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Fiscal Policy Switching in Japan, the U.S., and the U.K.

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February 4, 2011

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Fiscal Policy Switching in Japan, the U.S., and the U.K.

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Abstract

This paper estimates fiscal policy feedback rules in Japan, the United States, and the United Kingdom for more than a century, allowing for stochastic regime changes. Estimating a Markov-switching model by the Bayesian method, we find the following: First, the Japanese data clearly reject the view that the fiscal policy regime is fixed, i.e., that the Japanese government adopted a Ricardian or a non-Ricardian regime throughout the entire period. Instead, our results indicate a stochastic switch of the debt-GDP ratio between stationary and nonstationary processes, and thus a stochastic switch between Ricardian and non-Ricardian regimes. Second, our simulation exercises using the estimated parameters and transition probabilities do not necessarily reject the possibility that the debt-GDP ratio may be nonstationary even in the long run (i.e., globally nonstationary). Third, the Japanese result is in sharp contrast with the results for the U.S. and the U.K. which indicate that in these countries the government’s fiscal behavior is consistently characterized by Ricardian policy.

JEL Classification Numbers: E62

Keywords: fiscal policy rule; fiscal discipline; Markov-switching regression

1 Introduction

Recent studies about the conduct of monetary policy suggest that the fiscal policy regime has important implications for the choice of desirable monetary policy rules, particularly, monetary policy rules in the form of inflation targeting (Sims (2005), Benigno and Woodford (2007)). It seems safe to assume that fiscal policy is characterized as “Ricardian” in the terminology of Woodford (1995), or “passive” in the terminology of Leeper (1991), if the government shows strong fiscal discipline. If

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this is the case, we can design an optimal monetary policy rule without paying any attention to fiscal policy. However, if the economy is unstable in terms of the fiscal situation, it would be dangerous to choose a monetary policy rule independently of fiscal policy rules. For example, some researchers argue that the recent accumulation of public debt in Japan is evidence of a lack of fiscal discipline on the part of the Japanese government, and that it is possible that government bond market participants may begin to doubt the government’s intention and ability to repay the public debt. If this is the case, we may need to take the future evolution of the fiscal regime into consideration when designing a monetary policy rule.

Against this background, the purpose of this paper is to estimate fiscal policy feedback rules for Japan, the United States, and the United Kingdom for a period spanning more than a century, so as to gain a deeper understanding of the evolution of fiscal policy regimes. One of the most important features of recent studies on fiscal policy rules is the recognition that fiscal policy regimes are not fixed over time, but evolve in a stochastic manner. For example, Favero and Monacelli (2005) and Davig and Leeper (2007) estimate fiscal policy rules for the United States during the postwar period under the assumption that there are two alternative fiscal regimes, i.e. a “passive” and an “active” regime, and that stochastic fluctuations between the two regimes may be characterized by a Markov process. They find that fiscal regime switching occurred fairly frequently: Davig and Leeper (2007) report that there were twelve fiscal regime changes during the period of 1948-2004, while Favero and Monacelli (2005) found that fiscal policy was even more unstable than monetary policy.

However, these pioneering works still have some shortcomings. First, they do not make an empirical distinction between locally and globally Ricardian policy rules. For example, Favero and Monacelli (2005) specify a locally Ricardian rule and ask whether the U.S. government has followed this rule or deviated from it. However, as pointed out by Bohn (1998) and Canzoneri et al. (2001), the transversality condition may be satisfied even if the debt-GDP ratio does not follow a stationary process, or equivalently, even if a government deviates from a locally Ricardian policy rule. Second, the studies by Davig and Leeper (2007) and Favero and Monacelli (2005) do not pay much attention to governments’ tax smoothing behavior. As pointed out by Barro (1986) and Bohn (1998), tax-smoothing

---

1A comprehensive list of recent empirical studies on fiscal policy rules is provided by Afonso (2008).
2These studies are in sharp contrast with research on fiscal sustainability initiated by Hamilton and Flavin (1986) about two decades ago, which typically investigates whether fiscal variables such as the debt-GDP ratio are characterized by a stationary or a nonstationary process without any break (Trehan and Walsh (1988, 1991), Wilcox (1989), Ahmed and Rogers (1995)).
behavior may create a negative correlation between public debt and the primary surplus. Without properly controlling for such behavior when estimating a government’s reaction function, researchers may easily obtain biased estimates of fiscal policy reactions to a change in public debt. Third, the empirical approach of these studies is based on maximum likelihood estimation and implicitly assumes that the debt-GDP ratio is stationary at least in the long run (i.e., that it is “Harris recurrent”). This condition is satisfied if, for example, the debt-GDP ratio switches between two AR \((p)\) processes, one stationary and the other nonstationary, but the nonstationary regime is not visited too often or for too long (Francq and Zakoian (2001)). However, there is no a priori reason to believe that this condition is indeed satisfied for the debt-GDP ratio; it is possible that a non-Ricardian regime is visited frequently and/or for a long time, depending on the transition probabilities. If this is the case, maximum likelihood estimators will fail to follow a standard normal distribution even asymptotically (Douc et al. (2004)).

We derive an estimating equation based on a model of optimal tax smoothing, paying particular attention to differences between locally and globally Ricardian rules, and then estimate the equation by the Bayesian method. The main findings of the paper are as follows. First, the Japanese data set, covering the period 1885-2004, clearly rejects the view that the fiscal policy regime was fixed throughout the sample period, i.e., that the Japanese government adopted only one policy stance - Ricardian or non-Ricardian - throughout the entire period. Rather, our empirical results suggest that the fiscal policy regime evolved over time in a stochastic manner, and that the debt-GDP ratio is well described by a Markov switching model with two or three states. Specifically, Japanese fiscal policy is characterized by a locally Ricardian rule in 1885-1925 and 1950-1970. The former roughly corresponds to the period when Japan had adopted the gold standard, under which the government was forced to maintain a balanced budget. Japan left the gold standard in 1917. The latter period corresponds to the period of fiscal restructuring just after WWII, when the Japanese government, under the direction of the Supreme Commander for Allied Powers (SCAP) introduced a balanced budget system as part of the so-called “Dodge Line” in order to stop runaway inflation. On the other hand, Japanese fiscal policy is characterized by non-Ricardian rules in 1930-1950 and 1970-2004, suggesting that the Japanese government abandoned fiscal discipline not only during WWII, but also in the most recent period starting in 1970. These empirical results are confirmed as being quite robust to changes in empirical specifications.
Second, given that the Japanese debt-GDP ratio switches between stationary and nonstationary processes, one may wonder to what value the debt-GDP ratio goes to in the long run. To address this question, we conduct stochastic simulation exercises using the estimated transition probabilities, and find that the debt-GDP ratio is quite likely to increase over the next 20 years, but will start declining after that and finally converge to zero. This implies that the debt-GDP process is “globally stationary” (i.e., stationary across regimes), although it may not necessarily be locally stationary (i.e., stationary within each regime). However, we also find that this result is not very robust to changes in the specification of the estimating equation, such as the number of possible “states,” and in some cases, we find global nonstationarity.

Third, we apply our methodology to U.S. and U.K. data sets to find that the fiscal behavior of the U.S. government throughout the entire sample period, 1840-2005, may be described as switching between locally Ricardian policy rules, while the behavior of the U.K. government during the entire sample period, 1830-2003, can be characterized as switching between globally Ricardian policy rules. Thus, the U.S. and U.K. results are in sharp contrast with the result for Japan. The U.S. result is consistent with Bohn (1998, 2008), but differs from Favero and Monacelli (2005) who report that U.S. government behavior deviated from Ricardian policy for most of their sample period, 1961-2002.

The remainder of this paper is organized as follows. Sections 2 and 3 explain our empirical approach, while Section 4 explains our data set. Section 5 presents the regression results. Section 6 concludes the paper. A detailed explanation of our data set is provided in Appendix.

2 Ricardian fiscal policy

2.1 The government’s budget constraint

We start by looking at the government’s budget constraint. Let us denote the nominal amount of public debt and base money at the end of period $t$ by $B_t$ and $M_t$. Also, we denote the one-period nominal interest rate starting in period $t-1$ by $i_{t-1}$, the nominal government expenditure (excluding interest payments) and the nominal tax revenue in period $t$ by $G_t$ and $T_t$. Then the consolidated flow budget constraint of the government and the central bank takes the following form:

$$M_t + B_t = (1 + i_{t-1})B_{t-1} + M_{t-1} + (G_t - T_t).$$

3The recent behavior of investors in the Japanese government bond market seems to be consistent with this as they show no hesitation to purchase government bonds even though the government is lacking fiscal discipline and rapidly accumulating public debt.
Dividing both sides of this equation by nominal GDP, \( Y_t \), we obtain:

\[
m_t + b_t = \frac{1 + i_{t-1}}{1 + n_t} b_{t-1} + \frac{1}{1 + n_t} m_{t-1} - s_t,
\]

where \( m_t, b_t, s_t \), and \( n_t \) are defined by

\[
m_t \equiv \frac{M_t}{Y_t}; \quad b_t \equiv \frac{B_t}{Y_t}; \quad s_t \equiv \frac{T_t - G_t}{Y_t}; \quad n_t \equiv \frac{Y_t - Y_{t-1}}{Y_{t-1}}.
\]

Denoting the total consolidated liabilities by \( w_t(\equiv m_t + b_t) \), the transition equation for \( w_t \) can be expressed as:

\[
w_t - w_{t-1} = \frac{i_{t-1}}{1 + n_t} w_{t-1} - \frac{n_t}{1 + n_t} w_{t-1} = \left[ \frac{i_{t-1}}{1 + n_t} m_{t-1} + s_t \right]. \tag{1}
\]

Note that \( \frac{n_t}{1 + n_t} m_{t-1} \) represents seignorage and that an increase in the primary surplus \( s_t \) or seignorage reduces total liabilities. Also note that an increase in the nominal growth rate \( n_t \) contributes to lowering total liabilities through the second term on the right-hand side, \( -\frac{n_t}{1 + n_t} w_{t-1} \), which is sometimes called the “growth dividend” (Bohn (2008)).

Equation (1) can be rewritten as

\[
w_t = q_t [w_{t+1} + s_{t+1}] + \frac{i_t}{1 + i_t} m_t, \tag{2}
\]

where \( q_t \) represents a discount factor that is defined by

\[
q_t \equiv \frac{1 + n_{t+1}}{1 + i_t}.
\]

Integrating equation (2) forward from the current period and taking expectations conditional on information available in period \( t \), we obtain a present-value expression of the budget constraint:

\[
w_t = s_t + E_t \left( \sum_{j=1}^{T} \left( \prod_{k=0}^{j-1} q_{t+k} \right) s_{t+j} \right) + E_t \left( \sum_{j=0}^{T-1} \left( \prod_{k=0}^{j-1} q_{t+k} \right) \left( \frac{i_{t+j}}{1 + i_{t+j}} \right) m_{t+j} \right) + E_t \left( \prod_{k=0}^{T-1} q_{t+k} \right) w_{t+T}.
\]

This implies that the transversality condition is given by

\[
\lim_{T \to \infty} E_t \left( \prod_{k=0}^{T-1} q_{t+k} \right) w_{t+T} = 0. \tag{3}
\]

### 2.2 Locally Ricardian policy rules

Woodford (1995) proposes that a fiscal policy commitment be called “Ricardian” if it implies that the transversality condition, equation (3), necessarily holds for all possible paths of endogenous variables

The first type, which is referred to as “locally Ricardian,” can be expressed as

\[ s_t + \frac{i_{t-1}}{1 + n_t} m_{t-1} = \left[ \lambda_t + \frac{i_{t-1}}{1 + n_t} \right] w_{t-1} + \nu_t, \quad (4) \]

where \( \lambda_t \) is a time-varying parameter satisfying \( 0 < \lambda_t \leq 1 \), which represents the government’s responsiveness to changes in total liabilities, and \( \nu_t \) is an exogenous stationary variable. Note that the left-hand side of equation (4) represents the sum of the primary surplus and seignorage. Equation (4) requires the government to create a surplus in period \( t \) great enough to cover its interest payment in that period, \( \frac{n_{t-1}}{1 + n_t} w_{t-1} \).

By substituting (4) into (1), we can fully characterize the dynamics of \( w_t \):

\[ w_t = \left[ 1 - \lambda_t - \frac{n_t}{1 + n_t} \right] w_{t-1} - \nu_t. \quad (5) \]

Under the assumption that \( n_t \) is an exogenous process (i.e., the government treats \( n_t \) as exogenously given when making a fiscal decision in period \( t \)),\(^4\) this equation implies that \( w_t \) would be a stationary process and thus satisfies the transversality condition if the sum of \( \lambda_t \) and \( \frac{n_t}{1 + n_t} \) lies between zero and unity.\(^5\) Note that the assumption of a locally Ricardian policy requires that \( 1 - \lambda_t \) is smaller than unity, while stationarity of \( w \) requires that the coefficient on \( w_{t-1} \) in (5) is less than unity. These two conditions are closely related but not identical except for the case of \( n_t = 0 \).

An alternative specification to equation (4) would be:

\[ s_t + \frac{i_{t-1}}{1 + n_t} m_{t-1} + \frac{n_t}{1 + n_t} w_{t-1} = \left[ \hat{\lambda}_t + \frac{i_{t-1}}{1 + n_t} \right] w_{t-1} + \nu_t. \quad (6) \]

Note that \( \hat{\lambda}_t \equiv \lambda_t + \frac{n_t}{1 + n_t} \). Now the government seeks to adjust the sum of the primary surplus, seignorage, and the growth dividend in response to changes in total liabilities. An important difference from equation (4) is that the government reduces the primary surplus when the growth dividend is positive, for example, due to high inflation, and increases it when the growth dividend is negative; on

\(^4\)It is possible that \( n_t \) could be an endogenous variable in the sense that the government’s fiscal behavior could have non-negligible consequences on the path of \( n_t \). For example, as argued by Woodford (2001) among others, it might be possible that if the government does not react at all to changes in total liabilities (that is, \( \lambda_t = 0 \)), then inflation endogenously emerges (\( n_t > 0 \)), and consequently the coefficient on \( w_{t-1} \) in equation (5) becomes less than unity.

