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<td>Author(s)</td>
<td>Kim, Jaehong</td>
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<tr>
<td>Citation</td>
<td>Hitotsubashi Journal of Economics, 51(2): 43-58</td>
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<tr>
<td>Issue Date</td>
<td>2010-12</td>
</tr>
<tr>
<td>Type</td>
<td>Departmental Bulletin Paper</td>
</tr>
<tr>
<td>Text Version</td>
<td>publisher</td>
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<tr>
<td>URL</td>
<td><a href="http://doi.org/10.15057/18779">http://doi.org/10.15057/18779</a></td>
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OPTIMALITY OF ENTRY REGULATION UNDER INCOMPLETE INFORMATION

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Received June 2009; Accepted July 2010

Abstract

The lack of complete information of the government has been considered as a barrier to the optimal regulation, as it is well-known in price regulations literature. However, it is not true for the entry regulation: This paper shows that the performance of the entry regulation under incomplete information can be better than that under complete information. Under incomplete information, the incumbent firm would deviate from the monopoly behavior to signal itself as an efficient type and to trigger entry regulation which prevents excess entry in case that the incumbent is efficient. As a result, social welfare can be even higher than under complete information, since not only the optimal post-entry market structure is achieved as under complete information but the pre-entry price is even lower than that under complete information.

Keywords: incomplete information, excess entry, entry regulation, signaling

JEL Classification: D82, L51

I. Introduction

Since the seminal works by Akerlof (1970), Rothschild and Stiglitz (1976), Spence (1973), and some others, incomplete information has been considered as a major cause of market failure. Adverse selection and moral hazard problems either make it impossible for a market under incomplete information to reach an equilibrium, or make the market equilibrium suboptimal compared to the case of complete information.

Consider Spence (1973)’s job market signaling model as an example. The equilibrium in Spence’s model is inefficient because of the excess education by the worker. In Spence, education is socially wasteful since it doesn’t increase worker’s productivity, but only plays the role of signaling. If information were complete, the socially costly education would not be necessary. The only reason for a worker to invest into education is to signal himself under incomplete information.

However, what if the market signaling is through donation, something that contributes to the social welfare, instead of the wasteful education? There is no difference between donation...
and education in terms of signaling effect since any costly action can be a signaling device. However, if workers use donation, instead of education, as the signaling device, the equilibrium in the Spence’s job market signaling model may not be suboptimal any longer. Actually if we assume that one-dollar donation contributes more than one-dollar to the other members in the society, then excess donation, not excess education, will make social welfare under incomplete information even higher than that under complete information.\(^2\)

The possibility of welfare improvement under incomplete information is already demonstrated, for example, in Milgrom and Roberts (1982). In Milgrom and Roberts, particularly in the separating equilibrium, the more efficient type of incumbent firm chooses a lower price than the monopoly level to transmit information to the potential entrant and to discourage its entry. Since there would have also been no entry under complete information, the social welfare improves under incomplete information due to the lowered monopoly price in the pre-entry stage.\(^3\) Even though Milgrom and Roberts do not stress the possibility of welfare improvement under incomplete information as the main result, they surely point out this important issue in relation to the public policy.\(^4\)

Can we expect the same result in government regulation? In other words, can the government do better under incomplete information than under complete information in regulations? Contrary to the free market interactions, the issue in regulations is maximizing social welfare without a constraint or with a constraint by the benevolent dictator, where the constraint is the lack of complete information. Therefore, it seems trivial that the government can surely do better without the constraint of incomplete information. The theory of optimal price regulation under incomplete information confirms such a prediction.\(^5\) The standard trade-off between rent extraction and incentive provision implies that the optimal regulation under incomplete information can only be the second-best, not the first-best.

However, such a pessimism regarding optimality of the government intervention under incomplete information is not suitable for entry regulations contrary to the case of price regulations. Assume that the government lets the informed party, the incumbent firm, signal first against entry regulation. Then the incumbent firm will make costly expenditure to identify (or to misidentify) its own type to induce government’s entry regulation and to protect its monopoly position. In such case, if the signal by the incumbent firm is a donation to the whole society, the government’s entry regulation will do better under incomplete information than under complete information. Actually we can expect such a result if price is the signaling device and the firm has an incentive to send a low price signal against a regulation to induce entry regulation and to deter potential competitors. A low price is clearly a donation to the

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\(^1\) Even though education may also contribute to the social welfare in reality, we just follow the assumption in Spence. Donation refers to anything that improves social welfare, not just a transfer from the donator to the society. Education investment which contributes to the whole society more than the education costs, contrary to the case of Spence, can be interpreted as donation.

