CCES Discussion Paper Series
Center for Research on Contemporary Economic Systems

Graduate School of Economics
Hitotsubashi University

CCES Discussion Paper Series, No.28
March 2010

The Efficiency of Voluntary Pollution Abatement when Countries can Commit

Robin Boadway
(Queen's University)
Zhen Song
(Central University of Finance and Economics)
Jean-François Tremblay
(University of Ottawa)

Naka 2-1, Kunitachi, Tokyo 186-8601, Japan
Phone: +81-42-580-9076   Fax: +81-42-580-9102
URL: http://www.econ.hit-u.ac.jp/~cces/index.htm
E-mail: cces@econ.hit-u.ac.jp

The Efficiency of Voluntary Pollution Abatement when Countries can Commit

by

Robin Boadway, Queen's University, Canada

Zhen Song, Central University of Finance and Economics, China

Jean-François Tremblay, University of Ottawa, Canada

August, 2009

Abstract: We characterize a mechanism for reducing pollution emissions in which countries, acting non-cooperatively, commit to match each others' abatement levels and may subsequently engage in emissions quota trading. The mechanism leads to an efficient level of emissions, and if the matching abatements process includes a quota trading stage, the marginal benefits of emissions are also equalized across countries. Given equilibrium matching rates, the initial allocation of emission quotas (before trading) reflects each country's marginal valuation for lower pollution relative to its marginal benefit from emissions. These results hold for any number of countries, in an environment where countries have different abatement technologies and different benefits from emissions, and even if the emissions of countries are imperfect substitutes in each country's damage function. In a dynamic two-period setting, the mechanism achieves both intra-temporal and inter-temporal efficiency. We extend the model by assuming that countries are voluntarily contributing to an international public good, in addition to undertaking pollution abatements, and find that the level of emissions may be efficient even without any matching abatement commitments, and the marginal benefits of emissions may be equalized across countries even without quota trading.

Keywords: Voluntary pollution abatement, matching commitments, emissions quota trading; **JEL classification:** H23, H41, H87

Acknowledgments: We have benefited from useful comments by Vidar Christiansen, Shulin Liu, Ying Liu and Todd Sandler. Support of the Social Sciences and Humanities Research Council of Canada is gratefully acknowledged.

1 Introduction

International agreements on pollution reduction targets are difficult to achieve and sustain in the absence of a central authority with the ability to enforce the abatement objectives of national governments. Cooperative initiatives also require that countries be able to agree on the overall objectives of emissions reduction and on how abatement efforts should be distributed across countries. This is particularly challenging given that the costs and benefits of pollution abatement vary considerably across countries. Without cooperative agreements, emission reductions rely essentially on the voluntary contributions of countries.

In this paper, we show that voluntary pollution abatement by countries behaving non-cooperatively can lead to efficient outcomes provided that countries can commit to matching the abatement efforts of each other at some announced rates. The efficiency of voluntary contributions to international public goods when countries can commit has been established by Guttman (1978), Danziger and Schnytzer (1991), Varian (1994) and Boadway, Song and Tremblay (2007). We show how similar reasoning can be adapted to the case of international pollution abatement when countries have different abatement technologies and may be able to engage in emissions quota trading. Remarkably, we also find that efficiency can occur even in the absence of commitment provided that countries are also contributing to an international public good.

Recently, a number of papers have proposed mechanisms for implementing efficient contributions by countries to international public goods, such as pollution abatement. In particular, Gersbach and Winkler (2007) and Gerber and Wichardt (2009) have proposed schemes in which countries make up-front payments to a neutral institution as a way of pre-committing to contributions. The payments are eventually refunded, at least in part, if countries provide their intended contributions. The neutral institution's ability to deny refunds induces countries to act according to prior commitments. In principle, these mechanisms can be designed to implement any desired emission reduction objectives, although they require some prior cooperative agreement to establish such objectives, and to decide how to distribute the surplus across countries. In contrast, we take the commitment ability of countries as given, but focus on a non-cooperative mechanism that can emerge and induce full efficiency in emission abatement when countries are making commitments voluntarily and are acting in their own self-interest.

Altemeyer-Bartscher, Rübbelke and Sheshinski (2009) consider another form of commitment mechanism whereby each of two countries voluntarily makes a take-it-or-leave-it offer of a payment to the other country conditional on the tax rate that the latter imposes on a polluting good. They show that such a mechanism can induce the efficient level of pollution. While their mechanism is based on side-payments between countries, the mechanism we characterize relies on matching abatement commitments and, crucially, may allow emissions quota trading. Both mechanisms can lead to efficient allocations, although they do not generally result in the same distribution of net benefits across countries. Moreover, as Altemeyer-Bartscher, Rübbelke and Sheshinski recognize, their mechanism does not easily generalize to more than two countries, since any given country would receive take-it-or-leave-it offers from all other countries simultaneously.

Our static one-period base case without quota trading resembles the case analyzed by Guttman and Schnytzer (1992) who demonstrate the existence of a Pareto efficient equilibrium in a mechanism where two individuals are matching each others' externality-producing activities. The pollution reduction case that we study has some features that go beyond the simple externality case. Countries have access to different pollution abatement technologies, which gives rise to the issue of the optimal allocation of emissions across countries. In this case, the possibility of emissions quota trading provides an instrument for achieving that optimal allocation alongside the use of a matching mechanism to influence to aggregate level of abatements. We also extend the mechanism both to a multi-country setting and to a dynamic two-period setting where emissions in one period determine the initial stock of pollution in the next period.

Specifically, the pollution abatement process we consider works as follows. Each country simultaneously (and non-cooperatively) announces a rate at which it will match the abatement efforts of the other countries. Countries then choose their direct abatement efforts simultaneously, taking the previously announced matching rates as given. After these two stages of decisions, countries are committed to achieving a total emissions quota equal to their business-as-usual emissions minus the sum of their direct and matching abatement efforts. However, these commitments may be contingent in the sense that once they are determined, countries may trade emissions quotas at the competitively determined price.

The analysis shows that the subgame perfect equilibrium of this emission abatement process is efficient. The efficient level of pollution abatement is achieved, and if the mechanism allows for quota trading, the marginal benefits of emissions are equalized across countries. The equilibrium displays other interesting properties. For one, the effective marginal cost to a country from inducing an increase in world abatements is the same whether they do so directly through their own abatements or indirectly through matching the other country's abatements. For another, in equilibrium, a country's total abatements, both direct and matching, just equals its marginal valuation of pollution abatement times total world abatements. Thus, the countries' effective costs of abatement are the analogs of Lindahl prices in the context of this model.

As mentioned, the non-cooperative mechanism that we consider is easily applicable to a setup with any number of countries. Under a matching rate mechanism, the simultaneous offers of several countries readily add up to an aggregate matching rate applying to the abatement effort of an individual country. We also consider a dynamic two-period extension and find that the mechanism achieves intra-temporal and inter-temporal efficiency: the total level of emissions is efficient as well as its allocation between periods. And, we extend the model by adding an international public good provided by the voluntary contributions of countries. If contributions to the public good are made after the pollution abatement process, we find that the level of emissions and their allocation across countries are efficient even in the absence of matching abatement commitments and quota trading.

In the next section, we describe the main features of the model. We then characterize the abatement process equilibrium in a simple two-country case. Various extensions of the basic model are considered in Section 4, while contributions to an international public good are added in Section 5. In Section 6, we show that the mechanism leads to efficient levels of emissions even if countries' emissions are imperfect substitutes in each countries' damage function. Concluding remarks are provided in the last section.

2 The Basic Two-Country Model

There are two countries denoted by $i, j = 1, 2$. In the absence of any abatement effort, the business-as-usual level of emissions by country i is equal to $\overline{e}_i$. Both countries can undertake costly abatements which will reduce actual emissions. In the basic model, country i chooses a level of direct abatements a_i, as well as committing to match the direct abatement of country j at a rate m_i.[1] Therefore, country i's total choice of abatements equals $A_i = a_i + m_i a_j$. In some extensions of the basic model, we allow for emissions quota trading. In these cases, the initial choice of abatements A_i is contingent since countries can then trade emission quotas at market price p. In these cases, we can interpret county i's initial choice of emissions $\overline{e}_i - A_i$ as its pre-trade emissions quota. The number of emission quotas purchased by country i is denoted by q_i, where $q_1 = -q_2$. Given the number of quotas traded, the actual emissions of country i are $e_i = \overline{e}_i - A_i + q_i$. Note that aggregate emissions by both countries are equal to the sum of their initial commitments before quota trading. The latter simply reallocates emissions from one country to another.

The benefits of actual emissions to country i is given by the function $B_i(e_i)$, where $B_i' > 0$ and $B_i'' < 0$.[2] The damage to country i is a function of the total emissions of both countries, $D_i(e_1 + e_2)$, with $D_i' > 0$ and $D_i'' > 0$. Hence, the emissions of both countries are assumed to be perfect substitutes, although we later relax this assumption in an extension of the basic model.

The analysis will characterize the equilibrium levels of abatement in a number of cases, starting with the basic case where there are matching abatements but no emissions quota trading. We then consider the various extensions of the basic model mentioned in the Introduction.

---

1 In the real world, abatements a_i will be chosen by private agents, and the government will influence them indirectly by a tax or tradeable permit scheme. For simplicity, we suppress the private sector from our analysis and let the government choose abatements directly. Nothing of substance is lost by this simplification.

2 The marginal benefits of emissions can be viewed as the negative of a marginal cost of abatement function, $B'(e) = -C'(A)$. A cost of abatement function has been used by Roberts and Spence (1976), for example.

Before turning to the basic case without quota trading, it is useful to characterize the social optimum. To do so, we solve a Pareto-optimizing problem whereby the emissions of both countries are chosen to maximize the net benefits of one country, say country 1, subject to the constraint that the net benefits of country 2 equals some fixed level $\overline{\Pi}_2$:

$$\max_{\{e_1,e_2\}} \; B_1(e_1) - D_1(e_1+e_2) + \lambda\left[B_2(e_2) - D_2(e_1+e_2) - \overline{\Pi}_2\right]$$

The first-order conditions can be written as:

$$\frac{D_1'}{B_1'} + \frac{D_2'}{B_2'} = 1 \tag{1}$$

This condition is the analog of the Samuelson condition for public goods, but in the context of a public bad. It says that efficient emissions in each country are such that the sum of the two countries' ratios of marginal damages to marginal benefits is equal to unity.

The social optimum just defined is restrictive in the sense that the only instruments for redistributing between countries are emissions e_1 and e_2. To understand the implications of this, suppose we allow the possibility of a transfer T from country 2 to country 1, where $T \gtreqless 0$. The above Pareto optimizing problem then becomes:

$$\max_{\{e_1,e_2,T\}} \; B_1(e_1) - D_1(e_1+e_2) + T + \lambda\left[B_2(e_2) - D_2(e_1+e_2) - T - \overline{\Pi}_2\right]$$

and the first-order conditions would give (1) above as well as:

$$B_1'(e_1) = B_2'(e_2) \tag{2}$$

In effect, while (1) characterizes the efficient level of total emissions, (2) characterizes their efficient allocation across countries. We can think of the solutions to the latter problem for various values of $\overline{\Pi}_2$ as tracing out the first-best utility possibilities frontier, while the solution to the problem without transfers traces out a restricted utility possibilities frontier. The two would only coincide where $T = 0$ solves the latter problem. In the basic model to which we turn next, it is the restrictive problem that is relevant. However, as we shall see, outcomes on the first-best frontier can be achieved in some of our extensions.

