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Mergers in Fiscal Federalism*

Marie-Laure Breuillé† and Skerdilajda Zanaj‡

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Abstract

This paper analyzes mergers of regions in a two-tier setting with both horizontal and vertical tax competition. The merger of regions induces three effects on regional and local tax policies, which are transmitted both horizontally and vertically: i) an alleviation of tax competition at the regional level, ii) a rise in the regional tax base, and iii) a larger internalization of tax externalities generated by cities. It is shown that the merger of regions increases regional tax rates while decreasing local tax rates. This Nash equilibrium with mergers is then compared with the Nash equilibrium with coalitions of regions.

Keywords: Mergers, Tax Competition, Fiscal Federalism

JEL classification: H73, H25

1 Introduction

As part of an ongoing process of regionalization in Europe, several European countries decided to reduce the number of their regions (Dexia Crédit Local, 2008) with the aim of improving the management of public services, realizing scale economies or reducing bureaucracy. Several recent examples include Poland, where the number of "voïvodies" was cut from 49 to 16 in 1999, and more recently, Denmark, where the territorial reform implemented in 2007 replaced the 13 "amter" with 5 regions. Several other countries – including France, Hungary and Romania – are also considering merging regions.

The effect of a merger of jurisdictions on the horizontal externalities that arise from tax competition among these jurisdictions is well known in a one-tier structure. In a setting borrowed from Wildasin (1988), Hoyt (1991) demonstrated that tax rates on mobile capital – and thus public goods provision – increase as the number of jurisdictions decreases. This result comes from the reduction in the externality produced by a jurisdiction that changes

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its tax rate, where the externality corresponds to the capital inflow in other jurisdictions that become more attractive when a jurisdiction increases its tax rate. Decreasing the number of jurisdictions reduces the capital movement; thus, increasing its tax rate is less harmful for a jurisdiction. Considering the possibility of asymmetric mergers, Bucovetsky (2008) also concluded that any merger leads to a higher average tax rate for the federation as a whole due to higher tax rates in jurisdictions that do not belong to the merger.

When the mobile tax base is shared with a higher level of jurisdiction, this leads to vertical tax externalities, in addition to horizontal tax externalities (Wrede, 1997, Keen, 1998, Hoyt, 2001, Keen and Kotsogiannis, 2002). Excessive taxation at both levels results from the fact that jurisdictions ignore the depressive effect that a rise in their tax rate has on the common tax base shared with the other level. With horizontal externalities causing inefficiently low tax rates and vertical externalities causing inefficiently high tax rates, the equilibrium tax rates at the bottom-tier can be either inefficiently low or high. In a model of tax competition on capital between a unique top-tier jurisdiction and an arbitrary number of bottom-tier jurisdictions, Keen and Kotsogiannis (2002) showed that whether horizontal or vertical externalities dominate depends – apart from the tax rate on rents – on the elasticities of the capital demand and the savings supply. The effect on tax competition of a change in the number of bottom-tier jurisdictions in this two-tier setting with a unique top-tier jurisdiction has been studied by Keen and Kotsogiannis (2004). Although the authors were unable to determine whether an increase in the number of bottom-tier jurisdictions would increase or decrease the equilibrium tax rates, they showed that it unambiguously deteriorates welfare because the fiercer tax competition worsens tax externalities. Wrede (1997) compares tax competition in a two-tier country composed of n top-tier jurisdictions with several bottom-tier jurisdictions inside each top-tier jurisdiction, and tax competition in a one-tier country composed of n jurisdictions. In other words, he analyzes the impact of an overall merger of bottom-tier jurisdictions with their higher-tier jurisdiction. It should be noted that bottom-tier governments disregard the impact of their tax policy on the budget constraint of their top-tier government and vice-versa, contrary to Keen and Kotsogiannis (2002, 2004).

The effect on tax competition of a change in the number of top-tier jurisdictions in a two-tier setting with several top-tier jurisdictions is, however, unknown, although corporate and personal income tax bases are often shared by several tiers of sub-national jurisdictions in OECD countries (Joumard and Kongsrud, 2003). Our paper addresses this issue.

Addressing this issue requires building a tax competition model in a two-tier structure composed of several top-tier jurisdictions, e.g., regions, and several bottom-tier jurisdictions, e.g., cities. We take the standard model of horizontal tax competition among local governments developed[1] by Wilson (1986) and Zodrow and Mieszkowski (1986) and superimpose an additional layer composed of several regional governments, in contrast to the papers by Keen and Kotsogiannis (2002, 2004). Both levels of governments, which are assumed to be benevolent, provide public goods that are financed through a tax on mobile capital invested in their territory. The Nash game for tax rates becomes more

[1] Specific assumptions are needed about citizens' preferences and the production technology to derive closed form solutions.

complex as a consequence. Tax externalities at work are thus the following: i) horizontal tax externalities among regions that compete to attract mobile capital, ii) horizontal tax externalities among cities that also compete to increase the amount of capital invested in their territory and iii) bilateral vertical externalities, i.e., top-down and bottom-up, that arise because tax decisions taken at any level affect the shared tax base. In this framework, we consider an exogenous symmetric merger of regions – or, equally, a reduction in the number of regions – and analyze its impact on tax policies. While the total number of cities in the country remains stable, the number of cities inside each region increases accordingly. Therefore, contrary to Wrede (1997) who addresses vertical mergers, we analyze horizontal mergers of regions.

In this framework, we also aim at pointing out the difference between a merger of regions and a coalition of regions, where a coalition of regions is an agreement among these regions to set a common tax rate. Our analysis of coalitions, which relates to papers in a one-tier setting by Burbidge, DePater, Myers and Sengupta (1997) and Konrad and Schjelderup (1999), is particularly simplified because several symmetric coalitions of regions are exogenously formed, thus having all regions involved in coalitions.

We show that the merger of regions generates three effects on the tax game played by regions and cities. The first effect is the *alleviation of tax competition at the regional level*, which reduces horizontal tax externalities at the regional level, as shown in the literature (Hoyt, 1991), as well as top-down vertical tax externalities. The second effect is the *rise in the regional tax base* or, equally, in the regional population. These two effects are regional and exert both a direct upward pressure on regional tax rates and an indirect downward pressure on local tax rates. The third effect is the *larger internalization of tax externalities generated by cities*. This effect is local and exerts a direct upward pressure on local tax rates and an indirect downward pressure on regional tax rates. In other words, the vertical transmission of these effects to the other level of jurisdictions – either top-down for the first two effects or bottom-up for the last one – tends to counteract the horizontal increasing trend following the merger. From the relative magnitude of these three effects, we determine regional and local tax strategies. We show that the merger of regions always increases regional tax rates while decreasing local tax rates.

The equilibrium tax rates after the merger are then compared with the equilibrium tax rates after the coalition of regions. For a low number of regions after the merger or equally a low number of coalitions, regional tax rates are lower and local tax rates are higher with the merger than with the coalition. For a high number of regions after the merger or equally a high number of coalitions, opposite results appear. For an intermediate level of mergers/coalitions, both regional and local tax rates are higher with the merger than with the coalition.

Our paper thus contributes to the significant theoretical literature on both horizontal (Wilson, 1986, Zodrow and Mieszkowski, 1986, Wildasin, 1988, Bucovetsky, 1991, among others) and vertical tax competition (Wrede, 1997, Besley and Rosen, 1998, Keen, 1998, Keen and Kotsogiannis, 2002, 2003, 2004) by studying tax competition in a two-tier framework with more than one top-tier jurisdiction. In addition to the analysis of the impact of the merger of top-tier jurisdictions in this framework, our work bridges a gap in the understanding of the difference between mergers (Hoyt, 1991, Bucovetsky, 2008, Keen and

Kotsogiannis, 2004) and coalitions of jurisdictions (Burbidge et al., 1997, Konrad and Schjelderup, 1999). A parallelism can also be made with the industrial organization literature on mergers of firms, for which a great deal has been produced (e.g., Salant et al, 1983, Spengler, 1950, Gaudet and Van Long, 1996, Ordover et al, 1990 for some seminal papers). For vertical mergers alone, more than 500 papers exist (see Rey and Tirole, 2007, for a survey). This attention of scholars is justified by the complexity of this practical issue, both to the extent of the determinants of mergers and to the range of effects of mergers on consumer prices. Surprisingly, even though mergers of jurisdictions are frequent, few papers have dealt with their ins and outs in fiscal federalism. Our paper is thus a contribution to the theoretical literature of fiscal federalism that sheds light on this merger issue.

The paper is organized as follows. Section 2 presents an original model of tax competition in a two-tier setting with several top-tier jurisdictions. Section 3 determines the benchmark outcome before the merger of regions. Section 4 proceeds to the analysis of the impact of the merger of regions on the tax game played by the regional and local players. Section 5 compares the equilibrium tax rates after the merger and after the coalition of regions. Concluding comments are provided in section 6.

2 The basic framework

Regional and local jurisdictions Consider a country with two levels of subnational jurisdictions, that is, $n > 1$ identical regions indexed by i and, within each region, $m > 1$ identical cities indexed by j, with nm local/city jurisdictions altogether. Note that the central/federal government plays no role, e.g., no central/federal transfers are granted to sub-national jurisdictions.

