

# Foreign Monopoly and Trade Policy under Segmented and Integrated Markets

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## 1. Introduction

Since the beginning of 1980s, trade policy has extensively been analyzed under imperfect competition. Characteristics observed from the existing literature include the following<sup>1)</sup>. First, in the (segmented) multi-markets model, marginal costs (MCs) are almost always assumed to be constant. In other words, non-constant MCs are mostly assumed when only one market (say, domestic market) is focused on and the other markets are out of consideration. The assumption of constant MCs is imposed to eliminate the complication that firm's choices in different markets are connected through the dependence of MCs on the total output<sup>2)</sup>.

It is somewhat surprising that there are few rigorous analyses which explicitly take account of this spillover effect through MCs<sup>3)</sup>. One may argue that the effects of trade policy are qualitatively the same even with non-constant MCs. This may be correct as far as one is concerned with the effects only on the economy in question under constant MCs. However, this argument obviously becomes nonsense once we are particularly interested in how trade policies affects other markets through MCs.

Second, the number of researches conducted under integrated markets is very small relative to that under segmented markets<sup>4)</sup>. Moreover, a number of analyses under integrated markets assume free entry and/or linear demand. This is probably because of the complication associated with integrated markets. In particular, there are two different notions of market integration, which sometimes causes confusion<sup>5)</sup>. We refer to those two notions as the "strong" version and the "weak" version<sup>6)</sup>.

In the strong version, there are many competitive independent wholesalers. Firms

must sell all output to the wholesalers at a single price with no notion of where the output will eventually be retailed. There will then be a single producer price for all output. That is, firms can control only their total supplies and their allocation between markets is determined such that producer prices are equalized across markets.

In the weak version, firms control wholesale and can make direct contracts with domestic and foreign retailers. However, there are independent arbitragers that can buy in one market and sell in the other. Under free trade without transportation costs, the two notions result in the same equilibrium where the consumer prices as well as producer prices are equalized across markets. In the presence of trade taxes and/or transportation costs, however, the producer prices across markets could be different in the weak version. That is, firms may absorb some of the taxes and/or transportation costs.

In this paper, we examine the effects of various trade policies on domestic, foreign and world economies under both segmented and integrated markets in a single model with general demand and cost specifications. We consider import and export taxes/subsidies and import and export quotas as trade policies. With respect to the effects of those policies under segmented markets, the present study explicitly deals with non-constant MCs so as to analyze the spillover effects. For the analysis under integrated markets, we consider both strong and weak versions of market integration.

To accomplish the purpose of the paper, we focus on a case where a foreign monopolist produces a good in the foreign country and supplies it to both domestic and foreign markets. We deal with a monopoly model, because it can provide a systematic treatment regarding market structures with general

demand and cost functions. In particular, a monopolist makes the analysis much simpler, since there is no interaction among firms. Regardless of the absence of this aspect, we still obtain some new insights into trade policy under imperfect competition.

It should be mentioned that Markusen and Venables (1988) provide a single oligopoly model and analyze the effects of trade taxes on welfare under four different types of market structure generated by no entry versus free entry, and segmented markets versus integrated markets. For the sake of tractability, however, they also impose some restrictive assumptions, i.e., linear demand, constant MCs and Cournot conjectures.

The rest of the paper is organized as follows. Section 2 discusses the existing literature which is closely related to our study. Section 3 examines the effects of trade policies under segmented markets. The analysis under segmented markets is also necessary for that under the integrated markets in section 4. Section 5 concludes the paper.

## 2. Related Literature

There are a number of analyses similar to ours. However, our study provides fairly general structures (except for the monopoly aspect) and makes the systematic comparisons among various trade policies under various circumstances possible. In particular, no single article has analyzed trade policies under both versions of market integration. In this section, we briefly discuss some previous literature closely related to the present study.

Domestic tariffs against a foreign monopolist have been examined by a number of studies such as Katrak (1977), Brander and Spencer (1984) and Jones and Takemori (1989). Katrak (1977) and Brander and Spencer (1984) show that in the presence of foreign monopoly, domestic import taxes or subsidies can raise domestic welfare under the segmented markets. Our analysis of tariff under segmented markets is an extension of Katrak (1977) and Brander and Spencer (1984) to the multi-markets with general demand cost specifications<sup>7</sup>. Jones and Takemori (1989) examine the optimal tariff of a small open economy (SOE) with foreign monopoly in the weak version of market integration. However, their framework of SOE also abstracts from the spillover effects of

domestic tariffs on foreign markets.

Auquier and Caves (1979) and Katrak (1980), among others, have analyzed the optimal domestic export taxes in the presence of domestic monopoly with general cost specifications. Those studies are concerned with the optimal policies from a point of view of domestic welfare alone. The present study is concerned with not only domestic welfare but also foreign and world welfare. Katrak (1980) derives the optimal combinations of export taxes with another policy such as production subsidies. In order to raise domestic welfare with domestic monopoly, the domestic government intends to reduce the monopolistic distortion in the domestic market but keep the market power in the foreign market. Auquier and Caves (1979) consider export taxes under both segmented and under integrated markets (the strong version). In their analysis, the demand elasticities, which are assumed to be constant, are crucial for both analysis and result. In our analysis, constant demand elasticities are not necessarily assumed. We classify the results by using the elasticity of the slope of the inverse demand function<sup>8</sup>.

The examination of quotas with foreign monopoly under integrated markets has been done by Krishna (1990, 1991)<sup>9</sup>. Although some of our results are comparable to hers, her focus is different from ours. Krishna is concerned with auctioning quota licenses (particularly, license revenues). We are interested in more general effects of quotas and consider simpler implementation of quotas which is widely used: that is, a quota is based on "first come, first served". This allows us to avoid complicated issues associated with quotas implemented by auctioning quota licenses as in Krishna (1990, 1991): how licenses are allocated and/or auctioned off and what the market structure of licenses is. We assume that the government first sets the level of a quota and then the monopolist decides the quantity of exports. If the quota is not fully used by the monopolist, there is room for arbitrage. Our analysis of quotas under integrated markets can be regarded as an extension of Krishna (1990, 1991) to a different implementation scheme with general cost functions<sup>10</sup>.

### 3. Segmented Markets

#### 3.1 The Model

We consider a foreign monopolist that supplies a good to both domestic and foreign markets. The demand functions are given by

$$x = D(p); D' < 0, \quad (1)$$

$$x^* = D^*(p^*); D^{*'} < 0, \quad (2)$$

where  $x$  and  $p$  are, respectively, the demand and consumer price of the good. Foreign variables, parameters and functions are denoted by “\*”. We define the elasticity of the slope of the inverse demand function for the following analysis:

$$\varepsilon \equiv \frac{DD'}{(D')^2}; \varepsilon^* \equiv \frac{D^*D^{*'}}{(D^{*'})^2}. \quad (3)$$

The (inverse) demand curve is concave if  $\varepsilon \leq 0$  and convex if  $\varepsilon \geq 0$ .

In this section, we examine the effects of domestic tariffs and import quotas and foreign export taxes and export quotas (i.e., VERs) on both domestic and foreign economies under segmented markets. For simplicity, transportation costs are assumed away. Under the assumption of segmented markets, the monopolist can independently vary prices in each market. The profit function is

$$\Pi^*(p, p^*; \tau) = (p - \tau)D(p) + p^*D^*(p^*) - C^*(D(p) + D^*(p^*)), \quad (4)$$

where  $\tau$  denotes a specific trade tax (i.e., a domestic tariff,  $t$ , or a foreign export tax,  $t^*$ ) and  $C^*(\cdot)$  is the cost function with  $C^{*'} > 0$ . The MC is increasing, decreasing, or constant. The first-order conditions of the profit maximization are then

$$\frac{\partial \Pi}{\partial p} = D + (p - \tau - C^{*'})D' = 0, \quad (5)$$

$$\frac{\partial \Pi^*}{\partial p^*} = D^* + (p^* - C^{*'})D^{*'} = 0. \quad (6)$$

We assume that the second-order sufficient conditions are satisfied:

$$D'(2 - \varepsilon) - C^{*''}(D')^2 < 0, \quad (7)$$

$$D^{*'}(2 - \varepsilon^*) - C^{*''}(D^{*'})^2 < 0, \quad (8)$$

$$\begin{aligned} [D'(2 - \varepsilon) - C^{*''}(D')^2] \\ [D^{*'}(2 - \varepsilon^*) - C^{*''}(D^{*'})^2] \\ - (C^{*''}D'D^{*'})^2 \equiv \Omega > 0. \end{aligned} \quad (9)$$

We should note that  $\varepsilon < 2$  and  $\varepsilon^* < 2$  are necessary with  $C^{*''} \leq 0$  from (7) and (8) and that  $\varepsilon = \varepsilon^* = 2$  is not the case from (9)<sup>11</sup>.

Solving the first-order conditions with  $\tau = 0$ , we can obtain the free-trade prices under segmented markets<sup>12</sup>:

$$p^{Sf} = \frac{\theta}{\theta - 1} C^{*'}; p^{*Sf} = \frac{\theta^*}{\theta^* - 1} C^{*'}, \quad (10)$$

We let  $x^{Sf}$  and  $x^{*Sf}$  denote the supplies to domestic and foreign countries under segmented markets with  $\tau = 0$ . Similarly, we can obtain the equilibrium prices and supplies with  $\tau \neq 0$ , which are, respectively, denoted by  $p^{S\tau}$ ,  $p^{*S\tau}$ ,  $x^{S\tau}$  and  $x^{*S\tau}$ .

