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Abstract. Exchange economies in which preferences of some consumers are possibly satiated are considered. In a general model of an atomless exchange economy, the equivalence between the 'rejective' core and the set of dividend equilibrium allocations is proved by applying Liapunov's theorem in multi-dimensions.

Keywords: rejective core, dividend equilibrium, core equivalence.

1. Introduction

In general equilibrium theory, non-satiation of consumers' preferences is an essential assumption and is used in proving the existence of the competitive equilibrium and the equivalence between the core and the competitive equilibrium. However, in recent analyses, it has been tried to relax the non-satiation assumption and to define several new notions of equilibrium for economies with satiation. Among them, the dividend equilibrium that was originally defined by Aumann=Drèze [4] has been focused as a most natural and general notion.

The dividend equilibrium originates in the concept of 'coupons equilibrium' defined by Drèze=Müller [7] in the analysis of fixed price economies. In markets with price rigidities or in the more general context of markets with possibly satiated consumers, a Walrasian equilibrium may fail to exist. This led to a revision of the equilibrium concept and the dividend equilibrium was introduced. Aumann=Drèze [4] showed an equivalent relation between the dividend equilibrium and the Shapley value allocation. Thereafter, economies with possibly satiated consumers have been considered in many literatures. Mas-Colell [11] defined the notion of 'Walrasian equilibrium with slack' and considered the existence and the Pareto optimality of the equilibrium in general economies with possibly satiated consumers. By introducing 'paper money' to exchange economies, Kajii [9] proved the existence of the equilibrium under general assumptions of consumers' preferences. Allouch=Le-Van=Page [1] considered unbounded exchange economies with satiation and proved the existence of a competitive equilibrium under the assumption of weak non-satiation. Moreover, Allouch=Le-Van [2, 3] introduced an additional commodity to an economy with satiation and proved the existence of a dividend equilibrium for the original economy under a weak non-satiation assumption.

On the other hand, as a generalized concept of the core, the notion of the 'rejective' core was introduced by Konovalov [10] and it was proved that, in a large economy with finite types of consumers, an allocation is in the dividend equilibrium if and only if it belongs to the rejective core. In this paper, in a general model of an atomless economy we will prove the identity of the rejective core with the set of dividend equilibrium allocations. Thus, the purpose of this paper is to establish a general equivalence theorem on the rejective core and the dividend equilibrium.

Under the assumption of non-satiation of consumers' preferences, the core equivalence theorem was proved by Aumann [5] in atomless economies. Since the dividend equilibrium is a generalized notion of the competitive equilibrium, our theorem is a generalized version of the core equivalence theorem which includes the case of economies with satiation. Konovalov [10] considered economies with finite types of consumers and proved an equivalence theorem by using effectively the techniques of Debreu=Scarf [6] and Aumann [5]. The proof of our theorem is a modification of the arguments of Aumann [5]. His ingenious proof dispenses with

Liapunov's theorem on non-atomic measures. However, as we inevitably use Liapunov's theorem in multi-dimensions, it seems that the theorem is indispensable to our proof.

2. The model and the main theorem

We consider an exchange economy including L commodities and infinitely many consumers (continuum of consumers). The set of all consumers is denoted by a unit interval, T=[0, 1]. Each consumer $t \in T$ is characterized by a consumption set X_t, a preference relation $\succ_t$, and an initial endowment $\boldsymbol{w}(t) \in X_t$. We assume that $X_t = \mathbf{R}_+^L$ for every t and that function $\boldsymbol{w} : T \to \mathbf{R}_+^L$ is integrable.

An *assignment* is an integrable function $\boldsymbol{x} : T \to \mathbf{R}_+^L$ in which $\boldsymbol{x}(t)$ denotes the consumption bundle allocated to consumer $t \in T$. We call a fixed assignment, $\boldsymbol{w} : T \to \mathbf{R}_+^L$, the *initial assignment*. An *allocation* is an assignment $\boldsymbol{x} : T \to \mathbf{R}_+^L$ such that $\int_T \boldsymbol{x} = \int_T \boldsymbol{w}$. [1/]

We assume the following conditions for each consumer $t \in T$.

(A1) $\boldsymbol{w}(t)$ is in interior of $\mathbf{R}_+^L$.
(A2) $\succ_t$ is irreflexive.
(A3) for each $x \in \mathbf{R}_+^L$, set $\{ y \in \mathbf{R}_+^L \mid y \succ_t x \}$ is open in $\mathbf{R}_+^L$.