\(^2\) The possibility of the multiplying effect of donation is the counterpart of the distortionary tax in regulation theory. See Laffont and Tirole(1993) for the regulation analysis under the assumption of distortionary public fund; the social cost of one-dollar tax is more than one-dollar. If donation saves public spending, that is, if it reduces tax, then donation is surely welfare-increasing.

\(^3\) In the pooling equilibrium, the social welfare can also be higher under incomplete information since the less efficient incumbent type would produce more than its monopoly level to mimic the low cost type. However, in this case, since entry, which would have occurred under complete information, is limited, we have to compare welfare increment due to low pre-entry price and welfare loss due to less competition in the future.
whole society since the social welfare is increasing as the price goes down. This is the motivation of the paper.

In this paper, in a simple model a la Milgrom and Roberts (1982), we will show that the performance of entry regulation can be even better under incomplete information than under complete information. If the incumbent’s and the entrant’s production costs are not correlated, the entry, which incurs some fixed cost, will be socially desirable if the incumbent firm is inefficient, and it will be socially wasteful if the incumbent is efficient. Therefore, to induce entry regulation, the low cost type incumbent will choose a lower price than the monopoly level to signal itself against the high cost type (separating equilibrium). The low price by the incumbent monopolist is the donation to the whole society because one-dollar loss in firm’s profit due to the lowered price brings more than one-dollar benefit to the consumer surplus. The incentive of the high cost incumbent to mimic the low cost type can also be welfare-increasing by the same logic (pooling equilibrium).

The structure of the paper is as follows. Section II describes a simple signaling model of an entry regulation under incomplete information. Section III derives the perfect Bayesian equilibrium of the signaling game which passes the Intuitive Criterion by Cho and Kreps (1987). Section IV compares the performance of the entry regulation under incomplete information with that under complete information, and shows that the former is better than the latter in most cases. Section V allows the government to move first and choose the best entry regulation which is conditional on the incumbent’s first period price. Since ex-ante screening can generate any ex-post signaling outcome, such a screening mechanism by the government will clearly make entry regulation under incomplete information more efficient than that under complete information. Section VI concludes the paper with some remarks.

II. A Signaling Model of Entry Regulation

Consider a two-stage three-person entry game as in Figure 1. In the beginning of the game, nature $N$ selects firm 1’s type, the constant marginal cost $c_1$, which is either 0 or 2 with probability $\alpha$ and $(1-\alpha)$ respectively, $\alpha \in (0,1)$. Firm 1 knows its own cost, however, a potential entrant and the government only knows the probability distribution.

At $t_1$, firm 1 is the monopolist and chooses a price $p^M$, which may be different from the single-period monopoly profit maximizing price $p^{M*}$ for the strategic entry deterrence. At $t_2$, firm 2 makes a decision on entry. If firm 2 decides to enter (IN), the benevolent government $G$ implements an entry regulation and either allows (Y) or regulates (N) firm 2’s entry. If firm 2
does not enter (OUT) or the government regulates firm 2’s entry, then firm 1 maintains its monopoly position and chooses $p^M*$ because there is no further strategic reason to choose other prices than the static monopoly price.

To highlight the efficiency of entry regulation under incomplete information, we assume that the monopoly price is not regulated. If firm 2’s entry is allowed by the government, then firm 1 and firm 2 play a Cournot competition game. Firm 2’s unit cost, which is assumed to be 2 without loss of any generality, and the entry cost $F$ are common knowledge known to firm 1 and to the government. We assume the same inverse demand function in both periods such as $p = 10 - X$, where $p$ is the market price and $X$ is the total production level. Finally, players do not discount future payoffs.

In Figure 1, payoffs of the firm 1, firm 2, and the government are represented in this sequence both in the monopoly subgame $M$ and in the duopoly subgame $D$. Firm 1’s payoff is the two-period total profit, which is equal to $\pi^M(p^M) + \pi^M*$ in case of no entry and equal to $\pi^M(p^M) + \pi^M*$ in case of entry. Firm 2’s payoff is the Cournot Nash equilibrium profit net of entry cost in case of entry, and 0 in case of no entry. The payoff of the government is the second period social welfare, which is the sum of consumer surplus and profits, net of entry cost in case of entry.9

Table 1 summarizes market outcomes for both periods depending on the market structure.

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8 We can introduce price regulations on the monopoly and/or post-entry duopoly markets without affecting the main results of the paper.