#### 3.2 The Effects of Trade Taxes

In this subsection, we analyze the effects of domestic tariffs and foreign export taxes on the supplies, profits, prices, domestic welfare, foreign welfare and world welfare. To this end, we first totally differentiate (5) and (6) and obtain:

$$\begin{pmatrix} D'(2 - \varepsilon) - C^{*''}(D')^2 & -C^{*''}D'D^{*'} \\ -C^{*''}D'D^{*'} & D^{*'}(2 - \varepsilon^*) - C^{*''}(D^{*'})^2 \end{pmatrix} \begin{pmatrix} dp \\ dp^* \end{pmatrix} = \begin{pmatrix} D' \\ 0 \end{pmatrix} d\tau \quad (11)$$

with the solution

$$\begin{pmatrix} dp \\ dp^* \end{pmatrix} = \frac{1}{\Omega} \begin{pmatrix} D'(2 - \varepsilon) - C^{*''}(D')^2 & C^{*''}D'D^{*'} \\ C^{*''}D'D^{*'} & D^{*'}(2 - \varepsilon^*) - C^{*''}(D^{*'})^2 \end{pmatrix} \begin{pmatrix} D' \\ 0 \end{pmatrix} d\tau. \quad (12)$$

In view of (7), thus, the effects of a change in  $\tau$  on consumer price in each market are given by

$$\begin{aligned} \frac{dp}{d\tau} &= \frac{[D^{*'}(2 - \varepsilon^*) - C^{*''}(D^{*'})^2]D'}{\Omega} > 0 \\ \frac{dp^*}{d\tau} &= \frac{C^{*''}D^{*'}(D')^2}{\Omega}. \end{aligned} \quad (13)$$

The supply to the foreign market rises (resp. falls) if and only if  $C^{*''} > 0$  (resp.  $C^{*''} < 0$ ). This confirms a well-known result that a domestic tariff raises domestic consumer price and decreases the domestic demand, but does not affect the foreign market at all as long as  $C^{*''} = 0$ . If  $C^{*''} > 0$ , a trade tax increases the supply to the foreign market and benefits foreign consumers. This is because a decrease in the supply to the domestic market caused by a trade tax reduces the MC and hence the monopolist has an incentive to increase the supply to the other market, i.e., the foreign market. If  $C^{*''} < 0$ , on the other hand, a trade tax harms foreign consumers as well as domestic consumers.

The effect on the total supply can easily be obtained from (13). Obviously, the total supply falls with  $C^{*''} \leq 0$ . When  $C^{*''} > 0$ ,

$$\frac{d(x + x^*)}{d\tau} = \frac{(D')^2 D^{*'}(2 - \varepsilon^*)}{\Omega} \quad (14)$$

holds. Thus, we have:

$$\frac{d(x + x^*)}{d\tau} \geq 0 \Leftrightarrow \varepsilon^* \geq 2. \quad (15)$$

If  $C^{*''} > 0$ , thus, whether the total supply rises or falls depends on the curvature of the foreign demand curve. A trade tax increases the total supply if the foreign demand function is very convex.

**Proposition 1** *A domestic tariff or a foreign export tax (i. e., an increase in  $\tau$ ) necessarily decrease the supply to the domestic market; and increase the supply to the foreign market if and only if  $C^{**} > 0$ . With  $C^{**} > 0$ , the total supply falls if and only if  $\varepsilon^* < 2$ .*

Next we analyze the effects of trade taxes on domestic, foreign and world welfare. Domestic welfare is measured by the sum of consumers' surplus and tariff revenue:

$$W \equiv \int_p^\infty D(u) du + tD(p). \quad (16)$$

It is easy to see that the foreign export tax deteriorates domestic welfare. For this, we set  $t \equiv 0$  and differentiate (16) with respect to  $t^*$  to obtain

$$\frac{dW}{dt^*} = -D \frac{dp}{dt^*}. \quad (17)$$

As was shown by Brander and Spencer (1984), by using a tariff, the domestic government could extract some of the monopoly rent and hence raise domestic welfare. Differentiating (16) with respect to  $t$  and evaluating it at  $t=0$ , we obtain

$$\left. \frac{dW}{dt} \right|_{t=0} = D \left( 1 - \frac{dp}{dt} \right) \Big|_{t=0}. \quad (18)$$

Thus, a small tariff raises domestic welfare as long as  $(dp/dt)|_{t=0} < 1$  (i. e., an increase in the domestic consumer price caused by a tariff is less than the size of the tariff). This is actually the same condition as Brander and Spencer (1984) have derived. However, the value of  $[1 - (dp/dt)]$  in our model is different from theirs, because the monopolist in their model serves only the domestic market<sup>13)</sup>.

It is interesting to compare the two values. In Brander and Spencer (1984),  $(1 - dp/dt) = [D'(1 - \varepsilon) - C^{**}(D')^2] / [D'(2 - \varepsilon) - C^{**}(D')^2]$ , where the denominator is negative from the second-order (sufficient) condition. With  $C^{**} > 0$ ,  $\varepsilon < 1$  is sufficient for domestic welfare to improve; with  $C^{**} = 0$ ,  $\varepsilon < 1$  is necessary and sufficient; and with  $C^{**} < 0$ ,  $\varepsilon < 1$  is necessary. In our model, using (13), we obtain:

$$1 - \frac{dp}{dt} = \frac{D'(1 - \varepsilon)[D^*(2 - \varepsilon^*) - C^{**}(D^*)^2] - C^{**}(D')^2 D^*(2 - \varepsilon^*)}{\Omega}, \quad (19)$$

where the value in the brackets is negative from the second-order condition. With  $C^{**} > 0$ ,  $\varepsilon < 1$  is not a sufficient condition any more. Because of the spillover effect, the curvature of the foreign demand curve as well as that of the domestic demand curve enters in our value. When  $C^{**} > 0$ , a sufficient condition is  $\varepsilon < 1$  and  $\varepsilon^* < 2$  which hold with concave

demand curves. When  $C^{**} = 0$ ,  $\varepsilon < 1$  is necessary and sufficient because the two values become identical. When  $C^{**} < 0$ , noting  $\varepsilon^* < 2$ ,  $\varepsilon < 1$  is still a necessary condition.

The following should be noted. With  $C^{**} > 0$  and  $\varepsilon^* < 2$ , a tariff decreases the MC, since the total output falls [recall Proposition 1]. This implies that an increase in the domestic consumer price caused by the tariff is mitigated relative to the case with  $C^{**} = 0$ . With  $C^{**} < 0$ , on the other hand, the increase is magnified.

**Proposition 2** *The domestic country loses from a foreign export tax (i. e., an increase in  $t^*$ ). The domestic government may be able to raise domestic welfare by imposing a small tariff:*

- (i) If  $C^{**} > 0$ , then  $\varepsilon < 1$  and  $\varepsilon^* < 2$  is a sufficient condition for welfare improvement;
- (ii) If  $C^{**} = 0$ , then  $\varepsilon < 1$  is a necessary and sufficient condition;
- (iii) If  $C^{**} < 0$ , then  $\varepsilon < 1$  is a necessary condition.

Foreign welfare is measured by the sum of the profits, consumers' surplus and tax revenues:

$$W^* \equiv \Pi^*(p, p^*; \tau) + \int_{p^*}^\infty D^*(u) du + t^* D^*(p). \quad (20)$$

Using the envelop theorem, we have

$$\frac{d\Pi^*}{d\tau} = -D < 0, \quad (21)$$

which implies that a trade tax harms the foreign monopolist.

Thus, the effects of a domestic tariff on foreign welfare is as follows. If  $C^{**} \leq 0$ , a domestic tariff necessarily reduces foreign welfare, because the tariff does not lower the foreign price. With  $C^{**} > 0$ , however, foreign welfare may not deteriorate. That is, the foreign country benefits from a domestic tariff only if  $C^{**} > 0$ . Differentiating (20) with respect to  $t$ , we have

$$\begin{aligned} \frac{dW^*}{dt} &= - \left( D + \frac{dp^*}{dt} D^* \right) \\ &= - \frac{D'D(2 - \varepsilon)[D^*(2 - \varepsilon^*) - C^{**}(D^*)^2] + C^{**}D^*D(D')^2\varepsilon^* + C^{**}D^*(D')^2(D^* - 2D)}{\Omega}. \end{aligned} \quad (22)$$

With  $C^{**} > 0$ , thus, foreign welfare improves if  $\varepsilon > 2$ ,  $\varepsilon^* \geq 0$  and  $x^* > 2x$  hold but deteriorates if  $\varepsilon < 2$ ,  $\varepsilon^* \leq 0$  and  $x^* < 2x$ . The intuition why the relative market size enters in the conditions is straightforward. Since  $x^* > 2x$  means that the domestic market is relatively small, the loss of the monopolist is small



relative to the gain of foreign consumers and foreign welfare is likely to improve.

