Research Unit for Statistical and Empirical Analysis in Social Sciences (Hi-Stat)

Institution-Induced Productivity Differences and Patterns of International Capital Flows

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March 2011
Institution-Induced Productivity Differences and Patterns of International Capital Flows\footnote{This paper has a long gestation lag. Yet, I was lucky enough to be given the opportunity to present the idea at very preliminary stages under different titles without any written draft at the following places: Bank of Japan, UPF/CREI, DBJ-RICF, FRB of Chicago, GRIPS, Hitotsubashi, Keio/GSEC, KIER, Harvard, MIT, Tokyo, and Zurich. I would like to thank seminar participants at these places for listening to me and giving me constructive comments, which helped me to write up this paper.}

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Last revised: March 14, 2011

Abstract

This paper presents a stylized model of the world economy to study how the cross-country differences in the institutional quality (IQ) of the domestic credit markets shape the patterns of international capital flows when such IQ differences also cause productivity differences across countries. Institution affects productivity by changing the composition of credit across heterogeneous investment projects with different productivity. Such institution-induced productivity differences are shown to have effects on the investment and capital flows that are opposite of exogenous productivity differences. This implies that the overall effect of IQ could generate $U$-shaped responses of the investment and capital flows, which means, among other things, that capital flows out from middle-income countries and flows into both low-income and high-income countries, and that, starting from a very low IQ, a country could experience both a growth and a current account surplus after a successful institutional reform. More generally, it provides some cautions when interpreting the empirical evidence on the role of productivity differences and institutional differences on capital flows.

Keywords: Institution-dependent productivity-agency cost trade-off, Endogenous productivity through the composition of credit across heterogeneous investment projects, Pledgeability approach to modeling credit market imperfections, Reverse capital flows, Chains of comparative advantage in intertemporal trade; Strict log-submodularity, Envelope Theorem
1. Introduction

It is now well established that capital often flows “upstream,” i.e., from poor to rich countries, contrary to the prediction of the standard textbook neoclassical model. To explain such reverse flows, one needs to abandon the central tenet of the neoclassical paradigm; countries differ in per capita income due to the differences in the capital/labor ratios. In reality, of course, many other factors can account for the difference in per capita income. For example, some countries may be richer than others because they are more productive. Then, capital would flow upstream, because the lenders would get higher return in the rich countries. Or, some countries may be richer due to their superior credit market institutions. Then capital would flow upstream, because rich countries do better jobs protecting the interest of lenders. Indeed, a simple theoretical model can be used to show how exogenous cross-country variations in productivity or in institutional quality can generate reverse capital flows (as will be demonstrated in section 3). One might think intuitively that this logic should carry over even if the rich countries are more productive owing to their superior institutions. This paper aims to show theoretically that productivity differences that arise endogenously due to institutional differences have effects on capital flows that are opposite of exogenous productivity differences, and that institutional differences might have non-monotonic effects on capital flows through their effects on productivity.

In the model presented below, countries differ in the institutional quality (IQ) of their domestic credit markets. Saving flows freely across countries, equalizing the rate of return. In each country, entrepreneurs have access to a variety of heterogeneous investment projects with productivity-agency cost trade-off: a more productive project comes with a bigger agency cost. As entrepreneurs compete for funding, credit goes to the projects that generate the highest return to the lenders (net of agency cost), which are not the most productive ones. The key feature of

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2 See, for example, Gourinchas and Jeanne (2007) and Prasad, Rajan, and Subramanian (2007).
3 To economize language, we simply say “exogenous productivity differences” to mean productivity differences caused by factors unrelated to the institutional quality of domestic credit markets, which also include endogenous productivity differences, for example, due to human capital externalities discussed by Lucas (1990).
4 Thus, this is not a model of international financial market imperfections. A country’s poor IQ reduces the rate of return to lending to its entrepreneurs, regardless of the nationality of lenders. All lenders from all countries earn the same rate of return. In reality, of course, there might be a significant cost of lending to foreigners. However,
the model is that the agency cost of each project depends not only on the nature of each project, but also on the country’s IQ. More productive projects, due to their bigger agency problems, are more affected by the country’s IQ. In this setup, productivity differences arise endogenously due to IQ differences, because IQ affects the productivity-agency cost trade-off, hence the types of projects financed in each country. And it is shown, perhaps counter-intuitively, that an institution-induced productivity improvement, though it leads to a higher output and a higher wage just like an exogenous one, leads to a lower investment and a current account surplus (i.e., capital outflow), unlike an exogenous one.

Why do investment and capital flows respond differently to endogenous productivity changes? Let me try to offer verbally an intuition to this rather counterintuitive result. (Later, this will be shown more formally, which is precisely one of the main goals of the model.)

Although it is often overlooked, higher productivity generally has two effects that work in the opposite directions. The first effect is that more output can be produced with less investment. The second effect is that a higher rate of return makes the lender willing to finance more investment. In the exogenous case, both effects operate. However, under the relatively mild assumption, satisfied for example when the production function is Cobb-Douglas, the second effect dominates the first, which means that higher productivity leads to a higher investment, and hence to a current account deficit (i.e., capital inflow). When productivity rises in response to a better IQ in our model, it is because the composition of credit shifts toward more productive projects, which come with bigger agency problems. This offsets any effect on the rate of return to the lender (i.e., net of the agency cost) that the resulting productivity improvement might have.\(^5\) This means that the first effect dominates the second, hence a lower investment and a current account surplus (i.e., capital outflow).

Note also that this makes an overall effect of IQ on capital flows generally ambiguous, because two effects work in the opposite directions. First, holding productivity constant, a better

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\(^5\)The alert reader might notice that this is nothing but the envelope theorem, as will be shown more formally later. The market always selects the project that maximizes the rate of return to the lender, holding IQ constant. When a change in IQ changes the project selection, the effect on productivity is of the first-order, but the effect on the rate of return to the lender is of the second-order.
IQ causes to a current account deficit (i.e., capital inflow), because it makes the country a more attractive place to invest. Second, induced productivity improvement causes a current account surplus (i.e., capital outflow), because the country needs less investments to produce more output. This means that, even if the rich are more productive and have better IQ than the poor, there is no reason to expect large capital flows in either direction. Or the lack of such capital flows should not be interpreted as the prima facie evidence for the presence of significant barriers for international capital flows.