In the absence of international corrective action, country emissions would satisfy $B_1' = D_1'$ and $B_2' = D_2'$, and would not be optimal. A world government could achieve the restricted social optimum by imposing Pigouvian taxes on the emissions in each country at the tax rates $t_1 = D_2'$ and $t_2 = D_1'$. In the absence of transfers, these tax rates would generally differ. If the world government could also make inter-country transfers, the optimal Pigouvian tax would be uniform across countries.[3] Our analysis explores commitment mechanisms as a way of achieving efficiency in the absence of a world government.

In what follows, we focus on the case where the socially optimal abatements of the two countries are both interior. That is, the levels of emissions e_1^* and e_2^* corresponding with the solution to the social optimum satisfy $0 < e_1^* < \bar{e}_1$ and $0 < e_2^* < \bar{e}_2$.

3 Matching Abatements without Quota Trading

In this section, we examine the basic case where two countries can commit to matching the abatement efforts of each other, and where there is no quota trading. The timing of decisions is the following. In Stage 1, both countries simultaneously choose the rate m_i at which they will match the direct abatements of the other country. Countries then simultaneously choose direct abatement levels a_i in Stage 2. We characterize the subgame perfect equilibrium of this two-stage process by backward induction.[4]

Stage 2: Choosing Direct Abatements $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$

Taking (m_1, m_2) as given from Stage 1, country 1 chooses a_1 to solve the following:

$$\max_{\{a_1\}} \mathit{\Pi}_1 = B_1\left(\bar{e}_1 - a_1 - m_1 a_2\right) - D_1\Big(\bar{e}_1 - (1+m_2)a_1 + \bar{e}_2 - (1+m_1)a_2\Big)$$

The first-order condition, assuming an interior solution, is:

$$F^1(a_1, a_2, m_1, m_2) \equiv -B_1'(\cdot) + (1+m_2)D_1'(\cdot) = 0 \quad \text{or} \quad \frac{D_1'(\cdot)}{B_1'(\cdot)} = \frac{1}{1+m_2} \tag{3}$$

[3] This point is made by Sandmo (2006). He emphasizes the distinction between carbon prices in high-and low-income countries when there are limited international transfers.

[4] Multi-stage processes of matching contributions to public goods have been analyzed in Guttman (1978), Danziger and Schnytzer (1991), Varian (1994) and Boadway, Song and Tremblay (2007), among others.

The solution to this first-order condition is country 1's reaction function $a_1(a_2; m_1, m_2)$. For any a_2, country 1 will choose its level of abatements such that the ratio of marginal damage to marginal benefit equals the effective cost at which it can increase world abatements by one unit, $1/(1+m_2)$. Differentiating (3), we have:

$$\begin{aligned} F^1_{a_1} &= B_1'' - (1+m_2)^2 D_1'' < 0, \quad F^1_{a_2} = m_1 B_1'' - (1+m_1)(1+m_2) D_1'' < 0, \\ F^1_{m_1} &= a_2 B_1'' - a_2(1+m_2) D_1'' < 0, \quad F^1_{m_2} = -a_1(1+m_2) D_1'' + D_1' \gtrless 0 \end{aligned} \tag{4}$$

The problem of country 2 is analogous. Its reaction function is $a_2(a_1; m_1, m_2)$, and expressions similar to (4) apply. The slopes of the two countries' reaction curves are $da_2/da_1 = -F^1_{a_1}/F^1_{a_2} < 0$ for country 1 and the analog for country 2, $-F^2_{a_1}/F^2_{a_2} < 0$.

The simultaneous solution to both reaction functions gives the Nash equilibrium abatements as functions of matching rates, $a_1(m_1, m_2)$ and $a_2(m_1, m_2)$ (with some abuse of notation). For an interior Nash equilibrium in abatements to be stable, the slope of country 2's reaction curve in (a_1, a_2)–space must be less negative than that of country 1, that is, $-F^2_{a_1}/F^2_{a_2} > -F^1_{a_1}/F^1_{a_2}$. Equivalently, if we define $H \equiv F^1_{a_1} F^2_{a_2} - F^2_{a_1} F^1_{a_2}$, stability requires that $H > 0$. Using (4) and its analog for country 2, we can derive

$$H = (1 - m_1 m_2)\Big[B_1'' B_2'' - (1+m_1) B_1'' D_2'' - (1+m_2) B_2'' D_1''\Big] \tag{5}$$

Since the expression in the square brackets in (5) is positive, we have

$$H \gtreqless 0 \quad \Longleftrightarrow \quad 1 - m_1 m_2 \gtreqless 0$$

We can characterize different types of outcomes in Stage 2 with reference to the sign of H:

Case i) $H > 0$, $m_1 m_2 < 1$

In this case, the Nash equilibrium, which we assume is interior, will be stable.

Case ii) $H < 0$, $m_1 m_2 > 1$

An interior Nash equilibrium will be unstable, and any deviation from equilibrium would tend to a stable corner equilibrium with either $a_1 = 0, a_2 > 0$ or $a_1 > 0, a_2 = 0$.

Case iii) $H = 0$, $m_1 m_2 = 1$

In this case, the slopes of the two reaction curves are identical. Moreover, using (4), the slope of country 1's reaction curve when $m_1 m_2 = 1$ becomes

$$\frac{da_2}{da_1} = -\frac{F^1_{a_1}}{F^1_{a_2}} = -\frac{B_1'' - (1+m_2)^2 D_1''}{m_1 B_1'' - (1+m_1)(1+m_2) D_1''} = -\frac{1+m_2}{1+m_1}$$

An analogous calculation for country 2 reveals that the slope of its reaction curve is the same. Thus, when $m_1 m_2 = 1$, reaction curves are linear and parallel. There are three possible equilibria in this case, depending on the values of m_1 and m_2. First, country 2's reaction curve might be outside country 1's as shown in the dotted lines in Figure 1. In this case, only country 2 undertakes abatements. Alternatively, county 1's reaction curve is outside country 2's so only the former abates. Finally, the two reaction curves could overlap, in which case a_1 and a_2 are indeterminate.

The latter case where the reaction curves overlap will be of special interest, so it is worth mentioning a couple of relevant properties. First, there will be unique values of m_1 and m_2 such that the reaction curves coincide.[5] Second, since the Stage 2 reaction curves coincide, direct abatements (a_1, a_2) are indeterminate. However, total abatements by each country

[5] To see this, note that in an interior solution (including at the boundary), country 1's reaction curve in $(a_1, a_2)-$space has the following properties (using the expressions in (4)):

$$\frac{da_1}{dm_1} = -\frac{F^1_{m_1}}{F^1_{a_1}} \leqslant 0 \quad (= 0 \text{ at } a_2 = 0); \quad \frac{da_1}{dm_2} = -\frac{F^1_{m_2}}{F^1_{a_1}} \gtrless 0 \quad (= -D_1'/F^1_{a_1} > 0 \text{ at } a_1 = 0)$$

Analogous properties hold for country 2's reaction curve. Consider an initial situation in which which $m_1 m_2 = 1$ and reaction curves coincide. Now suppose we first increase m_1 by a small amount, holding m_2 constant. The properties above and the expression for the slopes of reaction curves imply that 1) the reaction curves become flatter in the (a_1-a_2)–space, 2) the intercept of country 1's reaction curve is unchanged along the a_1–axis, 3) the intercept of country 2's reaction curve along the a_1–axis moves right, so the reaction curves are unambiguously further apart, although the intercept of country 2's reaction curve along the a_2–axis can either go up or down. Next, starting with these new reaction curves, consider decreasing m_2 by a small amount, holding m_1 constant. 1) The reaction curves again become flatter, 2) the intercept of country 2's reaction curve is unchanged along the a_2–axis, 3) the intercept of country 1's reaction curve along the a_2–axis goes down so the reaction curves again go further apart unambiguously, although the intercept of country 1's reaction curve along the a_1–axis can increase or decrease. The opposite will occur if we increase m_2 and decrease m_1. These imply that there is only one pair of m_1 and m_2 such that $m_1 m_2 = 1$ and the two reaction curves coincide.

are determinate. That is, all combinations of a_1 and a_2 along the common reaction curve yield the same levels of total abatement A_1 and A_2.[6] The implication is that net benefits for each of the two countries are constant along the common reaction curves:

$$\Pi_i = B_i\Big(\overline{e}_i - A_i\Big) - D_i\Big(\overline{e}_1 - A_1 + \overline{e}_2 - A_2\Big) \qquad i = 1, 2$$

Note finally that when $m_1 m_2 = 1$, each country's effective costs of direct and matching abatements are equal. To see this, note first that the effective cost to country 1 of increasing world abatement by one unit through an increase in its direct abatement a_1 is equal to $1/(1+m_2)$, as mentioned earlier. On the other hand, the effective cost to country 1 of a one unit increase in world abatement induced by an increase in the direct abatement of country 2 is $m_1/(1+m_1)$.[7] When $m_1 m_2 = 1$, it follows that $1/(1+m_2) = m_1/(1+m_1)$, so the effective costs of direct and matching abatements are equal. The same holds for country 2.

Stage 1: Choosing Matching Rates $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$

At this stage, both countries anticipate the subsequent Nash equilibrium choices of direct abatements. Country 1 chooses its matching rate m_1 to maximize its net benefit, taking as given m_2 and taking account of the Nash equilibrium solution $a_1(m_1, m_2), a_2(m_1, m_2)$. Country 1's net benefit can be written as:

$$\Pi_1 \equiv B_1\Big(\overline{e}_1 - a_1(m_1, m_2) - m_1 a_2(m_1, m_2)\Big)$$
$$-D_1\Big(\overline{e}_1 - (1+m_2)a_1(m_1, m_2) + \overline{e}_2 - (1+m_1)a_2(m_1, m_2)\Big)$$

---

[6] Thus, along the common reaction curves, $\Delta a_2/\Delta a_1 = -(1+m_2)/(1+m_1)$. Since $A_1 = a_1 + m_1 a_2$ and using $m_1 m_2 = 1$,

$$\Delta A_1 = \Delta a_1 + m_1 \Delta a_2 = \left(1 - m_1 \frac{1+m_2}{1+m_1}\right) \Delta a_1 = 0$$

The same demonstration applies for A_2.

[7] To see this, note that since $A = (1+m_2)a_1 + (1+m_1)a_2$, a change of a_2 equal to $\Delta a_2 = 1/(1+m_1)$ will cause an increase in A of $\Delta A = 1$. The cost to country 1 will be $m_1 \Delta a_1 = m_1/(1+m_1)$.

