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Competition for the International Pool of Talents: Education Policy with Student Mobility

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Competition for the International Pool of Talents: Education Policy with Student Mobility

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Abstract

The paper presents a model of two countries competing for the international pool of talented students from the rest of the world. To relax tuition-fee competition, countries differentiate their education systems in equilibrium. While one country offers high education quality at high charges for students – the most talented ones study in this country – the other one provides lower quality and charges lower tuition fees. The regional quality-differentiation increases with the size of the international pool of talents, with the stay rate of foreign students in the host countries upon graduation and with the degree of development of the sending countries of foreign students. Compared to the welfare-maximizing education-policy, the decentralized solution is likely to imply an inefficient allocation of foreign students to the two host countries, as well as an inefficient quality differentiation.

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1 Introduction

The ongoing internationalization of higher education implies a significant challenge for national education policies within the OECD-area. The number of international students (i.e., students enrolled abroad) has grown considerably over the last thirty years and growth has been accelerated especially over the last couple of years. Since the year 2000, the number of foreign students within OECD countries has increased by more than 50 percent. The four top-destinations, namely the U.S., the UK, Germany and France host about half of the entire international student body. Besides Korea and Japan, France and Germany are also the largest sending countries. Overall, Asia is by far the largest sending region of origin of foreign students. Apart from students from OECD members Korea and Japan, especially students from China and India largely contribute to the group of international students. With 15.4 (China) and 5.4 percent (India), they represent the largest group of students from OECD partner countries enrolled within the OECD.¹

The present paper analyzes an oligopolistic competition under quality differentiation with two developed (OECD) countries competing for a pool of students from ‘the rest of the world (ROW)’, by which we especially mean less developed (non-OECD) countries. The two host countries can choose education quality and tuition fees to maximize the rent from educating foreign students. In equilibrium, they are demonstrated to differentiate education qualities in order to relax tuition-fee competition. The regional quality differentiation increases with the size of the international pool of talents, with the stay rate of foreign students in the host countries upon graduation and with the degree of development of the sending region of foreign students. A brief welfare analysis shows that the allocation of students to the two host countries and the regional quality differentiation are probably inefficient. The cost of providing education quality plays an important role for the welfare analysis.

In principle, a country might be interested in attracting students from abroad for example in order to overcome national bottlenecks in finding qualified students, raise additional tuition-fee revenue, benefit from research output by foreign graduate

students or positive spillovers from foreign to domestic students, to the university or to the society as a whole. Furthermore, given that part of foreign students stay on in their host country as graduates (see e.g., Lowell, Bump and Martin, 2007; Rosenzweig, 2006; Dreher and Poutvaara; 2005; Finn, 2003), the acquisition of students represents a strategy to attract high-skilled human capital. The fact that several OECD countries actually take measures to promote foreign students’ national labor-market access upon graduation (see e.g., Tremblay, 2005; Chaloff and Lemaitre, 2009), indicates that countries are aware of this option. Within the model, the positive effect of students staying on in the host country as graduates is represented by income-tax revenue. Immigration policy is exogenous.

The analysis contributes to the literature on local public-education policy with student mobility. In a fiscal-competition setting, Del Rey (2001) finds that countries tend to underinvest in public education if foreign students can free-ride the local education system, especially as they are all assumed to return to their country of origin upon graduation and therefore do not pay any income taxes in the host country. Buettner and Schwager (2004) state that positive external effects on non-resident students may cause local underprovision if policy makers only consider native students’ utility when deciding on education quality. This underinvestment justifies a tuition fee which is set on the federal level and which effectively raises the incentive to provide quality in order to attract students who pay these fees. A contribution coming closer to our model is presented by Boadway, Marceau and Marchand (1996). They analyze the competition of two private schools with quality investments and tuition fees. In a symmetric equilibrium, these institutions may spend an inefficiently large amount of resources in order to attract students. While we also consider competition both in prices (i.e., tuition fees) and quality, our focus is on public higher education, implying that decision makers (i.e., politicians, governments) also account, for example, for expected benefits in the form of income-tax revenue from graduates staying on in the host country upon graduation.

An important difference between the present approach and the studies mentioned so far, is that the two countries in our model compete for students from a third country (ROW). If ROW students do not have any ex ante country-specific preferences for one of the potential host countries and if both countries are exactly identical, students actually have to be regarded as perfectly mobile when it comes to their decision on the location of education. They will then only consider regional quality differences and tuition-fee differences. As a consequence, a symmetric equilibrium.

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will finally not exist. One country provides higher quality and charges higher tuition fees than the other country; thereby also attracting the most talented students. The reason is that quality differentiation effectively prevents fierce tuition-fee competition for the perfectly mobile pool of international students. The differentiation is in some analogy to Kemnitz’s (2007) finding of differentiated teaching qualities and tuition fees in the context of competition among autonomous universities.

The paper is organized as follows. Section 2 sets up the basic model and analyzes host countries’ competition in an oligopolistic model under quality differentiation. This section also presents the comparative statics. Section 3 presents the welfare-maximizing solution and evaluates the decentralized equilibrium accordingly. Section 4 briefly discusses some implications of the results for the sending countries of foreign students. Section 5 concludes.

2 The model

2.1 Basic setting

This section sets the stage for the analysis of the competition of two host countries for foreign students in a duopoly model with vertical product differentiation, i.e. differentiation of the quality of education. On the demand side it presents foreign students’ preferences and migration decisions, and on the supply side it presents host countries’ objectives.

The market size, or rather total demand from ROW for one of two (ex ante identical) developed host countries of education is exogenous and denoted by $N$. Students from this ‘pool of international talents’ are heterogeneous with respect to ability, denoted by $a$, which is uniformly distributed over the unit interval and which captures an individual’s capacity to exploit education quality. Students allocate themselves to the host countries, such that their expected net benefit from studying abroad is maximized. Thereby, they consider expected net labor-income as skilled workers in the future and tuition fees for higher education. Net labor-income is returns to education abroad net of income taxes. The return to education in one of the developed countries consists of some base salary $w$ and an education premium $aq_i \geq 0$, where $q_i \geq 0$ is quality of education in country $i$ and $a \in [0,1]$ is individual talent to acquire human capital. Talent and university quality are complementary in the production of the education premium. Labor income is taxed at rate $\tau \in [0,1]$ in countries 1 and 2 and at rate $\tau_{ROW} \in [0,1]$ in ROW.
Although labor incomes in the western countries possibly exceed those in ROW, there are usually non-economic reasons for foreign students to return to their home countries as graduates. These are represented by an exogenous repatriation rate \((1 - p)\), with \(p \in [0, 1]\) as a graduate’s stay rate in the host country (which is the probability that a foreign student stays on upon graduation). Repatriation motives are for example failure of social integration in the host country, private (e.g., family) issues in the country of origin, homesickness, problems with regard to the change of status from student to permanent immigrant in the host country, or labor market frictions.\(^3\) Repatriates earn a fraction \(\gamma \in [0, 1]\) of western labor income in their home countries. At the student migration stage, individuals already anticipate that they will stay on in the host country only with probability \(p\), however information on whether they belong to the group of repatriates is only revealed after graduation.\(^4\)

Expected net labor-income of a graduate with ability \(a\) then is

\[
E\{w_a\} = \varrho(w + aq), \quad \varrho := p(1 - \tau) + (1 - p)(1 - \tau_{ROW})\gamma.
\]

As ROW is supposed to be a developing region, the ROW net-income of a graduate from a university in one of the host countries never exceeds this graduate’s net-income when staying on in one of the (developed) countries:

**Assumption 1** \((1 - \tau) - (1 - \tau_{ROW})\gamma \geq 0.\)

A student’s choice of the location of education is determined by expected income, given the quality levels of the education systems in both countries and tuition fees (denoted by \(t_i\)). We do not restrict tuition fees to be positive, but perceive \(t_i\) as a

\(^3\)See for example Baruch, Budhwar and Khatri (2007) for a questionnaire survey on return/non-return determinants of foreign students in the U.S. and the UK.