9 Note that the payoffs of the players actually depend on firm 1’s type, the true unit cost.
Under monopoly, at $t_1$ as well as at $t_2$ in case of no entry, $\pi^M(p^M|c_1)$ is the monopoly firm’s profit with unit cost $c_1$ when it chooses $p^M$, and $W^M(p^M|c_1)$ is the social welfare of the monopoly market measured at price $p^M$. Meanwhile, $\pi^{M*}$ and $W^{M*}$ are the monopoly profit and the social welfare of the monopoly market respectively both measured by $p^{M*}$. Finally, the market outcomes of the duopoly in $t_2$ with entry are all measured by the Cournot-Nash equilibrium price.

**Assumption 1.** $0.5 \leq F \leq 4$.

Figure 2 will help us understand why we focus on some intermediate values of the entry cost as in Assumption 1. If entry cost is sufficiently low, that is if $F<0.5$, then entry is desirable not only to firm 2 but to the whole society, regardless of the type of the incumbent monopolist. This is not the interesting case and so will be ignored, since there is no strategic issue regarding entry regulation. On the other hand, if entry cost is sufficiently high, that is if $F>4$, then entry is blocked by the efficient incumbent type, and so there will be no strategic issue related to the government entry regulation, either. In this case, the inefficient type incumbent might have incentive to mimic the efficient type, however, if it succeeds, entry will also be blocked and the government’s entry regulation becomes redundant. Since our interest is the strategic interaction between the informed incumbent and the uninformed government, we will also exclude such case without losing any points.

Finally, if entry cost belongs to the range in Assumption 1, entry is desirable to the potential entrant regardless of the incumbent’s type and so cannot be blocked even by the efficient type incumbent monopolist. However, entry is socially excessive in case that the incumbent is of the efficient type (and it is socially desirable in case of the inefficient incumbent). We will focus on such case where the inefficient type incumbent has incentive to mimic efficient type and the efficient type has incentive to distinguish itself from the inefficient type both to induce entry regulation and protect its monopoly position. The uninformed government then use incumbent’s price as a signal about the true type of the monopolist and will decide on entry permission.

If entry cost belongs to the range in Assumption 1, firm 2 will always choose to enter since $\pi^{D*}_{1}(c_1) \geq F$ regardless of the true value of $c_1$, and so we can focus on the strategic

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<td><strong>Monopoly</strong> $(t_1, t_2)$</td>
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<tr>
<td>$\pi^M(p^M</td>
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<tr>
<td>$W^M(p^M</td>
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<tr>
<td>$p_{1*}^M=\frac{10+c_1}{2}$</td>
</tr>
<tr>
<td>$\pi^{M*}=\frac{(10-c_1)^2}{4}$</td>
</tr>
<tr>
<td>$W^{M*}=\frac{3(10-c_1)^2}{8}$</td>
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<tr>
<td><strong>Duopoly</strong> $(t_2)$</td>
</tr>
<tr>
<td>$p^{D*}(c_1)=\frac{(12+c_1)}{3}$</td>
</tr>
<tr>
<td>$\pi^{D*}_{1}(c_1)=\frac{(12-2c_1)^2}{9}$</td>
</tr>
<tr>
<td>$\pi^{D*}_{2}(c_1)=\frac{(6+c_1)^2}{9}$</td>
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<tr>
<td>$W^{D*}(c_1)=\pi^{D*}<em>{1}(c_1)+\pi^{D*}</em>{2}(c_1)+CS^{D*}(c_1)=\frac{(684-108c_1+11c_1^2)}{18}$</td>
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interaction between the informed monopolist and the uninformed government regarding entry regulation under incomplete information. Because there is no strategic links between pre-entry and post-entry market demands, and so the strategic entry deterrence by firm 1 itself is not feasible, the market structure at \( t_2 \) is solely determined by the government’s entry regulation. Therefore, the original entry game can be simplified to a two-person game with incomplete information between the incumbent and the government. The incumbent firm’s price \( p^M \) at \( t_1 \) plays the role of signaling about \( c_1 \). Firm 1 will choose an optimal \( p^M \) which maximizes two period total profit under entry regulation, and the benevolent government implements an optimal entry regulation under incomplete information, both given firm 2’s willingness to enter.