To see the effects of a foreign export tax on foreign welfare, we differentiate (20) with respect to  $t^*$  and evaluate it at  $t^*=0$  to obtain

$$\left. \frac{dW^*}{dt^*} \right|_{t^*=0} = - \left. \frac{dp^*}{dt^*} D^* \right|_{t^*=0}. \quad (23)$$

This implies that the loss of the monopolist is offset by the tax revenue. Thus, the change in foreign welfare caused by a small export tax depends only on that in consumers' surplus. When a small export tax is introduced, the foreign country gains or loses according as  $C^{**} > 0$  or  $C^{**} < 0$ <sup>14</sup>. It should be noted that both countries lose from an export tax with  $C^{**} < 0$ . In other words, an export subsidy certainly makes both countries better off with  $C^{**} < 0$ .

**Proposition 3** *A trade tax (i. e., an increase in  $\tau$ ) reduces the profits of the foreign monopolist. Foreign country gains from a small export tax if and only if  $C^{**} > 0$ . A domestic tariff (i. e., an increase in  $t$ ) deteriorates foreign welfare with  $C^{**} \leq 0$  but may improve it with  $C^{**} > 0$ . The foreign country is more likely to gain from a domestic tariff if the domestic market is small relative to the foreign market and if both demand curves are more convex.*

An import tax and an export tax set at the same levels result in the identical effects except for the effects on domestic and foreign welfare. The welfare effects crucially hinge on which government obtains the tax revenue. When we examine the effects on the total world welfare which is defined by the sum of domestic and foreign welfare, however, it does not matter which government obtains the tax revenue. That is, the world-welfare effect of an import tax is identical to that of an export tax.

It can be shown that a small trade tax could raise world welfare. In view of (17), (18), (22) and (23), the effect of a small trade tax on world welfare is given by

$$\left. \frac{d(W+W^*)}{d\tau} \right|_{\tau=0} = - \left( \left. \frac{dp}{d\tau} D + \left. \frac{dp^*}{d\tau} D^* \right|_{\tau=0} \right). \quad (24)$$

While the first term in the parentheses is positive, the second term is positive, zero or negative according as  $C^{**} < 0$ ,  $C^{**} = 0$  or  $C^{**} > 0$ . If the second term is negative and dominates the first term, the total effect is positive and hence world welfare improves. When the domestic market is small relative to the for-

ign market, for example, the second term is likely to dominate the first term.

**Proposition 4** *An import tax and an export tax set at the same levels lead to the same effects on world welfare. A small trade tax improves world welfare only if it lowers the foreign consumer price (i. e.,  $C^{**} > 0$ ).*

### 3.3 The Effects of Quotas

In this subsection, we consider the effects of domestic import quotas and foreign export quotas (i. e., VERs). To begin with, we should note the following, which was originally pointed out by Shibata (1968). Under segmented markets, the monopolist can get all of the quota rent whether a quota is imposed by the domestic government or the foreign government. The point is that the monopolist is the only exporter and hence, even with a domestic import quota, it can always choose the price so that the quota rent accrues entirely to the monopolist as its profits. As a result, the foreign monopolist sells the good to domestic consumers at the consumer price with a domestic import quota as well as with a foreign export quota. That is, a domestic import quota and a foreign export quota set at the same levels are equivalent. Since domestic welfare is measured by consumers' surplus alone, a quota less than free-trade level deteriorates domestic welfare.

We next examine if a quota is actually binding. Suppose that it is binding. When a quota is set at the level of  $q$ , the profit function (4) is replaced by

$$\Pi^*(p^*; q) = D^*(p^*)p^* + qp^q - C^*(D^*(p^*) + q), \quad (25)$$

where  $p^q \equiv D^{-1}(q)$ . Differentiating (25) with respect to  $q$  and using the envelop theorem, we have

$$\frac{d\Pi^*}{dq} = \frac{\partial \Pi^*}{\partial q} = p^q + \frac{q}{D'} - C^{*'} \geq 0, \quad (26)$$

where inequality holds for  $q < x^{sf}$  and equality holds at  $q = x^{sf}$  from (5). This implies that if the monopolist chooses a supply to the domestic market less than  $q$ , the profits become smaller than with  $q$ . Thus, when a quota is set less than the free-trade level,  $x^{sf}$ , it is binding.

In the presence of a quota, we have only one first-order condition (6) and only one second-order condition (8). To see the effects of quotas on supplies, we totally differentiate (6) with respect to  $q$ :

$$\frac{dx^*}{dq} = \frac{C^{*''}(D^{*'})^2}{D^{*'}(2 - \varepsilon^*) - C^{*''}(D^{*'})^2},$$

$$\frac{d(q+x^*)}{dq} = \frac{D^{*'}(2-\varepsilon^*)}{D^{*'}(2-\varepsilon^*) - C^{*''}(D^{*'})^2}. \quad (27)$$

As in the case of tariff, a quota (i.e., a decrease in  $q$ ) increases the supply to the foreign market if  $C^{*''} > 0$ ; does not change it if  $C^{*''} = 0$ ; and decreases it if  $C^{*''} < 0$ . Regarding the total supply with  $C^{*''} > 0$ , we obtain:

$$\frac{d(q+x^*)}{dq} \geq 0 \Leftrightarrow \varepsilon^* \leq 2. \quad (28)$$

In view of (15),  $d(q+x^*)/dq < 0$  if and only if  $d(x+x^*)/dt > 0$ .

**Proposition 5** *A quota (i.e., a decrease in  $q$ ) increases the supply to the foreign market if and only if  $C^{*''} > 0$ . With  $C^{*''} > 0$ , the total supply falls if and only if  $\varepsilon^* < 2$ .*

The effect on the profits can be seen from (26). A quota set less than the free trade level decreases the profits. The effect on foreign welfare is given by

$$\frac{dW^*}{dq} = \frac{d\Pi^*}{dq} - \frac{dp^*}{dq} D^*, \quad (29)$$

the sign of which is non-negative with  $C^{*''} \leq 0$ . However, it is generally ambiguous with  $C^{*''} > 0$ . In particular, evaluating (26) at  $q = x^{sf}$ , the sign becomes negative. Thus, as long as the level of a quota is close to the free-trade level, the quota improves foreign welfare.

**Proposition 6** *The monopolist loses from quotas (i.e., a decrease in  $q$ ). A quota lowers foreign welfare if  $C^{*''} \leq 0$ , but improves it if  $C^{*''} > 0$  and if the level of a quota is close to the free-trade level.*

The effect on the total world welfare is given by

$$\frac{d(W+W^*)}{dq} = -\left(D \frac{dp}{dq} + D^* \frac{dp^*}{dq}\right) + \frac{d\Pi^*}{dq}. \quad (30)$$

This is positive if  $C^{*''} \leq 0$ , but could be negative  $C^{*''} > 0$ . When evaluating (30) at  $q = x^{sf}$ , the last term vanishes. Thus, if the level of a quota is close to the free-trade level, the world-welfare effect of a quota is similar to that of a small trade tax [see (24)].

**Proposition 7** *A quota (i.e., a decrease in  $q$ ) improves world welfare only if it lowers the foreign consumer price (i.e.,  $C^{*''} > 0$  holds).*

#### 4. Integrated Markets

In this section, we examine the effects of trade taxes and quotas under integrated markets. In particular, we consider two notions of market integration: the "strong" and "weak" versions. As will be seen below, the spillover effects could arise even with constant MCs. For simplicity, thus, we

assume that  $C^{*''} \geq 0$  in this section<sup>15)</sup>.

##### 4.1 The Effects of Trade Taxes

In the strong version, there are many competitive independent wholesalers. The monopolist must sell all output to the wholesalers at a single price with no notion of where the output will eventually be retailed. There will then be a single producer price for all output. In the case of trade tax, thus, the monopolist maximizes its profits (4) subject to the following constraint:

$$p = p^* + \tau \quad (31)$$

In the weak version, the monopolist controls wholesale and can make direct contracts with domestic and foreign retailers. However, there are independent arbitragers that can buy in one market and sell in the other. In the case of trade tax, arbitrage results in

$$p^* \leq p \leq p^* + \tau. \quad (32)$$

Thus, the monopolist maximizes its profits under this constraint.

##### 4.1.1 The Case of the Strong Version (The Case of $p = p^* + \tau$ )

In the strong version, the profit function can be modified as follows:

$$\Pi^*(p^*; \tau) = [D(p^* + \tau) + D^*(p^*)]p^* - C^*(D(p^* + \tau) + D^*(p^*)). \quad (33)$$

The first-order condition of the profit maximization is

$$\frac{d\Pi^*}{dp^*} = (D + D^*) + (p^* - C^*)(D' + D^{*'}) = 0 \quad (34)$$

The second-order sufficient condition is assumed to be satisfied:

$$2(D' + D^{*'}) - \frac{D + D^*}{D' + D^{*'}}(D'' + D^{*''}) - C^{*''}(D' + D^{*'})^2 \equiv \Gamma < 0. \quad (35)$$

Solving (34) with  $\tau = 0$ , we obtain the free-trade price under integrated markets:

$$p^{lf} = p^{*lf} = \frac{\Theta}{\Theta - 1} C^{*'} \quad \text{where} \quad \Theta \equiv \lambda\theta + (1-\lambda)\theta^* \quad \text{and} \quad \lambda \equiv D/(D + D^*). \quad (36)$$

We let  $(x^{lf}, x^{*lf})$  and  $(x^{lr}, x^{*lr})$  be pairs of supplies to the domestic and foreign markets with free trade and with a trade tax, respectively.