\(^4\)We ignore the possibility that a foreign-born graduate leaves the host country of education in order to work in the other developed country. There are good reasons to believe that this is not too restrictive: (i) spending several years within the host country usually means that people have built up some social- (maybe even family-) ties and therefore have some attachment to the country; furthermore, foreign students are usually (at least to some extent) integrated in the local society of the host country, while they would have to start the integration process anew in the other country (which can be quite demanding, especially the larger the cultural difference between the host country and the new location of residence); (ii) the graduate can be integrated in the host country’s labor market much more easily, because he is familiar with the country’s culture (including its language) and has acquired some country-specific human capital; in addition, the host country might facilitate visas/work-permits if the applicant has successfully graduated from a domestic university (e.g., Germany allows foreign graduates from a German university to stay on in the country for one year in order to find a job and exempts applicants from the labor-market test; see Chaloff and Lemaitre, 2009, for similar procedures in other OECD countries).
net measure of tuition fees and subsidies per student. The student who is exactly indifferent between studying in one of the host countries has ability \( \hat{a} \), which is determined by

\[
\varrho(w + \hat{aq}_1) - t_1 = \varrho(w + \hat{aq}_2) - t_2 \iff \hat{a} = \frac{t_2 - t_1}{\varrho \Delta q},
\]

where \( \Delta q = q_2 - q_1 \geq 0 \) denotes the regional quality differential. Whenever we consider differentiated higher-education systems, we refer to country 2 as the high-quality country. Highly talented students (i.e., those with \( a \geq \hat{a} \)) go for high-quality education in country 2, while all others allocate to region 1.\(^5\)

The number of students in the low(er)-quality country 1 then is

\[
N_1 = N \times \begin{cases} \hat{a} & \text{if } \hat{a} \in [0, 1], \\ 1 & \text{if } \hat{a} > 1, \\ 0 & \text{if } \hat{a} < 0, \end{cases}
\]

where \( N \) is the total size of the pool of talents. The number of students in country 2 is \( N_2 = N - N_1 \).

For identical quality levels in both countries, i.e. \( \Delta q = 0 \), the size of the foreign student body in each country can no longer be determined by indifference condition (1). As students do not have any country-specific preferences, for equal qualities, all students would study in the country with lower tuition fees. If both countries offer identical education qualities and tuition fees, students allocate themselves randomly in a way that both countries end up with an overall number of foreign students of \( N/2 \) and face equal demand from all ability types in the distribution of talents. I.e. for \( \Delta q = 0 \),

\[
N_i|_{\Delta q=0} = \begin{cases} 0 & \text{if } t_i > t_j, \\ \frac{N}{2} & \text{if } t_i = t_j, \\ N & \text{if } t_i < t_j. \end{cases}
\]

Host country governments are maximizing net benefits or rather rents from offering an international study program. On the benefit side, foreign students pay tuition

\(^5\)The migration model relies on some implicit assumptions: (i) ex ante, foreign students do not have any ‘attachment’ to one of the two regions (e.g., in the sense of country-specific preferences, existing social networks, language and geographical/cultural distance); (ii) all students in the pool of talents can afford paying tuition fees when studying abroad (either because there are no credit constraints or because their initial endowment is sufficiently large); (iii) studying abroad is always preferred to studying/working in the country of origin.
fees and students who stay on in the country of education as graduates generate tax revenue (income is proportionally taxed at rate $\tau$). On the cost side, there are variable costs (i.e., costs of providing quality per student) $c(q_i) = \alpha q_i$, $\alpha \in [0, 1]$, and fix costs, which are represented by a continuous function $F(q_i)$ with $\partial F/\partial q_i > 0$, $\partial^2 F/\partial q_i^2 > 0$ and $F(0) = 0$.

If education systems are differentiated, the objective function of government 1 reads

$$R_1 = \tau W_1 + N_1[t_1 - c(q_1)] - F(q_1),$$

where the wage sum or rather the (foreign-born) tax base is

$$W_1 = pN \int_0^a (w + aq_1)da = pN_1 \left[ w + \frac{1}{2} \frac{(t_2 - t_1)}{\varrho \Delta q} q_1 \right],$$

so that the rent from educating foreign students can be decomposed into a variable part which depends on the number of students and into fix costs:

$$R_1 = N_1 \left\{ p\tau w + \frac{p\tau}{2} \frac{(t_2 - t_1)}{\varrho \Delta q} q_1 + t_1 - c(q_1) \right\} - F(q_1).$$

The product $p\tau$ basically represents a country’s effective rate of return to a marginal increase in foreign students’ incomes. Analogously, the objective function in country 2 is

$$R_2 = N_2 \left\{ p\tau w + \frac{p\tau}{2} \left( 1 + \frac{t_2 - t_1}{\varrho \Delta q} \right) q_2 + t_2 - c(q_2) \right\} - F(q_2),$$

where we used

$$W_2 = pN \int_0^1 (w + aq_2)da = pN_2 \left[ w + \frac{1}{2} \left( 1 + \frac{t_2 - t_1}{\varrho \Delta q} \right) q_2 \right].$$

2.2 Quality and tuition fee competition

The two host countries engage in a two-stage Nash-type competition. At the first stage, both regions simultaneously choose quality levels $q_i$, while tuition fees $t_i$ are determined at a second stage. The timing is in analogy to Boadway, Marceau and Marchand (1996) and Kemnitz (2007). Students then allocate to host countries and either stay on or leave their host country upon graduation. The game is solved recursively.
Stage 2 competition: tuition fees  When competing over tuition fees, the outcome of the first stage is already known. In principle, two situations have to be considered: (i) countries have chosen different quality levels at the first stage ($\Delta q > 0$); (ii) countries have chosen identical quality levels ($\Delta q = 0$). The respective outcomes at the second stage of the game are presented one after another.

Each country $i$ chooses tuition fees $t_i$ to maximize rents $R_i$, taking the other country’s policy and quality levels $(q_1, q_2)$, which were already determined at the first stage, as given. The corresponding optimization captures the tradeoff between the marginal costs and benefits of charging tuition fees, considering the direct revenue effect and the effect on the number of students and therefore also the number of graduates, who are potential tax payers in the host country. The equilibrium tuition fees $(t_1^*, t_2^*)$ simultaneously solve $t_1^* = t_1''(t_2^*; q_1, q_2)$ and $t_2^* = t_2''(t_1^*; q_1, q_2)$, where $t_i''(t_j; q_1, q_2)$ represents country $i$’s best-response function (please refer to the Appendix for the derivation):

$$t_1^* = \frac{\varrho[q\Delta q - p\tau q_1 + \alpha(q_2 + 2q_1)]}{p\tau + 3\varrho} - p\tau w,$$

$$t_2^* = \frac{\varrho[2q\Delta q - p\tau q_1 + \alpha(q_1 + 2q_2)]}{p\tau + 3\varrho} - p\tau w. \quad (7)$$

The tuition-fee differential

$$\Delta t^* := t_2^* - t_1^* = \frac{\varrho[c(q_2) - c(q_1) + \varrho\Delta q]}{(p\tau + 3\varrho)} = \frac{\varrho\Delta q(\alpha + \varrho)}{(p\tau + 3\varrho)} > 0,$$

reflects the fact that the high-quality country charges higher tuition fees. First of all, this is because the country with the higher quality has greater market power, which allows to charge higher fees, since for given tuition fees, the demand for an education system increases with its quality. Second, the higher fees in country 2 reflect the higher costs per student which are (partially) passed on to students. The larger $\alpha$, the more relevant becomes this effect and the larger the tuition-fee differential.