III. Equilibrium

Let \( \Delta W(c_1) = W^{D*}(c_1) - W^{M*}(c_1) \) be the welfare increment due to entry at \( t_2 \) as a function of \( c_1 \). Then the optimal entry regulation is allowing entry if \( \Delta W(c_1) \geq F \) and disallowing entry if \( \Delta W(c_1) < F \). Figure 2 describes \( \Delta W(c_1) \) in comparison with \( \pi_i^{D*}(c_1) \) and \( F \). Figure 2 confirms that, under Assumption 1, firm 2 always wants to enter the market, however, entry is socially desirable when \( c_1 = 2 \) and it is excessive when \( c_1 = 0 \).

Entry may be socially excessive when the new entry incurs business-stealing effect. The business-stealing effect is represented by \( \pi_i^{D*}(c_1) > \Delta W(c_1) \) in Figure 2, which implies that a new entry is more attractive to the entrant than to the whole society. This is the situation that the new entrant steals some profit from the incumbent, and so the welfare increment due to a new entry is less than the entrant’s profit. The business-stealing effect is the key factor which justifies the entry regulation by the benevolent government.

If information is complete, entry regulation improves social welfare by preventing firm 2’s entry in case of \( c_1 = 0 \) and allowing entry when the incumbent’s cost is high, that is, \( c_1 = 2 \).

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10 See Mankiw and Whinston (1986) for the business-stealing effect of an entry.
However, since the government has only incomplete information in our model, we need to solve the signaling game to find out an optimal entry regulation.

In this section, we will derive a full set of perfect Bayesian equilibria which pass Cho and Kreps’ Intuitive Criterion. Let \((p^M, E, B)\) be a perfect Bayesian equilibrium, where \(p^M\) is the strategy of the incumbent firm, \(E\) is the government’s entry regulation, and \(B\) is the government’s belief about incumbent’s type.

**Separating Equilibrium**

Consider a separating equilibrium first. In the separating equilibrium, if it exists, the low cost incumbent would set the price in \(t_1\) low enough to make the high cost unable to mimic. The high cost incumbent will then choose the monopoly price \(p^M\) in \(t_1\) since it cannot prevent entry in \(t_2\) by mimicking the low cost type. Proposition 1 summarizes such a separating equilibrium.

**Proposition 1.** There exists a unique separating equilibrium which passes the Intuitive Criterion by Cho and Kreps as follows;

\[
p^M: p^M_0 = 3.019, \quad p^M_1 = 6
\]

\(E\): disallow entry \(\forall p^M \in [0, 3.019]\) and allow entry \(\forall p^M \in (3.019, \infty)\)

\(B\): \(\mu(c_1 = 0 | p^M) = 1 \forall p^M \in [0, 3.019] \) and \(\mu(c_1 = 0 | p^M) = 0 \forall p^M \in (3.019, \infty),\)

where \(p^M_0 = p^M(c_1 = 0)\) and \(p^M_1 = p^M(c_1 = 2)\) are the strategies of the incumbent firms with low and high cost respectively, and \(\mu(c_1 | p^M)\) is the government’s belief that firm 1 who sends signal \(p^M\) is of type \(c_1\).

**Proof** Refer to Figure 3 for proof. Assume that the government has a belief system such that \(\mu(c_1 = 0 | p) = 1, \forall p \in [0, \bar{p}]\) and \(\mu(c_1 = 0 | p) = 0, \forall p \in (\bar{p}, \infty).\) Note that \(\bar{p} < p^M_0 = 5.\) This is because, otherwise, both types of incumbent will choose \(p^M_0^* = 5\) and then there will be no separating equilibrium. First consider the efficient type of incumbent. If it would choose a price in the interval \((\bar{p}, \infty)\), it will choose \(p^M_1 = 5,\) and the subsequent two period total profit will be \(\pi^M_0^* + \pi^M_1^*\). If it would choose a price in the interval \([0, \bar{p}]\), it would choose \(\bar{p}\), and the subsequent two period total profit will be \(\pi^M_0(\bar{p}) + \pi^M_1^*\). For the efficient type incumbent to choose a price which is consistent with government’s belief, it must hold that \(\pi^M_0 + \pi^M_1^* \leq \pi^M_0(\bar{p}) + \pi^M_1^*\), that is, \(\bar{p} \geq 2\) should hold. Now consider the behavior of the inefficient type. By the same logic as above, for the inefficient type to choose a price which is consistent with government’s belief, it should hold that \(\pi^M_0(\bar{p}) + \pi^M_1^* \leq \pi^M_1 + \pi^M_0^*\), which implies that \(\bar{p} \leq 3.019.\) Combining both types’ optimal strategies, we have \(2 \leq \bar{p} \leq 3.019.\) However, by Cho and Kreps’ intuitive criterion, all prices in the interval \(2 \leq \bar{p} \leq 3.019\) are equilibrium dominated for the inefficient incumbent, and therefore, it should be that \(\bar{p} = 3.019.\) Finally, it is easy to confirm that all the players’ strategies are sequentially rational and the government’s belief is consistent.