Each regional government i provides a regional public good in quantity G_i, which is financed by a tax τ_i levied on the amount of capital $K_i \equiv \sum_{j=1}^{m} K_{ij}$ invested in its region. The regional budget constraint is thus given by:

$$G_i = \tau_i \sum_{j=1}^{m} K_{ij}. \tag{1}$$

Each local government ij provides a local public good in quantity g_{ij}, which is financed by a tax t_{ij} levied on the amount of capital K_{ij} invested in its city. The local budget constraint is thus given by:

$$g_{ij} = t_{ij} K_{ij}. \tag{2}$$

We rule out public goods spillovers. Both regional and local governments are utilitarian and benevolent.

The representative citizen Citizens are assumed to be identical[2] and immobile. The representative citizen of the city ij derives a utility $v\left[g_{ij}\right]$ from the provision of the local public good g_{ij}, a utility $V\left[G_i\right]$ from the provision of the regional public good G_i and a utility c_{ij} from the consumption of a private good in quantity c_{ij} (to be defined below). The utility function of the representative citizen located in ij is thus given by:

$$U\left[c_{ij}, g_{ij}, G_i\right] = c_{ij} + v\left[g_{ij}\right] + V\left[G_i\right],$$

where the utility function $v\left[.\right]$ (resp. $V\left[.\right]$) is increasing in its argument, twice differentiable and concave.

To simplify the analysis, we assume that $v"\left[.\right] = 0$ and $V"\left[.\right] = 0$. Under this assumption, the utility functions $v\left[.\right]$ and $V\left[.\right]$ are linear[3]. The marginal utility derived from the local public good g_{ij} is proportional to the one derived from the regional public good G_i, that is, $v\prime = \alpha V'$, where α is a strictly positive parameter. In other words, the marginal value of an additional dollar of local (resp. regional) tax revenue is a constant. As a consequence, local and regional public goods are perfect substitutes since the marginal rate of substitution is constant. Alternative public policies to reduce emissions of CO_2 or to ensure the security of citizens may be examples of public goods that are perfect substitutes. Therefore, the utility function of the representative citizen is a linear combination of c_{ij}, g_{ij} and G_i.

The capital market Consider a unique firm in each city ij, which is owned by the representative citizen of the city. Firms are also assumed to be identical and immobile. The firm in ij makes a profit $\Pi_{ij} = F\left[K_{ij}\right] - r_{ij}K_{ij}$, which in its entirety is transferred to the representative citizen, where $F\left[.\right]$ is an increasing, thrice-differentiable and concave production function. We assume[4] that $F'''\left[.\right] = 0$, which will ensure that the net return on capital is linear w.r.t. $\boldsymbol{\tau}$ and $\mathbf{t_i}$ $\forall i$.

Capital K_{ij} used to produce the output is borrowed in the domestic capital market[5] and remunerated at a gross return r_{ij}. Firm profit maximizing behavior implies the familiar condition of remuneration at the marginal productivity of capital, that is, $F'\left[K_{ij}\right] = r_{ij}$ $\forall i, \forall j$. The resulting demand for capital $K_{ij}\left[r_{ij}\right]$ and profit $\Pi_{ij}\left[r_{ij}\right]$ are decreasing functions of the interest rate r_{ij}, *i.e.*, $K'_{ij}\left[r_{ij}\right] = \frac{1}{F"} < 0$ and $\Pi'_{ij}\left[r_{ij}\right] = -K_{ij} < 0$ $\forall i, \forall j$. Note that as a consequence of the assumption $F'''\left[.\right] = 0$, the demand for capital is a linear function of the interest rate r_{ij}.

Let $nm\overline{k}$ be the total amount of capital available in the country, where $\overline{k}$ is the exogenous amount of capital – i.e., the exogenous income – with which each citizen is initially endowed. This capital can be invested in a firm in any city ij to earn a net return on capital, denoted by ρ_{ij}, which is equal to the return after local and regional taxes.

[2] Admittedly, symmetry is a stark assumption; however, it allows us to simplify our analysis and to rule out any redistributive effects.

[3] As in Bucovetsky (2008), this specification is needed to derive closed-form solutions for equilibrium tax rates.

[4] The quadratic assumption is used by several papers on tax competition, including Grazzini and van Ypersele (2003), Devereux, Lockwood and Redoano (2008), Bucovetsky (2009).

[5] The capital market works in autarchy, as both lenders and borrowers reside in the country.

The private consumption of the representative citizen located in ij, denoted by c_{ij}, thus amounts to the sum of the profit of the firm and the net remuneration of the capital endowment:

$$c_{ij} = \Pi_{ij}\left[r_{ij}\right] + \rho_{ij}\overline{k}.$$

Capital is perfectly mobile in the country. It moves across cities and thus across regions to be located in the city where the net return is the most attractive. Since the net return on capital ρ_{ij} decreases when the cumulative tax rate $\tau_i + t_{ij}$ increases, the location choice of capital crucially depends on both regional and local tax choices, which generates i) horizontal tax externalities at both regional and local levels and ii) both bottom-up and top-down vertical tax externalities[6]. At the equilibrium, the net return on capital is the same everywhere, i.e.,

$$\rho = r_{ij} - \tau_i - t_{ij} \qquad \forall i, \forall j.$$

Given that $r_{ij} = \rho + \tau_i + t_{ij}$ $\forall i, \forall j$, the capital market-clearing condition $\sum_{i=1}^{n}\sum_{j=1}^{m} K_{ij}\left[\rho + \tau_i + t_{ij}\right] = nm\overline{k}$ implicitly defines the equilibrium value of the net return on capital, $\rho\left(\boldsymbol{\tau}, \mathbf{t_1}, ..., \mathbf{t_i}, ..., \mathbf{t_n}\right)$ with $\boldsymbol{\tau} = (\tau_1, ...\tau_n)$ and $\mathbf{t_i} = (t_{i1}, ..., t_{ij}, ..., t_{im})$ $\forall i$.

Differentiating the market-clearing condition yields, at the symmetric equilibrium:

$$\begin{aligned}
\frac{\partial \rho}{\partial \tau_i} &= -\frac{\sum_{j=1}^{m} K_{ij}'}{\sum_{i=1}^{n}\sum_{j=1}^{m} K_{ij}'} = -\frac{1}{n}, \quad \frac{\partial r_{ij}}{\partial \tau_i} = 1 + \frac{\partial \rho}{\partial \tau_i} = \frac{n-1}{n}, \quad \frac{\partial r_{ij}}{\partial \tau_{-i}} = \frac{\partial \rho}{\partial \tau_{-i}} = -\frac{1}{n}, \\
\frac{\partial \rho}{\partial t_{ij}} &= -\frac{K_{ij}'}{\sum_{i=1}^{n}\sum_{j=1}^{m} K_{ij}'} = -\frac{1}{nm}, \quad \frac{\partial r_{ij}}{\partial t_{ij}} = 1 + \frac{\partial \rho}{\partial t_{ij}} = \frac{nm-1}{nm}, \\
\frac{\partial r_{ij}}{\partial t_{-ij}} &= \frac{\partial r_{ij}}{\partial t_{i,-j}} = \frac{\partial \rho}{\partial t_{-ij}} = \frac{\partial \rho}{\partial t_{i,-j}} = -\frac{1}{nm}.
\end{aligned}$$

Let $\varepsilon_{\tau_i} = \frac{\partial(\sum_{j=1}^{m} K_{ij})}{\partial \tau_i}\frac{\tau_i}{\sum_{j=1}^{m} K_{ij}} < 0$ denote the elasticity of capital invested in region i with respect to region i's tax rate and $\varepsilon_{t_{ij}} = \frac{\partial K_{ij}}{\partial t_{ij}}\frac{t_{ij}}{K_{ij}} < 0$ denote the elasticity of capital invested in city ij with respect to city ij's tax rate. In line with empirical findings[7], we postulate that these elasticities belong to the interval $]-1, 0[$, which implies

[6] In Keen and Kotsogiannis (2002, 2004), there are only bottom-up vertical tax externalities. The absence of top-down vertical tax externalities is explained by the fact that the unique top-tier government maximizes the sum of all bottom-tier jurisdictions and therefore perfectly internalizes the top-down externalities.

[7] See Chirinko, Fazzari, and Meyer (1999) for instance.

$\frac{\partial \tau_i \sum_{j=1}^{m} K_{ij}}{\partial \tau_i} = (1 + \varepsilon_{\tau_i}) \sum_{j=1}^{m} K_{ij} > 0$ and $\frac{\partial t_{ij} K_{ij}}{\partial t_{ij}} = \left(1 + \varepsilon_{t_{ij}}\right) K_{ij} > 0$. Therefore, tax revenues of a jurisdiction always increase when its tax rate rises.

The timing of the game Sub-national jurisdictions play a Nash game. Regional governments simultaneously select their tax policy to maximize the welfare of the representative citizen residing within their region, taking as given tax policies chosen by the other regions and cities. Simultaneously, local governments select their tax policy to maximize the welfare of the representative citizen residing within their city, taking as given tax policies chosen by the other cities and regions. Regional and local public goods are determined as residuals after taxes are collected. Given these tax policies, firms determine the amount of capital that maximizes their profits, and production then takes place. Finally, profits are distributed, and citizens enjoy the consumption of both private and public goods. These two last stages are implicitly introduced in our analysis. Regional and local governments take into account the reaction of the capital demand when choosing their tax strategy and citizens' preferences guide the choices of the governments, as both are benevolent.

