi University, Naka 2-1, Kunitachi, Tokyo 186-0004, Japan. Phone: (81)-42-580-8354, Fax: (81)-42-580-8333. E-mail: yosihara@ier.hit-u.ac.jp

[‡]Department of Economics, Queen Mary University of London, Mile End Road, London E1 4NS, UK. E-mail: r.veneziani@qmul.ac.uk

# 1 Introduction

In a recent contribution, Fujimoto and Opocher (2009; henceforth, FO) have proposed an interesting approach to the determination of values, or commodity contents, which aims to generalise Fujimoto and Fujita (2008).[1] The key feature is the complete symmetry between goods and different types of labour in the definition of values: for any two commodities (including various types of labour) $i$ and $j$ produced in von Neumann-Morishima economies, it is possible to identify the commodity $i$ value of commodity $j$, denoted as $\lambda_j^{(i)}$, and thus the whole vector of commodity $i$ values $\Lambda^{(i)}$. According to FO, this approach allows one to define values (including labour values) in a more general context than the standard one and also to analyse a number of issues traditionally neglected in classical approaches. For example, a value-theoretic definition of bads, and of skilled vs. unskilled labour, can be provided based solely on supply-side data.

The framework is innovative and formally sophisticated, but its theoretical foundations and some of the key arguments are not entirely compelling. In particular, FO do not provide a thorough theoretical defense of their definition of values (FO, 2009, Definition 1, p.4), whose main advantage, according to them, is to generalise standard definitions and to establish a complete symmetry between (all types of) labour and other goods. Yet, a purely formalistic approach to the definition of values leaves a number of possibly problematic theoretical issues unanswered. For example, it is rather unclear why the value of good $j$ in terms of good $i$ should be *defined* as the amount of good $i$ directly or indirectly consumed "in order to make possible the production of one unit of [$j$] *as net product*" (FO, 2009, p.2, emphasis added), over and above the workers' consumption bundle. This is at odds with the standard definition of labour values in a Leontief system. Even more puzzlingly, why should the economy be required to produce a net output *of labour* in order to define values? And what does it mean, from a theoretical viewpoint, to say that one unit of labour is *produced as net product*? Furthermore, it is quite unclear that the activity vector solving the optimisation programme (**AG**) (FO, 2009, p. 6) has any economic meaning, or even any *direct* relevance to the definition of values, as suggested instead by FO.

Even setting aside the above definitional issues, the broader implications

[1] Throughout the paper, the terms "value" and "commodity content" are used interchangeably, following standard practice and FO's (2009, fn.2) suggestion. No argument in the paper depends on the specific terminology adopted.

of the formal framework proposed by FO (2009) for classical theory are unclear. In the rest of this comment, sections 2 and 3, respectively, highlight a number of shortcomings of the value-theoretic definitions of bads, and of skilled vs. unskilled labour, whose discussion takes up almost half of the substantive analysis in FO (2009). Section 4 analyses FO's claims on the relation between productivity and exploitation. Although exploitation theory is not the main focus of FO, it is a central topic in classical approaches and the framework proposed by FO has interesting implications for the so-called "Generalised Commodity Exploitation Theorem" (e.g., Roemer, 1982; henceforth, GCET) which directly contradict some of FO's arguments.

# 2 Bads

According to FO, their approach shows that "von Neumann-Morishima models can be useful in the distinction between goods and bads" (FO, 2009, p.9). To be precise, bads are defined in terms of their commodity $i$ content.

**Definition 1** (FO, 2009, Definitions 2 and 3, p.8)**:** *Bads of the first category are those commodities which have zero content for any type of labour as the standard. Bads of the second category are those bads of the first category which have a zero own commodity content.*

Definition 1 is interesting and original, but ultimately unconvincing. Theoretically, the intuitive concepts of goods and bads are arguably linked to some notion of *human welfare*, welfare functions, or similar. Definition 1, instead, focuses only on production data, without specifying any link between the intuitive notion of bads in terms of welfare and their commodity content. Thus, it can capture, *at best*, necessary conditions for a bad, but certainly not sufficient conditions. It *might* be reasonable to say that if a commodity is a bad, then it has zero labour (or own commodity) content. But it is arguably false to say that every commodity with zero labour value is definitionally a bad. Definition 1 seems to misleadingly identify bads with some kind of free-goods (in the sense that no labour is necessary in the production process). Formally, this can be seen by noting that whenever at the solution of the dual (**AG**) output of good $i$ strictly exceeds the replacement of inputs and workers' consumption, then the value of good $i$ in the primal (**VG;** FO, 2009, p.5) must be zero.