Differentiating this expression with respect to m_1 gives:

$$\frac{d\Pi_1}{dm_1} = -B_1'\left[\frac{\partial a_1}{\partial m_1} + a_2 + m_1\frac{\partial a_2}{\partial m_1}\right] + D_1'\left[(1+m_2)\frac{\partial a_1}{\partial m_1} + a_2 + (1+m_1)\frac{\partial a_2}{\partial m_1}\right] \tag{6}$$

Assume first an interior solution in abatements in Stage 2, so $F^1(\cdot) = 0$ and $F^2(\cdot) = 0$ by (3). Differentiating these expressions, we obtain:

$$\left.\frac{\partial a_1}{\partial m_1}\right|_{m_2} = \frac{-F^1_{m_1}F^2_{a_2} + F^2_{m_1}F^1_{a_2}}{H}, \quad \text{and} \quad \left.\frac{\partial a_2}{\partial m_1}\right|_{m_2} = \frac{-F^1_{a_1}F^2_{m_1} + F^2_{a_1}F^1_{m_1}}{H} \tag{7}$$

Using (3), (7) and the expressions for $F^i_{a_i}$, $F^i_{a_j}$, $F^i_{m_i}$, $F^i_{m_j}$ in (4), (6) can be written:

$$\frac{d\Pi_1}{dm_1} = -\frac{(1 - m_1m_2)D_1'D_2'F^1_{a_1}}{H} \tag{8}$$

This implies that $d\Pi_1/dm_1 > 0$ if $m_1m_2 \neq 1$ and the Stage 2 equilibrium is interior. The same holds for country 2.

Using this result and the characterization of Stage 2 above, it can be shown that the subgame-perfect equilibrium will be such that $m_1m_2 = 1$ and the two reaction curves coincide. As long as at least one country contributes in the absence of any matching contributions, this Nash equilibrium in matching rates will be unique. The demonstration is provided in the Appendix. As explained above, there will be unique values of m_1 and m_2 for which $m_1m_2 = 1$ and reaction curves coincide, the values of a_1 and a_2 will be indeterminate along the common reaction curve but total abatements, A_1 and A_2, will be uniquely determined.

Properties of the Equilibrium

A few other properties of the equilibrium are noteworthy. First, the equilibrium is efficient. The two Stage 2 first-order conditions together give:

$$\frac{D_1'(\bar{e}_1 - A_1 + \bar{e}_2 - A_2)}{B_1'(\bar{e}_1 - A_1)} + \frac{D_2'(\bar{e}_1 - A_1 + \bar{e}_2 - A_2)}{B_2'(\bar{e}_2 - A_2)} = \frac{1}{1+m_2} + \frac{1}{1+m_1} = 1 \tag{9}$$

using $m_1m_2 = 1$ in the last step. This is condition (1) characterizing the efficient levels of emissions by the two countries derived in Section 2.

Second, the direct cost at which country 1 can abate emissions, $1/(1+m_2)$, which is equal to D_1'/B_1' by the first-order condition in Stage 2, is the analog of a Lindahl price in the context considered here: it is the price per unit of abatement that country 1 would be willing to pay for the total abatements $A_1 + A_2$. To see this, simply note that the product of this price and total world abatements equals the total direct and matching abatement of country 1 (using $m_1 = 1/m_2$):

$$\frac{1}{1+m_2}(A_1 + A_2) = \frac{(1+m_2)a_1 + (1+m_1)a_2}{1+m_2} = a_1 + \frac{1+m_1}{1+m_2}a_2 = a_1 + m_1 a_2 = A_1$$

Thus, country 1's direct and matching abatements A_1 equals its marginal valuation for reduced pollution relative to its marginal valuation of the benefits of emissions, D_1'/B_1', applied to the world's total abatements, $(A_1 + A_2)$. The same applies for country 2. Thus, the total abatement each country makes can be seen as its quasi-Lindahl abatement effort.[8]

Finally, in equilibrium, countries 1 and 2 are indifferent between making direct abatements and matching abatements. As explained earlier, the effective cost to country 1 of direct abatements is $1/(1+m_2)$, whereas its effective cost of matching abatements is $m_1/(1+m_1)$. When $m_1 m_2 = 1$, $1/(1+m_2) = m_1/(1+m_1)$ and $1/(1+m_1) = m_2/(1+m_2)$. Thus, the cost to either country of reducing the world's pollution by one unit through direct abatement efforts or through matching abatement efforts are equal. If country 1 were to increase its matching rate, starting from an equilibrium with $m_1 m_2 = 1$, it would be reducing emissions indirectly at a cost higher than the cost at which it can reduce emissions directly. The same would apply for country 2. Therefore, neither country would want to increase their matching rate beyond $m_1 m_2 = 1$. By the same token, when $m_1 m_2 < 1$, $1/(1+m_2) > m_1/(1+m_1)$. It will be cheaper for country 1 to match the abatement of country 2 than to reduce emissions through its own direct abatements, so it will increase m_1. The same holds for country 2.

[8] Danziger and Schnytzer (1991) have shown that the Lindahl equilibrium in a public good contributions game can be implemented through a process where players can voluntarily subsidize the contributions of each other. Recently, Nishimura (2008) characterized the properties of the Lindahl equilibrium in the context of international emissions reduction, and examined different implementation mechanisms.

The main results of this section are summarized in the following proposition.

Proposition 1. *Assuming $a_1 > 0$ and/or $a_2 > 0$ when $m_1 = m_2 = 0$, the subgame perfect equilibrium of the abatement process is unique and has the following properties:*

i. Direct abatements (a_1, a_2) are indeterminate, but matching rates (m_1, m_2) and total abatements (A_1, A_2) are uniquely determined;

ii. Matching rates satisfy $m_1 m_2 = 1$ and countries are indifferent between direct and indirect contributions to abatements;

iii. The levels of emissions are Pareto efficient; and

iv. The effective cost of abatement faced by each country is the analog of a Lindahl price.

4 Extensions to the Basic Case

In this section, we consider three extensions to the basic case. First, we investigate the consequences of adding emissions quota trading. Then, we extend our model to a setting with more than two countries. Finally, we characterize the equilibrium of the abatement process in a dynamic two-period setting. In each case, the analysis is a straightforward extension of the basic case, so detailed analysis is not necessary.

4.1 The Mechanism with Emissions Quota Trading

With quota trading, the abatement mechanism involves three stages. The matching rates and the direct abatements chosen in the first two stages determine the emission quotas to which countries are committed. In the third stage, countries can trade these quotas at the equilibrium price, which we assume is competitively determined.[9] Note that in the absence of a central government with the authority to administer a quota trading system, the three-stage abatement process with quota trading requires a stronger form of commitment from countries than the two-stage process of the previous section. Again, we

[9] Although we are considering a two-country model, we assume that countries take the price of quotas as given so as to abstract from issues of market power which is not the focus of our analysis. The model is extended to a multi-country setting in the next section.

characterize the subgame perfect equilibrium by backward induction, starting with Stage 3.

Stage 3: Emissions Quota Trading

Direct abatements (a_1, a_2) and matching rates (m_1, m_2), and therefore total abatement commitments (A_1, A_2), have been determined in the previous two stages. The demand for emission quotas by country 1 at price p solves (assuming an interior solution and assuming that both countries are price-takers):

$$\max_{\{q_1\}} \; B_1(\overline{e}_1 - A_1 + q_1) - pq_1$$

where, recall, $A_1 = a_1 + m_1 a_2$. Since the total level of emission abatements for the two countries is fixed, the damage function can be left out of the problem. The first-order condition to this problem gives $B_1'(\overline{e}_1 - A_1 + q_1) = p$, whose solution is the demand for emissions quotas, $q_1(p, A_1)$. Differentiating the first-order condition $B_1'(\cdot) = p$ yields

$$\frac{\partial q_1}{\partial a_1} = \frac{\partial q_1}{\partial A_1} = 1, \qquad \frac{\partial q_1}{\partial m_1} = a_2, \qquad \frac{\partial q_1}{\partial a_2} = m_1 \tag{10}$$

Similarly, the demand for quotas by country 2 satisfies $B_2'(\overline{e}_2 - A_2 + q_2) = p$ and is denoted by $q_2(p, A_2)$. In equilibrium, $q_1(p, A_1) + q_2(p, A_2) = 0$, and the price satisfies

$$p(A_1, A_2) = B_1'(\overline{e}_1 - A_1 + q_1) = B_2'(\overline{e}_2 - A_2 + q_2)$$

Therefore, quota trading leads to an equalization of the marginal benefits of emissions, which is condition (2) for an efficient allocation of abatements across countries.

Stage 2: Choosing Direct Abatements $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$

We assume that countries correctly anticipate the price of quotas in Stage 3 and take it as given when making their abatement commitments. Given (m_1, m_2) from Stage 1, the problem of country 1 is:

$$\max_{\{a_1\}} \quad \Pi_1 = B_1\Big(\overline{e}_1 - A_1 + q_1(p, A_1)\Big) - D_1\Big(\overline{e}_1 + \overline{e}_2 - A_1 - A_2\Big) - pq_1(p, A_1)$$

The first-order condition, using $p = B_1'$, and assuming an interior solution, is:

$$F^1(a_1, a_2, m_1, m_2) \equiv -B_1'\Big(\bar{e}_1 - A_1 + q_1(p, A_1)\Big) + (1+m_2)D_1'\Big(\bar{e}_1 + \bar{e}_2 - A_1 - A_2\Big) = 0 \quad (11)$$

or,

$$\frac{D_1'(\cdot)}{B_1'(\cdot)} = \frac{1}{1+m_2}$$

Condition (11) has the same form as condition (3) characterizing the choice of direct abatement in the previous case without quota trading. Its solution gives country 1's reaction function, $a_1(a_2; m_1, m_2)$, and an analogous derivation for country 2 gives $F^2(\cdot)$ and $a_2(a_i; m_1, m_2)$.

In an interior solution (including at the boundary), differentiating $F^1(\cdot)$ and $F^2(\cdot)$ and using (10) yields the following properties of the two countries' reaction functions:

$$\frac{da_1(a_2, m_1, m_2)}{da_2} = -\frac{1+m_1}{1+m_2}, \quad \frac{da_2(a_1; m_1, m_2)}{da_1} = -\frac{1+m_2}{1+m_1}$$

Thus, the reaction curves for the two countries are linear and their slopes in $(a_1, a_2)-$space are the same regardless of the values of m_1 and m_2. That is, Figure 1 applies for all values of m_1 and m_2. The fact that reaction curves are parallel for any matching rates implies that either there will be a corner solution in Stage 2, or the curves will overlap in the interior so the solution is indeterminate.

Stage 1: Choosing Matching Rates $\boldsymbol{m}_1$ and $\boldsymbol{m}_2$

In Stage 1, both countries simultaneously choose their matching rates, m_1 and m_2, anticipating the outcomes of Stages 2 and 3. The equilibrium has the same form as in the case without quota trading, so there is no need to go through its derivation in detail. Equilibrium matching rates will be such that $m_1 m_2 = 1$ and Stage 2 reaction curves will coincide. In contrast to the case without quota trading, since Stage 2 reaction curves are parallel for any set of matching rates in this case, the equilibrium in direct abatements will be a corner solution whenever the reaction curves do not coincide. However, it is straightforward to show that if either $m_1 m_2 < 1$ or $m_1 m_2 > 1$, countries would have incentives to change their matching rates in ways that would cause the Stage 2 reaction curves to move closer together until they coincide and $m_1 m_2 = 1$. The demonstration that such matching rates constitute an equilibrium in the presence of a quota trading system is provided in the Appendix.

The equilibrium has all the properties of the equilibrium without quota trading. In particular, using the first-order conditions of the countries' Stage 2 problem, and $m_1 m_2 = 1$, we obtain $D'_1/B'_1 + D'_2/B'_2 = 1$, so the levels of emissions are Pareto efficient. However, with quota trading, the allocation of emissions across countries is also such that the marginal benefits of emissions are equalized, $B'_2 = B'_1$, which does not necessarily hold in the absence of quota trading. In terms of our discussion of the social optimum earlier, quota trading combined with matching abatement commitments results in an allocation along the first-best utility possibilities frontier, unlike with matching abatements in the absence of quota trading.