The second order conditions for optimal tuition fees in the two countries are

$$p\tau q_1 - 2\varrho\Delta q < 0, \quad -p\tau q_2 - 2\varrho\Delta q < 0. \quad (10)$$

The equilibrium tuition fees determine the equilibrium allocation of students

$$\hat{a}^* := \hat{a}(t_1^*, t_2^*) = \frac{\alpha + \varrho}{p\tau + 3\varrho}, \quad (11)$$

which follows directly from using the tuition-fee differential (9) in indifference condition (1).
If the two countries had chosen identical education qualities \( q_2 = q_1 = q \) at the first stage, they would face fierce tuition-fee competition for the entire pool of international students. For undifferentiated quality levels, the variable rent (i.e., the part of the rent depending on the number of foreign students) amounts to

\[
r_i|_{\Delta q=0} = \begin{cases} 
\tau W + N(t_i - c(q)) & \text{if } t_i < t_j, \\
\frac{1}{2} [\tau W + N(t_i - c(q))] & \text{if } t_i = t_j, \\
0 & \text{if } t_i > t_j, 
\end{cases}
\]

where \( W = pN \int_0^1 (w + aq) da = pN(w + q/2) \). The fix costs of providing quality are already sunk and therefore irrelevant for tuition-fee competition. Countries would have an incentive to undercut their competitor in order to attract all foreign students as long as \( r_i \) is still non-negative, thereby engaging in a race-to-the-bottom with tuition fees \( t_1 = t_2 = \alpha q - p\tau(w + q/2) \), \( r_i = 0 \) and overall rents \( R_i = -F(q) \).

**Stage 1 competition: education quality**  At the first stage, each country \( i \) decides on quality investments to maximize \( R_i \) for given quality investments abroad and subject to the non-negativity constraint \( q_i \geq 0 \). Thereby, countries anticipate the outcome of tuition-fee competition at the second stage. Given the equilibrium on stage 2, countries’ objective functions are

\[
R_i(q_1, q_2) = \begin{cases} 
 r_i(q_1, q_2; t^*_1, t^*_2) - F(q_i) & \text{if } q_i \neq q_j \\
- F(q_i) & \text{if } q_i = q_j, \quad i, j \in \{1, 2\}
\end{cases}
\]

where \( r_i(q_1, q_2; t^*_1, t^*_2) \) denotes the variable part of country \( i \)’s rent from educating foreign students, given \( t^*_1 \) and \( t^*_2 \) as of (7) and (8). With \( q_1 = q_2 \), this part of the rent is zero due to fierce tuition-fee competition.

As can be directly inferred from (12), a situation with undifferentiated education quality would imply local quality choice \( q_1 = q_2 = 0 \) and would leave both countries with a zero-rent \( (R = 0) \) from educating foreign students.

When choosing quality levels in a scenario with differentiated education qualities, decision makers consider not only direct quality effects, but also the consequences of an increased number of students and therefore graduates on the benefit side as well as on the cost side (cet. par. higher tax revenue and tuition-fee revenue vs. higher variable costs of tuition) and the effect on tuition fees \( t^*_i(q_i) \) that can be charged in price competition on the subsequent stage of the game. The Kuhn-Tucker conditions
for the optimal quality level in low-quality country 1 are
\[
\frac{\partial R_1}{\partial q_1} = N_1(t_1^*, t_2^*) \left( \frac{p\tau}{2} \hat{a}(t_1^*, t_2^*) + \frac{\partial t_1^*}{\partial q_1} - \frac{\partial c}{\partial q_1} \right) - \frac{\partial F}{\partial q_1} \leq 0
\]
\[
q_1 \geq 0 \quad \text{and} \quad q_1 \frac{\partial R_1}{\partial q_1} = 0.
\] (13)

Rent $R_1$ is downward-sloping and convex in $q_1$:
\[
\frac{\partial R_1}{\partial q_1} = -N_2 \left( \frac{p\tau + 2\varrho}{2} \right) \hat{a}^2 - \frac{\partial F}{\partial q_1} < 0,
\]
\[
\frac{\partial^2 R_1}{\partial q_1^2} = -\frac{\partial^2 F}{\partial q_1^2} < 0.
\] (14)

Therefore, $q^*_1 = 0$ maximizes $R_1$ for $0 \leq q_1 < q_2$.

The Kuhn-Tucker conditions for optimal quality of education in country 2 are
\[
\frac{\partial R_2}{\partial q_2} = N_2(t_1^*, t_2^*) \left\{ \frac{p\tau}{2} \left[ 1 + \hat{a}(t_1^*, t_2^*) \right] + \frac{\partial t_2^*}{\partial q_2} - \frac{\partial c}{\partial q_2} \right\} - \frac{\partial F}{\partial q_2} \leq 0
\]
\[
q_2 \geq 0 \quad \text{and} \quad q_2 \frac{\partial R_2}{\partial q_2} = 0.
\] (15)

An interior solution for the quality level in country 2 ($q^*_2 > q^*_1$) is then implicitly determined by
\[
\frac{\partial R_2}{\partial q_2} = \frac{N}{2} (p\tau + 2\varrho)(1 - \hat{a}^2) - \frac{\partial F}{\partial q_2} = 0.
\] (16)

The second order condition for a maximum holds due to $\frac{\partial^2 F}{\partial q_2^2} > 0$.

The following Lemma states that the equilibrium of the game is asymmetric.

**Lemma 1** In equilibrium, host countries of foreign students differentiate their education quality ($q^*_1 = 0, q^*_2 > 0$) to relax tuition-fee competition. One country (country 2) provides higher education quality and charges higher tuition fees. The high-quality country attracts the brightest students from the international talent pool.\(^6\)

**Proof.** Please refer to the Appendix. ■

The intuition for this result is in analogy to the rationale for vertical product-differentiation in oligopolistic competition, known from the IO-literature (Shaked and Sutton, 1982, is one of the standard references; Tirole, 1998, ch. 7.5.1, provides a plain textbook model): firms differentiate product qualities in order to relax price

\(^6\)In principle, there are two asymmetric equilibria: one in which country 2 provides the high quality education and one in which country 1 provides the higher quality.
competition. Kemnitz (2007) presents a similar result in the context of competition among autonomous universities.

2.3 Comparative statics

2.3.1 Size of the pool of international students

In the light of the increasing trend of international student mobility (as reported for example by the OECD, 2008, ch. C3), the question arises how an enlarged pool of international talents affects the degree of international differentiation of education systems. We state the following proposition.

**Proposition 1** An increase in the size of the international talent pool raises the regional differentiation of higher education.

