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Abstract

A model of heterogeneous firms with multiple products and two production factors (labor and capital) is used to study how trade liberalization affects firms' choices through both product and factor markets. Trade liberalization is shown to always redistribute capital toward more efficient firms and always to improve an industry's total factor productivity. However, it may reduce capital prices and cause labor productivity to drop. Low efficiency firms are affected mainly by changes in the factor market, while high efficiency firms are affected mainly by changes in the product market. In response to trade liberalization, low efficiency firms always reduce their product scope, but high efficiency firms may expand their scope. The model demonstrates the importance of the interplay between product and factor markets.

Keywords: firm heterogeneity, trade liberalization, multiproduct, multifactor, firm structure, scale, scope, mergers and acquisitions

JEL Code: F12, F13, F15, L11, L25

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1 Introduction

Recent studies of international trade have focused on how trade liberalization improves overall efficiency by redistributing resources across heterogenous firms.[1] These studies have shown that international trade exerts its impacts through intensified competition in the factor market or in the product market. In his seminal paper, Melitz (2003) demonstrated how, by bidding up the wage rate, trade liberalization forces the least efficient firms to quit. The intensified competition in the product market, however, does not play any role, as a CES preference was used.[2] This is changed in Melitz and Ottaviano (2008), who used a linear demand function and showed that exit of the least efficient firms was caused by tougher competition in the product market. But in that model, the factor market was dormant because the wage rate was assumed to be fixed.

In this research, we studied the *joint* impacts of intensified competition in both the product and factor markets. New results and new forces emerge when both markets are at work. In particular, we found that intensified competition in the product market may reduce the factor price, and substitution between factors may cause labor productivity to drop. Firms respond to changes in the product and factor markets in a non-uniform way: less efficient firms are affected mainly by changes in the factor market, while more efficient firms are affected mainly by changes in the product market. In response to trade liberalization, low efficiency firms always shrink the scope of their products, but high efficiency firms may expand it.

In our model, heterogeneous firms produce differentiated products using two inputs, labor and capital. While labor is readily available at an exogenous wage rate, industry-specific capital is in fixed supply and the capital price is endogenized. Demand is linear *a la* Melitz and Ottaviano (2008), so competition in the product market also affects firms' choices. Firms play a two-stage game. In the first stage, they trade capital in a perfectly competitive acquisition market before allocating capital to the production of multiple products. Each firm chooses the number of products (i.e., its scope) and the amount of capital applied to each product (i.e., the scale). In the second stage, all products compete monopolistically in the product market. Each firm's structure (i.e., scope and scale) and the distribution of inputs and outputs across firms will be determined by the tradeoff between the product and acquisition markets.

We start with two identical countries originally in autarky due to, say, prohibitively high trade costs. A shock then completely wipes out all the trade costs. This trade liberalization affects the acquisition market both directly, and indirectly through the product market. The exporting opportunity induces firms to produce more, pushing up the aggregate demand for capital. This is the direct effect. The increased competition in each country's product market reduces demand for each product, depressing the aggregate demand for capital. This is the indirect effect. We have shown that the relative strength of the two effects depends crucially on the substitutability between products. When the products are close substitutes, the reduction in product demand can be so great that the indirect effect dominates, in which case the capital price will drop as a result of trade liberalization.

[1] See the survey by Helpman (2006).

[2] "In the isoelastic case, the demand level has no effect on the cutoff because any shift in demand is offset by entry, in contrast to the case of linear demand." (Helpman, 2006, p.603)

For individual firms, trade liberalization brings exporting opportunities as well as the challenges of intensified product competition. Although all firms face the same opportunities and challenges, they respond differently. Low efficiency firms can make only small profits from each product; having exporting opportunity will not add much. They are affected mainly by the intensified competition, so they sell capital and reduce their scope and scale. High efficiency firms, in contrast, are more capable of making use of the exporting opportunity; they buy capital and expand their scope and/or scale. Capital moves from low to high efficiency firms.

Along with capital, labor will also flow toward more efficient firms. The redistribution improves industry total factor productivity as measured by the joint contribution of labor and capital. For labor productivity, however, there is an extra effect due to the substitution between the two inputs. Since both inputs are optimally chosen by each firm and all firms face the same wage rate and capital price, they all choose the same labor/capital ratio. The ratio will be affected by trade liberalization, which changes the capital price but not the wage rate. When capital becomes more expensive, firms will substitute labor for capital, more labor will be employed, and labor productivity at the firm level will drop. Such a substitution effect (within each firm) is distinct from the resource reallocation effect (across firms), so even when firm-level labor productivity drops, industry-level labor productivity may still improve.

This analysis clarifies the fundamental tradeoff that shapes firms' choices. In models where the impact of trade liberalization is through the factor market, the tradeoff is between product profits and various fixed costs (Melitz, 2003). In models where the impact is through increased competition in the product market, the tradeoff is between product profits and variable costs (Melitz and Ottaviano, 2008). In both cases, trade liberalization has a uniform impact on firms (i.e., firms' product market profits all move in the same direction), and all adjustments take place at the margin: The marginal exporters expand, reducing the product demand or pushing up the factor price, which in turn forces the marginal producers to quit.[3] In our model, both product and factor markets are at work, and the fundamental tradeoff faced by heterogeneous firms is better viewed as the one between products and resources.[4] Firms differ in their efficiency in turning resources into products. While a resource is equally costly to all, it generates different revenues in the product market depending on a firm's efficiency. Trade liberalization brings shocks to both markets and changes the tradeoff between products and resources. Further, the change depends on a firm's efficiency, leading to non-uniform responses to trade liberalization.

While the study of heterogeneous firms started with single product firms, attention has recently turned to multiproduct firms. According to Bernard, Jensen and Schott (2005) and Bernard, Redding and Schott

[3] Given the constant marginal cost of production assumed in all the models, domestic sales and exports are independent and additive activities governed by different fixed or variable costs. Firms self select into different activities: those with the lowest efficiencies exit; those with moderate efficiencies produce only for the domestic market; and those with the highest efficiencies produce for both their domestic market and the export. When trade costs are lowered, exporting profits increase, so more firms export. This will intensify product competition or bid up the factor price. Because the marginal producers do not export, they bear the burden of intensified competition in the product or factor market without reaping the benefits of increased exporting profits, so they are forced to quit.

[4] Such a view is not in conflict with the two approaches taken in previous studies, as fixed costs and variable costs are all costs of resources needed in production. Between the two approaches, the one by Melitz and Ottaviano (2008) is closer to ours because of the linearity of demand and the absence of fixed costs.

(2008), only 41 percent of U.S. manufacturing firms produce more than a single product, but these firms account for 91 percent of U.S. manufacturing output and 95 percent of U.S. exports. Theoretical modelling of multiproduct firms extended heterogeneity to products within a firm. The basic force remains the same: trade liberalization eliminates the marginal products, so all firms reduce scope uniformly (Bernard, Redding and Schott, 2009; Eckel and Neary, 2009; Mayer, Melitz and Ottaviano, 2009).[5] In our model, products are homogeneous within a firm, and the scope is proportional to each variety's profits. We have shown that a firm's choice of product scope in response to trade liberalization then depends on its efficiency.

Empirical studies of multiproduct firms have turned up mixed results about product scope. Although Bernard, Redding and Schott's (2009) theory predicts uniform reduction in scope, their own (2008) empirical investigation of U.S. manufacturing firms between 1987 and 1997 had different findings: High efficiency firms increased scope while low efficiency firms reduced scope, which is consistent with our theory. Baldwin and Gu (2006) found that both exporters and non-exporters in Canada reduced product diversification from 1973 to 1997, but the reduction does not seem to have been related to tariff cuts, even though Canada underwent two rounds of trade liberalization during that period. Iacovone and Javorcik (2008) found that Mexican firms invested in developing new products for export as a response to trade liberalization. Goldberg et al (2009) found that Indian firms did not reduce their product scopes during the 1989-2003 period, which included profound trade and other reforms.[6] Our model shows the possibility of and the conditions for expansions in scope after trade liberalization.

Capital trading was modeled explicitly in this study.[7] Such trading enables firms to add or delete production facilities, and thus can be regarded as a partial merger or acquisition. Mergers and acquisitions (M&As) constitute a major method of industrial restructuring (UNCTAD, 2000) and are the quickest and least costly way to respond to external shocks such as trade liberalization. Waves of mergers have been documented as a consequence of trade liberalization and other such shocks (Mitchell and Mulherin, 1996). Breinlich (2008) found that the Canada-United States Free Trade Agreement of 1989 increased domestic Canadian M&A activity by over 70%. Using data on Swedish firms for the period 1980-1996, Greenaway, Gullstrand and Kneller (2008) have shown that intensified international competition induced M&As. Maksimovic and Phillips (2001) contended that "industry shocks alter the value of the assets and create incentives for transfers to more productive uses", and they showed that productive assets tend to move from less efficient to more efficient firms when an industry undergoes a positive demand shock. Our analysis has now provided a theoretical explanation for these empirical findings.

We will first present the model and analyze the autarky equilibrium. The equilibrium after trade liberalization will then be found and compared with the autarky case. All the proofs are collected in Appendix

[5] Baldwin and Gu (2006) reached the same reduced scope conclusion assuming homogeneous products within each firm. Within-firm homogeneity has also been assumed by Nocke and Yeaple (2006), who further assumed an exogenous tradeoff between product scope and firm-level productivity. They predicted that trade liberalization would induce high efficiency firms to shrink their product lines and low efficiency firms to expand them.

[6] Goldberg et al (2009) attributed the discrepancy between their findings and the predictions of prevailing theories to regulations in India, which prevented the optimal allocation of resources.

[7] Spearot (2008) also allowed firms to trade capital, but assumed single-product firms and capital transactions in single unit. His conclusion that only moderately efficient firms acquire capital is thus very different from our predictions.

I. Appendix II analyzes unilateral trade liberalization in which a country faces competition from imports of differentiated products but is not allowed to export this industry's products.

2 Autarky

2.1 Production and inputs

Consider a world with two identical countries, between which there initially is no trade due to prohibitively high trade costs. In each country, consumption consists of a numeraire good and differentiated products produced by a continuum of firms in an industry. From a uniform distribution on $[0, 1]$, each firm draws its efficiency, φ, which is the only parameter that distinguishes firms.