Before analyzing the effects of trade taxes and quotas under integrated markets, the following lemma should be introduced<sup>16)</sup>.

**Lemma 1** *The free-trade price under integrated markets is between the free-trade prices under segmented markets. That is, either  $p^{sf} \leq p^{lf} = p^{*lf} \leq p^{*sf}$  or  $p^{*sf} \leq p^{lf} = p^{*lf} \leq p^{sf}$  holds.*

We first examine the effects of a trade tax on supplies. Using the implicit function

theorem, the effect on the foreign consumer price (i. e., the producer price) is given by

$$\frac{dp^*}{d\tau} = - \frac{\partial^2 \Pi / \partial p^* \partial \tau}{\partial^2 \Pi / \partial p^{*2}} = - \frac{D'[1 - (D' + D^*)C^{**}] - D''(D + D^*) / (D' + D^*)}{\Gamma} \quad (37)$$

Noting the second-order condition (35) (i. e.,  $\Gamma < 0$ ), we obtain the following condition:

$$\frac{dp^*}{d\tau} \geq 0 \Leftrightarrow D'' \geq \frac{D'(D' + D^*)[1 - (D' + D^*)C^{**}]}{D + D^*} \quad (38)$$

As in the case under segmented markets, a trade tax could affect the foreign consumer price. That is, the spillover effects could arise. In sharp contrast to the case under segmented markets, however, this is the case even if the MC is constant. A sufficient condition for  $dp^*/d\tau < 0$  is  $D'' \leq 0$ . That is, if the domestic demand function is concave, a trade tax lowers the foreign price<sup>17</sup>.

The effect of a trade tax on the domestic consumer price is given by

$$\frac{dp}{d\tau} = \frac{d(p^* + \tau)}{d\tau} = \frac{(D' + 2D^*) - D^{**}(D' + D^*)C^{**} - D^{**}(D + D^*) / (D' + D^*)}{\Gamma} \quad (39)$$

Thus, we have

$$\frac{dp}{d\tau} \geq 0 \Leftrightarrow D^{**} \leq \frac{(D' + 2D^*)[(D' + 2D^*) - D^{**}(D' + D^*)C^{**}]}{D + D^*} \quad (40)$$

This implies that a trade tax may or may not raise the domestic consumer price. A trade tax raises the domestic consumer price if the foreign demand function is concave (i. e.,  $D^{**} \leq 0$ ). When  $D^{**} > 0$ , however, the domestic consumer price may fall by a trade tax.

It is instructive to see the equilibrium with the aid of Figure 1 where  $RR$  and  $R^*R^*$ , respectively, shows the first-order conditions (5) and (6) on the price plane. Using the implicit function theorem, the slopes of  $RR$  and  $R^*R^*$  are, respectively, given by

$$\left. \frac{dp^*}{dp} \right|_{RR} = \frac{D'(2 - \varepsilon) - C^{**}(D')^2}{D'D^{**}C^{**}}, \quad (41)$$

$$\left. \frac{dp^*}{dp} \right|_{R^*R^*} = \frac{D'D^{**}C^{**}}{D^{**}(2 - \varepsilon') - C^{**}(D')^2}. \quad (42)$$

Since the numerator of (41) and the denominator of (42) are negative from the second-order conditions, the signs of the slopes depend on  $C^{**}$ . With  $C^{**} > 0$ , the loci are downward sloping. The second-order conditions imply that  $RR$  is steeper than  $R^*R^*$ . When the MC is constant,  $RR$  is vertical and  $R^*R^*$  is horizontal. Iso-profit contours are circular and their slopes are zero along  $RR$  and are infinite along  $R^*R^*$ .

The equilibrium under segmented mar-

kets is indicated by  $S$  where  $RR$  and  $R^*R^*$  intersect to each other. That is, with a given  $\tau$ , the largest profits are realized at  $S$ . The equilibrium in the strong version of market integration must be located on (31). Suppose that  $TT$  shows (31) in the figure. Then, the equilibrium in the strong version is given by  $I$  where an iso-contour is tangent to  $TT$ .

When  $\tau$  increases, both  $RR$  and  $TT$  shift to the right. The effects of raising  $\tau$  on prices can be seen from how the new tangent point between the shifted  $TT$  and one of the new iso-profit contours moves. As was analyzed above, it is not obvious where  $I$  moves when both  $RR$  and  $TT$  shift.

A change in the total output is given by

$$\begin{aligned} \frac{d(x + x^*)}{d\tau} &= (D' + D^*) \frac{dp^*}{d\tau} + D' \\ &= \frac{(D + D^*)(D'D^{**} - D'D^{**}) + D'(D' + D^*)^2}{(D' + D^*)\Gamma}. \end{aligned} \quad (43)$$

The second term of the numerator is negative but the first term takes either sign. If the first term is positive, the sign of (43) could be positive. This is likely to arise when  $D''$  is small (i. e., the domestic demand curve is very concave) and  $D^{**}$  is large (i. e., the foreign demand curve is very convex). If the demand functions (1) and (2) are linear, a trade tax decreases the supply to the domestic market but increases the foreign market. Nevertheless the total supply falls.

**Proposition 8** *A trade tax (i. e., an increase in  $\tau$ ) may or may not decrease the supplies.*

(i) *The supply to the foreign market rises if the domestic demand function is concave.*

(ii) *The supply to the domestic market falls if the foreign demand function is concave.*

(iii) *The total supply falls if the domestic demand function is convex and the foreign function is concave.*

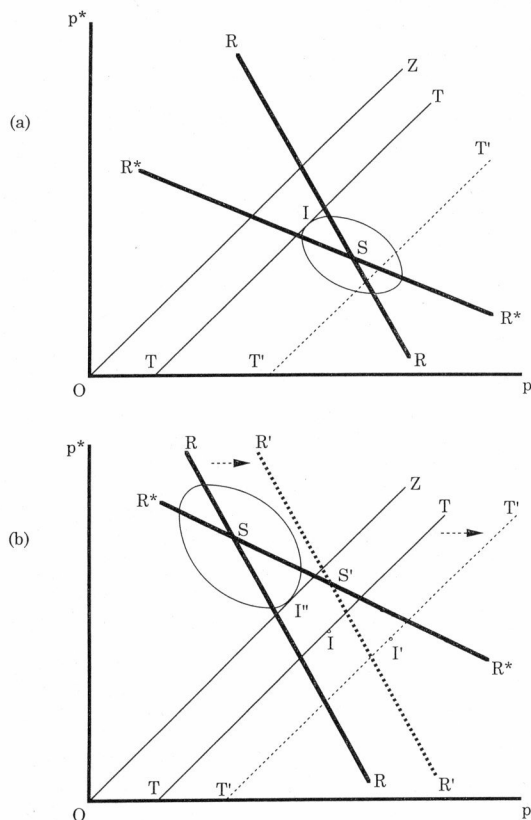
We next examine the effect of a trade tax on the profits and welfare. Using the envelop theorem and (34), we obtain

$$\frac{d\Pi^*}{d\tau} = \frac{\partial \Pi^*}{\partial \tau} = - \frac{D'(D + D^*)}{D' + D^*} < 0. \quad (44)$$

Thus, the monopolist loses from a trade tax.

The effects of a domestic tariff on foreign welfare is as follows. If a tariff raises the foreign price, foreign welfare necessarily deteriorates. On the other hand, if the foreign price falls, foreign welfare may improve. The effects of a tariff on foreign welfare is given by

Figure 1. Trade Taxes under Integrated Markets



$$\begin{aligned} \frac{dW^*}{dt} &= \frac{d\Pi^*}{dt} - D^* \frac{dp^*}{dt} = -\frac{D'(D+D^*)}{D'+D^{**}} - D^* \frac{dp^*}{dt} \\ &= -\frac{D+D^*}{D'+D^{**}} \left[ D' + \frac{D^*}{D+D^*} (D'+D^*) \frac{dp^*}{dt} \right] \quad (45) \end{aligned}$$

the sign of which is generally ambiguous.

Comparing the second line of (45) with (43), it is known that  $dW^*/dt > 0$  only if  $d(x+x^*)/dt > 0$ . In particular, when  $D$  is very small relative to  $D^*$ , a condition for  $dW^*/dt > 0$  is almost identical to that for  $d(x+x^*)/dt > 0$ .

When a foreign export tax is imposed instead of a domestic tariff, we can obtain the effect by differentiating (20) with respect to  $t^*$  and evaluating it at  $t^*=0$ :

$$\begin{aligned} \left. \frac{dW^*}{dt^*} \right|_{t^*=0} &= \left( \frac{d\Pi^*}{dt^*} - D^* \frac{dp^*}{dt^*} + D \right) \Big|_{t^*=0} = \left( \frac{DD'^* - D'D^*}{D'+D^{**}} - D^* \frac{dp^*}{dt^*} \right) \Big|_{t^*=0} \\ &= \left( \left( -\frac{D^*}{D'+D^{**}} \left[ D' + (D'+D^*) \frac{dp^*}{dt^*} \right] + \frac{DD'^*}{D'+D^{**}} \right) \right) \Big|_{t^*=0} \quad (46) \end{aligned}$$

the sign of which is generally ambiguous. If  $(D'/D)|_{t^*=0} > (D'^*/D^*)|_{t^*=0}$  (i. e.,  $\theta|_{t^*=0} > \theta^*|_{t^*=0}$ ) and  $(dp^*/dt^*)|_{t^*=0} < 0$  hold, a small foreign export tax improves foreign welfare.

