

Welfare Enhancing Tariff War in Duopoly with Product Differentiation*

Yasuhito Tanaka

I. Introduction

This paper examines the welfare of countries involved in a tariff war between two countries in an international duopoly with differentiated products. Tariff war describes a situation where both countries impose optimum tariffs on imported goods. The optimum tariff is the tariff level in the country which maximizes its welfare given the tariff level in the other country. Tariff war means a Nash equilibrium of a non-cooperative tariff game. Employing a model with linear demand and cost functions, I shall show that if, and only if, the duopolistic goods are substitutes, and the value of the transportation cost satisfies certain conditions, the welfare of both countries in an equilibrium of a non-cooperative tariff game with positive tariffs is higher than in a free-trade equilibrium. That is, a tariff war situation with positive tariffs is Pareto superior to free trade.

Johnson(1953) and Hamilton and Whalley(1983), for some numerical examples, illustrate the possibility that one country, not both, is better off in a Nash equilibrium of a non-cooperative tariff game than in a free-trade equilibrium under

perfect competition. Gros(1987) examines a Nash equilibrium of a non-cooperative tariff game in a monopolistic competition model, according to Krugman(1980). He shows that, also using a numerical example, one country, not both, may benefit in a Nash equilibrium of a non-cooperative tariff game, but does not consider the possibility that a tariff war could be Pareto superior to free trade.

On the other hand, Brander and Krugman(1983) show that in an intra-industry trade model with two countries under duopoly, prohibitive tariffs may be beneficial for both countries if transportation costs are high.

In the next section, I will present the model, and consider the optimum tariff for each country. In section III, I shall show that if the duopolistic goods are substitutes, and the value of the transportation cost satisfies certain conditions, a tariff war situation with positive tariffs is Pareto superior to free trade. In section IV, I shall show that the main cause of the welfare-enhancing effects of a tariff war is savings in transportation costs due to contraction of the volume of trade through tariffs. Section V concludes this paper.

II. The Model and the Optimum Tariffs

I consider an intra-industry trade model in a differentiated duopoly with linear demand and cost functions which is analogous to the models in recent studies by Dixit(1988) and Cheng(1988)¹⁾. Such a

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model is specific. But with general demand and cost functions, we cannot explicitly compare the welfare of countries in a free-trade equilibrium and a Nash equilibrium of a non-cooperative tariff game.

Suppose a world consisting of two countries, called home and foreign, and a duopolistic industry with one firm in each country. The two countries and the two firms are symmetrical. Consumers in both countries have the same utility functions, and the firms have the same cost functions, and they are Nash-Cournot duopolists.

I assume that the markets in the two countries are segmented, as assumed by Brander(1981), Brander and Krugman (1983), Brander and Spencer(1984), Dixit (1984), and Venables(1985). Markets are segmented when firms are permitted to select strategies for each national market. Under segmented markets, prices of goods in one country are not necessarily equal to those in the other country²⁾.

Denote the prices of the good of the home firm (called the home good) and the good of the foreign firm (called the foreign good) in the home country by, respectively, p and q . The inverse demand functions for the home and foreign goods in the home country are represented as, respectively,

$$p = a - b(x + ky)$$

and

$$q = a - b(kx + y)$$

where x and y are the quantities of the home and foreign goods in the home country.

The utility function which yields these demand functions is

$$u(x, y) = a(x + y) - \frac{1}{2}b(x^2 + 2kxy + y^2)$$

k denotes the degree of product differentiation, and $-1 \leq k \leq 1$. The marginal utility of the home good is

$$\frac{\partial u}{\partial x} = a - b(x + ky)$$

If $0 < k \leq 1$, an increase (or a decrease) in y lowers (or raises) the marginal utility of the home good. On the other hand if $-1 \leq k < 0$, an increase (or a decrease) in y raises (or lowers) the marginal utility of the home good. So if $0 < k \leq 1$, the goods are substitutes; and in particular, if $k=1$, they are homogeneous. If $-1 \leq k < 0$, they are complements; and if $k=0$, they are independent.