\(^5\)Note that, from an econometric point of view, \( w_t \) is a stationary process if the coefficient on \( w_{t-1} \) in equation (5) lies between -1 and 1 (-1 < \( 1 - \lambda_t - \frac{n_t}{1 + n_t} \) < 1). However, it seems safe to rule out the possibility that \( w \) converges over time to a constant value with oscillation, so that we can concentrate on the condition that the coefficient lies between 0 and 1 (0 \( \leq 1 - \lambda_t - \frac{n_t}{1 + n_t} \) < 1).
the other hand, equation (4) requires the government to create a primary surplus independently of
the level of the growth dividend. It can be easily seen that the transition equation corresponding to
(5) is now given by
\[ w_t = \left[ 1 - \lambda_t \right] w_{t-1} - \nu_t, \tag{7} \]
and that \( w_t \) is a stationary process if \( \lambda_t \) satisfies the condition that \( 0 < \lambda_t \leq 1 \).

Favero and Monacelli (2005) adopt a policy reaction function very close to equation (6). According
to their definition, a government with fiscal discipline seeks to keep the primary deficit lower than the
“debt-stabilizing deficit”, which is given by
\[ - \left[ \frac{i_{t-1}}{1 + n_t} - \frac{n_t}{1 + n_t} \right] w_{t-1}. \]
Given this definition, the debt-stabilizing deficit becomes positive if \( n_t \) takes a sufficiently large positive
value, implying that the government can run a deficit.

2.3 Globally Ricardian policy rules

The idea that the government should maintain a surplus large enough to at least cover interest
payments seems to be a useful one from a practical point of view, but the transversality condition
does not necessarily require it. Specifically, as shown by Bohn (1998) and Canzoneri et al. (2001),
the transversality condition could be satisfied even if the government reacts to an increase in total
liabilities by less than the amount needed to cover its interest payments. This is the second type of
Ricardian policy, which is referred to as “globally Ricardian.”

Globally Ricardian policy can be expressed as
\[ s_t + \frac{i_{t-1}}{1 + n_t} m_{t-1} = \gamma_t w_{t-1} + \nu_t, \tag{8} \]
where \( \gamma_t \) is a time-varying parameter satisfying \( 0 < \gamma_t \leq 1 \). Note that equations (4) and (6) require
the government to generate a primary surplus that is sufficient to cover its interest payments in each
period. Here, however, the government can now issue additional debt to pay interest on the existing
debt at the beginning of that period. Under this policy rule, the dynamics of \( w_t \) are now given by
\[ w_t = \left[ 1 - \gamma_t - \frac{n_t}{1 + n_t} + \frac{i_{t-1}}{1 + n_t} \right] w_{t-1} - \nu_t \tag{9} \]

If we rewrite equation (4) as \( s_t - \frac{i_{t-1}}{1 + n_t} m_{t-1} = \lambda_t w_{t-1} + \nu_t \), we see that the rule requires that not the primary surplus
but the traditional fiscal surplus (i.e., primary surplus less interest payment) be adjusted in response to a change in
total liabilities, which is the idea underlying the Maastricht Treaty and the Stability and Growth Pact. See Woodford (2001) for more on this issue.
or

\[ w_t = \left( \frac{1}{\gamma_t} - \gamma \right) w_{t-1} - \nu_t, \]  

which implies that the transversality condition (equation (3)) is satisfied if \( 0 < \gamma_t \leq \frac{1}{\nu_t}. \) Note that this condition does not necessarily guarantee that \( w_t \) is a stationary process; in fact, it allows \( w_t \) to grow forever, but at a rate lower than the interest rate in each period. In that sense, a globally Ricardian rule imposes a weaker condition on government behavior than a locally Ricardian rule.

Bohn (1998, 2008) adopts a policy reaction function very close to equation (8) and looks at U.S. data to determine whether \( \gamma_t \) is positive.\(^8\) Equation (8) is an appropriate estimating equation when the government adopts a globally Ricardian policy or when it actually adopts a locally Ricardian policy but interest rates do not fluctuate much during the sample period. In the latter case, we would be able to empirically distinguish between a locally and a globally Ricardian policy just by looking at whether the estimated coefficient on \( w_{t-1} \) is greater than the sample average of the nominal interest rate. However, if the government adopts a locally Ricardian policy and fluctuations in interest rates are not small, then Bohn’s specification may not be appropriate. For example, the estimated coefficient on \( w_{t-1} \) may become biased towards zero if fluctuations in interest rates are quite large during the sample period while those in public debt are negligibly small.

3 Estimation method

3.1 Estimating equations

Given the two definitions of Ricardian fiscal policy above, we estimate an equation of the form

\[
    b_t = \begin{cases} 
        \mu_0 + (\alpha_0 + \eta_t)b_{t-1} + u_{0t}, & \text{if } S_t = 0 \\
        \mu_1 + (\alpha_1 + \eta_t)b_{t-1} + u_{1t}, & \text{if } S_t = 1 
    \end{cases} 
\]  

(11)

where \( u_{it} = \varepsilon_{it} - \nu_t \) with \( \varepsilon_{it} \sim i.i.d. N(0, \sigma_i^2). \)\(^9\) \( \{S_t \in (0, 1)\} \) is a two-state Markov chain with transition probabilities \( p_{ij} = \Pr(S_t = j \mid S_{t-1} = i). \) Note that, given that the current regime is \( i, \)

\(^7\)Again, we rule out the possibility that the coefficient on \( w_{t-1} \) in (9) or (10) is below zero.

\(^8\)However, Bohn (1998, 2008) does not consider the possibility that the fiscal regime evolves over time in a stochastic manner.

\(^9\)It is assumed that the error term, \( \varepsilon_{it} \), is uncorrelated with \( b_{t-1} \). A usual justification of this assumption is that \( b_{t-1} \) is predetermined. In this paper, we will employ this orthogonality assumption following the previous studies such as Bohn(1998). However, it should be noted that \( b_{t-1} \) may not necessarily be a predetermined variable but a forward-looking variable in some models. If this is the case, the coefficient on \( b_{t-1} \) is no longer consistently estimated because of the presence of endogeneity problem. See Li (2010) for more on this issue.
the expected average duration of staying in the same regime is \((1 - p_{ii})^{-1}\). Also, note that we use public debt issued by the government \(b_t\) as the dependent variable rather than the total liabilities \(w_t\), assuming that the amount of base money \(m_t\) is small relative to the public debt and that fluctuations play a much less important role in seignorage than in the primary surplus.

We specify four different estimation equations based on different definitions of the observable variables, \(\eta_t\) and \(\nu_t\).

**Specification 1** \(\eta_t = \nu_t = 0\): This is the benchmark case in which no exogenous variables are included. Hence, \(b_t\) follows a simple Markov-switching AR(1) process.

**Specification 2** \(\eta_t = 0\), and \(\nu_t = -g_t^m\): This is a case in which government tax smoothing behavior is incorporated through \(g_t^m\) (military expenditures relative to GDP). As pointed out by Barro (1986) and Bohn (1998), the government’s tax-smoothing behavior may create a negative correlation between public debt and the primary surplus. To illustrate this, consider a situation in which the government increases its expenditures, but only temporarily (such as in the case of a war). The government could increase taxes simultaneously by the same amount as the increase in expenditures, but it is costly to change marginal tax rates over time, since doing so increases the excess burden of taxation. Recognizing this, an optimizing government would seek to smooth marginal tax rates over time. This implies that a temporary increase in government expenditures would lead to a decrease in the primary surplus and an increase in public debt. Bohn (1998) argues that such a negative correlation between the primary surplus and public debt should be properly controlled for when estimating the government’s reaction function; otherwise researchers may easily obtain imprecise estimates of fiscal policy reactions to an increase in public debt. Bohn (1998, 2008) shows that empirical results for the U.S. sharply differ depending on whether or not temporary government expenditures are included as an independent variable, while Iwamura et al. (2006) report a similar finding for Japan during the post-war period.

**Specification 3** \(\eta_t = -\frac{n_t}{1 + n_t}\), and \(\nu_t = -g_t^m\): This specification corresponds to equation (5) with \(\alpha_i = 1 - \lambda_i\). Note that when \(n_t\) is very close to 0, specification 3 reduces to specification 2. This condition might hold in a very stable economy without any experience of high inflation, but unfortunately, this is not the case for Japan, which experienced three-digit inflation rates just after
the end of WWII. Of course, Japan is not an exception, and one can easily find other examples in
which the accumulation of public debt leads to uncontrollably high inflation. For such countries,
specifications 2 and 3 are not identical.

**Specification 4** \( \eta_t = -\frac{n_t}{1+n_t} + \frac{i_t}{1+n_t} \), and \( \nu_t = -g_t^m \): This corresponds to equation (9) with \( \alpha_i = 1 - \gamma_i \). This specification differs from specification 3 in that interest payments, \( \frac{i_t}{1+n_t} \), are included in \( \eta_t \), reflecting the fact that the government is not required to create surplus to cover its
interest payments. Note that a globally Ricardian policy requires \( \alpha_i \) to be less than unity, implying
that, when \( n_t \) is always equal to zero, \( b_t \) could continue to grow forever, but at a rate lower than the
borrowing cost in each period.

Later in Section 5, we will estimate each of the four equations shown above; we will pay a particular
attention to the specifications 3 and 4, each of which describes the fiscal behavior of a government
with fiscal discipline (i.e. locally or globally Ricardian) and tax smoothing motivation.

### 3.2 Estimation

We estimate equation (11) by employing a Bayesian approach via the Gibbs sampler instead of a
classical approach based on maximum likelihood estimation. The Bayesian approach has the following
advantages. First, the maximum likelihood estimator (MLE) has the potential disadvantage that
inference on \( S_t \) is conditional on the estimates of the unknown parameters. We estimate the parameters
of the model and then make inferences on \( S_t \) conditional on the estimates of the parameters as if we
knew for certain the true values of the parameters. In contrast, the Bayesian approach allows both
the unknown parameters and \( S_t \) to be random variables. Therefore, inference on \( S_t \) is based on the
joint distribution of the parameters and \( S_t \) (see Kim and Nelson (1999)).

Second, for the Markov switching models, the likelihood is often not uni-modal but multi-modal.
Therefore, numerical algorithms such as Expectation Maximization (EM) and Newton-Rapson algo-
rithms sometimes converge to a local maximum on the likelihood surface. This is a typical problem
encountered with data in practice, regardless of which optimization algorithms are used. Maddala and
Kim (1998) argue that the maximum likelihood estimation method is fragile as multiple local maxima
are often found.

Third, MLE follows a non-standard limiting distribution when the process is nonstationary in the
long run (or globally nonstationary). To our knowledge, such limiting distributions have not been derived for Markov-switching models. On the other hand, the Bayesian method can approximate the joint and marginal distributions of the parameters and $S_t$ via a Markov chain Monte Carlo (MCMC) simulation method such as the Gibbs sampler. The method is valid even when the observed process exhibits non-stationarity (or explosive) behavior in the long run (see Sims (1988)). To illustrate this point, let us suppose there are two fiscal policy regimes: one is a stable regime in which the debt-GDP ratio is characterized by a stationary process, and the other one is an unstable regime in which the debt-GDP ratio is characterized by a nonstationary process. Note that the mere existence of an unstable regime does not necessarily imply global instability: The system could still be globally stable if the unstable regime is not visited too often or for too long. In this sense, the transition probabilities of the Markov chain are important determinants of global stability or instability. On the other hand, as shown by Francq and Zakoian (2001), it is possible that the system is globally unstable even when both of the two regimes are stable. An important point to be emphasized here is that it would not be appropriate to employ MLE if it is uncertain whether the system is globally stable.

3.3 MCMC simulation

The first time the Gibb sampler was used in a Bayesian analysis of Markov switching models was in the study by Albert and Chib (1993). The Gibbs sampler is used to approximate the joint and marginal distributions of the parameters of interest from the conditional distributions of the subsets of parameters given the other parameters (see Kim and Nelson (1999) for an introduction to Gibbs sampling). It is useful in this case because the joint distributions are difficult to obtain.

We follow Kim and Nelson (1999) to estimate a model of the form:

\[
\begin{align*}
\eta_t^* = \begin{cases} 
\mu_0 + \alpha_0 \eta_{t-1} + \epsilon_t, & \text{if } S_t = 0 \\
\mu_1 + \alpha_1 \eta_{t-1} + \epsilon_t, & \text{if } S_t = 1
\end{cases}
\]

where $b_t^* = b_t - \eta_t b_{t-1} + \nu_t$ and $\epsilon_t \sim i.i.d. N(0, \sigma^2_i)$ for $i = 0, 1$ with $\sigma^2_{\epsilon_i} = \sigma^2_i (1 + h_1 S_t)$ and $h_1 > 0$. \{\$S_t \in (0, 1)$\} is a two-state Markov chain with transition probabilities $p_{ij} = \Pr(S_t = j \mid S_{t-1} = i)$.

An alternative empirical framework to study fiscal regime shifts would be to use the methodology proposed by Bai and Perron (1998), in which a multiple linear regression model with $l$ breaks (or $l+1$ regimes) is examined within the classical framework. However, this approach requires the process to be weakly stationary in each regime. Therefore, their method cannot be applied in our context.
Note that the two states are assumed to be identified not by $\alpha_{S_t}$, but by $\sigma^2_{S_t}$, simply because we want to know if there is any difference between the two states in terms of $\alpha_{S_t}$.