A firm can produce multiple products by incurring a management cost, mv^2, where $m > 0$ and v is the number of varieties that the firm produces. Production of the numeraire good requires two units of labor for one unit of output. Labor supply to this industry is perfectly elastic, so the wage rate, w, is equal to $\frac{1}{2}$. Producing differentiated products requires both labor and capital as inputs. The production function of a single variety is

$$q = (\varphi x l)^{\frac{1}{2}},$$

in which q is the output of the variety, x is its capital input and l is its labor input. In the short run, a plant's capital is fixed, and thus its variable cost is $\min_l(wl)$ subject to $q = (\varphi x l)^{\frac{1}{2}}$, which gives rise to a labor cost of $\frac{w}{\varphi x}q^2$, or

$$c(q|\varphi, x) = \frac{q^2}{2\varphi x}.$$

Each firm is endowed with one unit of capital. With this endowment and knowing their efficiency levels, firms play a two-stage game. In the first stage (the acquisition stage), firms buy and sell capital in a perfectly competitive acquisition market. If a firm sells all of its capital, it becomes inactive in the product market. An inactive firm still keeps its φ and may choose to re-enter the product market later by buying capital in the acquisition market. Entry or exit incurs no extra cost. After the capital trading, every active firm chooses its number of varieties and the allocation of capital to each plant. A firm's *structure* therefore consists of its product *scope* (the number of varieties) and plant-level capital *scale* (the amount of capital used in each plant). For expositional convenience, scale and scope will be treated as continuous variables. Capital is assumed to be perfectly divisible, and there is no minimum capital requirement for running a plant. In the second stage (the production stage), production is carried out and all varieties compete in a monopolistically competitive product market.[8]

[8]We assume away any cannibalization producing one variety has on the profitability of other varieties. Schwartz and Thompson (1986) and also Baye, Crocker and Ju (1996) have shown that, in an effort to gain market share from competing companies or forestall entry, many companies instruct their divisions to act as independent firms despite cannibalization. If this assumption is relaxed to accommodate cannibalization, our major conclusions still hold. Feenstra and Ma (2008) have demonstrated that

In this model, capital represents industry-specific physical assets that are needed in production. Unlike financial assets, after the initial trading round, capital must be acquired through acquisition or merger rather than from financial institutions. This implies that the supply of capital is perfectly inelastic at the industry level. This is justified if the total amount of physical assets cannot quickly be increased, but the results still hold even if new capital can be generated in response to an increase in the capital price. Assuming a perfectly competitive acquisition market implies that capital is homogeneous and acquisitions can be partial. That is, instead of acquiring a stand-alone target firm, the acquirer buys some productive assets from other firms and uses them along with its own assets. Jovanovic and Rousseua (2002) have argued that transactions in the used-capital market work just like those in a M&A market. Maksimovic and Phillips (2001) report that about half of U.S. M&A transactions are partial acquisitions or divestitures by multi-product conglomerates.

Consider first the product market, then the acquisition market (also referred to as the factor market), and finally the industry equilibrium.

2.2 The product market

Assume L identical consumers in each country. Each consumer has a quasi-linear preference (à la Melitz and Ottaviano, 2008) for the numeraire good and all varieties in the industry:

$$U = Q_0 + \alpha \int_{i\in\Omega} q_i di - \frac{1}{2}\beta \left(\int_{i\in\Omega} q_i di \right)^2 - \frac{1}{2}\gamma \int_{i\in\Omega} q_i^2 di,$$

where $\alpha, \beta, \gamma > 0$, Q_0 is the consumption of the numeraire good, Ω is the set of all varieties, and q_i is the consumption of variety i. A consumer maximizes his utility subject to a budget constraint. As a result, the market demand for variety i from all L consumers is $p_i = \alpha - \frac{\beta}{L} \int_{j\in\Omega} q_j dj - \frac{\gamma}{L} q_i$. For a given γ, when β is larger, other varieties' outputs reduce the demand for variety i by a larger amount, meaning that the substitution between varieties is stronger. Therefore, β measures substitutability between varieties: larger β means stronger substitution.

In monopolistic competition, the seller of variety i regards itself as a monopolist, and the competition between products is captured only in the vertical intercept of the demand function. In equilibrium, the demand function for variety i is:

$$p_i = A - bq_i, \text{ where } A = \frac{\alpha\gamma + \beta P}{\beta M + \gamma} \text{ and } b = \frac{\gamma}{L}. \tag{1}$$

In this demand function, p_i is the price of variety i, M is the measure of Ω, and $P = \int_{i\in\Omega} p_i di$ is the aggregate price of all varieties. The slope b is exogenous, but the intercept A is endogenous, depending

cannibalization would not change the major findings of Melitz (2003).

on both the degree of product substitution (captured in β) and the degree of product market competition (captured in the endogenous P and M).

Each firm takes A as given when choosing its output. If a firm with efficiency φ has amount x_i of capital in its plant to produce variety i, it chooses output q_i to maximize its profit from this variety:

$$\max_{q_i \geq 0} \ \pi_i \equiv (A - bq_i)q_i - \frac{q_i^2}{2\varphi x_i}. \tag{2}$$

The resulting quantity, price and profit for this variety are, respectively,

$$q_i(x_i) = \frac{\varphi x_i A}{2\varphi b x_i + 1}, \ p_i(x_i) = \frac{(\varphi b x_i + 1)A}{2\varphi b x_i + 1}, \text{ and } \pi_i(x_i) = \frac{\varphi x_i A^2}{2(2\varphi b x_i + 1)}. \tag{3}$$

Greater demand (i.e., a larger A) leads to more output, a higher price and more profit for each variety. Since a variety's profit, $\pi_i(x_i)$, is increasing and concave in x_i, a firm will always allocate its total capital among its varieties equally. Consequently, the subscript i can be dropped in (3).

2.3 The acquisition market

Let R be the market price of capital. If a firm chooses scope v and scale x, its capital cost is $(vx - 1)R$, where $vx - 1$ is the firm's net demand for capital, which can be negative (meaning that the firm is selling capital). The firm's optimization problem in the acquisition market is

$$\max_{x \geq 0, \, v \geq 0} \Pi(v, x) \equiv v\pi(x) - (vx - 1)R - mv^2 = v\tilde{\pi}(x) + R - mv^2,$$

where $\tilde{\pi}(x) \equiv \pi(x) - xR$ is the profit of each single plant, taking into account the capital cost but not the management cost. Given the expression for $\pi(x)$, a plant's profit is

$$\tilde{\pi}(x) = \frac{\varphi x A^2}{2(2\varphi b x + 1)} - xR.$$

It is as if the firm first sells its endowment of unit capital and then chooses how much capital to buy for each of its plants. Since there is no transaction cost, selling and buying capital is fully reversible.

Given the above decomposition of the profit function, a firm's optimization problem can be solved in two steps: The optimal scale is $x^* = \arg\max_x \tilde{\pi}(x)$, which is independent of the choice of v, and the optimal scope is then $v^* = \arg\max_v \Pi(v, x^*)$.

2.3.1 Plant-level capital scale

Define

$$y \equiv \frac{A}{\sqrt{2R}}. \tag{4}$$

The variable y reflects the connection between the product and factor markets. $\pi(x)$ is proportional to $\frac{A^2}{2}$, so $y^2 = \frac{A^2}{2R}$ reflects the value of capital (in the product market) relative to the cost of capital (in the factor market).

The first-order condition $\frac{\partial \tilde{\pi}}{\partial x} = 0$ leads to the optimal scale for each plant (the asterisk is dropped for simplicity of notation):

$$x = \begin{cases} 0 & \text{for } \varphi \leq \varphi^0, \\ \frac{y\sqrt{\varphi}-1}{2\varphi b} & \text{for } \varphi > \varphi^0, \end{cases} \tag{5}$$

where $\varphi^0 \equiv \frac{1}{y^2}$. The second-order condition is always satisfied. It will later become clear that $y > 1$ in equilibrium and therefore $\varphi^0 < 1$. Expression (5) says that very inefficient firms will not operate in the product market. When φ is small, a plant's profit from the product market will also be small, and the firm will earn a better payoff by selling all its capital and discontinuing production.[9] Note that the cutoff point for exit, φ^0, depends solely on y. When the product market is more profitable relative to the capital price, even a low efficiency firm will choose to operate, so more firms will be active in the product market. For an active plant, the scale increases with y: When the product market becomes more profitable relative to the capital price, each plant will operate on a larger scale. This is because the marginal benefit of capital is proportional to $\frac{A^2}{2}$, while the marginal cost of acquiring capital is R.

Interestingly, more efficient firms do not necessarily have larger scale:

$$\frac{\partial x}{\partial \varphi} = \frac{2 - y\sqrt{\varphi}}{4\varphi^2 b} \begin{cases} > 0 & \text{for} \quad \varphi \in (\varphi^0, 4\varphi^0), \\ < 0 & \text{for} \quad \varphi > 4\varphi^0. \end{cases}$$

Plant scale increases with a firm's efficiency when the efficiency is low, but if $4\varphi^0 < 1$ (or equivalently, $y > 2$), the scale decreases with efficiency when the efficiency is sufficiently high, generating an inverse U-shape. The reason is that when φ is small, each variety is sold at a high price, where demand is highly elastic. Increasing x will bring a large benefit. But when φ is large, each variety is already sold at a low

[9] In many formulations, least efficient firms exit due to fixed production costs (e.g., Melitz, 2003). Without fixed costs, exit happens with a linear demand function (Melitz and Ottaviano, 2008), as the constant marginal costs that some firms draw turn out to be larger than the intercept of the demand. In our model, demand is linear as in Melitz and Ottaviano (2008) and there is no fixed production cost. A firm's marginal cost can be arbitrary small (when its output is close to zero) even if it is very inefficient. It will always generate some profit, however small, in the product market. A firm exits only because it can earn a better payoff by selling its capital in the factor market. The tradeoff between product and factor markets is the driving force here.

price, where the demand is barely elastic. Increasing x will not bring much benefit. Note that the inverse U relationship does not exist with CES preferences.

2.3.2 Product scope

Given the optimal scale, a firm's profit from each variety (after paying for capital) is $\tilde{\pi} = \frac{R(y\sqrt{\varphi}-1)^2}{2\varphi b}$ for $\varphi > \varphi^0$, which is strictly increasing in φ. Given $\tilde{\pi}$, the first-order condition for optimal scope $\frac{\partial \Pi}{\partial v} = \tilde{\pi} - 2mv = 0$ leads to $v = \frac{\tilde{\pi}}{2m}$ or

$$v = \begin{cases} 0 & \text{for } \varphi \leq \varphi^0, \\ \frac{R(y\sqrt{\varphi}-1)^2}{4\varphi bm} & \text{for } \varphi > \varphi^0. \end{cases} \tag{6}$$

The second-order condition is always satisfied. Notice that

$$\frac{\partial v}{\partial \varphi} > 0 \text{ for all } \varphi > \varphi^0.$$

Thus, more-efficient firms maintain larger scopes. The marginal benefit from adding a variety is $\tilde{\pi}$, while the marginal cost is $2vm$. Since the marginal benefit increases with φ, the optimal scope should be larger when φ is higher.