Prior to the analysis with the merger of regions, we first present the outcome of tax competition in our two-tier setting. This outcome will serve as a benchmark for comparison purposes to highlight the impact of the merger of regions on the budgetary decisions at the symmetric equilibrium.

3 Tax competition before the merger of regions

3.1 The regional government's problem

Each regional government selects the tax rate that maximizes the welfare of citizens located in its region. In doing so, it takes as a given the tax rates chosen by cities and other regions. The program of the government of the region i is thus:

$$
\begin{aligned}
& \underset{\tau_i}{Max} \quad \sum_{j=1}^{m} \left(c_{ij} + v\left[g_{ij}\right] + V\left[G_i\right]\right) \\
& s.t. \\
c_{ij} &= \Pi_{ij}\left[r_{ij}\right] + \rho \overline{k} \\
g_{ij} &= t_{ij} K_{ij}\left[r_{ij}\right], \\
G_i &= \tau_i \sum_{j=1}^{m} K_{ij}\left[r_{ij}\right].
\end{aligned}
$$

The first-order condition is:

$$\sum_{j=1}^{m}\left(\Pi'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}+\frac{\partial \rho}{\partial \tau_i}\overline{k}+v'\left(t_{ij}K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\right)+V'\left(1+\varepsilon_{\tau_i}\right)\sum_{j=1}^{m}K_{ij}\right)=0, \tag{4}$$

which determines the regional government's reaction function $\{\widehat{\tau}_i(\mathbf{t_1},...,\mathbf{t_i},...,\mathbf{t_n};\boldsymbol{\tau}_{-i})\}_i$. Note that at the symmetric equilibrium, distortive effects – through the net return on capital – on private consumption compensate each other, i.e., $\Pi'_{ij}\frac{\partial\rho}{\partial\tau_i}+\frac{\partial\rho}{\partial\tau_i}\overline{k}=0$ since $\Pi'_{ij}=-K_{ij}$ and $K_{ij}=\overline{k}\ \forall i,j$, implying that $\frac{\partial c_{ij}}{\partial\tau_i}=\Pi'_{ij}<0$. According to (4), each region i determines its tax rate to equalize the marginal costs of a reduction in private consumption and local public good provision, that is, $\frac{\partial c_{ij}}{\partial\tau_i}+v'\frac{\partial g_{ij}}{\partial\tau_i}<0$, and the marginal benefit of a rise in regional public good provision, that is, $V'\frac{\partial G_i}{\partial\tau_i}=V'\left(1+\varepsilon_{\tau_i}\right)\sum_{j=1}^{m}K_{ij}>0$, following an increase in τ_i.

The concavity of the regional government's problem is ensured when the following second-order condition is satisfied:

$$\left(-\frac{\partial r_{ij}}{\partial \tau_i}+2mV'\right)K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\leq 0, \tag{5}$$

which we assume in the sequel.

3.2 The local government's problem

Each local government chooses the tax rate that maximizes the utility of the representative citizen located in its city, given the tax rates chosen by regions and other cities. The program of the government of the city ij is thus:

$$\begin{aligned}
&\underset{t_{ij}}{Max}\quad c_{ij}+v\left[g_{ij}\right]+V\left[G_i\right]\\
&s.t.\\
c_{ij}&=\Pi_{ij}\left[r_{ij}\right]+\rho\overline{k}\\
g_{ij}&=t_{ij}K_{ij}\left[r_{ij}\right],\\
G_i&=\tau_i\sum_{j=1}^{m}K_{ij}\left[r_{ij}\right].
\end{aligned}$$

From the first-order condition,

$$\Pi'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}+\frac{\partial \rho}{\partial t_{ij}}\overline{k}+v'\left(1+\varepsilon_{t_{ij}}\right)K_{ij}+V'\tau_i\left(\sum_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)=0, \tag{7}$$

we determine the local government's reaction function $\{\widehat{t}_{ij}(\boldsymbol{\tau};\mathbf{t_1},...,\mathbf{t_{i,-j}},...,\mathbf{t_n})\}_{ij}$. Again note that at the symmetric equilibrium, distortive effects – through the net return on capital – on private consumption compensate each other, i.e., $\Pi'_{ij}\frac{\partial\rho}{\partial t_{ij}}+\frac{\partial\rho}{\partial t_{ij}}\overline{k}=0$ since

$\Pi'_{ij} = -K_{ij}$ and $K_{ij} = \overline{k}$ $\forall i,j$, implying that $\frac{\partial c_{ij}}{\partial t_{ij}} = \Pi'_{ij} < 0$. Furthermore, the marginal demands for capital are identical, i.e., $K'_{ik} = K'_{ij}$ $\forall i,j,k$, implying that $\frac{\partial G_i}{\partial t_{ij}} = \tau_i \left((m-1) K'_{ij} \frac{\partial \rho}{\partial t_{ij}} + K'_{ij} \frac{\partial r_{ij}}{\partial t_{ij}} \right) = \tau_i \left(m \frac{\partial \rho}{\partial t_{ij}} + 1 \right) K'_{ij}$. The tax rate chosen by the local government ij is such that it equalizes the marginal costs of a reduction in private consumption and regional public good provision, that is, $\frac{\partial c_{ij}}{\partial t_{ij}} + V' \frac{\partial G_i}{\partial t_{ij}} < 0$, and the marginal benefit of a rise in local public good provision, that is, $v' \frac{\partial g_{ij}}{\partial t_{ij}} = v' \left(1 + \varepsilon_{t_{ij}}\right) K_{ij} > 0$, following an increase in t_{ij}.

To ensure of the concavity of the local government's problem, the following second-order condition is assumed:

$$\left(-\frac{\partial r_{ij}}{\partial t_{ij}} + 2v' \right) K'_{ij} \frac{\partial r_{ij}}{\partial t_{ij}} \leq 0. \tag{8}$$

Solving the first-order conditions for all cities and regions simultaneously determines the Nash equilibrium levels of tax rates.

4 Tax competition after the merger of regions

4.1 The capital market after the merger

Suppose now that regions merge so that the total number of regions decreases from n to $\widetilde{n}$, i.e., $n > \widetilde{n}$. The merger is exogenously decided, i.e., we do not make explicit the forces that lead to the merger. In our symmetric setting, the merger resembles a territorial reorganization where former regions are broken up to constitute new regions, which are fewer in number. The number of cities inside each region changes accordingly; that is, it increases from[8] m to $\frac{nm}{\widetilde{n}}$. As a consequence, the regional tax base expands from $\sum_{j=1}^{m} K_{ij}$ to $\sum_{j=1}^{nm/\widetilde{n}} \widetilde{K}_{ij}$. Note that the total number nm of cities, frontiers of cities and thus local tax bases do not change.

We still consider nm firms in the country, each one owned by the representative citizen of the city. As before, capital relocates until the net return on capital is the same everywhere. The capital market-clearing condition becomes $\sum_{i=1}^{\widetilde{n}} \sum_{j=1}^{nm/\widetilde{n}} \widetilde{K}_{ij} [r_{ij}] = nm\overline{k}$, which implicitly defines the equilibrium value of the net return on capital[9], $\widetilde{\rho} \left(\widetilde{\boldsymbol{\tau}}, \widetilde{\mathbf{t}}_{\mathbf{1}}, ..., \widetilde{\mathbf{t}}_{\mathbf{i}}, ..., \widetilde{\mathbf{t}}_{\widetilde{n}} \right)$ with $\widetilde{\boldsymbol{\tau}} = (\widetilde{\tau}_1, ...\widetilde{\tau}_{\widetilde{n}})$ and $\widetilde{\mathbf{t}}_{\mathbf{i}} = (\widetilde{t}_{i1}, ..., \widetilde{t}_{ij}, ..., \widetilde{t}_{i\frac{nm}{\widetilde{n}}})$ $\forall i$. Differentiating the market-clearing condition yields, at the symmetric equilibrium:

[8] We assume that $\frac{nm}{\widetilde{n}}$ is an integer.