Some parametric examples also suggest that improving IQ, while monotonically increasing the capital stock and wages, leads initially to a lower investment and a current account surplus (i.e., capital outflow) and then to a higher investment and a current account deficit, (i.e., capital inflow). Such an U-shaped response of capital flows to IQ implies that, if countries inherently differ only in IQ, middle-income countries run a current account surplus (i.e., capital outflow), while high-income and low-income countries run a current account deficit (i.e., capital inflow). However, these countries experience capital inflows for different reasons. High-income countries experiences inflows because they do better jobs protecting the interest of lenders, while low-income countries experiences inflows because they are less efficient in allocating the investment. It also suggests that, starting from very low IQ, an institutional reform would help low-income countries to experience both a growth and a current account surplus at the same time.

Even if the indirect effect of IQ through productivity is not large enough to offset its direct effect, hence unable to generate non-monotonic effects, the prediction that institution-induced productivity differences have the effects on the patterns of capital flows opposite from productivity differences due to other factors should provide some cautions when interpreting the empirical evidence. For example, imagine that the rich countries are more productive partly due to their better IQ and partly for other reasons, say, human capital externalities. Then, one’s failure to properly separate the two sources of productivity differences could lead one to overestimate the effects on capital flows of IQ differences and to underestimate those of productivity differences due to human capital externalities.

Many studies have already examined the effects of domestic credit market imperfections on international capital flows. In models of Gertler and Rogoff (1990) and Matsuyama (2004,
countries do not differ in their institutional quality, but the presence of credit market imperfections give advantage to those entrepreneurs with higher net worth when competing for credit, which could cause reverse capital flows. In models of Sakuragawa and Hamada (2001), Caballero, Farhi, and Gourinchas (2008), and Ju and Wei (2010), among others, reverse flows occur because countries differ in their IQ, but productivity does not respond to the IQ.\(^6\)

Productivity responds endogenously through a change in the composition of credit across projects with different productivity in the closed economy dynamic macro model of Matsuyama (2007), but the reason why the composition changes is due to an endogenous movement of borrower net worth over time, not due to a change in IQ.\(^7\)

The non-monotonic patterns of capital flows implied by the \(U\)-shaped response to IQ, i.e., current account surpluses of the middle-income countries finance current account deficits of the low- and high-income countries, might be somewhat reminiscent of the empirical finding by Gourinchas and Jeanne (2007), who called it the “allocation puzzle.” It should be noted, however, that the primary goal of this paper is not to account for the observed patterns of capital flows. Rather, it is to clarify a mechanism (previously unknown, to the best of my knowledge), through which the quality of domestic financial markets affects productivity, the aggregate investment, and the patterns of capital flows. To this end, the model developed below deliberately abstracts from many other factors that affect the patterns of capital flows. In particular, the model is set up in such a way that the aggregate saving does not respond to changes in IQ nor in productivity. In this respect, the recent studies by Song, Storesletten, and Zilibotti (2011) and Buera and Shin (2010) are noteworthy. Partly motivated by the allocation puzzle, they have shown that, when the economy starts booming after an economic reform that triggers the process of reallocation from the old and less efficient to the new and more efficient sectors, it experiences a current account surplus (i.e., capital outflow) because the aggregate saving grows faster than the aggregate investment, under the assumption that the new and more efficient sectors are borrowing-constrained due to the domestic credit market imperfections. This might lead some to

\(^6\)Strictly speaking, reverse flows in the Caballero-Farhi-Gourinchas model occurs because some countries have a limited supply of saving vehicles. However, they suggested that this could be attributed to the country’s institutional problem.
suspect that their mechanisms would be weakened, if the institutional quality of domestic credit markets improves as a result of the very economic reform that triggers the boom. The result obtained here, however, suggests that such an improvement could even magnify capital outflows generated by the economic reform.\(^8\) In this sense, the present study is complementary to their studies.

The rest of the paper is organized as follows. After setting up the model in section 2, section 3 briefly discusses the patterns of international capital flows when both productivity differences and IQ differences are exogenous. Section 4 shows how productivity responds to IQ through its effect on the composition of credit and explains why such endogenous productivity differences have opposite implications on the investment and capital flows. Section 5 looks at the patterns of international capital flows when countries differ only in IQ but IQ differences causes productivity differences, first for the case with two projects and then for the case with a continuum of projects. From section 2 through section 5, the model is described as a two-sector, two-period model for the ease of presentation. However, the model can also be given a one-sector, and infinite-period interpretation, as explained in Section 6. Section 7 concludes.

2. The Setup; A Two-sector, Two-period Interpretation

There are two periods: \(t = 0\), “today” and \(t = 1\), “future.” (Section 6.2 shows how this two-period setup can be reinterpreted as an infinite period model within an overlapping generations framework.) In \(t = 0\), the endowment is allocated between consumption in \(t = 0\) and investment projects. In \(t = 1\), these investment projects generate capital, \(K\), which are combined with labor, \(L\), available in fixed supply, to produce the consumption good with CRS technology, \(Y = F(K, L) \equiv f(k)L\), where \(k \equiv K/L\) is the capital-labor ratio and \(f(k)\) is output per labor, satisfying the usual properties, \(f'(k) > 0 > f''(k)\) and \(f'(0) = \infty\).

\(^7\) Also related is Buera, Kaboski, and Shin (2010), which shows that productivity responds endogenously to IQ by affecting the sectoral composition in a two-sector model.

\(^8\) Of course, it is important that the reform will not completely eliminate the credit market imperfections for this to be true. As stressed in Matsuyama (2008), credit market imperfections have generally non-monotonic effects, so that one should not study the effects of improving the credit market imperfections by making a binary comparison between a model with the perfect credit market and a model without.
The world economy consists of a finite number of countries, indexed by $c \in C$. (For the moment, however, we suppress the country index to keep the notation simple.) In each country, there are two types of agents. First, there is a continuum of *savers/workers* with measure $L$, each of whom has $\omega$ units of endowment in $t = 0$ and supplies one unit of labor and earns $w(k) \equiv f(k) - kf'(k)$ in $t = 1$. They seek to maximize the quasi-linear preferences of the form:

$$U^s = V(C^s_0) + C^s_1, \quad V' > 0 > V''$$

subject to the budget constraint,

$$C^s_1 = r(\omega - C^s_0) + w(k),$$

where $r$ is the (gross) market rate of return on their saving. From the first-order condition, $V'(C^s_0) = r$, each saver/worker consumes $C^s_0 = (V')^{-1}(r)$ in $t = 0$, so that their total saving is equal to:

$$S^s(r) \equiv [\omega - (V')^{-1}(r)]L.$$

Note that the saving schedule is independent of the wage rate and hence the production side of the economy. This feature of the model helps us to focus on the primary goal of the analysis, i.e., to understand how IQ or productivity changes affect the investment side of capital flows, by removing the saving channel.

