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ANTI-LIMIT PRICING*

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Abstract

Extending Milgrom and Roberts (1982), we analyze an infinite horizon entry model where an incumbent may use its current price to signal its strength, in order to deter entry. In contrast with conventional limit pricing, we show that due to the importance of entrants' types on the post-entry duopoly/oligopoly profits, the incumbent may want to signal its weakness to invite the entry of weaker firms. We also provide necessary and sufficient conditions for this phenomenon to arise in equilibrium, in the benchmark cases that no second entry is profitable.

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I. *Introduction*

Since the seminal work of Spence (1973) the motive of informational signaling has been applied fruitfully to explain economic behavior in various contexts such as education, advertising, entry deterrence, corporate finance, etc.[1] The core argument of these explanations is that an agent with more favorable information would incur a cost to signal the information, if the market's inference on the favorable information were to lead to an outcome that would

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[1] A survey paper by Riley (2001) has an excellent discussion on this literature.

compensate for the signaling cost. In many situations each signaling need and its effect appear to be self-contained and hence may be analyzed in one-shot settings, as has been done in most signaling studies hitherto. In certain circumstances, however, signaling needs are recurrent and can evolve over time.

In a limit pricing context, in particular, an incumbent may face entry threats recurrently rather than once, and its response to new threats would change as the industry landscape evolves owing to entry. In such cases, signaling may have a long-lasting effect and hence needs to be evaluated from a more dynamic perspective. In this paper we conduct such an analysis and discover a new insight: By signaling its own weakness/inefficiency, an incumbent may want to invite more entry than there would have been otherwise. The basic logic behind this is that allowing such entries (which are relatively weak contenders) benefits the incumbent in the long-run by discouraging future entry by stronger contenders. We refer to this phenomenon as *anti-limit pricing* due to its contrast with conventional limit pricing in its motive (to promote, rather than deter, entry), and consequently, in the direction of the distortion of the signaling act.

A brief review of the limit pricing literature may be useful. Bain (1949) introduced the notion of limit price as the highest price that incumbents can charge without inducing entry, and establishes that "it is... consistent with such profit maximization by the established seller(s) that price will be held at the limit level *continually through time*" (p.455, emphasis added). His discussion remains informal, however, as to why entry decisions are influenced by pre-entry prices, and consequently, also on the determination of the limit price.

Several authors formalized this link, thereby rationalizing limit pricing. Milgrom and Roberts (1982) show that the incumbent may price below the myopic optimal level in order to signal its low cost and deter entry. Matthews and Mirman (1983) and Bagwell and Ramey (1990) illustrate similar pricing behavior to deter entry by signaling unfavorable market demand. Harrington (1986) shows that an incumbent may distort its monopoly pricing upwards to deter entry if the potential entrant does not know its own cost but knows that it is strongly correlated with the incumbent's cost.[2] Extending the analysis to allow for multiple incumbents, Harrington (1987) finds uncoordinated entry deterrence possible when the entrant observes only the market price, while Bagwell and Ramey (1991) find that no distortion is the only robust outcome if the entrant is able to observe the individual choices of the incumbents. In a quality-signaling model of experience goods, Overgaard (1994) shows that potential entry exacerbates the upward price distortion which had been known to prevail in such markets even in the absence of potential entry by, among others, Bagwell and Riordan (1991).

In all these papers incumbents distort price to *deter* potential entry, by way of *signaling adverse market conditions* for the entrant. Note that the price distortion from this motive can be upward, rather than downward, depending on the specific environments considered, i.e., in Harrington (1986) and Overgaard (1994).[3] The anti-limit pricing also exhibits an upward price distortion, however, we stress that it results from a fundamentally different motive, namely, to *induce/promote* entry of a weak entrant who would not enter otherwise, by way of *signaling accommodating market conditions* for the entrant.

[2] If the correlation is weak, the standard limit pricing obtains.

[3] Bagwell and Ramey (1990) also considers a special case in which the incumbent may distort price upward when the firms engage in Bertrand competition after entry, but this happens essentially to "trick" the entrant into pricing high, rather than to influence the entry decision.

This contrast in the underlying motives is attributable, at least partly, to the fact that in all previous studies mentioned above the analysis was conducted in a two-period context[4]: If it is successful in deterring entry in the first period, the incumbent enjoys monopoly profit in the second period without any further entry threat. Two-period models illustrate the main effects of signaling very clearly. However, they do not capture a more dynamic effect of limit pricing, which arises because the price needs to be held at the limit level *continually* as Bain observed in the quote above. This constitutes the main force behind the anti-limit pricing as explained below.

With recurring entry threats, the incumbent cannot initiate the monopoly price even after it has succeeded in deterring entry, because it needs to deal with the new threat. However, this does not mean that limit pricing cannot be compensated because the compensation comes not from the incumbent's profit being equal to the monopoly profit after deterring entry, but from it being higher than what it would have been if there had been an entry. Hence, a limit price can play dual roles, namely, that of signaling strength, and that of compensating for the signaling loss by delaying a lower, post-entry profit. A potential downside of this which is absent in two-period models is that entry, when it happens, brings in a tougher competitor due to the signalled strength of the incumbent, which reduces the incumbent's post-entry profits. If this effect is large, an incumbent may want to signal weakness rather than strength.

In our model, a new potential entrant arrives in each period with a private type. As in Milgrom and Roberts (1982), a weak firm would enter only if it infers the incumbent to be weak, hence can be deterred by proper signaling, but a strong firm would enter regardless of the incumbent's type. Continued limit pricing therefore delays entry but eventually results in a strong entry after which the incumbent will be left with but a small duopoly profit due to tougher competition. Alternatively, if the incumbent appears to be weak by pricing high, even a weak entrant would enter, after which the incumbent's duopoly profit would be larger than that after a strong entry. In effect, this alternative behavior replaces the anticipated entry with an earlier yet weaker one on average.[5] If *i*) an arrived entrant is neither too likely to be strong nor too likely to be weak so that this replacement effect is significant, and *ii*) the incumbent is patient enough that the post-entry profit is important, then it pays off for the incumbent to signal weakness by pricing high, and thereby invite weak entrants in, for the sake of enhancing post-entry profits, which results in anti-limit pricing. On the other hand, conventional limit pricing tends to arise when the incumbent is less patient and the entrant is not too likely to be strong.

Note that both the direction of distortion and the impact of signaling are reversed between anti-limit pricing and conventional limit pricing. This means that the incumbents may take fundamentally different deterrence decisions when faced with recurrent entry threats rather than a single threat, potentially leading to vastly different policy implications. This is a caution also emphasized by Bernheim (1984), although informational issues make the nature of our analysis quite different from his.[6]

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[4] The only exception we know (in limit pricing literature) is Harrington (1984) who provide an infinite-horizon extension of the result reported in Harrington (1986).

[5] Hence, the decision whether to signal strength or weakness becomes a choice between a tough competitor later and a weak competitor now. In this sense anti-limit pricing can be regarded as a selection towards a weak type and early entry, as suggested by the referee.

[6] Specifically, he shows that policies making deterrence activity more costly, may have a perverse effect of

The observation that an incumbent might want to induce entry by a weak firm to preempt that by a strong firm has been documented: Rockett (1990) reports that Du Pont licensed its polyester, cellophane and nylon patents selectively to weaker potential competitors shortly before the patents expired, and Comanor (1964) cites similar motives among pharmaceutical firms. It has also been theoretically shown that in certain environments an incumbent, in the face of potential threat of a strong entrant, would find it optimal to induce a weak entrant that would have been deterred otherwise. Specifically, in complete information models in which a weak firm and a strong firm make entry decisions in a predetermined order, Rockett (1990) characterizes when it is optimal to license only to a weak firm to deter entry of a strong firm when the patent expires, and Ashiya (2000) characterizes when it is optimal for an incumbent to position itself in a differentiated market to allow entry of a weak firm, in order to deter a strong entrant that arrives later by overcrowding the market. In these studies, an incumbent can induce a weak entry by committing to an action that changes the continuation game to one that is accommodating for a weak firm. Thus, their arguments do not apply to environments in which such commitments are not available, which is the case in settings of repeated games with possibility of entry as in our model. In particular, in such settings it may not be possible for an incumbent to influence the entry decision of a weak firm as long as the information is complete. Under incomplete information, on the other hand, we show that an incumbent can induce entry by influencing the entrant's "perception" of the continuation game towards one that is more accommodating for a weak entrant, i.e., weak entry may be enticed via signaling.

A handful of papers exist on dynamic signaling. Noldeke and Van Damme (1990), Swinkels (1999) and Kremer and Skrzypacz (2007) study informed sellers who, due to lack of commitment ability, may incur costs for long enough to signal effectively to uninformed agents, who decide when as well as how to respond. Kaya (2009) examines the least costly way of signaling over time in a repeated setting with public history. Our work differs from these studies in that it emphasizes that the current signal influences not only the current response but how the entire market evolves afterwards, and derives new insights on the signaling behavior driven by the latter impact.