Q.E.D.

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11 The Intuitive Criterion is as follows: The inefficient type will never send a signal in the interval of \(2 \leq \bar{p} < 3.019\) because the maximum profit by choosing a price in this interval is smaller than the profit by choosing the equilibrium price.
In the unique separating equilibrium with refinement, the low cost type incumbent deviates from the monopoly behavior and chooses a lower price than $p^*_M = 5$ and the high cost type incumbent chooses $p^*_M = 6$. The entry is regulated when the incumbent is of low cost type, and it is allowed when the incumbent is of high cost type.

If the equilibrium is separating, since the government can tell the true cost of the incumbent just by observing the first period market price, it can implement the optimal entry regulation as under complete information, that is, allowing entry if and only if the incumbent is of high cost type.

**Pooling Equilibrium**

Now consider a pooling equilibrium, where the incentive of the high cost incumbent to mimic the low cost type dominates low cost incumbent’s separating incentive. In the pooling equilibrium, the high cost incumbent chooses the same price as that chosen by the low cost incumbent and successfully maintains its monopoly position through entry regulation, which would not be possible under complete information.

**Proposition 2.** If $\alpha < \frac{80 - 18F}{71}$, then there exists no pooling equilibrium.

**Proof** In any pooling equilibrium, the government cannot distinguish the low cost type from the high cost type, and so it maximizes the expected social welfare at $t_2$, which is equal to $\alpha W^M(c_1 = 0) + (1 - \alpha) W^M(c_1 = 2) = \frac{27\alpha + 28}{2}$ in case entry being regulated, and equal to $\alpha W^D(c_1 = 0) + (1 - \alpha) W^D(c_1 = 2) = \frac{86\alpha + 256}{9} - F$ in case entry being allowed. Therefore, the optimal strategy of the government is to allow entry if and only if $\alpha < \frac{80 - 18F}{71}$. In this case, since the
government always allows entry regardless of the first period price, there is no incentive for any type incumbent to deviate from its monopoly profit maximizing price. This means that there doesn’t exist any pooling equilibrium where the high cost type mimics the pricing behavior of the low cost type incumbent. Q.E.D.

Proposition 2 says, when the chance that the incumbent monopolist is an efficient type is small, there exists no pooling equilibrium. The intuition behind Proposition 2 is as follows; when the high and the low cost type choose the same price, if the incumbent monopolist is more likely a high cost type, then the optimal entry regulation should be allowing entry, therefore the high cost type will not mimic the low cost type’s pricing.

Proposition 2 also implies that when the probability of being an efficient type is sufficiently high, there exists a pooling equilibrium. However, as usual in many signaling games, the pooling equilibrium may not be unique. Actually we have a continuum of pooling equilibria, some of which might be unreasonable since they are based on unreasonable beliefs on the off-the-equilibrium paths. We want to delete such unreasonable pooling equilibria by applying Cho and Kreps’ Intuitive Criterion, and the refined pooling equilibrium is summarized in Proposition 3.

**Proposition 3.** If \( \alpha \geq \frac{80 - 18F}{71} \), then there exist pooling equilibria which pass the Intuitive Criterion by Cho and Kreps as follows;

- \( \mu^H: p^H = p^2 = p_r \in [3.019, 5] \)
- \( E: \text{disallow entry } \forall p^H \in [0, p_r] \) and allow entry \( \forall p^H \in (p_r, \infty) \)
- \( B: \mu(c_1 = 0 | p^H) = \alpha \) for \( \forall p^H \in [0, p_r] \) and \( \mu(c_1 = 0 | p^H) = 0 \) for \( \forall p^H \in (p_r, \infty) \).