For $\varphi > \varphi^0$, the scope can be rewritten as

$$v = \frac{\left(A\sqrt{\varphi} - \sqrt{2R}\right)^2}{8\varphi bm}. \tag{7}$$

The numerator relates a firm's product scope to the product and acquisition markets, or more precisely to the value of capital in the product market vis-a-vis the cost of capital in the acquisition market. The value of capital is positively related to φ, while the cost of capital is independent of φ. Therefore, if there is any change to the two markets due to, say, trade liberalization, firms will respond differently depending on their efficiencies even though they face the same changes in A and R. In particular, high efficiency firms are affected mainly by the value of capital and thus by changes in the product market, while low efficiency firms are affected mainly by the cost of capital and thus by changes in the acquisition market.

2.3.3 Firm-level capital

A firm's capital is xv. For an active firm,

$$\frac{\partial xv}{\partial \varphi} = \frac{R(y\sqrt{\varphi}-1)^2}{16\varphi^3 b^2 m}(4 - y\sqrt{\varphi}) \begin{cases} > 0 & \text{for } \varphi < 16\varphi^0, \\ < 0 & \text{for } \varphi > 16\varphi^0. \end{cases}$$

Because scope increases with φ while scale is inverse U-shaped, firm capital is also inverse U-shaped with efficiency. Note that the turning point for firm capital (at $16\varphi^0$) is larger than that for plant scale (at $4\varphi^0$). In any case, the important message is that more efficient firms do not necessarily require more capital. Also note that the envelope theorem indicates that a firm's profit always increases with φ even if its capital scale does not.

2.3.4 Outputs

With the optimal choice of x, a plant's output is

$$q = \frac{A}{2b}\left(1 - \frac{1}{y\sqrt{\varphi}}\right),$$

while a firm's total output is

$$qv = \frac{A^3}{16b^2 m}\left(1 - \frac{1}{y\sqrt{\varphi}}\right)^3.$$

Both outputs increase with efficiency.

2.3.5 Productivity

Three measures of productivity will be calculated: labor productivity, total factor productivity (or TFP) of labor and capital, and overall productivity that takes into consideration management costs.

Labor productivity is the ratio between output and labor input. At the plant level, labor input is $l = \frac{q^2}{\varphi x}$. Given the optimal choices of x and q, the plant-level labor productivity is:

$$\lambda^\varphi \equiv \frac{q}{l} = \sqrt{\frac{\varphi}{2R}}.$$

Since all plants of a firm are identical, the firm-level labor productivity is the same. Capital price R matters for labor productivity because it affects the relative price between labor and capital and thus the optimal combination of the two inputs in production.

In this model, both labor and capital are used in production, so we need a measure that relates outputs to the two inputs. Given the production function at the plant level, $q = \sqrt{\varphi x l}$, define total factor productivity as

$$\omega^{\varphi} \equiv \frac{q}{\sqrt{xl}} = \sqrt{\varphi},$$

which is the same at both the plant and firm levels. Unlike labor productivity, TFP does not depend on market conditions such as input prices, substitutability between products, or degree of competition. To the extent that firm-level productivity should reflect technologies only and should be independent of changes in market conditions, TFP is a more useful measure than labor productivity. Although fixed at the firm level, TFP at the industry level will change as labor and capital is redistributed among firms, thus providing a measure of the impact of trade liberalization.

The two productivity measures defined above ignore the management costs of maintaining multiple products.[10] To take that into consideration, define overall productivity as the reciprocal of average cost. Plant-level cost consists of labor and capital costs, and the overall productivity is

$$\mu^{p} \equiv \frac{q}{wl + xR} = \sqrt{\frac{\varphi}{2R}},$$

which equals the plant-level labor productivity.[11] At the firm level, management cost is included and the overall productivity is

$$\mu^{\varphi} \equiv \frac{vq}{vwl + vxR + mv^2} = \frac{4}{A\left(1 + \frac{3}{y\sqrt{\varphi}}\right)}.$$

The overall productivity increases with φ at both the plant and firm levels, and the difference between the two reflects the impact of management cost.

To summarize:

Proposition 1. *Given A and R, capital scale at both the plant and firm levels increases with a firm's efficiency φ when φ is small, but may decrease with φ when φ is large. Scope, output and productivity all increase with φ.*

Since a firm's scope and each plant's output are both increasing in φ, the two are positively correlated. That is, intensive margins (i.e., each variety's output) and extensive margins (i.e., the number of varieties) are positively correlated, confirming what Bernard, Redding and Schott (2009) have found.

[10]We view the management cost as a labor cost, but this labor is skilled labor, unlike the (unskilled) labor that is an input to production. More specifically, v^2 can be viewed as the physical units of skilled labor needed to manage v varieties, and m is the exogenous wage rate for skilled labor. Throughout this discussion, labor refers to unskilled labor.

[11]This is not an coincidence. Given the Cobb-Douglas production function, the expenditure on the two inputs will be equalized when labor and capital are chosen optimally. Then $wl + xR = 2wl = l$ given that $w = \frac{1}{2}$.

2.3.6 Capital price

Now to determine the equilibrium capital price R for a given A. Market clearing in the acquisition market requires total capital supply to equal capital demand at the industry level. Since each firm is initially endowed with one unit of capital, the total capital supply in the industry is $\int_0^1 1 d\varphi = 1$. The total capital demand is

$$K \equiv \int_0^1 xv d\varphi = \frac{R}{8b^2m}\rho, \text{ where } \rho = 2y^3 + 3y^2(1 - 2\ln y) - 6y + 1.$$

Given A, it can be proved (see Appendix I) that there exists a unique equilibrium capital price R satisfying the equilibrium condition $K(R) = 1$, from which R can be derived as a function of y:

$$R(y) = \frac{8b^2m}{\rho}. \tag{8}$$

Note that this expression is not a reduced form solution for R, as y is defined on R.

2.4 Industry equilibrium

The measure of varieties is

$$M \equiv \int_{\varphi^0}^1 v d\varphi = \frac{R}{4bm}\psi, \text{ where } \psi = y^2 - 4y + 3 + 2\ln y. \tag{9}$$

Given the choice of x, each single plant's price and quantity are:

$$p = \frac{A}{2}\left(1 + \frac{1}{y\sqrt{\varphi}}\right) \text{ and } q = \frac{A}{2b}\left(1 - \frac{1}{y\sqrt{\varphi}}\right)$$

for all $\varphi > \varphi^0$. Therefore, the aggregate price is

$$P \equiv \int_{\varphi^0}^1 vp d\varphi = \frac{AR}{8bm}\frac{\phi}{y}, \text{ where } \phi = y^3 - 2y^2 - 2 + 3y - 2y\ln y. \tag{10}$$

By the definition of A in (1): $A = \frac{\alpha\gamma+\beta P}{\beta M+\gamma}$, and using (9) and (10) we have

$$A = \frac{\alpha}{1 + \frac{\beta}{L}\frac{\eta}{y\rho}}, \tag{11}$$

where $\eta = y^3 - 6y^2 + 2 + 3y + 6y \ln y$. The three equilibrium values A, R and y are jointly determined by equations (4), (8) and (11). Use (8) and (11) in (4) to yield the following equation expressed in terms of y only:

$$Z(y) = 0, \quad \text{where } Z(y) \equiv \frac{\rho^3}{\left(y\rho + \frac{\beta}{L}\eta\right)^2} - \frac{16b^2m}{\alpha^2}. \tag{12}$$

It can be shown that $Z'(y) > 0$, $Z(1) < 0$, and $Z(y) > 0$ when y is sufficiently large. Therefore, there exists a unique $y > 1$ satisfying (12). Once y is determined, R is determined from (8) and A is determined from (4) or (11). As a result,

Proposition 2. *A unique equilibrium A and R exist in autarky.*

The industry-level productivity can now be calculated. The industry's total output is

$$Q \equiv \int_{\varphi^0}^{1} vqd\varphi = \frac{AR}{8b^2m}\frac{\eta}{y},$$

and the industry's total labor input is

$$L_b \equiv \int_{\varphi^0}^{1} v\frac{q^2}{\varphi x}d\varphi = \frac{A^2R}{8b^2m}\frac{\rho}{y^2}.$$

So the industry's labor productivity is

$$\lambda \equiv \frac{Q}{L_b} = \frac{y\eta}{A\rho},$$

and industry TFP is

$$\omega \equiv \frac{Q}{\sqrt{KL_b}} = \frac{y\eta}{\rho^{\frac{3}{2}}},$$

where $K = 1$ is the industry's capital input. Overall productivity at the industry level is

$$\mu \equiv \frac{Q}{R + \int_{\varphi^0}^{1}(vwl)d\varphi + \int_{\varphi^0}^{1} mv^2 d\varphi} = \frac{4y}{A}\frac{\eta}{\theta}, \text{ where } \theta = y^4 + 12y^2(1 - \ln y) - 16y + 3.$$

3 Trade Liberalization: The Impacts of Imports and Exports

Now suppose that trade liberalization reduces all trade costs (fixed and variable) to zero.[12] Such a trade liberalization gives producers in each country both the opportunity to export and the challenge of intensified competition. We are interested in how firms respond to trade liberalization by adjusting their capital structure, and how productivity changes as a result of resource redistribution. Cross-border M&A is excluded.[13]

3.1 Equilibrium after trade liberalization

Since the two countries are symmetrical, there must be a symmetric equilibrium. Consider one of the countries and define

$$y_t \equiv \frac{A_t}{\sqrt{2R_t}} \text{ and } \varphi_t^0 \equiv \frac{1}{y_t^2},$$

where the subscript t stands for trade. Variables in autarchy are denoted by subscript a. In what follows, we will use f_i to denote $f(y_i)$ for any given function $f(y)$, where $i = a, t$.

Given A_t in each country, a firm chooses the quantity of each variety, q_t, which is allocated equally to the domestic and export markets to maximize the variety's total profit from the two markets:

$$\max_{q_t \geq 0} \pi_t \equiv 2\left(A_t - b\frac{q_t}{2}\right)\frac{q_t}{2} - \frac{q_t^2}{2\varphi x_t}. \tag{13}$$

The optimal output, price and profit are, respectively,

$$q_t = \frac{\varphi x_t A_t}{\varphi b x_t + 1}, \ \ p_t = \frac{(\varphi b x_t + 2)A_t}{2(\varphi b x_t + 1)}, \ \text{ and } \ \pi_t = \frac{\varphi x_t A_t^2}{2(\varphi b x_t + 1)}.$$

A firm chooses its optimal scale for each plant to maximize the plant-level profit $\tilde{\pi}_t = \pi_t - x_t R_t$. The solution is

$$x_t = \begin{cases} 0, & \text{if } \varphi \leq \varphi_t^0, \\ \frac{y_t\sqrt{\varphi}-1}{\varphi b}, & \text{if } \varphi > \varphi_t^0. \end{cases} \tag{14}$$

The firm chooses its scope, v_t, to maximize its total profit $v_t\tilde{\pi}_t(x_t) - mv_t^2 + R_t$, yielding the following

[12] If trade costs are positive but small enough to allow for trade, the qualitative results will not change.