Second, there is a continuum of *borrowers/entrepreneurs* with measure $E$, each of whom may be endowed with (small) $\omega^b \geq 0$ units in $t = 0$. They consume only in $t = 1$ and hence save all of $\omega^b$ in $t = 0$. Each entrepreneur has access to a set of projects, $J$. A type-$j$ ($\in J$) project converts $m_j$ units of the endowment to $R_j m_j$ units of “physical capital,” by borrowing $m_j - \omega^b$ at the market rate of return, $r$. They aim to maximize period-1 consumption. By running a

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9 The two assumptions are responsible for this feature. First, the quasi-linearity ensures that all the income effect is entirely absorbed by period-1 consumption. (In addition, the quasi-linearity rules out the possibility of a backward-bending saving schedule.) Second, the wage income is not earned in $t = 0$. Considering that $t = 1$ should be interpreted to include all future periods, these assumptions are not so unreasonable. However, they are not necessary to make the saving schedule independent of the wage rate. Alternatively, this can be achieved, for example, by separating the identity of the savers and the workers, and assuming that the workers do not save.

10 For the moment, we treat the scale of investment $m_j$ as a fixed parameter. It should become clear later that entrepreneurs would not gain even if they were allowed to invest less than $m_j$. In contrast, it is necessary to assume that they face the upper-bound in order to make their problem well-defined.
project-\( j \), they can obtain
\[
R_j m_j f'(k) - r(m_j - \omega^b) = [R_j f'(k) - r]m_j + r \omega^b,
\]
which is greater than or equal to \( r \omega^b \) (the amount obtained by lending instead of borrowing to running any project) iff
\[
(\text{PC-}j): \quad R_j f'(k) \geq r,
\]
where (\text{PC-}j) stands for Profitability Constraint for a Type-\( j \). This constraint implies that
\( R_j f'(k) \) is the maximal rate of return that they are willing to offer to the lender by running a type-\( j \) project. Furthermore, each entrepreneur has access to any project type \( j \in J \). This means that, in a world with the perfect credit market, competition among entrepreneurs would drive up the market rate of return to
\[
r \equiv \max_{j \in J} R_j f'(k)
\]
and only the most productive projects, \( \arg\max_{j \in J} R_j f'(k) \), would be funded.

However, the credit market is imperfect in this world. The imperfections are introduced by the assumption that borrowers/entrepreneurs can pledge no more than a fraction \( 0 < \lambda_j < 1 \) of the project-\( j \) revenue for the repayment.\(^{11} \)

This condition can be stated as:
\[
(\text{BC-}j): \quad \lambda_j R_j m_j f'(k) \geq r(m_j - \omega^b),
\]
where (\text{BC-}j) stands for Borrowing Constraint for a Type-\( j \). By combining (\text{PC-}j) and (\text{BC-}j), we may define the maximal rate of return that an entrepreneur could credibly offer to the lender by running a type-\( j \) project as follows:
\[
(\text{PC-}j)+(\text{BC-}j): \quad r_j \equiv \frac{R_j}{\max\left\{1, (m_j - \omega^b)/\lambda_j m_j\right\}} f'(k).
\]

Again, recall that every entrepreneur has access to any project type \( j \in J \). Hence, competition among entrepreneurs ensure that, in equilibrium, the credit goes only to the projects with the highest \( r_j \), so that
\[
r = \max_{j \in J} r_j = \max_{j \in J} \left\{ \frac{R_j}{\max\left\{1, (m_j - \omega^b)/\lambda_j m_j\right\}} f'(k) \right\}
\]
\(^{11} \) See Tirole (2005) for the pledgeability approach for modeling credit market imperfections, and see Matsuyama (2008) for a variety of applications in macroeconomics. Although various stories of agency problems can be told to justify the assumption that only a fraction is pledgeable, its main appeal is the simplicity, which makes it suitable for studying general equilibrium implications of credit market imperfections.
Note that the ranking of projects may depend on \( \{R_j,m_j;\lambda_j\}_{j=1}^J \), but not on \( k \) nor \( r \). This means that only one project is funded by the credit market.\(^{12}\) By denoting such a project by \( j^* \equiv \text{Arg max}_{j \in J} \{r_j\} \),

\[
  r = \text{Max}_{j \in J} \{r_j\} = \left\{ \frac{R^*}{\text{Max} \{ \lambda_j(m_j - \omega^*)/\lambda_j m_j \} } \right\} f' \left( \frac{R^* I}{L} \right),
\]

where \( I \) is the aggregate investment, i.e., the total amount of the endowment left unconsumed and allocated to the investment projects in \( t = 0 \), which are transformed into capital in \( t = 1 \), at the rate equal to \( R^* \), because \( K = (R^* m^*_j)(I/m^*_j) = R^* I \).\(^{13}\)

In what follows, we focus on the case where \( BC-j \) is more stringent than \( PC-j \). This can be achieved by setting \( \omega^b = 0 \).\(^{14}\) Then, the above expression is simplified to:

\[
  r = \text{Max}_{j \in J} \{r_j\} = \text{Max} \{ \lambda_j R_j \} f' \left( k \right) = (\lambda_j R_j) f' \left( \frac{R^* I}{L} \right).
\]

In words, the credit goes to the projects that generate the highest pledgeable rate of return.\(^{15}\) This expression can be inverted to obtain the *Aggregate Investment Schedule*, which is decreasing in \( r \):

\[
  I(r) = \frac{L}{R^*} \left( f'^{-1} \left( \frac{r}{\lambda_j R^*_j} \right) \right) \quad I' < 0.
\]

Since the *Aggregate Saving Schedule* is an increasing function of \( r \),

\[
  S(r) \equiv \omega^b E + \left[ \omega - (V'^{-1}(r)) L \right] = \left[ \omega - (V'^{-1}(r)) \right] L, \quad S' > 0
\]

the *Current Account Schedule*, the difference between the aggregate saving and investment, is also increasing in \( r \):

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\(^{12}\) In the case of a tie, the entrepreneurs would pick the project type that is more productive.