Denote the prices of the home and foreign goods in the foreign country by, respectively, p^* and q^* . The inverse demand functions for the home and foreign goods in the foreign country are, respectively,

$$p^* = a - b(x^* + ky^*)$$

and

$$q^* = a - b(kx^* + y^*)$$

where x^* and y^* are the quantities of the home and foreign goods in the foreign country.

The profit of the home firm with a specific tariff imposed by the foreign country, t^* , is represented as

$$\begin{aligned} \pi &= px + p^*x^* - c(x + x^*) - rx^* - t^*x^* \\ &= [a - b(x + ky)]x \\ &\quad + [a - b(x^* + ky^*)]x^* \\ &\quad - c(x + x^*) - rx^* - t^*x^* \end{aligned}$$

where c is a marginal production cost, and r is a transportation cost, respectively, per unit of output³⁾.

Similarly the profit of the foreign firm with a specific tariff imposed by the home country, t , is

$$\begin{aligned} \pi^* &= qy + q^*y^* - c(y + y^*) - ry - ty \\ &= [a - b(kx + y)]y + [a - b(kx^* \\ &\quad + y^*)]y^* - c(y + y^*) - ry - ty \end{aligned}$$

Under the assumption of Cournot behavior, the first order conditions of profit maximization for the home and foreign firms in the home country are, respectively,

$$\frac{\partial \pi}{\partial x} = a - 2bx - bky - c = p - c - bx = 0$$

and

$$\begin{aligned} \frac{\partial \pi^*}{\partial y} &= a - 2by - bky - c - r - t \\ &= q - c - r - t - by = 0 \end{aligned}$$

From these equations the equilibrium quantities of the home and foreign goods in the home country are obtained as follows,

$$x = \frac{1}{(4-k^2)b} [(2-k)(a-c) + k(r+t)] \quad (1)$$

and

$$y = \frac{1}{(4-k^2)b} [(2-k)(a-c) - 2(r+t)] \quad (1)'$$

The responses of x and y to a change in t are

$$\frac{dx}{dt} = \frac{k}{(4-k^2)b}$$

and

$$\frac{dy}{dt} = -\frac{2}{(4-k^2)b}$$

The first order conditions for the home and foreign firms in the foreign country are, respectively,

$$\begin{aligned} \frac{\partial \pi}{\partial x^*} &= a - 2bx^* - bky^* - c - r - t^* \\ &= p^* - c - r - t^* - bx^* = 0 \end{aligned}$$

and

$$\begin{aligned} \frac{\partial \pi^*}{\partial y^*} &= a - 2by^* - bky^* - c \\ &= q^* - c - by = 0 \end{aligned}$$

The equilibrium quantities of the home and foreign goods in the foreign country are

$$x^* = \frac{1}{(4-k^2)b} [(2-k)(a-c) - 2(r+t^*)] \quad (2)$$

and

$$y^* = \frac{1}{(4-k^2)b} [(2-k)(a-c) + k(r+t^*)] \quad (2)'$$

The welfare of the home country is represented by

$$\begin{aligned} w &= u(x, y) - px - qy + px + p^*x^* \\ &\quad - c(x+x^*) - rx^* - t^*x^* + ty \\ &= u(x, y) - qy + p^*x^* - c(x+x^*) \\ &\quad - rx^* - t^*x^* + ty \end{aligned}$$

This is the sum of (1) consumers' surplus, $u(x, y) - px - qy$, (2) net-of-tariff profit of the home firm, $px + p^*x^* - c(x+x^*) - rx^* - t^*x^*$, and (3) tariff revenue.

Next, I examine the optimum tariff for each country. The optimum tariff is the tariff level imposed by each country which maximizes its welfare given the tariff level in the other country.