Section II presents an infinite-horizon model and an equilibrium concept. Section III focuses on environments where no more than one entry is viable and fully characterizes when anti-limit pricing arises, and Section IV provides an illustrative example. Section V demonstrates that anti-limit pricing is a phenomenon that arises in a wider class of environments and in a variety of forms. Section VI contains some concluding remarks. Some technical details are collected in Appendix.

II. *Model*

We use an infinite horizon version of the two-type model of Milgrom and Roberts (1982). A monopoly firm produces a product from period 0 onwards. At each future period $t=1, 2, \cdots$, one potential entrant arrives in the market with probability $\theta \in (0, 1)$, in which case it observes the market situation (to be detailed below), and decides whether to enter or not. If it does not enter, it leaves the market for good. No potential entrant arrives with probability $1-\theta$ in each

discouraging entry by lowering the value of entry due to more entries anticipated in the future.

period $t=1, 2, \cdots$.

Each firm i, either the incumbent or a potential entrant, has either an efficient production technology (a "strong" type, denoted by s) or an inefficient one (a "weak" type, denoted by w). A strong type is more efficient in the sense that its marginal cost is no higher than that of a weak type, i.e.,

$$c_w'(q) \geq c_s'(q) \geq 0, \ \ \forall q \geq 0, \tag{1}$$

where $c_z(\cdot)$ is the cost function of type $z \in \{s, w\}$. We assume that there is no fixed cost, i.e., $c_s(0)=c_w(0)=0$, which is mainly for expositional ease as explained at the end of Section III. The types are private and independent random variables that assume s and w with probabilities m_s and $m_w=1-m_s$, respectively, for each entrant arriving in each period, and with probabilities μ_s and $\mu_w=1-\mu_s$, respectively, for the incumbent.

We assume that the market demand remains the same across periods: we denote the inverse demand function by $p(q)$ and assume that it is continuously differentiable and $p'(q)<0$ for all $q>0$. If entry occurs in any period, the entrant pays a fixed entry/setup cost $K>0$ and, to be consistent with existing papers on limit pricing, it is assumed that the types of the firms in the market become commonly known between them (but not to future potential entrants) so that they engage in Cournot competition under complete information in that period and onwards, until another firm enters and joins Cournot competition in the same manner.[7] Each firm is maximizing its expected δ-discounted sum of profit stream net of any entry cost, where $\delta \in (0, 1)$ is the common discount factor.

Depending on how much of the past history each arriving potential entrant observes, the details of analysis change. To allow for any scope of signaling, we should assume that the entrant arriving in each period observes the market price of the immediately preceding period (as well as the number of existing firms). Alternatively, we may assume either that the entrant observes the number of periods that have passed, in addition to the price level of the last period; or that it observes the full history of price levels. Since observing the price level is equivalent to observing the (total) output level, we use the latter expression which proves useful.

For expositional clarity, we present the main analysis assuming that each potential entrant, upon arrival, only observes the last period's output level. In this case, since the continuation game from each period is undistinguished so long as what happened in the last period is the same, the equilibrium has the Markov property. In the alternative cases mentioned above, the equilibrium is more complex because the entrant updates its belief on the incumbent's type based on the observable history. Nonetheless, the main results of the paper carry through in these cases as explained in the latter part of Section III.

Since the arrived entrants are assumed to observe only the last period's output level without knowing how many periods have passed, they face the same game upon arrival provided no entry has occurred by then: In particular, they possess the same belief on the incumbent's types, placing probability μ_s for it being strong as of the beginning of the previous period. In other words, each entrant perceives the game as described above with itself arriving

[7] We do not consider the possibility of collusion which, we think, is a separate issue that goes beyond the purpose of this paper. Furthermore, the incentive to induce a weak entry appears robust to the collusion possibility because the incumbent's bargaining share of the collusion outcome would be larger when the partner is weak than when strong.

in period $t=1$. Conceptually, this model captures a market that has been a monopoly for an unspecified length of time.

In this section we define equilibrium presuming that once the market reaches duopoly no further entry is profitable. This is the case when the entry cost K is not recouped by even the best possible profit stream for a potential third firm in the market. If we use the notation $\pi_i^T(z_1, z_2, z_3)$ to denote the Cournot equilibrium profit of firm i when there are 3 firms in the market and firm i's type is $z_i \in \{s, w\}$, $i=1, 2, 3$, then this condition can be written as

$$\pi_3^T(w, w, s) < (1-\delta)K. \tag{2}$$

Let $\pi_z(q) := p(q)q - c_z(q)$ denote the monopoly profit of an incumbent of type $z \in \{s, w\}$ when it produces output q. To avoid inessential complications and facilitate analysis, we make the following standard assumptions.

1. The optimal (myopic) monopoly output levels, denoted by q_s and q_w for a strong and a weak type, respectively, are unique and $q_s > q_w > 0$.

2. The reaction curves are negatively sloped and the Cournot duopoly equilibrium is unique for every possible type configuration of a duopoly.[8]

Whether an incumbent exercises (anti-)limit pricing or not can be ambiguous without the first assumption. The second assumption eliminates coordination problem between duopoly firms, which is inessential for the purpose of this paper. These properties hold in familiar cases such as the one with a linear demand and constant marginal costs. We may now let $\pi_i^D(z_1, z_2)$ denote the (one-period) Cournot equilibrium duopoly profit of an incumbent ($i=1$) of type z_1 and that of an entrant ($i=2$) of type z_2. Since an s-type firm has a more efficient technology, it follows that

$$\pi_2^D(s, w) < \pi_2^D(w, w) < \pi_2^D(s, s) < \pi_2^D(w, s). \tag{3}$$

In period $t=0$ the incumbent chooses an output level q and earns a monopoly profit $\pi_{z1}(q)$. If entry occurs in the next period, the two firms earn duopoly profits $\pi_1^D(z_1, z_2)$ and $\pi_2^D(z_1, z_2)$ from that period onwards; If entry does not occur the incumbent maintains its monopoly position, and the continuation game is the same as the original game.

Since there will be no further entry once the market reaches duopoly due to (2), the continuation value (i.e., the discounted sum of the expected profit stream) of a z_1-type incumbent after a z_2-type firm's entry, and that of this entrant are, respectively, $\pi_1^D(z_1, z_2)/(1-\delta)$ and $\pi_2^D(z_1, z_2)/(1-\delta)$. Hence, according to (3), the entry decision of each entrant of either type is trivial regardless of the incumbent's type if $(1-\delta)K$ exceeds $\pi_2^D(w, s)$, is in between $\pi_2^D(s, s)$ and $\pi_2^D(w, w)$, or falls short of $\pi_2^D(s, w)$, leaving no scope of signaling. For the case that $\pi_2^D(s, s) < (1-\delta)K < \pi_2^D(w, s)$, on the other hand, only standard limit-pricing may arise because a weak potential entrant would never enter. Consequently, we focus on the case that

$$\pi_2^D(s, w) < (1-\delta)K < \pi_2^D(w, w) \tag{4}$$

so that a weak entrant may or may not enter depending on the probability that the incumbent is strong. This is the environment in which anti-limit pricing arises as will be elaborated in

[8] Sufficient conditions for this can be found in Vives (1996), pp.96-98, for example.

Section III and illustrated in Section IV.

Recall that, once an entry occurs, the Cournot duopoly outcome prevails in all future periods due to (2). By (3) and (4), therefore, a strong entrant always enters in equilibrium if there was no previous entry, which we take for granted below. A strategy of a weak entrant is a function $\tau(\cdot)$: $\mathbf{R}_+ \rightarrow [0, 1]$ where $\tau(q)$ is the probability that this entrant enters upon observing the monopoly output q in the previous period.

Given a weak entrant's strategy $\tau(\cdot)$, consider an optimal strategy of the incumbent, denoted by $\mathbf{q}=(q_0, q_1, \cdots)$, producing an output q_t in period $t=0, 1, \cdots$, provided that no entry has occurred by then. Being an optimal strategy, $\mathbf{q}$ is no worse than $\mathbf{q}_{\geq 1}=(q_1, q_2, \cdots)$, i.e., producing q_{t+1} in period $t=0, 1, \cdots$, so long as no entry has occurred. In addition, since $\mathbf{q}_{\geq 1}$ is optimal once period 1 is reached with no entry, $\mathbf{q}_{\geq 1}$ is no worse than $\mathbf{q}$ because $\mathbf{q}$ is also feasible in the continuation game. Hence, $\mathbf{q}$ and $\mathbf{q}_{\geq 1}$ are equivalent in period 1 if no entry has occurred. Applying the same logic repeatedly, we deduce that it is also optimal to employ $\mathbf{q}$ in each period until an entry occurs, i.e., always producing q_0 while a monopolist. By the same token, $\mathbf{q}$ is also equivalent to always producing q_t for any fixed $t=1, 2, \cdots$, while a monopolist. For our purpose of examining the incumbent's output levels while a monopolist, therefore, it suffices to consider Markov production strategies that do not vary across periods as long as no entry has occurred: A strategy of an incumbent of type $z \in \{s, w\}$ is a probability distribution $\sigma(\cdot; z)$ over $\mathbf{R}_+$, where $\sigma(A; z)$ denotes the probability that this incumbent chooses an output level in $A \subset \mathbf{R}_+$ while a monopolist.