\(^{13}\) Given that each entrepreneur face the upper-bound, \( m_j^* \), it is necessary to assume that there are sufficiently many entrepreneurs, \( E > I/m_j^* \), to ensure the interior solution for \( I \). This is the only role that the parameter \( E \) plays in the model.

\(^{14}\) In addition, setting \( \omega^b = 0 \) for all countries eliminates the reverse capital flows mechanism that operate through the cross-country differences in the borrower net worth, already studied by Gertler and Rogoff (1991) and Matsuyama (2004, 2005) and others.

\(^{15}\) Since \( r = (\lambda_j R_j) f' \left( k \right) < R_j f' \left( k \right) \), \( PC-j^* \) holds with strict inequality, hence the entrepreneurs have an incentive to invest as much as possible. For this reason, it is necessary to impose the upper-bound. On the other hand, entrepreneurs would not gain from investing project-\( j \) less than \( m_j \).
These schedules are illustrated by Figure 1, the Metzler diagram. If a country were in autarky, its domestic market rate of return would adjust to equate its aggregate saving and the aggregate investment, so that

\[
CA(r^A) \equiv S(r^A) - I(r^A) = 0,
\]

where \(r^A\) is the country's autarky market rate of return, given at the intersection of its aggregate saving and investment schedule.\(^{16}\)

Instead, imagine that this country can lend and borrow at the rate \(r = r^*\), determined at the world financial market. More specifically, suppose that period-0 endowment is (intertemporally) tradeable at the price \(r^*\) for a unit of period-1 consumption good, while the capital stock generated by the project and labor are not tradeable. Then, if \(r^A < r^*\), as depicted in Figure 1, this country runs a current account surplus (i.e., capital outflow) in \(t = 0\).\(^{17}\) If \(r^A > r^*\), this country runs a current account deficit (i.e., capital inflow) in \(t = 0\).

To determine \(r^*\), let us suppose that saving can flow freely across borders to equate the rates of return everywhere. Since the world as a whole is a closed economy, the equilibrium rate of return is given by the condition:

\[
\sum_{c \in C} S^c(r^*) = \sum_{c \in C} I^c(r^*) \quad \iff \quad \sum_{c \in C} CA^c(r^*) = 0,
\]

where superscript \(c \in C\), the country index, is now made explicit. Recall that \(CA^c(r) \equiv S^c(r) - I^c(r)\) is strictly increasing in \(r\). Thus, the autarky rates of returns, \(\{r^A\}_{c \in C}\), dictate "chains of comparative advantage" in intertemporal trade, i.e., the patterns of capital flows. If we list the countries from the left to the right in the increasing order of their autarky rates of return, we can draw a line somewhere in the middle such that all the countries on the left (right) side of the line experience current account surpluses (deficits). In other words, if a

\(^{16}\) Note that \(r^A\) is independent of \(L\), a feature that turns out to be convenient later in section 6.2.

\(^{17}\) Recall that, in the balance-of-payment accounting, a surplus in the current account, \(CA^c(r) = S^c(r) - I^c(r) > 0\) is a deficit in the financial account (used to be called the “capital account”), \(FA^c(r) = I^c(r) - S^c(r) < 0\), and hence a capital outflow.
country’s autarky rate of return is higher than another country’s autarky rate of return, the former would experience a current account surplus (i.e., capital outflow) whenever the latter experiences a current account surplus (i.e., capital outflow), and the latter would experience a current account deficit (i.e., capital inflow) whenever the former experiences a current account deficit (i.e., capital inflow). In particular, if \( C = 2 \), the country with the higher autarky rate runs a surplus (or capital outflow) and the other country runs a deficit (i.e., capital inflow).

3. **Patterns of Capital Flows with Exogenous Productivity and IQ**

First, let us consider the case where there is one type of the project, hence IQ cannot possibly affect the composition of credit, and hence productivity of projects funded. By dropping the project index, \( j \), the aggregate investment schedule, eq.(1), becomes:

\[
I(r) = \frac{L}{R} \left( f' \right)^{-1} \left[ \frac{r}{\lambda R} \right],
\]

which can be used to examine the effects of IQ and exogenous productivity.

For example, a higher \( \lambda \) captures the effect of a better IQ. This unambiguously shifts the investment schedule to the right in Figure 1, which leads to a higher \( r^A \). Hence, if countries differ only in IQ, those with better IQs become richer (measured in the wage and per capita income) and run current account deficits (i.e., capital inflows), while those with worse IQs are poorer and run current account surpluses (i.e., capital outflows), generating the reverse patterns of capital flows.

In contrast, productivity parameter, \( R \), appears twice in the equation. The aggregate investment is decreasing in the first \( R \), while increasing in the second \( R \). They capture the two effects of (exogenously) higher productivity. On one hand, more output can be produced with less investment. On the other hand, the higher rate of return makes the lenders willing to finance more investment. Simple algebra shows \( d \log(I) / dR = 1/\eta(k) - 1 \), where \( \eta(k) \equiv -kf''(k)/f'(k) > 0 \). Thus, higher productivity leads to a higher investment if and only if \( \eta(k) < 1 \), the condition satisfied, for example, for the Cobb-Douglas case, \( f(k) = A(k)^\alpha \) since \( \eta(k) = 1 - \alpha \). In what follows, we will focus on the case where this condition holds. Then, a higher \( R \) shifts the investment schedule to the right in Figure 1, leading to a higher \( r^A \). Thus, if countries differ only in exogenous productivity, those with higher \( R \)s are richer (measured in the wage and per capita income).
income) and run current account deficits (i.e., capital inflows), while those with lower $R_s$ are poorer and run current account surpluses (i.e, capital outflows), generating the reverse patterns of capital flows.