**Proof.** Follows directly from (14) and (16).

A marginal quality increase reduces the variable rent in country 1, while it raises the variable rent in country 2. As the marginal rents’ absolute value increases with the size of the pool of talents, this implies that the degree of quality differentiation between both countries increases with $N$, i.e. $\partial \Delta q^*/\partial N > 0$. More intuitively, while

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7An application of vertical product-differentiation to public finance, which is partially comparable to the present approach, was recently presented by Zissimos and Wooders (2008). In a two-country model, they analyze a two-stage Nash competition for firm settlements by means of production-cost reducing public-good provision and tax policy. If firms only differ in technology but do not have any ex ante country-specific location preferences, the decentralized equilibrium is characterized by differentiated public-good policy and tax policy. An undifferentiated public-good provision would imply fierce tax competition leaving both countries worse off. There are further related contributions in the education literature. In a model with imperfectly mobile households and capital mobility, Hoyt and Jensen (2001) provide a rationale for two cities to offer differentiated public-school quality which is financed by property-tax revenue: the quality differentiation increases individuals’ attachment to their residence and reduces competition between cities, making both of them better off. De Fraja and Iossa (2002) analyze the competition of two ex ante identical universities within a country, which receive a fixed budget by the central government and try to maximize their institution’s ‘prestige’ by setting student admission standards. Only with low student mobility, a symmetric solution will exist. For high student mobility, if there is an equilibrium at all, it will be asymmetric implying one university becoming an elite institution, setting higher standards and attracting only the best students.

8The negative marginal rent of an increase in $q_1$ is finally due to a relatively small latitude to increase tuition fees in the competitive environment and the lower average abilities of country 1 graduates (implying a lower marginal effect of education quality on the wage sum and therefore tax revenue in country 1) in comparison with the marginal cost of the quality investment.
a marginal increase in education quality always produces the same fix costs which are independent of the number of students, a rising demand implies higher variable rents for each quality level in country 2, which finally implies an incentive to raise \( q_2 \). As of (9), the increased quality differentiation goes along with more differentiated tuition fees, i.e. \( \partial \Delta t^*/\partial N > 0 \).

### 2.3.2 Stay rate of foreign students

The stay rate of foreign students in their host countries affects the equilibrium allocation of foreign students, the quality differentiation between the host countries and the tuition fee differential. The following Proposition summarizes.

**Proposition 2** An increase in the stay rate of foreign students upon graduation in the host countries of education

(i) raises the share of foreign students who study in the high-quality country, i.e. \( \partial(1 - \hat{a}^*)/\partial p \geq 0 \),

(ii) raises the quality differential, i.e. \( \partial \Delta q^*/\partial p \geq 0 \),

(iii) has an ambiguous effect on the tuition fee differential:

\[
\frac{\partial \Delta t^*}{\partial p} \geq 0 \iff \epsilon_{\Delta q p} + \epsilon_{\varrho p} \geq |\epsilon_{\hat{a}^* p}|
\]

where \( \epsilon_{\Delta q p} := (\partial \Delta q/\partial p)(p/\Delta q) > 0 \), \( \epsilon_{\varrho p} := (\partial \varrho/\partial p)(p/\varrho) \geq 0 \) and \( \epsilon_{\hat{a}^* p} := (\partial \hat{a}^*/\partial p)(p/\hat{a}^*) \leq 0 \).

**Proof.** Please refer to the Appendix.

The allocation of students With Assumption 1, an increase in stay rate \( p \) raises student’s expected benefit from studying abroad. Therefore, for given \( \Delta t/\Delta q \), the allocation of students shifts unambiguously towards country 2 (see indifference condition (1)). This effect largely explains part (i) of the Proposition.

The quality differential Given the equilibrium derived in the section above (especially \( q_1^* = 0 \)), the effect of a rising \( p \) on quality differentiation \( \Delta q \) is equivalent to the effect on \( q_2^* \). The stay rate of foreign students affects both marginal revenues and marginal costs of providing education quality in high-quality country 2. Overall, however, \( \partial q_2^*/\partial p \geq 0 \) (and therefore \( \partial \Delta q^*/\partial p \geq 0 \)). One main driving force is the higher
total tax revenue which can be generated from a marginal quality investment when
the number of tax payers increases and which provides (ceteris paribus) a higher in-
centive to invest in quality \( (\partial[\partial r W^*_2/\partial q_2]/\partial p) = \tau N[(1 - \hat{a}^*^2)/2 - \hat{p} \hat{a}^*(\partial \hat{a}^*/\partial p)] > 0) \).

The tuition-fee differential  The effect of a rising stay rate \( p \) on the tuition-
fee differential is ambiguous. Given the identity \( \hat{a}^*(p) \equiv \frac{1}{\varrho(p)} \frac{\Delta t^*(p)}{\Delta q(p)} \) (see (1)) and
parts (i) and (ii) of Proposition 2, the intuition for the inequality in part (iii) is
straightforward as an equilibrium result. The inequality does, however, not really
elucidate why countries alter tuition fees in a way that finally implies a change in the
tuition-fee differential. As the effect of the stay rate \( p \) on actual choices of tuition
fees is quite complex, we make use of some simplifications to highlight the main
insights.

Suppose \( \tau_{ROW} = \tau \) and \( \gamma = 1 \), such that \( \varrho = 1 - \tau \). The parameter \( \varrho \) is then
independent of \( p \) (and therefore also \( \epsilon_{\varrho p} = 0 \)), i.e. the direct effect of the stay rate
on the allocation of students \( (1/\varrho \) is the proportionality constant of the relation
\( \hat{a} \propto \Delta t/\Delta p \) as of equation (1)) is eliminated, so that the focus is exclusively on the
stay rate’s effect on equilibrium policies and its indirect effect on student allocation
through a policy change.\(^9\) Then, using (9),
\[
\frac{\partial \Delta t^*}{\partial p} \bigg|_{\tau_{ROW}=\tau,\gamma=1} = \frac{\varrho(\alpha + \varrho)}{(p\tau + 3\varrho)} \frac{\partial \Delta q}{\partial p} - \frac{\varrho(\alpha + \varrho)}{(p\tau + 3\varrho)^2} \tau \Delta q. \tag{17}
\]
The overall effect can be decomposed into two components. First of all, the tuition-
fee differential (to some extent) goes along with the rising quality differential, i.e.,
a higher differentiation of qualities allows for higher differentiation of tuition fees.
This very intuitive effect is represented by the first term in (17).

The second term reflects the more direct effects of a change in the stay rate on
the incentives to raise tuition fees in the two regions and thereby also considers the
relevance of income-tax policy. Two effects can be identified. First, the marginal
cost of raising tuition fees due to deterring students away (in terms of foregone
tax revenue in the future) is higher in the country in which the marginal student

\(^9\)The auxiliary assumption \( \gamma = 1 \) in this section also implies that a foreign-born graduate from
a university in one of the two developed host countries earns the same labor income when staying
on in the host country and when returning to his (less-developed) home country. This specification
can be justified by recognizing that it is also the relative prize-level (which is usually lower in
less-developed countries) and therefore the real income that finally matters for the worker’s utility.
Furthermore, Baruch, Budhwar and Khatri (2007) point out that, for example, Chinese and Indian
students with a foreign university degree have excellent career opportunities back in their home
countries, implying a respective living standard.
earns higher income as a graduate (which is the high-quality country 2). An increase in the stay rate $p$ implies that the difference in these marginal costs between the two countries increases, resulting in a relatively reduced incentive to raise tuition fees in country 2. The tuition-fee differential would decrease. The second effect is directly opposed. An increase in tuition fees in country 2 increases the average wage-income of foreign-born graduates and therefore average tax-revenues in this country. The reason is that a marginal increase of $t_2$ only deters away the least productive students from the group of ROW-students in country 2. The average income of the remaining students in the future is therefore higher. In country 1, however, an increase in $t_1$ deters away the students with the highest productivity within the group of foreign students in this country, so that the average income of graduates in country 1 decreases. As a rising stay rate of graduates implies an increase in the relevance of this tax-revenue related aspect within the governments’ objective functions, the incentive to raise tuition fees in the high-quality country increases, while it decreases in the low-quality country. The tuition-fee differential would increase. This last effect via the composition of the student body and therefore average wage-incomes is a second-order effect compared to the first-mentioned effect through the marginal cost of raising tuition fees. Therefore, the second term in (17) is finally negative. The higher the relevance of tax-revenue for local governments, the more important becomes this effect.