Comparing the last line of (46) with (43),

it is known that  $(dW^*/dt^*)|_{t^*=0} > 0$  if  $d(x+x^*)/dt^* > 0$ . Thus, in the case of foreign export taxes,  $d(x+x^*)/dt^* > 0$  is sufficient for foreign welfare improvement, while in the case of domestic tariffs,  $d(x+x^*)/dt > 0$  is necessary.

**Proposition 9** A trade tax (i. e., an increase in  $\tau$ ) harms the foreign monopolist but may raise foreign welfare. An increase in the total supply is necessary for the foreign country to benefit from a domestic tariff, while it is sufficient for the foreign country to benefit from a small foreign export tax.

As in the case of segmented markets, we can show that a small tariff can raise domestic welfare. In fact, (18) is independent of market structure and hence still holds under integrated markets. Noting that  $(1 - dp/dt) = -dp^*/dt$  holds in the strong version, it is obvious that  $(dW/dt)|_{t=0} > 0$  if and only if  $(dp^*/dt)|_{t=0} < 0$ .

In the case of foreign export taxes, noting that there is no tax revenue in the domestic country, it is obvious that domestic welfare depends whether the domestic consumer price rises or falls.

**Proposition 10** A small domestic tariff raises domestic welfare if and only if it decreases the foreign consumer price. A foreign export tax (i. e., an increase in  $t^*$ ) deteriorates domestic welfare if and only if it makes the domestic consumer price higher.

The effect on world welfare is as follows. Noting that  $(1 - dp/dt) = -dp^*/dt$ , it is known from (45) and (46) that the effect of a small trade tax on world welfare is given by

$$\begin{aligned} \left. \frac{d(W+W^*)}{dt} \right|_{\tau=0} &= -\left( \frac{D+D^*}{D'+D^{**}} \left[ D' + (D'+D^*) \frac{dp^*}{dt} \right] \right) \Big|_{\tau=0} \\ &= -\left( \frac{D+D^*}{D'+D^{**}} \left[ \frac{d(x+x^*)}{dt} \right] \right) \Big|_{\tau=0} \quad (47) \end{aligned}$$

where the last equation is obtained from (43).

**Proposition 11** A small trade tax raises the total world welfare if and only if it increases the total supply.

#### 4.1.2 The Case of the Weak Version

There are three cases to analyze in the case of the weak version. We find the three cases using Figure 1. In the figure, one of the constraints (32),  $p^* \leq p$ , is satisfied in the area on and below the 45 degree line labeled  $OZ$ . The other constraint,  $p \leq p^* + \tau$ , is satisfied in the area on and above the line, (31), labeled  $TT$  (or  $T'T'$ ). That is, the constraint is satisfied in the area between  $OZ$  and (31),

which is referred to as Region I. Moreover, the area below (31) and the area above  $OZ$  are, respectively, called Region II and Region III.

Depending on the location of  $S$ , the following three cases must be considered<sup>18</sup>.

**Case I:** Suppose that (31) is given by  $T'T'$  in Figure 1 (a) and  $S$  is in Region I. Then, the equilibrium under the segment markets satisfies (32) and hence no arbitrage exists even if arbitrage is allowed. In this case, the equilibrium under segmented markets coincides with that under integrated markets. Thus, the effects of trade taxes are the same as those in the case under segmented markets.

**Case II:** Suppose that (31) is given by  $TT$  in Figure 1 (a) and  $S$  is in Region II. Then  $p^{*sr} + \tau < p^{sr}$  holds and hence arbitrage from the foreign market to the domestic market is profitable. Thus, the monopolist sets the price such that  $p = p^*$ , which does not lead to any arbitrage. In the figure, the equilibrium is given by  $I$  where an iso-profit contour is tangent to  $TT$ . In fact, this is the case of the strong version, which has been examined above.

**Case III:** Suppose that the equilibrium under segmented markets is located above the 45 degree line as in Figure 1 (b) and  $S$  is in Region III. In this case, the good is priced by the monopolist such that  $p = p^*$ . In the figure, the equilibrium is given by  $I''$  where an iso-profit contour is tangent to the 45 degree line. This case (i. e., the case of  $p = p^*$ ) is analyzed below.

Before examining Case III, however, we should note that as a trade tax rises,  $S$  could shift from one region to others<sup>19</sup>. For example, even if  $S$  is originally in Region III (i. e., Case III originally holds), raising  $\tau$  eventually shifts  $S$  to Region I and hence Case I arises. This is because an increase in trade tax raises the domestic consumer price under segmented markets but either decreases or does not change the foreign consumer price under the segmented markets [see (13)].

This may be seen in Figure 1 (b) more easily. As  $\tau$  increases, both  $RR$  and  $TT$  shift to the right. Thus, a high trade tax certainly leads  $S$  to move from Region III to Region I. In the figure, the new equilibrium is  $S'$ . A further increase in the trade tax could lead  $S'$  to move from Region I to Region II and hence

Case II could arise. This could be the case when the shift of  $RR$  is larger than that of  $TT$ .

Whether a further increase in the trade tax results in Case II may be seen from the sign of (19). If it is negative for any  $t$ , that is, if an increase in the domestic consumer price is greater than an increase in the trade tax,  $S$  eventually moves to Region II. If the sign of (19) is positive for any  $t$ , on the other hand,  $S$  remains to be in Region I. In this case, however, we should note that whether  $S$  is originally in Region II or Region III, raising  $\tau$  eventually results in Case I.

**Lemma 2** *Increasing a trade tax eventually results in either Case I or Case II.*

In Case III (i. e., the case of  $p = p^*$ ), the profit function (4) is modified as follows:

$$\Pi^*(p^*; \tau) = (p^* - \tau)D(p^*) + p^*D^*(p^*) - C^*(D(p^*) + D^*(p^*)). \quad (48)$$

Then the first-order condition of the profit maximization is

$$\begin{aligned} \frac{d\Pi^*}{dp^*} &= (D + D^*) + D'(p^* - \tau - C^*) \\ &\quad + D^{*'}(p^* - C^*) = 0. \end{aligned} \quad (49)$$

The second-order sufficient condition is assumed to be satisfied:

$$\begin{aligned} 2(D' + D^{*'}) + D''(p^* - \tau - C^*) \\ + D^{*''}(p^* - C^*) - C^{*''}(D' + D^{*'})^2 \\ \equiv \Lambda < 0. \end{aligned} \quad (50)$$

Using the implicit function theorem, the effect on the consumer price is given by

$$\frac{dp^*}{d\tau} = -\frac{\partial^2 \Pi / \partial p^* \partial \tau}{\partial^2 \Pi / \partial p^{*2}} = \frac{D'}{\Lambda} > 0. \quad (51)$$

**Proposition 12** *A trade tax (i. e., an increase in  $\tau$ ) decreases the supplies to both markets and the total supply in Case III (i. e., with  $p = p^*$ ).*

We next examine the effect of a trade tax on the profits and welfare. Using the envelop theorem and (49), we can show that the monopolist loses from a trade tax:

$$\frac{d\Pi^*}{d\tau} = \frac{\partial \Pi^*}{\partial \tau} = -D < 0. \quad (52)$$

Thus, in view of Proposition 12, a domestic tariff deteriorates foreign welfare. In the case of foreign export taxes, we have

$$\frac{dW^*}{dt^*} = -D^* \frac{dp^*}{dt^*} + t^* \frac{dD}{dt^*} < 0, \quad (53)$$

where both terms are negative. When (53) is evaluated at  $t^* = 0$ , the second term vanishes.

**Proposition 13** *Trade taxes (i. e., an increase in  $\tau$ ) harm the monopolist and foreign consumers. Foreign welfare deteriorates*



with  $p=p^*$ .

In view of (18), domestic welfare in the case of domestic tariffs improves if the following holds:

$$\left(1 - \frac{dp^*}{dt}\right) \Big|_{t=0} = \frac{(D'+2D'') + D''(p^*-t-C'') + D''(p^*-C'') - C''(D'+D'')^2}{\Lambda} \Big|_{t=0} > 0. \quad (54)$$

A sufficient condition for welfare improvement is that  $D'' \leq 0$  and  $D^{*''} \leq 0$ , that is, both demand curves are concave. Moreover, since  $p=p^*$ , it is known from (51) that a foreign export tax is harmful for the domestic country.

**Proposition 14** *A small domestic tariff improves domestic welfare if both demand curves are concave. A foreign export tax (i. e., an increase in  $t^*$ ) reduces domestic welfare with  $p=p^*$ .*

Both countries lose from a small foreign export tax. Thus, a small export tax reduces the total world welfare. Moreover, a small export tax and a small import tax have the same effect on world welfare.

**Proposition 15** *A small trade tax necessarily reduces world welfare.*

## 4.2 The Effects of Quotas

In this subsection, we examine the effects of import and export quotas under integrated markets. Under integrated markets, the effects of quotas crucially depend on how quotas are implemented. We consider quotas on the basis of "first come, first served". We assume that the government sets a quota and then the monopolist decides the amount of exports. If the quota is not fully used by the monopolist, room for arbitrage arises.