**Proof** Without loss of generality, suppose that the government holds a reasonable belief system \( B \) such that \( \mu(c_1 = 0 | p) = \alpha \) for \( \forall p \in [0, p_r] \) and \( \mu(c_1 = 0 | p) = 0 \) for \( \forall p \in (p_r, \infty) \). With such belief, the government will allow entry for \( \forall p \in (p_r, \infty) \) and reject entry for \( \forall p \in [0, p_r] \), given \( \alpha \geq \frac{80 - 18F}{71} \). In order for such belief by the government to be consistent with optimal behavior of the incumbent monopolist, the following sequential rationality conditions for both types should be satisfied in a pooling equilibrium. First note that if \( p_r < 3.019 \) then the inefficient type will never mimic efficient type’s behavior. Therefore for the pooling equilibrium, \( p_r \geq 3.019 \) should hold. Meanwhile, if \( p_r > p^H_{\alpha^*} = 5 \), then the efficient type will choose \( p_2^H = 5 \) and the inefficient type will choose a price greater that \( p_2^H = 5 \), which means no pooling equilibrium. Therefore, it should be true that \( p_r \leq p^H_{\alpha^*} = 5 \). Finally, for \( 3.019 \leq p_r \leq 5 \), both types will choose \( p_r \). We have infinitely many pooling equilibria, and they all pass the Intuitive Criterion. Q.E.D.

In a pooling equilibrium, both types choose the same price in the interval of \([3.019, 5]\), which includes \( p^H_{\alpha^*} = 5 \), and entry is regulated regardless of the incumbent’s type. If \( \alpha \) is high enough, the government will regulate entry even when it cannot tell the true type of the incumbent monopolist. Therefore the high cost type incumbent should have incentive to mimic
low cost type’s pricing behavior to induce entry regulation and protect its monopoly position, which were impossible under complete information and/or under separating equilibrium.

IV. Efficiency of Entry Regulation under Incomplete Information

Now, let us show that signaling by the incumbent firm makes entry regulation under incomplete information more efficient than under complete information.

**Proposition 4. (Optimality of the Unique Separating Equilibrium)** At the unique separating equilibrium which passes Cho and Kreps’ Intuitive Criterion, entry regulation is more efficient under incomplete information than under complete information.

**Proof** Incomplete information generates the same optimal market structure at $t_2$ as under complete information. Furthermore, since the efficient type incumbent deviates from the monopoly pricing to signal its identity, the first period’s social welfare is enhanced under incomplete information. Therefore, the outcome under incomplete information is strictly better than under complete information; the same optimal market structure in the future and the lower (same) market price today in case that the incumbent is of efficient (inefficient) type. Q.E.D.

Proposition 4 implies that the incomplete information is not necessarily a cause of inefficiency, rather it can generate supra-optimal outcome as in the case of government’s entry regulation. We already show that, while there always exists a unique separating equilibrium, the pooling equilibrium doesn’t exist when $\alpha < \frac{80-18F}{71}$. Therefore, in case of $\alpha < \frac{80-18F}{71}$, Proposition 4 becomes stronger and we can certainly predict that entry regulation is clearly more efficient under incomplete information than under complete information.

Meanwhile, if $\alpha \geq \frac{80-18F}{71}$ then there also exist pooling equilibria, and so we may not guarantee the supra-optimality of entry regulation under incomplete information. However we can show that, even at the pooling equilibria, the efficiency of the entry regulation under incomplete information also holds almost surely.

**Proposition 5. (Ex-post Efficiency of the Pooling Equilibrium)** When the incumbent is of low cost type, the pooling equilibrium under incomplete information is always more efficient than the complete information equilibrium. When the incumbent is of high cost type, pooling equilibria with $p_r \in [3.019, 4.848]$ are more efficient than the complete information equilibrium and those with $p_r \in (4.848, 5]$ are more efficient than the complete information equilibrium for $F \in (\hat{F}, 4]$, where $\hat{F} = \frac{9p_r^2 - 36p_r - 28}{18}$.

**Proof** Let $\Delta SW(c_1) = SW^P(c_1) - SW^C(c_1)$ for $c_1 = 0, 2$, where the superscripts $P$ and $C$ represent pooling (under incomplete information) and complete information respectively, and $SW$ is the two period total social welfare. Then, $\Delta SW(c_1 = 0) = \sum_i \left[ \int_0^{X_i} p(s)ds \right] - 75$ for $X_1 = 10 - p_r$ and $X_2$...
\[
= 5, \text{ and } \Delta SW(c_1 = 2) = \sum_i \left[ \int_0^{X_i} p(s) ds - 2X_i \right] - (52.44 - F) \text{ for } X_1 = 10 - p_P \text{ and } X_2 = 4.
\]

First note that \( \Delta SW(c_1 = 0) \geq 0 \) is always satisfied for all \( p_P \leq 5 \). Therefore, if the incumbent is the efficient type, then the incomplete information always brings a higher efficiency than the complete information in government’s entry regulation. Next, if the incumbent is the inefficient type, \( \Delta SW(c_1 = 2) \geq 0 \) is also guaranteed for all \( p_P \leq 4.848 \). Meanwhile if \( p_P > 4.848 \) then, \( \Delta SW(c_1 = 2) \geq 0 \) holds for \( F \in (\hat{F}, 4] \) where \( \hat{F} = \frac{9p_P^2 - 36p_P - 28}{18} \). Q.E.D.