A priori, the overall effect of the stay rate on the tuition-fee differential is ambiguous. When the tax-revenue argument becomes sufficiently strong, the following interesting scenario could emerge: while an increase in the stay rate of foreign students upon graduation raises differentiation in education quality ($\partial \Delta q / \partial p > 0$), tuition fees in the two countries actually converge ($\partial \Delta t^* / \partial p < 0$).

### 2.3.3 Degree of development of the sending region

A basic feature of the present analysis is the asymmetry of host countries and sending countries of foreign students. While host countries are developed countries, ROW is a less developed region. This section briefly considers the ROW reaching a higher degree of development and therefore catching up with the developed countries. Analytically, we can analyze the effect of a marginal increase in $\gamma$, implying a narrowing wage gap between ROW and the developed countries. The following Proposition summarizes.
Proposition 3  An increase in the degree of development of the sending region of foreign students raises the host countries’ differentiation of education quality \((\partial \Delta q^*/\partial \gamma > 0)\). The effect on the allocation of students and tuition fees is ambiguous.

Proof. Please refer to the Appendix. ■

Ceteris paribus, the degree of development of the sending region raises a student’s expected return to education abroad. For given tuition fees and quality levels in the host countries (more precisely, for given \(\Delta t/\Delta q\)), this increase in returns implies an increase in the share of students who decide to study in high-quality country 2 (see equation (1)). In other words, country 2 enhances market power relative to low-quality country 1. For some given allocation of students, a marginal increase in education quality \(q_2\) then implies increased latitude to raise tuition fees at the price-competition stage.\(^{10}\) Ceteris paribus, country 2 has an incentive to increase quality. This effect largely explains the increased regional quality differentiation if the degree of development of the sending region increases.

The increase in the quality differential is likely to go along with a rising tuition fee differential. A sufficient condition is that the number of students in the high-quality country increases in equilibrium, as can be seen from the proof of Proposition 3.

3 Welfare considerations

This section analyzes whether the outcome of competitive/decentralized education policy deviates from a welfare maximum. An allocation of students and the quality levels in the two host countries are supposed to be first best if the talent pool’s aggregate gross income net of education costs is maximized. Graduates earn wage income either in the host countries of education or in the home region ROW. The aggregate welfare function then is

\[
W^* = N[p + (1 - p)\gamma] \left[ \int_{\hat{a}^0}^{\hat{a}^1} (w + aq_1^\circ)da + \int_{\hat{a}^0}^{\hat{a}^1} (w + aq_2^\circ)da \right] \\
- N\alpha[q_2^\circ - \hat{a}^\circ(q_2^\circ - q_1^\circ)] - \sum_{i \in \{1,2\}} F(q_i^\circ),
\]

(18)

\(^{10}\)This can bee see from (8):

\[
\left. \left[ \frac{\partial}{\partial \gamma} \left( \frac{\partial t^*}{\partial q_2} \right) \right] \right|_{da^* = 0} = 2\hat{a}^*(1 - p)(1 - \tau_{ROW}) \geq 0.
\]
where the first line is aggregate gross income and the second line comprises variable and fixed costs of providing education quality in the two host countries.

The first order condition for an interior/boundary solution of \( \hat{a}^\circ \) is

\[
\frac{\partial W^\circ}{\partial \hat{a}^\circ} = N\{\alpha - [p + (1 - p)\gamma]\hat{a}^\circ\}(q^\circ_2 - q^\circ_1) = 0,
\]

such that the first best allocation of students is characterized by

\[
\hat{a}^\circ = \frac{\alpha}{p + (1 - p)\gamma}.
\]

If there are no variable costs of providing education quality (i.e., \( \alpha = 0 \)), the first best is characterized by an allocation of the entire pool of international students to the high-quality country 2 (boundary solution \( \hat{a}^\circ = 0 \)). The reason is that wage incomes increase with quality, which is higher in country 2. Allocating students to region 1 would therefore reduce welfare. With strictly positive variable costs (i.e., \( \alpha > 0 \)), however, allocating some students to country 1 becomes worthwhile, because a lower quality also implies lower costs per student.\(^{11}\) The interior solution as of (20) balances both effects at the margin.

The Kuhn-Tucker conditions for education qualities \( q^\circ_1 \) and \( q^\circ_2 \) are

\[
\frac{\partial W^\circ}{\partial q^\circ_1} = N\hat{a}^\circ\left\{[p + (1 - p)\gamma]\frac{\hat{a}^\circ}{2} - \alpha\right\} - \frac{\partial F}{\partial q^\circ_1} \leq 0
\]

\[q^\circ_1 \geq 0 \text{ and } q^\circ_1 \frac{\partial W^\circ}{\partial q^\circ_1} = 0,\]

\[
\frac{\partial W^\circ}{\partial q^\circ_2} = N(1 - \hat{a}^\circ)\left\{[p + (1 - p)\gamma]\frac{1 + \hat{a}^\circ}{2} - \alpha\right\} - \frac{\partial F}{\partial q^\circ_2} \leq 0
\]

\[q^\circ_2 \geq 0 \text{ and } q^\circ_2 \frac{\partial W^\circ}{\partial q^\circ_2} = 0.\]

Using (20) in (21) and (22) yields first best quality levels: \( q^\circ_1 = 0 \) and \( q^\circ_2 > 0 \) which is implicitly determined by

\[
\frac{N}{2}[p + (1 - p)\gamma](1 - \hat{a}^\circ)^2 - \frac{\partial F}{\partial q^\circ_2} = 0.
\]

With (20), the Hessian matrix of \( W^\circ = W(\hat{a}^\circ, q^\circ_1, q^\circ_2) \) is negative-definite, i.e. the solution actually maximizes aggregate welfare.

\(^{11}\)As can be seen from (18) and \( \int_0^{\hat{a}^\circ}(W + aq^\circ_1)da + \int_{\hat{a}^\circ}^1(W + aq^\circ_2)da = \hat{a}^\circ q^\circ_2/2 - \hat{a}^\circ 2(q^\circ_2 - q^\circ_1)/2 \), the welfare loss of allocating students to country 1 if \( \alpha = 0 \), is captured by \( N[p + (1 - p)\gamma]\hat{a}^\circ 2(q^\circ_2 - q^\circ_1)/2 \). The cost saving of allocating students to country 1 if \( \alpha > 0 \), is captured by \( N\alpha\hat{a}^\circ(q^\circ_2 - q^\circ_1) \).
The equilibrium allocation of students and the differentiation of education qualities in the competition for the international pool of talents as of Section 2.2 are likely to deviate from the first best.