Hence, we have the following proposition:

Proposition 2. *With emissions quota trading, the equilibrium of the abatement process satisfies the properties in Proposition 1, and the marginal benefits of emissions are equalized across countries,* $B'_2 = B'_1$.

4.2 The Mechanism with More than Two Countries

In this section, we show that all the results of the basic two-country model with quota trading can be generalized to the case where there are more than two countries. To do so, let us now assume that there are n countries denoted by $i, j = 1, ..., n$, and let m_{ij} be the matching rate offered by country i on the direct abatement commitment of country j. Thus, countries can commit to matching the direct abatements of all other countries at different rates. As in the two-country case, countries simultaneously choose their matching rates in Stage 1, then set their direct abatement commitments in Stage 2. Finally, countries trade emission quotas in Stage 3.

Stage 3: Emissions Quota Trading

At this stage, the total abatement commitment of country i is $A_i = a_i + \sum_{j=1}^{n} m_{ij} a_j$.

The demand for emission quotas by country i maximizes $B_i\left(\overline{e}_i - A_i + q_i\right) - pq_i$. The first-order condition is $B_i'\left(\overline{e}_i - A_i + q_i\right) = p$, and the solution is country i's demand for emission quotas $q_i(p, A_i)$, for $i, j = 1, ..., n$ and $i \neq j$. In equilibrium, $\sum_i q_i(p, A_i) = 0$, and the price is such that $p(A_i, ..., A_n) = B_i'(\overline{e}_i - A_i + q_i)$ for all i. Quota trading therefore leads to an equalization of the marginal benefits of emissions across all n countries.

Stage 2: Choosing Direct Abatements $\boldsymbol{a}_i$

Matching rates are determined at this stage, and all countries take the price of quotas as given. The direct abatement commitment of country i solves the following:

$$\max_{\{a_i\}} \ \mathit{\Pi}_i = B_i(\overline{e}_i - A_i + q_i(p, A_i)) - D_i\Big(\sum_{j=1}^{n}(\overline{e}_j - A_j)\Big) - pq_i(p, A_i)$$

The first-order condition, using $p = B_i'$ from the emissions quota trading equilibrium, is:

$$F^i(a_1, ..., a_n, m_{i1}, ..., m_{in}) \equiv -B_i'(\cdot) + \Big(1 + \sum_{j \neq i}^{n} m_{ji}\Big) D_i'(\cdot) = 0, \text{ or } \frac{D_i'(\cdot)}{B_i'(\cdot)} = \frac{1}{1 + \sum_{j \neq i}^{n} m_{ji}} \tag{12}$$

The effective cost at which country i can increase world abatements by one unit depends on the total rate at which its direct abatement will be matched by all other countries. Country i chooses a_i to equalize this effective cost to the ratio of marginal damages and marginal benefits of emissions.

Stage 1: Choosing Matching Rates $\boldsymbol{m_{ij}}$

The equilibrium matching rates turn out to satisfy similar properties as in the two-country case. In fact, with n countries, matching rates are such that $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij}m_{jk} = 1$ (or equivalently $m_{ki}m_{ij} = m_{kj}$). Since the equilibrium is analogous to that in the two-country case, we need not go through its full derivation. We show in the Appendix that a set of matching rates satisfying these conditions constitute an equilibrium in Stage 1.

The other properties of the equilibrium matching rates derived in the two-country case apply here as well. In particular, with matching rates satisfying $m_{ij}m_{ji} = 1$ and

$m_{ki}m_{ij}m_{jk} = 1$, it is also the case that

$$\sum_{i=1}^{n} \frac{1}{1+\sum_{j\neq i}^{n} m_{ji}} = \sum_{i=1}^{n} \frac{D_i'(\cdot)}{B_i'(\cdot)} = 1$$

Thus, equilibrium abatements are efficient and the marginal benefits of emissions are equalized across all countries.

The total abatements of each country are again quasi-Lindahl abatement efforts. To see this, note that country i's quasi-Lindahl price D_i'/B_i' is equal to $1/(1+\sum_{j\neq i}^{n} m_{ji})$ by (12). Using $A_i = a_i + \sum_{j=1}^{n} m_{ij}a_j$, $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, country i's quasi-Lindahl abatement effort simplifies to the following after straightforward simplification:

$$\frac{1}{1+\sum_{j\neq i}^{n} m_{ji}}(A_1^* + \cdots + A_n^*) = a_i + \sum_{j\neq i}^{n} m_{ij}a_j = A_i^*$$

Thus, country i's marginal rate of substitution, $1/(1+\sum_{j\neq i}^{n} m_{ji})$, multiplied by the world's total abatements, $\sum_{j=1}^{n} A_j^*$, equals its total abatement before quota trading, A_i^*.

Finally, when $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, $1/(1+\sum_{j\neq i}^{n} m_{ji}) = m_{ik}/(1+\sum_{j\neq k}^{n} m_{jk})$ for all i and k. Each country faces equal direct and indirect costs of reducing the world's emissions by one unit. Each country is therefore indifferent between making direct abatements or matching abatements.

The analysis of this section leads to the following:

Proposition 3. *When there are n countries that can commit to matching the abatement efforts of each other at country-specific rates, and emissions quota trading exists, the equilibrium matching rates satisfy $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij}m_{jk} = 1$ for $i, j, k = 1, ..., n$, parts i, iii and iv of Proposition 1 hold, and the marginal benefits of emissions are equalized across all n countries.*

4.3 A Two-Period Model

In this section, we extend the analysis to a two-period setting and show that the three-stage abatement process can induce full efficiency even in a dynamic context where current emissions increase the stock of pollution that will exist in the future. For simplicity, we return to the two-country case. We assume that, in each period, countries can offer to match each other's abatement commitments in the current period before engaging in emissions quota trading. Matching rates and direct abatement commitments determine the number of period-specific emission quotas that each country holds. Trading takes place in each period and countries are not permitted to transfer emission quotas across periods. Therefore, the three-stage process of the one-period model is undertaken in each period, and in the first period, both countries anticipate the impact of their decisions on the second-period equilibrium.

In what follows, superscripts will denote time periods and subscripts will denote countries. We normalize the initial stock of pollution to S^0. In period 1, the actual emissions of country 1 and country 2 are e_1^1 and e_2^1, respectively, while the initial stock S^0 decays at the rate γ, with $0 < \gamma < 1$. Therefore, the stock of pollution at the end of period 1 is:

$$S^1 = (1-\gamma)S^0 + e_1^1 + e_2^1$$

Similarly, the stock of pollution at the end of period 2 is:

$$S^2 = (1-\gamma)S^1 + e_1^2 + e_2^2 = (1-\gamma)\left[(1-\gamma)S^0 + e_1^1 + e_2^1\right] + e_1^2 + e_2^2$$

The levels of emissions in the absence of any abatements are assumed to be constant in both periods and equal to $\overline{e}_1$ and $\overline{e}_2$. Before characterizing the equilibrium of the abatement process, let us briefly examine the social optimum in this two-period case.

The Social Optimum

The socially efficient levels of emissions of each country in each period can be characterized by maximizing the discounted sum of country 1's benefits net of damages over both periods, subject to the constraint that the discounted sum of country 2's net benefits equals some given level $\overline{\Pi}_2$. As in Section 2, we allow lump-sum transfer of T from country 2 to country 1 in order to characterize efficient points along the first-best utility possibilities frontier. The transfer is assumed to take place in the first period, although it makes no difference. The social optimum is the solution to the following problem:

$$\max_{\{e_1^1, e_2^1, e_1^2, e_2^2, T\}} \quad B_1(e_1^1) - D_1(S^1) + T + \delta(B_1(e_1^2) - D_1(S^2))$$

$$+\lambda\left[B_2(e_2^1)-D_2(S^1)-T+\delta B_2(e_2^2)-\delta D_2(S^2)-\overline{\Pi}^2\right]$$

where δ is the common discount factor and S_1 and S_2 are given by the expressions defined above. The first-order conditions imply the following:

$$\frac{D_1'(S^2)}{B_1'(e_1^2)}+\frac{D_2'(S^2)}{B_2'(e_2^2)}=1,\quad \frac{D_1'(S^1)+\delta(1-\gamma)D_1'(S^2)}{B_1'(e_1^1)}+\frac{D_2'(S^1)+\delta(1-\gamma)D_2'(S^2)}{B_2'(e_2^1)}=1$$

$$B_1'(e_1^1)=B_2'(e_2^1),\qquad B_1'(e_1^2)=B_2'(e_2^2)$$

In period 2, efficient emissions are such the sum of the ratios of marginal damages to marginal benefits of the two countries is equal to one. Efficient emissions in period 1 are such that the sum of the two countries ratios of period 1 marginal damages plus the discounted and decay-adjusted period 2 marginal damages, over period 1 marginal benefits, is equal to one. As well, marginal benefits of emissions are equalized across countries in each period.

The Two-Period Equilibrium

In period 1, countries 1 and 2 offer matching rates m_1^1 and m_2^1 and make direct abatement commitments a_1^1 and a_2^1, and similarly in period 2. Their actual emissions are:

$$e_1^1\equiv\overline{e}_1-a_1^1-m_1^1a_2^1+q_1^1,\quad e_2^1\equiv\overline{e}_2-a_2^1-m_2^1a_1^1+q_2^1$$

$$e_1^2\equiv\overline{e}_1-a_1^2-m_1^2a_2^2+q_1^2,\quad e_2^2\equiv\overline{e}_2-a_2^2-m_2^2a_1^2+q_2^2$$

These actual emissions result in stocks of pollution in each period given by:

$$S^1\equiv(1-\gamma)S^0+e_1^1+e_2^1=(1-\gamma)S^0+(\overline{e}_1-a_1^1-m_1^1a_2^1)+(\overline{e}_2-a_2^1-m_2^1a_1^1)$$

$$S^2\equiv(1-\gamma)S^1+e_1^2+e_2^2=(1-\gamma)[(1-\gamma)S^0+e_1^1+e_2^1]+e_1^2+e_2^2$$

$$=(1-\gamma)[(1-\gamma)S^0++(\overline{e}_1-a_1^1-m_1^1a_2^1)+(\overline{e}_2-a_2^1-m_2^1a_1^1)]+(\overline{e}_1-a_1^2-m_1^2a_2^2)+(\overline{e}_2-a_2^2-m_2^2a_1^2)$$

We characterize the two-period equilibrium by backward induction starting with period 2.

Period 2

Since emission quotas cannot be transferred across periods, the decisions in the first period (a_1^1, a_2^1, m_1^1, m_2^1) will only affect the period 2 equilibrium through their effects on the pollution stock at the end of the first period, S^1. It is straightforward to see that, for a given level of S_1, the three-stage abatement process that countries face in period 2 is essentially the same as in the basic one-period case, and the equilibrium will have the same characteristics. In particular, the equilibrium in period 2 will be fully efficient, given the pollution stock S_1. Denote the efficient total abatements in the second period by A_1^{2*} and A_2^{2*}, where $A_1^{2*} = a_1^2 + m_1^2 a_2^2$ and $A_2^{2*} = a_2^2 + m_2^2 a_1^2$. The demand for quotas by countries 1 and 2 satisfy $B_1'(\bar{e}_1 - A_1^{2*} + q_1^2) = p^2$ and $B_2'(\bar{e}_2 - A_2^{2*} + q_2^2) = p^2$, with $\partial q_i^2/\partial A_i^2 = 1$ and $\partial q_i^2/\partial A_j^2 = 0$ for $i, j = 1, 2$, and can be written as $q_1^2(p^2, A_1^{2*})$ and $q_2^2(p^2, A_2^{2*})$. In equilibrium, $q_1^2(\cdot) + q_2^2(\cdot) = 0$ and $p^2(A_1^{2*}, A_2^{2*}) = B_1'(\bar{e}_1 - A_1^{2*} + q_1^2) = B_2'(\bar{e}_2 - A_2^{2*} + q_2^2)$.