To find the optimum tariff for the home country, differentiating w with respect to t ,

$$\begin{aligned} \frac{dw}{dt} &= (p-c) \frac{dx}{dt} - y \frac{dq}{dt} + t \frac{dy}{dt} + y \\ &= (p-c) \frac{dx}{dt} - y \left(\frac{\partial q}{\partial x} \frac{dx}{dt} + \frac{\partial q}{\partial y} \frac{dy}{dt} \right) \\ &\quad + t \frac{dy}{dt} + y \\ &= bx \frac{k}{(4-k^2)b} + y \frac{bk^2}{(4-k^2)b} \\ &\quad - by \frac{2}{(4-k^2)b} - t \frac{2}{(4-k^2)b} + y \\ &= 0 \end{aligned}$$

The optimum tariff is obtained as

$$t_m = \frac{1}{2} b(kx + 2y) = \frac{1}{3} (a - c - r)$$

The welfare of the foreign country is

$$\begin{aligned} w^* &= u(x^*, y^*) - p^*x^* - q^*y^* + qy \\ &\quad + q^*y^* - c(y+y^*) - ry \\ &\quad - ty + t^*x^* \\ &= u(x^*, y^*) - p^*x^* + qy - c(y+y^*) \\ &\quad - ry - ty + t^*x^* \end{aligned}$$

By analogous procedures, we obtain the optimum tariff for the foreign country, as follows,

$$t_m^* = \frac{1}{3} (a - c - r) = t_m$$

We need $r < a - c$ so that t_m and t_m^* are positive.

III. Welfare-enhancing Tariff War

Because of the symmetric structure of

the model, we have a symmetric equilibrium, both under free trade and tariff war. In the symmetric equilibria we have $x = y^*$, $x^* = y$, $p = q^*$, $p^* = q$, $t_m = t_m^*$ and $w = w^*$. The welfare of both countries in the symmetric equilibria is represented as

$$\begin{aligned} w = w^* &= u(x, y) - qy + qy - c(x + y) \\ &\quad - ry - ty + ty = u(x, y) - c(x + y) \\ &\quad - ry = (a - c)(x + y) - ry - \frac{1}{2}b \\ &\quad (x^2 + 2kxy + y^2) \end{aligned} \quad (3)$$

I compare the welfare level of the countries in the equilibrium of a non-cooperative tariff game and the free-trade equilibrium. Substituting $t = t^* = 0$ into (1) and (1)', or into (2) and (2)', we obtain

$$x_f = \frac{1}{(4 - k^2)b} [(2 - k)(a - c) + kr] \quad (4)$$

and

$$y_f = \frac{1}{(4 - k^2)b} [(2 - k)(a - c) - 2r] \quad (4)'$$

x_f is the quantity of the home good in the home country (or the quantity of the foreign good in the foreign country) in the free-trade equilibrium. y_f is the quantity of the home good in the foreign country (or the quantity of the foreign good in the home country) in the free-trade equilibrium. r must be smaller than $\frac{2 - k}{2}(a - c)$

for y_f to be positive.

The welfare of both countries in the free-trade equilibrium is obtained by substituting x_f and y_f into (3) as follows,

$$\begin{aligned} w_f &= \frac{1}{2(4 - k^2)^2 b} [2(2 - k)^2(3 + k) \\ &\quad (a - c)^2 - 2(2 - k)^2(3 + k)r(a - c) \\ &\quad + (12 - k^2)r^2] \end{aligned} \quad (5)$$

Derivation of (5) is in Appendix (1).

Substituting $t = t^* = t_m$ into (1) and (1)', or into (2) and (2)', we obtain

$$x_t = \frac{2}{3(4 - k^2)b} [(3 - k)(a - c) + kr] \quad (6)$$

and

$$y_t = \frac{1}{3(4 - k^2)b} [(4 - 3k)(a - c) - 4r] \quad (6)'$$

x_t is the quantity of the home good in the home country (or the quantity of the foreign good in the foreign country) in the equilibrium of a non-cooperative tariff game. y_t is the quantity of the home good in the foreign country (or the quantity of the foreign good in the home country) in the equilibrium of a non-cooperative tariff game. r must be smaller than $\frac{(4 - 3k)}{4}(a - c)$ for y_t to be positive.

Now we can show.

Proposition 1

“(1) When the goods are substitutes, that is, $0 < k \leq 1$, we need $r < \frac{4 - 3k}{4}(a - c)$ so that y_f and y_t are positive. Then the optimum tariffs are positive.

(2) When the goods are independent, that is, $k = 0$, we need $r < a - c$ so that y_f and y_t are positive. Then the optimum tariffs are positive.