**Proposition 4** Comparing the equilibrium of the competition for the international pool of talents to the first best, one can distinguish two cases:

(i) If $p + 2 \varrho > p + (1 - p) \gamma$,

(a) $(1 - \hat{a}^*) \gtrless (1 - \hat{a}^\circ)$ \iff\hspace{1cm} $\alpha \gtrless \frac{q[p+(1-p)\gamma]}{p\tau+3\varrho-[p+(1-p)\gamma]}$,

(b) $(1 - \hat{a}^*) \geq (1 - \hat{a}^\circ)$ involves $q_2^* > q_2^\circ$ ($\Delta q^* > \Delta q^\circ$),

(c) $(1 - \hat{a}^*) < (1 - \hat{a}^\circ)$ can in principle involve $q_2^* \leq q_2^\circ$ ($\Delta q^* \geq \Delta q^\circ$) as well as $q_2^* < q_2^\circ$ ($\Delta q^* < \Delta q^\circ$). If $\exists \tilde{\alpha} \in [0, \frac{q[p+(1-p)\gamma]}{p\tau+3\varrho-[p+(1-p)\gamma]}]$ such that $q_2^\circ = q_2^*$,

$\quad q_2^\circ \gtrless q_2^*$ ($\Delta q^\circ \gtrless \Delta q^*$) \iff \alpha \lessgtr \tilde{\alpha}.$

(ii) If $p + 2 \varrho \leq p + (1 - p) \gamma$,

(a) $(1 - \hat{a}^*) \nless (1 - \hat{a}^\circ),$

(b) $(1 - \hat{a}^\circ) > (1 - \hat{a}^*)$ involves $q_2^\circ > q_2^*$ ($\Delta q^\circ > \Delta q^*$).

**Proof.** Please refer to the Appendix.
While the equilibrium education quality in country 1 is welfare maximizing (i.e., \( q_1^* = q_1^o = 0 \)), education quality in country 2 (and therefore also the quality differential \( \Delta q \)) is likely to deviate from the \textit{first best}. Case (i) in Proposition 4 includes all cases in which for given and identical allocations of students in the decentralized solution and the \textit{first best} (i.e., \( \hat{a}^* = \hat{a}^o \)), the marginal benefit of investing in education quality is higher from the rent-maximizing perspective of country 2 in the competitive setting than from the welfare maximizing perspective (see (16) and (23)). While country 2 in the decentralized setting considers the effect of education quality on the local tax base and tuition-fee revenue, the aggregate welfare-maximizing solution considers the effect of education quality on aggregate income. With more students studying in country 2 in the decentralized solution compared to the \textit{first best}, quality level \( q_2^* \) unambiguously exceeds \textit{first best} level \( q_2^o \) (part (i)-(b) of Proposition 4). In other words, the competition for the pool of talents wastes resources compared to aggregate welfare. With the number of country-2 students in the decentralized equilibrium falling short of the \textit{first best}, \( q_2^* \) can either be smaller, larger or equal to the welfare maximizing level. The higher the variable cost-parameter \( \alpha \), the smaller the number of country-2 students in the \textit{first best} relative to the decentralized setting (i.e., the smaller the ratio \( (1 - \hat{a}^o)/(1 - \hat{a}^*) \)) and the more likely the competition for the pool of talents implies local underinvestment in education quality (\( q_2^* < q_2^o \)).

Part (ii) of Proposition 4 deals with the case where, for given and identical allocations of students in the decentralized solution and the \textit{first best}, the marginal benefit of investing in education quality is smaller from the rent-maximizing perspective of country 2 in the competitive setting than from the welfare maximizing perspective. This scenario is only consistent with a larger number of country-2 students in the \textit{first best} compared to the decentralized equilibrium (part (ii)-(a)). Competition for the pool of talents then implies an underinvestment in education quality (part (ii)-(b)).

4 Some implications for sending countries

The positive effect of the size of the pool of talents on quality differentiation (2.3.1) has an implication for the brain-drain/brain-gain discussion, which usually takes the perspective of a less-developed source region (ROW in our case) and analyzes the consequences of (high-skilled) emigration. Especially if domestic education prospects are rather poor, ROW probably has a vital interest in obtaining the high(er) skills.
of native students who have been trained in a developed country.\footnote{The idea here is that human capital not only has a quantitative but also a qualitative component. The endogenous-growth theory identifies skilled human capital as a crucial determinant of economic growth (e.g., Lucas, 1988; Romer, 1990).} Then, for a given stay rate $p$, the share of high-skilled graduates in ROW who have been educated abroad increases with $N$ (the share of return migrants within the ROW workforce is $\psi := (1 - p)N/(\overline{N} - pN)$, where $\overline{N} > N$ is the total number of ROW-born individuals; $d\psi/dN = (1 - p)\overline{N}/(\overline{N} - pN)^2 > 0$). This increase is what we might call a \textit{quantitative} brain-gain effect. In addition, an increase in $N$ alters the competition of the host countries of ROW-born foreign students: country 2 now offers higher education quality $q_2$, while education quality in country 1 remains unchanged ($q_1 = 0$); the allocation of students to the host regions remains unchanged too, because it is independent of both $N$ and $\Delta q$ (see (11)). Therefore, return migrants from the high-quality country 2 are more productive now, implying what we might call a \textit{qualitative} brain-gain effect.

A qualitative brain-gain effect also plays an important role when looking at the stay rate of foreign students in their host countries. First of all, an increase in the stay rate $p$ reduces the share of internationally educated graduates in ROW ($d\psi/dp = N(N - \overline{N})/(\overline{N} - pN)^2 < 0$), which can be called a quantitative brain drain. At the same time, an increase in $p$ alters competition between host countries of foreign students (2.3.2). As a result, the allocation of ROW-born students to host countries changes: the share of the pool of talents being educated in the high-quality country 2 increases ($d(1 - \hat{a}^*)/dp > 0$). In addition, education quality in country 2 increases, while $q_1 = 0$ remains unchanged. Therefore, with a rising stay rate of foreign students in the host regions, ROW suffers from a \textit{quantitative} brain drain effect, but benefits from a \textit{qualitative} brain gain effect in terms of (i) a larger share of return migrants who have been educated in the high-quality country 2, and (ii) a better education (and therefore higher productivity) of graduates who return from the high-quality country. As the focus of the present paper is on host countries of foreign students and not on sending regions, we do not carry on this brain-gain idea in more detail. Haupt, Krieger and Lange (2010) take on the basic idea and show that the qualitative brain-gain effect can cause both aggregate and per-capita human capital to increase in the sending country of foreign students, as long as the stay rate of students in the host country of education is not too large.
5 Conclusion

The present paper starts from the observation that a relatively small number of top-destinations for international students hosts a considerable share of students from countries like China and India going for higher education in one of the western developed countries. The model reduces this observation to the competition of two developed countries for the international pool of talents from a third region (ROW). There are good reasons for host countries to attract those students by means of their education system. Especially the prospect of thereby attracting future high-skilled workers if some of the international students stay on in their host countries deserves special attention. The equilibrium in our model is characterized by differentiated education policy in the sense of one country offering a high-quality-high-price education for the most talented students, while the other country charges lower tuition fees for a low(er)-quality education, attracting less talented students. The regional differentiation is actually the result of competition and not due to an ex ante asymmetry of countries: countries relax tuition-fee competition through quality differentiation. The differentiation of education quality between host countries increases with the size of the international talent pool, with the stay rate of foreign students in the host countries upon graduation and with the degree of development of the sending region of foreign students.