### 4.2.1 The Case of the Weak Version

In order to examine the effects of quotas on economies, we first consider the effects of a quota imposed at the level of free trade and then the effects of reducing the quota level.

We let  $p^{*Sq}$  denote the foreign price that maximizes the profits when the domestic price is fixed at  $p^q \equiv D^{-1}(q)$ . When a quota is imposed at the free-trade level, we have  $q = x^{If}$  and  $p^q = p^{If}$ . First, suppose that  $p^{*Sf} < p^{Sf}$ . This case is shown in Figure 2 (a). The free-trade equilibrium under segmented markets is given by S. Noting  $p=p^*$  under integrated markets, the free-trade equilibrium under integrated markets is indicated by I where an iso-profit contour is tangent to the 45 degree line. We should note that  $p^{*Sq}$  is given by  $R^*R^*$ . When a quota is imposed at the free-trade level, the monopolist chooses the prices indicated by Q. Noting that the slopes of an

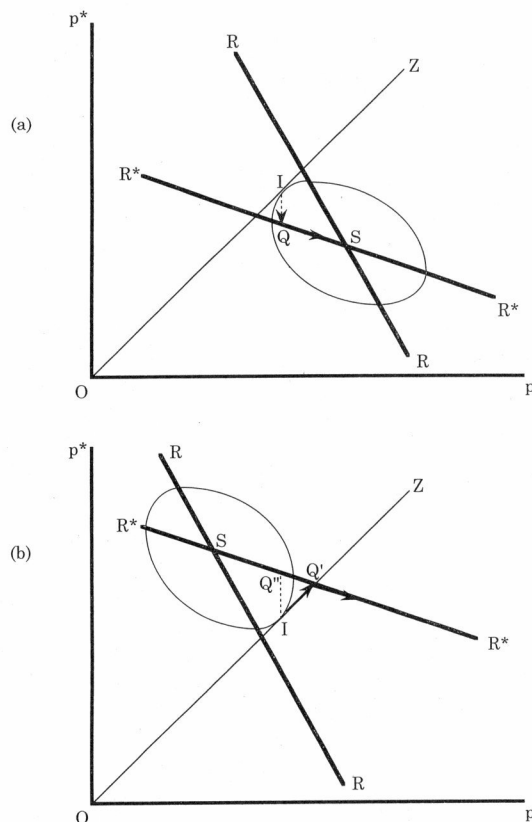
iso-profit contour are zero along  $RR$  and infinite along  $R^*R^*$ ,  $p^{*Sq} < p^{*If}$  holds. If the monopolist sets the foreign price at  $p^{*Sq}$ , then arbitrage from the foreign market to the domestic market is profitable. However, the arbitrage cannot arise because of the quota. In this case, thus, the monopolist sets different prices between the domestic and foreign markets. That is, the monopolist sets the domestic price such that the quota is just binding and the foreign price such that the profits are maximized with  $p^q$ . It should be noted that although there is no effect on the domestic country, both monopolist and foreign consumers clearly gain from the quota and hence foreign welfare rises.

Next suppose that  $p^{*Sf} > p^{Sf}$ . This case is shown in Figure 2 (b). In this case,  $p^{*Sq} > p^{*If}$  holds. When a quota is imposed at the free-trade level, the best prices for the monopolist is given by  $Q''$ . If the monopolist sets these prices, however, arbitrage from the domestic market to the foreign market actually arises. The largest profits without arbitrage is still indicated by I and hence the monopolist imposes a single consumer price,  $p^{If}$ , in both markets.

**Proposition 16** *In the weak version of market integration, a quota imposed at the free-trade level has no effects at all if  $p^{*Sf} \geq p^{Sf}$ ; but lowers the foreign price alone and benefits both monopolist and foreign consumers if  $p^{*Sf} < p^{Sf}$ .*

Next, we examine the effects of reducing the quota level from the free-trade level. First, suppose that  $p^{*Sf} < p^{Sf}$  (i. e.,  $x^{Sf} < x^{If}$ ). It is known from (27) that a decrease in the quota level increases the supply to the foreign market if  $C^{*''} > 0$  but does not change it (which is actually equal to  $x^{*Sf}$ ) if  $C^{*''} = 0$ . In Figure 2 (a), as the quota level decreases from the free-trade level,  $p^q$  rises and hence the equilibrium moves along  $R^*R^*$  from Q toward S. Thus, domestic consumers lose, while foreign consumers gain with  $C^{*''} > 0$  and remain indifferent with  $C^{*''} = 0$ . In view of (28), the total supply rises if and only if both  $C^{*''} > 0$  and  $\epsilon^* > 2$  hold. The profit function is given by (25) in this case. Using the envelop theorem, we have  $d\Pi^*/dq = \partial\Pi^*/\partial q < 0$  for  $q > x^{Sf}$  but  $d\Pi^*/dq = \partial\Pi^*/\partial q > 0$  for  $q < x^{Sf}$ . In terms of Figure 2 (a), between Q and S, reducing the quota raises the profits. Once the equilibrium point goes beyond S, however,

Figure 2. Quotas under Integrated Markets:  
the Weak Version



the profits start to fall. Thus, as long as the quota level is close to the free-trade level, the monopolist gains from the quota. It should be noted that once  $q = x^{sf}$  holds, the effects of a further decrease in the quota level is identical to those in the case of segmented markets.

Second, suppose that  $p^{*sf} > p^{sf}$  (i. e.,  $x^{sf} > x^{lf}$ ). Then, as long as  $p^{*sq} > p^q$ , the prices in both markets are equal to  $p^q$ . In terms of Figure 2 (b), this corresponds to the movement of equilibrium point from  $I$  toward  $Q'$  along the 45 degree line. The total supply decreases as the quota level falls. As far as both consumer prices are equalized, the profits are maximized under free trade. Thus, the profits also decrease as the quota level falls. Once  $p^{*sq} = p^q$  holds, that is, once the equilibrium point reaches  $Q'$ , it is better for the monopolist to set the prices along  $R^*R^*$ , because arbitrage cannot arise below  $OZ$  and  $R^*R^*$  gives the largest profits for any given  $p^{20}$ . However, since the equilibrium becomes

further from  $S$ , the profits continue to decrease. It can be checked from (28) whether the total supply rises or falls.

Whether  $p^{*sf} < p^{sf}$  or  $p^{*sf} > p^{sf}$  is crucial for the results.

**Proposition 17** Suppose that  $p^{*sf} < p^{sf}$ . As the quota level lowers, foreign consumers become better off with  $C^{**} > 0$  but remain indifferent with  $C^{**} = 0$ ; and the profits of the monopolist increase until  $q = x^{sf}$  (i. e.,  $p^q = p^{sf}$ ) holds and then start to decrease. Unless the quota is very restrictive, the foreign country gains from reducing the quota level. Suppose that  $p^{*sf} > p^{sf}$ . As the quota level lowers, the monopolist becomes worse off; and the foreign consumer price rises until  $p^q = p^{*sq}$  holds and then starts to fall (resp. remains constant) with  $C^{**} > 0$  (resp. with  $C^{**} = 0$ ). Unless the quota is very restrictive, the foreign country loses from reducing the quota level.

To see the effects on domestic welfare, we should note that the domestic price is equal to the price charged by the monopolist (i. e., the producer price) in the above analysis. That is, the domestic country cannot capture any rent of import quota (as well as export quota) with foreign monopoly and hence the equivalence between import and export quotas is still valid.

With respect to the effects on world welfare of reducing quota level from the free-trade level, (30) still holds here. The first term in the parentheses is always negative. If  $p^{*sf} < p^{sf}$  holds, the last term is negative for  $q < x^{sf}$ . In comparison with the case under segmented case, this makes an improvement of world welfare more likely<sup>21</sup>. If  $p^{*sf} > p^{sf}$ , on the other hand, the last term is always positive and, regardless of  $C^{**} > 0$ , the second term in the parentheses is negative for  $p^q < p^{*sq}$ . Thus, world welfare deteriorates if a quota level is close to the free trade level.

**Proposition 18** With  $p^{*sf} < p^{sf}$ , a decrease in the quota level raises world welfare only if  $C^{**} > 0$ . With  $p^{*sf} > p^{sf}$ , a decrease in the quota level reduces world welfare unless it is very restrictive.

#### 4.2.2 The Case of the Strong Version

In the strong version, the monopolist sells the good to competitive independent wholesalers at a single price. Although the implementation of quotas in our study is different from that in Krishna (1991), our analysis in the strong version is basically the same as

hers. This is partly because the price of a quota license, which is the main focus of her analysis, equals zero unless the quota is very restrictive; and partly because, in contrast to the case of the weak version, non-constant MCs are not very crucial for the results. In our analysis, however, we provide a diagrammatic method different from Krishna (1990)<sup>22</sup>.

First, suppose that the level of quota is simply added to the foreign demand and the monopolist sets a single price,  $p^*$ . Then the profit function is given by

$$\Pi^*(p^*; q) = [D^*(p^*) + q]p^* - C^*(D^*(p^*) + q). \quad (55)$$

The first-order condition for this profit maximization is

$$\frac{d\Pi^*}{dp^*} = (D^* + q) + (p^* - C^*)D^* = 0. \quad (56)$$

The relationship between  $p^*$  and  $q$  obtained from (56) is drawn in the second quadrant of Figure 3. Point  $M$  corresponds to the monopoly price when the monopolist supplies to the foreign market alone<sup>23</sup>.