Furthermore, we need the next condition for mathematical treatments.

(A4) For any two assignments $\boldsymbol{x} : T \to \mathbf{R}_+^L$ and $\boldsymbol{y} : T \to \mathbf{R}_+^L$, set $\{ t \in T \mid \boldsymbol{y}(t) \succ_t \boldsymbol{x}(t) \}$ is measurable.

Now, let us define the dividend equilibrium and the rejective core.

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[1] The integral of any assignment $\boldsymbol{x} : T \to \mathbf{R}_+^L$ is denoted by $\int_T \boldsymbol{x}$.

Definition 1 An allocation $\boldsymbol{x}: T \to \mathbf{R}_+^L$ is a *dividend equilibrium allocation* if there exist a price vector $p \in \mathbf{R}^L$ and a measurable function $\boldsymbol{d}: T \to \mathbf{R}_+$ such that for almost every $t \in T$,

(i) $p \cdot \boldsymbol{x}(t) \le p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t)$,

(ii) if $y \succ_t \boldsymbol{x}(t)$, then $p \cdot y > p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t)$.

Let us denote by $\mathscr{W}_d$ the set of dividend equilibrium allocations. In dividend equilibrium allocations, the surplus created by satiated consumers is distributed among non-satiated consumers as dividends which is denoted by function $\boldsymbol{d}: T \to \mathbf{R}_+$. In Definition 1, if $\boldsymbol{d}(t)=0$ for all $t \in T$, a dividend equilibrium allocation $\boldsymbol{x}: T \to \mathbf{R}_+^L$ is said to be a *competitive equilibrium allocation*. By $\mathscr{W}$, we denote the set of competitive equilibrium allocations. By definition, we have that $\mathscr{W} \subset \mathscr{W}_d$.

For a measurable subset S of [0, 1], by $\lambda(S)$ we denote the Lebesgue measure of set S. When S is a measurable set of consumers whose Lebesgue measure is positive, we call it a *coalition*.

Definition 2 A coalition S *rejects* an allocation $\boldsymbol{x}: T \to \mathbf{R}_+^L$ if and only if there are a measurable partition (S_1, S_2) of S and another allocation $\boldsymbol{y}: T \to \mathbf{R}_+^L$ such that

(i) $\int_S \boldsymbol{y} = \int_{S_1} \boldsymbol{w} + \int_{S_2} \boldsymbol{x}$,

(ii) $\boldsymbol{y}(t) \succ_t \boldsymbol{x}(t)$ for almost every $t \in S$,

(iii) $\boldsymbol{w}(t) \not\succ_t \boldsymbol{y}(t)$ for almost every $t \in T \setminus S$.

Condition (1) allows each consumer t in coalition S to provide either his own initial endowment $\boldsymbol{w}(t)$ or assignment $\boldsymbol{x}(t)$ allotted to him in attaining a new allocation $\boldsymbol{y}: T \to \mathbf{R}_+^L$. Condition (2) means that the new allocation $\boldsymbol{y}: T \to \mathbf{R}_+^L$ must be better than allocation $\boldsymbol{x}: T \to \mathbf{R}_+^L$ for all consumers in coalition S. On the other hand, condition (3) ensures that any individual consumer outside coalition S has no incentives to withdraw his initial endowment.

The *rejective core* is the set of all allocations that are not rejected by any coalition. Let us denote the rejective core by $\mathscr{C}_r$.

In Definition 2, when $\lambda(S_2)=0$, allocation $\boldsymbol{x}: T \to \mathbf{R}_+^L$ is said to *be improved upon* by coalition S in the standard definition of the core. The *core* is defined to be the set of all allocations that are not be improved upon by any coalition and is denoted by $\mathscr{C}$. By definition, if an allocation is improved upon by a coalition, then the coalition rejects the allocation. Thus, the rejective core is a subset of the core, that is, $\mathscr{C}_r \subset \mathscr{C}$.

Without its proof, we state a proposition on the relation between $\mathscr{W}_d$ and $\mathscr{C}_r$ that is due to Konovalov [10].

Proposition 1 *Any dividend equilibrium allocation belongs to the rejective core, that is,* $\mathscr{W}_d \subset \mathscr{C}_r$.

Thus, in general, we have $\mathscr{W} \subset \mathscr{W}_d \subset \mathscr{C}_r \subset \mathscr{C}$. It is well known (see Aumann [5] and Hildenbrand [8]) that the equivalence theorem on the core and the set of competitive equilibrium allocations holds under the following assumption of local non-satiation of consumers' preferences.