### 3.3.1 Prior distributions

Next we describe the choice of priors for the unknown parameters. Let $\tilde{h}_1 = 1 + h_1$ with $h_1 > 0$. Then the priors are the following:

\[
\begin{align*}
\mu_i & \sim N(\psi, \omega^{-1}), \quad \alpha_i \sim N(\phi, c^{-1}), \\
\sigma_0^2 & \sim IG(\frac{v}{2}, \frac{\delta}{2}), \quad \tilde{h}_1 \sim IG(\frac{v}{2}, \frac{\delta}{2})_{1(\tilde{h}_1 > 1)}, \\
p_{i1} & \sim \text{beta}(u_{11}, u_{10}), \quad p_{00} \sim \text{beta}(u_{00}, u_{01})
\end{align*}
\]

The parameters used are $\psi = 0, \omega = 25, \phi = 0, c = 1, (v, \delta) = (0, 0), u_{00} = u_{11} = 8$, and $u_{10} = u_{01} = 2$. Hence the prior of $\sigma^2_i$ is non-informative. The other parameters are chosen so that the priors are informative but relatively diffused. The means and standard deviations of the prior distributions are presented in the following table.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Distribution</th>
<th>Mean</th>
<th>Std. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_i$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.20</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>Normal</td>
<td>0.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$p_{i1}$</td>
<td>Beta</td>
<td>0.80</td>
<td>0.12</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>Inverted Gamma</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$h_1$</td>
<td>Inverted Gamma</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

### 3.3.2 Computational algorithm

The needed posterior conditional distributions for implementing Gibbs sampling are easily obtained from the priors and the assumptions of the data generating process. The following steps 1 through 5 are iterated to obtain the joint and marginal distributions of the parameters of interest.

**Step 1:** Generate $p_{11}$ and $p_{00}$ conditional on $\tilde{S}_T = (S_1, ..., S_T)$. Let $n_{ij}$ refer to the total number of transitions from state $i$ to $j$, which can be counted from $\tilde{S}_T$. Then

\[
\begin{align*}
p_{11} \mid \tilde{S}_T & \sim \text{beta}(u_{11} + n_{11}, u_{10} + n_{10}) \\
p_{00} \mid \tilde{S}_T & \sim \text{beta}(u_{00} + n_{00}, u_{01} + n_{01})
\end{align*}
\]
Step 2: Generate $\mu_i$ conditional on $\tilde{S}_T$, $\sigma_i^2$, and $\alpha_i$. We have the regression $y_t = \mu_i + \varepsilon_t$ where $y_t = b_t^* - \alpha_i b_{t-1}$ for $t \in \{t : S_t = i\}$. Hence, the posterior distribution is $\mu_i \sim N(\psi_*, \omega_*^{-1})$ where

$$\omega_* = \sum_{t \in \{t : S_t = i\}} 1/\sigma_i^2 + \omega, \psi_* = \omega_*^{-1} \left[ \sum_{t \in \{t : S_t = i\}} y_t / \sigma_i^2 + \omega \psi \right]$$

Step 3: Generate $\alpha_i$ conditional on $\tilde{S}_T$, $\sigma_i^2$, and $\mu_i$. Let $d_t^* = b_t^* - \mu_i$, then we have the regression $d_t^* = \alpha_i b_t + \varepsilon_t$ for $t \in \{t : S_t = i\}$. Hence, the posterior distribution is $\alpha_i \sim N(\phi_*, c_i^{-1})$ where

$$c_* = \sum_{t \in \{t : S_t = i\}} b_{t-1}^2 / \sigma_i^2 + c, \phi_* = c_*^{-1} \left[ \sum_{t \in \{t : S_t = i\}} b_{t-1} d_t^* / \sigma_i^2 + c \phi \right]$$

Step 4: Generate $\sigma_0^2$ and $\sigma_1^2$ conditional on $\tilde{S}_T$, $\mu_i$, and $\alpha_i$. We first generate $\sigma_0^2$ conditional on $h_1$ and then generate $\tilde{h}_1 = 1 + h_1$ to indirectly generate $\sigma_1^2$. Conditional on $h_1$, the posterior distribution of $\sigma_0^2$ is as follows:

$$\sigma_0^2 \sim IG \left( \frac{v_{0*}}{2}, \frac{\delta_{0*}}{2} \right)$$

where

$$v_{0*} = v + T$$

$$\delta_{0*} = \delta + RSS_0 + RSS_1 / (1 + h_1)$$

with $RSS_i = \sum_{t \in \{t : S_t = i\}} (b_t^* - \mu_i - \alpha_i b_{t-1})^2$. Conditional on $\sigma_0^2$, the posterior distribution of $\tilde{h}_1 = 1 + h_1$ is as follows:

$$\tilde{h}_1 \sim IG \left( \frac{v_{1*}}{2}, \frac{\delta_{1*}}{2} \right)_{1(\tilde{h}_1 > 1)}$$

where

$$v_{1*} = v + T_1$$

$$\delta_{1*} = \delta + RSS_1 / \sigma_0^2$$

with $T_1 = \sum_{t=1}^T S_t$. Once $\tilde{h}_1$ is obtained, we can calculate $\sigma_1^2$.

Step 5: Generate $\tilde{S}_T = (S_1, ..., S_T)$ conditional on the other parameters. This is conducted using multi-move Gibbs sampling, which was first introduced by Carter and Kohn (1994) in the context of a state-space model. Here the procedure for generating $\tilde{S}_T$ using the multi-move Gibbs-sampling is the same as that in Kim and Nelson (1999).
We iterate steps 1 through 5 $M + N$ times and discard the realizations of the first $M$ iterations but keep the last $N$ iterations to form a random sample of size $N$ on which statistical inference can be made. $M$ must be sufficiently large so that the Gibbs sampler converges. Also, $N$ must be large enough to obtain the precise empirical distributions. Taking these aspects into consideration, we set $M = 5000$ and $N = 10000$.

4 Data

We construct a data set covering the period 1885-2004 for Japan, 1840-2005 for the United States, and 1830-2003 for the United Kingdom.\(^{11}\)

4.1 Japan

Public debt Public debt is defined as the amount of gross debt issued by the central and local governments at the end of each fiscal year.\(^{12}\) To convert the figures reported in various budget documents into a format consistent with the SNA, we make adjustments by excluding the amount of debt issued under the Colonial Special Accounts and the Public Enterprise Special Accounts, both of which are outside the general government according to the SNA definition.\(^{13}\)

Nominal GDP A single data set covering the entire sample period is not available, so that we collect data from various sources and link them in a consistent way. For the period after FY1936, we use a data set produced by the Japanese government (various versions of the SNA), while for the period before FY1935, we basically use Ohkawa et al. (1974). However, since data are completely missing for the final stage of WWII (FY1944 and 1945), we estimate the real GDP in these two years by using the index of industrial production and the index of agriculture, forestry and fishery

\(^{11}\)See Appendix for details. All data we use are available upon request. All data we use are available upon request.


\(^{13}\)We use various definitions of the general government: For 1885-1954, we use the definition by the Economic Counsel Board, for 1995-1969, the OLD SNA, for 1970-1979, the 68SNA, and for 1980-2004, the 93SNA. Note that these definitions slightly differ from each other, because special accounts held by the central government and business accounts held by local governments are sometimes classified as part of the general government and sometimes not.
production,\textsuperscript{14} and the GDP deflator by using the agricultural price index, the production goods price index, and the consumer price index.

**Government interest payments** The data for government interest payments for FY1885 to 1929 are taken from Emi and Shionoya (1966) for FY1885 to 1929, while those for FY1952 to 2004 are from various documents published by the government, including the “White Paper on National Income,” the “Annual Report on National Income Statistics,” and the “Annual Report on National Accounts.” As for the period between FY 1930 and FY1951, we estimate interest payments closely following the methodology adopted by Emi and Shionoya (1966).

**Military expenditure** For the years after FY1947, we use the figures referred to as “National Defense and Related Affairs” in various issues of the “Settlement of General Account Revenues and Expenditures” published by the Ministry of Finance. The data for FY1946 are taken from Economic Counsel Board (1954), while for the years before FY1946, we use the data from Emi and Shionoya (1966).

As for military spending during wartime, that is, FY1937 to FY1945, we define this as expenditures spent only by the forces at home, and do \textbf{not} include expenditures spent by the forces overseas. This is consistent with our definition of public debt in which those debts issued under the five Colonial Special Accounts (namely, the Chosen Government, Taiwan Government, Kwantung Office, Karafuto Office, and Nanyo Office) are \textbf{not} included.\textsuperscript{15}

### 4.2 The U.S. and the U.K.

For the United States, the data are taken from the “Historical Statistics of the United States” (Carter et al. (2006)) and the “Historical Tables, Budget of the United States Government” published by the Office of Management and Budget. For the United Kingdom, the data sources are the “British

\textsuperscript{14}This methodology closely follows the one used by the Japanese central bank in its various publications on financial and economic activities around the end of the WWII (see, for example, Bank of Japan (1950)).

\textsuperscript{15}However, as one might imagine, a non-negligible portion of expenditures spent by the forces \textit{overseas} was financed by the central government through the issue of public debt, especially at the final stage of WWII. Ideally, this portion should be included in our definition of military expenditure, but we do not do so because reliable figures for that portion are not available. However, to see how sensitive our empirical results are to this treatment of military expenditures, we created an alternative series of military expenditures using a tentative estimate by Emi and Shionoya (1966) for military spending by the forces overseas that were financed by the central government through the Colonial Special Accounts, and repeated the same empirical exercise as in Section 5. We were able to confirm that the basic empirical findings are not sensitive to the definition of military spending.

5 Empirical results

5.1 Preliminary analysis

The trend in the debt-GDP ratio for Japan, the U.S., and the U.K. is shown in Figure 1. We see that there are three major periods of debt accumulation in Japan. The first period, 1904 to 1905, is the period of the Russo-Japanese War (1904-1905). Reflecting a substantial increase in military expenditure, the debt-GDP ratio increased to over 50 percent at the end of 1905; however, it started to decrease again right after the end of the war and the decline continued until, in 1918, the debt-GDP ratio had returned to the pre-war level. Given that there was no remarkable growth dividend during this period (the nominal growth rate in 1906-1915 was 5.4 percent per year on average), one can see that this downward trend mainly came from fiscal reconstruction, including substantial spending reductions.\textsuperscript{16} As pointed out by many researchers, the government during this period had a strong political will to restore budget balance so as to avoid the risk of a massive outflow of gold under the gold standard system.

The second phase of debt accumulation was 1920 to 1944, i.e., the period that includes WWII. The increase in the debt-GDP ratio accelerated following the outburst of war with China in 1937, and the ratio eventually reached 1.8 when the war ended in 1945. However, as can be seen in Figure 1, the debt-GDP ratio dropped precipitously right after the end of the war, all the way to a level very close to zero. This is an episode of inflationary erosion of the debt, or “partial default,” due to hyper-inflation during this period.\textsuperscript{17}

Finally, the most recent phase of debt accumulation started in the early 1970s and continues until today. A series of reforms in the social security system, including the introduction of indexation in the public pension system, have been implemented since the Tanaka administration declared a changeover to the welfare state in 1973. This accumulation of debt continued until the government finally started fiscal reconstruction in the latter half of the 1980s, including a substantial cut in spending and the introduction of a consumption tax in 1989. However, the debt-GDP ratio started to increase again in

\textsuperscript{16}Although Japan won the war, it received no war reparations from Russia.

\textsuperscript{17}The rate of inflation in terms of the GDP deflator was 273 percent in 1945, 175 percent in 1946, and 154 percent in 1947.
the 1990s, at least partially due to the collapse of the asset price bubble in the early 1990s.

In Table 1, we decompose changes in the debt-GDP ratio into four components: the contribution of primary deficit; the contribution of interest charge; the contribution of real GDP growth; and the contribution of inflation. Focusing on the three phases in which the debt-GDP ratio declined, we see the following. The first phase, 1906 to 1916, is the period immediately after the end of the Russo-Japanese war. The debt-GDP ratio declined by 4.3 percent per year. Importantly, it came mostly from a reduction of primary deficit, implying that the fiscal reconstruction during this period was a successful one. On the other hand, the decline of the debt-GDP ratio in 1945 to 1948 was much larger than that in the preceding phase, but this came not from a reduction of primary deficit but from inflation. Finally, the debt-GDP ratio declined in 1987 to 1990 by 2 percent per year. The contribution of primary deficit was 3.9 percent, suggesting that fiscal reconstruction was going on during this period. However, public debt had already reached at a very high level at this time, thus the contribution of interest charge was high. As a result, the contribution of deficit with interest, which is defined as the sum of the contribution of primary deficit and the contribution of interest charge, was positive, contributing to an increase (rather than a decrease) in the debt-GDP ratio. This is sharply contrasted with what happened during the fiscal reconstruction in 1906-1916. The debt-GDP ratio did decline in 1987-1990, but it mainly came from high economic growth.

Turning to the U.S. and the U.K., we see that the main cause of debt accumulation was increases in military expenditures during wartime. Specifically, the U.S. debt-GDP showed a rapid and substantial increase in 1861-66, 1916-19, and 1941-46, respectively corresponding to the Civil War, WWI, and WWII periods. The debt-GDP ratio for the U.K. is also characterized by three spikes, created by the Napoleonic War, WWI, and WWII. A notable difference with the Japanese data is that in both of these countries there was no major inflation comparable to Japan’s hyper-inflation in 1945-47. It should also be noted that the U.S. and the U.K. have never experienced an uncontrollable accumulation of public debt during peacetime, which again is in sharp contrast with the Japanese experience since the early 1970s.

Table 2 presents a standard ADF test for the debt-GDP ratio in Japan, the U.S., and the U.K. Specifically, we run an AR ($p$) regression of the form

$$b_t = \mu + \alpha \Delta b_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta b_{t-j} + \epsilon_t$$

(12)
for the entire sample period with no break. We repeat this with various lag lengths \((p = 1 \text{ to } 10)\) to find that the estimates of \(\alpha\) are very close to unity, and thus the null hypothesis \(H_0: \alpha = 1\) cannot be rejected for each of the three countries.

But does this necessarily imply that the debt-GDP ratio follows a unit root process throughout the entire sample period? In order to examine this, we conduct a rolling regression of equation (12) with a window of 40 years; for example, the estimated value for 1925 is from a regression conducted over the period 1885-1925. The lag length is set to \(p = 2\). The results are shown in Figure 2 and show for Japan that the estimate of \(\alpha\) fluctuates substantially in a range from 0.6 to 1.1, suggesting that the time-series properties of the debt-GDP ratio changed significantly at least several times. Specifically, the estimate of \(\alpha\) shows a sharp rise during WWII, indicating that the Japanese government abandoned fiscal discipline during this period. On the other hand, \(\alpha\) shows a sharp decline during the period just after the war, probably reflecting the fact that hyper-inflation during that period quickly reduced the real value of public debt. It should also be noted that the value of \(\alpha\) has stayed very close to (or slightly above) unity since the latter half of the 1980s, suggesting that the debt-GDP ratio has been following a unit root process or even an explosive process during this most recent period.