[13] Because the two countries are symmetrical, cross-border trading of capital will not happen even if it is allowed. Cross-border M&A will matter if the two countries are asymmetric or if the trading in the product market is one directional.

optimal scope

$$v_t = \begin{cases} 0, & \text{if } \varphi \leq \varphi_t^0, \\ \frac{R_t\left(y_t\sqrt{\varphi}-1\right)^2}{2\varphi bm}, & \text{if } \varphi > \varphi_t^0. \end{cases} \tag{15}$$

The aggregate demand for capital is $K_t \equiv \int_{\varphi_t^0}^1 x_t v_t d\varphi = \frac{R_t}{2b^2m}\rho_t$. Market clearing in the acquisition market, $K_t = 1$, leads to

$$R_t(y_t) = \frac{2b^2m}{\rho_t}. \tag{16}$$

The aggregate variety produced in a country is $M_t \equiv \int_{\varphi_t^0}^1 v_t d\varphi = \frac{R_t\psi_t}{2bm}$, while the aggregate variety consumed in each country is $2M_t$. The aggregate price is $P_t \equiv 2\int_{\varphi_t^0}^1 v_t p_t d\varphi = \frac{A_t R_t \phi_t}{2bmy_t}$. Plug P_t, M_t and R_t into $A_t = \frac{\alpha\gamma+\beta P_t}{2\beta M_t+\gamma}$ to yield

$$A_t(y_t) = \frac{\alpha}{1+\frac{\beta}{L}\frac{\eta_t}{y_t\rho_t}}. \tag{17}$$

Equations (16) and (17) and the definition of y_t jointly determine the three unknowns A_t, R_t and y_t as the equilibrium after trade liberalization.

In fact, instead of going through these steps, there is an alternative and much simpler way to obtain the equilibrium. Note that the plant's maximization problem (13) can be rewritten as

$$\max_{q_t} \pi_t = \left(A_t - \frac{b}{2}q_t\right) q_t - \frac{q_t^2}{2\varphi x_t}.$$

This problem is identical to that in the autarky case, (2), except that the demand slope b is replaced by $\frac{b}{2}$. In other words, if the equilibrium solutions are $p(A, \varphi, x, b)$, $q(A, \varphi, x, b)$ and $\pi(A, \varphi, x, b)$ under autarky, then after trade liberalization the solutions will be $p_t = p(A, \varphi, x, \frac{b}{2})$, $q_t = q(A, \varphi, x, \frac{b}{2})$ and $\pi_t = \pi(A, \varphi, x, \frac{b}{2})$. It is straightforward to reach the following conclusion: if $y_a = y(b)$, $A_a = A(b)$ and $R_a = R(b)$ are the equilibria in autarky, then the trade equilibria are $y_t = y\left(\frac{b}{2}\right)$, $A_t = A\left(\frac{b}{2}\right)$, and $R_t = R\left(\frac{b}{2}\right)$. This connection between the equilibria before and after trade liberalization proves very convenient in studying the impact of trade liberalization.

These two approaches are not only mathematically equivalent, but also economically isomorphic. Without any trade cost, a firm views the two product markets (domestic and export) as identical. At the same time, a consumer is assumed not to treat domestic varieties differently from imported varieties. Compared to autarky, therefore, a firm's demand under trade liberalization is doubled (population increases from L to $2L$), which is captured by the change of the slope from b to $\frac{b}{2}$ (because $b = \frac{\gamma}{L}$). From the firm's point of view, for given A and R, the change in the slope of the demand curve is the only difference between

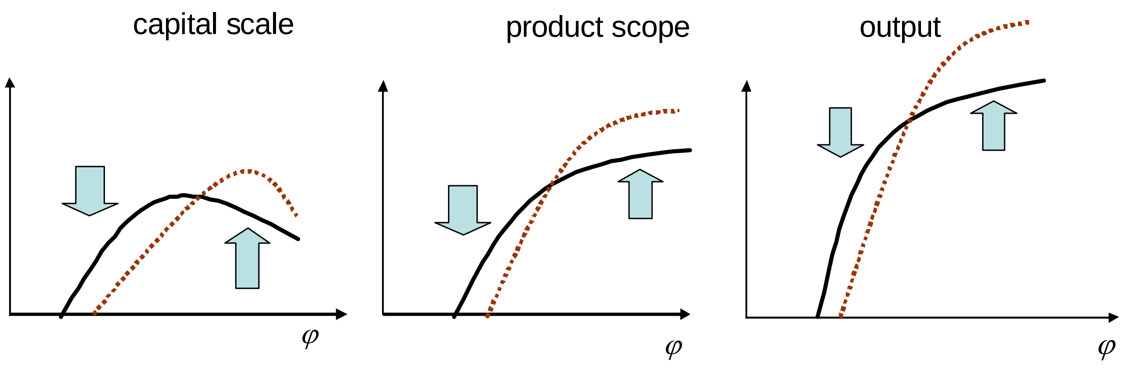

Figure 1: Impacts of Trade Liberalization

autarky and liberalized trade, so its optimal scale and scope are doubled after trade. Of course, A, R and consequently y will be different from those in autarky.

3.2 Equilibrium comparison

A proof Appendix I shows that $y_t < y_a$. The following proposition summarizes the effects of trade liberalization.

Proposition 3. *After trade liberalization, the following changes happen:*

(1) Product market: the demand in each country drops: $A_t \in (\frac{A_a}{2}, A_a)$*;*

(2) Capital redistribution: capital moves from low to high efficiency firms; the least efficient firms exit: $\varphi_t^0 > \varphi_a^0$*;*

(3) Firm structure: low efficiency firms reduce scope and scale, while high efficiency firms expand scope and/or scale;

(4) Aggregate scope: the total number of varieties consumed in each country increases: $2M_t > M_a$*;*

(5) Outputs: low efficiency firms reduce plant and firm outputs, while high efficiency firms increase these outputs; industry output increases.

Proof. See Appendix I.

Some of the changes are illustrated in Figure 1, where the solid lines represent autarky and the dotted lines represent the situation after liberalization. After trade liberalization, there is more competition in the product market. More products are sold in each country, so the demand for each variety, represented by A, drops. Later it will be shown that the capital price R may rise or drop. In either case, due to the increased competition, the product market in each country becomes less profitable relative to the capital price, i.e., y drops.

Plant capital scale is affected by trade liberalization through two effects. First, because product markets become less profitable relative to capital prices, each plant will reduce its scale (recall that scale increases with y). Second, because each variety is sold in both countries, plant scale will be doubled. Low efficiency firms have small scales; doubling the scale will not add much. So the first effect dominates for low efficiency firms, and they reduce their scales after trade liberalization. Firms with very low efficiency will sell their capital and exit. For high efficiency firms, the second effect dominates, so they expand their scale. Depending on the parameters, it is possible that all firms reduce scale, but if a firm expands its scale, all firms with higher efficiency will also expand their scale.

Product scope is proportional to each plant's profit, and it is affected by trade liberalization in the same way as scale. Because the product market becomes more competitive, the profit will drop. Because each variety is sold in both countries, the profit will double. Again, the first effect dominates for low efficiency firms, so they reduce scope; the second effect dominates for high efficiency firms, so they expand scope. It is possible that all firms reduce scope, but if a firm expands its scope, all firms with higher efficiency will also expand their scope.

Now consider the redistribution of capital. Low efficiency firms reduce both scale and scope, so they must sell capital; the least efficient firms sell all of their capital and exit. High efficiency firms, by contrast, expand scale and/or scope, and they buy capital. It is possible that all firms reduce scale, or all firms reduce scope,[14] but not both at the same time. Otherwise all firms would sell capital, which cannot happen at equilibrium.

The effect of trade liberalization on the aggregate scope (i.e., the total number of varieties) produced in each country is unclear, as low efficiency firms reduce scope while high efficiency firms may or may not expand it. Nevertheless, the aggregate scope consumed in each country, produced by both countries, is higher after trade liberalization. The effect on outputs is not surprising: low efficiency firms reduce plant and firm outputs, while high efficiency firms increase these outputs even when they reduce scale or scope. Total industry output increases.

The overall picture is clear: trade liberalization brings both exporting opportunities and intensified competition. Facing the same opportunities and challenges, different firms respond differently. High efficiency firms are more capable of turning inputs into outputs, but in autarky they are constrained by the limited demand in the product market. Trade liberalization enlarges the markets for their products, and they take the opportunity to expand. Low efficiency firms, by contrast, have limited ability to take advantage of the exporting opportunity and are affected mainly by the increased competition. Different product/factor trade-offs lead to different choices: low efficiency firms reduce scale, scope, outputs and even exit, while high efficiency firms do the opposite. Capital moves from low to high efficiency firms.

A prediction of many previous multiproduct models has been that all firms reduce product scope after

[14] When $\frac{\beta}{L}$ is larger or $\frac{\gamma^2 m}{L^2 \alpha^2}$ is smaller, the cutoff point of φ for scale moves to the left (i.e., more firms increase scale) while the cutoff point of φ for scope moves to the right (i.e., fewer firms expand scope).

trade liberalization (Bernard, Redding and Schott, 2009; Eckel and Neary, 2009; Mayer, Melitz and Ottaviano, 2009). The conclusion of this analysis is different: high efficiency firms may expand their scope. Previous models considered products as heterogeneous within a firm, and the impact of trade liberalization worked through either the factor market or the product market, but not both. The export opportunity bids up labor price or raises product market competition, forcing each firm to drop its marginal products, just as trade liberalization forces marginal firms to exit in single product models. This study assumes products are homogeneous within a firm and the optimal scope is proportional to each plant's profit. Trade liberalization changes a firm's product/factor tradeoff differently depending on the firm's efficiency, leading to different responses in terms of scope. Efficient firms can make better use of exporting opportunities. Their profits may increase, leading to scope expansion. This points to the possibility that high efficiency firms may expand their scope after trade liberalization. Furthermore, they are more likely to do so when products are less valuable (small α) or more differentiated (small β), the market size is small (small L), or managing varieties is more costly (large m). It is therefore an empirical question as to whether or not high efficiency firms expand their scope after trade liberalization and, if so, under what conditions.

