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Cumulative Effect of Inequality: A Computational Study of Conflict Models

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Abstract

This paper examines the relationship between inequality and economic growth using conflict models and computational simulations. I construct a dynamic sequential conflict model that allows us to observe the cumulative effect of inequalities in wealth and ability in a single framework. The computational simulation illustrates the dynamics of the wealth level of players. The main findings are as follows: (1) if the conflict is not intensive, then the equilibrium that achieves equal distribution of wealth is unique and stable; (2) if the conflict is intensive, the equal distribution equilibrium becomes an unstable saddle point; and furthermore (3) when the productivity of one player is lower than the other, the less productive player exploits the other through the conflict process.

Keywords: conflict, conflict model, inequality, war.

JEL Classifications: D74, D72, D63

1 Introduction

The aim of conflict models is to formalize the conflict between two (or more) groups in a society and to analyze its general characteristics. This paper uses the conflict model and computer simulations to examine the relationship between the inequality and the economic growth.

The pioneer research on conflict models of Boulding (1962) was first formalized by Bush and Mayer (1974). They constructed a game theoretical model without assuming that property rights are given. In their model, players need to fight for their own property rights against each other and they also can exploit the property of another player. Since the

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fighting activity is unproductive, if they could cooperate and keep themselves from fighting, they would achieve higher welfare. However, every one has the incentive to exploit others and thus the cooperation cannot be equilibrium. The game was known as The Prisoner's Dilemma.

Skaperdas (1992) characterized three types of equilibria in a two-player conflict model; both players devote all endowments to the production (full cooperation); one of them still inputs all of the endowment to the production while the other exploits it (partial cooperation); and both of them fight and exploit against each other (conflict). The model has two parameters. One of them is the discrepancy in the productivity and the other is the intensity of conflict. The two-dimensional parameter space is divided into four regions of different equilibria.

The two parameters that Neary (1997) incorporates are the discrepancy in the players' initial endowments and the aggregate level of the endowments. Neary shows that there are four types of equilibria: full cooperation, partial cooperation, conflict, and banditry. In the bandit equilibrium, the player with less endowment invests nothing in the production and spends all for the fighting effort. This banditry occurs only when the aggregate level of endowments is sufficiently high and the discrepancy is also sufficiently large. That is, the hopeless poor player puts all of his endowment into the fighting activity.

In these two models, there are four parameters: the inequality in the initial endowments among players, the gap in the productivity among players, the total wealth level of the society, and the intensity of the conflict. Appealing to a computer simulation model, I incorporate all of these parameters into one framework. So, we could observe the cumulative effect of the inequalities in several aspects.

2 The Model

Suppose that there are two agents in the economy and that each of them divides his resource endowment into two efforts respectively: productive effort z and "fighting" effort y. The payoff of agents is

$$\pi_1(z_1, y_1, z_2, y_2) = p(y_1, y_2)F(z_1, z_2), \tag{1}$$

$$\pi_2(z_1, y_1, z_2, y_2) = (1 - p(y_1, y_2))F(z_1, z_2) \tag{2}$$

where $F(z_1, z_2)$ is the production and $p(y_1, y_2)$ is the share of agent 1. That is, in one hand, two agents combine their productive efforts so as to generate a pool of income available to themselves and on the other hand they determine their share by the function . In the literature of conflict models, p is often called the *Contest Success Function* (CSF). There are two specific forms of CSF proposed in the literature,

$$\text{Hirshleifer (1991):} \quad p(y_1, y_2) = \frac{\exp(y_1)^k}{\exp(y_1)^k + \exp(y_2)^k} \tag{3}$$

$$\text{Tullock (1980):} \quad p(y_1, y_2) = \frac{y_1^k}{y_1^k + y_2^k}. \tag{4}$$