In contrast with the Japanese result, the estimates of \(\alpha\) for the U.S. and the U.K. do not show large fluctuations. Basically, the estimates stay below unity, except that the U.S. estimate shows a sharp rise during WWII.\(^{18}\)

### 5.2 Empirical results for Japan

Table 3 presents the regression results for Japan obtained from a two-state model. Panel A of the table shows a benchmark regression in which no exogenous variables are included (namely, specification 1). The estimate of \(\alpha\) in regime 0 is 0.517, indicating that the debt-GDP ratio is characterized by a stationary process that converges to its mean quite quickly. On the other hand, the estimate of \(\alpha\) in regime 1 is 1.116. Since its lower bound (1.067) exceeds unity, we cannot reject the null that the debt-GDP ratio follows an explosive process.\(^{19}\) Figure 3 presents the estimated probability of regime 1 in each year of the sample period, as well as the estimated coefficient on \(b_{t-1}\), which is calculated

\(^{18}\)In addition, \(\alpha\) shows a sharp decline during high inflation periods (the 1920s in the U.S. and the U.K., and the 1950s in the U.K.), again probably reflecting the fact that the real value of public debt quickly decreased due to high inflation.

\(^{19}\)Note that the estimate of the constant terms \(\mu\) is below zero, implying that \(b\) explodes to negative infinity if \(b\) starts at an extremely low level. However, this is unlikely to occur, given that the estimate of \(\mu\) is very close to zero, and not significantly different from zero.
as a weighted average of the coefficients in regimes 0 and 1, with the estimated probabilities of each regime being used as a weight. The shaded area represents the 95 percent confidence interval. Figure 3 shows that the years except 1945-1970 fall under regime 1 and that the coefficient on $b_{t-1}$ exceeds unity except during the period 1945-1970.

Panel B of Table 3 shows the results of a similar regression, but this time we added military expenditures as an exogenous variable (specification 2). Again, the debt-GDP ratio is characterized by a stationary process for regime 0 and an explosive process for regime 1. The estimated coefficient on $b_{t-1}$, shown in Figure 3, looks quite similar to the previous case, except that the coefficient is now lower than unity in 1890-1905 (the period of the Sino-Japanese and the Russo-Japanese Wars) and 1915-1920 (the period of WWI).

Panel C of Table 3 reports the regression result for the case in which military expenditure and the growth dividend, $-\frac{n_t}{1+n_t} b_{t-1}$, are included as exogenous variables (specification 3). Again, we see that regime 0 is characterized by a stationary process and regime 1 by an explosive process. But a notable difference from the previous two specifications is that the estimate of $\alpha$ in regime 0 is now much closer to unity, indicating that convergence to its mean is much slower. Specifically, the estimate of $\alpha$ in specification 1 (0.5177) implies that the debt-GDP ratio declines to half of its initial value after about 1.05 years, while the one in specification 3 (0.9178) implies that the half-life is 8.08 years. The surprisingly quick decline in the debt-GDP ratio found in specifications 1 and 2 mainly reflects the fact that the debt-GDP ratio fell very quickly during the hyper-inflation period in 1945-47. This problem is now fixed by properly controlling for the growth dividend. Figure 3 now shows that the probability of regime 1 is close to unity in 1930-1950 and 1970-2004, while the probability of regime 0 is high in 1885-1925 and 1950-1970. These results suggest that the former periods are characterized by a lack of fiscal discipline, while the latter periods are characterized by a locally Ricardian rule.

Finally, Panel D of Table 3 reports the results for the case in which military expenditure and $(\frac{n_{t-1}}{1+n_{t-1}} - \frac{n_t}{1+n_t}) b_{t-1}$ are included as exogenous variables (specification 4). The results are basically the same as those for specification 3, except that the estimates of $\alpha$ in regimes 0 and 1 are both lower, confirming that the assumption of a globally Ricardian policy is weaker than that of a locally Ricardian policy.

In sum, we find that the Japanese government made several large changes with respect to its fiscal behavior over the past 120 years. Specifically, Japanese fiscal policy is characterized by a locally
Ricardian rule in 1885-1925 and 1950-1970. The former largely corresponds to the period in which Japan had adopted the gold standard under which the government was forced to maintain a balanced budget until Japan left the gold standard in 1917, following the same move by the core countries of the system.\textsuperscript{20} The second period follows the fiscal restructuring ushered in in December 1948, when SCAP instructed the Japanese government to implement a balanced budget in order to stop runaway inflation.\textsuperscript{21} On the other hand, Japanese fiscal policy is characterized by a non-Ricardian rule in 1930-1950 and 1970-2004, suggesting that the Japanese government abandoned fiscal discipline not only during WWII, but also in the most recent period starting in 1970.\textsuperscript{22}

5.3 Sensitivity analysis

AR (2) model The baseline regressions reported in Table 3 assume that the government adjusts the primary surplus in period $t$ in response to a change in public debt at the beginning of period $t$. Given that we use annual data, this seems to be a good approximation to actual policy making. However, as often pointed by researchers and practitioners, it usually takes more than one year before fiscal decisions are finally made. If this is the case, our baseline specification may not be appropriate. To address this potential problem, we change the lag structure of our estimating equation (equation (11)) to

\[
\begin{align*}
\beta_t &= \begin{cases} 
\mu_0 + \alpha_0 \sum_{k=1}^{K} \omega_{0k} b_{t-k} + \eta_t b_{t-1} + u_{0t}, & \text{if } S_t = 0 \\
\mu_1 + \alpha_1 \sum_{k=1}^{K} \omega_{1k} b_{t-k} + \eta_t b_{t-1} + u_{1t}, & \text{if } S_t = 1
\end{cases}
\end{align*}
\]

where $\omega_{0k}$ and $\omega_{1k}$ are parameters satisfying $\sum_{k=1}^{K} \omega_{ik} = 1$ and representing the lag structure of fiscal decision making.\textsuperscript{23} The estimating equation is now given by

\[
\begin{align*}
\beta_t &= \begin{cases} 
\mu_0 + (\alpha_0 + \eta_t) b_{t-1} + \sum_{k=1}^{K-1} \theta_{0k} \Delta b_{t-k} + u_{0t}, & \text{if } S_t = 0 \\
\mu_1 + (\alpha_1 + \eta_t) b_{t-1} + \sum_{k=1}^{K-1} \theta_{1k} \Delta b_{t-k} + u_{1t}, & \text{if } S_t = 1
\end{cases}
\end{align*}
\]

\textsuperscript{20}See Shizume (2001) for more on the Japanese government’s fiscal behavior during the gold standard period.
\textsuperscript{21}For details on the “Dodge Line,” see, for example, Cohen (1950) and Yamamura (1967).
\textsuperscript{22}Markov switching models, including our model given by equation (11), are based on the assumption of recurring states. This may not be an appropriate way to describe the evolution of fiscal policy rules. For example, the Ricardian rule in 1885-1925 may not necessarily be identical to the one in 1950-70. It is beyond the scope of this paper to fully address this issue, but we estimated a random coefficient model, in which the estimating equation is given $\beta_t = \mu + (\alpha_t - \frac{\alpha_{t-1}}{1+m}) b_{t-1} + \eta_t b_{t-1} + \epsilon_t$, where $\alpha_t$ follows a random walk process ($\alpha_t = \alpha_{t-1} + \text{disturbance}$). The regression result shows that $\alpha$ exceeds unity (i.e. non-Ricardian rule) in 1945, 1965-1985, and 1995-2004, and stays below unity for the other years. This is basically the same as our baseline result.
\textsuperscript{23}Note that this specification differs from a partial adjustment model, such as the one adopted by Favero and Monacelli (2005), in that the coefficient on lagged values of $b$ depends only on the current regime (and not on past regimes).
where $\theta_{ik} \equiv -\alpha_i \omega_{i,k+1}$. We conducted a lag search to end up with $K = 2$. The results of the regressions using this equation are reported in Table 4 and Figure 4 and are basically the same as before. In addition, the coefficient on $\Delta b_{t-1}$, denoted by $\theta$ in Table 4, is very close to zero in each regime, indicating that the AR (1) specification is not a binding constraint.

**Net public debt.** The baseline regressions use *gross* public debt issued by the central and local governments rather than *net* debt. This is based on the assumption that governments own only a small amount of financial assets and that fluctuations in the amount of financial assets over time are insubstantial. However, as pointed out by Broda and Weinstein (2005), Japan’s public sector, through its social security funds, holds non-negligible amounts of financial assets. According to their estimate, net debt held by the Japanese public sector at the end of FY2002 was equivalent to 64 percent of GDP, while the corresponding gross figure was 161 percent. Obviously, the difference is not trivial.

To evaluate how sensitive the baseline results are, we re-estimate our equations replacing gross debt with net debt. The net debt data we use here are the data published by the Economic and Social Research Institute (ESRI), Cabinet Office, which cover the general government, including the central and local governments and the social security funds. Unfortunately, however, the ESRI data cover only the postwar period starting in 1955, so that the estimation is conducted only for this shorter sample period.

Figure 5 compares Japanese general government gross and net debt. Although the difference between the two in terms of the vertical distance is indeed substantial, we still see a common long-term trend: Namely, both start to increase around 1970 and basically continue to rise over the next 35 years. Comparing the estimation result reported in Table 5 with the baseline result (Table 3), we see no change in that the debt-GDP ratio is characterized by a stationary process in regime 0 and a nonstationary process in regime 1. We may therefore safely conclude that our baseline results are not particularly sensitive to the definition of public debt.

Somewhat interestingly, however, if one looks closely at Figure 6, one can see a substantial decline in the probability of regime 1 during the latter half of the 1980s. Correspondingly, the coefficient on $b_{t-1}$ fell below unity during this period in specification 3 and more clearly in specification 4. The latter half of the 1980s famously is a period of fiscal reconstruction during which the Japanese government
intensively cut expenditure to achieve the target of “no net issuance of government bonds.”

One may interpret the decline in the coefficient on $b_{t-1}$ during this period as reflecting the restoration of fiscal discipline. However, the coefficient on $b_{t-1}$ started to increase again in the early 1990s and has remained very close to unity since.

**Automatic stabilizers** Recent empirical studies on fiscal policy rules emphasize the importance of automatic stabilizers in explaining fluctuations in the fiscal surplus/deficit (see Taylor (2000), Auerbach (2003), and Bohn (1998)). For example, Taylor (2000), using U.S. data for 1960-1999, finds that the cyclical surplus was highly correlated with fluctuations in the output gap but this was not necessarily the case for the structural surplus. To control for this effect in our regression exercise, we add the output gap to the estimating equations. Specifically, we closely follow Barro (1986) and Bohn (1998) by introducing a new variable, $YVAR_t$, which is defined as $YVAR_t = (1 - Y_t/Y_t^*)(G_t^*/Y_t)$, where $Y_t$ is the real GDP, $Y_t^*$ is its trend component estimated by HP filter, and $G_t^*$ is the trend component of real government spending. The regression results presented in Table 6 show that the coefficient on $YVAR$ is around 0.7 and significantly different from zero in both specifications, indicating that automatic stabilizers did play an important role even in the Japanese case. However, the coefficient of main interest to us, $\alpha$, is almost the same as before, suggesting that the baseline result is not sensitive to whether we control for the output gap or not.

**No restriction on the coefficient on interest payments** As we can see from equations (4) and (8), the sole difference between locally and globally Ricardian rules is what kind of restriction we impose on the coefficient on interest payments $\frac{\mu_{t-1}}{1+n_0} w_{t-1}$. Locally Ricardian rules impose the restriction that the coefficient should be equal to unity, while globally Ricardian rules impose the restriction that it should be zero. The former corresponds to specification 3, while the latter corresponds to specification 4. An important implication of these restrictions, whether the coefficient should be zero or unity, is that these specifications allow a switching only between locally Ricardian rules and other rules (i.e., rules that do not belong to locally Ricardian rules) in the case of specification 3, and a

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24See, for example, Ihori et al. (2001) for more on fiscal reform efforts during this period.

25Figure 6 shows that the coefficient on $b_{t-1}$ during the latter half of the 1980s is slightly below unity but not statistically different from unity in specification 3, while it is significantly smaller than unity in specification 4. This implies that the fiscal regime during this period is characterized not by a locally Ricardian but a globally Ricardian rule. This result is perfectly consistent with the fact that the government indeed aimed at “no net issuance of government bonds” but had little intention of going further than that, i.e., it had no intention to reduce the debt-GDP ratio to a lower level or even zero.
switching only between globally Ricardian rules and the other rules in the case of specification 4. These specifications would be inappropriate if, for example, policy switching occurs between locally and globally Ricardian rules.

To deal with this potential problem, we conduct a similar regression as before but now do not impose an a priori restriction on the coefficient on interest payments. Specifically, we add a new independent variable \( \frac{\eta_{t-1}}{1 + \eta_t} \) to equation (11) with

\[
\eta_t = -\frac{n_t}{1 + n_t}; \quad \nu_t = -\frac{\eta_t}{\eta_t}.
\]

The coefficient on the new independent variable should be close to zero if the true rule is well approximated by a locally Ricardian rule, and it should be unity in the case of a globally Ricardian rule. The results are shown in Table 7 and indicate that the estimated coefficient is 0.628 in regime 0 (the stationary regime) and 0.506 in regime 1 (the nonstationary regime). More importantly, the lower bound in regime 0 is 0.235, rejecting the null of zero, while the upper bound in regime 0 is slightly lower than unity (0.990), again rejecting the null of unity. This means that the true rule is not well approximated by the two extremes (i.e., locally and globally Ricardian rules) but is located between them. The same results can be seen for regime 1. However, the estimated values of \( \alpha \) in Table 7 tend to fall between those obtained in specifications 3 and 4 of Table 3, confirming that the main results regarding fiscal policy behavior in Table 3 hold without any substantial modifications.