4. **Modeling Endogenous Response of Productivity to Institutional Quality**

Let us now go back to the world where entrepreneurs have access to heterogeneous investment projects. Up to now, we have merely assumed that projects differ both in productivity and in pledgeability, but have not specified how these differences are related to each other. We now impose more structures to introduce institution-dependent productivity-agency cost trade-off. More concretely, suppose that pledgeability of project $j$ in country $c$ is decomposed into two components, as follows:

$$
\lambda_j^c = \Lambda(R_j)^\theta_c.
$$

First, $0 < \Lambda(R_j) < 1$ is the project-specific component, which is common across countries. It represents the agency problem associated with each project-type, and $\Lambda(\bullet)$ is strictly decreasing, which captures the trade-offs between productivity and the agency problem. Second, $\theta_c > 0$ is the country-specific component, which represents the degree of credit market imperfections in country $c$, thus the (inverse) measure of its IQ. A bigger $\theta$ makes pledgeability smaller, exacerbating the agency problem. Furthermore, as the credit market becomes perfect, $\theta_c \to 0$, $\lambda_j^c \to 1$ for all $j$, so that all projects become fully pledgeable, and hence the credit would go to the most productive projects. Note that the assumed functional form satisfies the property of *strict log-submodularity* in $R$ and $\theta$.\(^{18}\) In words, a more productive project, with its bigger agency problem, suffers disproportionately more in a country with a bigger institutional problem.

Under this specification, the country’s IQ affects productivity of projects funded. To see why, recall that the credit goes to the projects that generate the highest pledgeable rate of return. In other words, the market solves,

\(^{18}\text{More generally, let } \tilde{\lambda}_j = \tilde{\Lambda}(R_j; \theta^c) < 1. \text{ Then, } \tilde{\Lambda} \text{ is strictly log-submodular in } R \text{ and } \theta \text{ i ff } \tilde{\partial}^2 \log \tilde{\Lambda} / \tilde{\partial}R \tilde{\partial} \theta < 0. \text{ The assumed functional form in (5) is not crucial, as long as it satisfies strict log-submodularity. Nevertheless, eq. (5) is algebraically simple and more convenient because it can be decomposed into the project-specific and country-specific components to discuss them separately.} \)
\[
\max_{j=1} \lambda_j, R_j = \max_R \{ \Lambda(R_j) R_j \}. 
\]

Figure 2 illustrates this maximization problem. As IQ deteriorates (a bigger \( \theta \)), the graph, \( \lambda_j = [\Lambda(R_j)]^0 \), shifts down. Furthermore, with strict log-submodularity, this negative effect is disproportionately larger for more productive projects with bigger agency problems, which is illustrated by a tilting movement of the graph. As a result, the credit shifts towards to less productive projects with smaller agency problems. In other words, the solution, \( R(\theta) \), is decreasing in \( \theta \).\(^{19}\)

By inserting \( R(\theta) \), the aggregate investment schedule, eq. (1), can be now rewritten as:

\[
I(r; \theta) = \frac{L}{R(\theta)} (f')^{-1} \left( \frac{r}{[\Lambda(R(\theta))]^0 R(\theta)} \right). 
\]

Note that \( R(\theta) \) appears three times in the equation. An increase in the first \( R(\theta) \) reduces the investment. This effect, i.e., less investment is needed to produce more output, is of the first-order. In contrast, the remaining effects are of the second-order, because a change in the second \( R(\theta) \) and a change in the third \( R(\theta) \) offsets each other. This is because \( R(\theta) \) is chosen to maximize \( [\Lambda(R)]^0 R \). When \( R(\theta) \) changes due to a change in \( \theta \), it is because the composition of credit shifts towards projects that are not only more productive but also subject to bigger agency problems. As a result, it has negligible effects on the pledgeable rate of return, which eliminates the usual effect of making the lenders willing to finance more investment. This is nothing but the envelope theorem. The credit market always selects the best project for the lenders under the institutional constraint. Hence, an improvement in IQ has only negligible effects on the lenders. For this reason, an increase in \( R(\theta) \) through a change in \( \theta \) reduces the investment, unlike an exogenous increase in \( R \).\(^{20}\)

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\(^{19}\) If there is a continuum of projects with the range of \( R \), being an interval, as in the case studied in Section 5.2, this can also be shown by differentiating \( \Phi(R; \theta) = \log \Lambda(R; \theta) + \log R \) with respect to \( R \) to obtain \( \Phi_R(R(\theta); \theta) = 0 \) and \( \Phi_{RR}(R(\theta); \theta) < 0 \) and then by applying the implicit function theorem to obtain \( R'(\theta) = -\Phi_{Rg}(R(\theta); \theta) / \Phi_{Rg}(R(\theta); \theta) \), and hence \( \text{sgn} R'(\theta) = \text{sgn} \Phi_{Rg}(R(\theta); \theta) = \text{sgn} \frac{\partial}{\partial \theta} \log \Lambda / \partial \theta < 0 \).

\(^{20}\) It is worth noting that the statement, an increase in \( R(\theta) \) through a change in \( \theta \) reduces the investment, is quite general. It does not depend on \( J \), \( \Lambda(\bullet) \), nor \( f(\bullet) \). Nor does it depend on the assumption of strict log-submodularity, which only plays a role in determining how \( R(\theta) \) depends on \( \theta \).
The above paragraph is concerned with the indirect effect of IQ on the investment through its effect on productivity. In addition, there is the direct effect of improving IQ (a lower $\theta$), which increases the investment. The combined effect on the investment is generally ambiguous, so we need to look at some specific examples. In contrast, improving IQ (a lower $\theta$) ambiguously increases $R(\theta)I(r, \theta)$, and hence the country’s wage and per capita income.21

5. Patterns of International Capital Flows with Endogenous Productivity

We now look at several examples to understand how exogenous IQ differences across countries shape the patterns of international capital flows, when IQ differences also cause productivity differences.