Given that the outcome in period 2 is fully efficient, the marginal effect of the period 1 pollution stock on total abatements in period 2 can be derived from the condition that characterizes the social optimum:

$$f(\cdot) \equiv \frac{D_1'(S^2)}{B_1'(e_1^2)} + \frac{D_2'(S^2)}{B_2'(e_2^2)}$$

$$\equiv \frac{D_1'\left((1-\gamma)S^1 + \bar{e}_1 - A_1^{2*} + \bar{e}_2 - A_2^{2*}\right)}{B_1'\left(\bar{e}_1 - A_1^{2*} + q_1^2(\cdot)\right)} + \frac{D_2'\left((1-\gamma)S^1 + \bar{e}_1 - A_1^{2*} + \bar{e}_2 - A_2^{2*}\right)}{B_2'\left(\bar{e}_2 - A_2^{2*} + q_2^2(\cdot)\right)} = 1$$

Differentiating the above and using $\partial q_i^2/\partial A_i^2 = 1$ and $\partial q_i^2/\partial A_j^2 = 0$ for $i, j = 1, 2$, we have:

$$f_{A_1^2} = f_{A_2^2} = -\frac{D_1''(S^2)}{B_1'(e_1^2)} - \frac{D_2''(S^2)}{B_2'(e_2^2)}, \quad f_{S^1} = \frac{(1-\gamma)D_1''(S^2)}{B_1'(e_1^2)} + \frac{(1-\gamma)D_2''(S^2)}{B_2'(e_2^2)}$$

from which we obtain:

$$\frac{\partial A_i^{2*}}{\partial S^1} = -\frac{f_{S^1}}{f_{A_i^{2*}}} = 1 - \gamma$$

Consequently, the change in the net benefit of country 1 in period 2 resulting from a change in the stock of pollution at the end of period 1 is given by (using $p^2 = B_1'(e_1^2)$):

$$\frac{d(B_1(e_1^2) - D_1(S^2) - p^2 q_1^2)}{dS^1} = -(1-\gamma)\left[B_1'(e_1^2) - D_1'(S^2)\right] < 0$$

A similar expression holds for country 2. An increase in the stock of pollution in period 1, of which a proportion $(1-\gamma)$ will remain in period 2, will induce an increase in the total abatement of country 1 in period 2, reducing the period 2 net benefit of country 1 by an amount equal to the difference between its benefit from emission and its own damages from pollution $\left(B_1'(e_1^2) - D_1'(S^2)\right)$. Let $\Pi_1^2(S^1)$ and $\Pi_2^2(S^1)$ denote the second period net benefits of countries 1 and 2, respectively.

Period 1

Since quota trading in the third stage does not affect the stock of pollution at the end of period 1, the quota trading process has no impact on the second period. Therefore, the quota trading equilibrium has the same properties as in the static one-period case, and there is no need to characterize it again.

In Stage 2, country 1 chooses its direct abatement a_1^1, taking matching rates (m_1^1, m_2^1) and country 2's direct abatement a_2^1 as given and anticipating the effect of a_1^1 on the second-period equilibrium, in order to maximize the discounted sum of its net benefits over both periods. Thus, it solves the following:

$$\max_{\{a_1^1\}} \; B_1(e_1^1) - D_1(S^1) - p^1 q_1^1 + \delta \Pi_1^2(S^1)$$

for which the first-order condition is

$$F(\cdot) \equiv -B_1'(e_1^1) + (1+m_2^1) D_1'(S^1) + \delta(1-\gamma)\left[-B_1'(e_1^2) + D_1'(S^2)\right]\left[-(1+m_2^1)\right] = 0$$

This condition can be written as

$$\frac{D_1'(S^1)}{B_1'(e_1^1)} - \frac{\delta(1-\gamma)\left[D_1'(S^2) - B_1'(e_1^2)\right]}{B_1'(e_1^1)} = \frac{1}{1+m_2^1}$$

The second term in the expression above is the discounted reduction in country 1's second-period net benefits resulting from higher first-period pollution as a ratio of the marginal benefit of first period emissions. Country 1 chooses its level of direct abatement such that the sum of this discounted cost and of the ratio of first-period marginal damages to marginal benefits of emissions equals the effective cost to country 1 of reducing world emissions by one unit, given that its own abatements are matched at the rate m_2^1 by country 2. The solution to this condition gives the reaction function of country 1, which can be shown to satisfy the following:

$$\frac{da_1^1}{da_2^1} = -\frac{F_{a_2^1}}{F_{a_1^1}} = -\frac{1+m_1^1}{1+m_2^1}$$

The analog holds for country 2. Hence, as in the one-period case, reaction curves are linear and parallel in the (a_1, a_2)–space for any matching rates (m_1^1, m_2^1). As in the one -period model, we could again show that the equilibrium matching rates in Stage 1 are such that $m_1^1 m_2^1 = 1$, and that the subgame perfect equilibrium has same properties as in the one-period case. Hence, the equilibrium replicates the social optimum derived earlier, so both intra-temporal efficiency and inter-temporal efficiency are achieved. Total emissions are efficient, and they are efficiently allocated across periods.

The results of this section are summarized below.

Proposition 4. *In a two-period setting where both countries can commit to match each others abatements and engage in emissions quota trading in both periods, the subgame perfect equilibrium is such that:*

i. The properties listed in Proposition 1 apply in each period;

ii. The marginal benefits of emissions are equalized across countries in both periods;

iii. Inter-temporal efficiency is achieved: emissions are efficiently allocated across periods.

5 Adding Contributions to an International Public Good

In this section, we explore how the introduction of an international public good provided through the voluntary contributions of countries will affect the pollution abatement process. For ease of exposition, we return to the basic one-period two-country case. Let the level of provision of the international public good be denoted by G and the contributions of each country by g_1 and g_2. Contributions are assumed to be perfect substitutes, so $G = g_1 + g_2$.

Utility in country i is $u^i(G, x_i)$, where x_i is private consumption net of the benefits and damages of emissions. Utility is increasing and quasi-concave in both arguments. Both G and x_i are assumed to be normal, and the latter is given by

$$x_i = w_i - g_i + B_i\Big(\bar{e}_i - a_i - m_i a_j + q_i\Big) - D_i\Big(\bar{e}_1 - (1+m_2)a_1 + \bar{e}_2 - (1+m_1)a_2\Big) - pq_i$$

where w_i is the initial endowment of country i. This formulation assumes that the benefits of emissions, net of damages, as well as the revenues from emissions quota trading are perfect substitutes for consumption.

The timing of decisions is important. We assume that countries choose their level of pollution abatement first, and then contribute to the international public good. With this order of decisions, we find that even without matching commitments and quota trading, the levels of emissions are efficient and the marginal benefits of emissions are equalized across countries. Although we will not go through the analysis of the case where contributions to the public good are determined first, it is straightforward to show that, in this case, the equilibrium of the abatement process will only be efficient if countries are making matching rate commitments and are engaging in emission quota trading, as in the basic case without contributions to a public good.

As will become apparent, with contributions to the public good determined after abatement decisions, commitments to matching abatements and emission quota trading turn out to be irrelevant so we can ignore them. The sequence of decisions is then simply as follows. In Stage 1, the two countries simultaneously choose emission abatements a_i. Both countries then set their contributions to the international public good g_i in Stage 2. We consider Stage 2 first.

Stage 2: Choosing Contributions to the International Public Good g_i

At the beginning of this stage, the wealth of the two countries are $w_1 + B_1(\bar{e}_1 - a_1) - D_1(\bar{e}_1 - a_1 + \bar{e}_2 - a_2)$ and $w_2 + B_2(\bar{e}_2 - a_2) - D_2(\bar{e}_1 - a_1 + \bar{e}_2 - a_2)$, given the levels of abatements (a_1, a_2) chosen in the previous stage. Country i chooses its contribution to maximize $u^i(g_1+g_2, w_i - g_i + B_i(\cdot) - D_i(\cdot))$, taking the contribution of the other country as given. Assuming an interior solution to public good contributions, g_i is such that $u^i_G/u^i_x =$

1. The provision of the public good is inefficiently low given that efficient contributions would satisfy $u_G^1/u_x^1 + u_G^2/u_x^2 = 1$. More importantly, the well-known Neutrality Theorem (Shibata, 1971; Warr, 1983; Bergstrom, Blume, and Varian, 1986) implies that the net private consumptions of the two countries x_1 and x_2 and the level of public good provision G will depend only on aggregate wealth, and not on its distribution across the two countries. Aggregate wealth here is $w_1 + w_2 + I$, where

$$I \equiv B_1(\bar{e}_1 - a_1) - D_1(\bar{e}_1 - a_1 + \bar{e}_2 - a_2) + B_2(\bar{e}_2 - a_2) - D_2(\bar{e}_1 - a_1 + \bar{e}_2 - a_2)$$

Thus, the two countries' utilities after the second stage can be written as $u^1[G(I), x_1(I)]$ and $u^2[G(I), x_2(I)]$, since $w_1 + w_2$ is constant. Given that G, x_1, and x_2 are normal goods, and that utilities are increasing in both arguments, maximizing I will also maximize the utility of each country. As a result, the objectives of the two countries in Stage 1 will be perfectly aligned.

Stage 1: Choosing Emission Abatements a_i

In this stage, the countries choose their abatement efforts, anticipating the outcome of Stage 2. The problem of country i consists in choosing a_i, given a_j, to maximize $u^i[G(I), x_i(I)]$, and the first-order condition implies that

$$-B_i'(\bar{e}_i - a_i) + D_i'(\bar{e}_1 - a_1 + \bar{e}_2 - a_2) + D_j'(\bar{e}_1 - a_1 + \bar{e}_2 - a_2) = 0, \quad i, j = 1, 2$$

It is immediately clear that the first-order conditions for the two countries taken together coincide with the condition characterizing the social optimum derived in Section 2, i.e. $D_1'/B_1' + D_2'/B_2' = 1$, as well as the condition that $B_1' = B_2'$. Remarkably, the equilibrium is such that the levels of emissions are efficient and the marginal benefits of emissions are equalized across countries, despite the fact that countries do not commit to match each other's abatements and there is no emission quota trading. Moreover, it can readily be shown that even if countries are able to commit to matching the abatement efforts of each other, they cannot derive any gain from making such commitments.

The main results of this section are stated below.

Proposition 5. *If countries make voluntary contributions to pollution abatement and then contribute voluntarily to an international public good, the equilibrium has the following properties:*

i. If contributions to the public good are strictly positive for both countries, the levels of emissions are efficient and the marginal benefits of emissions are equalized across countries without any matching rate commitments and quota trading;

ii. Countries cannot gain by offering strictly positive matching rates;

ii. Contributions to the public good are inefficient.