$DD'$  in the third quadrant shows the relationship:  $p = D(q)$ . If the domestic consumer price is higher than  $D(q)$ , the quota is not fulfilled. Thus,  $QQ'$  in the first quadrant gives the combination of the domestic and foreign consumer prices which is consistent with the maximization of (55) and the full utilization of quota.

As in the weak version, arbitrage arises from the domestic country to the foreign country if the consumer prices are located above the 45 degree line  $OZ$ . Thus, the combination of the domestic and foreign consumer prices must be located on or below  $OZ$ . Noting this, we need to consider two cases. One is the case where the level of quota is greater than  $q_c$  which is determined by the intersection between  $QQ'$  and  $OZ$ . The other is the case where it is less than  $q_c$ .

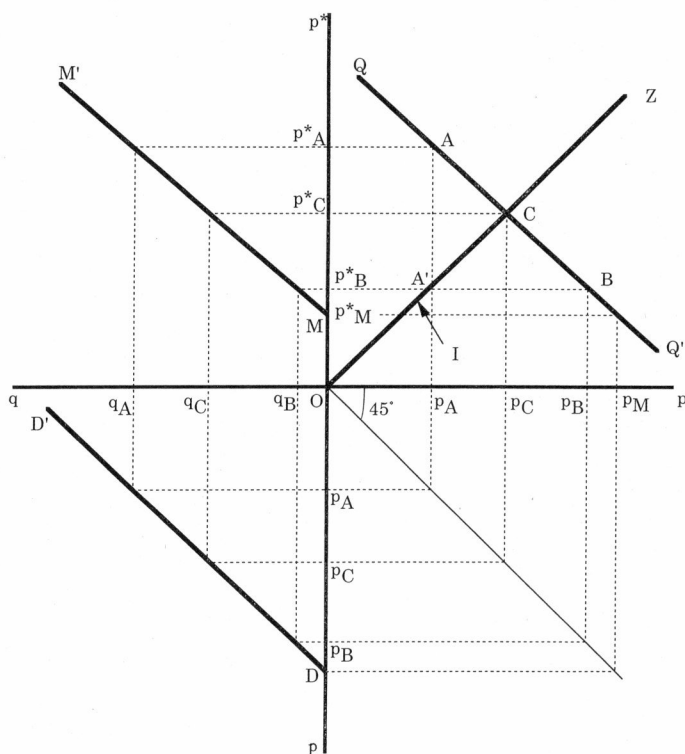
First, suppose that the

level of quota is less than  $q_c$ , say,  $q_A$ . Then, the monopolist wants to charge the producer price at  $p^*_A$ . However, the domestic consumer price which fulfills the quota is  $p_A$ . Since  $A$  is located above  $OZ$ , the profits are not maximized in the presence of arbitrage. With  $p = p_A$ , the profits decrease as  $p^*$  falls from  $p^*_A$ . Taking arbitrage into account, thus,  $A'$  (i.e.,  $p^*_B$ ) gives the largest profits. In this case, the producer price is equal to the domestic consumer price as well as the foreign consumer price.

Next suppose that the level of quota is  $q_B$ . Then  $B$  gives the optimal combination of the producer price and the domestic consumer price. In this case, the domestic consumer price is higher than the producer price. That is, the quota rent which is equal to the gap between  $p^*_B$  and  $p_B$  does not accrue to the monopolist. It should be emphasized that the presence of this quota rent is not observed in the weak version of market integration as well as in the segmented-markets case.

In sum, as the quota becomes tighter from the free-trade level, the producer and the

Figure 3. Quotas under Integrated Markets: the Strong Version



**Table 1. The Effects of Trade Taxes under Segmented Markets**

		tariffs	export taxes	quotas
$x$		↓	↓	↓
$x^*$	$C^{**} > 0$	↑	↑	↑
	$C^{**} = 0$	0	0	0
	$C^{**} < 0$	↓	↓	↓
$x + x^*$	$C^{**} > 0$	↑ iff $\varepsilon^* > 2$	↑ iff $\varepsilon^* > 2$	↑ iff $\varepsilon^* > 2$
	$C^{**} = 0$	↓	↓	↓
	$C^{**} < 0$	↓	↓	↓
$W$	$C^{**} > 0$	↑ if $\varepsilon < 1$ & $\varepsilon^* < 2$	↓	↓
	$C^{**} = 0$	↑ iff $\varepsilon < 1$	↓	↓
	$C^{**} < 0$	↑ only if $\varepsilon < 1$	↓	↓
$\Pi^*$		↓	↓	↓
$W^*$	$C^{**} > 0$	↑ if $\varepsilon > 2$ , $\varepsilon^* \geq 0$ & $x^* > 2x$	↑	↑
	$C^{**} = 0$	↓	0	0
	$C^{**} < 0$	↓	↓	↓
$W + W^*$		↑ only if $C^{**} > 0$	↑ only if $C^{**} > 0$	↑ only if $C^{**} > 0$

Notes) The effects of a tariff on domestic welfare are evaluated at  $t=0$ .

The effects of an export tax on foreign welfare are evaluated at  $t^*=0$ .

The effects of trade taxes on world welfare are evaluated at  $\tau=0$ .

The effects of a quota on foreign welfare are evaluated at the free-trade level.

**Table 2. The Effects of Trade Taxes under Integrated Markets**

	$p = p^* + \tau$		$p = p^*$	
	tariffs	export taxes	tariffs	export taxes
$x$	↓ if $\varepsilon^* \leq 0$	↓ if $\varepsilon^* \leq 0$	↓	↓
$x^*$	↑ if $\varepsilon \leq 0$	↑ if $\varepsilon \leq 0$	↓	↓
$x + x^*$	↓ if $\varepsilon \geq 0$ & $\varepsilon^* \leq 0$	↓ if $\varepsilon \geq 0$ & $\varepsilon^* \leq 0$	↓	↓
$W$	↑ iff $x^* \uparrow$	↑ iff $x \uparrow$	↑ if $\varepsilon, \varepsilon^* \leq 0$	↓
$\Pi^*$	↓	↓	↓	↓
$W^*$	↑ only if $(x + x^*) \uparrow$	↑ if $(x + x^*) \uparrow$	↓	↓
$W + W^*$	↑ iff $(x + x^*) \uparrow$	↑ iff $(x + x^*) \uparrow$	↓	↓

Notes) The effects of a tariff on domestic welfare are evaluated at  $t=0$ .

The effects of an export tax on foreign welfare are evaluated at  $t^*=0$ .

The effects of trade taxes on world welfare are evaluated at  $\tau=0$ .

foreign consumer prices first rise along  $OZ$  from the free trade equilibrium  $I$  to  $C$  and then fall along  $OZ$  until  $p^* = p_M^*$  holds. The domestic consumer price coincides with the producer price up to point  $C$  and then moves long  $CQ'$  until  $p = p_M$  holds (i.e., domestic demand becomes zero).

**Proposition 19** *As the quota level falls from the free-trade level, the domestic consumer price monotonically rises. The producer price and the foreign consumer price first rise and then start to fall.*

Any quota harms domestic consumers. Foreign consumers could gain if a quota

makes the foreign consumer price lower. The monopolist loses, because with a single producer price, the profits are maximized at the free-trade equilibrium. Thus, as long as the quota level is close to the free-trade level, no one gains from decreasing  $q$ . When the quota level is small (in the sense that it is less than  $q_c$  in Figure 3), those wholesalers who export the good to the domestic country gain. If those wholesalers are foreigners, that is, if all quota rent accrues to the foreign country, the foreign country may gain from reducing  $q$  but the domestic country necessarily loses.

If some of the quota rent accrues to the domestic country, on the other hand, the domestic country may gain<sup>24</sup>. World welfare necessarily deteriorates because the total output necessarily lowers.

**Proposition 20** *As the quota level falls from the free-trade level, domestic consumers and the monopolist necessarily lose; and foreign consumers first lose and then start to gain. When wholesalers can get some quota rent, a decrease in the quota level may benefit one of the countries. World welfare never improves.*

## 5. Concluding Remarks

Using a single model with general demand and cost specifications, we have examined the effects of trade taxes/subsidies and quotas on domestic, foreign and world economies under both segmented and integrated markets. Although we have focused on a monopoly model, our study has provided fairly general structures and made the systematic comparisons among various trade policies under various circumstances possible. In particular, two versions of market integra-

**Table 3. The Effects of Quotas under Integrated Markets :  
the Weak Version**

	quota imposed at free trade level	a small decrease in quota from free-trade level
$x^*$ $p^{sf} > p^{*sf}$ $p^{sf} < p^{*sf}$	$\uparrow$ 0	$\uparrow$ wiht $C^{**} > 0$ ; 0 wiht $C^{**} = 0$ $\downarrow$
$x + x^*$ $p^{sf} > p^{*sf}$ $p^{sf} < p^{*sf}$	$\uparrow$ 0	$\uparrow$ iff $\epsilon^* > 2$ wiht $C^{**} > 0$ ; $\downarrow$ wiht $C^{**} = 0$ $\downarrow$
$W$	0	$\downarrow$
$\Pi^*$ $p^{sf} > p^{*sf}$ $p^{sf} < p^{*sf}$	$\uparrow$ 0	$\uparrow$ $\downarrow$
$W^*$ $p^{sf} > p^{*sf}$ $p^{sf} < p^{*sf}$	$\uparrow$ 0	$\uparrow$ $\downarrow$
$W + W^*$ $p^{sf} > p^{*sf}$ $p^{sf} < p^{*sf}$	$\uparrow$ 0	$\uparrow$ only if $C^{**} > 0$ $\downarrow$

**Table 4. The Effects of Quotas under  
Integrated Markets : the Strong  
Version**

	a (small) decrease in quota level from $q^*$
$x^*$ $q^* > q_c$ $q^* < q_c$	$\downarrow$ $\uparrow$
$x + x^*$	$\downarrow$
$W$ $q^* > q_c$ $q^* < q_c$	$\downarrow$ ?
$\Pi^*$	$\downarrow$
$W^*$ $q^* > q_c$ $q^* < q_c$	$\downarrow$ ?
$W + W^*$ $q^* > q_c$ $q^* < q_c$	$\downarrow$ $\downarrow$

tion have been examined under integrated markets. The results are summarized in Tables 1-4.