A major purpose of studying heterogeneous firms is to investigate how trade liberalization changes industry productivity. We have defined three measures of productivity.

Proposition 4. *After trade liberalization,*

(1) Total factor productivity improves: $\omega_t > \omega_a$*;*

(2) Labor productivity has the following properties: $\frac{\lambda_t^\varphi}{\lambda_a^\varphi} = \sqrt{\frac{R_a}{R_t}}$*;* $\frac{\lambda_t}{\lambda_a} > \frac{\lambda_t^\varphi}{\lambda_a^\varphi}$*;*

(3) Overall productivity has the following properties: $\frac{\mu_t^p}{\mu_a^p} = \sqrt{\frac{R_a}{R_t}}$*;* $\frac{\mu_t^\varphi}{\mu_a^\varphi} > \frac{\mu_t^p}{\mu_a^p}$*;* $\frac{\mu_t}{\mu_a} > \frac{\mu_t^p}{\mu_a^p}$*. In addition,* $\frac{\mu_t^\varphi}{\mu_a^\varphi}$ *increases with* φ*.*

Proof. See Appendix I.

TFP is fixed at $\sqrt{\varphi}$ at both the plant and firm level, and therefore does not change after trade liberalization. However, because capital, labor and output redistribute to more efficient firms, the industry's TFP improves.

Labor productivity at the plant and firm level is inversely related to the capital price—trade liberalization improves labor productivity if and only if capital becomes cheaper. This inverse relationship is easy to understand. When capital is cheaper, firms will substitute capital for labor, so for the same output, the labor input will decrease, improving labor productivity. Note that the proportional change in labor productivity is the same for all firms. This is because, in this two-stage game, all firms choose the optimal combinations of labor and capital. Because they face the same wage rate and capital price, the combination is the same for all firms. On the industry level, because labor moves to more efficient firms, the improvement of labor productivity is greater than at the individual firm level.

Overall productivity at the plant, firm and industry levels are all inversely related to the capital price. That is simply because capital is part of the cost, so more expensive capital affects overall productivity inversely. At the plant level, the proportional change in overall productivity depends only on the proportional change in the capital price and is therefore the same for all firms. The improvement in firm productivity is greater than that at the plant level ($\frac{\mu_t^{\varphi}}{\mu_a^{\varphi}} > \frac{\mu_t^p}{\mu_a^p}$), indicating rationalization within firms. Because the two measures of overall productivity differ only in the management cost, the rationalization indicates that trade liberalization is conducive to the management of multiple products. Furthermore, the rationalization is greater for more efficient firms ($\frac{\mu_t^{\varphi}}{\mu_a^{\varphi}}$ increases with φ). If overall productivity improves for firm φ', then it also improves for all firms with $\varphi > \varphi'$. The improvement in overall industry productivity is also greater than that at the plant level.

The three measurements, TFP best captures the redistribution of resources among firms, labor productivity reflects substitution between labor and capital, and overall productivity captures the joint effectiveness of managing varieties and the production of each variety. In the latter two cases, although all the measures depend on the capital price, which may rise or drop, rationalization is still evident in comparisons between the measures. Therefore, the general picture is that trade liberalization induces more efficient use of resources.

The two inputs, labor is supplied perfectly elastically, so the wage rate is fixed. Capital supply, however, is perfectly inelastic and the capital price is endogenized. So, we have seen, the effect of trade liberalization depends crucially on changes in the capital price, which may increase or decrease after trade liberalization.

Proposition 5. *Capital price increases when $\frac{\beta}{L}$ is small, and decreases when $\frac{\beta}{L}$ is large while $\frac{\gamma^2 m}{L^2\alpha^2}$ is small.*

Proof. See Appendix I.

Whether or not the capital price increases depends crucially on product substitutability represented by β. Recall that the demand for each variety in autarky is $p_i = A - bq_i$, where $A = \frac{\alpha\gamma+\beta P}{\beta M+\gamma}$ and $b = \frac{\gamma}{L}$. Trade liberalization brings two changes to product demand: A drops, meaning that competition in the product market is intensified; and b is halved, meaning that the market size for each product is doubled. If β is very small, for example, imagine $\beta = 0$ (each variety is a monopoly), then $A = \alpha$, and trade liberalization will not reduce A. In that case, every firm produces for a larger market with no more competition. The extra production raises demand for capital, which bids up the capital price. Conversely, when β is large, products are close substitutes. Because competition between products is strong, trade liberalization will intensify competition greatly, leading to a much smaller A. This may dominate the double-market-size effect and lead many firms to sell capital, in which case the aggregate demand for capital drops and the capital price declines. Note that for the capital price to drop, in addition to a large β, another condition is a small $\frac{\gamma^2 m}{L^2\alpha^2}$, which implies that y must be small both before and after trade liberalization.[15] That is, capital must be scarce in the general economy.

In this model, the only use of capital is for production, so the capital price relative to the value of the product produced can be a measure of capital scarcity. On that measure, the effect of trade liberalization is unambiguous: capital becomes more expensive relative to the demand in the product market as $y \equiv \frac{A}{\sqrt{2R}}$ decreases.

[15]Recall that y_a is solved from equation (12) and y_t is solved from the same equation with b being replaced with $\frac{b}{2}$. In both cases, y is increasing in $\frac{\gamma^2 m}{L^2\alpha^2}$.

The finding that factor prices may drop in response to trade liberalization is a surprise. In all previous studies in which factor price was not fixed (Melitz, 2003; Bernard, Redding and Schott, 2009), trade liberalization was invariably found to raise factor prices. There are two reasons for the difference. Melitz (2003) derived demand from CES preference and thus had a constant and exogenous markup, and there was free entry (i.e., repeated drawing of efficiencies) so that any disturbance to the market demand was exactly offset by entry. In this analysis, by contrast, demand is linear, and there is no free entry to restore A, the aggregate variable representing the degree of market competition. Note that even when the capital price drops, the least efficient firms still exit, which was impossible in previous formulations.

Combining propositions 4 and 5, we have

Corollary:

(1) When $\frac{\beta}{L}$ is large while $\frac{\gamma^2 m}{L^2\alpha^2}$ is small, labor productivity and overall productivity improve at all levels, plant, firm and industry.

(2) When $\frac{\beta}{L}$ is small, labor productivity drops at the plant and firm levels, but may rise at the industry level. Overall productivity drops at the plant level, but may rise at the firm and industry levels.

As has been discussed, trade liberalization always improves industry TFP, as resources move from low to higher efficiency firms. Other measurements depend on the capital price. The corollary says that when the capital price declines, all efficiency measures improve, indicating a more efficient distribution of resources both across and within firms. Even when the capital price increases, rationalization is evident in the fact that the improvement at the firm and industry levels is greater than the improvement at the plant level.

4 Concluding Remarks

This study looked at heterogeneous firms and the tradeoff between products and resources. Both product and factor markets are at work in shaping firms' choices. Consistent with prior work in this area, we have shown that trade liberalization improves industry productivity by redistributing resources toward more efficient firms. However, the joint consideration of product and factor markets produces new results. Factor prices may drop as a result of trade liberalization, which is impossible if the two markets are considered separately. Although all firms face the same changes in the product and factor markets, they respond differently depending on their efficiencies. While capital is homogenous and affects firms uniformly (i.e., the cost of acquiring capital is the same for all firms), the product market affects more efficient firms more heavily than it affects less efficient firms.

Most prior work has assumed a single production factor such as labor, and hence has focused on labor productivity.[16] Firms in our model use two inputs, and the analysis demonstrates that labor productivity

[16] An exception is the work of Bernard, Redding and Schott (2007), who investigated the impact of trade liberalization in a general equilibrium framework with different relative factor abundance across countries. But they used CES demand functions and therefore trade liberalization affected firms' choices only through the factor markets.

depends crucially on the substitution between labor and capital. This represents a new channel through which productivity can be affected by trade liberalization. Furthermore, we have demonstrated that the substitution between factors depends on the relative factor prices, which in turn depends on the degree of product substitution in consumers' demand function.

The bilateral trade liberalization considered here brings exporting opportunities and intensified competition. To further demonstrate how the tradeoff between product and factor markets shapes firms' choices, we study in Appendix II a different scenario in which firms face intensified competition without enjoying the benefit of exporting. This will be the case if trade liberalization is unilateral, i.e. a country imports the industry's products but cannot export them. Then the direct effect disappears; trade liberalization affects the acquisition market only indirectly through the product market. Because imports reduce the demand for each domestic product, the aggregate demand for capital drops unambiguously, reducing the capital price. In fact, we show that the capital price must drop more than the demand does so that the product actually becomes more profitable relative to the capital price. All plants expand scale, but the change in product scope is again non-uniform. High efficiency firms are affected mainly by changes in the product market, so they reduce scope and sell capital. Low efficiency firms are affected mainly by changes in the acquisition market, so they increase scope and buy capital.

To allow the product market to affect firms' choices, we have followed Melitz and Ottaviano (2008) in assuming a linear demand function. An alternative is the CES demands assumed by Melitz (2003) and most subsequent studies.[17] A CES demand has the peculiar feature of exogenous and constant markups that are independent of costs, demands or market sizes. This is not only unrealistic (Helpman, 2006), but also leads to the special property that firm choices depend only on the ratio between demand and the factor price, leading to uniform impacts of trade liberalization even when product and factor markets are both considered. With linear demand, firms' choices will depend on both demand and the factor price rather than the ratio between the two. Trade liberalization causes non-uniform responses from firms because only the product market impact depends on a firm's efficiency. Admittedly, a linear demand function is highly specific, but a CES function is equally so. Our model demonstrates that some of the fine details of the implications drawn from previous models based on CES functions will not survive under alternative forms of demand.

We have assumed that production requires two inputs, labor and capital, which are treated asymmetrically: Labor is supplied perfectly elastically at a fixed wage rate, while capital is supplied perfectly inelastically at a price that is determined endogenously. Our qualitative results will not change if the wage rate is also endogenized, or if the supply of capital is elastic, as long as capital price is endogenous.

Since two inputs are used, it seems that rationalization between the two inputs may provide another channel for trade liberalization to improve productivity. However, this turns out not to be the case in this formulation. The substitution between the two inputs in the production function is assumed to be the same for all firms. Given the optimal choice of capital and labor in the two-stage game and the same wage rate and capital price faced by all firms, the labor/capital ratio is uniform across firms. In fact, a composite input (at the optimal labor/capital ratio) can be used in place of the two inputs to generate the same results. Although

[17]CES preference has also been used by Bernard, Redding and Schott (2009) and Feenstra and Ma (2008), while the linear demand was used by Baldwin and Gu (2005), Eckel and Neary (2009), and Nocke and Yeaple (2006).

trade liberalization changes the capital price and thus the labor/capital ratio, the same ratio applies to all firms, so there is no rationalization between labor and capital. Nevertheless, such a rationalization will be possible if firms differ in the labor/capital substitutability of their production functions. This is a refinement worth further investigation.