We can explain the optimality of the pooling equilibrium in most cases in the following four steps: First, any pooling equilibrium is more efficient than the complete information equilibrium if the incumbent is of low cost type. Second, when the incumbent is of high cost type, all the pooling equilibria with \( p_P \in [3.019, 4.848] \) are also more efficient than the complete information equilibrium. Third, for those pooling equilibria with \( p_P \in (4.848, 5] \), the optimality of the pooling equilibrium is also guaranteed for \( F \in (\hat{F}, 4] \), where \( \hat{F} = \frac{9p_P^2 - 36p_P - 28}{18} \). Finally, even in case of the high cost incumbent, the range of supra-optimality of the pooling equilibrium is substantially large as is shown in Figure 4.

Since ex-post efficiency under incomplete information holds for all pooling equilibria in the efficient incumbent case and it holds for almost all pooling equilibria in the inefficient incumbent case, and furthermore since the pooling equilibria more likely exist when the incumbent is of low cost type, the ex-ante social welfare \( E(SW) = \alpha SW(c_1 = 0) + (1 - \alpha) SW(c_1 = 2) \) under the pooling equilibrium should be necessarily higher than that under complete information. Proposition 6 summaries such an ex-ante optimality of the pooling equilibrium of the government’s entry regulation under incomplete information.
Proposition 6. (Ex-ante Optimality of the Pooling Equilibrium) Entry regulation in the pooling equilibrium under incomplete information is more efficient than that under complete information.

**Proof** The ex-ante welfare differential between the pooling equilibrium and the complete information equilibrium \( \alpha \Delta SW(c_1 = 0) + (1 - \alpha) \Delta SW(c_1 = 2) \) is greater than zero if and only if \( \alpha \geq 1 + \frac{M^2/2 + 37.5 - 10M}{2M - F - 9.06} \), where \( M = 10 - p_r \). Note that for all \( p_r \) in region A in Figure 4, that is \( \forall p_r \in [3.019, 4.848) \) or \( \forall p_r \in (4.848, 5) \) and \( F \in (\hat{F}(p_r), 4) \), it is trivially true that \( \alpha \Delta SW(c_1 = 0) + (1 - \alpha) \Delta SW(c_1 = 2) \geq 0 \). But \( \forall p_r \in (4.848, 5) \) and \( F \in (0.5, \hat{F}(p_r)) \), that is for those prices belonging to the region B in Figure 4, we can show that \[ 1 + \frac{M^2/2 + 37.5 - 10M}{2M - F - 9.06} < \frac{80 - 18F}{71} \]. Therefore, if there exists any pooling equilibrium, then \( \alpha \geq \frac{80 - 18F}{71} \) and so the ex-ante social welfare under the pooling equilibrium is strictly higher than that under the complete information. Q.E.D.

V. A Screening Model of Entry Regulation

In this paper, we model entry regulation under incomplete information as a signaling game, where the informed party, the incumbent monopolist, moves first. However, we might also model the same entry regulation as a screening game where the uninformed government moves first by offering entry condition as a function of the price chosen by the incumbent monopolist. In this section, we will derive an optimal entry regulation under such a screening game, rather than under a signaling game, and confirms the optimality of the entry regulation under incomplete information.

Note that in a screening game of entry regulation, the government can always replicate any equilibrium outcome of the signaling game. Furthermore, in a screening game, since we don’t have to take the belief of the uninformed party into considerations, all signaling equilibria without any refinements are the possible candidates for the government’s choice. The benevolent government will choose the best separating equilibrium in case that pooling equilibrium does not exist. In this case as we see the entry regulation is more efficient under incomplete information than under complete information.

Meanwhile the government will compare the best separating equilibrium with the best pooling equilibrium in case that the latter exists, and then choose the better one as the optimal entry regulation condition. Since we already show that both the best separating and the best pooling equilibrium dominate the complete information equilibrium, the entry regulation in a screening game also clearly shows a better performance than under complete information. Proposition 7 summarizes such an optimal entry regulation in a screening game.