6 Imperfect Substitutability of Emissions in Damage Functions

In this section, we show that the matching mechanism may also achieve efficiency in abatements even if the emissions of each country are not perfect substitutes in the damage functions. To do so, we go back to the basic setting of Section 3, but assume that the damage function of country i is given by $D^i(e_1, e_2)$. We first characterize the social optimum in this case.

The Social Optimum

Efficient emissions will solve the following Pareto optimization problem:

$$\max_{\{e_1,e_2\}} \; B^1(e_1) - D^1(e_1,e_2) + \lambda \left[B^2(e_2) - D^2(e_1,e_2) - \overline{\Pi}_2\right]$$

Combining the first-order conditions, we get

$$\frac{D_1^1(e_1,e_2)}{B_1^1(e_1)} + \frac{D_2^2(e_1,e_2)}{B_1^2(e_2)} + \frac{D_2^1(e_1,e_2)D_1^2(e_1,e_2) - D_1^1(e_1,e_2)D_2^2(e_1,e_2)}{B_1^1(e_1)B_1^2(e_2)} = 1 \tag{13}$$

The last term on the left side of the condition above will be negative if countries suffer higher marginal damages from their own emissions than from the emissions of the other country, and will tend to zero as emissions become perfect substitutes.

The Decentralized Equilibrium

In Stage 2, each country chooses its level of abatements, taking matching rates as given. The problem of country 1 is the following:

$$\max_{\{a_1\}} \ \Pi^1 = B^1(\bar{e}_1 - a_1 - m_1 a_2) - D^1(\bar{e}_1 - a_1 - m_1 a_2, \bar{e}_2 - a_2 - m_2 a_1)$$

The first-order condition is

$$F^1 \equiv -B^1_1(\bar{e}_1 - A_1) + D^1_1(\bar{e}_1 - A_1, \bar{e}_2 - A_2) + m_2 D^1_2(\bar{e}_1 - A_1, \bar{e}_2 - A_2) = 0 \tag{14}$$

The problem of country 2 is analogous. In an interior solution (including at the boundary), country 1's reaction curve in $(a_1, a_2)-$space satisfies the following (differentiating the first-order condition above):

$$\frac{da_1}{da_2} = -\frac{F^1_{a_2}}{F^1_{a_1}} = -\frac{m_1 B^1_{11} - m_1 D^1_{11} - (1 + m_1 m_2) D^1_{12} - m_2 D^1_{22}}{B^1_{11} - D^1_{11} - 2m_2 D^1_{12} - m_2^2 D^1_{22}}$$

Note that if $m_1 m_2 = 1$, we have $F^1_{a_1} = m_2 F^1_{a_2}$, and:

$$\frac{da_1}{da_2} = -\frac{1 + m_1}{1 + m_2}$$

Similarly, when $m_1 m_2 = 1$, the slope of country 2's reaction function is:

$$\frac{da_2}{da_1} = -\frac{1 + m_2}{1 + m_1}$$

Therefore, the slopes of the two reaction curves in $(a_1, a_2)-$space are the same when $m_1 m_2 = 1$. As in the basic model analyzed in Section 3, matching rates for which Stage 2 reaction curves coincide and $m_1 m_2 = 1$ will constitute a subgame perfect equilibrium, provided that

$$H \gtreqless 0 \quad \Longleftrightarrow \quad 1 - m_1 m_2 \gtreqless 0.$$

The condition under which this will hold is derived in the Appendix.

We can readily verify that, when $m_1 m_2 = 1$ and reaction curves coincide, the first-order conditions from the Stage 2 problems of both countries together yield condition (13) characterizing the Pareto efficient levels of emissions. Moreover, the properties of the subgame perfect equilibrium derived in Section 3 will all apply in the current case where emissions are imperfect substitutes in the damage functions.

7 Concluding Remarks

Our purpose in this paper has been to characterize a process of pollution emissions reduction in which countries can commit to match each others' abatement efforts and may subsequently engage in emissions quota trading. The mechanism that we considered is non-cooperative in the sense that each country, acting in its own self-interest, voluntarily offers to match the emission abatements of the other country's at some announced rates, anticipating the subsequent abatement equilibrium and the outcome of emissions quota trading. The analysis has shown that this mechanism leads to an efficient outcome. The level of emissions is efficient, and quota trading leads to an equalization of the marginal benefits of emissions across countries. This result holds independently of the number of countries involved, and in an environment where countries have different abatement technologies as well as different benefits from emissions. Efficient levels of emissions also occur even if countries' emissions are imperfect substitutes in the damage function of each country. In a dynamic setting where the quality of the environment depends on cumulative emissions over two periods, the mechanism is found to achieve both intra-temporal and inter-temporal efficiency.

The mechanism also has appealing distributional implications. The initial allocation of emission quotas across countries (before trading) emerges endogenously without central coordination and reflects each country's net marginal benefits from reducing pollution. This result also implies that all countries will find it in their own interest to participate. Countries with relatively low net marginal valuations for pollution reduction will face relatively low effective costs of abatement, given the set of equilibrium matching rates.

We extended the model by considering the case where countries are voluntarily contributing to an international public good in addition to undertaking pollution abatement. We found that if public good contributions are determined after abatement efforts, the level of emissions is efficient even in the absence of any matching abatement commitments. In fact, the incentive for countries to match the abatements of each other vanishes entirely. Moreover, the marginal benefits of emissions are equalized across countries even in the absence of emissions quota trading.

Throughout, our analysis has assumed that all countries were able to commit to match the other countries' abatements. It would be interesting to extend the analysis to characterize the pollution abatement process when only a subset of countries are able to commit. In this case, different forms of commitment could emerge as well as different distributions of the gains from achieving more efficient allocations. What determines the commitment ability of countries remains an open question. The recent papers of Gersbach and Winkler (2007) and Gerber and Wichardt (2009) suggest potential mechanisms to address that issue.

Appendix

Equilibrium Matching Rates in the Two-Country Case

We show here that the subgame-perfect equilibrium in the basic two-country case will be such that $m_1 m_2 = 1$ and the two stage 2 reaction curves coincide. We consider separately outcomes with $m_1 m_2 <, >, = 1$, in each case allowing for both corner and interior solutions in stage 2. When corner solutions apply, we consider the case where country 2's reaction curve is outside 1's so $a_2 > 0, a_1 = 0$, with $D_2'/B_2' = 1/(1+m_1)$ and $D_1'/B_1' < 1/(1+m_2)$. It is apparent that the same results extend to the case where $a_1 > 0, a_2 = 0$.

Case i) $m_1 m_2 < 1$

If the stage 2 equilibrium is interior, $m_1 m_2 < 1$ implies that $H > 0$ by (5). By (8), both countries will want to increase m_1 so this cannot be an equilibrium.

Suppose the equilibrium is at a corner with $a_2 > 0, a_1 = 0$. There is no cost to country 2 of increasing m_2 until country 1's stage 2 first-order condition is just binding. At this point we have an interior solution. If this occurs while $m_1 m_2 < 1$, both countries will want to increase m_i. Thus, $m_1 m_2 < 1$ cannot be an equilibrium. If country 1's stage 2 first-order condition does not bind before $m_1 m_2 > 1$, then the outcome falls into Case ii) below.

Case ii) $m_1 m_2 > 1$

If the stage 2 equilibrium is interior, $m_1 m_2 > 1$ implies that $H < 0$ by (5), so again both countries will want to increase m_i by (8). Note that, since interior stage 2 equilibria are unstable, an increase in either matching rate will lead to a stable corner solution in which only one country commits to $a_i > 0$. Therefore, an interior stage 2 equilibrium with $m_1 m_2 > 1$ cannot occur.

Suppose the equilibrium is at a corner with $a_2 > 0, a_1 = 0$. Then, $D_1'/B_1' < 1/(1+m_2) < m_1/(1+m_1)$, where the latter inequality follows from $m_1 m_2 > 1$. Country 1 wants to reduce abatements, and since it cannot do so directly, it will do so indirectly by decreasing m_1. Formally, abatements are determined by the first-order condition on country 2's choice of a_2, given m_1. Using $A = (1+m_1)a_2$, we can write this as $D_2'(\bar{e}_1 + \bar{e}_2 - A)/B_2'(\bar{e}_2 -$

$A/(1+m_1)) = 1/(1+m_1)$. Differentiating with respect to m_1, we obtain:

$$\frac{dA}{dm_1} = \frac{(1+m_1)B' - AB''}{(1+m_1)^2 D'' - (1+m_1)B''} > 0$$

Country 1's net benefit can be written in terms of A as $\Pi_1 = B_1(\bar{e}_1 - m_1 A/(1+m_1)) - D_1(\bar{e}_1 + \bar{e}_2 - A)$. Differentiating with respect to m_1:

$$\frac{d\Pi_1}{dm_1} = \left(-\frac{m_1}{1+m_1}B_1' + D_1'\right)\frac{dA}{dm_1} - B_1'\frac{A}{(1+m_1)^2} < 0$$

since as we mentioned $D_1'/B_1' < m_1/(1+m_1)$. Therefore, country 1 will want to reduce m_1 so this cannot be an equilibrium.

Case iii) $m_1 m_2 = 1$

Begin with the case where there is a corner solution with $a_2 > 0, a_1 = 0$. Using $m_1 m_2 = 1$, we have $D_1'/B_1' < 1/(1+m_2) = m_1/(1+m_1)$. Country 1 would like to reduce its contribution, and can do so only by decreasing m_1. Using the same argument as in Case ii) above, a reduction in m_1 when country 2 is the only contributor will reduce A and increase Π_1. Specifically, the above expression again yields $d\Pi_1/dm_1 < 0$ since $D_1'/B_1' < m_1/(1+m_1)$. Thus, this cannot be an equilibrium.

Finally, consider the case where reaction curves overlap with $m_1 m_2 = 1$. We can show that neither country will want to change its matching rate. Suppose first that country 1 increases m_1 by a small amount. Country 2's reaction curve will then be everywhere outside that of country 1, so $a_1 = 0, a_2 > 0$. This is so because, first, from (4) and similar expressions for country 2, the a_1-intercept of country 1's reaction curve does not change (since $da_1/dm_1|_{F^1=0} = 0$ at $a_2 = 0$), while the a_1-intercept of country 2's reaction curve increases (since $da_2/dm_1|_{F^2=0} > 0$ at $a_2 = 0$). Second, with $m_1 m_2 > 1$, at any interior solution the slope of 2's reaction curve is steeper than that of 1's, because $H < 0$ in (5) and an interior solution is unstable. Therefore, given that 2's intercept on the horizontal axis is greater than 1's, the reaction curves cannot cross in the interior (even though the reaction curves are no longer linear).

Next, we can show in three small steps that country 1 is worse off than before the deviation. First, to compare country 1's payoffs before and after the unilateral increase in m_1, we can think of country 1 as starting from the corner where $a_1 = 0$, $a_2 > 0$, $m_1 m_2 = 1$ and the reaction curves overlap and increasing its matching rate by a small amount. This is because, when $m_1 m_2 = 1$ and the reaction curves overlap, each combination of a_1 and a_2 along the common reaction curve yields the same payoff to a country, as we have shown before, and it does not matter for the purpose of welfare comparison which combination we choose as a reference. At this corner allocation, to examine the effect of an increase in m_1 on a_2, we can use *only* country 2's first-order condition to do comparative statics: a_1 is already zero in this corner allocation, where $F^1 = 0$ just holds, and a_1 remains zero as m_1 increases and F^1 becomes negative; however, a_2 is always positive and $F^2 = 0$ always holds as m_1 increases. In other words, at the corner where $a_1 = 0$, $a_2 > 0$, $m_1 m_2 = 1$ and the reaction curves overlap, and where country 1 is considering an increase in m_1, m_1 already has no effect on a_1.