Then, given τ, the incumbent's expected payoff from producing q as a monopolist is

$$\Pi_z(q, \tau(q))=\pi_z(q)+\delta\Bigg[\theta\left(m_s\frac{\pi_1^D(z, s)}{1-\delta}+m_w\tau(q)\frac{\pi_1^D(z, w)}{1-\delta}\right) \\ +\Big(1-\theta+\theta m_w(1-\tau(q))\Big)\Pi_z(q, \tau(q))\Bigg],$$

because a duopoly profit of $\pi_1^D(z, z_2)$ accrues to the incumbent in all future periods if an entrant of type $z_2 \in \{s, w\}$ arrives and enters, while the incumbent will face the same continuation game in the next period if no entry occurs. By rearranging the equation above, we obtain

$$\Pi_z(q, \tau(q))=\frac{\pi_z(q)+\delta\theta(m_s\pi_1^D(z, s)+m_w\tau(q)\pi_1^D(z, w))/(1-\delta)}{1-\delta\Big(1-\theta+\theta m_w(1-\tau(q))\Big)}. \tag{5}$$

Let $\beta : \mathbf{R}_+ \rightarrow [0, 1]$ denote a belief function that specifies a probability, $\beta(q)$, that an entrant attaches to the incumbent being strong upon observing the monopoly output q. We are now ready to define equilibrium when no more than one entry is viable, i.e., when (2) holds. It can be generalized without difficulty to the cases of more than one viable entry (Jun and Park, 2005).

Definition 1: A strategy profile (σ, τ) and a belief function β constitute a *perfect Bayesian equilibrium* (PBE) if

1) $\sigma(Q_z^*; z)=1$ and $\sigma(\mathbf{R}_+\backslash Q_z^*; z)=0$ where $Q_z^*=\arg\max_q \Pi_z(q, \tau(q)) \subset \mathbf{R}_+$ for $z=s, w$;
2) $\tau(q)=1$ (0, resp.) if $\beta(q)\pi_2^D(s, w)+(1-\beta(q))\pi_2^D(w, w)>(1-\delta)K(<(1-\delta)K$, resp.) for all $q \geq 0$,;

3) β is consistent with the Bayes' rule whenever possible.

Signaling in the current context is about influencing the potential entrant to believe that the incumbent is more (less) likely to be strong by producing a larger (smaller) output, with an aim to discourage (encourage) entry. It therefore seems most natural for the incumbent to be believed to be more likely to be strong after a larger output, and for the incumbent's output to affect entry decision through the belief it generates, but not directly: i.e.,

$$\beta(q) \leq \beta(q') \text{ if } q < q', \text{ and } \tau(q) = \tau(q') \text{ if } \beta(q) = \beta(q'). \tag{6}$$

We refer to such a PBE as a *monotone PBE* and focus on such equilibria in this paper.[9]

III. *Characterization When Only One Entry Is Viable*

In this section we analyze the cases that once the market reaches duopoly, no further entry is profitable owing to (2). Our focus is on the equilibria in which an incumbent sometimes distorts its output *downward*, or equivalently, its price *upward*, from the myopic optimum level, in order to promote entry. For a stark contrast with conventional limit pricing behavior, we say that anti-limit pricing arises if distortion never arises in the other direction.[10] Formally,

Definition 2: *Anti-limit pricing* arises in an equilibrium (σ, τ, β) if

$$\sigma([0, q_s); s) + \sigma([0, q_w); w) > 0, \text{ and } \sigma((q_s, \infty); s) = \sigma((q_w, \infty); w) = 0. \tag{7}$$

All entry is detrimental to the incumbent in the short-term. Any incentive to promote entry, therefore, comes from potential long-term benefits: inducing entry by a weak firm prevents entry by a strong firm in the future (which would take place if weak entry was deterred), which benefits the incumbent in the long-run because the duopoly profit is larger against a weak competitor than a strong one. For such long-term benefits to exist, it is necessary that a weak firm recovers the entry cost K when it enters against a weak incumbent, i.e., the second inequality of (4) holds, for otherwise a weak firm would never enter. Note that this and condition (2) may hold simultaneously if and only if $\pi_3^T(w, w, s) < \pi_2^D(w, w)$. We establish below that as long as this is the case the anti-limit pricing is the only sensible equilibrium outcome for an open set of environments.

At the center of the anti-limit pricing is a strong incumbent who desires to appear weak by imitating a weak incumbent's output to reap the aforementioned long-term benefits. If a weak entrant were to enter against an incumbent of unknown type (but not against a strong incumbent), then a weak incumbent would not mind such imitation because it would not affect the entry prospects that a weak incumbent faces.[11] As a result, a pooling equilibrium arises in which the two types of incumbent pool by producing the myopic optimum output level of a weak incumbent. Then, the first potential entrant that arrives enters regardless of its type, and the incumbent and the entrant earn the Cournot duopoly profits thereafter without further entry.

[9] All our results are true when the second condition of (6) is weakened as: $\tau(q) \geq \tau(q')$ if $\beta(q) = \beta(q')$ and $q < q'$.

[10] If we allow distortion in the other direction, the anti-limit pricing phenomenon arises in a wider class of environments, sometimes for a reason unrelated to promoting entry.

[11] Note that a strong firm would enter whenever a weak firm would.

A separating equilibrium results if a weak entrant were to enter against a weak incumbent but not against an incumbent of unknown type: Since producing a weak incumbent's output level would not prompt entry by a weak firm when the imitation by a strong incumbent is foreseen, to dissuade such imitation and achieve separation, a weak incumbent distorts its output below the myopic optimum and thereby induce entry by both types. A strong incumbent sticks with its own short-run optimum output and faces entry by only a strong type. If the incumbent is weak, therefore, a duopoly forms as soon as an entrant arrives regardless of its type. If the incumbent is strong, monopoly is maintained as long as the arrived potential entrant is weak; the first strong entrant that arrives will enter, forming a duopoly that will remain thereafter. Note that the anti-limit pricing is exercised by a strong incumbent in a pooling equilibrium, but by a weak incumbent in a separating equilibrium.

We now formally characterize the environments in which the anti-limit pricing arises. Suppose the following inequalities hold:

$$\max\{\pi_3^T(w, w, s), \pi_2^D(s, w)\} < (1-\delta)K < \pi_2^D(w, w). \tag{8}$$

Then,

(i) a weak type would enter against a weak incumbent but not against a strong incumbent;
(ii) a strong type would enter regardless of the incumbent's type;
(iii) no second entry would take place.

In a separating equilibrium with the anti-limit pricing as described earlier, a strong incumbent produces q_s, its one period monopoly output, and faces entry only by a strong type (i.e., $\tau(q_s)=0$), whilst a weak incumbent produces $q^* < q_w$ to induce entry by either type of entrant (i.e., $\tau(q^*)=1$). If the weak type produces any larger output[12], it would invite imitation by a strong incumbent and thereby discourage weak entry. Such an equilibrium exists, as verified below, if the following conditions are satisfied:

$$\Pi_s(0, 1) < \Pi_s(q_s, 0) < \Pi_s(q_w, 1), \text{ and} \tag{9}$$
$$\Pi_w(q_w, 0) < \Pi_w(q^*, 1) \text{ for some } q^* \in [0, q_M^*], \tag{10}$$

where $\Pi_z(q, r)$ is as defined in (5) with $\tau(q)=r$ and $q_M^* := \min\{q>0 \mid \Pi_s(q, 1) = \Pi_s(q_s, 0)\} \in (0, q_w)$. Since $\Pi_s(q, r)$ is continuous in q given r, (9) ensures that q_M^* is well-defined.

By definition, $q_M^* (< q_w)$ is the smallest monopoly output level such that a strong incumbent is indifferent between producing q_M^* and facing entry by both types, and producing q_s and facing entry by strong type only. Hence, a strong incumbent would find it optimal to produce q_s if a weak entrant would enter when $q \leq q_M^*$ and would not enter otherwise. In addition, given the same entry strategy, (10) implies that a weak incumbent would prefer producing some output level not exceeding q_M^* to producing any output larger than q_M^*. Since $\Pi_w(q, 1)$ is continuous in q, we may let q^* to be a solution to $\max_{0 \leq q \leq q_M^*} \Pi_w(q, 1)$.[13] Then, a weak incumbent would find it optimal to produce q^*, again given the same entry strategy. Since this entry strategy is optimal (as per (i)-(iii) above), we have constructed a desired

[12] To be fully precise, "any larger output" means "any larger output that generates a higher monopoly profit," as elaborated below.