The results have some implication for the ongoing brain-drain/brain-gain discussion. While an increase in the stay rate of students in the host countries implies a quantitative brain drain from the (poor) source countries’ perspective, the induced increase in the equilibrium quality-differentiation between host countries and a shift of the student allocation towards the high-quality country finally contrasts the quantitative brain-drain effect with a qualitative brain-gain effect through return migrants. Depending on whether the brain-drain or brain-gain effect dominates, the source regions of the international talent pool will either lose or gain from a reduced return rate of their human capital trained in the western world. So far, the recent literature on a ‘beneficial brain drain’ has mainly emphasized the role of additional incentives to acquire skills in a less-developed country when there is an option to migrate to a developed country upon graduation in order to earn higher wages (e.g., Mountford, 1997; Stark, Helmenein and Prskawetz, 1997, 1998; Vidal, 1998; Beine, Docquier and Rapoport, 2001, 2008; Stark and Wang, 2002; Mayr and Peri, 2009; Eggert, Krieger and Meier, 2010).

Furthermore, we argued that the allocation of students to the two host countries as well as the degree of regional quality differentiation are likely to deviate from the
aggregate welfare-maximizing solution.

We should mention that the assumption of perfect student mobility might not hold in reality. If students in the international pool of talents had some country-specific preferences implying imperfect mobility, competition would be less fierce and the quality differentiation might be less extreme. However, compared to a two-country setting in which each country tries to attract students from the other country, students from a third country (developing country), as in our model, going for education in Europe, North America or Australia, should have much weaker country-specific preferences in location choice. Pure two-country models with student migration usually feature imperfect student mobility (e.g., Boadway, Marceau and Marchand, 1996; Buettner and Schwager, 2004; Gérard, 2007; Lange, 2009; Krieger and Lange, 2010).

The analysis points to some issues for future research. While we have assumed simultaneous moves, for example, there could also be sequential choice of quality levels or rather entrance in the competition for international students (e.g., by launching international study programs). Countries then have an incentive to spend resources to lead the way and obtain a first-mover advantage by choosing the more profitable quality level. Furthermore, it would be worthwhile considering an endogenous immigration policy which targets the stay rates of graduates. Countries could try to support the success of social integration and exert some effort to facilitate graduates’ labor-market access (e.g., by promoting permanent residency). More and more OECD countries already make use of this option and it could be interesting to elaborate more on the strategic aspects of immigration policy in the context of the competition for the international pool of talents. Including admission standards to the choice set of countries, like for example in De Fraja and Iossa (2002), may also enrich further research.
Appendix

Tuition-fee competition (Section 2.2)

Country 1 chooses $t_1$ to maximize $R_1$, taking $t_2$ and quality levels $(q_1, q_2)$ as given. The corresponding first order condition for given $\Delta q > 0$ is

$$t_1 \left( \frac{p\tau q_1}{\varrho \Delta q} - 2 \right) - t_2 \left( \frac{p\tau q_1}{\varrho \Delta q} - 1 \right) - p\tau w + c(q_1) = 0,$$

from which one can directly derive the best-response function $t_1 = t_1^{br}(t_2; q_1, q_2)$:

$$t_1 = \theta_1 t_2 + \frac{p\tau w - c(q_1)}{\frac{p\tau q_1}{\varrho \Delta q} - 2}; \quad \theta_1 := \frac{\frac{p\tau q_1}{\varrho \Delta q} - 1}{\frac{p\tau q_1}{\varrho \Delta q} - 2}. \tag{24}$$

The first order condition for tuition fees chosen by country 2 and the best-response function $t_2 = t_2^{br}(t_1; q_1, q_2)$ can analogously be determined as

$$t_1 \left( \frac{p\tau q_2}{\varrho \Delta q} + 1 \right) - t_2 \left( \frac{p\tau q_2}{\varrho \Delta q} + 2 \right) - p\tau w + c(q_2) + \varrho \Delta q = 0$$

and

$$t_2 = \theta_2 t_1 + \frac{\varrho \Delta q + c(q_2) - p\tau w}{\frac{p\tau q_2}{\varrho \Delta q} + 2}; \quad \theta_2 := \frac{\frac{p\tau q_2}{\varrho \Delta q} + 1}{\frac{p\tau q_2}{\varrho \Delta q} + 2}. \tag{25}$$

Combining (24) and (25) yields equilibrium tuition fees

$$t_1^* = \frac{1}{1 - \theta_1 \theta_2} \left[ \theta_2 \frac{p\tau w - c(q_1)}{\frac{p\tau q_1}{\varrho \Delta q} - 2} + \theta_1 \frac{\varrho \Delta q + c(q_2) - p\tau w}{\frac{p\tau q_2}{\varrho \Delta q} + 2} \right],$$

$$t_2^* = \frac{1}{1 - \theta_1 \theta_2} \left[ \theta_2 \frac{p\tau w - c(q_1)}{\frac{p\tau q_1}{\varrho \Delta q} - 2} + \theta_1 \frac{\varrho \Delta q + c(q_2) - p\tau w}{\frac{p\tau q_2}{\varrho \Delta q} + 2} \right],$$

which finally can be reduced to (7) and (8).

Proof Lemma 1

With undifferentiated education quality, both countries were demonstrated to generate a zero-rent from educating the international pool of talents (i.e., $R_1 = R_2 = 0$). The fact that both countries can earn strictly positive rents ($R_1, R_2 > 0$) with differentiated qualities finally proves the Lemma.

To this end, we first of all prove that variable rents are strictly positive for an interior solution of the allocation of foreign students $\hat{a}^*$, i.e. we prove that $r_i(q_1, q_2) \equiv \ldots$
\[
\tau W_i + N_i[t_i - c(q_i)] > 0, \ i \in \{1, 2\}. \text{ Variable rents are }
\]
\[
r_1(q_1, q_2) = N\hat{a}^* \left\{ \frac{p\tau}{2} \hat{a}^* q_1 + t_1^* - \alpha q_1 \right\},
\]
\[
r_2(q_1, q_2) = N(1 - \hat{a}^*) \left\{ \frac{p\tau}{2} (1 + \hat{a}^*) q_2 + t_2^* - \alpha q_2 \right\}. \]

Using equilibrium values \( t_1^* \), \( t_2^* \) and \( \hat{a}^* \) as of (7), (8) and (11), and for strictly positive demand for both education systems (i.e., \( 0 < \hat{a}^* < 1 \)), we find
\[
r_1(q_1, q_2) > 0 \text{ if } p\tau q_1 - 2\rho \Delta q < 0,
\]
\[
r_2(q_1, q_2) > 0 \text{ if } \left( \frac{p\tau}{2} q_2 + \rho \Delta q \right) (p\tau + 2\rho - \alpha) > 0. \]

While the second order condition for the optimal \( t_1^* \) guarantees \( r_1(q_1, q_2) > 0 \), the strictly positive demand for education in country 2 (see that \( 1 - \hat{a}^* = (p\tau + 2\rho - \alpha)/(p\tau + 3\rho) \)) ensures \( r_2(q_1, q_2) > 0 \). With undifferentiated education quality, a race-to-the-bottom in tuition fees would drive this rent down to zero.

With \( q_1^* = 0 \) and \( r_1(q_1, q_2) > 0 \), as can be seen from (12), country 1 generates a strictly positive rent \( R_1 > 0 \) from educating foreign students. The reason is that country 1 does not incur any costs from educating foreign students but nevertheless generates some (tax/tuition-fee) revenue from those students who cannot afford to study in country 2.

Country 2 also generates a strictly positive rent \( R_2 \). As \( \lim_{q_2 \to 0} R(q_2) = 0 \),
\[
q_2^* = \arg \max R_2(q_2) > 0 \iff R_2 > 0.
\]

The equilibrium allocation of students is \( \hat{a}^* \). As of (1), all individuals with ability \( a \geq \hat{a}^* \) study in the high-quality country 2, while all students with \( a < \hat{a}^* \) study in country 1.