[9] Let "$\sim$" be the notation used for the model after the merger of regions to differentiate from the absence of notation for the model before the merger of regions.

$$\frac{\partial \widetilde{\rho}}{\partial \tau_i} = -\frac{\sum_{j=1}^{nm/\widetilde{n}} \widetilde{K}'_{ij}}{\sum_{i=1}^{\widetilde{n}} \sum_{j=1}^{nm/\widetilde{n}} \widetilde{K}'_{ij}} = -\frac{1}{\widetilde{n}}, \quad \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} = 1 + \frac{\partial \widetilde{\rho}}{\partial \tau_i} = \frac{\widetilde{n}-1}{\widetilde{n}}, \quad \frac{\partial \widetilde{r}_{ij}}{\partial \tau_{-i}} = \frac{\partial \widetilde{\rho}}{\partial \tau_{-i}} = -\frac{1}{\widetilde{n}},$$

$$\frac{\partial \widetilde{\rho}}{\partial t_{ij}} = -\frac{\widetilde{K}'_{ij}}{\sum_{i=1}^{\widetilde{n}} \sum_{j=1}^{nm/\widetilde{n}} \widetilde{K}'_{ij}} = -\frac{1}{nm}, \quad \frac{\partial \widetilde{r}_{ij}}{\partial t_{ij}} = 1 + \frac{\partial \widetilde{\rho}}{\partial t_{ij}} = \frac{nm-1}{nm},$$

$$\frac{\partial \widetilde{r}_{ij}}{\partial t_{-ij}} = \frac{\partial \widetilde{r}_{ij}}{\partial t_{i,-j}} = \frac{\partial \widetilde{\rho}}{\partial t_{-ij}} = \frac{\partial \widetilde{\rho}}{\partial t_{i,-j}} = -\frac{1}{nm}.$$

We first note that the distortionary effect of local taxation is identical to the one before the merger of regions, i.e., $\frac{\partial \widetilde{\rho}}{\partial t_{ij}} = \frac{\partial \rho}{\partial t_{ij}} = -\frac{1}{nm}$; consequently, $\frac{\partial r_{ij}}{\partial t_{ij}} = \frac{\partial \widetilde{r}_{ij}}{\partial t_{ij}} = \frac{nm-1}{nm}$ $\forall i, \forall j$. Indeed, as the number of cities remains stable and equal to nm, the merger of regions has no impact on the fierceness of horizontal tax competition at the local level, i.e., $\frac{\partial}{\partial \widetilde{n}}\left(\frac{\partial \widetilde{\rho}}{\partial t_{ij}}\right) = \frac{\partial}{\partial \widetilde{n}}\left(\frac{\partial \widetilde{r}_{ij}}{\partial t_{ij}}\right) = 0$. Due to the assumption of fixed supply of capital, we will show that the allocation of capital among cities does not change.

In contrast, the distortionary effect of regional taxation decreases after the merger of regions, i.e., $\frac{\partial \widetilde{\rho}}{\partial \tau_i} = -\frac{1}{\widetilde{n}} < \frac{\partial \rho}{\partial \tau_i} = -\frac{1}{n}$; consequently, $\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} < \frac{\partial r_{ij}}{\partial \tau_i}$ $\forall i, \forall j$, where $\frac{\partial \rho}{\partial \tau_i} = -1$ and $\frac{\partial r_{ij}}{\partial \tau_i} = 0$ when capital is completely inelastic or equally without regional tax competition for capital. In other words, the market share of each region, which is equal to $\frac{1}{\widetilde{n}}$, rises with the merger. The merger of regions thus induces a reduction in the responsiveness of both the net return on capital and the interest rate to an increase in the regional tax rate, i.e., $\frac{\partial}{\partial \widetilde{n}}\left(\frac{\partial \widetilde{\rho}}{\partial \tau_i}\right) = \frac{\partial}{\partial \widetilde{n}}\left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right) = \frac{1}{\widetilde{n}^2}$. Horizontal tax competition for capital at the regional level becomes less fierce.

To improve the readability of the paper, we will use the notation "$\sim$" only when the response is different from the one before the merger of regions. Therefore, $\widetilde{K}_{ij}$, $\frac{\partial \widetilde{r}_{ij}}{\partial t_{ij}}$ and $\frac{\partial \widetilde{\rho}}{\partial t_{ij}}$ will respectively be written as K_{ij}, $\frac{\partial r_{ij}}{\partial t_{ij}}$ and $\frac{\partial \rho}{\partial t_{ij}}$.

Let $\widetilde{\varepsilon}_{\tau_i} = \frac{\partial (\sum_{j=1}^{nm/\widetilde{n}} K_{ij})}{\partial \tau_i} \frac{\tau_i}{\sum_{j=1}^{nm/\widetilde{n}} K_{ij}} < 0$ denote the elasticity of capital invested in region i with respect to region i's tax rate. Note that $\widetilde{\varepsilon}_{t_{ij}} = \varepsilon_{t_{ij}}$.

In summary, the merger of regions increases each regional tax base since the fixed national supply of capital is equally divided into fewer regions at the symmetric equilibrium. The merger of regions also affects the responsiveness of the net return on capital only with respect to regional tax rates.

Note that due to the linearity of $V[.]$ w.r.t. G_i, the marginal utility derived by citizens from the regional public good provision is not affected by the merger. In addition, to simplify the analysis, we rule out economies of scale in the regional public good provision following the increase in the size of each region.

Subsequently, we study the impact on local and regional tax choices of these changes.

4.2 The regional government's problem

Each regional government chooses the tax rate τ_i that maximizes the utility of citizens located in its region, taking as given the tax choices of other regions and cities. It solves the problem:

$$\begin{aligned}
& \underset{\tau_i}{Max} \quad \sum_{j=1}^{nm/\widetilde{n}} \left(c_{ij} + v\left[g_{ij}\right] + V\left[G_i\right]\right) \\
& s.t. \\
c_{ij} &= \Pi_{ij}\left[r_{ij}\right] + \rho\overline{k} \\
g_{ij} &= t_{ij} K_{ij}\left[r_{ij}\right], \\
G_i &= \tau_i \sum_{j=1}^{nm/\widetilde{n}} K_{ij}\left[r_{ij}\right],
\end{aligned}$$

which leads to the following first-order condition:

$$\sum_{j=1}^{nm/\widetilde{n}} \left(\Pi'_{ij} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + \frac{\partial \widetilde{\rho}}{\partial \tau_i}\overline{k} + v' t_{ij} K'_{ij} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + V'\left(1 + \widetilde{\varepsilon}_{\tau_i}\right) \sum_{j=1}^{nm/\widetilde{n}} K_{ij} \right) = 0, \tag{10}$$

and second-order condition:

$$\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + 2\frac{nm}{\widetilde{n}} V' \right) K'_{ij} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} \leq 0. \tag{11}$$

To ensure the concavity of the regional government's problem, we subsequently assume that:

Assumption CONC1: $2\frac{nm}{\widetilde{n}} V' \geq \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}$.

The tradeoff that faces each regional government in setting its tax rate is the same as the one before the merger, except that it now considers i) the widening of the regional tax base, i.e., $\sum\limits_{j=1}^{nm/\widetilde{n}} K_{ij} > \sum\limits_{j=1}^{m} K_{ij}$, and ii) the change in the responsiveness of the net return on capital – and thus on the interest rate – to an increase in its regional tax rate via $\frac{\partial \widetilde{\rho}}{\partial \tau_i}$, $\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}$ and $\widetilde{\varepsilon}_{\tau_i}$.

4.3 The local government's problem

Simultaneously, each local government chooses the tax rate t_{ij} that maximizes the utility of the representative citizen located in its city, taking as given the tax choices of other cities and regions. It solves the problem:

$$
\begin{array}{rcl}
& & \underset{t_{ij}}{Max} \quad c_{ij} + v\left[g_{ij}\right] + V\left[G_i\right] \\
& & s.t. \\
c_{ij} & = & \Pi_{ij}\left[r_{ij}\right] + \rho\overline{k} \\
g_{ij} & = & t_{ij} K_{ij}\left[r_{ij}\right], \\
G_i & = & \tau_i \sum\limits_{j=1}^{nm/\widetilde{n}} K_{ij}\left[r_{ij}\right],
\end{array}
$$

which yields the following first-order condition:

$$
\Pi'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}} + \frac{\partial \rho}{\partial t_{ij}}\overline{k} + v'\left(1+\varepsilon_{t_{ij}}\right)K_{ij} + V'\tau_i\left(\sum_{k\neq j} K'_{ik}\frac{\partial \rho}{\partial t_{ij}} + K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right) = 0, \qquad (13)
$$

and second-order condition:

$$
\left(-\frac{\partial r_{ij}}{\partial t_{ij}} + 2v'\right)K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}} \leq 0. \qquad (14)
$$

The SOC requires to subsequently assume that:

Assumption CONC2: $2v' \geq \frac{\partial r_{ij}}{\partial t_{ij}}$.

As seen before, a reduction in t_{ij} induces capital to flee other cities to relocate in the city ij, which becomes more attractive. This strategic game at the local level will affect the regional tax base and thus regional tax receipts intended for the regional public good provision. Following the widening of the regional tax base due to the merger, the local government now internalizes for more cities the negative externalities it generates by raising its tax rate. However, the local government located in city ij does not internalize the negative externalities it generates on cities that do not belong to region i.