[13] Note that $q^* = q_M^*$ as long as $\pi_w(q) < \pi_w(q_M^*)$ for $q < q_M^*$, e.g., when $\pi_w(\cdot)$ is single-peaked.

equilibrium, formally described as

$$\text{[SE]}\ \sigma(\{q_s\};s)=\sigma(\{q^*\};w)=1;\ \tau(q)=1\ \ \forall q\in[0,q_M^*],\ \tau(q)=0\ \ \forall q>q_M^*;$$
$$\beta(q)=0\ \ \forall q\in[0,q_M^*],\ \beta(q)=1\ \ \forall q>q_M^*.$$

Next, we examine the anti-limit pricing as arises in a pooling equilibrium. As explained earlier, this is possible when a weak incumbent does not mind being imitated by its strong counterpart because that does not deter weak entry. Such an equilibrium exists, as verified below, if a strong incumbent prefers the output-entry prospect pair $(q_w, 1)$ to $(q_s, 0)$, and a weak incumbent prefers $(q_w, 1)$ to $(q_w, 0)$, the latter being the best it may achieve by deterring weak entry, i.e., if

$$\Pi_s(q_s,0)<\Pi_s(q_w,1)\ \text{and}\ \Pi_w(q_w,0)<\Pi_w(q_w,1). \tag{11}$$

There are two cases to consider. First, if a weak entrant would enter under the prior belief on the incumbent's type (because $(1-\delta)K\leq\mu_s\pi_2^D(s,w)+(1-\mu_s)\pi_2^D(w,w)$), then the following is easily verified to be an equilibrium: a strong incumbent always imitates its weaker counterpart by producing q_w, and a weak entrant enters if $q\leq q_w$ provided no entry took place previously and do not otherwise, formally described as

$$\text{[PE]}\ \sigma(\{q_w\};s)=\sigma(\{q_w\};w)=1;\ \tau(q)=1\ \ \forall q\in[0,q_w],\ \tau(q)=0\ \ \forall q>q_w;$$
$$\beta(q)=\mu_s\ \ \forall q\in[0,q_w],\ \beta(q)=1\ \ \forall q>q_w.$$

In the alternative case that a weak entrant would not enter under the prior belief on the incumbent's type (because $(1-\delta)K>\mu_s\pi_2^D(s,w)+(1-\mu_s)\pi_2^D(w,w)$), a partial pooling equilibrium exists in which a strong incumbent mixes between producing q_w and q_s, and a weak entrant mixes between entering and not entering after q_w. To see this, note that $\Pi_s(q_w,r^*)=\Pi_s(q_s,0)$ for some $r^*\in(0,1)$ since $\Pi_s(q_w,r)$ is continuous with respect to r and $\Pi_s(q_w,0)<\Pi_s(q_s,0)<\Pi_s(q_w,1)$ where the latter inequality is from (11). Hence, if a weak incumbent enters with probability r^* after an incumbent's output of q_w or lower but with probability 0 otherwise, then it would be optimal for a strong incumbent to mix between producing q_w and producing q_s,[14] say with probabilities σ^* and $1-\sigma^*$, respectively, and for a weak incumbent to produce q_w. If the ensuing posterior belief satisfies

$$\beta(q_w)=\frac{\mu_s\sigma^*}{\mu_s\sigma^*+1-\mu_s}=\beta^*:=\frac{\pi_2^D(w,w)-(1-\delta)K}{\pi_2^D(w,w)-\pi_2^D(s,w)} \tag{12}$$

so that a weak firm would be indifferent between entering and not after q_w, then the postulated mixed entry behavior of a weak firm would be justified and the partial pooling equilibrium confirmed. We can ensure this to be the case by finding $\sigma^*\in(0,1)$ that solves (12), which exists because $\beta^*\in(0,\mu_s)$ since a weak firm would enter against a weak incumbent but not under the prior belief in the current case. Thus, we have constructed a desired equilibrium which is formally described as

$$\text{[PPE]}\ \sigma(\{q_s\};s)=1-\sigma^*,\ \sigma(\{q_w\};s)=\sigma^*;\ \sigma(\{q_w\};w)=1;\ \tau(q)=r^*\ \ \forall q\in[0,q_w],$$

[14] From (5) it is obvious that $\Pi_s(q_s,0)>\Pi_s(q,0)$ for all $q>q_w$, $q\neq q_s$. Note that $\pi_s(q)<\pi_s(q_w)$ for all $q<q_w$: Otherwise, say $\pi_s(\tilde{q})\geq\pi_s(q_w)$ for some $\tilde{q}<q_w$, then $\pi_w(\tilde{q})\geq\pi_w(q_w)$ because $0\leq c_w(\tilde{q})-c_s(\tilde{q})\leq c_w(q_w)-c_s(q_w)$ due to (1), contradicting the unique optimality of q_w. Hence, $\Pi_s(q,\tau^*)<\Pi_s(q_w,\tau^*)$ holds by (5) for any $q<q_w$.

$$\tau(q)=0 \;\; \forall\, q>q_w;\; \beta(q)=\beta^* \;\; \forall\, q\in[0,\, q_w],\; \beta(q)=1 \;\; \forall\, q>q_w.$$

Notice that (11) is implied by (9) and (10) because $\Pi_w(q,\, 1)\leq \Pi_w(q_w,\, 1)$ for any q. Hence the anti-limit pricing arises if (11) holds, as summarized in Theorem 1 below. In fact, the anti-limit pricing *necessarily* arises in *all* monotone PBE if (11) holds, as is stated in Theorem 1 and proved in the Appendix.

Theorem 1: Suppose $\pi_3^T(w,\, w,\, s)<\pi_2^D(w,\, w)$ and fix $\theta\in(0,\, 1)$. Let Ψ_S be the set of all $(K,\, m_s,\, \delta)\in \mathbf{R}_+\times(0,\, 1)\times(0,\, 1)$ for which (8), (9), and (10) are satisfied; Let Ψ be the set of all $(K,\, m_s,\, \delta)$ for which (8) and (11) are satisfied. Then, $\Psi\supset\Psi_S\neq\emptyset$, and the anti-limit pricing arises in a separating monotone PBE for all $(K,\, m_s,\, \delta)\in\Psi_S$ and in a pooling monotone PBE for all $(K,\, m_s,\, \delta)\in\Psi$. Furthermore, the anti-limit pricing arises in all monotone PBE if $(K,\, m_s,\, \delta)\in\Psi$.

Proof: See Appendix.

This theorem identifies a sufficient condition for the anti-limit pricing. This condition is not a necessary condition as it stands, because output reduction may occur in a broader set of circumstances due to unnatural off-equilibrium beliefs. For example, think of a situation in which the two types of incumbent separate themselves by producing their respective myopic optima, i.e., $\Pi_s(q_w,\, 1)<\Pi_s(q_s,\, 0)$ and $\Pi_w(q_s,\, 0)<\Pi_w(q_w,\, 1)$, and a weak firm would enter only against a weak incumbent. In this situation the following would also be an equilibrium: a strong incumbent produces q_s and a weak incumbent produces slightly less than q_w, say q'_w, so long as $\Pi_w(q_s,\, 0)<\Pi_w(q'_w,\, 1)$, supported by the posterior belief $\beta(q)=0$ if $q\leq q'_w$ and $\beta(q)=1$ otherwise. The latter equilibrium is supported by unnatural off-equilibrium beliefs and can be eliminated by an argument in the spirit of the Intuitive Criterion of Cho and Kreps (1987). The next theorem states that the sufficient condition in Theorem 1 is also a necessary condition for the anti-limit pricing if an appropriate version of the Intuitive Criterion is imposed (under a mild technical condition).

Since the Intuitive Criterion of Cho and Kreps is defined for a static setting, we modify it for our dynamic context as formalized below. The core idea is: If one incumbent type cannot benefit by producing a non-equilibrium output level regardless of how the other party responds, whilst an incumbent of the other type may and indeed does benefit from any optimal response of the other party as long as the other party places no posterior probability on the former type, then the equilibrium is deemed not to be robust.

Definition 3: Given a PBE, denoted by $(\sigma,\, \tau)$ and β, let Π_s^* and Π_w^* be the equilibrium payoffs of a strong and weak incumbent, respectively. This PBE *satisfies the Intuitive Criterion* if there do not exist a non-equilibrium output level $\tilde{q}\geq 0$ and a type $\tilde{z}\in\{s,\, w\}$ such that *i)* $\Pi_z^*>\max_{0\leq r\leq 1}\Pi_z(\tilde{q},\, r)$ for $z\neq\tilde{z}$, and *ii)* $\Pi_{\tilde{z}}(\tilde{q},\, r^{\tilde{z}})>\Pi_{\tilde{z}}^*$ where $r^s=0$ and $r^w=1$.

Condition *i*) says that an incumbent of a type other than $\tilde{z}$ cannot benefit by producing $\tilde{q}$ (once or repeatedly) regardless of how a weak firm may change its entry decision in the sequel.[15] Condition *ii*) implies that, if the arrived entrant infers, upon observing output of $\tilde{q}$

[15] In the current context, the only strategic interaction is the impact of the incumbent's current output on the entry decision of a weak firm in the next period. Therefore, we maintain that a potential entrant always undertakes the strictly dominant strategies of entering into a monopoly market if it is of a strong type, and not entering into any duopoly market, and that multiple firms in the market play the Cournot equilibrium.

unexpectedly, that the incumbent cannot be of a type other than $\tilde{z}$ due to *i*), then a $\tilde{z}$-type incumbent would indeed improve upon the supposed equilibrium by producing $\tilde{q}$, which in turn justifies the arrived entrant's inference and corresponding optimal response. If there exist such $\tilde{q}$ and $\tilde{z}$ that satisfy *i*) and *ii*), therefore, a $\tilde{z}$-type incumbent would deviate and upset the supposed equilibrium.