(A5) for every $t \in T$, for any $x \in \mathbf{R}_+^L$ *and* $\varepsilon>0$, there exists $y \in \mathbf{R}_+^L$ such that $\|x-y\|<\varepsilon$ and $y \succ_t x$.

Thus, we have the following proposition.

Proposition 2 *Under Assumptions (A1)-(A5),* $\mathscr{W}=\mathscr{W}_d=\mathscr{C}_r=\mathscr{C}$.

In case that some consumers are satiated, the rejective core is strictly smaller than the core, that is, $\mathscr{C}_r \subset \mathscr{C}$ and $\mathscr{C}_r \neq \mathscr{C}$. Such examples are shown by Konovalov [10]. Therefore, in economies with satiation, the equivalence of two sets $\mathscr{W}$ and $\mathscr{C}$ does not hold. However, when it comes to two sets $\mathscr{W}_d$ and $\mathscr{C}_r$, the following equivalence theorem holds even in economies with satiation, which is the main theorem of this paper.

Theorem *Under Assumptions (A1)-(A4), An allocation is a dividend equilibrium if and only if it belongs to the rejective core, that is,* $\mathscr{W}_d=\mathscr{C}_r$.

3. Proof of the main theorem

In order to prove the main theorem, it suffices, by Proposition 1, to show that any allocation in the rejective core is a dividend equilibrium allocation.

Now, let $\boldsymbol{x}: T \to \mathbf{R}_+^L$ be an allocation in the rejective core. For each $t \in T$, let us define

$$G_w(t) := \{z \in \mathbf{R}^L \mid z + \boldsymbol{w}(t) \succ_t \boldsymbol{x}(t)\},$$

$$G_x(t){:=}\{z \in \mathbf{R}^L \mid z + \boldsymbol{x}(t) \succ_t \boldsymbol{x}(t)\}.$$

Also, for each $z \in \mathbf{R}^L$, define

$$G_w^{-1}(z) := \{t \in T \mid z \in G_w(t)\} = \{t \in T \mid z + \boldsymbol{w}(t) \succ_t \boldsymbol{x}(t)\},$$

$$G_x^{-1}(z) := \{t \in T \mid z \in G_x(t)\} = \{t \in T \mid z + \boldsymbol{x}(t) \succ_t \boldsymbol{x}(t)\}.$$

Note that, by assumption (A4), $G_w^{-1}(z)$ and $G_x^{-1}(z)$ are both measurable.

Next we define

$$N_w{:=}\{r \in \mathbf{Q}^L \mid \lambda(G_w^{-1}(r)) = 0\},$$

$$N_x{:=}\{r \in \mathbf{Q}^L \mid \lambda(G_x^{-1}(r)) = 0\},$$

$$U := T \diagdown \left\{ \left(\bigcup_{r \in N_w} G_w^{-1}(r) \right) \cup \left(\bigcup_{r \in N_x} G_x^{-1}(r) \right) \right\},$$

where $\mathbf{Q}^L$ is the set of all rational points in $\mathbf{R}^L$. N_w and N_x are both countable, so $\lambda\left(\bigcup_{r \in N_w} G_w^{-1}(r) \right) = \lambda\left(\bigcup_{r \in N_x} G_x^{-1}(r) \right)$=0, and therefore $\lambda(U)$=1.

Finally, we define a subset of $\mathbf{R}^L$ by

$$\Delta{:=}\ \mathrm{co} \bigcup_{t \in U} [\mathbf{Q}^L \cap (G_w(t) \cup G_x(t) \cup \{\mathbf{0}\})].$$ [2/]

Lemma Set Δ is a non-empty convex subset of $\mathbf{R}^L$ such that $\mathbf{0} \notin \mathrm{int}\,\Delta$. [3/]

Proof. Since $\mathbf{0} \in \Delta$, Δ is non-empty. The convexity of Δ follows from its definition.

Now, suppose that $\mathbf{0} \in \mathrm{int}\,\Delta$. Then, by definition of Δ, there exist $t_i \in U$ and $r_i \in \mathbf{Q}^L \cap (G_w(t_i) \cup G_x(t_i) \cup \{\mathbf{0}\})$ (i=1,$\cdots$, k) such that $\mathbf{0} \in \mathrm{int}\ \mathrm{co}\,\{r_1, \cdots, r_k\}$. Without loss of generality, we can assume that $r_i \neq \mathbf{0}$ and $r_i \in \mathbf{Q}^L \cap (G_w(t_i) \cup G_x(t_i))$ for each

[2] "co" means convex hull.