4.4 Tax rates at the symmetric equilibrium and conditions of positivity

At the symmetric equilibrium, the amount of capital K_{ij} invested in each city ij is equal to the exogenous amount of capital $\overline{k}$ each representative citizen is initially endowed with. Using $K_{ij} = \overline{k}\ \forall i, \forall j$, we know that $\frac{\partial c_{ij}}{\partial \tau_i} = \frac{\partial c_{ij}}{\partial t_{ij}} = \Pi'_{ij} = -\overline{k}$ and $K'_{ij} = \frac{1}{F"\left[\overline{k}\right]}\ \forall i, \forall j$. The FOCs (10) and (13) thus reduce to:

$$
\begin{cases}
-\overline{k} + v' t_{ij} K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + V'\left(\frac{nm}{\widetilde{n}}\overline{k} + \tau_i\frac{nm}{\widetilde{n}}K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right) = 0, \\
-\overline{k} + v'\left(\overline{k} + t_{ij}K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right) + V'\tau_i\left(\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}} + 1\right)K'_{ij} = 0
\end{cases}
$$

Solving this system of FOCs for all regions and cities simultaneously, we derive the tax rates chosen by regions and cities at the symmetric Nash equilibrium, hereafter denoted as $\widetilde{\tau}_i^*$ and $\widetilde{t}_{ij}^*$ $\forall i, \forall j$:

$$\widetilde{\tau}_i^* = \frac{\left(\left(\frac{nm}{\widetilde{n}}V' - 1\right) \frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} - (v' - 1) \right)}{-V'K'_{ij}\left(\frac{nm}{\widetilde{n}} - 1\right)} \overline{k}, \tag{15}$$

$$\widetilde{t}_{ij}^* = \frac{\left(\frac{nm}{\widetilde{n}}V' - 1\right) - \left(\left(\frac{nm}{\widetilde{n}}V' - 1\right) \frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} - (v' - 1) \right) \frac{\frac{nm}{\widetilde{n}}}{\left(\frac{nm}{\widetilde{n}} - 1\right)} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}}{-v'K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} \overline{k}. \tag{16}$$

As tax rates must be positive by assumption to ensure that public goods are provided, the following conditions of positivity are required[10]:

Assumption POSτ_i: $\left(\frac{nm}{\widetilde{n}}V' - 1\right) \frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} \geq (v' - 1)$.

Assumption POSt_{ij}: $(v' - 1) \geq \left(\frac{nm}{\widetilde{n}}V' - 1\right) \left(\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} - \frac{\frac{nm - \tilde{n}}{nm}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} \right)$.

In the remainder of the paper, results are derived provided that assumptions CONC1, CONC2, POSτ_i and POSt_{ij} are satisfied.

Using $v' = \alpha V'$, we derive:
Lemma 1: $\alpha > 1$
Proof: See Appendix A

Lemma 2: $\alpha V' > 1$
Proof: See Appendix B

Therefore, the marginal utility derived from the local public good must be both higher than the utility derived from the regional public good and higher than one to ensure that tax rates are positive. In particular, this rules out the case $v' = V' = 1$.

4.5 Implications of the merger of regions

We now use the comparative statics to examine in more detail the role of the merger of the n regions into $\tilde{n}$ regions on regional and local taxes. Differentiating the first-order conditions after the merger, i.e., (10) and (13), with respect to τ_i, t_{ij} and $\widetilde{n}$ yields the following system of equations in matrix form[11]:

[10] The need for conditions of positivity is explained by the linearity of the utility derived from local and regional public goods.

[11] Contrary to the subsection 4.4., where equilibrium tax rates were calculated at the symmetric equilibrium, we here consider that K_{ij} is endogenous as a function of r_{ij}.

$$
\begin{bmatrix}
\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+2V'\frac{nm}{\widetilde{n}}\right)K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} & \begin{array}{c} -K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+v'K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} \\ +V'\left(\sum\limits_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)\end{array} \\
\begin{array}{c} -K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+v'K'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} \\ +V'\left(\sum\limits_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)\end{array} & \left(-\frac{\partial r_{ij}}{\partial t_{ij}}+2v'\right)K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}
\end{bmatrix}
\begin{bmatrix} \frac{d\tau_i}{d\widetilde{n}} \\ \frac{dt_{ij}}{d\widetilde{n}} \end{bmatrix}
$$

$$
= \begin{bmatrix}
-v't_{ij}K'_{ij}\frac{\partial\left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}}-V'\frac{\partial}{\partial \widetilde{n}}\left((1+\widetilde{\varepsilon}_{\tau_i})\sum\limits_{j=1}^{nm/\widetilde{n}}K_{ij}\right) \\
-V'\tau_i\frac{\partial}{\partial \widetilde{n}}\left(\left(\sum\limits_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)\right)
\end{bmatrix}
$$

Using Cramer's rule gives:

$$
\frac{d\tau_i}{d\widetilde{n}} = \frac{\det B}{\det A}, \tag{17a}
$$

$$
\frac{dt_{ij}}{d\widetilde{n}} = \frac{\det C}{\det A}, \tag{17b}
$$

where[12]

$$
\det A = K'^2_{ij}\left(\begin{array}{c}\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+2V'\frac{nm}{\widetilde{n}}\right)\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+2v'\right)\frac{\partial r_{ij}}{\partial t_{ij}} \\ -\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+v'\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+V'\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right)^2\end{array}\right),
$$

$$
\det B = K'^2_{ij}\left(\begin{array}{c}
-\frac{\partial\left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}}\left(v't_{ij}+V'\frac{nm}{\widetilde{n}}\tau_i\right)\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+2v'\right)\frac{\partial r_{ij}}{\partial t_{ij}} \\
+\frac{\partial\left(\frac{nm}{\widetilde{n}}\right)}{\partial \widetilde{n}}\frac{V'}{(-K'_{ij})}\left(K_{ij}+\tau_iK'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+2v'\right)\frac{\partial r_{ij}}{\partial t_{ij}} \\
+\frac{\partial\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \widetilde{n}}V'\tau_i\left(-\frac{\partial r_{ij}}{\partial t_{ij}}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+v'\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+V'\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right)
\end{array}\right)
$$

and

$$
\det C = K'^2_{ij}\left(\begin{array}{c}
\frac{\partial\left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}}\left(v't_{ij}+V'\frac{nm}{\widetilde{n}}\tau_i\right)\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+v'\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+V'\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right) \\
+\frac{\partial\left(\frac{nm}{\widetilde{n}}\right)}{\partial \widetilde{n}}\frac{V'}{K'_{ij}}\left(K_{ij}+\tau_iK'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+v'\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+V'\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)\right) \\
-\frac{\partial\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \widetilde{n}}V'\tau_i\left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}+2V'\frac{nm}{\widetilde{n}}\right)\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}
\end{array}\right).
$$

[12] Invoking symmetry, i.e. $K_{ij}=\overline{k}$ and $K'_{ij}=\frac{1}{F"\left[\overline{k}\right]}$ $\forall i, \forall j$, we simplify the following expressions:

$\frac{\partial}{\partial \widetilde{n}}\left((1+\widetilde{\varepsilon}_{\tau_i})\sum\limits_{j=1}^{nm/\widetilde{n}}K_{ij}\right)=\frac{\partial\left(\frac{nm}{\widetilde{n}}\right)}{\partial \widetilde{n}}\left(K_{ij}+\tau_iK'_{ij}\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)+\sum\limits_{j=1}^{nm/\widetilde{n}}\tau_iK'_{ij}\frac{\partial\left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}}$

and $\left(\sum\limits_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)=\left(1+\frac{nm}{\widetilde{n}}\frac{\partial \rho}{\partial t_{ij}}\right)$.

Let us first determine the sign of $\det A$.

Lemma 3: $\det A > 0$
Proof: See Appendix C

The sign of the impact of the merger of regions on regional tax rates, i.e., $\frac{d\tau_i}{d\widetilde{n}}$, is thus the one of $\det B$ and the sign of the impact of the merger of regions on local tax rates, i.e., $\frac{dt_{ij}}{d\widetilde{n}}$, is the one of $\det C$. The signs of $\det B$ and $\det C$ result from the interplay of three effects generated by the merger of regions, which are transmitted either horizontally or vertically. To simplify the exposition of the three effects and their transmission, let[13]:

$$\begin{aligned}
E1 &= -\left(v' t_{ij} + V' \frac{nm}{\widetilde{n}} \tau_i\right) K'_{ij} \frac{\partial \left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}} > 0, \\
E2 &= -V' \frac{\partial \left(\frac{nm}{\widetilde{n}}\right)}{\partial \widetilde{n}} \left(K_{ij} + \tau_i K'_{ij} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right) > 0, \\
E3 &= -V' \tau_i K'_{ij} \frac{\partial \left(1 + \frac{nm}{\widetilde{n}} \frac{\partial \rho}{\partial t_{ij}}\right)}{\partial \widetilde{n}} > 0, \\
HORIREG &= \left(-\frac{\partial r_{ij}}{\partial t_{ij}} + 2v'\right) K'_{ij} \frac{\partial r_{ij}}{\partial t_{ij}} < 0, \\
HORILOC &= \left(-\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + 2V' \frac{nm}{\widetilde{n}}\right) K'_{ij} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} < 0, \\
VERTI &= \left(-\frac{\partial r_{ij}}{\partial t_{ij}} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + v' \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + V' \left(1 + \frac{nm}{\widetilde{n}} \frac{\partial \rho}{\partial t_{ij}}\right)\right) K'_{ij} < 0,
\end{aligned}$$

so that the expressions of $\det B$ and $\det C$ can be rewritten as follows:

$$\det B = \underbrace{E1 * HORIREG}_{<0} + \underbrace{E2 * HORIREG}_{<0} \underbrace{- E3 * VERTI}_{>0},$$

and

$$\det C = \underbrace{-E1 * VERTI}_{>0} \underbrace{- E2 * VERTI}_{>0} + \underbrace{E3 * HORILOC}_{<0}.$$

The first effect, denoted by $E1$, results from the *alleviation of horizontal tax competition at the regional level* following the merger. The merger of regions reduces the distortionary effect of regional taxation, i.e., $-\frac{\partial \left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}} < 0$, thus lowering the incentive for regional governments to set inefficiently low tax rates, as in Hoyt (2001). The race to the bottom among regions consequently slows down.