5.1. A Two-Project Case:

Consider first the case with two projects, $J = \{0, 1\}$, with $R_0 < R_1$ and $1 \geq \Lambda(R_0) \equiv \Lambda_0 > \Lambda(R_1) \equiv \Lambda_1$. Thus, a type-1 project is more productive than a type-0 project, but it is more subject to the agency problem. Hence, the pledgeable rate of return declines faster for type-1 projects than for type-0 projects when IQ deteriorates (a bigger $\theta$), as shown in Figure 3a. Only type-0 projects are financed when $\theta > \hat{\theta}$ and only type-1 projects are financed when $\theta \leq \hat{\theta}$, where the switch occurs at $\hat{\theta} \equiv \log(R_1 / R_0) / \log(\Lambda_0 / \Lambda_1)$. Figure 3b shows $R(\theta)$. Note that productivity, $R(\theta)$, jumps at $\theta = \hat{\theta}$, but the pledgeable rate of return, $[\Lambda(R(\theta))'] R(\theta)$, changes smoothly at $\theta = \hat{\theta}$. Thus, when productivity changes as $\theta$ crosses $\hat{\theta}$, the first effect of productivity improvement, --less investment is needed to produce more output--., dominates the second effect, --lenders are willing to finance more investment. Hence, the investment schedule shifts to the left, when productivity increases at $\theta = \hat{\theta}$. With the fixed upward-sloping saving schedule, this translates into a nonmonotone response of $r^A$ to a change in $\theta$, as depicted in Figure 3c.22

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21To see this, for a fixed $r = [\Lambda(R(\theta))]^y R(\theta) f'(k)$, a lower $\theta$ increases $[\Lambda(R(\theta))]^y R(\theta)$ and hence $k = R(\theta)I/L$, $y = f(k)$, and $w = f(k) - kf'(k)$.

22Figure 3c is drawn for $\Lambda_0 = 1$, which implies that the graph is flat for $\theta > \hat{\theta}$. For $\Lambda_0 < 1$, the graph would have a negative slope after it jumps upward at $\theta = \hat{\theta}$. The assumption, $\eta(k) < 1$, ensures that the graph starts at
5.1.1: A Two-Country World

Now suppose that there are two countries, $C = \{N, S\}$, where $N$ stands for the rich North and $S$ for the poor South, with $\theta^N < \theta^S$. Let us assume that the two countries are identical in all other dimensions.

Figure 4a depicts the case of $\theta^N < \theta^S < \bar{\theta}$. This means that $r^AN > r^AS$, which implies $CA^N < 0 < CA^S$. Thus, capital flows from $S$ to $N$. In this case, both countries use the same technologies, but $N$’s superior institution causes the reverse flows, because the interest of lenders is better protected in $N$ than in $S$.

Figure 4b depicts the case of $\theta^N < \bar{\theta} < \bar{\theta} < \theta^S$. Again, $r^AN > r^AS$, which implies $CA^N < 0 < CA^S$. Thus, capital flows from $S$ to $N$. In this case, countries differ both in productivity and in institutional quality. However, the institutional quality difference is the cause for the reverse flows. Although $N$ is more productive than $S$, it is false to attribute the reverse capital flows to the productivity difference. Indeed, in this case, endogenous response of productivity partially offsets the effect of institutional difference on the capital flows. Figure 4c depicts the case of $\bar{\theta} < \theta^N < \bar{\theta} < \theta^S$. This means that $r^AN < r^AS$, which implies $CA^N > 0 > CA^S$. Hence, capital flows from $N$ to $S$. However, the logic behind these capital flows from the rich to the poor is quite different from the standard neoclassical logic. In this case, $S$ is less productive due to its inferior institution, and hence it needs to borrow from abroad. Thus, the causality runs from the underdevelopment to foreign borrowing. It is false to interpret this case as showing that “foreign capital” somehow undermines South’s development.

Now, imagine that, starting from the case depicted in Figure 4c, $S$ manages to improve its institution and succeed improving its productivity, but does not catch up with $N$. This thought experiment is illustrated in Figure 4d. Capital flows are reversed. $S$’s current account turns from a deficit to a surplus. (That is, capital starts flowing out, instead of flowing in.) This illustrates one scenario in which a poor country can experience both a rapid growth and a capital outflow after the reform.
5.1.2: A Three-Country World:

Now suppose that there are three countries, $C = \{N, M, S\}$, with $\theta^N < \theta^M < \theta^S$. Again, assume that these countries are identical in all other dimensions.

Figure 5a depicts the case of $\theta^N < \bar{\theta} < \theta^M < \hat{\theta} < \theta^S$. This means that $r^N < r^S < r^M$, which implies $CA^N < 0 < CA^M$, so that capital flows into $N$, and out of $M$, hence reverse flows between $N$ and $M$. Furthermore, among developing countries, capital flows from the more successful $M$ to the less successful $S$, which is reminiscent of the allocation puzzle.

Figure 5b depicts the case of $\bar{\theta} < \theta^N < \hat{\theta} < \theta^M < \theta^S$. This means that $r^N < r^S = r^M$, which implies that $CA^N > 0 > CA^M, CA^S$, so that capital flows from $N$ to $M$ and $S$. This is because (and not despite that) the most developed $N$ has higher productivity than $M$ or $S$.

Figure 5c depicts the case of $\bar{\theta} < \theta^N < \hat{\theta} < \theta^M < \theta^S$. This implies that capital flows into $S$ and out of $M$. Again, among developing countries, capital flows from the more successful to the less successful among developing countries, because (and not despite that) the more successful is more productive.

Now, consider the thought experiment, in which some developing countries, represented by $M$, succeeded in improving their institutions, while other developing countries, represented by $S$, are left behind. This is illustrated by Figure 5d, which shows that $M$’s current account turns from a deficit to a surplus (capital starts flowing out, instead of flowing in). Thus, $M$ experiences both a rapid growth and a capital outflow after the reform. Furthermore, $N$’s current account could turn from a surplus to a deficit as a result of $M$’s growth.

5.2 A Continuum of Projects Case

One might think that the $U$-shaped patterns obtained above may be driven by the two features of the set of available technologies assumed, $J = \{0, 1\}$. First, its discrete nature means that the autarky rate of return jumps when the switch occurs. Second, its finiteness means the presence of the most productive technology, type-1, so that, once a country’s IQ becomes sufficient good, a further improvement in IQ could not improve productivity. To show that the $U$-shaped patterns can arise more generally, this subsection considers the case where there is a continuum of available projects with no upper-bound on the productivity.
More concretely, suppose \( R_j \in [R_0, \infty) \) for \( j \in J = [0, \infty) \), with

\[
\Lambda(R_j) = \exp \left[ \frac{1}{\gamma} - \frac{1}{\gamma} \left( \frac{R_j}{R_0} \right)^\gamma \right] \quad \text{with} \quad \gamma > 0.
\]

Note that \( \Lambda(R_0) = 1; 0 < \Lambda(R_j) < 1 \) for \( R_j > R_0 \) and \( \Lambda(R_j) \) is decreasing in \( R_j \). This captures the trade-off between productivity and the agency problem.