[3] "int" means interior

i=1,···, k. Therefore, there exists $\alpha_i > 0$ for each i=1,···, k such that

(3.1) $\sum_{i=1}^{k} \alpha_i = 1$, and $\sum_{i=1}^{k} \alpha_i r_i = \mathbf{0}$.

Each t_i belongs to U, so $t_i \notin \bigcup_{r \in N_w} G_w^{-1}(r)$ and $t_i \notin \bigcup_{r \in N_x} G_x^{-1}(r)$. If $r \in N_w$, then $t_i \notin G_w^{-1}(r)$, i.e., $r \notin G_w(t_i)$. Therefore, $N_w \cap G_w(t_i) = \phi$. Similarly, we have $N_x \cap G_x(t_i) = \phi$. Since $r_i \in G_w(t_i) \cup G_x(t_i)$, it follows that $r_i \notin N_w$ or $r_i \notin N_x$. This implies that $\lambda(G_w^{-1}(r_i)) > 0$ or $\lambda(G_x^{-1}(r_i)) > 0$. Therefore there exist a number $\delta > 0$ and A_i (i=1,···, k) such that $A_i \subset G_w^{-1}(r_i)$ or $A_i \subset G_x^{-1}(r_i)$, and

(3.2) $\lambda(A_i) = \delta\alpha_i$, $A_i \cap A_j = \phi$ (if $i \neq j$).

Rearranging i, we can say, without loss of generality, that for some integer i_0 ($1 \leq i_0 \leq k$) such that

$$A_i \subset G_w^{-1}(r_i) \text{ for each } i=1,\cdots, i_0-1,$$

$$A_i \subset G_x^{-1}(r_i) \text{ for each } i=i_0,\cdots, k.$$

Now, for each i=1,···, k, consider a vector measure defined by

$$\nu_i(B) := \left(\int_B \boldsymbol{w}, \int_B \boldsymbol{x}, \int_B r_i\right) \quad \text{for } B \subset A_i.$$

By Liapunov's theorem on the ranges of vector measures, for each i=1,···, k, we have a set $B_i \subset A_i$ such that

$$\text{(3.3)} \quad \nu_i(B_i) = \left(\int_{B_i} \boldsymbol{w}, \int_{B_i} \boldsymbol{x}, \int_{B_i} r_i\right)$$

$$= \left(\int_{A_i \setminus B_i} \boldsymbol{w}, \int_{A_i \setminus B_i} \boldsymbol{x}, \int_{A_i \setminus B_i} r_i\right)$$

$$= \frac{1}{2}\left(\int_{A_i} \boldsymbol{w}, \int_{A_i} \boldsymbol{x}, \int_{A_i} r_i\right)$$

Furthermore, consider a vector measure defined by

$$\nu(B) := \left(\int_B \boldsymbol{w}, \int_B \boldsymbol{x}\right) \quad \text{for } B \subset T \setminus \bigcup_{i=1}^{k} A_i.$$

Then, similarly we have a measurable partition of $T \setminus \bigcup_{i=1}^{k} A_i$, say V and W, such that

$$(3.4) \qquad \nu(V) = \left(\int_V \boldsymbol{w},\ \int_V \boldsymbol{x}\right)$$

$$= \left(\int_W \boldsymbol{w},\ \int_W \boldsymbol{x}\right)$$

$$= \frac{1}{2}\left(\int_{T\setminus\bigcup_{i=1}^{k} A_i} \boldsymbol{w},\ \int_{T\setminus\bigcup_{i=1}^{k} A_i} \boldsymbol{x}\right)$$

Using these sets we define an assignment $\boldsymbol{y}: T \to \mathbf{R}_+^L$ by

$$\boldsymbol{y}(t) := \begin{cases} r_i + \boldsymbol{w}(t) & t \in B_i \text{ for } i = 1,\cdots,i_0 - 1 \\ r_i + \boldsymbol{x}(t) & t \in B_i \text{ for } i = i_0,\cdots,k \\ \boldsymbol{w}(t) & t \in V \cup \bigcup_{i=i_0}^{k} A_i \setminus B_i \\ \boldsymbol{x}(t) & t \in W \cup \bigcup_{i=1}^{i_0-1} A_i \setminus B_i \end{cases}$$