Beyond the reduction of horizontal tax externalities at the regional level, the alleviation of regional tax competition also reduces top-down tax externalities. Indeed, the reduction

[13] We know that $HORILOC < 0$ from **CONC1**, $HORIREG < 0$ from **CONC2** and $VERTI = \left(-\frac{\partial r_{ij}}{\partial t_{ij}} \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + v' \frac{\partial \widetilde{r}_{ij}}{\partial \tau_i} + V' \left(1 + \frac{nm}{\widetilde{n}} \frac{\partial \rho}{\partial t_{ij}}\right)\right) K'_{ij} = \left(-\frac{\partial r_{ij}}{\partial t_{ij}} + v' + V'\right) \left(\frac{\widetilde{n}-1}{\widetilde{n}}\right) K'_{ij} < 0$ from **Lemma 2.**

in the distortionary effect of regional taxation lowers the sensitivity of the demand for capital to the regional tax rate $K'_{ij}\frac{\partial\left(\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}\right)}{\partial \widetilde{n}}$ in each city j inside the region i due to the tax-base sharing.

The second effect, denoted by $E2$, is the *rise in the regional tax base* following the merger or, equally, the increase of the regional population. Consequently, the regional tax rate – and the tax revenues because $1 + \widetilde{\varepsilon}_{\tau_i} > 0$ – increases; thus, more regional public goods are provided.

The third effect, denoted by $E3$, captures the *larger internalization of tax externalities generated by any city ij* following the merger. Each city ij neglects the impact its tax choice t_{ij} has on the provision of local public goods in other cities; however, it considers the externalities generated by a change in its tax rate on the regional tax base and thus on the regional public good provided inside its region. The reduction in the number of regions, which increases the regional tax base, thus favors the rise in the local tax rate because each city internalizes tax externalities for a larger amount of capital.

The first two effects are regional, as they measure the impact of the merger on tax choices made by regions whereas the third one is local, as it measures the impact of the merger on tax choices made by cities. These three effects are both horizontally transmitted, i.e., at the regional (resp. local) level if they are regional (resp. local) at a weight $HORIREG$ (resp. $HORILOC$), and vertically transmitted, i.e., at the regional (resp. local) level if they are local (resp. regional) at a weight $VERTI$. The effects $E1$, $E2$ and $E3$ all tend to favor an increase of tax pressure following the merger of regions when they are horizontally transmitted. Indeed, the *alleviation of horizontal tax competition at the regional level* ($E1$) and the *rise in the regional tax base* ($E2$) both drive regional tax rates upwards, while the *larger internalization of tax externalities generated by any city ij* ($E3$) yields to higher local tax rates, following a decrease in $\widetilde{n}$. However, due to the overlapping structure, these effects are also vertically transmitted, either top-down or bottom-up, where they counteract the increase of tax pressure. Indeed, the *alleviation of horizontal tax competition at the regional level* ($E1$) and the *rise in the regional tax base* ($E2$) both tend to reduce local tax rates, while the *larger internalization of tax externalities generated by any city ij* ($E3$) encourages regions to lower their tax rate, following the merger of regions.

Therefore, the overall impact of the merger on tax rates is a mix of composite effects, which are transmitted both horizontally and vertically. Note that for $K'_{ij} = 0\ \forall j$, capital becomes inelastic to a change in the gross return on capital, and all these effects vanish.

The relative magnitude of these three effects determine the signs of $\det B$ and $\det C$. It follows that:

Proposition 1: *The merger of regions always increases regional tax rates.*
Proof: See Appendix D

Proposition 2: *The merger of regions always decreases local tax rates.*
Proof: See Appendix E

The merger of regions has an unambiguous impact on both regional and local tax rates. Following the territorial reorganization, regions – in fewer number – reduce their tax rates whereas cities increase their tax rates. We learn from Proposition 1 and Proposition 2 that the *alleviation of horizontal tax competition at the regional level* and the *rise in the regional tax base*, i.e., $E1$ and $E2$ (when transmitted at a weight $HORIREG$ at the regional level and $VERTI$ at the local level), overcome the *larger internalization of tax externalities generated by any city* ij, i.e., $E3$ (when transmitted at a weight $VERTI$ at the regional level and $HORILOC$ at the local level).

5 Mergers of regions versus coalitions of regions

This section analyzes the difference between a merger and a coalition of regions. The merger of regions induces a complete reorganization of jurisdictions because former regions disappear to make up new and bigger – composed of more cities and thus of more inhabitants – regions, which has the three effects detailed above: i) the *alleviation of horizontal tax competition at the regional level* ($E1$), ii) the *rise in the regional tax base* ($E2$) and iii) the *larger internalization of tax externalities generated by any city* ij ($E3$). In contrast, the coalition of regions, which coordinate to jointly choose their tax rate, does not change the territorial organization, that is, the number of regions and the number of cities inside each region remain stable. The three effects $E1$, $E2$, $E3$ are thus absent. In particular, the distortionary effect of regional taxation does not change after the coalition of regions, i.e., $\frac{\partial \rho}{\partial \tau_i} = \frac{1}{n}$.

To stress the difference between a coalition and a merger, we first define the problem faced by regional and local governments when regions collude. Contrary to Konrad and Schjelderup (1999), we partition all regions into coalitions, i.e., we consider $\widetilde{n}$ exogenous coalitions of $\frac{n}{\widetilde{n}}$ regions. These $\widetilde{n}$ coalitions play noncooperatively. We then compare the Nash equilibrium with mergers and the one with coalitions.

5.1 The regional government's problem with coalitions

In the economy described in section 2, we suppose that $\tilde{n}$ coalitions of $\frac{n}{\tilde{n}}$ regions are exogenously formed. In each coalition, the $\frac{n}{\tilde{n}}$ regions jointly choose a common tax rate that applies to all members, taking as given the tax rates chosen by other coalitions, to maximize the utility of the citizens located in their territory, that is,:

$$\begin{aligned} & \underset{\tau_i}{Max} \quad \sum_{i=1}^{n/\widetilde{n}} \sum_{j=1}^{m} \left(c_{ij} + v\left[g_{ij} \right] + V\left[G_i \right] \right) \\ & s.t. \\ c_{ij} &= \Pi_{ij}\left[r_{ij} \right] + \rho \overline{k} \\ g_{ij} &= t_{ij} K_{ij}\left[r_{ij} \right], \\ G_i &= \tau_i \sum_{j=1}^{m} K_{ij}\left[r_{ij} \right]. \end{aligned}$$

which yields to the following first-order condition[14]:

$$\sum_{j=1}^{m}\left(-K_{ij}+v' t_{ij}K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}+V'\left(\sum_{j=1}^{m}K_{ij}+\tau_i\sum_{j=1}^{m}K'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}\right)\right)+\sum_{l\neq i}^{n/\tilde{n}}\sum_{j=1}^{m}\left(v' t_{lj}K'_{lj}\frac{\partial \rho}{\partial \tau_i}+V'\tau_l\sum_{j=1}^{m}K'_{lj}\frac{\partial \rho}{\partial \tau_i}\right)=0. \tag{19}$$

The first term of the regional FOC when regions collude is the FOC with no coalition, i.e., the FOC (4). The second term captures the fact that horizontal and vertical tax externalities previously generated by each one of the $\frac{n}{\tilde{n}}$ regions are now internalized inside the coalition.