(3) When the goods are complements, that is, $-1 \leq k < 0$, we need $r < \frac{2 - k}{2}(a - c)$ so that y_f and y_t are positive. In this case the optimum tariffs may be negative.”

Proof:

(1) When $0 < k \leq 1$, we have $\frac{4 - 3k}{4}$

$$(a - c) < \frac{2 - k}{2}(a - c) < a - c.$$

(2) When $k = 0$, we have $\frac{4 - 3k}{4}(a - c)$

$$= \frac{2 - k}{2}(a - c) = a - c.$$

(3) When $-1 \leq k < 0$, we have $\frac{4 - 3k}{4}$

$$(a - c) > \frac{2 - k}{2}(a - c) > a - c.$$

Then $r < \frac{2 - k}{2}(a - c)$ does not imply $r < a - c$, and hence the optimum tariffs may

be negative. (Q. E. D.)

The welfare of both countries in the equilibrium of a non-cooperative tariff game is obtained by substituting x_t and y_t into (3) as follows,

$$w_t = \frac{1}{18(4-k^2)^2b} [(18k^3-21k^2-120k+188)(a-c)^2 - (18k^3-24k^2-120k+160)r(a-c) + (80-12k^2)r^2] \quad (7)$$

Derivation of (7) is in Appendix (2).

Comparing w_t and w_f ⁴,

$$w_t - w_f = -\frac{1}{18(4-k^2)^2b} [r - (a-c)] [(3k^2+28)r - (3k^2-24k+28)(a-c)] \quad (8)$$

For (8) to be positive we need

$$\frac{3k^2-24k+28}{3k^2+28}(a-c) < r < a-c \text{ when } 0 <$$

$$k \leq 1, \text{ and } a-c < r < \frac{3k^2-24k+28}{3k^2+28}(a-c)$$

when $-1 \leq k < 0$ ⁵. But in the latter case tariffs are negative. We have $\frac{2-k}{2} <$

$$\frac{3k^2-24k+28}{3k^2+28} \text{ when } -1 \leq k < 0 \text{ because}$$

$$\frac{2-k}{2} - \frac{3k^2-24k+28}{3k^2+28} = \frac{k(20-3k^2)}{2(3k^2+28)} < 0.$$

$$\text{Hence if } -1 \leq k < 0 \text{ and } a-c < r < \frac{2-k}{2}$$

$(a-c)$, a tariff war is Pareto superior to free trade, but tariffs are negative.

$$\text{Therefore, if, and only if, } 0 < k \leq 1 \text{ and } \frac{3k^2-24k+28}{3k^2+28}(a-c) < r < \frac{4-3k}{4}(a-c),$$

a tariff war with positive tariffs is Pareto superior to free trade. Since $\frac{4-3k}{4}$

$$- \frac{3k^2-24k+28}{3k^2+28} = \frac{3k(4-3k^2)}{4(3k^2+28)} > 0 \text{ when}$$

$0 < k \leq 1$, there exists r which is positive, and makes w_t larger than w_f when $0 < k \leq 1$.

Summarizing the results,

Proposition 2

"If and only if the goods are substitutes,

and the value of the transportation cost, r , satisfies the relation,

$$\frac{3k^2-24k+28}{3k^2+28}(a-c) < r < \frac{4-3k}{4}(a-c)$$

a tariff war with positive tariffs is Pareto superior to free trade."

IV. Mechanism of a Welfare-enhancing Tariff War

In this section, I examine the welfare-enhancing mechanism of a tariff war. Two possible causes for the welfare-enhancing effects of a tariff war are (1) output expansion by tariffs and (2) savings in transportation costs due to contraction of the volume of trade through tariffs. Under imperfect competition, the prices of the goods are higher than the marginal costs, and the goods are under-produced. If production is increased due to tariffs, the welfare levels of the countries may rise. I shall show, however, that this is not the case.