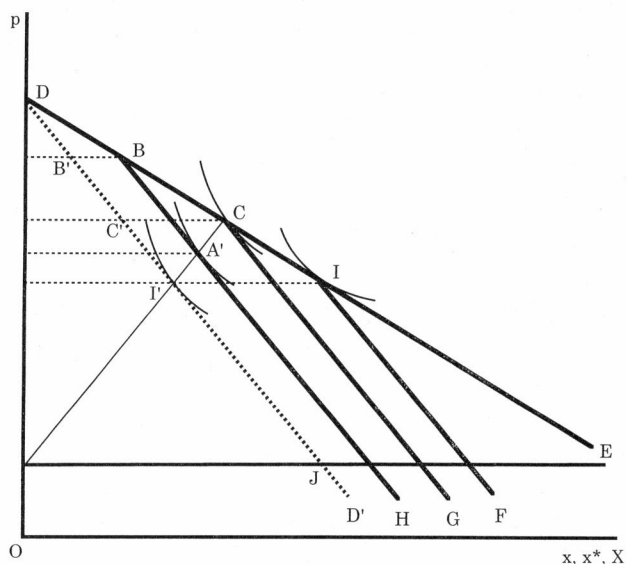
General conclusions obtained in our analysis are as follows. First, the curvature of demand curves is crucial for many results. This has not been pointed out in the previous literature that is closely related to the present study. Second, the domestic government may be able to extract some monopoly rents by using tariffs but is likely to fail to capture quota rents. From the point of view of domestic welfare, thus, trade taxes are more likely to be favorable than quotas. Finally, trade policies taken by one country may benefit the other country and/or the world economy. Since most of the existing works ignore the spillover effects, few studies have examined

this point.

The analysis under integrated markets is more complicated than that under segmented markets. In contrast to the segmented-markets case, the integrated markets lead to the spillover effects even with constant MCs; and also makes the effects of trade taxes different from those of quotas.

Furthermore, the results in the strong version of market integration contrast with those the weak version. The case of trade taxes in the strong version is included in that in the weak version. With

**Figure 4. Alternative Diagrammatic Method**



respect to the effects of quotas, it is shown that in the strong version, quota could lead to a wedge between the producer price and the domestic consumer price and hence quota rents could accrue to the domestic country; and that a quota could benefit the monopolist in the weak version.

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#### Appendix

This appendix provides an alternative diagrammatic method (Figure 4) to examine the effects of quota in the strong version of market integration.

For simplicity, Figure 4 is drawn under assump-



tions that two countries have the identical linear demand functions and that the MC is constant. In the figure, the demand curve for each market is  $DD'$  and hence the total demand is given by  $DE$ . To find the free-trade equilibrium, we draw iso-profit curves on the diagram. With a constant MC,  $c$ , the iso-profit curve which gives a profit level  $\Pi_0$  is defined by the following hyperbola:

$$p = c + \frac{\Pi_0}{X}; \text{ where } X \equiv x + x^*$$

The free-trade equilibrium is given by  $I$  where an iso-profit curve is tangent to the total demand curve. First, we consider a case where a quota is imposed at the free-trade level that is equal to  $II'$ . The quota results in a kink on the total demand curve at  $I$ . That is, the total demand curve becomes  $DIF$ . It is obvious that the iso-profit curve that goes through  $I$  still gives the largest profits along  $DIF$ . Thus, it remains optimal for the monopolist to choose the total supply and the price indicated by  $I$ .

Next we consider reducing the quota from the free-trade level. As the quota level falls, the point of kink moves towards  $D$  along  $DE$ .  $C$  is the point where an iso-profit curve is tangent to  $CG$  extended. It should be noted that the level of quota  $CC'$  corresponds to  $q_c$  in Figure 3. If the quota is reduced further, the largest profits are given by a point on the steeper segment of the total demand curve. If the total demand curve is  $DBH$ , for example,  $A'$  (on  $BH$ ) gives the largest profits. It should be noted that the quota level with the total demand  $DBH$  is  $BB'$ . Thus, the domestic consumer price given by  $B$  is higher than the producer price and the foreign consumer price given by  $A'$ . The difference between the price charged by the monopolist and the domestic consumer price accrues to the wholesalers as rents.

When the quota level is zero,  $I'$  (where an iso-profit curve is tangent to the foreign demand curve,  $DD'$ ) is the equilibrium point. Since both countries have the identical linear demand and the MC is constant, the price charged by the monopolist at  $I$  is equal to that at  $I'$ .

In sum, if the quota level is between  $II'$  and  $CC'$ , the monopolist chooses the total supply and the price indicated by the point of kink. In this case, the domestic and foreign consumer prices are equal and rise as the quota level decreases. If the quota level is less than  $CC'$ , on the other hand, the monopolist chooses the total supply and the price indicated by the point where an iso-profit curve is tangent to the steeper segment of the total demand curve. As the quota is reduced further, the domestic consumer price becomes higher but the foreign consumer price becomes lower. The locus of points chosen by the monopolist is  $ICI'$ . The level of the quota is given by the horizontal distance between  $ICI'$  and the foreign

demand curve,  $DD'$ .

## Notes

\* I would like to thank Kazuharu Kiyono and the seminar participants at Hitotsubashi University and Niigata University for comments and suggestions. All remaining errors are my own responsibility.

1) The literature is surveyed in Helpman and Krugman (1989), and Brander (1995), among others.

2) If entry and exit are allowed, the spillover effect arises even with constant MCs. See Venables (1985).

3) A notable exception is Krugman (1984) which shows that import protection may promote exports with decreasing MCs. Okuguchi and Serizawa (1996) have extended his analysis.

4) Studies under integrated markets include Auquier and Caves (1979), Markusen and Venables (1988), Jones and Takemori (1989), and Krishna (1990, 1991).

5) I am grateful to Jim Markusen for pointing me out the two versions.

6) Auquier and Caves (1979), Markusen and Venables (1988), Krishna (1990), and Tanaka (1991, 1992) adopt the strong version, while Jones and Takemori (1989) and Krishna (1991) adopt the weak version.

7) Katrak (1977) assumes linear demand and cost functions. Brander and Spencer (1984) use general demand and cost functions but focus on the domestic market.

8) With respect to the relationship between the two elasticities, see footnote 12.

9) As far as I know, Krishna (1990, 1991) and Tanaka (1991) are the only studies that investigate quotas under imperfect competition and integrated markets. Tanaka's analysis seems unsatisfactory, because he simply assumes that the imposition of VERs makes integrated markets segmented ones.

10) Constant MCs are assumed in Krishna (1990, 1991).

11)  $\epsilon > 2$  implies that the slope of the marginal revenue curve is positive.

12) Note that  $\theta$  is not necessarily assumed to be constant in this paper. When  $\theta$  is constant as in Auquier and Caves (1979),  $\epsilon = 1 + 1/\theta$  holds. Thus,  $\epsilon$  is greater than 1 if  $\theta$  is assumed to be constant.

13) Brander and Spencer (1984b) have extended Brander and Spencer (1984a) to an oligopoly model with cross-hauling. However, constant MCs are assumed.

14) This result is also obtained in Auquier and Caves (1979).

15) Ishikawa (2000) deals with the case of  $C^{**} < 0$  in a different context.

16) The proof of Lemma 1 is found in Ishikawa

(2000).

17) In an oligopoly model with free entry, Tanaka (1992) shows that a specific tariff lowers the producer price if the domestic demand function is *strictly* concave.

18) Which case actually arises depends on the demand elasticities, the size of tax and MCs. Jones and Takemori (1989) consider this question when the producer price (i.e., the foreign consumer price) is constant but the income effect exists.

19) It is implicitly assumed that an increase in the trade tax does not result in the prohibitive level.

20) For any quota less than this level,  $p^* = p^{*sf}$  holds if the MC is constant.

21) It can easily be verified that world welfare deteriorates if  $C^* = 0$  and hence the foreign consumer price remains constant.

22) Another diagrammatic method, is given in Appendix. See also Ishikawa (1997).

23) If the MC is constant, this price equals  $p^{*sf}$ .

24) In Krishna (1990), the quota rent always accrues to the domestic government, because quota is auctioned off by the domestic government.

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