Second, as we showed in the corner equilibrium under Case ii) above, total abatements rise with m_1: $dA/dm_1 > 1$.

Third, country 1 is made worse off from an increase in m_1 starting at $m_1 m_2 = 1$ and $a_1 = 0$. As above, country 1's net benefit can be written, using $m_1 a_2 = A m_1/(1 + m_1)$ as $\mathit{\Pi}_1 = B_1(\bar{e}_1 - m_1 A/(1 + m_1)) - D_1(\bar{e}_1 + \bar{e}_2 - A)$. Differentiating with respect to m_1, we obtain, using $m_1/(1 + m_1) = 1/(1 + m_2)$,

$$\frac{d\mathit{\Pi}_1}{dm_1} = \left(-\frac{B_1'}{1 + m_2} + D_1'\right)\frac{dA}{dm_1} - B_1'\frac{A}{(1 + m_1)^2} = -B_1'\frac{A}{(1 + m_1)^2} < 0$$

since $B_1'/(1 + m_2) = D_1'$ at the starting point. Therefore, an upward deviation makes country 1 worse off.

Finally, consider a reduction in m_1 by country 1. First, note that by reasoning similar to the above, country 2's reaction curve will be everywhere inside that of country 1, so $a_1 > 0, a_2 = 0$ after a downward deviation. To see this, note that with $m_1 m_2 < 1$, at any interior solution the slope of 2's reaction curve is flatter than that of country 1. Therefore, given that 2's intercept on the horizontal axis is smaller than 1's, the reaction curves cannot cross in the interior (even though they are nonlinear). After the deviation, country 1's direct abatement remains the same, since the a_1-intercept of its reaction curve does not change. Country 1's total abatement $A_1 = a_1$ and the world's total abatement $A = (1 + m_2)a_1$ are the same as before. Thus, country 1 is just as well off as before the deviation.

Therefore, country 1 has no incentive to change m_1 starting from an allocation with $m_1 m_2 = 1$ and overlapping Stage 2 reaction curves. The same will apply for country 2. The allocation is therefore an equilibrium. Since there are unique values on m_1 and m_2 for which $m_1 m_2 = 1$ and the reaction curves overlap, as explained in the characterization of Stage 2 in Section 3, the equilibrium is unique.

Equilibrium Matching Rates with Quota Trading

Analogously to the basic setting without quota trading, we now show that matching rates for which $m_1 m_2 = 1$ and reaction curves coincide also constitute an equilibrium in the presence of a quota trading system. To see this, consider first a small increase in m_1 starting with $m_1 m_2 = 1$ and overlapping reaction curves. By differentiating (11) and the analogous condition for country 2, the a_1-intercept of country 1's reaction curve remains unchanged, since $da_1/dm_1|_{F^1=0} = -F^1_{m_1}/F^1_{a_1} = -a_2/(1 + m_1)$ which is equal to zero at $a_2 = 0$, and the a_1-intercept of country 2's reaction curve moves right, since $da_2/dm_1|_{F^2=0} = -F^2_{m_1}/F^2_{a_2} = -a_2/(1 + m_1) - D'_2/F^2_{a_2}$ $(= -D'_2/F^2_{a_2} > 0$ at $a_2 = 0)$. Given that the reaction curves remain parallel, country 2's reaction curve necessarily moves outside that of country 1.

As in the case without quota trading, when $m_1 m_2 = 1$, each country's net benefit is the same for any combination of a_1 and a_2 along the common reaction curve. Therefore, we can assume that we are initially at an allocation with $a_1 = 0$ and $a_2 > 0$, so that $A = (1+m_1)a_2$ and abatements are determined by country 2's Stage 2 first-order condition. In this case, country 1's level of abatement is $m_1 a_2 = Am_1/(1 + m_1)$ and its net benefit is

$$\Pi_1 = B_1\left(\bar{e}_1 - \frac{m_1 A}{1 + m_1} + q_1(p, A_1)\right) - D_1\left(\bar{e}_1 + \bar{e}_2 - A\right) - pq_1(p, A_1)$$

Differentiating with respect to m_1, using $\partial q_1/\partial m_1 = a_2$ and $p = B'_1$ from the quota trading equilibrium in Stage 3, and $B'_1/(1+m_2) = D'_1$ from Stage 2 where $m_1/(1+m_1) =$

$1/(1+m_2)$, we obtain,

$$\frac{d\Pi_1}{dm_1} = \left(-\frac{m_1 B_1'}{1+m_1} + D_1'\right)\frac{dA}{dm_1} - B_1'\frac{A}{(1+m_1)^2} = \frac{d\Pi_1}{dm_1} = -B_1'\frac{A}{(1+m_1)^2} < 0$$

The increase in country 1's matching rate will reduce its net benefit.

If country 1 were to reduce m_1, starting with $m_1 m_2 = 1$ and overlapping reaction curves, its reaction curve would move outside that of country 2 and both reaction curves would remain parallel. The Stage 2 equilibrium would be such that $a_1 > 0$ and $a_2 = 0$, with $A = (1+m_2)a_1$. Moreover, as shown above, a change in m_1 leaves unchanged the a_1-intercept of country 1's reaction curve. As a result, the reduction in m_1 by country 1 would have no effect on either countries' total abatements or net benefits.

This demonstration also holds for a change in country 2's matching rate. Therefore, when $m_1 m_2 = 1$ and reaction curves coincide, neither country has any incentive to change its matching rate and the allocation is an equilibrium.

The Multi-Country Case

Start by examining how changes in one country's matching rates affect all countries' reaction functions. Differentiating country i's Stage 2 first-order condition (12), we obtain:

$$\left.\frac{da_i}{da_k}\right|_{F^i=0} = -\frac{1+\sum_{j\neq k} m_{jk}}{1+\sum_{j\neq i} m_{ji}}, \qquad \left.\frac{da_i}{dm_{ik}}\right|_{F^i=0} = -\frac{a_k}{1+\sum_{j\neq i} m_{ji}} \ (= 0 \text{ at } a_k = 0)$$

$$\left.\frac{da_i}{dm_{ki}}\right|_{F^i=0} = -\frac{D_i' - a_i(1+\sum_{j\neq i} m_{ji})D_i''}{(1+\sum_{j\neq i} m_{ji})^2 D_i''} \ (> 0 \text{ at } a_i = 0)$$

For expositional convenience, consider the case of three countries. Suppose that matching rates satisfy $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, for $i, j, k = 1, 2, 3$, and that reaction functions coincide. Figure 2 depicts the countries' reaction functions. They coincide in the interior and correspond to the triangle PQS. Analogously to the two-country case with quota trading, reaction functions are parallel planes in the interior for any set of matching rates. If matching rates are such that reaction functions do not coincide, the country with the reaction function that is furthest from the origin will be the only one to undertake direct abatement, which will equal the own-axis intercept of its reaction function.

Starting with matching rates for which $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$ and reaction functions coincide, suppose country 1 increases one of its matching rates, say m_{12}. For country 1's reaction function, $da_1/da_2|_{F^1=0}$ increases and $da_1/da_3|_{F^1=0}$ remains unchanged; $da_1/dm_{12}|_{F^1=0} = 0$ at $a_2 = 0$, so the segment PS in Figure 2 remains part of country 1's reaction function. Thus, country 1's new reaction function will move to a position such as $PQ''S$ in the interior. For country 2, $da_2/da_1|_{F^2=0}$ and $da_2/da_3|_{F^2=0}$ both decrease; $da_2/dm_{12}|_{F^2=0} > 0$ at $a_2 = 0$, so country 2's reaction function shift outwards from segment PS. The own-axis intercept of country 2's reaction function may increase or decrease. Thus, country 2's new reaction function will move to a position such as $P'\overline{Q}S'$ or $P'\widetilde{Q}S'$ in the interior. Finally, for country 3, $da_3/da_2|_{F^3=0}$ increases and $da_3/da_1|_{F^3=0}$ remains unchanged; $da_3/dm_{12}|_{F^3=0} = 0$ at $a_2 = 0$, so the segment PS is still part of the reaction function. In fact, country 3's new reaction function continues to coincide with that of country 1, at a position such as $PQ''S$. Overall, the increase in m_{12} results in country 2's reaction function moving above those of countries 1 and 3, while all three remain parallel. The new Stage 2 equilibrium will be at country 2's new a_2-intercept where $a_1 = a_3 = 0$.

Note that if country 1 were to increase m_{13} as well, reaction functions would shift in an analogous fashion. In this case, the country for which the reaction function would shift out the furthest (i.e. the one that would face the largest matching rate increase) would be the country undertaking direct abatements in the subsequent Stage 2 equilibrium.

Suppose now that country 1 decreases m_{12}. The effects on the countries' reaction functions would be the opposite to those described above. The reaction functions of countries 1 and 3 would continue to coincide and would move to a position such as $PQ'S$, while country 2's reaction function would move to a position such as either $P''\widetilde{Q}S''$ or $P''\overline{Q}S''$. The new Stage 2 equilibrium could be anywhere on segment PS, along which $a_2 = 0$ and the abatements and net benefits of each country are the same as before the change in m_{12}. If country 1 were to reduce m_{13} as well, then the reaction functions of countries 2 and 3 would move everywhere below country 1's reaction function. The Stage 2 equilibrium would be at the a_1-intercept of country 1's reaction function where the abatements and net benefits of all countries remain unchanged.

Finally, if country 1 were to increase one matching rate, say m_{12}, and reduce the other, m_{13}, country 2's reaction function would move everywhere above that of country 1 while country 3's reaction function would move everywhere below. The Stage 2 equilibrium would move to the new a_2-intercept of country 2's reaction function with $a_1 = a_3 = 0$.

As in the two-country case, no country can make itself better off by changing any matching rate when reaction functions coincide. To see this in the n-country case, note first that the Stage 2 equilibrium is indeterminate when $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$. Therefore, let us characterize the effect of an increase in m_{ik} by using as an initial allocation the case where only country k is making direct abatements, so $a_k > 0$ and $a_i = 0$ for all $i = 1, ..., n$, $i \neq k$. As shown in the three-country case, country k will remain the only one making direct abatements after the increase in m_{ik}. In this case, country i's abatement is $m_{ik}a_k$ and total world abatement is $A = \left(1 + \sum_{j \neq k}^{n} m_{jk}\right) a_k$. The abatement of country k can therefore be written as $a_k = A/\left(1 + \sum_{j \neq k}^{n} m_{jk}\right)$.