**Proof Proposition 2**

First of all,
\[
\frac{\partial \varrho(p)}{\partial p} = (1 - \tau) - (1 - \tau_{\text{row}}) \gamma \geq 0
\]
can be signed unambiguously by Assumption 1. Furthermore,
\[
\frac{\partial \hat{a}^*}{\partial p} = -\frac{\tau[(1 - \tau_{\text{row}})\gamma + \alpha] + 3\alpha \frac{\partial \varrho}{\partial p}}{(p\tau + 3\rho)^2} \leq 0
\]
and therefore \( \partial(1 - \hat{a}^*)/\partial p \geq 0 \), which proves part (i) of the proposition.
Part (ii) follows from
\[ \frac{\partial \Delta q^*}{\partial p} \geq 0 \quad (q^* \approx 0) \Rightarrow \frac{\partial q^*}{\partial p} \approx 0 \quad (16) \Rightarrow \frac{\partial}{\partial p} \left[ \frac{N}{2} (p \tau + 2 \varrho)(1 - \hat{a}^*)^2 \right] \approx 0 \]
and
\[ \frac{\partial}{\partial p} \left[ \frac{N}{2} (p \tau + 2 \varrho)(1 - \hat{a}^*)^2 \right] = \frac{N(1 - \hat{a}^*)}{2} \left[ (\tau + 2 \frac{\partial \varrho}{\partial p})(1 - \hat{a}^*) - 2(p \tau + 2 \varrho) \frac{\partial \hat{a}^*}{\partial p} \right] \geq 0. \]

Considering the tuition-fee differential (9) and the equilibrium allocation of students (11),
\[ \frac{\partial \Delta t^*}{\partial p} = \frac{\partial [g(p)\Delta q(p)\hat{a}^*(p)]}{\partial p} \approx 0 \]
\[ \Leftrightarrow \hat{a}^* \frac{\partial \Delta q}{\partial p} + \Delta q \left( \frac{\partial \varrho \hat{a}^* + \hat{a}^* \frac{\partial \varrho}{\partial p}}{\partial q(p)\hat{a}^*(p)} \right) < 0 \]
\[ \Leftrightarrow \frac{\partial \Delta q(p)}{\partial p} \frac{p}{\Delta q} + \frac{\partial \varrho p}{\partial p} + \frac{\partial \hat{a}^* p}{\partial p} \hat{a}^* < 0, \]
which proves part (iii) of the proposition.

**Proof Proposition 3**

The first part follows from
\[ \frac{\partial \Delta q^*}{\partial \gamma} \approx 0 \quad (q^* \approx 0) \Rightarrow \frac{\partial q^*}{\partial \gamma} \approx 0 \quad (16) \Rightarrow \frac{\partial}{\partial \gamma} \left[ \frac{N}{2} (p \tau + 2 \varrho)(1 - \hat{a}^*)^2 \right] \approx 0 \]
\[ (26) \]
and
\[ \frac{\partial}{\partial \gamma} \left[ \frac{N}{2} (p \tau + 2 \varrho)(1 - \hat{a}^*)^2 \right] > 0 \quad \Leftrightarrow \quad 3g \rho \tau + 6 \varrho^2 + 2 \alpha \rho \tau + 3 \alpha \varrho > 0, \]
which always holds. Second, with \( \frac{\partial \varrho}{\partial \gamma} > 0 \ \forall \gamma \tau_{row}, \rho \in [0, 1], \)
\[ \frac{\partial (1 - \hat{a}^*)}{\partial \gamma} \approx 0 \quad \Leftrightarrow \quad \frac{3 \alpha - p \tau}{(p \tau + 3 \varrho)^2} \approx 0. \]
Third,
\[ \frac{\partial \Delta t^*}{\partial \gamma} \approx 0 \quad \Leftrightarrow \quad \epsilon_{\Delta t \gamma} + \epsilon_{\varrho \gamma} + \epsilon_{\hat{a}^* \gamma} \approx 0, \]
where \( \epsilon_{\Delta q \gamma} := (\partial \Delta q / \partial \gamma)(\gamma / \Delta q) > 0, \epsilon_{\varrho \gamma} := (\partial \varrho / \partial \gamma)(\gamma / \varrho) \geq 0 \) and \( \epsilon_{\hat{a}^* \gamma} := (\partial \hat{a}^* / \partial \gamma)(\gamma / \hat{a}^*) \approx 0. \)
Proof Proposition 4

Part (i)-(a) of the Proposition follows from comparing (11) and (20).

Comparing (16) and (23),

\[ q_2^\circ \gtrless q_2^* \iff \frac{1 - \hat{a}^\circ}{1 - \hat{a}^*} \gtrless \sqrt{\frac{p \tau + 2 \varrho}{p + (1 - p) \gamma}}. \]  

(27)

Part (i)-(b) follows immediately.

Part (i)-(c) takes on (27) and uses the fact that \( (1 - \hat{a})/(1 - \hat{a}^*) \) decreases monotonically in \( \alpha \) if \( p \tau + 2 \varrho > p + (1 - p) \gamma \):

\[
\frac{\partial}{\partial \alpha} \left( \frac{1 - \hat{a}^\circ}{1 - \hat{a}^*} \right) = \frac{p \tau + 3 \varrho}{[p + (1 - p) \gamma](p \tau + 2 \varrho - \alpha)} \left( \frac{p + (1 - p) \gamma - \alpha}{p \tau + 2 \varrho - \alpha} - 1 \right) < 0
\]

\[ \iff p + (1 - p) \gamma - p \tau - 2 \varrho < 0. \]

Therefore, if there exists an \( \tilde{\alpha} \in ]0, \frac{\varrho \gamma + (1 - p) \gamma}{p \tau + 3 \varrho - p (1 - p) \gamma}[ \) for which \( q_2^\circ = q_2^* \iff \frac{1 - \hat{a}^\circ}{1 - \hat{a}^*} = \sqrt{\frac{p \tau + 2 \varrho}{p + (1 - p) \gamma}} \), it follows that

\[ q_2^\circ \gtrless q_2^* \iff \alpha \lesssim \tilde{\alpha}. \]

Simplifying the analysis for example by assuming \( \tau = \tau_{row} \), using (27) one finds that \( q_2^\circ > q_2^* \) if \( \alpha = 0 \). Then, with part (i)-(b) and \( (1 - \hat{a}^\circ)/(1 - \hat{a}^*) \) decreasing in \( \alpha \), there will always exist a unique \( \tilde{\alpha} \in ]0, \frac{\varrho \gamma + (1 - p) \gamma}{p \tau + 3 \varrho - p (1 - p) \gamma}[ \).

Part (ii)-(a) follows from the fact that \( p \tau + 2 \varrho > p + (1 - p) \gamma \) is a necessary condition for \( (1 - \hat{a}^*) \geq (1 - \hat{a}^\circ) \) if \( \varrho \neq 0 \): with \( p \tau + 3 \varrho > p + (1 - p) \gamma \) (which is a necessary condition for \( \hat{a}^\circ \geq \hat{a}^* \)), \( (1 - \hat{a}^*) \geq (1 - \hat{a}^\circ) \) if \( p \tau + 2 \varrho < p + (1 - p) \gamma \).

Part (ii)-(b) finally follows immediately from (27).

See that \( q_2^\circ \gtrless q_2^* \) always implies \( \Delta q^\circ \gtrless \Delta q^* \), as \( q_1^* = q_1^\circ = 0 \).
References