Consider a tariff war when the goods are substitutes, that is, $0 < k \leq 1$. Comparison of x_t and x_f gives us⁶

$$\Delta x = x_t - x_f = \frac{k}{3(4-k^2)b}(a-c-r) > 0 \quad (9)$$

On the other hand, comparison of y_t and y_f gives us⁷

$$\Delta y = y_m - y_f = -\frac{2}{3(4-k^2)b}(a-c-r) < 0 \quad (10)$$

The inequalities in (9) and (10) are obtained since $r < a-c$ in the equilibrium of a non-cooperative tariff game when the goods are substitutes. From (9) and (10) we have $|\Delta y| > |\Delta x|$, which implies that the total output of the goods in a tariff war is smaller than that under free trade. Therefore, the welfare-enhancing effects of a tariff war does not arise from output expansion.

The second term of the welfare of the

countries, $-ry$, in (3) denotes the transportation costs of the goods, which reduce the welfare of the countries through reduction of the firms' profits.

The welfare of both countries, including transportation costs, under a tariff war is described as follows⁸⁾,

$$\bar{w}_t = w_t + ry_t = \frac{1}{18(4-k^2)^2b} [(18k^3 - 21k^2 - 120k + 188)(a-c)^2 + (48k - 64)r(a-c) + (12k^2 - 16)r^2]$$

And the welfare of both countries, including transportation costs, under free trade is⁹⁾

$$\bar{w}_f = w_f + ry_f = \frac{1}{2(4-k^2)^2b} [2(2-k)^2(3+k)(a-c)^2 - 2(2-k)^2r(a-c) + (3k^2-4)r^2]$$

Comparing \bar{w}_t and \bar{w}_f ¹⁰⁾,

$$\bar{w}_t - \bar{w}_f = \frac{1}{18(4-k^2)^2b} [r - (a-c)] [5(4-3k^2)r + (3k^2-24k+28)(a-c)] \quad (11)$$

We have $3k^2-24k+28 > 0$ and $4-3k^2 < 0$ from $0 < k \leq 1$. Therefore (11) is negative since $r < a-c$ when the goods are substitutes. This means that if we ignore the welfare reduction through transportation costs, the welfare of the countries in the equilibrium of a non-cooperative tariff game cannot be higher than that in the free-trade equilibrium. As described in (10), the quantity of the home good in the foreign country (or the quantity of the foreign good in the home country) in a tariff war is smaller than that under free trade. Thus we can save transportation costs by tariffs. This savings in transportation costs due to contraction of the volume of trade through tariffs is the main cause of the welfare-enhancing effects of a tariff war. Indeed, as shown in the previous section, if transportation costs are sufficiently low, that is, $r <$

$\frac{3k^2-24k+28}{3k^2+28}(a-c)$, a tariff war is not Pareto superior to free trade.

V. Concluding Remarks

In this paper I have shown the possibility that in an equilibrium of a non-cooperative tariff game (tariff war) between two countries, the welfare of both countries is higher than in a free-trade equilibrium in an international duopoly with product differentiation. I assumed that demand and cost functions are linear. But, as stated in the Introduction, with general demand and cost functions we cannot explicitly compare the welfare of the countries in a free-trade equilibrium and a Nash equilibrium of a non-cooperative tariff game.

I do not assert that the imposition of tariffs by countries is generally desirable for the world. But in the modern economy in which imperfect competition prevails, free trade may not be optimal, and protection policies by the countries may have some merit. Krugman(1987) says "Free trade is not passè, but it is an idea that has irretrievably lost its innocence. Its status has shifted from optimum to reasonable rule of thumb."

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Appendices

(1) Derivation of (5)

From (4) and (4)' we obtain

$$x_f^2 = \frac{1}{(4-k^2)^2b^2} [(2-k)^2(a-c)^2 + 2k(2-k)r(a-c) + k^2r^2] \quad (A.1)$$

$$y_f^2 = \frac{1}{(4-k^2)^2b^2} [(2-k)^2(a-c)^2 - 4(2-k)r(a-c) + 4r^2] \quad (A.2)$$