### Three-state model

The robustness of the findings in Table 3 are examined in Table 8 by extending the analysis to a three state model. Panel A, which reports the regression results for specification 3, shows that regime 0 is characterized by a stationary process (\( \alpha = 0.926 \)), regime 1 by an explosive process (\( \alpha = 1.081 \)), and regime 2 by another highly explosive process (\( \alpha = 1.313 \)). Figure 9 shows that the periods falling under regime 1 in Figure 3 are again classified as regime 1, suggesting that the number of regimes allowed in Table 3 (namely, two regimes) is not an inappropriate description of the true model. These results, together with the results for specification 4, more or less confirm the earlier findings: (1) the periods 1885-1920 and 1950-1970 fall under regime 0 (a regime with fiscal discipline); (2) the period 1920-1950 falls under regime 1 (a regime without fiscal discipline).

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26These results suggest that neither empirical studies focusing only on locally Ricardian rules nor those focusing only on globally Ricardian rules employ an appropriate estimating equation.

27The exceptions are 1944 and 1970-1980, years in which the debt-GDP ratio recorded an extremely high growth rate, so that they are classified as regime 2.
5.4 Are debt ratios globally stationary or nonstationary?

The regression analysis in this section seeks to determine whether the debt-GDP ratio follows a stationary process within a regime. However, as we discussed earlier, even if the ratio is stationary within a regime, this does not necessarily imply that it is stationary in the long run. This is simply because regime changes occur stochastically in accordance with transition probabilities. Thus, what we need to know is where the debt-GDP ratio is headed in the long run given the estimated transition probabilities, or, put differently, we need to know whether its distribution converges over time to a certain distribution. A process is said to be globally stationary if the distribution converges to a certain distribution over time, while stationarity within a regime is called local stationarity. Global stationarity implies that the effect of policy shocks on the debt-GDP ratio becomes smaller and smaller over time and finally disappears in the long run. Investors in government bonds markets are interested in whether this global stationarity is satisfied or not, and policymakers, especially central banks, are interested in this property when designing monetary policy rules.

Francq and Zakoian (2001) obtain a result regarding the relationship between local and global stationarity that is of some interest in the present context, namely that local stationarity is neither a necessary nor a sufficient condition for global stationarity. For example, suppose there are two regimes and one satisfies local stationary while the other does not. Even in this combination, the process could be globally stationary. On the other hand, even if each of the two regimes satisfies local stationarity, this does not necessarily imply global stationarity.\(^{28}\)

As we saw in Table 3, the regression results using a two-state model show that one regime satisfies (local) stationarity while the other one does not. Also, as we saw in Table 8, the regression results using a three-state model indicate that one regime satisfies (local) stationarity, but the other two do not. Given these results, one may wonder if they imply global stationarity or nonstationarity. To address this issue, we conduct the following simulation exercise. We generate a time series of \(b_t\) using

\[
\begin{cases}
\hat{\mu}_0 + (\hat{\alpha}_0 - \frac{n_t}{1+n_t}) b_{t-1} + u_{0t}, & \text{if } S_t = 0 \\
\hat{\mu}_1 + (\hat{\alpha}_1 - \frac{n_t}{1+n_t}) b_{t-1} + u_{1t}, & \text{if } S_t = 1
\end{cases}
\]

for a two-state model and the corresponding equation for a three-state model. Here, parameters with

\(^{28}\)Gali (2007) provides a clear and interesting discussion of the implications of Francq and Zakoian’s (2001) result on the determinacy of an equilibrium in a monetary economy.
a hat represent the values estimated in the earlier regressions. More specifically, we randomly draw policy shocks and policy regimes using the parameters and transition probabilities obtained from the regression of specification 3 and generate a replication for the time series of the debt-GDP ratio over 1000 years for various paths of the nominal growth rate \( (n_t) \) that are exogenously determined. We repeat this process 5000 times to obtain a distribution of the debt-GDP ratio in every year of the 1000 years. We can say that the debt-GDP process is globally stationary if this distribution is stable over time; otherwise it is globally nonstationary.

Table 9 reports the first, second, and third quartiles of the simulated distribution with \( T = 500 \) (500 years later) and \( T = 1000 \) for the two-state and three-state models. In Panel A it is assumed that the initial regime is a stationary one (i.e., \( S_0 = 0 \)), and that the debt-GDP ratio in period 0 is zero. On the other hand, in Panel B, it is assumed that the initial regime is a nonstationary one \( (S_0 = 1) \) and that the initial debt-GDP ratio is unity (100 percent). The simulation results from the two-state model show that the distribution is stable over time, irrespective of the initial conditions and the assumed values of nominal growth rates \( (n) \), clearly indicating that the debt-GDP ratio satisfies global stationarity. On the other hand, the results from the three-state model show that the distributions with \( T = 500 \) and \( T = 1000 \) differ significantly for the case of \( n = 0.00, 0.03, \) and \( 0.06, \) implying that the process is globally nonstationary.29

Figure 10 presents a similar simulation conducted to forecast the future path of the debt-GDP ratio over the next 100 years. To make the initial condition as close to the current situation in Japan as possible, we assume that the initial regime is \( S_0 = 1 \) and that the debt-GDP ratio in period 0 is 1.7, which is the actual figure at the end of 2004. According to the result from the two-state model with 3 percent nominal growth, the “third quartile” line goes up until it reaches 3 with \( T = 20 \), indicating that a further increase in the debt-GDP ratio is quite likely to occur over the next 20 years. After that, however, the debt-GPD ratio enters a declining trend as a result of the switch to a stationary regime and then converges to a quite narrow (and probably tolerable) band within 100 years. On the other hand, the result from the three-state model with 3 percent nominal growth shows that the median of the distribution increases quite quickly to reach an unrealistic and intolerable level within 50 years, and that its variance increases over time, clearly indicating global nonstationarity.

29 However, when \( n \) goes up to 0.10, the distributions with \( T = 500 \) and \( T = 1000 \) become identical, suggesting that sufficiently high nominal growth could make the debt-GDP ratio globally stationary. The threshold for nominal growth rates is about 8 percent, which is lower than the sample average (13.7 percent).
5.5 Empirical results for the U.S.

Table 10 presents the regression results for the United States using a two state model. Results for specification 3, presented in Panel A, indicate that each of the regimes, 0 and 1, is characterized by a stationary process. This implies that the U.S. government’s fiscal behavior during the sample period can be described as a switching between locally Ricardian policy rules. If we turn to the results for specification 4, presented in Panel B, they again indicate that each regime, 0 and 1, satisfies stationarity, implying that U.S. fiscal policy is characterized by a switching between globally Ricardian rules.

These results suggest that the U.S. government’s fiscal behavior consistently has been very close to locally Ricardian policy throughout the entire sample period. In fact, the estimated coefficient on $b_{t-1}$, presented in Figure 11, consistently and statistically significantly remains below unity. If we compare these results with those reported in previous studies on U.S. fiscal policy, we find some similarities. Bohn (2008), for example, regressed the U.S. primary surplus on public debt for a sample period from 1793 to 2003 and reports that the OLS estimate of the coefficient on public debt is positive and significantly different from zero when tax smoothing effects are properly controlled for. Bohn interprets this result as providing evidence for a globally Ricardian rule; but since the estimated coefficient is typically greater than the average interest rate level, this could be interpreted as suggesting a rule that is even locally Ricardian. Bohn (1998) conducts a similar exercise using data for 1916-1995 and finds that the coefficient on public debt is significantly positive not only for the entire sample period, but also for five sub-sample periods, including the postwar period. These results reported by Bohn (1998, 2008) are consistent with ours.

Favero and Monacelli (2005) estimate an equation that is very close to our specification 1 (equation (6)) using the maximum likelihood method and report that U.S. government behavior has been deviating from Ricardian policy for most of the entire sample period (1961-2002), except that it was close to a locally Ricardian rule during the period of 1995-2001. Although their results cannot be directly compared to ours because the empirical methodologies differ in several respects, we still attempt to do so by adjusting our sample period to theirs. Panels C and D of Table 10 show the results of a regression that is similar to that underlying Panels A and B, but that now uses data for the postwar period. The regression results indicate that the estimate of $\alpha$ in regime 0 is less than unity, suggesting that it is a stationary regime as before, but that the upper bound of $\alpha$ in regime 1 slightly exceeds
unity, so that we fail to reject the null of a unit root. Fluctuations in the estimated coefficient on $b_{t-1}$, presented in Figure 11, show that it has been slightly higher than unity since 1975, implying the possibility that the U.S. government started to deviate from Ricardian policy around 1975. However, the figure clearly shows that the estimated coefficient on $b_{t-1}$ is consistently less than unity during the period before 1975 and that there is no evidence for the return to Ricardian policy around 1995 that Favero and Monacelli (2005) detected. Thus, there are certain inconsistencies between their results and ours.  

Given that U.S. fiscal policy is characterized by switching between stationary regimes, we may apply a model with multiple breaks, as proposed by Bai and Perron (1998), to the U.S. data. This model does not require researchers to assume that policy regime switching is a recurrent phenomenon, and that it has a Markov property. This is an important advantage, but on the other hand it requires the debt process to be weakly stationary in each regime, so that we cannot apply it to the Japanese data. The regression results reported in Table 11 show that regime changes occur four times (i.e., there are five different regimes) with both specifications 3 and 4. According to the result for specification 4, the estimate of $\alpha$ is slightly higher than unity during the wartime period (regime 3, 1917-1943) but is significantly smaller than unity in the other four regimes. These results may be interpreted as confirming our earlier results obtained from the Markov switching regression.

5.6 Empirical results for the U.K.

Table 12 presents the regression results for the United Kingdom using a two-state model. Results for specification 3 indicate that regime 0 is characterized by a stationary process, while regime 1 is characterized by a unit root process (the upper bound of $\alpha$ slightly exceeds unity). On the other hand, results for specification 4 indicate that both regime 0 and regime 1 are characterized by a stationary process, implying that the U.K. government’s fiscal behavior is characterized by switching between globally Ricardian rules.

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30 We also estimated specification 1, which is very close to the estimating equation employed by Favero and Monacelli (2005), for the entire sample period as well as for the postwar period, but found that both regimes are stationary ones.  
31 However, the results for specification 3 are not very informative since $\alpha$ exceeds unity in three out of the five regimes. This result may be interpreted as evidence against applying the Bai-Perron method even to the U.S. data.
6 Conclusion

This paper estimated fiscal policy feedback rules in Japan, the United States, and the United Kingdom for more than a century and allowing for stochastic regime changes. By estimating a Markov switching model by the Bayesian method, we arrived at the following findings. First, the Japanese data clearly reject the view that the fiscal policy regime has been fixed, i.e., that the Japanese government has adopted a regime that is either Ricardian or non-Ricardian throughout the entire period. Rather, our results indicate a stochastic switch of the debt-GDP ratio between stationary and nonstationary processes and thus a stochastic switch between Ricardian and non-Ricardian regimes. Specifically, Japanese fiscal policy was characterized by a locally Ricardian rule in 1885-1925 and 1950-1970 but by a non-Ricardian rule in 1930-1950 and 1970-2004. Second, through simulation exercises using the estimated parameters and transition probabilities, we showed that the debt-GDP ratio may be nonstationary even in the long run (i.e., globally nonstationary). Third, the Japanese result stands in sharp contrast with the results for the U.S. and the U.K., which indicate that in these countries, government fiscal behavior has been consistently characterized by Ricardian policy.
References


Appendix   Data description

A.1 Japan

The definition of “general government” varies over time and there are four such definitions for the time span of our sample period: the definition by the Economic Counsel Board (ECB) for 1885-1954, the OLD SNA for 1955-1969, the 68SNA for 1970-1979, and the 93SNA for 1980-2004. The ECB’s definition is described in Economic Counsel Board (1954) and in the Kokumin Shotoku Hokoku [Report on National Income] published by the ECB. A detailed classification table of government organizations in the OLD SNA and the 68SNA is provided in Economic Planning Agency (1978). Finally, the 93SNA definition is available from the Economic and Social Research Institute, Cabinet Office website.

A.1.1 Public debt

Public debt is defined as the amount of gross debt issued by the central and local governments at the end of each fiscal year (or calendar year). The data for the debt issued by the central government are taken from various documents published by the Ministry of Finance, including “Japanese Government Bonds Statistics,” “The Financial History of the Meiji and Taisho Period in Japan,” the “Annual Report on Japanese Government Bonds Statistics,” and “Budget Statistics.” The data for the debt issued by local governments are collected from documents issued by the Ministry of Finance and the Ministry of Internal Affairs and Communications, including “The Financial History of the Meiji and Taisho Period in Japan,” “Local Government Bonds Statistics,” the “Annual Publication on Local Public Finance,” and the “Annual Report on Local Public Finance Statistics.” To convert the figures reported in various budget documents into a format consistent with the SNA, we make adjustments by excluding the amount of debt issued under the Colonial Special Accounts and the Public Enterprise Special Accounts, both of which are outside the general government according to the SNA definition.

Central government   Central government gross debt consists of government bonds, financing bills, borrowings, and temporary borrowings. For 1885-1899, figures for these are taken from a table (pp.31-32) in Ministry of Finance (1936). For 1900-1962, the figures are from Summary Table 1 (pp.4-5) in the Kokusai Tokei Nenpo [Annual Report on Japanese Government Bonds Statistics], FY1975, published by the Ministry of Finance. For 1963-2003, the data are obtained from Table 40 (pp.397) in the Zaisei Tokei [Budget Statistics], FY2005, also published by the Ministry of Finance. Finally, for 2004 they are from Summary Table 1 (pp.4-5) in the FY2004 edition of the Kokusai Tokei Nenpo.