Let $\mathbb{B}$ and $\mathbb{A}$ represent the output and input coefficient matrices, inclusive of household activities to produce labour in general. Consider the numerical example analysed by FO (2009, p.8). They suppose that good 1 is an ordinary commodity (corn), good 2 is a pollutant (manure), and good 3 is labour.

$$\mathbb{B} \equiv \begin{bmatrix} 11 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \mathbb{A} \equiv \begin{bmatrix} 1 & 0 & 1 \\ 0 & 10 & 0 \\ 1 & 1 & 0 \end{bmatrix},$$

Using labour (good 3) as the numeraire, FO (2009, p.8) prove that the optimal solutions of (**AG**) and (**VG**) are, respectively, $x^* = (1/9, 0, 10/9)'$ and $q^* = (1/9, 0, 10/9)$,[2] which implies that manure is a bad of the first type.

This example seems at best inconclusive. First, it is unclear that the example can provide valid support to Definition 1 from a methodological viewpoint: that good 2 is manure is assumed, and it is precisely this assumption that makes the example plausible. No argument is provided to suggest that *every* good commonly considered as a bad will have a similar production structure. Second, suppose that, instead of manure, the economy produces a good called 'oil' with a very inefficient production process (e.g. because the so-called 'peak oil' has been passed). Formally, the economy is identical to the above, except that the second columns of $\mathbb{B}$ and $\mathbb{A}$ are replaced, respectively, by $\mathbb{B}_2 = (0, 2, 0)'$ and $\mathbb{A}_2 = (0, 12, 1)'$. It is immediate to show that the solution vectors of the optimisation programmes are again $x^* = (1/9, 0, 10/9)'$ and $q^* = (1/9, 0, 10/9)$. But why should 'oil' be considered as a bad in any intuitively relevant sense?

Third, and perhaps more important, consider another economy which produces goods 1 and 3 as above, and another (properly unspecified) commodity according to $\mathbb{B}_2 = (0, 1, 0)'$ and $\mathbb{A}_2 = (0, 0.5, 1)'$. Again, if labour is chosen as the standard of value, then the solution vectors are $x^* = (1/9, 0, 10/9)'$ and $q^* = (1/9, 0, 10/9)$. And, again, by simply looking at the production side of the economy, it is quite unclear why good 2 should be defined as a bad, especially if one notes that the second process cannot be interpreted as a pure disposal process in any meaningful way.

The previous examples suggest that *perhaps* Definition 1 would be better derived as a formal result proving that, under some theoretically relevant definition of bads, the labour (or commodity) content of a bad is zero. Yet, as

[2] Note that the expression $x'$ represents the *transpose* of the vector $x$. If $x = (x_1, \ldots, x_m)$ is a row vector, then $(x_1, \ldots, x_m)'$ is a column vector.

already noted, it is unlikely that a theoretically satisfactory and intuitively appealing definition of bads can be obtained “on the sole basis of technological data” (FO, 2009, p.11) and independently of some notion of human welfare. Moreover, if the definition of bads does depend on some notion of human welfare, then it is not obvious that any commodity that is a bad in welfare terms must have zero labour (and possibly also zero own commodity) content. Not all bads are simply joint products of other production processes, which only need to be disposed of. Nuclear weapons, for example, might be regarded as bads, at least according to some notion of welfare, because there is a nonzero probability that they might lead to the extinction of human life. Yet, various types of capital and labour are used in the production process of nuclear weapons, which implies that the latter have nonzero labour value.

# 3 Skilled and Unskilled Labour

The definition proposed by FO (2009) aims to incorporate the intuition that the production of a unit of skilled labour requires more labour of any type than the production of a unit of unskilled labour. Formally:

**Definition 2** (FO, 2009, Definition 4, p.9)**:** Labour of type $i$ is less skilled than labour of type $j$ whenever $\lambda_i^{(i)} < \lambda_j^{(i)}$ and $\lambda_i^{(j)} < \lambda_j^{(j)}$.