Note that $p(y_1, y_2) = 0.5$ whenever $y_1 = y_2$. Both of these are S-shaped and the variable k parameterizes the intensity of the conflict process in the sense that k propagates the gap in the fighting efforts between the players. In one extreme case where $k \to 0$, it follows that $p \to 0.5$ regardless of fighting efforts. On the other extreme, if $k \to +\infty$, these functions will take on the so-called winner-take-all feature; the player who inputs more fighting effort than the other will take all of the production. The maximization problem of each agent is

$$\max_{y_1, z_1} \; p(y_1, y_2)F(z_1, z_2), \qquad \text{subject to } y_1 + z_1 \leq w_1$$

$$\max_{y_2, z_2} \; (1 - p(y_1, y_2))F(z_1, z_2), \qquad \text{subject to } y_2 + z_2 \leq w_2$$

where w_i is the initial endowment of agent i. By specifying the functions, this model can capture the nature of the social interactions between two groups of economic agents. For example, they might be two firms that jointly cultivate a new market and then compete for their shares; two employees contribute to the employer and then ask for the salary, etc. In the following subsection, I present a plausible application of the conflict model.

2.1 An Application of the Conflict Model

In this subsection, I present an application of the general conflict model. Let us assume that there are two groups of players, h and ℓ, and their wage rates are w_h and w_ℓ, respectively ($w_h > w_\ell$). They divide their endowment 1 into the productive activity L and the fighting activity y. The government imposes a linear income tax and redistributes the revenue to the two players evenly. Therefore, the consumption level of player i is

$$C_i = (1 - t)w_i L_i + G, \qquad \text{for } i = h, \ell, \tag{5}$$

where t is the tax rate and $G = 0.5t(w_h L_h + w_\ell L_\ell)$. The government is partially benevolent in the sense that it maximizes the following Cobb-Douglas social welfare function,

$$\max_t C_h^p C_\ell^{1-p}$$

and the weight p of this function is not given but determined by the fighting effort of two players, $p = p(y_h, y_\ell)$. The *optimal* tax that maximizes the social welfare for a given p is

$$t^*(y_h, y_\ell, L_h, L_\ell) = \frac{2w_h w_\ell - 2(w_h L_h + w_\ell L_\ell)p(y_h, y_\ell)}{w_h L_h - w_\ell L_\ell}. \tag{6}$$

By backward induction, the optimization problem of player i is

$$\begin{aligned} \max_{L_i} C_i \quad &= \quad \Big(1 - t^*(y_h, y_\ell, L_\ell, L_h)\Big) w_i L_i + \frac{1}{2} t^*(y_h, y_\ell, L_\ell, L_h)\big(w_h L_h + w_\ell L_\ell\big) \\ \text{s.t.} \quad & L_i + y_i = 1 \end{aligned}$$

Further calculation yields the following formulae:

$$\begin{aligned} \max_{L_h} C_h \quad &= \quad p(1 - L_h, 1 - L_\ell)\Big[w_h L_h + w_\ell L_\ell\Big] \\ \max_{L_\ell} C_h \quad &= \quad \Big\{1 - p(1 - L_h, 1 - L_\ell)\Big\}\Big[w_h L_h + w_\ell L_\ell\Big] \end{aligned}$$

These are exactly the same optimization problems that players solve in the general conflict models. Moreover, when p is Hirshleifer's (1991) CSF, it follows from Skaperdas (1992) that there is a unique Nash equilibrium and $p > 0.5$ at the equilibrium. It implies that the *optimal* tax rate is always positive and that the income is transferred from player h to player ℓ.

3 The Dynamics

Extension of the model of Neary (1997) into multi-round game shows the cumulative effect of inequality. In the following model, two players repeatedly interact within the same framework, but the endowments will differ. As the chart illustrates below, the payoff in the current round π_i^t becomes the endowment in the next round w_i^{t+1}. In this way, we can simulate the cumulative effect of inequality. The players are assumed to play Nash equilibrium strategy in each round independently of other rounds.

Definition (Long-run Equilibrium)**.** $\{w_1^t, w_2^t\}$ is a long-run equilibrium if the payoffs in round t, $\{\pi_1^t, \pi_2^t\}$, are equal to the initial endowment $\{w_1^t, w_2^t\}$.