However, we exclude the Colonial Special Accounts and Enterprise Special Accounts from any debt held by the central government, because neither is classified as part of the general government. There are five Colonial Special Accounts: the Chosen Government, the Taiwan Government, the Kwantung Office, the Karafuto Office, and the Nanyo Office. As for Enterprise Special Accounts, the number of accounts varies over time. The following is a list of special accounts that can be actually identified from government debt statistics tables:

For 1955-1969: Foodstuff Controls, Postal Service, Postal Savings, Alcohol Monopoly, Old Taiwan Foodstuff Controls, Old Chosen Foodstuff Controls, Silk Price Stabilization.


The data for government debt by account are obtained from Table 4 (pp.10-13) in the *Kokusai Tokei* [Japanese Government Bonds Statistics], 1907, published by the Ministry of Finance for 1890-1902.\(^1\) For 1903-2004, the data are from various issues of the *Kokusai Tokei Nenpo*, and Ministry of Finance (1936, 1954).

**Local government** Local government gross debt consists of ordinary accounts bonds, public enterprise bonds, and business bonds. While ordinary accounts bonds are entirely included in the general government debt, both public enterprise bonds and business bonds are not always included. This is because some public management business accounts sometimes are classified as part of the general government and sometimes not. In the ECB definition, no public management business accounts are included as part of the general government. Therefore, for 1885-1954, ordinary accounts bonds are only included in the general government debt. In the OLD SNA and the 68SNA, public enterprise accounts for hospital projects (HPs) and sewage enterprise services (SESSs) and other business accounts except for those of profit-making business as well as ordinary accounts are classified as part of the general government accounts. Thus, for 1955-1979 the local government debt is defined as the sum of ordinary accounts bonds, public enterprise bonds for HPs and SESSs, and business bonds for the national health insurance, public-university-affiliated hospitals (PUAHs), and public municipal pawnshops. In the 93SNA, the classification of HPs and PUAHs was switched from general government to non-financial public corporations. Therefore, for 1980-2004, all bonds issued under these business accounts are excluded from the local government debt.

\(^1\)For years before 1901, the statistics report no figures on government bonds held under the special account for the imperial railway service; therefore, the debt held under this special account is included in the debt held by general government during this period.
Moreover, we include public enterprise bonds carried under the ordinary account (PEBOAs) in the general government debt after 1974. However, in order to avoid the potentially severe problem of double-counting some public enterprise bonds issued by HPs and SESs in the local government debt, we need to subtract the amount of outstanding bonds belonging to these two accounts from the total amounts of PEBOAs. Because we cannot directly obtain the data on PEBOAs by business, we need to estimate them. We do so by first calculating the ratio of outstanding public enterprise bonds to the total sum by business account each year. Then, assuming that the percentage distribution of PEBOAs by business is equal to the percentage distribution in outstanding public enterprise bonds, we calculate the amount of PEBOAs held by HPs and SESs using this percentage share multiplied by the total amounts of PEBOAs.

The data source for ordinary accounts and public enterprise and business bonds for 1885-1899 is Table 1 (pp.732) in Ministry of Finance (1937); that for 1900-1911 is a table (pp.7) in the Chihosai Tokei [Local Government Bonds Statistics], FY1912, published by the Ministry of Home Affairs; that for 1912-1953 is the Chiho Zaisei Gaiyo [Annual Publication on Local Public Finance] also published by the Ministry of Home Affairs; and that for 1954-2004 is the Chiho Zaisei Tokei Nenpo [Annual Report on Local Public Finance Statistics] published by the Ministry of Internal Affairs and Communications. The 1945 value of ordinary accounts bonds is not available from the Chiho Zaisei Gaiyo and we therefore estimate it using the data on “general local government bonds” reported in Ministry of Finance (1983). The data source for PEBOAs is the Chiho Zaisei Yoran [Handbook of Local Public Finance] published by the Institute of Local Finance.

A.1.2 Interest payments

Turning to interest payments, the data source for 1885-1929 is Table 7a (pp.172-173) in Emi and Shionoya (1966). We calculate interest payments of the general government as the sum of interest payments made by the central government and local governments. For 1885-1900, we construct the calendar year series using the following calculation: (interest payments in the current fiscal year)×3/4+(interest payments in the previous fiscal year)×1/4. For 1930-1951, because we cannot obtain the data directly from Emi and Shionoya (1966) and various issues of the Kokumin Shotoku Hakusho [White Paper on National Income] published by the Economic Planning Agency, we estimate general government interest payments closely following Emi and Shionoya (1966). Figures for interest payments by the central government are taken from various issues of the Kokusai Tokei Nenpo. On the other hand, interest payments made by local governments are estimated by multiplying outstanding local government debt by the average interest rate. We can obtain data on outstanding local government debt for each level of interest rate from the Chiho Zaisei Gaiyo but not data on their weighted

\[\text{interest payments} = \text{central government interest payments} + \text{local government interest payments}\]

The detailed methodology and data sources are provided in the respective sections of the source references listed above.
average.\textsuperscript{3} Unfortunately, Emi and Shionoya (1966) do not provide an explanation of how they obtain the weighted average of interest rates. We therefore estimate interest payments by local governments using the values in the appendix table 1 as the average rate for each level of interest rate.\textsuperscript{4}

\begin{table}[h]
\centering
\caption{Weighted average of interest rates}
\begin{tabular}{llll}
\hline
\hline
Zero percent & 0.0000 & Under 3.5 percent & 0.0155 \\
Under 4.0 percent & 0.0162 & Under 4.0 percent & 0.0381 \\
Over 4.0 percent & 0.0480 & Under 5.0 percent & 0.0496 \\
Over 5.0 percent & 0.0536 & Under 6.0 percent & 0.0562 \\
Over 6.0 percent & 0.0638 & Under 7.0 percent & 0.0644 \\
Over 7.0 percent & 0.0738 & Under 8.0 percent & 0.0752 \\
Over 8.0 percent & 0.0832 & Under 9.0 percent & 0.0839 \\
Over 9.0 percent & 0.0958 & Under 10.0 percent & 0.0964 \\
Over 10.0 percent & 0.1084 & Over 10.0 percent & 0.1088 \\
\hline
\end{tabular}
\end{table}

However, for 1945 we cannot estimate interest payments by local governments because no data on outstanding local government debt for each level of interest rate are available. We estimate interest payments by the general government using the data on interest payments by the central government. For 1952-1954, the data source is Table 3 (pp.128) in the Kokumin Shotoku Hakusho, FY1962. For 1955-1969, it is Account 4 (pp.24-27) in the Kokumin Shotoku Tokei Nenpo [Annual Report on National Income Statistics], 1978, published by the Economic Planning Agency. The data source for 1970-2004 is an income and outlay account of general government in the Kokumin Keizai Keisan Nenpo [Annual Report on National Accounts] published by the Economic Planning Agency and the Economic and Social Research Institute.

\subsection{Military expenditure}

As for the pre-World War II period, the data source is Table 10 (pp.186-189) in Emi and Shionoya (1966). We define military expenditure during this period as the sum of regular defense expenditure and war expenditure (if any). Note that for the period 1904-1905, we revise the data on the expenditure by the extraordinary military special account (EMSA), which is included in the war expenditure. The EMSA expenditure for the period was almost entirely financed through bond issues. However, we can clearly observe a somewhat odd relationship between expenditure and bond issues in each year: For 1904, the expenditure is about 200 million yen larger than the issue of bonds, while for 1905 the issue of bonds is about 200 million yen larger than the expenditure. This is because total revenues and expenditures under the extraordinary military special account were sufficient to balance out not for

\textsuperscript{3}Figures on outstanding ordinary accounts bonds for each level of interest rate have been reported in the Chiho Zases Guoyo since 1950. For 1930-1949, we can obtain nothing but the outstanding data including ordinary accounts bonds as well as public enterprise bonds. Therefore, we use them here.

\textsuperscript{4}Our estimates are 93.1 for 1926, 107.3 for 1927, 116.7 for 1928 and 125.0 for 1929. The corresponding estimates by Emi and Shionoya are 94.1 for 1926, 108.2 for 1927, 117.1 for 1928 and 125.0 for 1929. For 1952, our estimated value is 13438.2 and the estimate by the Economic Planning Agency is 13352.0. The fact that the difference between our estimates and the estimates by Emi and Shionoya and the Economic Planning Agency is not large suggests that our estimates of interest payments by local governments are precise.
one fiscal year but for several fiscal years. This characteristic of the budget system prevents us from determining to what extent the increase of bond issuance is caused by a change in expenditure in one year. We revise the data on the EMSA expenditure by first calculating the ratio of the outstanding value of bonds issued to the total revenues for the period 1904-1905 in each year. We then multiply this ratio by the total EMSA expenditure for the period. Note also that for 1937-1945, we use the data labeled “Forces in the Home Land” in Table 10.

As for the postwar period, the data source for 1946 is Supplementary Table 25 (pp.283) in Economic Counsel Board (1954). For 1947-2004, the data are obtained from various issues of the Ippan Kaisai Sainyu Saishutsu Kessan [Settlement of General Account Revenues and Expenditures]. We define military expenditure as the sum of Boei Kankei-hi (Expenditures on national defense and related affairs) and Shusen Shorih-i (Expenditures related to the termination of WWII, until 1956).

A.1.4 Nominal GDP

For the period 1885-1954, we construct nominal GDP data series by calculating nominal GNP less net income abroad. For the period 1885-1935, the data on nominal GNP and net income abroad are taken from Ohkawa et al. (1974) and Ohkawa and Shinohara (1979), while for the period 1936-1950 these data are from the Kokumin Shotoku Hakusho, FY1963. Furthermore, for the period 1951-1954 they are from the Kokumin Shotoku Tokei Nenpo, 1978. Note that for the period 1885-1943, we convert the GDP data to a fiscal year series using the following calculation because Ohkawa et al. (1974), Ohkawa and Shinohara (1979), and the Kokumin Shotoku Hakusho provide only data on a calendar year basis: (nominal GDP in the current calendar year)×3/4+(nominal GDP in the following calendar year)×1/4. For the period after 1954, GDP data are directly obtained from the Kokumin Keizai Keisan Nenpo. Because official nominal GDP data in fiscal years are not available for the final stage of World War II (1944-1945), we estimate the real GDP and the GDP deflator in these two years and then calculate the nominal GDP by multiplying them together.

Real GDP for 1944-1945 We use the series of real GDP on a calendar year basis produced by Mizoguchi and Nojima (1993) to estimate the one on a fiscal year basis for the period 1944-1945. Specifically, we construct the series in fiscal years using the following calculation: for the fiscal years 1943 and 1946, (real GDP in the current calendar year)×3/4+(real GDP in the following calendar year)×1/4; for the fiscal year 1944, real GDP in the current calendar year; and for the fiscal year 1945, (real GDP in the current calendar year)×1/2+(real GDP in the following calendar year)×1/2. We then estimate real GDP in fiscal years for 1944-1945 using those data for 1943-1946 and official real GDP data for 1943 and 1946.

GDP deflator for 1944-1945 We estimate the GDP deflator for 1944-1945 using the data of an aggregate price index for 1943-1946. The data on the aggregate price index for 1943 and 1946 (on the basis of the period 1934-1936) are obtained from Supplementary Table 5 (pp.100-101) in the Kokumin
Note that because for years before 1946 the *Kokumin Shotoku* provides data only on a calendar year basis, for 1943 we convert the aggregate price index to a fiscal year basis using the following calculation: (index value in calendar 1943)×3/4+(index value in calendar 1944)×1/4. However, the data on the aggregate price index for 1944-1945 are not available. We produce the data on the aggregate price index for the fiscal year 1944-1945 closely following the methodology the Economic Planning Agency adopted in the *Kokumin Shotoku*.

The aggregate price index consists of three indices: the agricultural price index, the production goods price index, and the consumer price index. These indices are weighted using the following values, respectively: 0.300, 0.250 and 0.450. First, we use an index of commodities for family maintenance (on the basis of the 1937 calendar year) as the agricultural price index. The data are taken from Table 1 (pp.13) in the *Noson Bukka Chosa Hokoku* [Survey on Agricultural Prices], 1946-1948, published by the Ministry of Agriculture and Forestry. We change the base year from the 1937 calendar year to the period 1934-1936 by multiplying the index by the link coefficient, 1.180. Our calculated values are 3.655 for the fiscal year 1944 and 12.081 for the fiscal year 1945.

Second, we estimate the production goods price index using the data of the effective wholesale price index. The data in calendar years (on the basis of the period 1930-1934) are obtained from Supporting Table 2 (pp.100-101) in Bank of Japan (1948). However, we revise the index values for 1944 and January-August 1945. This is because the 1944 value is produced under the assumption that the level of output in 1944, being unknown at the time of estimating the indices, was nearly equal to that in 1943, and because the January-August 1945 value is produced using data for the period 1937-1944. We first construct the new value for 1944. Morita (1963), one of those who were in charge of producing the effective price index, points out that various economic statistics published after estimating the effective price index show a clear decrease in production in 1944 compared with 1943 and that it would be better to use not 115.0 but values around 110.0 for the index of transactions. Using 110.0 as the transactions index value, our revised effective wholesale price index is 3.951, which is 0.184 larger than the Bank of Japan index, 3.767. Then the revised effective wholesale price index for January-August 1945 is 4.749, which is 0.147 larger than the Bank of Japan index, 4.602. We change the base period from the period 1930-1934 to the period 1934-1936 by multiplying the index by the link coefficient, 0.91213. Our estimates of the production goods price index are 3.640 for the fiscal year 1944 and 9.599 for the fiscal year 1945.5

Third, we estimate the consumer price index using the data of the Bank of Japan effective retail price index. The data in calendar years (on the basis of the period 1930-1934) are taken from Supporting Table 3 (pp.103-104) in Bank of Japan (1948). The effective retail price index has the same

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5We convert the effective wholesale price index on a calendar year basis to a fiscal year series using the following calculation: for the fiscal year 1944, (index value in calendar 1944)×3/4+(index value for January-August 1945)×1/4; and for the fiscal year 1945, (index value for January-August 1945)×5/12+(sum of the seven months from September 1945 to March 1946)×1/12.
problem as the effective wholesale price index. In order to revise the data, we adopt the same method used for the Bank of Japan effective wholesale price index. As a result of the revision, the effective retail price index for 1944 is 4.504, which is 0.202 larger than the Bank of Japan index, 4.302, and that for January-August 1945 is 5.408, which is 0.137 larger than the Bank of Japan index, 5.271. We change the base period from the period 1930-1934 to the period 1934-1936 by multiplying the index by the link coefficient, 0.94251. Our estimates of the consumer price index are 4.012 for the fiscal year 1944 and 21.049 for the fiscal year 1945.\(^6\)

**A.1.5 Government revenue and expenditure**

A single data set covering the entire sample period is not available. We collect data from various sources and link them in a consistent way. For the period 1885-1929, we use the series produced by Emi and Shionoya (1966) in which they have estimated government revenues and expenditures following closely Economic Counsel Board (1954). For the period 1930-2004, the data are from various documents published by the Economic Planning Agency and the Economic and Social Research Institute.