We have assumed away the possibility of cross-border capital movement. When the two countries are symmetric, relaxing this assumption will not change any result for bilateral trade liberalization, as the capital price will be the same in any case. If the countries are asymmetric, or if trade liberalization is unilateral, cross-border M&A will allow an extra channel through which resources are rationalized. Such a setting will be particularly suitable for investigating trade liberalization between countries that are asymmetric in size, preferences, or labor/capital endowments, and represents a second direction for future work.

Appendix I: Proofs

Proof of the existence and uniqueness of R for given A

Since $R = \frac{A^2}{2y^2}$, we have $K = \frac{A^2}{16b^2m}\xi(y)$, where $\xi(y) = \frac{\rho(y)}{y^2}$ and $\xi'(y) > 0$. Hence,

$$\frac{\partial K}{\partial R} = \frac{A^2}{16b^2m}\xi'(y)\frac{\partial y}{\partial R} < 0.$$

When $R = \frac{A^2}{2}$, we have $y = 1$, $\xi = 0$ and so $K = 0$. When R is sufficiently close to zero, y is large, which means ξ can be large enough that $K > 1$. Thus, given A, there exists a unique equilibrium capital price, R, satisfying $K(R) = 1$.

Q.E.D.

Proof of Proposition 3

All variables will be expressed as functions of y. We first prove $y_t < y_a$. y_a is solved from equation (12): $Z_a(y_a) \equiv \frac{\rho_a^3}{\left(y_a\rho_a + \frac{\beta}{L}\eta_a\right)^2} - \frac{16b^2m}{\alpha^2} = 0$. y_t is solved from $Z_t(y_t) \equiv \frac{\rho_t^3}{\left(y_t\rho_t + \frac{\beta}{L}\eta_t\right)^2} - \frac{4b^2m}{\alpha^2} = 0$. Obviously, $Z_t(y) > Z_a(y)$. Then $Z_t(y_a) > Z_a(y_a) = 0 = Z_t(y_t)$. Because $Z_t'(\cdot) > 0$, we have $y_a > y_t$.

To compare a particular variable before and after trade liberalization, express the variable as a function of y independent of b. To do so, make use of the relations $R = \frac{8b^2m}{\rho}$, $A = \frac{\alpha}{1+\frac{\beta}{L}\frac{\eta}{y\rho}}$ (which does not contain b) or $A = y\sqrt{2R} = 4b\sqrt{m}\frac{y}{\sqrt{\rho}}$ (which is linear in b). A comparison is possible because the expression of a particular variable differs between the two cases only in the value of b. With an expression independent of b, we need only determine whether the expression is increasing or decreasing in y, as we already know that $y_t < y_a$.

(1) Product market:

$A(y) = \frac{\alpha}{1+\frac{\beta}{L}\frac{\eta}{y\rho}}$. It is straightforward to verify that $\frac{\eta}{y\rho}$ decreases in y. Therefore, $A_t = A(y_t) < A(y_a) = A_a$. On the other hand, $A = A(b, y) = 4b\sqrt{m}\frac{y}{\sqrt{\rho}}$, so $A_a = A(b, y_a)$ while $A_t = A(\frac{b}{2}, y_t) = \frac{A(b,y_t)}{2}$. Then $2A_t = A(b, y_t)$. Because $\frac{y}{\sqrt{\rho}}$ decreases in y, we have $2A_t > A_a$.

(2) Capital redistribution:

Since $y_t < y_a$, we have $\varphi_t^0 > \varphi_a^0$. Therefore, for $\varphi \in [\varphi_a^0, \varphi_t^0]$, $x_a v_a > 0$ while $x_t v_t = 0$, i.e., these firms sell capital.

A firm buys capital, i.e., $x_t(\varphi)v_t(\varphi) > x_a(\varphi)v_a(\varphi)$, if and only if $\frac{y_t\sqrt{\varphi}-1}{y_a\sqrt{\varphi}-1} > \left(\frac{R_a}{4R_t}\right)^{\frac{1}{3}}$. The left hand side of this inequality is increasing in φ because $y_t < y_a$, while the right hand side is independent of φ. Therefore, if the inequality holds for φ', it also holds for all $\varphi > \varphi'$. Because some firms (at least those who exit) sell capital, there must be some firms who buy capital. Thus, the threshold of φ must be strictly smaller than 1.

(3) Firm structure:

Scope: $v_t(\varphi) > v_a(\varphi)$ if and only if $\frac{y_t\sqrt{\varphi}-1}{y_a\sqrt{\varphi}-1} > \sqrt{\frac{R_a}{2R_t}}$. Thus, a firm expands scope if and only if its φ exceeds some threshold.

Scale: $x_t(\varphi) > x_a(\varphi)$ if and only if $(2y_t - y_a)\sqrt{\varphi} > 1$. If $2y_t < y_a$, all firms reduce scale. If $2y_t > y_a$, a plant expands scale if and only if its φ exceeds some threshold.

Note that the thresholds for x, v and xv are in general different, and that the thresholds for x and v may be 1, i.e., all plants reduce scale, or all firms reduce scope, but the two will not happen at the same time, as all firms would reduce capital, which cannot occur in equilibrium.

(4) Aggregate scope:

$M = M(b, y) = \frac{2b\psi}{\rho}$, so $M_a = M(b, y_a)$ and $M_t = M(\frac{b}{2}, y_t)$, which means $2M_t = M(b, y_t)$. Because $\frac{d(\frac{\psi}{\rho})}{dy} < 0$ and $y_t < y_a$, we have $2M_t > M_a$.

(5) Outputs:

Plant level: $q = \frac{A}{2b}\left(1 - \frac{1}{y\sqrt{\varphi}}\right) = 2\sqrt{m}\left(\frac{y}{\sqrt{\rho}} - \frac{1}{\sqrt{\rho}\sqrt{\varphi}}\right)$. Then $q_t > q_a$ if and only if $\left(\frac{1}{\sqrt{\rho_t}} - \frac{1}{\sqrt{\rho_a}}\right)\frac{1}{\sqrt{\varphi}} < \frac{y_t}{\sqrt{\rho_t}} - \frac{y_a}{\sqrt{\rho_a}}$. Because $\frac{d\rho}{dy} > 0$, $\frac{d\frac{y}{\sqrt{\rho}}}{y} < 0$, and $y_t < y_a$, we have $\frac{1}{\sqrt{\rho_t}} > \frac{1}{\sqrt{\rho_a}}$ and $\frac{y_t}{\sqrt{\rho_t}} > \frac{y_a}{\sqrt{\rho_a}}$. Then $q_t > q_a$ if and only if φ exceeds some threshold. Because firms with $\varphi \in [\varphi_a^0, \varphi_t^0]$ exit and therefore reduce their outputs, the threshold is greater than φ_t^0 (i.e., the least efficient firms that survive must reduce plant-level output). For the most efficient firm, $\varphi = 1$ and therefore $q = 2\sqrt{m}\frac{y-1}{\sqrt{\rho}}$. Because $\frac{d\frac{y-1}{\sqrt{\rho}}}{y} < 0$, we have $q_t > q_a$ for $\varphi = 1$, i.e., the most efficient firm must increase plant-level output.

Firm level: $qv = \frac{A^3}{16b^2m}\left(1 - \frac{1}{y\sqrt{\varphi}}\right)^3 = \left(y - \frac{1}{\sqrt{\varphi}}\right)^3 \frac{\alpha}{y\rho + \frac{\beta}{L}\eta}$. Then, if $q_t v_t > q_a v_a$ for φ', it must be true for all $\varphi > \varphi'$. So firm-level output increases if and only if φ exceeds some threshold. The least efficient firms that survive must reduce output. For the most efficient firm, $\varphi = 1$ and therefore $qv = \frac{\alpha(y-1)^3}{y\rho + \frac{\beta}{L}\eta}$. It can be shown that $\frac{(y-1)^3}{y\rho + \frac{\beta}{L}\eta}$ decreases with y for any positive value of $\frac{\beta}{L}$. Thus, firm-level output increases for the most efficient firm.

Industry level output: $Q = \frac{AR}{8b^2m}\frac{\eta}{y} = \frac{\alpha}{\frac{y\rho}{\eta} + \frac{\beta}{L}}$. Because $\frac{y\rho}{\eta}$ increases with y, industry output increases.

Q.E.D.

Proof of Proposition 4

(1) TFP: $\omega = \frac{y\eta}{\rho^{\frac{3}{2}}}$, which decreases with y. Because $y_t < y_a$, we have $\omega_t > \omega_a$.

(2) Labor productivity:

Plant and firm level: $\lambda^{\varphi} = \sqrt{\frac{\varphi}{2R}}$, so $\frac{\lambda_t^{\varphi}}{\lambda_a^{\varphi}} = \sqrt{\frac{R_a}{R_t}}$.

Industry level: $\lambda = \frac{y\eta}{A\rho} = \frac{\eta}{\rho}\sqrt{\frac{1}{2R}}$. Because $\frac{d\left(\frac{\eta}{\rho}\right)}{dy} < 0$, we have $\frac{\eta_t}{\rho_t} > \frac{\eta_a}{\rho_a}$. Then, $\frac{\lambda_t}{\lambda_a} = \frac{\eta_t}{\rho_t} \Big/ \frac{\eta_a}{\rho_a} \sqrt{\frac{R_a}{R_t}} > \sqrt{\frac{R_a}{R_t}} = \frac{\lambda_t^{\varphi}}{\lambda_a^{\varphi}}$.

(3) Overall productivity:

Plant level, $\mu^p = \sqrt{\frac{\varphi}{2R}}$, so $\frac{\mu_t^p}{\mu_a^p} = \sqrt{\frac{R_a}{R_t}}$.

Firm level, $\mu^{\varphi} = \frac{4}{A\left(1+\frac{3}{y\sqrt{\varphi}}\right)} = \frac{4}{\sqrt{2R}\left(y+\frac{3}{\sqrt{\varphi}}\right)}$. Then, $\frac{\mu_t^{\varphi}}{\mu_a^{\varphi}} = \sqrt{\frac{R_a}{R_t}}\frac{y_a+\frac{3}{\sqrt{\varphi}}}{y_t+\frac{3}{\sqrt{\varphi}}} > \sqrt{\frac{R_a}{R_t}} = \frac{\mu_t^p}{\mu_a^p}$. Because $\frac{\mu_t^{\varphi}}{\mu_a^{\varphi}} = \sqrt{\frac{R_a}{R_t}}\frac{y_a\sqrt{\varphi}+3}{y_t\sqrt{\varphi}+3}$ and $y_t < y_a$, $\frac{\mu_t^{\varphi}}{\mu_a^{\varphi}}$ increases with φ.