5.2 The local government's problem with coalitions

The problem faced by each local government is the same as the problem before the merger of regions because the formation of coalitions does not change the regional tax base and thus the provision of the regional public good. Therefore, the FOC condition that determines the tax choice of the city ij is the same as (7), i.e.,:

$$\Pi'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}+\frac{\partial \rho}{\partial t_{ij}}\overline{k}+v'\left(1+\varepsilon_{t_{ij}}\right)K_{ij}+V'\tau_i\left(\sum_{k\neq j}K'_{ik}\frac{\partial \rho}{\partial t_{ij}}+K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)=0, \tag{20}$$

5.3 Comparison of the Nash equilibria at the symmetric equilibrium

As in the subsection 4.4, we compute the tax rates chosen by regions and cities at the symmetric Nash equilibrium, hereafter denoted by τ_i^{C*} and t_{ij}^{C*} $\forall i,\forall j$. Using $K_{ij}=\overline{k}$ and $K'_{ij}=\frac{1}{F"\left[\overline{k}\right]}$ $\forall i,\forall j$, we solve the following system of the FOCs (19) and (20) for all regions and cities simultaneously:

$$\begin{cases} -\overline{k}+v' t_{ij}K'_{ij}\left(1+\frac{n}{\tilde{n}}\frac{\partial \rho}{\partial \tau_i}\right)+V'm\left(\overline{k}+\tau_i K'_{ij}\left(1+\frac{n}{\tilde{n}}\frac{\partial \rho}{\partial \tau_i}\right)\right)=0,\\ -\overline{k}+v'\left(\overline{k}+t_{ij}K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}\right)+V'\tau_i\left(m\frac{\partial \rho}{\partial t_{ij}}+1\right)K'_{ij}=0,\end{cases}$$

which yields to:

$$\tau_i^{C*} = \frac{\left(mV'-1\right)\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\left(1+\frac{n}{\tilde{n}}\frac{\partial \rho}{\partial \tau_i}\right)}-\left(v'-1\right)}{-V'K'_{ij}\left(m-1\right)}\overline{k}, \tag{21}$$

$$t_{ij}^{C*} = \frac{\left(v'-1\right)-\frac{\left(mV'-1\right)\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\left(1+\frac{n}{\tilde{n}}\frac{\partial \rho}{\partial \tau_i}\right)}-\left(v'-1\right)}{(m-1)}\left(m\frac{\partial \rho}{\partial t_{ij}}+1\right)}{-v'K'_{ij}\frac{\partial r_{ij}}{\partial t_{ij}}}\overline{k} \tag{22}$$

[14] At the symmetric equilibrium, we know that $\Pi'_{ij}\frac{\partial r_{ij}}{\partial \tau_i}+\frac{\partial \rho}{\partial \tau_i}\overline{k}=-K_{ij}$ and $\sum_{l\neq i}^{n/\tilde{n}}\sum_{j=1}^{m}\left(\Pi'_{lj}\frac{\partial r_{lj}}{\partial \tau_i}+\frac{\partial \rho}{\partial \tau_i}\overline{k}\right)=0$.

We then compare tax rates with coalitions and tax rates with mergers at the symmetric equilibrium. Results are summarized by Proposition 3:

Proposition 3: *Assuming $V' > 1$, the comparison at the symmetric equilibrium between tax rates with mergers of regions and tax rates with coalitions of regions reveals three cases:*

i) for $\widetilde{n} < \frac{(\alpha V' - 1)}{\left(\left(\alpha - \frac{nm-1}{nm}\right) V' - \frac{1}{nm} \right)}$, *then* $\widetilde{\tau}_i^* < \tau_i^{C*}$ *and* $\widetilde{t}_{ij}^* > t_{ij}^{C*}$;

ii) for $\frac{\left((\alpha V' - 1) + (m-1) V' \right)}{\left(\left(\alpha - \frac{n-1}{n}\right) V' - \frac{1}{nm} \right)} > \widetilde{n} > \frac{(\alpha V' - 1)}{\left(\left(\alpha - \frac{nm-1}{nm}\right) V' - \frac{1}{nm} \right)}$, *then* $\widetilde{\tau}_i^* > \tau_i^{C*}$ *and* $\widetilde{t}_{ij}^* > t_{ij}^{C*}$;

iii) for $\frac{\left((\alpha V' - 1) + (m-1) V' \right)}{\left(\left(\alpha - \frac{n-1}{n}\right) V' - \frac{1}{nm} \right)} < \widetilde{n}$, *then* $\widetilde{\tau}_i^* > \tau_i^{C*}$ *and* $\widetilde{t}_{ij}^* < t_{ij}^{C*}$.

Proof: See Appendix F

Assuming $V' > 1$, i.e., that regional public expenditures are valued at least as highly as private consumption, the comparison between mergers and coalitions crucially depends on the value of $\widetilde{n}$. For a low number of regions after the merger or, equally, a low number of coalitions, regional tax rates are lower, but local tax rates are higher, with mergers than with coalitions. In contrast, for a high number of regions after the merger or, equally, a high number of coalitions, regional tax rates are higher but local tax rates are lower, with mergers than with coalitions.

Note that without assuming any threshold on V', we can also identify three cases (see Appendix F). In particular, for $V' < 1$, we can show that $\widetilde{\tau}_i^*$ is always higher than τ_i^{C*}.

6 Conclusion

Beyond building a tax competition model in a two-tier setting with several top-tier jurisdictions, which generates i) horizontal tax externalities at both top and bottom tiers and ii) both top-down and bottom-up vertical tax externalities, our paper analyzes the impact of a merger of top-tier jurisdictions on tax policies. Two top-tier (or regional) effects and one bottom-tier (or local) effect, both of which are horizontally and vertically transmitted, result from the merger. The two regional effects are shown to overcome the local effect. Therefore, the merger of regions increases regional tax rates while decreasing local tax rates.

The merger of regions – which induces a territorial reorganization where former regions are broken up to constitute fewer new regions – differentiates from the coalition of regions – which is an agreement among regions to set a common tax rate – notably by the absence of the three effects detailed above. The comparison at the symmetric equilibrium of the impact on tax competition of these two different types of regional changes depends on the extent of the merger/cooperation of regions.

This work contributes to a better understanding of the consequences of a merger of regions on tax competition in a multilayer structure and therefore fuels the debate on territorial reorganization that has taken place in most OECD countries. Extensions in this two-tier framework are numerous. The merger of regions may be asymmetric in the sense that regions after the merger may differ in terms of population. The merger of regions may also be partial if only some regions merge. The determinants of the merger may also be made endogenous, which would raise new issues concerning the stability of mergers of jurisdictions. Finally, future research could take into consideration other types of territorial reforms such as the merger of jurisdictions that belong to two different tiers, i.e., the merger of a region and some cities.

7 Appendices

7.1 Appendix A: Proof of Lemma 1

Replacing v' by its value $\alpha V'$, **POS**t_{ij} reduces to $(\alpha - 1)\frac{nm}{\widetilde{n}}V' \geq \frac{nm}{\widetilde{n}} - 1$. Because $nm > \widetilde{n}$, we know that α can never be lower than 1. *Q.E.D.*

7.2 Appendix B: Proof of Lemma 2

From **POS**τ_i and **POS**t_{ij}, we know that $\left(\frac{nm}{\widetilde{n}}V' - 1\right)\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} \geq \left(\frac{nm}{\widetilde{n}}V' - 1\right)\left(\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} - \frac{\frac{nm-\widetilde{n}}{nm}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}}\right)$, which boils down to the following condition after substituting the value for $\frac{\partial r_{ij}}{\partial t_{ij}}$:

$$\left(\frac{nm}{\widetilde{n}}V' - 1\right)(nm - \widetilde{n}) \geq 0 \tag{23}$$

Because $nm > \widetilde{n}$, we deduce that $\frac{nm}{\widetilde{n}}V' > 1$. Therefore, from **POS**t_{ij}**,** i.e., $(v' - 1) \geq \left(\frac{nm}{\widetilde{n}}V' - 1\right)\frac{\widetilde{n}}{nm}$, we can infer that $v' > 1$ or equally $\alpha V' > 1$. *Q.E.D.*

7.3 Appendix C: Proof of Lemma 3

We proceed by contradiction. Assume that $\det A < 0$. Substituting the values for $\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}$, $\frac{\partial r_{ij}}{\partial t_{ij}}$, $\frac{\partial \rho}{\partial t_{ij}}$ and using $v' = \alpha V'$, we obtain that $\det A < 0$ iif[15]:

$$\widetilde{n} > 1 + 2(nm - 1)\frac{\left(2\alpha V' - \frac{nm-1}{nm}\right)}{\left((\alpha + 1)^2 V' - 2\frac{nm-1}{nm}\right)}. \tag{24}$$

POSτ_i, requires[16]:

$$\widetilde{n} < \frac{V'(nm - 1) + (\alpha V' - 1)}{\left(\alpha V' - \frac{1}{nm}\right)}, \tag{25}$$

[15] $(\alpha + 1)^2 V' - 2\left(\frac{nm-1}{nm}\right) > 0$ because it is the sum of two positive terms, i.e. $\left(\alpha^2 + 1\right)V' - \frac{nm-1}{nm}$, which is positive from **CONC2** and **Lemma 1** and $2\alpha V' - \frac{nm-1}{nm}$, which is positive from **CONC2**.

[16] $\left(\alpha V' - \frac{1}{nm}\right) > 0$ from **CONC2** and provided that $nm > 3$, which is always true.

to make sure that the regional tax rate is strictly positive.