Definition 2 is interesting, but not entirely convincing. Again, arguably, it can *at best* capture necessary conditions for skill differentials. In general, the property of being skilled or unskilled pertains to workers and it describes their characteristics as the final outcomes of processes of accumulation of knowledge, but also as the product of innate abilities. Therefore, although it might be true that skilled labour has normally a higher labour content than unskilled labour, it is rather unclear that every type of labour that has a higher labour content is necessarily more skilled. Innate abilities may make an untrained worker more skilled than a very educated one. Different types of labour should be defined in terms of their actual characteristics and not by the process of their ‘production’.

Even more puzzling is that Definition 2 holds even if no labour (and therefore no education or training) is spent on either type of labour. A type of labour can be defined as more skilled simply based on the labour content of the bundle of physical commodities used in its reproduction. To see this extremely doubtful property, consider the example by FO (2009, p.10):

$$\mathbb{B} \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0.9 \\ 0 & 11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \mathbb{A} \equiv \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 10 & 0 & 0 \\ 10 & 1 & 1 & 0 & 0.1 \\ 1 & 0.2 & 0.1 & 0 & 0 \end{bmatrix}.$$

The five rows represent, respectively, the following goods: a 'luxury' commodity, corn, manure, 'unskilled labour,' and 'skilled labour.' Solving the two linear optimisation programmes using goods 4 and 5 as a numeraire, respectively, FO (2009, p.10) prove that $\lambda_4^{(4)} < \lambda_5^{(4)}$ and $\lambda_4^{(5)} < \lambda_5^{(5)}$, and conclude that labour produced according to the fifth process is indeed more skilled than labour produced according to the fourth process. Both this conclusion and the example as a whole are quite puzzling.[3]

It is slightly odd that no labour of any type is used in the production of unskilled labour, given that the matrices $\mathbb{B}$ and $\mathbb{A}$ incorporate also reproduction activities. And it is rather peculiar that no skilled labour is used in the production of skilled labour, even though it seems reasonable to assume that education and training - an obvious type of skilled labour - play an important role in the production of skills.

The most puzzling feature of Definition 2, though, can be illustrated with a simple modification of the example. Consider the same economy as above but suppose that $\mathbb{A}_{45} = 0$ so that *no labour of any type is used in the production of any type of labour.* Taking 'unskilled labour' (good 4) as the numeraire, it can be shown that the vector of labour values is $\Lambda^{(4)} = (\frac{490}{43}, \frac{11}{86}, 0, \frac{11}{86}, \frac{60}{43})$. Similarly, taking 'skilled labour' (good 5) as the numeraire, the vector of labour values is $\Lambda^{(5)} = (\frac{11}{9}, \frac{1}{45}, 0, \frac{1}{45}, \frac{1}{6})$. Because $\lambda_4^{(4)} < \lambda_5^{(4)}$ and $\lambda_4^{(5)} < \lambda_5^{(5)}$, according to Definition 2, one should conclude that labour produced according to the fifth process is more skilled than labour produced according to the fourth, *even though no labour is used in the production of either type of labour.* This is extremely counterintuitive, especially if one notes that goods 1, 2 and 3 may just be consumption goods, with no relation whatsoever to the production of skills. But then, different types of labour would be defined as more or less skilled simply based on the amount of labour contained in their consumption bundles. From this viewpoint, Definition 2 at

[3]It is worth noting in passing that it is rather unclear in what sense the first commodity is a 'luxury good': this concept is undefined and it is by no means obvious how it might be defined based only on production data.

most implies that reproducing one unit of type $j$ labour needs *more resources* (measured by the units of type $i$ and type $j$ labour) than reproducing one unit of type $i$ labour. This is arguably not the essential feature of the notions of skilled and unskilled labour, but, if anything, it characterises high-paid and low-paid labour. Yet, since prices and wages are not explicitly included in the analysis, the latter conclusion can only be tentative.

The previous example also suggests that *at best* Definition 2 can capture necessary conditions for skill differentials. *Perhaps* one might provide a different definition of skilled and unskilled labour and then prove that the inequalities in Definition 2 must hold. Yet, as already noted, it is quite unlikely that a theoretically relevant and intuitively appealing definition of skilled labour can only focus on the inputs necessary to produce it, without mentioning its final characteristics. Further, in the framework proposed by FO, it is difficult to distinguish between differences in skills, as commonly interpreted, and heterogenous labour as understood in the classical framework. According to FO (2009, p.3), "any kind of labour services comes into our model just like ordinary commodities," which implies that labour of type $i$ and labour of type $j$ in Definition 2 can be regarded as two types of heterogenous labour, say intellectual labour and manual labour, respectively. However, in this case, it seems inappropriate to define these two types of labour in terms of skilled and unskilled, since it is unclear how one hour of intellectual labour can be regarded as being indifferent to some hours of manual labour, without price information.