We will show that assignment $\boldsymbol{y}: T \to \mathbf{R}_+^L$ can be a counter proposal to assignment $\boldsymbol{x}: T \to \mathbf{R}_+^L$. First, sum up the amounts of commodities that all consumers have in assignment $\boldsymbol{y}: T \to \mathbf{R}_+^L$. Then, we have

$$\int_T \boldsymbol{y} = \Sigma_{i=1}^{i_0-1}\left(\int_{B_i}(r_i + \boldsymbol{w}) + \int_{A_i\setminus B_i} \boldsymbol{x}\right) + \int_V \boldsymbol{w} + \Sigma_{i=i_0}^{k}\left(\int_{B_i}(r_i + \boldsymbol{x}) + \int_{A_i\setminus B_i} \boldsymbol{w}\right) + \int_W \boldsymbol{x}$$

$$= \Sigma_{i=1}^{i_0-1}\left(\frac{1}{2}\int_{A_i} r_i + \frac{1}{2}\int_{A_i} \boldsymbol{w} + \frac{1}{2}\int_{A_i} \boldsymbol{x}\right) + \frac{1}{2}\int_{T\setminus\bigcup_{i=1}^{k} A_i} \boldsymbol{w}$$

$$+ \Sigma_{i=i_0}^{k}\left(\frac{1}{2}\int_{A_i} r_i + \frac{1}{2}\int_{A_i} \boldsymbol{x} + \frac{1}{2}\int_{A_i} \boldsymbol{w}\right) + \frac{1}{2}\int_{T\setminus\bigcup_{i=1}^{k} A_i} \boldsymbol{x} \quad \text{(by (3.3) and (3.4))}$$

$$= \frac{1}{2}\left(\Sigma_{i=1}^{k} \lambda(A_i) r_i + \int_T \boldsymbol{w} + \int_T \boldsymbol{x}\right)$$

$$= \frac{1}{2}\left(\Sigma_{i=1}^{k} \delta\alpha_i r_i + \int_T \boldsymbol{w} + \int_T \boldsymbol{x}\right) \quad \text{(by (3.2))}$$

$$= \int_T \boldsymbol{w} \qquad \text{(by (3.1))},$$

which implies that $\boldsymbol{y}: T \to \mathbf{R}_+^L$ is an allocation.

Furthermore, define

$$S_1 := \bigcup_{i=1}^{i_0-1} B_i,\ S_2 := \bigcup_{i=i_0}^{k} B_i,\ \text{and } S := S_1 \cup S_2.$$

Sum up the amounts of commodities that consumers in S have. Then, we have

$$\int_S \boldsymbol{y} = \Sigma_{i=1}^{i_0-1} \int_{B_i} (r_i + \boldsymbol{w}) + \Sigma_{i=i_0}^{k} \int_{B_i} (r_i + \boldsymbol{x})$$

$$= \Sigma_{i=1}^{k} \frac{1}{2} \int_{A_i} r_i \quad + \Sigma_{i=1}^{i_0-1} \int_{B_i} \boldsymbol{w} + \Sigma_{i=i_0}^{k} \int_{B_i} \boldsymbol{x} \qquad \text{(by (3.3))}$$

$$= \frac{1}{2} \Sigma_{i=1}^{k} \lambda(A_i) r_i + \int_{S_1} \boldsymbol{w} + \int_{S_2} \boldsymbol{x}$$

$$= \frac{1}{2} \Sigma_{i=1}^{k} \delta \alpha_i r_i + \int_{S_1} \boldsymbol{w} + \int_{S_2} \boldsymbol{x} \quad \text{(by (3.2))}$$

$$= \int_{S_1} \boldsymbol{w} + \int_{S_2} \boldsymbol{x} \qquad \text{(by (3.1))},$$

which is condition (i) of Definition 2.

When $t \in S$, $t \in B_i \subset A_i$ for some *i*. Therefore, for some *i*, $t \in G_w^{-1}(r_i)$ or $t \in G_x^{-1}(r_i)$, i.e., $\boldsymbol{y}(t)=r_i+\boldsymbol{w}(t) \succ_t \boldsymbol{x}(t)$ or $\boldsymbol{y}(t)=r_i+\boldsymbol{x}(t) \succ_t \boldsymbol{x}(t)$. In any case, we have that $\boldsymbol{y}(t) \succ_t \boldsymbol{x}(t)$, which is condition (ii) of Definition 2.