Theorem 2: Suppose (2) holds and $c'_s(q_w) \neq c'_w(q_w)$. If the anti-limit pricing arises in a monotone PBE[16] that satisfies the Intuitive Criterion, then $(K, m_s, \delta) \in cl(\Psi)$ where $cl(\Psi)$ is the closure of Ψ.

Proof: See Appendix.

In the rest of this section we discuss the robustness of our main findings. Recall that we provided the analysis for the case that each potential entrant, upon arrival, observes only the last period's output level. First, we establish that the main insights of the paper extend to alternative cases.

We start with the case that each entrant observes the number of periods that have passed as well as the output level of the incumbent in the last period. In this case the separating equilibrium with anti-limit pricing described in [PE] continues to be an equilibrium with the following interpretation: The entrant arriving in period $t > 1$ calculates the posterior μ_s^{t-1} for the incumbent being a strong type at the point of choosing the output level q^{t-1} of period $t-1$ conditional on there having been no entry until then, based on the equilibrium strategies described in [SE]. Note that $\mu_s^{t-1} > \mu_s$ because, according to the equilibrium strategy profile, the posterior for the incumbent being strong gets enhanced by the fact that no entry occurred. Since the arguments leading to the specification of [SE] are independent of μ_s^{t-1},[17] the specified strategies remain to be optimal for all periods until an entry takes place.

The pooling equilibrium, [PE], is easily verified to be an equilibrium in this case as well because $\mu_s^{t-1} = \mu_s$ since the incumbent always produces q_w regardless of its type. The partial pooling equilibrium, [PPE], is also verified to be an equilibrium with the following modification: In periods 0 and 1 the strategies in [PPE] are followed. The entrant arriving in period 2 updates μ_s^1 by the Bayes' rule conditional on no entry having occurred in period 1, where $\mu_s^1 > \mu_s$ for the same reason as above. Since the arguments leading to the specification of [PPE] are valid as long as $\mu_s^1 > \beta^*$, which is the case because $\mu_s^1 > \mu_s > \beta^*$, the strategies in [PPE] are optimal for the incumbent in period 1 and for the entrant in period 2, with one change: σ^* solves $\dfrac{\mu_s^1 \sigma^*}{\mu_s^1 \sigma^* + 1 - \mu_s^1} = \beta^*$. Since an analogous logic applies to all future periods without a previous entry, the equilibrium [PPE] extends to this case. Consequently, the first claim of Theorem 1 extends to this case.

To be able to discuss the second claim of Theorem 1, we need to reinterpret the notion of *monotone* PBE as the condition (6) holding for every period. Yet, the second claim may not

[16] A strong firm is not guaranteed to enter regardless of the posterior belief by condition (2) alone. For this result, therefore, we naturally modify the definition of monotone PBE by requiring the condition (6) for the entry strategy of a strong potential entrant as well.

[17] The arguments hold even if $\mu_s^{t-1} < \beta^*$, but in this case a pooling equilibrium with anti-limit pricing appears more plausible.

hold as it stands in the current case, because the equilibrium to prevail in a continuation game can depend on the previous period's output in various sorts of ways, e.g., a continuation equilibrium that is better for a weak incumbent could ensue when the previous period's output is above q_w than when it is below q_w. Hence, we need to strengthen the notion of monotone PBE as: the entry strategy τ^t in every period t is determined by μ_s^{t-1}, i.e., $\tau^t(q)=\tau(q\,|\,\mu_s^{t-1})$ and $\tau(q\,|\,\mu_s)\geq\tau(q\,|\,\mu_s')$ if $\mu_s<\mu_s'$. Then, it is straightforward to see that the proof of the second claim of Theorem 1 in the Appendix extends to the current case.

Next, consider the other case that each potential entrant observes the full history of the output levels of the incumbent. In this case, too, the separating equilibrium, [SE], continues to be an equilibrium with μ_s^{t-1} calculated based on the full history of output levels (and no previous entry).[18] In addition, variants of [SE] may also constitute equilibria in which the weak incumbent's output level q_t^* in period t vary with t, supported by appropriate belief profiles, as long as a strong incumbent does not benefit by imitating q_t^* (up to a certain period or forever). A weak incumbent may have higher equilibrium payoffs in some of these variants because it has more scope for manipulating output levels: For instance, it may produce less than q^* in the initial period, then produce an output level closer to (but no higher than) q_w in all subsequent periods, without prompting imitation by its strong counterpart. Hence, anti-limit pricing arises in such variants of [SE]. The pooling equilibrium, [PE], and the partial pooling equilibrium, [PPE], can be verified to be equilibria in this case, too, by the same reasoning as before. Lastly, the second claim of Theorem 1 extends to the current case as well in the sense explained above.

Another point for discussion is that a weak entrant might not enter if it foresees itself exiting the market in the future due to an excessive number of strong firms that will enter. We preclude this possibility from the outset by assuming zero fixed cost. Although possibility of exit tends to make weak entrants less likely to enter/survive, we stress that our results do not hinge on the absence of exit possibility. For instance, suppose that the firms in our current model incur a fixed cost such that a weak firm would exit because it cannot recover the fixed cost if there are two or more strong firms that have entered as well. Note, however, that once a weak entrant enters and forms a duopoly with the incumbent, no two or more strong firms will enter subsequently because not all of them will be able to recover the entry cost (although once entered, their operating profits may be positive). Foreseeing this, not even one strong firm would enter because it would not recover the entry cost, either, given that the weak firm would stay. Thus, exit by a weak entrant is possible but does not materialize, and the anti-limit pricing still prevails.

[18] In this equilibrium, the posterior is $\mu_s^{t-1}=0$ after a history of $q_i=q^*$ for $i=0, \cdots, t-2$, but switches to $\mu_s^t=1$ after a history of $q_i=q^*$ for $i=0, \cdots, t-2$ and $q_{t-1}>q^*$. The latter history being on off-equilibrium-path, this is fine in PBE. Thus, the strategy of producing q^* for one period then q_w in all subsequent periods, which appears attractive when any signaling act is perfectly observable to all future entrants, is not necessarily viable especially if a strong incumbent finds imitating it more profitable than producing q_s all the time. However, producing q^* for sufficiently many periods before producing q_w can be effective because imitating such behavior is more costly for a strong incumbent.

IV. *Example When Only One Entry Is Viable*

In this section, for the sake of providing a guideline, we illustrate the areas Ψ_S and Ψ for a simple case of a linear demand $p(q)=1-q$ and constant marginal costs, namely, $c_s(q)=0$ for all $q\geq 0$ and $c_w(q)=\bar{c}q$. Since the signaling motives (either to deter or promote entry) are larger for higher θ, we present the limit case of $\theta=1$. Conditions (2) and (4) require $0<\bar{c}<0.1$ in this case. In Figure 1 we plot, for various values of $0<\bar{c}<0.1$, the areas of (m_s, δ) for which anti-limit pricing arises in a separating equilibrium, i.e., the projection of Ψ_S onto (m_s, δ), in diagram (a); and that corresponding to a pooling equilibrium, i.e., the projection of Ψ onto (m_s, δ), in diagram (b). Figure 1 (c) shows these areas for $\bar{c}=0.09$, along with the (lightly shaded) area for which conventional limit pricing arises in equilibrium. When the future is insignificant relative to the present, the motive to influence entry decision via signaling is limited; hence, Figure 1 presents the cases in which the current period's profit alone is not more important than all future periods' profits combined, i.e., $\delta>0.5$.

If an incumbent were to produce its myopic monopoly output, its type could be inferred from the output level and a strong incumbent would face entry only by a strong type. If it behaved as if it were a weak incumbent instead, it would face entry by both types of entrant. This mimicking behavior has three effects on the incumbent's profit stream: it reduces its monopoly profit; it shifts weight from the monopoly profit to post-entry, duopoly profits by reducing the expected duration of monopoly; and it increases the expected duopoly profits. The first two effects are negative, while the third is positive. If duopoly profits carry enough weight relative to the monopoly profits in the expected stream of profits, then the positive effect dominates and a strong incumbent would have incentive to mimic a weak incumbent's behavior, resulting in the anti-limit pricing.

Post-entry profits carry more weight for an incumbent when the expected duration of the market is longer[19] or new entry threats arise with a higher frequency, both of which are reflected as high δ in our model. Hence, anti-limit pricing tends to arise for higher δ, as depicted in Figure 1.