**For the period 1885-1929** For the period 1885-1900, we convert the series on a fiscal year basis to the one on a calendar year basis using the following calculation: \(\frac{3}{4}\times\text{(the figures in the current fiscal year)}+\frac{1}{4}\times\text{(the figures in the previous fiscal year)}\). On the expenditure side, all the series of public sector are from Table 7a (pp.172-173) in Emi and Shionoya (1966). Gross capital formation of public enterprises is defined as the sum of capital formations produced by public corporations inside the central and local governments. As for public corporations of the central government, we can obtain the data for the period 1912-1929 from Table A-2-(1) (pp.22) in Emi and Takamatsu (1962), but not the data for the period 1885-1911. We thus estimate them using gross capital formation of public sector, assuming that the fraction of gross capital formations of public corporations inside the central government in total capital formations of public sector for the period 1885-1911 is equal to the fraction average for five years from 1912 to 1916. As for public corporations of local governments, gross capital formation is defined as the sum of capital formations produced by the electricity supply and gas supply business. The data are from Table 6 (pp.243, pp.245) in Emi (1971).

On the revenue side, all the series of public sector are from Table 7b (pp.174-175) and 7d (pp.178). Total revenues are defined as the sum of the central and local governments’ revenues. Note that we subtract from total revenues grants and other transfers from the central government to local governments in order not to record redundantly revenues in the consolidated account of the central and local governments. Unfortunately, we cannot directly obtain the data on surpluses produced by public corporations from Table 7b and 7d, because Emi and Shionoya (1966) does not report a breakdown of “Surplus of government enterprises” in which not only surpluses of public corporations but also

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\(^6\)We use the same calculation to make a fiscal year series as the one used for the effective wholesale price index.
government interest receipts and rents are included. We thus collect the data on interests and rents received by the central government and then estimate surpluses of public corporations by calculating “Surplus of government enterprises” less interest receipts and rents. As for local governments, we do not perform the same work as the central government since Emi and Shionoya (1966) has not estimated surpluses produced by public corporations inside local governments due to data limitations. We construct the series of interests and rents received by the central government following Emi and Shionoya (1966). For the general accounts, interests, dividends and rents received by the Keijyo-bu (ordinary category) and the Rinji-bu (extraordinary category) are counted in interest receipts and rents. For the special accounts, interests, dividends and rents received by 38 special accounts are counted in.

The data are from the Sainyu Saishutsu So-kessan (or Genkeisho) [Settlement of Revenues and Expenditures], the Ippan Kaikei Sainyu Saishutsu Kessan, and the Tokubetsu Kaikei Sainyu Saishutsu Kessan [Settlement of Special Accounts Revenues and Expenditures], all of which were published by the Ministry of Finance.

For the period 1930-1954  The main series of public sector for the entire period are from Table 3 (pp.128-129) in the Koumin Shotoku Hakusho, FY1962. For the period 1930-1951, the data on surplus of public corporations are from Supplementary Table 24 (pp.281) in Economic Counsel Board (1954). For the fiscal year 1952, those are from Supporting Table 6 (pp.39) in the Kokumin Shotoku Houkoku, FY1953; for the fiscal year 1953, from Supporting Table 7 (pp.39) in the Koumin Shotoku, FY1954; and for the fiscal year 1954, from Supporting Table 8 (pp.34) in the Kokumin Shotoku, FY1955. As for gross capital formation of public corporations, the data for the whole period are from Supplementary Table 23 (pp.277-279) in Economic Counsel Board (1954), Supporting Table 14 (pp.51) in the Kokumin Shotoku Houkoku, FY1953, Supporting Table 15 (pp.53) in the Koumin Shotoku, FY1954, and Table 5 (pp.138) in the Koumin Shotoku Hakusho, FY1962.

Yet, the Economic Planning Agency does not report all the series of public sector for the fiscal year 1945 due to the difficulty in estimating them because of data limitations. On the expenditure side, we use the consolidated net government expenditures, which are reported in the item labeled “Net Total of General Government Expenditures” in Emi and Shionoya (1966, Table 6), to estimate total expenditures of general government. We estimate transfers from general government to households (social security transfers) following Economic Counsel Board (1954). Specifically, transfers are defined as the sum of pensions, health insurances, seamen’s insurances, relief liability insurances for worker’s accident, pensions for public servants, and welfare benefits. On the revenue side, we estimate general government revenues following Economic Counsel Board (1954) and Emi and Shionoya (1966). General government revenues are defined as the sum of tax revenues, social security contributions and property incomes. Tax revenues are defined as the sum of national and local taxes. Social security contributions are defined as the sum of those collected by the following two special accounts: the special account for welfare insurances and for relief liability insurances for worker’s accident. Finally, property incomes...
are defined as the sum of interests, dividends and rents received by general government. We perform the same work as Emi and Shionoya (1966) to estimate the data on property incomes. The data are mainly from the Ippan Kaitei Saigyo Saishutsu Kessan, the Tokubetsu Kaitei Saigyo Saishutsu Kessan and the Chiho Zaisei Gaiyo.

For the period 1955-1969  The main series of expenditures and revenues of public sector for the whole period are from Account 4 (pp.24-27) in the Kokumin Shotoku Tokei Nenpo, 1978. The data on surplus of public corporations are from Table 4 (pp.56-57, pp.60-61) in the report. As for gross fixed capital formation of public corporations, the data for the period 1965-1969 are from Supporting Table 5 (pp.370) in the report, and those for the period 1955-1964 from Supporting Table 7 (pp.238-239) in the Kaitei Kokumin Shotoku Tokei [Revised Report on National Income Statistics 1951-1967].

For the period 1970-2004  All the series for the whole period are from an income and outlay account of general government in the Kokumin Keizai Keisan Nenpo, which is available online on the website of the Economic and Social Research Institute.

A.2 The United States

Public debt  The data source for 1840-1939 is Table Ea584-587 in Carter et al. (2006), Volume 5, where we use “Public debt” (Ea587), while for 1940-2005, we use Table 7.1 in the Historical Tables, Budget of the United States Government, Fiscal Year, 2007, published by the Office of Management and Budget, where we use “Gross Federal Debt.” All figures are for the end of the fiscal year, that is, December 31 for 1840-1842, June 30 for 1843-1976, and September 30 after 1977.

Nominal GDP  The data source for 1840-1928 is Table Ca9-19 in Carter et al. (2006), Volume 3, where we use series Ca10, while that for 1929-1947 is Table 1.1.5 in the National Income and Product Accounts published by the Bureau of Economic Analysis, U.S. Department of Commerce. Because these sources for the period before 1948 provide only calendar year data, for 1843-1947 we convert the data to fiscal year data using the following calculation: for 1843, (nominal GDP in the first half of the year)×2; and for 1844-1947, (nominal GDP in the current year)×1/2+(nominal GDP in the previous year)×1/2. We obtain fiscal year data for 1948-2004 from Table 8.1 and for 2005 from Table 1.1.5 in the National Income and Product Accounts.

Interest payments and military expenditure  The data source for 1840-1970 is Table Ea636-643 in Carter et al. (2006), Volume 5. For the period 1840-1953, military expenditure is given by the sum of “Army” (Ea638), “Navy” (Ea639) and “Air Force” (Ea640) expenditure, while for 1954-1970 we use “Total (Department of Defense)” (Ea637). As for interest payments, we use “Interest on public debt” (Ea641) for the entire period. For 1971-2005, the data source for military expenditure and interest payments is Table 3.2 in the Historical Tables, Budget of the United States Government, Fiscal Year,
2007. We use “Subtotal, Department of Defense-Military” (series 051) for military expenditure and “Interest on Treasury debt securities (gross)” (series 901) for interest payments.

A.3 The United Kingdom

**Public debt**  The data source for 1830-1979 is Public Finance 7 in Mitchell (1988). We calculate public debt as $FUD + UFD$ for 1830-1868, $FUD + UFD + OIB$ for 1869-1939 and $TND + OIB$ for 1940-1979, where $FUD$ denotes funded debt, $UFD$ unfunded debt, $OIB$ outstanding investment borrowing, and $TND$ total national debt. The source for 1980-2003 is Table A10 in HM Treasury’s Public Sector Finances Databank. All figures are for the end of the fiscal year, that is, January 5 for 1830-1853 and March 31 after 1854.

**Nominal GDP**  The data source for 1830-1947 is National Accounts 5 in Mitchell (1988). For 1830-1853, we use the calendar year series, because the term of the fiscal year is almost identical to the calendar year. For 1854-1947, we construct fiscal year data using the following calculation: $(\text{nominal GDP in the current year}) \times 3/4 + (\text{nominal GDP in the following year}) \times 1/4$. The data source for 1948-2003 is Table A2 in the United Kingdom Economic Accounts published by the Office for National Statistics, where we use the series BKTL. For 1948-1954, we construct fiscal year data using the following calculation: $(\text{nominal GDP in the current year}) \times 3/4 + (\text{nominal GDP in the following year}) \times 1/4$.

**Interest payments and military expenditures**  The data source for interest payments for the period 1830-1966 is Public Finance 4 in Mitchell (1988). For 1830-1937, we use figures obtained by subtracting “Debt Charges (Terminable Annuities)” from “Debt Charges (Total)” and for 1938-1966 we use “Debt Charges (Total).” The data source for 1967-2003 is the Annual Abstract of Statistics published by the Office for National Statistics, from which we use the item labeled “Service of the National Debt.” On the other hand, the data source for military expenditure for 1830-1979 is Public Finance 4 in Mitchell (1988). We calculate military expenditure as $ARM + NAV + EEV$ for 1830-1904, $ARM + NAV + AIF + EEV$ for 1905-1937 and $ARM + NAV + AIF + CEN + EEV$ for 1938-1967, where $ARM$ denotes army and ordnance expenditure, $NAV$ navy expenditure, $AIF$ air force expenditure, $CEN$ central expenditure and $EEV$ gross expenditure on special expeditions, votes of credit, etc. For 1968-1979, we use figures from the item labeled “Defense.” The data source for 1980-2002 is the Annual Abstract of Statistics. We estimate military expenditure for 2003 using both the Annual Abstract of Statistics value for 2002 and the Treasury’s value for 2002-2003. The Treasury value is obtained from Table 3.2 in the Public Expenditure Statistical Analyses, 2006.
Appendix References


Table 1: Decomposition of Changes in Japan’s Debt-GDP Ratio

<table>
<thead>
<tr>
<th>Period</th>
<th>Change in Debt-GDP Ratio</th>
<th>Contribution of Primary Deficit</th>
<th>Contribution of Interest Charge</th>
<th>Contribution of Real Growth</th>
<th>Contribution of Inflation</th>
</tr>
</thead>
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<td>1886-2004</td>
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<td>2.4</td>
<td>1.7</td>
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<td>-3.1</td>
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<td>1.1</td>
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<td>-0.8</td>
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<tr>
<td>1906-1916</td>
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<td>2.4</td>
<td>-1.3</td>
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</tr>
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<td>6.3</td>
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Table 2: Unit Root Tests

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<th>U.S. α</th>
<th>t-stat</th>
<th>U.K. α</th>
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<td>-1.78</td>
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Note: We conduct the standard ADF tests, $b_t = \mu + \alpha b_{t-1} + \sum_{j=1}^{p-1} \phi_j \Delta b_{t-j} + u_t$, with various lag length. The null hypothesis is $\alpha = 1$ and the 10% critical value is -2.57 when the sample size is 100 (see Hamilton (1994)). The sample periods for Japan, the U.S. and the U.K. are 1885-2004, 1840-2005, and 1830-2003, respectively.
Table 3: Two-State Model for Japan

Panel A: Specification 1

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Panel C: Specification 3

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Note: The transition probability, \( p_{ij} \), represents \( \Pr(S_t = j \mid S_{t-1} = i) \). The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Table 4: AR(2) Model

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Panel B: Specification 4

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Note: The transition probability, $p_{ij}$, represents $Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Table 5: Net Public Debt

Panel A: Specification 3

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Panel B: Specification 4

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<td>0.7159</td>
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<td>0.9583</td>
<td></td>
</tr>
</tbody>
</table>

Note: The transition probability, $p_{ij}$, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Table 6: Automatic Stabilizers

Panel A: Specification 3

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>The coefficient on YVAR</td>
<td>0.4700</td>
<td>0.7765</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.0103</td>
<td>0.0031</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8696</td>
<td>0.9192</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0003</td>
<td>0.0005</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.8603</td>
<td>0.9397</td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.8903</td>
<td>0.9482</td>
</tr>
</tbody>
</table>

Panel B: Specification 4

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>The coefficient on YVAR</td>
<td>0.4811</td>
<td>0.8034</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.0073</td>
<td>0.0067</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8054</td>
<td>0.8505</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.8626</td>
<td>0.9423</td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.8796</td>
<td>0.9459</td>
</tr>
</tbody>
</table>

Note: The transition probability, $p_{ij}$, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Table 7: No Restriction on the Coefficient on Interest Payments

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>The coefficient on</td>
<td>0.2351</td>
<td>0.6283</td>
</tr>
<tr>
<td>interest payments</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.0088</td>
<td>0.0052</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8228</td>
<td>0.8772</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.8668</td>
<td>0.9420</td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.8826</td>
<td>0.9454</td>
</tr>
</tbody>
</table>

Note: The transition probability, $p_{ij}$, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Table 8: Three-State Model

Panel A: Specification 3

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
<td>UB</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-0.0448</td>
<td>-0.0018</td>
<td>0.0232</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.8518</td>
<td>0.9261</td>
<td>1.0657</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0006</td>
</tr>
<tr>
<td>( p_{00} )</td>
<td>0.7937</td>
<td>0.9111</td>
<td>0.9783</td>
</tr>
<tr>
<td>( p_{01} )</td>
<td>0.0018</td>
<td>0.0388</td>
<td>0.1067</td>
</tr>
<tr>
<td>( p_{10} )</td>
<td>0.0149</td>
<td>0.1353</td>
<td>0.3272</td>
</tr>
</tbody>
</table>

Panel B: Specification 4

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
<td>UB</td>
</tr>
<tr>
<td>( \mu )</td>
<td>-0.0002</td>
<td>0.0153</td>
<td>0.0262</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.7958</td>
<td>0.8283</td>
<td>0.8673</td>
</tr>
<tr>
<td>( \sigma^2 )</td>
<td>0.0001</td>
<td>0.0003</td>
<td>0.0004</td>
</tr>
<tr>
<td>( p_{00} )</td>
<td>0.7839</td>
<td>0.8994</td>
<td>0.9698</td>
</tr>
<tr>
<td>( p_{01} )</td>
<td>0.8498</td>
<td>0.9261</td>
<td>0.9739</td>
</tr>
</tbody>
</table>

Note: The transition probability, \( p_{ij} \), represents \( \Pr(S_t = j \mid S_{t-1} = i) \). The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
### Table 9: Globally Stationary or Nonstationary?