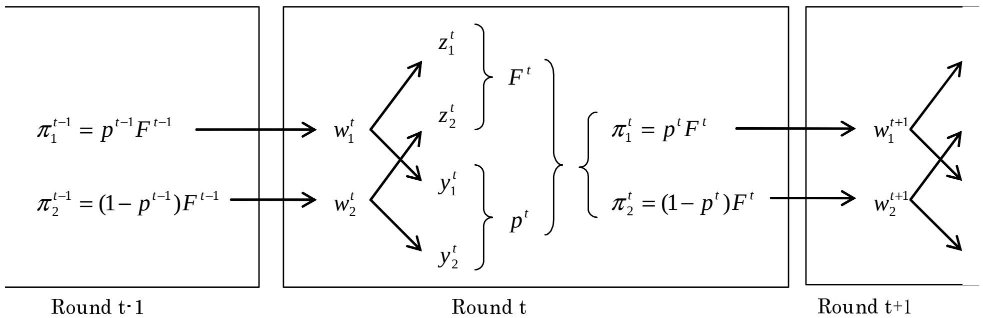

Figure 1: The sequential structure of the game

To run a simulation model, let us suppose that the production function is a Cobb-Douglas function and that CSF is Tullock's. Thus, the equilibrium is always an interior. The first-order conditions are

$$\partial\pi_1^t/\partial z_1^t = p^t F_1^t - p_1^t F^t = 0. \tag{7}$$

$$\partial\pi_2^t/\partial z_2^t = (1-p^t)F_2^t + p_2^t F^t = 0. \tag{8}$$

where F_i^t is the partial derivative of F^t with respect to z_i^t and p_i^t is the partial derivative with respect to the i-th argument. Rearranging the terms yields

$$(1-p^t)F^t = \frac{w_1^t - z_1^t}{k} F_1^t. \tag{9}$$

$$p^t F^t = \frac{w_2^t - z_2^t}{k} F_2^t. \tag{10}$$

Summing up these two equations, we obtain

$$F^t = \frac{1}{k}\left((w_1^t - z_1^t)F_1^t + (w_2^t - z_2^t)F_2^t\right).$$

Since the production function is first-order homogeneous, $z_1^t F_1^t + z_2^t F_2^t = w_1^{t+1} + w_2^{t+1}$, and thus

$$\frac{w_1^{t+1} + w_2^{t+1}}{w_1^t + w_2^t} = \frac{\sigma^t F_1^t + (1-\sigma^t)F_2^t)}{1+k}$$

where σ^t is the share of player 1's endowment to the total endowment, $\sigma^t = w_1^t/(w_1^t + w_2^t)$. Note that the left-hand side of the equation is unity at any long-run equilibrium. A proposition for the symmetric long-run equilibrium follows: .

	Productivity is same $F = z_1^{0.5} z_2^{0.5}$	Productivity is different $F = z_1^{0.52} z_2^{0.48}$
$k = 1.45$ moderate conflict	**Panel A** There is a unique long-run equilibrium at which the distribution is equal. Moreover, any initial endowment will eventually reach the long-run equilibrium.	**Panel B** The unique long-run equilibrium is located in a different position in favor of player 2 who has lower productivity.
$k = 1.70$ intensive conflict	**Panel C** There are 3 long-run equilibria: one saddle point on the 45 degree line and two sinking points.	**Panel D** The two of the three long-run equilibria in Panel C no longer exist. At the remaining long-run equilibrium, player 2 who has the lower productivity exploited player 1.

Table 1: The comparison of equilibria

Proposition 1. *If the two players are identical, then there is a unique symmetric long-run equilibrium, at which,* $\frac{\partial F}{\partial z_1} = 1 + k$.

This result implies that economic growth will end at the point where the marginal product diminishes down to $1 + k$. Note that the marginal product is decreasing in the level of production. Thus, the intensive conflict indicated by large k limits the sustainable growth of the economy. For players, the intensive conflict means that the marginal profit of fighting effort is not negligible, since the conflict propagates the small gap in the fighting effort and results in the large gap in the distribution of the production. To find other long-run equilibria, I run a computer simulation in the next section.

4 The Simulation

The dynamics has already captured the inequality in the initial endowment, $\sigma^t = w_1^t/(w_1^t + w_2^t)$, at each round and the level of the aggregated endowment $w_1^t + w_2^t$. Thus, the simulation examines the effect of the other two factors. Those are the discrepancy in the productivity and the intensity of the conflict.