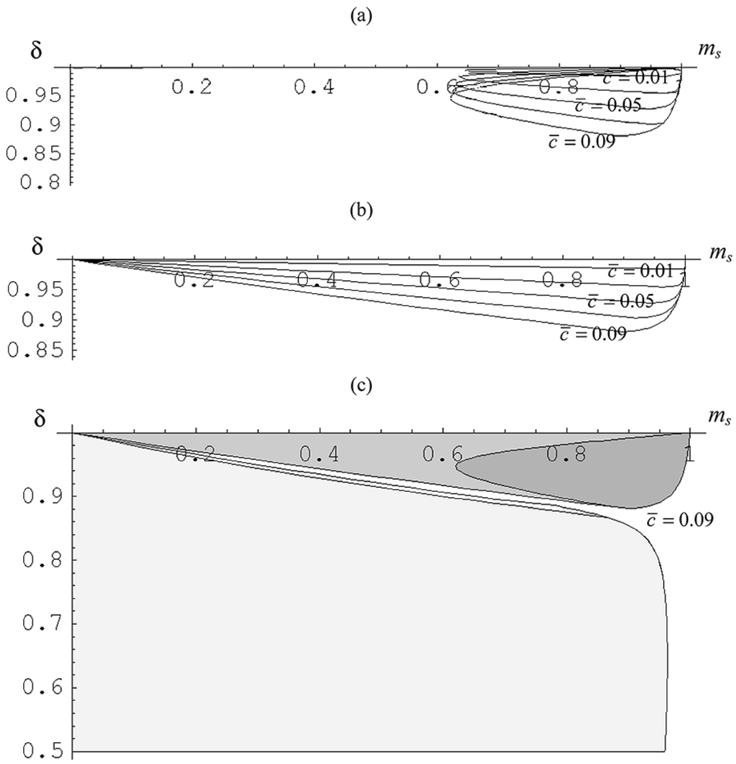
The benefit of anti-limit pricing comes from the prevention of a strong entry through earlier inducement of a weaker one. This benefit is small if either a strong entry is unlikely in any case, or the prospect of actually inducing a weak entry is low. Hence, anti-limit pricing does not arise if m_s is either too low or too high.

In a separating equilibrium, a weak incumbent has to distort monopoly output to induce weak entry. Such a distortion would not be worthwhile if the foregone monopoly profit stream were to be large due to the infrequency of strong entrants' arrivals. This is reflected in Figure 1: The anti-limit pricing areas have higher lower bounds of m_s for separating equilibria than for pooling equilibria.

As shown in Figure 1 (c), conventional limit pricing also arises in dynamic settings. For this to take place, in contrast to the anti-limit pricing, a weak incumbent must desire to appear strong by imitating a strong incumbent's output, and to thereby deter weak entry. This

[19] Anti-limit pricing may arise in long enough, yet finite horizon models. Our search for a 3-period example was unsuccessful, though. Hence, we opted to present it in an infinite horizon model which may be simpler to analyze conceptually.

FIG. 1



mimicking behavior has three effects on the weak incumbent's profit stream: it reduces its monopoly profit; it shifts weight to the monopoly profit from post-entry, duopoly profits; and it reduces the duopoly profits on average. The first and third effects are negative, while the second is positive. If the weight of duopoly profits is small relative to that of monopoly profits in the expected profit stream, then the positive effect dominates and limit pricing arises. Relative to the area of the anti-limit pricing, this tends to happen for lower δ (because monopoly profits are front-loaded in the profit stream) and for not too high m_s (otherwise, deterring weak entry would have a negligible effect), as shown in the diagram. It needs be noted, though, that the presented diagram is for the specific demand and cost functions selected for ease of calculation: the area of the anti-limit pricing can be larger or smaller for other demand and cost specifications.

V. *Examples When More Than One Entry Is Viable*

The previous sections illustrate the essential forces behind the anti-limit pricing: If entry is bound to occur eventually and the post-entry profit is important, the incumbent may prefer to have weaker firms enter because the higher post-entry profits from having a weaker competitor overcompensates the loss from allowing an early entry. Clearly, these forces are not limited to environments that allow only one entry, although the previous section focused on such environments for analytical ease. In this section we demonstrate that the anti-limit pricing arises in a wider class of environments in various forms. For instance, the anti-limit pricing may arise at multiple stages as the market structure evolves due to entry, possibly occurring in turns with conventional limit pricing.

1. Two Entry Case

We now provide an environment in which up to two entries indeed take place and the anti-limit pricing arises both when there is a single incumbent in the market and when two firms operate in the market after a first entry. We are confident that the anti-limit pricing can happen when more entries are viable as well, although it is an open question whether it is compatible with arbitrarily many viable entries. The problem becomes more complex as more entries are viable because the contingencies to consider increase exponentially, let alone the additional issues that crop up when multiple firms operate in the market, such as joint signaling and bargaining amongst them.

We continue with the case of a linear demand and constant marginal costs described in the last section. Recall market demand $p(q)=1-q$ and cost functions $c_s(q)=0$ and $c_w(q)=\overline{c}q$. Set $\overline{c}=0.05$, $m_s=\mu_s=0.8$, $\delta=0.972$ and $K=1.89$. A separating equilibrium in this environment has the following properties. (i) A strong incumbent produces its myopic monopoly output, 0.5, whereas a weak incumbent produces $q_M^*=0.471$,[20] below its monopoly output (0.475), to deter imitation by the strong counterpart. (ii) Consequently, an entrant of either type enters if the single incumbent's production last period was q_M^* or lower, but only a strong type enters otherwise. (iii) When there are two firms in the market, they produce the Cournot outputs unless both firms are weak; if both firms are weak they produce $Q_D^*=0.466$ (i.e., 0.233 each), less than their Cournot output (0.634), to deter imitation by different pairs of duopoly firms. (iv) Consequently, an entrant of either type enters if the duopoly output last period was $Q_D^*=0.466$ or lower, whereas only a strong type enters otherwise. (v) If there are three or more firms in the market they produce the Cournot outputs, and no additional entry takes place regardless of their total output.

Verifying this equilibrium requires checking, for various market structures, the incentive compatibility of the firms in the market (of various type-compositions) and the appropriate entry conditions of potential entrants. This is a straightforward, albeit lengthy, exercise which is available in an earlier version (2005) of this paper (and hence is omitted here). Two conditions worth noting here are: $q_M^*=0.471$ is the smallest distortion below a weak incumbent's myopic optimum, that prevents imitation by a strong incumbent; and $Q_D^*=0.466$ is the smallest distortion below the Cournot duopoly output of two weak firms, that prevents (noncooperative) imitation by firms in any other duopoly. (For referees only, this equilibrium is fully verified in Appendix X at the end of this paper.)

[20] All numerical values were calculated using *Mathematica* (a file is available from the authors) and are reported here as the three-digit approximations below the decimal point.

2. Anti-limit Pricing with the Prospect of Limit Pricing

Here we provide an example where (separating) anti-limit pricing arises when the industry is monopoly, but conventional limit pricing arises when the industry becomes duopoly. We continue with the case of a linear demand $p(q)=1-q$ and cost functions $c_s(q)=0$ and $c_w(q)=\overline{c}q$. In this senario anti-limit pricing can arise with $\overline{c}>0.1$. Set $\theta=1$, $\overline{c}=0.11$, $m_s=\mu_s=0.95$, $\delta=0.93$ and $K=1.15$. In the equilibrium: (i) An incumbent produces its myopic monopoly output, 0.5, if strong, but $q_M^*=0.398$ if weak (less than its myopic monopoly output, 0.445). (ii) Consequently, an entrant of either type enters if the single incumbent's production last period was q_M^* or lower, but only a strong type enters otherwise. (iii) When there are two firms in the market, they produce the Cournot outputs unless both firms are weak; in the latter case, each firm produces 0.315, a half of the Cournot duopoly output when one firm is strong and the other is weak, $Q_D^*=0.63$. (iv) Consequently, a strong firm would enter if the duopoly output were less than $Q_D^*=0.63$, while neither type would otherwise. (v) If there were three or more firms in the market they would produce the corresponding Cournot outputs, and, regardless of their total output, no additional entry would take place.

We now verify the equilibrium conditions. Let $\langle z_1, z_2\rangle$ denote a duopoly formed by an incumbent of type z_1 and an entrant of type z_2. Note that $\langle s, w\rangle$ cannot be formed because a weak entrant would not enter against a strong incumbent. Observing Q_D^*, therefore, a potential entrant infers that the market is either $\langle w, s\rangle$ or $\langle w, w\rangle$ with probability m_s and $1-m_s$, respectively. Given this posterior, a strong firm would not enter because the expected discounted sum of income stream, 1.112, is less than K. We postulate the same posterior belief for duopoly outputs exceeding Q_D^*, so that a strong firm would not enter after such outputs, either. On the other hand, for duopoly output less than Q_D^*, we postulate that the market is believed to be $\langle w, w\rangle$ for certain, so that a strong firm would enter. A weak firm would never enter into a duopoly market, because even if the market is $\langle w, w\rangle$ the expected discounted income stream when entered, 0.707, is less than K. Thus, we have verified the optimality of entry strategy after the market has reached duopoly.