Consider country i's net benefit which is given by the following:

$$\Pi_i = B_i\left(\overline{e}_i - \frac{m_{ik}A}{1 + \sum_{j \neq k}^{n} m_{jk}} + q_i(p, A_i)\right) - D_i\left(\sum_{j=1}^{n} \overline{e}_j - A\right) - pq_i(p, A_i)$$

Differentiating with respect to m_{ik} and using $p = B_i'$, we obtain:

$$\frac{d\Pi_i}{dm_{ik}} = \frac{-B_i'A\left(1 + \sum_{j \neq k}^{n} m_{jk} - m_{ik}\right)}{\left(1 + \sum_{j \neq k}^{n} m_{jk}\right)^2} + \left(\frac{-B_i'm_{ik}}{\left(1 + \sum_{j \neq k}^{n} m_{jk}\right)} + D_i'\right)\frac{dA}{dm_{ik}}$$

Noting that when $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, we have $m_{ik}/(1 + \sum_{j \neq k}^{n} m_{jk}) = 1/(1 + \sum_{j \neq i}^{n} m_{ji})$, the above can be rewritten as

$$\frac{d\Pi_i}{dm_{ik}} = \frac{-B_i'A\left(1 + \sum_{j \neq k}^{n} m_{jk} - m_{ik}\right)}{\left(1 + \sum_{j \neq k}^{n} m_{jk}\right)^2} + \left(\frac{-B_i'}{\left(1 + \sum_{j \neq i}^{n} m_{ji}\right)} + D_i'\right)\frac{dA}{dm_{ik}}$$

Given that (12) holds at the initial allocation, the above reduces to

$$\frac{d\Pi_i}{dm_{ik}} = \frac{-B_i'A\left(1 + \sum_{j \neq k}^{n} m_{jk} - m_{ik}\right)}{\left(1 + \sum_{j \neq k}^{n} m_{jk}\right)^2} < 0$$

Therefore, no country would have an incentive to increase its matching rate. Finally, as shown above for the three-country case, when $m_{ij}m_{ji} = 1$ and $m_{ki}m_{ij} = m_{kj}$, a reduction in any country's matching rate would leave each country's total abatement and net benefit unchanged. Hence, the set of matching rates satisfying these conditions constitute an equilibrium.

Imperfectly Substitutable Emissions

By differentiating equation (14), we can show that $H = F^1_{a_1}F^2_{a_2} - F^1_{a_2}F^2_{a_1} = (1 - m_1m_2)K$, where

$$K = B^1_{11}B^2_{11} - B^1_{11}D^2_{12} - m_1B^1_{11}D^2_{22} - D^1_{11}B^2_{11} + D^1_{11}D^2_{12} + m_1D^1_{11}D^2_{22} - m_2D^1_{12}B^2_{11}$$

$$-D^1_{12}D^2_{11} + (m_2 - m_1)D^1_{12}D^2_{12} + m_1m_2D^1_{12}D^2_{22} - m_2D^1_{22}D^2_{11} - m_1m_2D^1_{22}D^2_{12} \gtreqless 0$$

in general. If $K > 0$, H will have the same sign as $(1 - m_1m_2)$. Then, as in the basic case with perfectly substitutable emissions, interior Stage 2 equilibria will be stable if $m_1m_2 < 1$ and unstable if $m_1m_2 > 1$. In this case, the demonstration that we used earlier, to show that matching rates for which $m_1m_2 = 1$ and Stage 2 reaction curves coincide is the unique subgame perfect equilibrium in the basic case of Section 3, will also apply with emissions that are imperfectly substitutable in each country's damage function. However, if $K < 0$, that demonstration may not hold.

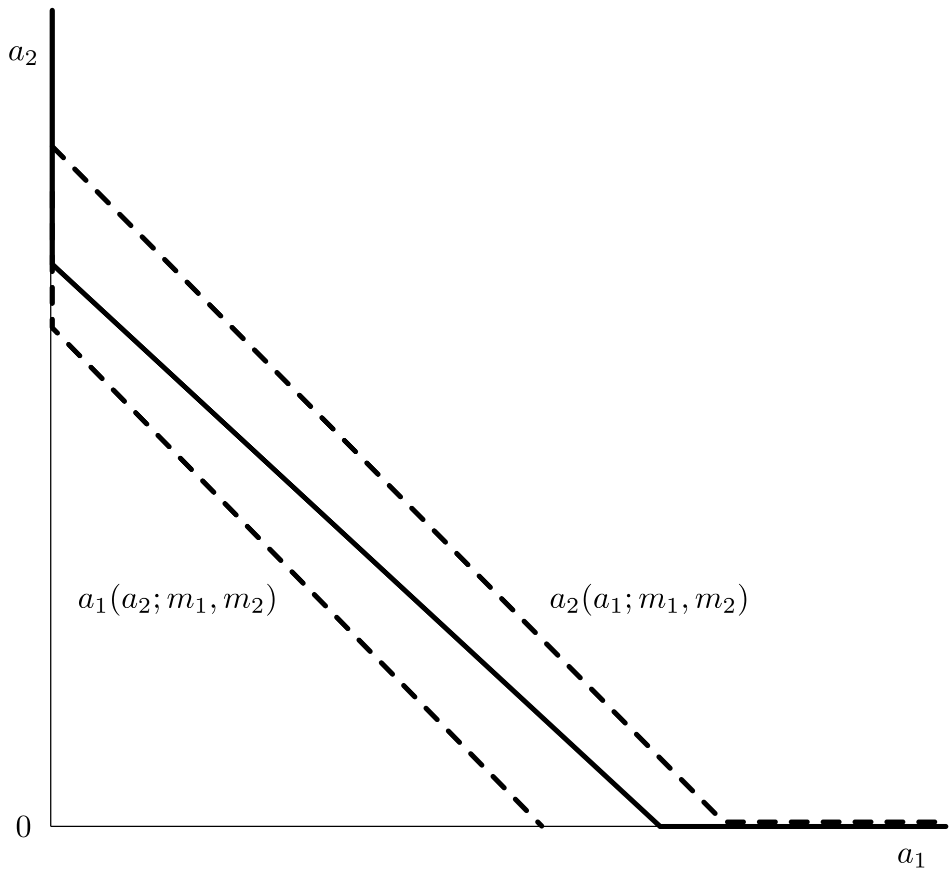

Figure 1. Stage 2 reaction curves

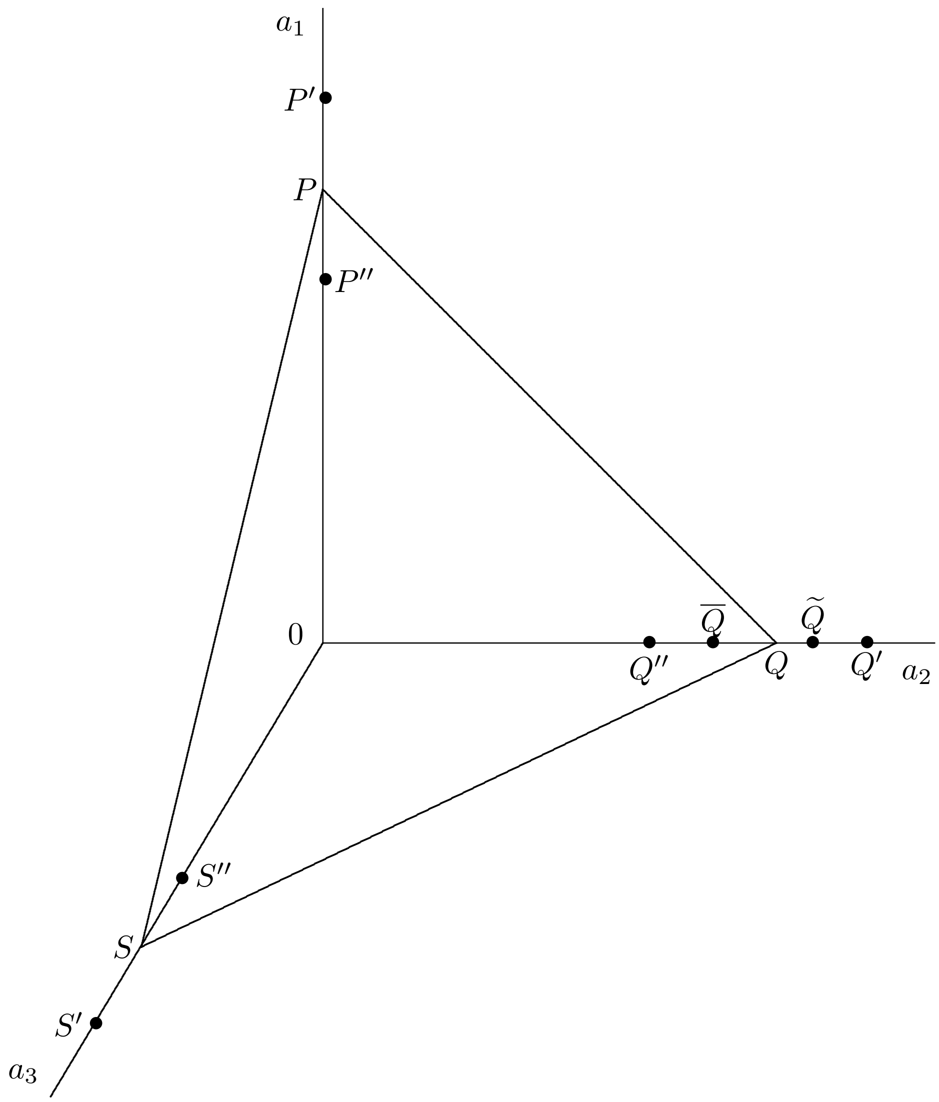

Figure 2. Stage 2 reaction curves with three countries

References

Altemeyer-Bartscher, M., D. Rübbelke and E. Sheshinski (2009), 'Environmental Protection and the Private Provision of International Public Goods', *Economica*, forthcoming.

Bergstrom, T., L. Blume and H. Varian (1986), 'On the Private Provision of Public Goods', *Journal of Public Economics* **29**, 25–49.

Boadway, R., Z. Song and J.-F. Tremblay (2007), 'Commitment and Matching Contributions to Public Goods', *Journal of Public Economics* **91**, 1664–83.

Danziger, L. and A. Schnytzer (1991), 'Implementing the Lindahl Voluntary-Exchange Mechanism', *European Journal of Political Economy* **7**, 55–64.

Gerber, A. and P.C. Wichardt (2009), 'Providing Public Goods in the Absence of Strong Institutions', *Journal of Public Economics* **93**, 429–39.

Gersbach, H. and R. Winkler (2007), 'On the Design of Global Refunding and Climate Change', *Center of Economic Research – ETH Zurich*, Working Paper no. 07/69.

Guttman, J. (1978), 'Understanding Collective Action: Matching Behavior', *American Economic Review* **68**, 251–55.

Guttman, J. and A. Schnytzer (1992), 'A Solution of the Externality Problem Using Strategic Matching', *Social Choice and Welfare* **8**, 73–88.

Nishimura, Y. (2008), 'A Lindahl Solution to International Emissions Trading', Working Paper No. 1177, Department of Economics, Queen's University.

Roberts, M. J. and M. Spence (1976), 'Effluent Charges and Licenses Under Uncertainty', *Journal of Public Economics* **5**, 193–208.

Sandmo, A. (2006), 'Global Public Economics: Public Goods and Externalities', Discussion Paper No. 32, Norwegian School of Economics and Business Administration, Bergen, Norway.

Shibata, H. (1971), 'A Bargaining Model of the Pure Theory of Public Expenditure', *Journal of Political Economy* **79**, 1–29.

Varian, H. (1994), 'Sequential Contributions to Public Goods', *Journal of Public Economics* **53**, 165–86.

Warr, P. (1983), 'The Private Production of a Public Good is Independent of the Distribution of Income', *Economic Letters* **13**, 207–11.