But the conditions (24) and (25) are mutually incompatible. Indeed, $1+2\left(nm-1\right)\frac{\left(2\alpha V'-\frac{nm-1}{nm}\right)}{\left((\alpha+1)^2V'-2\frac{nm-1}{nm}\right)}<\frac{V'(nm-1)+(\alpha V'-1)}{\left(\alpha V'-\frac{1}{nm}\right)}$ iif $\begin{pmatrix}\left((3\alpha+1)\,V'-2\right)nm\\ +(\alpha+1)\end{pmatrix}(\alpha-1)\left(nm-1\right)V'<0$, which is impossible from **Lemma 1** and **Lemma 2**. *Q.E.D.*

7.4 Appendix D: Proof of Proposition 1

Let $\det B'=K_{ij}'^{2}\begin{pmatrix}-\frac{\partial\left(\frac{\partial\widetilde{r}_{ij}}{\partial\tau_i}\right)}{\partial\widetilde{n}}\left(v't_{ij}+V'\frac{nm}{\widetilde{n}}\tau_i\right)\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+2v'\right)\frac{\partial r_{ij}}{\partial t_{ij}}\\ +\frac{\partial\left(1+\frac{nm}{\widetilde{n}}\frac{\partial\rho}{\partial t_{ij}}\right)}{\partial\widetilde{n}}V'\tau_i\begin{pmatrix}-\frac{\partial\widetilde{r}_{ij}}{\partial\tau_i}\frac{\partial r_{ij}}{\partial t_{ij}}+v'\frac{\partial\widetilde{r}_{ij}}{\partial\tau_i}\\ +V'\left(1+\frac{nm}{\widetilde{n}}\frac{\partial\rho}{\partial t_{ij}}\right)\end{pmatrix}\end{pmatrix}$

such that $\det B-\det B'=K_{ij}'^{2}\left(\frac{\partial\left(\frac{nm}{\widetilde{n}}\right)}{\partial\widetilde{n}}\frac{V'}{\left(-K_{ij}'\right)}\left(K_{ij}+\tau_iK_{ij}'\frac{\partial\widetilde{r}_{ij}}{\partial\tau_i}\right)\left(-\frac{\partial r_{ij}}{\partial t_{ij}}+2v'\right)\frac{\partial r_{ij}}{\partial t_{ij}}\right)$, which is always <0 from **CONC2** and given $\widetilde{\varepsilon}_{\tau_i}\in]-1,0[$. Demonstrating that $\det B'$ is negative ensures that $\det B$ is negative too. After replacing $\frac{\partial\widetilde{r}_{ij}}{\partial\tau_i}$, $\frac{\partial r_{ij}}{\partial t_{ij}}$ and $\frac{\partial\rho}{\partial t_{ij}}$ by their values and substituting v' by $\alpha V'$, the expression of $\det B'$ boils down to:

$$K_{ij}'^{2}\frac{\partial\left(\frac{\widetilde{n}-1}{\widetilde{n}}\right)}{\partial\widetilde{n}}V'\begin{pmatrix}-\left(\alpha t_{ij}+\frac{nm}{\widetilde{n}}\tau_i\right)\left(-\frac{nm-1}{nm}+2\alpha V'\right)\frac{nm-1}{nm}\\ +\tau_i\left(-\frac{nm-1}{nm}+(\alpha+1)\,V'\right)\frac{\widetilde{n}-1}{\widetilde{n}}\end{pmatrix}.$$

Because $K_{ij}'^{2}\frac{\partial\left(\frac{\widetilde{n}-1}{\widetilde{n}}\right)}{\partial\widetilde{n}}V'>0$, and $\left(-\frac{nm-1}{nm}+2\alpha V'\right)>\left(-\frac{nm-1}{nm}+(\alpha+1)\,V'\right)$ from **Lemma 1**, a sufficient condition to ensure that $\det B'$ is negative is that $-\left(\alpha t_{ij}+\frac{nm}{\widetilde{n}}\tau_i\right)\frac{nm-1}{nm}+\tau_i\frac{\widetilde{n}-1}{\widetilde{n}}<0$ or, equally, $\alpha t_{ij}\frac{nm-1}{nm}+\tau_i\frac{nm-\widetilde{n}}{\widetilde{n}}>0$, which is always true. Therefore $\det B'$ and consequently $\det B$ are negative. Because $\det A>0$ from **Lemma 3**, $\frac{d\tau_i}{d\widetilde{n}}=\frac{\det B}{\det A}<0$. *Q.E.D.*

7.5 Appendix E: Proof of Proposition 2

After replacing $\frac{\partial\widetilde{r}_{ij}}{\partial\tau_i}$, $\frac{\partial r_{ij}}{\partial t_{ij}}$ and $\frac{\partial\rho}{\partial t_{ij}}$ by their values and substituting v' by $\alpha V'$, the expression of $\det C$ becomes:

$$K_{ij}'^{2}V'\frac{\widetilde{n}-1}{\widetilde{n}}\begin{pmatrix}\frac{\partial\left(\frac{\widetilde{n}-1}{\widetilde{n}}\right)}{\partial\widetilde{n}}\left(\alpha t_{ij}+\frac{nm}{\widetilde{n}}\tau_i\right)\left(-\frac{nm-1}{nm}+(\alpha+1)\,V'\right)\\ -\frac{\partial\left(\frac{nm}{\widetilde{n}}\right)}{\partial\widetilde{n}}\frac{1}{-K_{ij}'}\left(K_{ij}+\tau_iK_{ij}'\frac{\widetilde{n}-1}{\widetilde{n}}\right)\left(-\frac{nm-1}{nm}+(\alpha+1)\,V'\right)\\ -\frac{\partial\left(\frac{\widetilde{n}-1}{\widetilde{n}}\right)}{\partial\widetilde{n}}\tau_i\left(-\frac{\widetilde{n}-1}{\widetilde{n}}+2V'\frac{nm}{\widetilde{n}}\right)\end{pmatrix}.$$

After simplification,

$$\det C=K_{ij}'^{2}V'\frac{\widetilde{n}-1}{\widetilde{n}}\frac{1}{\widetilde{n}^2}\begin{pmatrix}\frac{1}{\widetilde{n}}\tau_i\left(1+2nm\left(\alpha V'-1\right)\right)\\ +\left(\alpha t_{ij}+\frac{nm}{-K_{ij}'}\left(K_{ij}+\tau_iK_{ij}'\right)\right)\left(-\frac{nm-1}{nm}+(\alpha+1)\,V'\right)+\tau_i\end{pmatrix},$$

which implies that $\det C > 0$ for:

$$\widetilde{n}\left(\left(\alpha t_{ij}+\frac{nm}{-K'_{ij}}\left(K_{ij}+\tau_i K'_{ij}\right)\right)\left(-\frac{nm-1}{nm}+(\alpha+1)V'\right)+\tau_i\right) > -\tau_i\left(1+2nm\left(\alpha V'-1\right)\right). \tag{26}$$

We know from **Lemma 2** that the right-hand side is negative. Given $\widetilde{\varepsilon}_{\tau_i} \in]-1,0[$ and $\left(-\frac{nm-1}{nm}+(\alpha+1)V'\right) > 0$ from **Lemma 2**, the condition (26) is always true because $\widetilde{n} > 0$. Therefore $\det C$ is positive. Because $\det A > 0$ from **Lemma 3**, $\frac{dt_{ij}}{d\widetilde{n}} = \frac{\det C}{\det A} > 0$. *Q.E.D.*

7.6 Appendix F: Proof of Proposition 3

We first compare regional tax rates with mergers, $\widetilde{\tau}_i^*$, and regional tax rates with coalitions, τ_i^{C*}, from their expressions given by (15) and (21). Rearranging, we show that $\widetilde{\tau}_i^* > \tau_i^{C*}$ for $(v'-1) > (V'-1)\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}}$ and $\widetilde{\tau}_i^* < \tau_i^{C*}$ for $(v'-1) < (V'-1)\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}}$. Note that $\widetilde{\tau}_i^*$ is always higher than τ_i^{C*} for $V' < 1$ from **Lemma 2**.

We then compare local tax rates with mergers, $\widetilde{t}_{ij}^*$, and local tax rates with coalitions, t_{ij}^{C*}, from their expressions given by (16) and (22). Rearranging, we show that $\widetilde{t}_{ij}^* > t_{ij}^{C*}$ for $\begin{pmatrix} \left(\frac{nm}{\widetilde{n}}V'-1\right) \\ -\frac{mV'(nm-\widetilde{n})(n-1)}{\widetilde{n}(nm-1)} \end{pmatrix}\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} > (v'-1)$ and $\widetilde{t}_{ij}^* < t_{ij}^{C*}$ for $\begin{pmatrix} \left(\frac{nm}{\widetilde{n}}V'-1\right) \\ -\frac{mV'(nm-\widetilde{n})(n-1)}{\widetilde{n}(nm-1)} \end{pmatrix}\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} < (v'-1)$.

Assuming $V' > 1$, we can easily specify these conditions with respect to $\widetilde{n}$. The condition $(v'-1) > (V'-1)\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}}$ boils down to $\widetilde{n} > \frac{(\alpha V'-1)}{\begin{pmatrix} \left(\alpha-\frac{nm-1}{nm}\right)V' \\ -\frac{1}{nm} \end{pmatrix}}$ and $\begin{pmatrix} \left(\frac{nm}{\widetilde{n}}V'-1\right) \\ -\frac{mV'(nm-\widetilde{n})(n-1)}{\widetilde{n}(nm-1)} \end{pmatrix}\frac{\frac{\partial r_{ij}}{\partial t_{ij}}}{\frac{\partial \widetilde{r}_{ij}}{\partial \tau_i}} > (v'-1)$ boils down to $\widetilde{n} < \frac{\begin{pmatrix} (\alpha V'-1) \\ +(m-1)V' \end{pmatrix}}{\begin{pmatrix} \left(\alpha-\frac{n-1}{n}\right)V' \\ -\frac{1}{nm} \end{pmatrix}}$. From these conditions, we derive the three cases. *Q.E.D.*

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