We now check the strategies of duopoly firms. The continuation equilibrium payoff level of each firm in $\langle w, w\rangle$ is $\pi_i^D(w, w)/(1-\delta)=1.170$, which exceeds 0.590, the maximum payoff from deviation, i.e., from producing the short-run maximizing output given the other firm's output $Q_D^*/2$, after which the two firms produce the Cournot duopoly outcome until a strong firm enters and the three firms produce the Cournot triopoly outcome. For other duopolies, producing their respective Cournot outputs is clearly optimal given that there will be no further entry.

Next, to check the conditions when the market is a monopoly, we specify the posterior belief as: the incumbent's type is weak if the monopoly output is $q_M^*=0.398$ or lower, and it is strong if otherwise. Given the equilibrium strategies in duopoly markets, it is a lengthy yet routine calculation (hence omitted) to verify that a strong entrant's discounted sum of profit stream is 1.587 when the incumbent is strong, hence it would enter regardless of the incumbent's type; those of a weak entrant are 1.170 and 0.068 when the incumbent is weak and strong, respectively, hence it would enter precisely when the monopoly output is q_M^* or lower. Then, a weak incumbent's equilibrium payoff level is calculated as 1.103, which exceeds the maximum deviation payoff, 1.102, of it producing the myopic monopoly output and facing entry by a strong type only. Finally, a strong incumbent is indifferent between the equilibrium strategy and the most beneficial deviation (i.e., produce q_M^* and face entry by either type): $q_M^* = 0.398$ is obtained to ensure this indifference. This completes verification of the specified equilibrium.[21]

VI. *Concluding Remarks*

In this paper we have analyzed the anti-limit pricing and fully characterized the conditions under which it necessarily arises in equilibrium for the class of environments in which once there is an entry no further entry would be profitable. We believe that the key insight underlying the anti-limit pricing is conveyed more clearly in such environments, due to a relatively small number of future contingencies to consider strategically.

It was also emphasized that the anti-limit pricing is a phenomenon that arises more extensively and in a variety of forms. In particular, we showed via examples that it can arise when more than one entry takes place, and by oligopolists as well as by an incumbent monopolist; and that both anti-limit pricing and conventional limit pricing can occur in different stages along the same equilibrium path. A complete characterization of dynamic price signaling when more than one entry is viable appears challenging, due to various dynamic paths it may take and the complexity of strategies that are more forward-looking; and it is a task awaiting future research.

Situations exist in which the signaling needs are recurrent and interrelated. Examples include recurring entry threats (analyzed here) and job markets where the dimension of ability to signal may change as one moves up the corporate pyramid (e.g., from productive to managerial ability). The existing literature deals mainly with isolated signaling needs, and hence is not well-equipped to analyze dynamic signaling environments. Our exercise in this paper manifests that the implications of dynamic signaling can be drastically different from those of isolated signaling, and consequently, underscores the need for further study on dynamic signaling.

Appendix

Proof of Theorem 1: Recall that in the discussion preceding the Theorem 1 we have shown that (9) and (10) imply (11), which implies $\Psi \supset \Psi_S$; in addition, we have constructed a separating monotone PBE that exhibits anti-limit pricing for $(K, m_s, \delta) \in \Psi_S$ and a pooling monotone PBE for $(K, m_s, \delta) \in \Psi$. To complete the proof, we show below that $\Psi_S \neq \emptyset$ and that all monotone PBE exhibit anti-limit pricing if $(K, m_s, \delta) \in \Psi$.

To show $\Psi_S \neq 0$, first observe from (5) that

$$\Pi_z(q, 1) - \Pi_z(q, 0) = \frac{\delta\theta(1-m_s)}{(1-\delta)(1-\delta+\delta\theta)}\left(\pi_1^D(z, w) - \frac{\delta\theta m_s \pi_1^D(z, s) + (1-\delta)\pi^z(q)}{1-\delta+\delta\theta m_s}\right). \quad (13)$$

[21] In this equilibrium, a second entry is potentially viable but does not arise in equilibrium due to conventional limit pricing in the duopoly market.

Fix an arbitrary $m'_s \in (0, 1)$. Since $\pi_1^D(z, w) > \pi_1^D(z, s)$, (13) implies that one can choose $\hat{\delta} \in (0, 1)$ close enough to 1 so that both $\Pi_s(q, 1) - \Pi_s(q, 0)$ and $\Pi_w(q, 1) - \Pi_w(q, 0)$ are arbitrarily large; in particular, larger than $\frac{\pi_s(q_s) - \pi_s(q_w)}{1-\delta+\delta\theta} > 0$. Since the right hand side (RHS) of (13) approaches 0 as $m_s \to 1$ for given $\hat{\delta}$, one can find the smallest $\hat{m}_s > m'_s$ such that $\Pi_s(q_s, 1) - \Pi_s(q_s, 0) = \frac{\pi_s(q_s) - \pi_s(q_w)}{1-\delta+\delta\theta}$ when $(m_s, \delta) = (\hat{m}_s, \hat{\delta})$. Since $\pi_s(q_w) \geq \pi_w(q_w) > 0$ while $\pi_s(0) \leq 0$, due to continuity of Π_s, the following inequalities hold when $(m_s, \delta) = (\hat{m}_s - \varepsilon, \hat{\delta})$ for sufficiently small $\varepsilon > 0$:

$$\frac{\pi_s(q_s) - \pi_s(q_w)}{1-\delta+\delta\theta} < \Pi_s(q_s, 1) - \Pi_s(q_s, 0) < \frac{\pi_s(q_s) - \pi_s(0)}{1-\delta+\delta\theta}. \tag{14}$$

Furthermore, since $\Pi_w(q, 1) - \Pi_w(q, 0) > 0$ at $(\hat{m}_s, \hat{\delta})$ because the RHS of (13) stays positive as m_s increases from m'_s, the following also holds when $(m_s, \delta) = (\hat{m}_s - \varepsilon, \hat{\delta})$ for sufficiently small $\varepsilon > 0$:

$$\Pi_s(q_s, 1) - \Pi_s(q_s, 0) - (\Pi_w(q, 1) - \Pi_w(q, 0)) < \frac{\pi_s(q_s) - \pi_s(q_w)}{1-\delta+\delta\theta}. \tag{15}$$

From (5) it is immediate that

$$\Pi_z(q, 1) - \frac{\pi_z(q)}{1-\delta+\delta\theta} = \Pi_z(q', 1) - \frac{\pi_z(q')}{1-\delta+\delta\theta} \text{ for every } q, q' \geq 0. \tag{16}$$

Combining (16) for $(q, q', z) = (q_s, q_w, s)$ and the first inequality of (14), we deduce that $\Pi_s(q_s, 0) < \Pi_s(q_w, 1)$; Combining (16) for $(q, q', z) = (q_s, 0, s)$ and the second inequality of (14), we deduce that $\Pi_s(0, 1) < \Pi_s(q_s, 0)$. These two inequalities mean that (9) is satisfied at $(\hat{m}_s - \varepsilon, \hat{\delta})$. Furthermore, (10) is satisfied at $(\hat{m}_s - \varepsilon, \hat{\delta})$ as is verified below:

$$\begin{aligned}
&\Pi_w(q_M^*, 1) - \Pi_w(q_w, 0) \\
&= \Pi_w(q_w, 1) + \frac{\pi_w(q_M^*) - \pi_w(q_w)}{1-\delta+\delta\theta} - \Pi_w(q_w, 0) \\
&> \Pi_s(q_s, 1) - \Pi_s(q_s, 0) + \frac{\pi_s(q_w) - \pi_s(q_s) + \pi_w(q_M^*) - \pi_w(q_w)}{1-\delta+\delta\theta} \\
&= \frac{\pi_s(q_w) - \pi_s(q_M^*) + \pi_w(q_M^*) - \pi_w(q_w)}{1-\delta+\delta\theta} \geq 0
\end{aligned}$$

where the first equality follows from (16) for $(q, q', z) = (q_M^*, q_w, w)$; the first inequality from (15); the second equality from (16) and the definition of q_M^*, i.e., $\Pi_s(q_M^*, 1) = \Pi_s(q_s, 0)$; and the last inequality from that $\pi_s(q_w) - \pi_s(q_M^*) - \pi_w(q_w) + \pi_w(q_M^*) = c_w(q_w) - c_w(q_M^*) - c_s(q_w) + c_s(q_M^*) = \int_{q_M^*}^{q_w} [c'_w(q) - c'_s(q)]dq \geq 0$ due to (1). This proves that (9) and (10) are satisfied at $(\hat{m}_s - \varepsilon, \hat{\delta})$ for sufficiently small $\varepsilon > 0$ and, therefore, $\Psi_S \neq \emptyset$.