Industry level, $\mu = \frac{4y}{A}\frac{\eta}{\theta} = \frac{4}{\sqrt{2R}}\frac{\eta}{\theta}$. Because $\frac{d\left(\frac{\eta}{\theta}\right)}{dy} < 0$, we have $\frac{\eta_t}{\theta_t} > \frac{\eta_a}{\theta_a}$. Then, $\frac{\mu_t}{\mu_a} > \sqrt{\frac{R_a}{R_t}} = \frac{\mu_t^p}{\mu_a^p}$.

Q.E.D.

Proof of Proposition 5

Capital price $R(y) = \frac{A^2}{2y^2} = \frac{\alpha^2}{2\left(y+\frac{\beta}{L}\frac{\eta}{\rho}\right)^2}$. It can be easily verified that $y + \frac{\beta}{L}\frac{\eta}{\rho}$ increases with y when $\frac{\beta}{L}$ is small, but decreases with y when $\frac{\beta}{L}$ is sufficiently large and y is small, which happens when $\frac{\gamma^2 m}{L^2\alpha^2}$ is small. Here is a numerical example: When $\alpha = \beta = \gamma = L = m = 1$, $R_a = 0.0019$ and $R_t = 0.0079 > R_a$. When $\alpha = 20$, $\beta = 100$, and other parameters remain the same, $R_a = 0.034$ and $R_t = 0.032 < R_a$.

Q.E.D.

Appendix II: Unilateral Opening-up

Trade liberalization brings both opportunities and challenges for firms. This analysis has shown that high efficiency firms can make better use of exporting opportunities while low efficiency firms are affected mainly by the challenges of increased competition from imports. To better understand how firm respond differently depending on their efficiencies, consider unilateral trade liberalization: a country opens up to imports but cannot export to the other country. The one-way trading involves only differentiated products. To balance the trade, the numeraire good is assumed to be traded both ways. We focus on the importing country, which is also referred to as the home country, while the exporting country is referred to as the foreign country.

Firm structure

Let the subscripts h, f and e stand for home, foreign and export, respectively, and define the following notation:

$$y_h \equiv \frac{A_h}{\sqrt{2R_h}}, \;\; y_f \equiv \frac{A_f}{\sqrt{2R_f}}, \;\; y_e \equiv \frac{A_h}{\sqrt{2R_f}}, \tag{18}$$

with $\varphi_i^0 \equiv \frac{1}{y_i^2}$ for $i = h, f, e$.

Home firms face the same optimization problem as in autarky for any given A_h and R_h. Hence, the expressions of x_h and v_h can be obtained by replacing A, R and y with A_h, R_h and y_h in (5) and (6).

In the foreign country, each firm can sell its products in both the domestic and export markets. Suppose that a foreign firm sells q_f of a particular variety in its domestic market and q_e of the same variety in the export market. These quantities solve the following optimization problem:

$$\max_{q_f \geq 0,\, q_e \geq 0} \pi_f \equiv (A_f - bq_f)q_f + (A_h - bq_e)q_e - \frac{(q_f + q_e)^2}{2\varphi x_f}.$$

The interior solution (i.e., when the non-negative constraints are not binding) is

$$q_f = \frac{(2\varphi b x_f + 1)A_f - A_h}{4b(\varphi b x_f + 1)} \quad \text{and} \quad q_e = \frac{(2\varphi b x_f + 1)A_h - A_f}{4b(\varphi b x_f + 1)}. \tag{19}$$

A_f and A_h are endogenous, and it can be shown that $A_f > A_h$ at equilibrium. Hence q_f is always positive, while q_e is positive if and only if $\varphi x_f > \frac{1}{2b}\left(\frac{A_f}{A_h} - 1\right)$. That is, a firm exports if and only if its efficiency is sufficiently high. Thus, even without any (fixed or variable) trade costs, firms with low efficiency sell only to their domestic market without exporting.[18]

So a foreign firm with efficiency φ sells in both markets if $x_f > \frac{1}{2\varphi b}\left(\frac{A_f}{A_h} - 1\right)$, and the sales in the two markets are given by (19). The corresponding prices and a variety's total profit in the two markets are

$$\begin{aligned}
p_e &= \frac{A_h(2\varphi b x_f + 3) + A_f}{4(\varphi b x_f + 1)}, \qquad p_f = \frac{A_f(2\varphi b x_f + 3) + A_h}{4(\varphi b x_f + 1)}, \\
\pi_f &= \frac{2\varphi b x_f(A_h^2 + A_f^2) + (A_h - A_f)^2}{8b(\varphi b x_f + 1)}.
\end{aligned}$$

If $x_f \leq \frac{1}{2\varphi b}\left(\frac{A_f}{A_h} - 1\right)$, then $q_e = 0$. The firm makes its choices as if there were no exporting opportunity, and the results are the same as in a closed economy:

$$q_f = \frac{A_f \varphi x_f}{2\varphi b x_f + 1}, \;\; p_f = \frac{A_f(\varphi b x_f + 1)}{2\varphi b x_f + 1}, \text{ and } \pi_f = \frac{A_f^2 \varphi x_f}{2(2\varphi b x_f + 1)}.$$

Each plant's profit net of capital cost is $\tilde{\pi}_f = \pi_f - x_f R_f$, where π_f is given above depending on whether

[18] The intuition is the following. For a firm in the foreign country, the domestic demand (in the foreign country) is higher than the export demand (in the home country): $A_f > A_h$. The marginal benefit of selling to the domestic market is therefore higher than that of exporting if the sales in the two countries are the same. When φ (and consequently φx_f) is low, output is small. The marginal benefit of selling the last unit in the domestic market is still higher than the marginal benefit of selling the first unit in the export market, so the product is not exported. When φ is larger, output is larger. After selling some quantity in the domestic market, the marginal benefit of selling a further unit falls below the marginal benefit of selling the first unit in the export market, and the firm starts to export.

or not $x_f > \frac{1}{2\varphi b}\left(\frac{A_f}{A_h} - 1\right)$. The firm chooses x_f to maximize $\tilde{\pi}_f$, which yields

$$x_f = \begin{cases} 0, & \text{if } \varphi \leq \varphi_f^0, \\ \frac{y_f\sqrt{\varphi}-1}{2\varphi b}, & \text{if } \varphi_f^0 < \varphi \leq \varphi_e^0, \\ \frac{y_f\sqrt{\varphi}-1}{2\varphi b} + \frac{y_e\sqrt{\varphi}-1}{2\varphi b}, & \text{if } \varphi > \varphi_e^0. \end{cases} \tag{20}$$

Note that $\varphi_f^0 < \varphi_e^0$ because $A_f > A_h$.

Given this optimal scale, a firm chooses its scope, v_f, to maximize its profit $v_f\tilde{\pi}_f - mv_f^2 + R_f$. The optimal choice is

$$v_f = \begin{cases} 0, & \text{if } \varphi \leq \varphi_f^0, \\ \frac{R_f\left(y_f\sqrt{\varphi}-1\right)^2}{4\varphi bm}, & \text{if } \varphi_f^0 < \varphi \leq \varphi_e^0, \\ \frac{R_f\left[\left(y_f\sqrt{\varphi}-1\right)^2+\left(y_e\sqrt{\varphi}-1\right)^2\right]}{4\varphi bm}, & \text{if } \varphi > \varphi_e^0. \end{cases}$$

At the optimal scale, a foreign firm's domestic sales are $q_f = \frac{A_f}{2b}\left(1 - \frac{1}{y_f\sqrt{\varphi}}\right)$ at $p_f = \frac{A_f}{2}\left(1 + \frac{1}{y_f\sqrt{\varphi}}\right)$ for $\varphi > \varphi_f^0$. Its export sales are $q_e = \frac{A_f}{2b}\left(\frac{y_e}{y_f} - \frac{1}{y_f\sqrt{\varphi}}\right)$ at price $p_e = \frac{A_f}{2}\left(\frac{y_e}{y_f} + \frac{1}{y_f\sqrt{\varphi}}\right)$ for $\varphi > \varphi_e^0$. The firm's labor productivity is $\sqrt{\frac{\varphi}{2R_f}}$ for $\varphi > \varphi_f^0$. Thus, exporters are more efficient in production, produce more and earn more than firms that produce only for the domestic market, confirming well-established empirical findings.

Industry equilibrium

In the home country, the acquisition market equilibrium is derived for given A_h in exactly the same way as in autarky. Hence,

$$R_h = \frac{8b^2m}{\rho_h}. \tag{21}$$

Then $M_h \equiv \int_{\varphi_h^0}^{1} v_h d\varphi$ is the measure of all home varieties and $M_e \equiv \int_{\varphi_e^0}^{1} v_f d\varphi$ is the measure of all imported varieties. Substituting a firm's optimal scale in its equilibrium price and making use of $\frac{A_h}{A_f} = \frac{y_e}{y_f}$, we obtain $p_h = \frac{A_h}{2}\left(1 + \frac{1}{y_h\sqrt{\varphi}}\right)$ for $\varphi \geq \varphi_h^0$ in the home country and $p_e = \frac{A_h}{2}\left(1 + \frac{1}{y_e\sqrt{\varphi}}\right)$ for $\varphi \geq \varphi_e^0$ in the foreign country. The aggregate price in the home market is $P_h \equiv \int_{\varphi_h^0}^{1} v_h p_h d\varphi + \int_{\varphi_e^0}^{1} v_f p_e d\varphi$.

In the foreign country, the total capital demanded is $K_f \equiv \int_0^1 x_f v_f d\varphi$, and from $K_f = 1$ we obtain

$$R_f = \frac{8b^2 m}{\rho_f + \rho_e + 2\hat{\rho}}, \tag{22}$$

where $\hat{\rho} = 1+2y_e y_f - y_f^2 + 2y_e^2 + (y_e + y_f)(y_f y_e - 3) - 2\ln(y_e)(y_e^2 + 4y_e y_f + y_f^2)$. The total product varieties are $M_f \equiv \int_{\varphi_f^0}^1 v_f d\varphi$. Using x_f from (20) and the fact that $\frac{A_h}{A_f} = \frac{y_e}{y_f}$, the individual product price sold in the foreign market is $p_f = \frac{A_f}{2}\left(1 + \frac{1}{y_f\sqrt{\varphi}}\right)$ for $\varphi \in (\varphi_f^0, 1]$. The aggregate price is $P_f \equiv \int_{\varphi_f^0}^1 v_f p_f d\varphi$.