#### Panel A: $S_0 = 0$ and $b_0 = 0$

<table>
<thead>
<tr>
<th>Quartile</th>
<th>Two-State Model First</th>
<th>Median</th>
<th>Third</th>
<th>Three-State Model First</th>
<th>Median</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.1471</td>
<td>0.0006</td>
<td>0.1525</td>
<td>-1.1E+10</td>
<td>-3.1117</td>
<td>1.4E+10</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.1593</td>
<td>-0.0007</td>
<td>0.1497</td>
<td>-2.5E+21</td>
<td>-2.7E+07</td>
<td>2.6E+21</td>
</tr>
<tr>
<td>3% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0751</td>
<td>0.0005</td>
<td>0.0772</td>
<td>-3.6E+04</td>
<td>0.0414</td>
<td>4.9E+04</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0790</td>
<td>-0.0011</td>
<td>0.0786</td>
<td>-5.9E+09</td>
<td>0.6536</td>
<td>1.1E+10</td>
</tr>
<tr>
<td>6% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0569</td>
<td>0.0003</td>
<td>0.0574</td>
<td>-4.2741</td>
<td>-0.0016</td>
<td>4.939</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0591</td>
<td>-0.0012</td>
<td>0.0584</td>
<td>-16.665</td>
<td>0.0085</td>
<td>20.506</td>
</tr>
<tr>
<td>10% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0476</td>
<td>0.0001</td>
<td>0.0463</td>
<td>-0.1160</td>
<td>0.0008</td>
<td>0.1205</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0478</td>
<td>-0.0006</td>
<td>0.0466</td>
<td>-0.1175</td>
<td>0.0010</td>
<td>0.1192</td>
</tr>
<tr>
<td>13.7% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0423</td>
<td>-0.0003</td>
<td>0.0413</td>
<td>-0.0579</td>
<td>0.0013</td>
<td>0.0615</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0428</td>
<td>-0.0004</td>
<td>0.0403</td>
<td>-0.0598</td>
<td>-0.0001</td>
<td>0.0592</td>
</tr>
</tbody>
</table>

#### Panel B: $S_0 = 1$ and $b_0 = 1$

<table>
<thead>
<tr>
<th>Quartile</th>
<th>Two-State Model First</th>
<th>Median</th>
<th>Third</th>
<th>Three State Model First</th>
<th>Median</th>
<th>Third</th>
</tr>
</thead>
<tbody>
<tr>
<td>0% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.1476</td>
<td>0.0017</td>
<td>0.1623</td>
<td>6.1E+08</td>
<td>2.9E+11</td>
<td>8.9E+13</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.1524</td>
<td>-0.0028</td>
<td>0.1515</td>
<td>1.0E+19</td>
<td>5.2E+22</td>
<td>1.8E+26</td>
</tr>
<tr>
<td>3% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0800</td>
<td>-0.0007</td>
<td>0.0780</td>
<td>131.32</td>
<td>1.7E+05</td>
<td>7.2E+07</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0780</td>
<td>-0.0007</td>
<td>0.0751</td>
<td>4.2E+05</td>
<td>1.3E+10</td>
<td>9.2E+13</td>
</tr>
<tr>
<td>6% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0573</td>
<td>-0.0003</td>
<td>0.0582</td>
<td>-0.2212</td>
<td>0.4199</td>
<td>121.10</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0575</td>
<td>0.0006</td>
<td>0.0585</td>
<td>-1.8212</td>
<td>0.2312</td>
<td>359.11</td>
</tr>
<tr>
<td>10% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0481</td>
<td>-0.0007</td>
<td>0.0467</td>
<td>-0.1082</td>
<td>0.0011</td>
<td>0.1195</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0458</td>
<td>0.0007</td>
<td>0.0467</td>
<td>-0.1131</td>
<td>0.0006</td>
<td>0.1172</td>
</tr>
<tr>
<td>13.7% Growth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T=500</td>
<td>-0.0427</td>
<td>-0.0009</td>
<td>0.0416</td>
<td>-0.0603</td>
<td>-0.0009</td>
<td>0.0574</td>
</tr>
<tr>
<td>T=1000</td>
<td>-0.0400</td>
<td>0.0008</td>
<td>0.0413</td>
<td>-0.0573</td>
<td>0.0013</td>
<td>0.0600</td>
</tr>
</tbody>
</table>

Note: We randomly draw policy shocks and policy regimes using the parameters obtained from regressions of specification 3 and generate 5000 replications for the time series of the debt-GDP ratio (1000 years) for various paths of the nominal growth rate ($n_t$), which are exogenously determined. The figures in the table represent the first, second, and third quartiles of the simulated distribution with $T=500$ (i.e., 500 years later) and $T=1000$. The average growth rate over the entire sample was 13.7 percent.
Table 10: Two-State Model for the U.S.

Panel A: Specification 3, 1840-2005

<table>
<thead>
<tr>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-0.0039</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.8734</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>0.00003</td>
</tr>
<tr>
<td>(p_{11})</td>
<td>0.8448</td>
</tr>
<tr>
<td>(p_{00})</td>
<td>0.8635</td>
</tr>
</tbody>
</table>

Panel B: Specification 4, 1840-2005

<table>
<thead>
<tr>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-0.0031</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.8448</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>0.00003</td>
</tr>
<tr>
<td>(p_{11})</td>
<td>0.8101</td>
</tr>
<tr>
<td>(p_{00})</td>
<td>0.8637</td>
</tr>
</tbody>
</table>

Panel C: Specification 3, 1948-2004

<table>
<thead>
<tr>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-0.0426</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.8478</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>0.00005</td>
</tr>
<tr>
<td>(p_{11})</td>
<td>0.8250</td>
</tr>
<tr>
<td>(p_{00})</td>
<td>0.7771</td>
</tr>
</tbody>
</table>

Panel D: Specification 4, 1948-2004

<table>
<thead>
<tr>
<th>Regime 0</th>
<th>Regime 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>LB</td>
<td>Mean</td>
</tr>
<tr>
<td>(\mu)</td>
<td>-0.0522</td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.8492</td>
</tr>
<tr>
<td>(\sigma^2)</td>
<td>0.00005</td>
</tr>
<tr>
<td>(p_{11})</td>
<td>0.8379</td>
</tr>
<tr>
<td>(p_{00})</td>
<td>0.7636</td>
</tr>
</tbody>
</table>

Note: The transition probability, \(p_{ij}\), represents \(\Pr(S_t = j \mid S_{t-1} = i)\). The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Table 11: Multiple Break Tests for the U.S.

<table>
<thead>
<tr>
<th>Panel A: Specification 3</th>
<th>Panel B: Specification 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu$</td>
</tr>
<tr>
<td>Regime 1</td>
<td>-0.0093</td>
</tr>
<tr>
<td>1840-1872</td>
<td>( 0.0026 )</td>
</tr>
<tr>
<td>Regime 2</td>
<td>1.0009</td>
</tr>
<tr>
<td>1873-1916</td>
<td>( 0.0265 )</td>
</tr>
<tr>
<td>Regime 3</td>
<td>1.0469</td>
</tr>
<tr>
<td>1917-1943</td>
<td>( 0.0125 )</td>
</tr>
<tr>
<td>Regime 4</td>
<td>0.9071</td>
</tr>
<tr>
<td>1944-1972</td>
<td>( 0.0057 )</td>
</tr>
<tr>
<td>Regime 5</td>
<td>1.0135</td>
</tr>
<tr>
<td>1973-2004</td>
<td>( 0.0079 )</td>
</tr>
</tbody>
</table>

Note: The constant term is imposed to be identical across regimes. The maximum number of breaks is 5 with $\epsilon=0.15$. Figures in parentheses denote standard errors.
Table 12: Two-State Model for the U.K.

Panel A: Specification 3

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th></th>
<th>Regime 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
<td>UB</td>
<td>LB</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.0837</td>
<td>0.1026</td>
<td>0.1220</td>
<td>-0.0341</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.8043</td>
<td>0.8213</td>
<td>0.8369</td>
<td>0.9884</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0003</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0004</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.9128</td>
<td>0.9544</td>
<td>0.9817</td>
<td></td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.6603</td>
<td>0.8175</td>
<td>0.9248</td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Specification 4

<table>
<thead>
<tr>
<th></th>
<th>Regime 0</th>
<th></th>
<th>Regime 1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LB</td>
<td>Mean</td>
<td>UB</td>
<td>LB</td>
</tr>
<tr>
<td>$\mu$</td>
<td>-0.0033</td>
<td>0.0692</td>
<td>0.0952</td>
<td>-0.0411</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.7986</td>
<td>0.8202</td>
<td>0.8686</td>
<td>0.9640</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.0003</td>
<td>0.0004</td>
<td>0.0005</td>
<td>0.0003</td>
</tr>
<tr>
<td>$p_{11}$</td>
<td>0.9027</td>
<td>0.9474</td>
<td>0.9816</td>
<td></td>
</tr>
<tr>
<td>$p_{00}$</td>
<td>0.6323</td>
<td>0.7900</td>
<td>0.9199</td>
<td></td>
</tr>
</tbody>
</table>

Note: The transition probability, $p_{ij}$, represents $\Pr(S_t = j \mid S_{t-1} = i)$. The columns labeled “LB” and “UB” refer to the lower and upper bound of the 95% confidence interval and the columns labeled “Mean” refer to the mean of the marginal distribution of the parameter.
Figure 1: Public Debt (Relative to Nominal GDP)
Figure 2: Coefficient on $b_{t-1}$ Estimated from Rolling Regressions

Japan

U.S.

U.K.
Figure 3: Two-State Model for Japan

Specification 1

Probability of regime 1

Estimated coefficient on $b_{t-1}$

Specification 2

Probability of regime 1

Estimated coefficient on $b_{t-1}$

58
Figure 3: Two-State Model for Japan—Continued

Specification 3

Probability of regime 1

Estimated coefficient on $b_{t-1}$

Specification 4

Probability of regime 1

Estimated coefficient on $b_{t-1}$
Figure 4: AR(2) Model

Specification 3

Probability of regime 1


Estimated coefficient on $b_{t-1}$

Specification 4

Probability of regime 1


Estimated coefficient on $b_{t-1}$
Figure 5: Net Public Debt in Japan (Relative to Nominal GDP)

Figure 6: Net Public Debt

### Specification 3

- **Probability of regime 1**

### Specification 4

- **Probability of regime 1**

### Estimated coefficient on $b_{t-1}$
Figure 7: Automatic Stabilizers

Specification 3

Probability of regime 1

Estimated coefficient on $b_{t-1}$

Specification 4

Probability of regime 1

Estimated coefficient on $b_{t-1}$
Figure 8: No Restriction on the Coefficient on Interest Payments

[Graph showing the probability of regime 1 and the estimated coefficient on $b_{t-1}$ over time.]
Figure 9: Three-State Model for Japan

Specification 3

Probability of regime 0

Probability of regime 1

Probability of regime 2

Estimated coefficient on $b_{t-1}$
Figure 9: Three-State Model for Japan—Continued

Specification 4

Probability of regime 0

Probability of regime 1

Probability of regime 2

Estimated coefficient on $b_{t-1}$
Figure 10: Globally Stationary or Nonstationary?

Path of the debt-GDP ratio with 3.0 percent growth (Specification 3 of the two-state model)

Path of the debt-GDP ratio with 13.7 percent growth (Specification 3 of the two-state model)

Path of the debt-GDP ratio with 3.0 percent growth (Specification 3 of the three-state model)

Path of the debt-GDP ratio with 13.7 percent growth (Specification 3 of the three-state model)

Note: The data of size 120 are generated from Specification 3 using estimated values with $b_0 = 1.7$ and $S_0 = 1$. In all cases, we replicate this procedure 5000 times to compute the first, second, and third quartiles.
Figure 11: Two-State Model for the U.S.

Specification 3, 1840-2005

Probability of regime 1

Estimated coefficient on $b_{t-1}$

Specification 4, 1840-2005

Probability of regime 1

Estimated coefficient on $b_{t-1}$
Figure 11: Two-State Model for the U.S.—Continued

Specification 3, 1948-2004

Probability of regime 1

Estimated coefficient on $b_{t-1}$

Specification 4, 1948-2004

Probability of regime 1

Estimated coefficient on $b_{t-1}$
Figure 12: Two-State Model for the U.K.

Specification 3

Probability of regime 1

Estimated coefficient on $b_{t-1}$

Specification 4

Probability of regime 1

Estimated coefficient on $b_{t-1}$