# 4 Productivity and Exploitation

Consider standard von Neumann-Morishima economies with one type of homogeneous labour and no joint production, so that $\mathbb{B}$ and $\mathbb{A}$ are

$$\mathbb{B} \equiv \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix}, \mathbb{A} \equiv \begin{bmatrix} A & c \\ L & 0 \end{bmatrix},$$

where $A$ is the $n \times n$ matrix of intermediate inputs, $L$ is the $1 \times n$ vector of direct labour inputs, and $c$ is the $n \times 1$ workers' consumption vector. The following assumption guarantees the productivity of the system.

**(A1)**. *The matrix* $(\mathbb{B} - \mathbb{A})$ *satisfies the* Hawkins-Simon conditions.

For all $i = 1, ..., n+1$, the commodity $i$ values are given by:

(1) $\Lambda^{(i)} \equiv (\lambda_1^{(i)}, \lambda_2^{(i)}, ..., \lambda_{i-1}^{(i)}, \lambda_i^{(i)}, \lambda_{i+1}^{(i)}, ..., \lambda_{n+1}^{(i)}) = \Lambda_{[i]}^{(i)} \mathbb{A}$,

where $\Lambda_{[i]}^{(i)} \equiv (\lambda_1^{(i)}, \lambda_2^{(i)}, ..., \lambda_{i-1}^{(i)}, 1, \lambda_{i+1}^{(i)}, ..., \lambda_{n+1}^{(i)})$.

In this context, the standard GCET can be expressed as follows.

**Proposition 1:** *Under **A1**, $\lambda_i^{(i)} < 1$, for all $i$.*

According to FO, however, Proposition 1 "is misunderstood by some people as expressing the exploitation of commodities. ... Actually, this has nothing to do with exploitation but is simply an alternative expression of productiveness of an economy" (FO, 2009, p.7). This claim is arguably unwarranted and even in contradiction with FO's own framework.

First, the definition of the 'exploitation' of a commodity in the GCET is a natural generalisation of the standard definition of labour exploitation proposed by Okishio (1963) and Morishima (1973) in Leontief economies. Therefore, whenever the Okishio-Morishima approach is adopted, it is quite natural to conclude that the possibility of producing a surplus is equivalent to the exploitation of any commodity, including labour (and to the existence of positive profits). To be sure, the definition of the exploitation of a commodity is "an alternative expression of productiveness of an economy," but this is also true for labour exploitation. In the context of standard linear economies, if the Okishio-Morishima approach is adopted, the exploitation of a commodity (including labour) is just a numerical representation of the possibility of surplus production measured in the unit of this commodity.

Second, and perhaps more important for the analysis of FO (2009), the relation between Proposition 1 and the exploitation of commodities can be forcefully shown even *within FO's own framework*. In order to support their claim, FO (2009, p.7) refer to the argument developed by Fujimoto and Fujita (2008), whose model they aim to generalise. However, given the symmetry between labour and other goods which characterises FO's approach, the analysis developed in Fujimoto and Fujita (2008) can be naturally extended to contradict FO's claim. Thus, for any vector $x \in \mathcal{R}^{n+1}$, let $x_{(i)} \in \mathcal{R}^n$ denote the vector obtained from $x$ by removing the $i$-th entry. Fujimoto and Fujita (2008, pp.535-6) prove the following Proposition.

**Proposition 2:** *Assume **A1** and a positive wage rate. In the exchange between one unit of labour and the workers' consumption basket, their labour values are equal. Instead, the commodity $i$ value of labour is greater than that of the basket, provided $c_i > 0$.*

The first part of the statement follows immediately from the definition of labour values $\Lambda^{(n+1)}$, which implies $\lambda_{n+1}^{(n+1)} = \Lambda_{(n+1)}^{(n+1)}c$. The second part of the statement follows because under ***A1***, and assuming $c_i > 0$, it must be $\lambda_{n+1}^{(i)} = \Lambda_{[i](n+1)}^{(i)}c > \Lambda_{(n+1)}^{(i)}c$, where the equality follows by definition and the strict inequality follows by Proposition 1.