The market selects the project that generates the highest pledgeable rate of return, i.e., the project that solves \( \max \{ \lambda_j R_j \} = \max \{ \Lambda(R_j) \} R_j \). Thus,

\[
R(\theta) = R_0 / \theta^{1/\gamma} \quad \text{with} \quad [\Lambda(R(\theta))]^{\theta} R(\theta) = R_0 \left( e^{(\theta-1) / \theta} \right)^{1/\gamma} \quad \text{for} \quad 0 < \theta < 1,
\]

both of which are decreasing in \( \theta \). Thus, as the IQ deteriorates, the credit shifts towards less productive projects and the lenders obtain lower rate of returns. Furthermore, \( \lim_{\theta \to 0} R(\theta) = \infty \), so that productivity continues to improve as IQ improves. In contrast,

\[
R(\theta) = R_0 \quad \text{with} \quad [\Lambda(R(\theta))]^{\theta} R(\theta) = R_0,
\]

for \( \theta \geq 1 \), so that the credit goes to the least productive but fully pledgeable project.

Inserting the above expressions to

\[
[\Lambda(R(\theta))]^{\theta} R(\theta) f\left( \frac{R(\theta)L}{L} \right) = r,
\]

and differentiating with respect to \( \theta \) yield:

\[
\eta \frac{d \log I}{d \theta} = \frac{1}{\gamma} \left( 1 + \frac{\eta - 1}{\theta} \right), \quad \text{where} \quad \eta(k) \equiv - \frac{k f''(k)}{f'(k)}, \quad \text{for} \quad 0 < \theta < 1.
\]

If \( \eta(k) > 1 \), \( I(r; \theta) \) and hence \( r^\Lambda \) are increasing in \( \theta \). In this case, capital flows from the rich to the poor, simply because the more efficient rich needs less investment. This is not an interesting case, as it has nothing to do with the endogeneity of productivity. If \( \eta(k) < 1 \), \( I(r; \theta) \) and hence \( r^\Lambda \) are increasing in \( \theta > 1 - \eta(k) \), and decreasing in \( \theta < 1 - \eta(k) \).

For the Cobb-Douglas case, \( f(k) = Ak^\alpha \). Then, \( \eta(k) \) is constant and \( \eta = 1 - \alpha < 1 \), and the investment schedule is given by \( \log I(r; \theta) = \Omega(r) + \theta - 1 - \alpha \log \theta \) for \( \theta < 1 \), and \( = \Omega(r) \) for \( \theta > 1 \), where \( \Omega(r) \) is independent of \( \theta \). Note that

- \( I(r; \theta) \) is decreasing in \( \theta < \alpha \) and increasing in \( \alpha < \theta < 1 \).
• \( I(r; \theta) > I(r; 1) \) if \( \theta < \tilde{\theta} \) and \( I(r; \theta) < I(r; 1) \) if \( \tilde{\theta} < \theta < 1 \), where \( \tilde{\theta} \neq 1 \) is the second solution to \( h(\theta) = \theta - 1 - \alpha \log \theta = 0 \), and satisfies \( 0 < \tilde{\theta} < \alpha \).

With the fixed upward-sloping saving schedule, this translates into \textit{U-shaped} patterns of the (autarky) rate of return, as shown in Figure 6, despite that \( R(\theta) \) responds smoothly to \( \theta \) in the presence of a continuum of projects, and it is unbounded. Therefore, all the patterns of capital flows described for the two-projects case in the previous section can also occur for this case as well.

6. Alternative Interpretations

6.1 A One-Sector Interpretation

Up to now, the model has been given a two-sector interpretation. That is, the entrepreneurs run projects that produce tangible “physical capital,” in the capital goods sector, which is rented out to the consumption goods sector. Taken literally, this means that IQ affects the investment and capital flows through its impacts of the productivity of capital goods sector. This is consistent with the empirical evidence suggesting that the relative prices of capital goods to consumption goods are higher among less developed countries.\(^{23} \) Nevertheless, it is not an essential element of the argument, and the mechanism does not rely on the two-sector structure. To show this, this subsection offers a one-sector interpretation of the model.

Imagine that the economy produces the single consumption good, using the endowment and labor. In \( t = 0 \), entrepreneurs may invest \( m_j \) units of the endowment to set up a type-\( j \) firm. A type-\( j \) firm produces the consumption good in \( t = 1 \), using the labor input, \( n \), with a concave production function, \( y_j = \varphi_j(n) \). Each firm hires labor in the competitive labor market at the wage rate, \( w \), so that its employment would be determined by \( \varphi_j'(n_j) = w \). Hence, the profit that could be earned from running a type-\( j \) firm is \( \pi_j = \max\{\varphi_j(n) - wn\} = \varphi_j(n_j) - \varphi_j'(n_j)n_j \).

Suppose that \( \varphi_j(n) \equiv F(R_jm_j, n) = f(R_jm_j / n)n \), where \( R_j \) is a parameter. Then, for a given wage rate, \( w \), the employment and the profit by a type-\( j \) firm can be written as \( n_j = R_jm_j / k \)

and \( \pi_j = (R_j m_j) f'(k) \), where \( k \) is defined uniquely for each \( w \) by \( w = f(k) - k f'(k) \). If a type-\( j \) firm can pledge up to \( \lambda_j \) fraction of its profit, the pledgeable rate of return for lending to type-\( j \) firms would be \( \lambda_j \pi_j / m_j = \lambda_j R_j f'(k) \), so that the credit flows only to those firms with the highest \( \lambda_j R_j \). By denoting such firms by \( j^* = \text{Arg max}_{j \in J} \{ \lambda_j R_j \} \), the equilibrium rate of return earned by the lenders is
\[
r = \lambda_j^* R_j^* f'(k).
\]

Since each active firm hires \( n_j^* = R_j^* m_j / k \), summing up across all firms yield the labor market equilibrium condition,
\[
L = \sum n_j^* = R_j^* \sum m_j / k = R_j^* I / k,
\]
where \( I \) is the aggregate investment. By combining these expressions, we obtain
\[
r = \lambda_j^* R_j^* f'(k) = \lambda_j^* R_j^* f'\left( R_j^* I / L \right),
\]
from which eq.(1) and hence eq.(4) and eq.(6) will also follow. This alternative interpretation thus gives the same predictions on the relationship between IQ, the investment and capital flows.