When $t \in T \setminus S$, $\boldsymbol{y}(t)$ equals $\boldsymbol{w}(t)$ or $\boldsymbol{x}(t)$. Consider two sets defined by

$$\{t \in T \mid \boldsymbol{w}(t) \succ_t \boldsymbol{w}(t)\} \text{ and } \{t \in T \mid \boldsymbol{w}(t) \succ_t \boldsymbol{x}(t)\}.$$

The first set has measure zero by irreflexivity of $\succ_t$. If the second set has positive measure, then the coalition consisting of consumers in the set rejects allocation $\boldsymbol{x}$ via the initial assignment $\boldsymbol{w}$. Therefore, since $\boldsymbol{x}$ is an allocation in the rejective core, the second set cannot have positive measure. Thus, $\boldsymbol{w}(t) \not\succ_t \boldsymbol{y}(t)$ for almost every *t* in $T \setminus S$, which is condition (iii) of Definition 2.

Hence, we have shown that coalition *S* rejects an allocation $\boldsymbol{x}$, which is a contradiction. □

In what follows, we will show that allocation $\boldsymbol{x}$ is a dividend equilibrium allocation. By virtue of Lemma, we can apply a separating hyperplane theorem, and there is a vector $p \in \mathbf{R}^L \setminus \{\mathbf{0}\}$ such that $p \cdot z \geq 0$ for any $z \in \Delta$. Define a measurable function $\boldsymbol{d} : T \to \mathbf{R}_+$ by

$$\boldsymbol{d}(t) = \max\{0, \; p \cdot \boldsymbol{x}(t) - p \cdot \boldsymbol{w}(t)\}$$

Then, immediately we have $p \cdot \boldsymbol{x}(t) \leq p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t)$ for each *t*, which is condition (i) of Definition 1.

Take any $t \in U$. Then, by definition of Δ, $\mathbf{Q}^L \cap (G_w(t) \cup G_x(t)) \subset \Delta$, so $p \cdot z \geq 0$ for any $z \in \mathbf{Q}^L \cap (G_w(t) \cup G_x(t))$. Therefore, by continuity of $\succ_t$, we can easily show that $p \cdot z \geq 0$ for all $z \in G_w(t) \cup G_x(t)$. If follows from the definition of $G_w(t)$ and $G_x(t)$ that z + $\boldsymbol{w}(t) \succ_t \boldsymbol{x}(t)$ or z + $\boldsymbol{x}(t) \succ_t \boldsymbol{x}(t)$ implies $p \cdot z \geq 0$. Take a commodity vector $y \in \mathbf{R}^L$ such that $y \succ_t \boldsymbol{x}(t)$. Then, $y - \boldsymbol{w}(t) \in G_w(t)$ and $y - \boldsymbol{x}(t) \in G_x(t)$, so we have $p \cdot (y - \boldsymbol{w}(t)) \geq 0$ and $p \cdot (y - \boldsymbol{x}(t)) \geq 0$. Therefore, by definition of $\boldsymbol{d}(t)$, we have

$$p \cdot y \geq \max\{p \cdot \boldsymbol{w}(t), p \cdot \boldsymbol{x}(t)\} = p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t).$$

Suppose that $p \cdot y = p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t)$. Then, by assumptions (A1) and (A3), we can change y so slightly that $y \succ_t \boldsymbol{x}(t)$ and $p \cdot y' < p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t)$. This contradicts the above inequality. Therefore, $p \cdot y > p \cdot \boldsymbol{w}(t) + \boldsymbol{d}(t)$. This is condition (i) of Definition 1 of the dividend equilibrium. This completes the proof of the main theorem.

4. Concluding remarks

The strong core can be defined by a weaker notion of "improving upon", which requires that none of consumers in an improving coalition are worse off and some of the consumers with positive measure are better off. The strong core is called the *b-core*, while the (weak) core is called the *a-core*. As was pointed out by Aumann-Drèze [4], the *b*-core is too small in economies with satiation, while the *a*-core is too large. In contrast, the rejective core proposed by Konovalov [10] is "exact" in that the rejective core is equivalent to the dividend equilibrium that is a most natural notion of equilibrium for economies with satiation.

Aumann-Drèze [4] showed that the Shapley value allocations correspond to the dividend equilibrium allocations by replication of a market with finite types of traders. In markets with a continuum of traders, however, they showed that the value allocations are inappropriate in order to capture the feature of the dividend equilibrium allocations. On the other hand, their comment on the core approach seems to suggest that the notion of core should be revised. We believe that the rejective core is a solution to characterize the dividend equilibrium by a game-theoretic concept.

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