According to Fujimoto and Fujita (2008, pp.535-6), Proposition 2 proves that in the trade between labour and the wage bundle $c$, workers do not acquire any "surplus", whereas the combined commodity basket $c$ or its owners can obtain "surpluses" through exchanges with the workers.

To be sure, various doubts may be raised on the theoretical implications of Proposition 2, and on the underlying notions of surplus and exploitation. But *even if* all doubts on the theoretical foundations of the above analysis are set aside and Proposition 2 is taken to represent a proof of the exploitation of labour, then *within FO's own framework* a revised GCET can be proved, which forcefully shows the link between productivity, Proposition 1, and the 'exploitation' of every commodity, contrary to FO's claim.

As a preliminary point, note that $L_{n+1} = 0$ is a necessary condition for (the first part of) Proposition 2 to hold. This is a natural assumption in the standard framework, but not in FO (2009) where - consistently with the emphasis on the symmetry between labour and other goods - the $n+1$-th column of $\mathbb{A}$ is interpreted as describing the (re)production process of labour (see, e.g., the economy with skilled and unskilled labour above). Indeed, in FO's framework to assume $L_{n+1} = 0$ is theoretically as legitimate as it is to assume $\mathbb{A}_{ii} = 0$, for any $i$. Thus, let $\mathbb{A}_i$ denote the column of input requirements in the production of good $i$.

**Theorem** (Revised GCET)**:** *Assume* ***A1****. For all* $i = 1, ..., n+1$*, in the exchange between one unit of good* $i$ *and the basket of goods* $\mathbb{A}_i$ *necessary to produce it, their commodity* $i$ *values are equal whenever* $\mathbb{A}_{ii} = 0$*; whereas for any* $j \neq i$*, the commodity* $j$ *value of good* $i$ *is greater than that of* $\mathbb{A}_i$*, provided* $j$ *is contained in the basket, i.e.* $\mathbb{A}_{ji} > 0$*.*

**Proof.** By equation (1), and given $\mathbb{A}_{ii} = 0$, it follows that $\lambda_i^{(i)} = \Lambda_{[i]}^{(i)}\mathbb{A}_i = \Lambda_{(i)}^{(i)}\mathbb{A}_{i(i)} = \Lambda^{(i)}\mathbb{A}_i$. This proves the first part of the statement.

The second part of the Theorem can be proved noting that, by equation (1), it follows that $\lambda_i^{(j)} = \Lambda_{[j]}^{(j)}\mathbb{A}_i = \Lambda_{[j](i)}^{(j)}\mathbb{A}_{i(i)} + \lambda_i^{(j)}\mathbb{A}_{ii}$. But then, under ***A1***, and assuming $\mathbb{A}_{ji} > 0$, it follows that $\lambda_i^{(j)} > \Lambda_{(i)}^{(j)}\mathbb{A}_{i(i)} + \lambda_i^{(j)}\mathbb{A}_{ii}$. ■

In other words, under ***A1***, for *any* good $i = 1, ..., n+1,$ the combined commodity basket $\mathbb{A}_i$ or its owners can obtain "surpluses" through exchanges with the owners of $i$. Therefore, an interesting implication of the approach proposed by FO is that a revised version of the GCET can be proved and the relation between the productivity of the economy, Proposition 1, and the exploitation of any commodity is forcefully confirmed, contrary to FO's own claim. In the traditional Okishio-Morishima framework, there is no analytical basis for distinguishing labour exploitation from the exploitation of any other commodity. In FO's approach, this distinction is even less likely to be derived, given that 'The essence of our method is the complete *symmetry* in dealing with goods and labour' (FO, 2009, p.2).

# 5 References


Fujimoto, T. and Y. Fujita (2008): "A Refutation of Commodity Exploitation Theorem," *Metroeconomica* **59**, pp. 530-540.

Fujimoto, T. and A. Opocher (2009): "Commodity content in a general input-output model," *Metroeconomica*, forthcoming.

Morishima, M. (1973): *Marx's Economics*, Cambridge University Press, Cambridge.

Okishio, N. (1963): "A mathematical note on Marxian theorems," *Weltwirtschaftliches Archiv* **91**, pp. 287-299.

Roemer, J. E. (1982): *A General Theory of Exploitation and Class*, Harvard University Press, Cambridge.