Finally, by definition, A_h and A_f are jointly determined by

$$A_h = \frac{\alpha\gamma + \beta P_h}{\beta(M_h + M_e) + \gamma}, \tag{23}$$

$$A_f = \frac{\alpha\gamma + \beta P_f}{\beta M_f + \gamma}. \tag{24}$$

Having expressed R_h and R_f as functions of y_f, y_e and y_h (from (21) and (22)), we have five unknowns, A_f, A_h, y_f, y_e and y_h, and five equations, (23), (24) and the definitions of y_f, y_e and y_h (from (18)), which jointly determine the equilibrium.

Equilibrium comparison

From (23), $A_h < \frac{\alpha\gamma+\beta P_h}{\beta M_h + \gamma} = \frac{\alpha}{1+\frac{\beta}{L}\frac{\eta_h}{y_h \rho_h}}$. The equilibrium y_h is solved from $A_h^2 - 2y_h^2 R_h$, which means $Z(y_h) = \left(\frac{\alpha}{1+\frac{\beta}{L}\frac{\eta_h}{y_h\rho_h}}\right)^2 - 2y_h^2 R_h > 0$. Since $Z(y_a) = 0$ and $Z'(\cdot) > 0$, we have $y_h > y_a$.

$R(y) = \frac{8b^2 m}{\rho}$. Since ρ increases in y, we have $R_h = R(y_h) < R(y_a) = R_a$.

$A(y) = y\sqrt{2R} = 4b\sqrt{m}\frac{y}{\sqrt{\rho}}$. Because $\frac{y}{\sqrt{\rho}}$ decreases in y, we have $A_h < A_a$.

The comparison of all the other variables is easy, as the only difference between autarky and unilateral opening-up is that home firms in the latter case face competition from imports, reflected in different expressions for A: $A = \frac{\alpha}{1+\frac{\beta}{L}\frac{\eta}{y\rho}}$ in autarky but not after opening-up. All other expressions as functions of y are the same. Many proofs can thus be borrowed from the trade liberalization case except that y increases in unilateral opening-up but decreases in trade liberalization, leading to opposite conclusions. Care is needed only if a proof makes use of the expression $A = \frac{\alpha}{1+\frac{\beta}{L}\frac{\eta}{y\rho}}$, which is not applicable in unilateral opening-up. Unlike in the trade liberalization case, we do not need to worry about b.

A firm buys capital ($x_h v_h > x_a v_a$) if and only if $\frac{y_h\sqrt{\varphi}-1}{y_a\sqrt{\varphi}-1} > \left(\frac{R_a}{R_h}\right)^{\frac{1}{3}}$. The left hand side of the inequality is decreasing in φ because $y_h > y_a$, so if the inequality holds for φ', it also holds for all $\varphi < \varphi'$. That is, low efficiency firms buy capital while high efficiency firms sell capital.

Because $x = \frac{y\sqrt{\varphi}-1}{2\varphi b}$ and $y_h > y_a$, all firms expand their scale. A firm expands scope if and only if

$\frac{y_h\sqrt{\varphi}-1}{y_a\sqrt{\varphi}-1} > \sqrt{\frac{R_a}{R_h}}$, i.e., if and only if its φ is below some threshold. It is possible that all firms reduce scope.

Aggregate scope: $M = \frac{2b\psi}{\rho}$ and $\frac{d(\frac{\psi}{\rho})}{dy} < 0$, so $M_h < M_a$.

Plant output is $q = \frac{1}{2b}\left(A - \sqrt{\frac{2R}{\varphi}}\right)$ and firm output is $qv = \frac{1}{16b^2m}\left(A - \sqrt{\frac{2R}{\varphi}}\right)^3$. Both increase if and only if φ is below some threshold. Industry output is $Q = \frac{AR}{8b^2m}\frac{\eta}{y} = 4b\sqrt{m}\rho\eta^{-\frac{3}{2}}$, which decreases with y, so industry output decreases.

Total factor productivity: $\omega = \frac{y\eta}{\rho^{\frac{3}{2}}}$, which decreases with y, so $\omega_h < \omega_a$.

Labor productivity at the plant and firm level: $\lambda^{\varphi} = \sqrt{\frac{\varphi}{2R}}$, which increases after opening up as R drops. Industry labor productivity is $\lambda = \frac{y\eta}{A\rho} = \frac{1}{4b\sqrt{m}}\frac{\eta}{\sqrt{\rho}}$, which increases with y, so industry labor productivity improves.

Overall productivity: $\mu^p = \sqrt{\frac{\varphi}{2R}}$ at the plant level, and $\mu^{\varphi} = \frac{4}{A\left(1+\frac{3}{y\sqrt{\varphi}}\right)} = \frac{4}{A+3\sqrt{\frac{2R}{\varphi}}}$ at the firm level. Both increase. Overall industry productivity is $\mu = \frac{4}{\sqrt{2R}}\frac{\eta}{\theta} = \frac{1}{b\sqrt{m}}\frac{\eta\sqrt{\rho}}{\theta}$, which increases with y, so μ increases.

These comparisons can be summarized as follows:

Proposition 6. *After unilateral opening-up, the importing country undergoes the following changes:*

(1) Markets: the product market becomes more competitive ($A_h < A_a$); the capital price drops ($R_h < R_a$).

(2) Capital redistribution: capital moves from higher to lower efficiency firms.

(3) Firm structure: all plants expand scale; less efficient firms expand their scope while higher efficiency firms may reduce scope.

(4) Aggregate scope: the total number of varieties produced by home firms decreases ($M_h < M_a$).

(5) Outputs: plant and firm outputs increase for low efficiency firms and decrease for high efficiency firms. Industry output decreases.

(6) Productivity: total factor productivity decreases; labor and overall productivity increase on the plant, firm and industry levels.

Some of the changes are illustrated in Figure 2, where the solid lines represent autarky and the dotted lines represent the situation after opening-up.

Discussion

After unilateral opening up, foreign products enter the home market, so the demand for each variety drops (A is lower). Since home capital can only be used for home production, there will be less demand for capital, so capital becomes cheaper (R drops). In fact, the drop in R must be greater than the drop in A so that product market becomes more profitable relative to the capital price (y must rise), otherwise all firms would reduce both scale and scope, which cannot happen at equilibrium.

Because the product market becomes more profitable relative to the capital price, every plant expands scale. For product scope, recall that high efficiency firms are mainly affected by changes in the product market while low efficiency firms are affected mainly by changes in the acquisition market (see expression

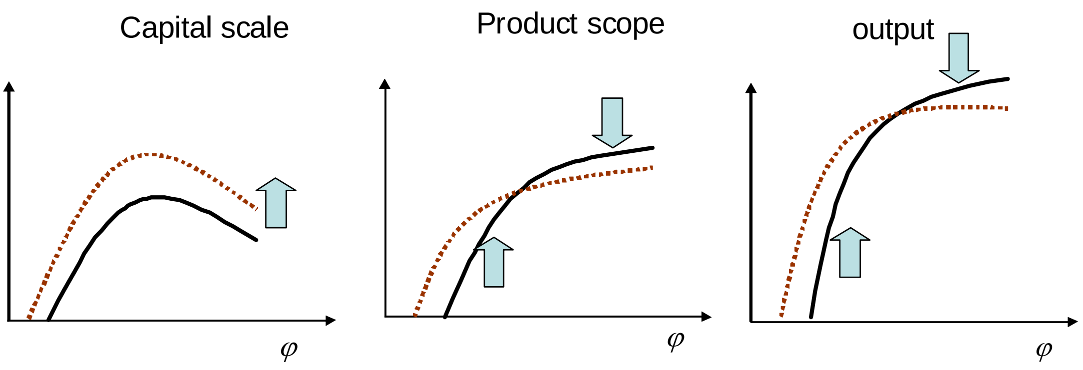

Figure 2: Impacts of Unilateral Trade Liberalization

(7)). After opening up, the product market is more competitive, which tends to reduce scope, while capital becomes cheaper, which tends to expand scope. As a result, high efficiency firms reduce scope while low efficiency firms expand it. In the end, capital redistributes toward less efficient firms. So does output. Firms are distributed more evenly in the industry in terms of capitalization and output.

Because capital becomes cheaper, all firms substitute capital for labor, raising labor productivity at the plant, firm and industry levels. Overall productivity, which is inversely related to average cost, also increases. Because inputs and outputs move from high to low efficiency firms, the total factor productivity of the industry drops. This last conclusion depends on the no free entry assumption, and no cross-border trading of capital even though capital is cheaper in the importing country than in the exporting country.

For the exporting country, analytical comparisons are intractable,[19] but numerical calculations confirm the intuition that almost every change is the opposite of what happens in the home country. Capital becomes more expensive. The least efficient firms exit. High efficiency firms buy capital and increase their scope while low efficiency firms sell capital and reduce scope, leading to a more skewed distribution of firm size measured by capital or output. Every firm reduces scale.

In both trade liberalization and unilateral opening-up, we have demonstrated that high and low efficiency firms respond differently in their choices of firms structure, even though they all face the same change in the product and factor markets. In the unilateral opening-up case, the mechanism is simple, because all changes in the importing country are induced by a negative demand shock in the product market.[20] And

[19]In the home country, although there are no closed-form solutions, analytical comparisons are possible because the expressions for scale and scope are exactly the same as in the closed economy, and we can prove that $y_h > y_a$ because (23) and (11) differ only by an extra term. In the foreign country, by contrast, the expressions for scale and scope are different from those in the closed economy, and the expressions contain y_f and y_e that cannot be easily compared with y_a.

[20]Given A and R, all expressions are the same as in autarky, and the aggregate demand for capital will also be the same, leading

the major conclusion is that high efficiency firms are affected mainly by changes in the product market, while low efficiency firms are affected mainly by changes in the acquisition market. With bilateral trade liberalization, there is an extra effect, because firms not only face increased competition but also enjoy increased exporting opportunities.[21] The major conclusion is that high efficiency firms are better able to make use of the exporting opportunities, while low efficiency firms are less able to do so.

Unilateral opening-up is a rather hypothetical setting. Real life tends to have general equilibrium effects with free entry, while this analysis is of partial equilibrium without entry. In the model, the product market in the importing country undergoes a negative demand shock, which induces a negative demand shock in the acquisition market. We can imagine the opposite, such as a positive taste shock that raises A, which in turn will induce a positive shock in the acquisition market. The foregoing discussion predicts that all of the conclusions will be reversed.

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to the same expression for the capital price. Therefore, the equilibrium is disturbed only through the reduction in A.

[21] For given A and R, a firm’s demand for capital will be different from that in autarky due to the exporting opportunity.

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