Table 1 summarizes the result of the simulations. For the gap in the productivity, I change the parameter of the Cobb-Douglas function. To observe the effect of the intensive conflict, I present two cases ($k = 1.45$ and 1.70) for illustration.

Figure 2 shows the trajectories of $\{w_1^t, w_2^t\}$ for $t = 1, 2, \ldots$. Each arrow connects two points from $\{w_1^t, w_2^t\}$ to $\{w_1^{t+1}, w_2^{t+1}\}$, the wealth of players before and after the game of round t. In Panel A (top-left), there is the unique long-run equilibrium that every trajectory eventually reaches. That is the symmetric long-run equilibrium, and at the long-run equilibrium the marginal product is equal to $1 + k$ or $\partial F^t / \partial z_1^t$. As seen in the figure, any path of economic growth converges to the equilibrium. This feature remains unchanged even after I introduce discrepancy in the productivity. In Panel B (top-right), in which player 1 has the higher productivity than player 2, the long-run equilibrium is located slightly upper-left and it is still a sinking point. It implies that player 2 exploits player 1 by investing her resource into the fighting effort. She does so, since the marginal profit to player 2 of fighting tends to be greater than that of productive effort.

Panel C shows an interesting result. Even though the productivities are identical, there could be another asymmetric long-run equilibrium. Moreover, the symmetric long-run equilibrium is no longer a sinking point, and the path to the symmetric long-run equilibrium is on the knife edge. It implies that any noise or turbulence in the distribution would lead the economy to one of the asymmetric long-run equilibria. Finally, let us observe the effect of the unequal productivity. Unlike the previous change from Panel A to Panel B, the simulation shows that the dispersion in the productivity is propagated by the conflict. Through the intensive conflict, player 2 could exploit player 1 further. I summarize these observations below.

Observations:

1. There is a unique symmetric long-run equilibrium when the players are identical. (Panel A and Panel C)
2. When the conflict is moderate, the symmetric long-run equilibrium is the global attractor, and thus it is automatically attained regardless of initial distribution of wealth. (Panel A)
3. The symmetric long-run equilibrium is, however, an unstable saddle point if the conflict is intensive. (Panel C)
4. When the conflict is moderate, the gap in the productivity among players does not have significant impact on the distribution of wealth at the long-run equilibrium. (Observe the difference between Panel A and Panel B)