Finally, to prove that all monotone PBE exhibit the anti-limit pricing if $(K, m_s, \delta) \in \Psi$, we first show that a strong incumbent does not produce $q > q_s$ in any monotone equilibrium because producing q_s is better. To do this, notice that if it produces $q > q_s$ instead of q_s, then i) the current period's profit is lower, and ii) the probability that a weak type enters in the next period is reduced, say by $\eta \geq 0$. The effect of ii) is that with probability η, instead of having $\pi_1^D(s, w)$ from next period onwards, the incumbent maintains monopoly in the next period. The value of the former is $\frac{\pi_1^D(s, w)}{1-\delta}$, while that of the latter is bounded above by $\max_{0\leq r\leq 1}\Pi_s(q_s, r)$. Since $\Pi_s(q_s, 1)-\Pi_s(q_s, 0)>0$ in Ψ (from (11) and $\Pi_s(q_s, 1)>\Pi_s(q_w, 1)$), therefore, the effect of ii) is negative for the incumbent because, as can be verified from (5) and (13),

$$\frac{\pi_1^D(s, w)}{1-\delta}-\Pi_s(q_s, r)=\frac{(1-\delta+\delta\theta)(1-\delta+\delta\theta m_s)(\Pi_s(q_s, 1)-\Pi_s(q_s, 0))}{\delta\theta(1-m_s)(1-\delta+\delta\theta(m_s+(1-m_s)r))}>0$$

for all $r\in[0, 1]$. Since the effect of i) is also negative, this proves that a strong incumbent does not produce more than q_s in any monotone equilibrium. By an analogous argument, one can prove from $\Pi_w(q_w, 1)-\Pi_w(q_w, 0)>0$ (see (11)) that a weak incumbent does not produce more than q_w in any monotone equilibrium if $(K, m_s, \delta)\in\Psi$. Since it is not viable that both types of the incumbent produce their respective myopic output levels if $(K, m_s, \delta)\in\Psi$ because $\Pi_s(q_s, 0)<\Pi_s(q_w, 1)$, it follows as desired that at least one of the types produces less than its short-run optimum level with a positive probability in all monotone equilibria. QED.

Proof of Theorem 2: We start with a few preliminary observations. Since

$$\frac{\partial\Pi_z(q, r)}{\partial r}=\frac{(1-\delta+\delta\theta)(1-\delta+\delta\theta m_s)(\Pi_z(q, 1)-\Pi_z(q, 0))}{(1-\delta+\delta\theta(m_s+(1-m_s)r))^2} \tag{17}$$

as verified by routine calculation, and $\Pi_z(q, 1)-\Pi_z(q, 0)$ assumes its minimum at $q=q_z$ as evident from (13),

[A] $\Pi_z(q, r)$ either monotonically increases, stays constant, or monotonically decreases in $r\in[0, 1]$;
[B] If $\Pi_z(q, r)$ monotonically increases in r for $q=q_z$, so it does for all $q\geq 0$.

In addition, from (5) and [B] we deduce that

[C] If $\Pi_z(q', r')\leq\Pi_z(q_z, r'')$ where $q'<q_z$ and $r'\geq r''$, then $r'>r''$ and $\Pi_z(q, r)$ monotonically increases in r for all $q\geq 0$.

To prove the theorem, we suppose that there exists a monotone PBE that satisfies the Intuitive Criterion, which we denote by $(\tilde{\sigma}, \tilde{\tau}, \tilde{\beta})$, in which the anti-limit pricing arises. Then, in the rest of proof we establish that $(K, m_s, \delta)\in cl(\Psi)$ must hold, or equivalently, we show that

(a) $\max\{\pi_3^T(w, w, s), \pi_2^D(s, w)\}\leq(1-\delta)K\leq\pi_2^D(w, w)$,
(b) $\Pi_w(q_w, 0)\leq\Pi_w(q_w, 1)$, and
(c) $\Pi_s(q_s, 0)\leq\Pi_s(q_w, 1)$.

First, to show (a) by contradiction, we suppose that either $\pi_2^D(s, w)>(1-\delta)K$ or $(1-\delta)K>\pi_2^D(w, w)$. Then, since there will be no second entry due to (2), a weak potential entrant either definitely enters or not enters regardless of the incumbent's type. If a strong potential entrant's decision is also independent of the incumbent's type, then there will be no distortion of monopoly output from the short-run optimum because monopoly output does not affect the entry decision of either type of entrant. Note that, given the supposition above, the only occasion that a strong entrant's decision may depend on the incumbent's type is when $\pi_2^D(w, s)\geq(1-\delta)K\geq\pi_2^D(s, s)$, in which case it would enter against a weak incumbent but not against a strong one. Since $\tilde{\tau}(q)=0$ for all q by (3) and $\tilde{\tau}(q)$ weakly decreases in q as per (6) in this case, a weak incumbent produces at least q_w because producing less would reduce monopoly output and weakly increase the probability of strong entry, both of which are detrimental. Since a strong incumbent produces at least q_s by an analogous reason, the anti-limit pricing does not arise in this case, either. Therefore, we conclude that $\pi_2^D(s, w)\leq(1-\delta)K\leq\pi_2^D(w, w)$ must hold for the anti-limit pricing to arise. Due to (2), this means that (a) must hold, which we assume below.

To establish (b) and (c), we consider two cases separately. First, consider the case that a strong incumbent produces no less than q_s in $(\tilde{\sigma}, \tilde{\tau}, \tilde{\beta})$, hence $\tilde{\sigma}(\{q_s\}, s)=1$ and a weak incumbent should produce some output level $q'<q_w$ as per (7). Then, by item 1) of Definition 1, $\Pi_w(q', \tilde{\tau}(q'))\leq\Pi_w(q_w, \tilde{\tau}(q_w))$. This, together with [C], implies (b).

To show (c), note that if $\Pi_s(q_s, 0)>\Pi_s(q_w, r)$ for all $r\in[0, 1]$, a strong incumbent would never benefit by producing q_w instead of q_s and, therefore, a weak incumbent would benefit by producing q_w instead of q' because such production would convince a weak entrant to enter. As this would fail the Intuitive Criterion, we conclude that $\Pi_s(q_s, 0)\leq\Pi_s(q_w, r)$ for some $r\in[0, 1]$. Since this cannot hold when $r=0$, it follows that $\Pi_s(q_w, r)$ increases in r in light of [A] and $\Pi_s(q_w, 0)<\Pi_s(q_s, 0)$. Hence, we have $\Pi_s(q_s, 0)\leq\Pi_s(q_w, 1)$, i.e., (c).

The other case to consider is one in which a strong incumbent produces some output level $q'<q_s$. Then, by item 1) of Definition 1,

$$\Pi_s(q', \tilde{\tau}(q'))\leq\Pi_s(q_s, \tilde{\tau}(q_s)). \tag{18}$$

This implies that $q'\leq q_w$ because, since $\tilde{\sigma}((q_w, \infty); w)=0$ as per (7), if $q'>q_w$ we would have $\tilde{\beta}(q)=1$ and $\tilde{\tau}(q)=0$ for all $q\geq q'$ by (6) and consequently, (18) would fail. Since $\pi_s(q')\leq\pi_s(q_w)$ for $q'<q_w$ (otherwise, q_w would not be the myopic optimum because $\pi_w(q')-\pi_w(q_w)=\pi_s(q')-\pi_s(q_w)+c_s(q')-c_w(q')-c_s(q_w)+c_w(q_w)>0$ by (1)), we have $\Pi_s(q', \tilde{\tau}(q'))\leq\Pi_s(q_w, \tilde{\tau}(q'))$ by (5). Furthermore, since $\Pi_s(q, r)$ monotonically increases in r for all $q\geq 0$ by [C] applied to (18), we have $\Pi_s(q_w, \tilde{\tau}(q'))\leq\Pi_s(q_w, 1)$ and $\Pi_s(q_s, 0)\leq\Pi_s(q_s, \tilde{\tau}(q_s))$. These three inequalities and (18) imply (c).

To show (b), note from [C] applied to (18) that $\tilde{\tau}(q')>\tilde{\tau}(q_s)\geq 0$, for which we need $\tilde{\beta}(q')<1$, i.e., a weak incumbent should produce q' as well. If $q'<q_w$, we must have $\Pi_w(q_w, 0)\leq\Pi_w(q_w, 1)$, i.e., (b), for otherwise, $\Pi_w(q_w, r)$ would decrease in r by [A], so a weak incumbent would be better off by producing q_w since $\Pi_w(q', \tilde{\tau}(q'))<\Pi_w(q_w, \tilde{\tau}(q'))\leq\Pi_w(q_w, \tilde{\tau}(q_w))$ would hold. Lastly, if $q'=q_w$ (in which case both types of incumbent produce q_w) then $\tilde{\tau}(q)$ cannot be a constant in any nonempty interval $[q_w, q_w+\varepsilon)$, for otherwise a strong incumbent would benefit by producing slightly more than q_w by (5) because i) the entry prospect would not change while ii) the monopoly profit would increase since the condition $c_s'(q_w)\neq c_w'(q_w)$ implies $c_s'(q_w)<c_w'(q_w)$ due to (1) and consequently, $\pi_s'(q_w)>\pi_w'(q_w)=0$. Given that $\tilde{\tau}(q)$ is a decreasing step function due to (6), this implies that $\tilde{\tau}(q_w)>\lim_{q\downarrow q_w}\tilde{\tau}(q)$. For a weak incumbent not to benefit by producing slightly above q_w in this case, $\Pi_w(q_w, r)$ must not increase when r falls, which implies (b) by [A]. This completes the proof. QED.

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