According to this interpretation, there is no separate investment good sector. Investment is an productivity-enhancing expenditure of the consumption goods sector; \( R_j^* = R(\theta) \) is the realized productivity parameter in the consumption good sector; and \( k \) represents the organizational (i.e., intangible) capital per worker, embodied in the firms that set up by entrepreneurs. The assumption of nontradeability of \( k \) might be more natural under this interpretation. Furthermore, for \( f(k) = A k^\alpha \), \( AR(\theta)^\alpha \) may be viewed as the TFP of the consumption goods sector firms.\(^{24}\)

### 6.2 An Infinite Period Interpretation in an OLG framework

Some readers may find the two-period setup developed above too restrictive, because each country’s intertemporal trade must be in balance due to the Walras’ Law, so that \( CA_0 = CA(r) > 0 \) in \( t = 0 \) implies \( CA_1 = -CA(r) < 0 \) in \( t = 1 \). Thus, taken literally, any country experiencing a capital outflow today will experience a capital inflow in the future. However, the
above setup can be given an infinite-period interpretation by embedding the structure into an overlapping generations framework.  

Imagine now that there is an infinite number of periods, extending from \( t = 0, 1, 2, \ldots \). In period \( t \), a continuum of savers/workers, with their total endowment \( \omega L_t \) and total labor supply \( L_t \), and a continuum of entrepreneurs of mass \( E_t \) are born and live for two periods. Those born in the same period interact with each other just as described above. Thus, the savers/workers born in period-\( t \) finance the projects run by the entrepreneurs born in period-\( t \), and the savers/workers born in period-\( t \) work with capital generated by the projects in their second period (period \( t+1 \)). In this setup, there is no interaction across different generations. This means that, from the intertemporal budget constraint of each agent, the current account of generation born in period-\( t \) (generation-\( t \)) in period \( t+1 \) must be equal to the negative of the current account of this generation-\( t \) in period \( t \).

Consider a particular country whose IQ is given by \( \theta \). Then, the investment in period \( t \) is

\[
I_t = \frac{L_t}{R(\theta)} \left( f^r \right)^{-1} \left( \frac{r_{t+1}}{\Lambda(R(\theta))^\prime R(\theta)} \right) = L_t I(r_{t+1}; \theta),
\]

which differs from eq.(6) only in that \( L_t \) may vary over time. Likewise, the saving by generation-\( t \) in period \( t \) can be written as:

\[
S'_t = L_t \left[ \omega - \left( V^r \right)^{-1} (r_{t+1}) \right] = L_t S(r_{t+1}),
\]

so that the current account by generation-\( t \) in period \( t \) can be written as:

\[
CA'_t \equiv L_t \left[ \omega - \left( V^r \right)^{-1} (r_{t+1}) - \frac{1}{R(\theta)} \left( f^r \right)^{-1} \left( \frac{r_{t+1}}{\Lambda(R(\theta))^\prime R(\theta)} \right) \right] = L_t CA(r_{t+1}; \theta)
\]

Since the current account by generation-(\( t-1 \)) in period \( t \) must be equal to the negative of the current account by this generation in period-(\( t-1 \)),

\[
CA'_{t-1} \equiv -L_{t-1} CA(r_t; \theta),
\]

the current account of this country in period \( t \) is equal to:

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\[24\] The idea that capital can be viewed as a parameter in the production function, representing productivity, with the investment being interpreted as any productivity-shifting expenditure, which includes but is not limited to manufacturing or purchasing capital goods, at least goes back to Uzawa (1969).

\[25\] See Obstfeld and Rogoff (1996, Ch.2) for a survey on overlapping generations models of the current account.
\[ CA_t = CA_{t-1} + CA'_t = -L_{t-1}CA(r_t; \theta) + L_tCA(r_{t+1}; \theta). \]

Suppose \( L_t = (1 + g)L_{t-1} \), where \( g > 0 \) is a constant rate of population (or Harrod-neutral productivity) growth, which is common across all countries. Then, in per capita term, this country’s current account is:

\[ ca_t = \frac{CA_t}{L_{t-1}} = -CA(r_t; \theta) + (1 + g)CA(r_{t+1}; \theta). \]

In this environment, the autarky equilibrium path of this country is characterized by a constant autarky rate of return, \( r^A \), given by \( ca_t = gCA(r^A; \theta) = 0 \). If this country has access to the world financial market where it could lend or borrow at \( r^* \),

\[ ca_t = gCA(r^*; \theta) > 0 \quad \text{if} \quad r^* > r^A; \quad ca_t = gCA(r^*; \theta) < 0 \quad \text{if} \quad r^* < r^A. \]

Thus, the country experiences a current surplus (deficit) and capital outflows (inflows) if its autarky rate is lower (higher) than the world rate, each period. This way, all of the results on the effects of IQ differences discussed in the two-period setup can be restated in an infinite period setup.

7. **Concluding Remarks**

This paper presented a stylized model of the world economy to study how the cross-country differences in the institutional quality (IQ) of the domestic credit markets shape the patterns of international capital flows when such IQ differences also cause productivity differences across countries. Institution affects productivity by changing the composition of credit across heterogeneous investment projects with different productivity. Such institution-induced productivity differences are shown to have effects on the investment and capital flows that are opposite of productivity differences due to other factors. This implies that the overall effect of IQ could generate \( U \)-shaped responses of the investment and capital flows, which means, among other things, that there is no reason to expect capital inflows when a country is more productive and has better institution protesting the interest of lenders, even if saving flows freely across borders to equalize the rates of return; that capital flows out from middle-income countries and flows into both low-income and high-income countries, and that, starting from a very low IQ, a
country could experience both a growth and a current account surplus after a successful institutional reform. More generally, it provides some cautions when interpreting the empirical evidence on the role of productivity differences and institutional differences on capital flows. Furthermore, certain features of the model, such as poor IQ preventing productive technologies from being adopted, institutional changes causing productivity change, etc., might have wider applications besides capital flows.
References:

Figure 1: Metzler Diagram

Figure 2: Endogenous Productivity Response to Institutional Quality Change
Figure 3: A Two-Project Case

Figure 3a

Figure 3b

Figure 3c
Figure 4: A Two-Country World

Figure 4a

Figure 4b

Figure 4c

Figure 4d
Figure 5: A Three-Country World

Figure 5a

Figure 5b

Figure 5c

Figure 5d
Figure 6: A Continuum of Projects Case