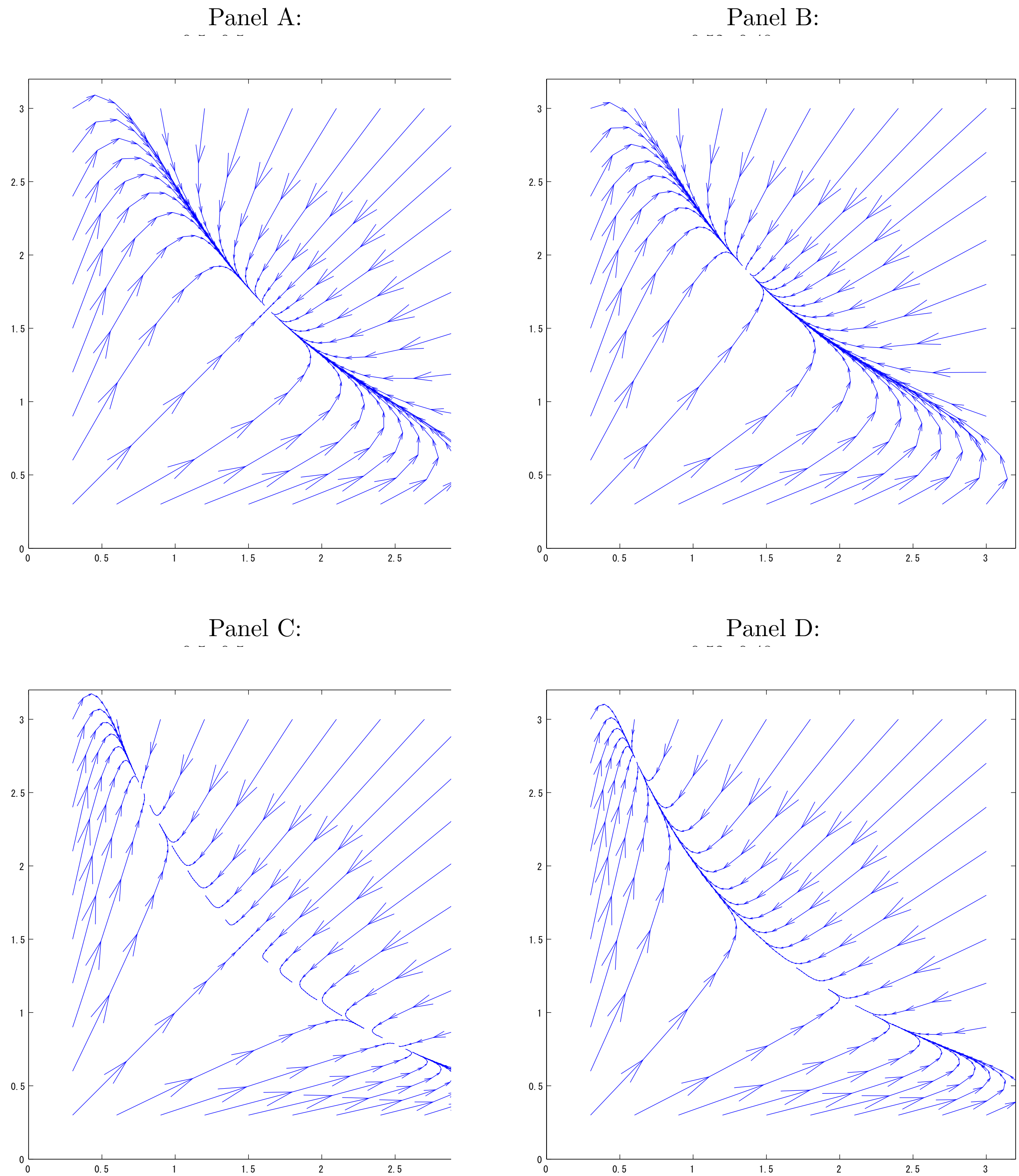

Figure 2: Simulations

Note. These figures illustrate the dynamics of sequential conflict games. Table 1 summarizes the result for each panel above.

5. When the conflict is intensive, the gap in the productivity among players has the impact on the significant distribution of wealth at the long-run equilibrium. (Observe the difference between Panel C and Panel D)

5 Concluding Remarks

I wish to conclude with two remarks. First, we should carefully interpret the "fighting" effort in this model. It does not necessarily mean physical fight, but it could be any form of social interaction that affects the distribution of wealth or resources such as politics, competition among employees, competition between firms, negotiation over salary, and so forth. As we live in the market mechanism and believe in it, we also believe that the distribution of wealth is basically determined by skills. Indeed, it is true. The skills, however, might include the ability of "fighting," which is characterized by the conflict model. Think of blue collar and white collar workers or executives. While the blue collar workers are skillful at making the products of a company, the executives are excellent in distributing the profit brought by the blue collar workers. Even though the executives are incapable of making any products of the company, they take a significant amount of the profit. Then, the simulation results appeal us, as Panel D shows that at the equilibrium in the dynamics player 2 who is less productive takes more than 80% of the total wealth. What we believe as the market mechanism might be part of conflict process.

Second, when society witnesses an increase in inequality, this tends to be attributed to the dispersion of productivity. The simulation result, however, suggests that the change in the nature of the conflict causes the change in the distribution of wealth. Suppose that the society has been around the symmetric long-run equilibrium in Panel A. Suppose that the society suddenly changes and the distribution process (or conflict) becomes more intensive. Then, the distribution of wealth departs from the 45 degree line and it will eventually reach either of the asymmetric long-run equilibria. Note that this could happen even if the production function and the productivity remain unchanged. Finally, I conjecture that these observations of the simulation can be fully or partially verified by theoretical analysis, which remains for future work.

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