Proposition 7. If \( \alpha \in \left[ \frac{80 - 18F}{71}, \frac{3.04 + F}{5.6 + F} \right] \), the optimal entry regulation is allowing entry for
all $p^M \in (3.019, \infty)$ and reject entry for all $p^M \in [0, 3.019]$. Under such entry regulation both efficient and inefficient types of incumbent monopolist choose the same price $p^M = 3.019$ and entry is prevented by the government (replicating the best pooling equilibrium). If either $\alpha < \frac{80-18F}{71}$ or $\alpha > \frac{3.04+F}{5.6+F}$, then the optimal entry regulation is allowing entry for all $p^M \in (2, \infty)$ and reject entry for all $p^M \in [0, 2]$. Under such entry regulation, the efficient type of incumbent monopolist chooses $p^M = 2$ with entry being rejected and the inefficient type chooses $p^M^* = 6$ with entry being allowed (replicating the best separating equilibrium). The government’s entry regulation in either case shows a better performance than under complete information.

**Proof** It is obvious that $(p_0^M = 2, p_2^M = 6, E, B)$ is the most efficient among all separating equilibria of the signaling game without any refinement by the Intuitive Criterion, since the level of price is minimal. Therefore the government chooses $(p_0^M = 2, p_2^M = 6)$ as the separating equilibrium which will be duplicated in the screening game. The ex-post social welfare for both types of incumbents are $SW^S(c_1 = 0) = 85.5$ and $SW^S(c_1 = 2) = 52.44 - F$, and so the ex-ante social welfare is $E^S(SW) = 33.06\alpha + 52.44 - (1 - \alpha)F$, where the superscript $S$ denotes separating. On the other hand, the government, if it chooses a pooling equilibrium, selects $(p_0^M = p_2^M = 3.019)$ as the one to be duplicated in the screening game, since $(p_0^M = p_2^M = p_F = 3.019, E, B)$ is the most efficient among all the pooling equilibria in the signaling game. At this pooling equilibrium $(p_0^M = p_2^M = 3.019)$, the ex-post social welfare for both types of incumbents are $SW^P(c_1 = 0) = 82.94$ and $SW^P(c_1 = 2) = 55.48$, and so the ex-ante social welfare is $E^P(SW) = 27.46\alpha + 55.48$. $E^S(SW) \leq E^P(SW)$ is satisfied if $\alpha \leq \frac{3.04+F}{5.6+F}$. Combining with the condition
for the existence of a pooling equilibrium \( \alpha \geq \frac{80 - 18F}{71} \), we know that the best choice by the benevolent government is duplicating the pooling equilibrium \((p^M_0 = p^M_2 = 3.019)\) if \( \alpha \in \left[ \frac{80 - 18F}{71}, \frac{3.04}{5.6 + F} \right] \), and it is duplicating the separating equilibrium \((p^M_0 = 2, p^M_2 = 6)\) otherwise. Q.E.D.

VI. Concluding Remarks

The main proposition of this paper is that, while the lack of complete information by the government is a barrier to the optimal price regulation, it improves the performance of the entry regulation. This result is based on the following basic observations: First, if incumbent's andentrant's costs are not correlated, excess entry can occur when the incumbent is more efficient. Second, the incumbent firm should have an incentive to signal itself as an efficient type to induce entry regulation. Finally, therefore, the incumbent firm will deviate from the monopoly pricing, which surely is a welfare-increasing donation to the whole society.

Note that if incumbent's and entrant's costs are positively correlated, the incumbent would choose a higher price than the monopoly price to signal a high cost of the new entrant to the government and so to induce entry regulation against excess entry. Since the incumbent's incentive under entry regulation is not consistent with the optimality of the government regulation, entry regulation under incomplete information becomes suboptimal contrary to the case of mutually independent costs.

Several issues can be brought up for further research. First, can we find other interesting cases where incomplete information is not the cause of the market failure but the cause of supra-efficiency? This is equivalent to asking if we can find other real examples under which the signaling costs can be donations to the whole economy.

Second, what's the optimal regulation if both entry and price regulations are implemented at the same time? The government might not be able to implement different mechanisms simultaneously, that is, incentive mechanism for price regulation and signaling mechanism for entry regulation. This is because the incentive price regulation may not be incentive compatible any longer if the regulated firm takes entry regulation into consideration at the same time. Likewise, the signaling effect in entry regulation may change against price regulation.

Finally, the supra-optimality result of entry regulation under incomplete information might be an example that, when more than one market failures coexist, one cause of market failure can mitigate the other market failure problem; the monopolist should deviate from the monopoly behavior because of the incomplete information problem. Analyzing multiple market failure problems together seems to be an interesting research issue.

References