and

$$x_f y_f = \frac{1}{(4-k^2)^2b^2} [(2-k)^2(a-c)^2$$

$$-(2-k)^2 r(a-c) - 2kr^2] \quad (\text{A. 3})$$

Substituting (4), (4)', (A. 1), (A. 2) and (A. 3) into (3), we obtain

$$\begin{aligned} (a-c)(x_f + y_f) - ry_f &= \frac{1}{(4-k^2)b} [2(2-k) \\ &\quad (a-c)^2 - 2(2-k)r \\ &\quad (a-c) + 2r^2] \\ \frac{1}{2}b(x_f^2 + 2kx_f y_f + y_f^2) &= \frac{1}{2(4-k^2)^2 b} [2(1+k) \\ &\quad (2-k)^2 (a-c)^2 \\ &\quad - 2(1+k)(2-k)^2 r \\ &\quad (a-c) + (4-3k^2)r^2] \end{aligned}$$

and

$$\begin{aligned} w_f &= \frac{1}{2(4-k^2)^2 b} [2(2-k)^2(3+k)(a-c)^2 \\ &\quad - 2(2-k)^2(3+k)r(a-c) + (12-k^2)r^2] \end{aligned}$$

(2) Derivation of (7)

From (6) and (6)' we obtain

$$\begin{aligned} x_t^2 &= \frac{4}{9(4-k^2)^2 b^2} [(3-k)^2(a-c)^2 + 2k(3-k) \\ &\quad r(a-c) + k^2 r^2] \quad (\text{A. 4}) \end{aligned}$$

$$\begin{aligned} y_t^2 &= \frac{1}{9(4-k^2)^2 b^2} [(4-3k)^2(a-c)^2 - 8(4-3k) \\ &\quad r(a-c) + 16r^2] \quad (\text{A. 5}) \end{aligned}$$

and

$$\begin{aligned} x_t y_t &= \frac{2}{9(4-k^2)^2 b^2} [(3-k)(4-3k)(a-c)^2 \\ &\quad - (3k^2 - 8k + 12)r(a-c) - 4kr^2] \quad (\text{A. 6}) \end{aligned}$$

Substituting (6), (6)', (A. 4), (A. 5) and (A. 6) into (3), we obtain

$$\begin{aligned} (a-c)(x_t + y_t) - ry_t &= \frac{1}{3(4-k^2)b} [5(2-k) \\ &\quad (a-c)^2 - (8-5k)r(a-c) + 4r^2] \\ \frac{1}{2}b(x_t^2 + 2kx_t y_t + y_t^2) &= \frac{1}{18(4-k^2)^2 b} \\ &\quad [(12k^3 - 39k^2 + 52)(a-c)^2 - (12k^3 - 24k^2 + 32)r \\ &\quad (a-c) + (16 - 12k^2)r^2] \end{aligned}$$

and

$$\begin{aligned} w_t &= \frac{1}{18(4-k^2)^2 b} [(18k^3 - 21k^2 - 120k + 188) \\ &\quad (a-c)^2 - (18k^3 - 24k^2 - 120k + 160)r \\ &\quad (a-c) + (80 - 12k^2)r^2] \end{aligned}$$

Notes

1) They examine the effects of trade policies in a duopoly with product differentiation, but do not consider a tariff war.

2) The alternative assumption for market structure in a trade model under imperfect competition is "integrated markets" under which producer prices of goods by arbitrage must be equal throughout the world. Horstman and Markusen (1986) and

Markusen and Venables (1988) examine the effects of trade policies in oligopolies under integrated markets. In another paper, Tanaka (1991), I show that a tariff war, with an ad-valorem tariff between two countries in free-entry oligopolies under integrated markets, is strictly Pareto superior to free trade in the case of linear demand and cost functions, even with zero transportation cost.

3) I ignore fixed cost which does not affect our analyses and I ignore profits of firms in a transportation service industry, or assume perfect competitiveness with zero profit in this industry.

$$\begin{aligned} 4) \quad w_t - w_f &= \frac{1}{18(4-k^2)^2 b} [-(3k^2 + 28)r^2 + (6k^2 \\ &\quad - 24k + 56)r(a-c) - (3k^2 - 24k \\ &\quad + 28)(a-c)^2] \\ &= -\frac{1}{18(4-k^2)^2 b} [r - (a-c)][(3k^2 \\ &\quad + 28)r - (3k^2 - 24k + 28)(a-c)] \end{aligned}$$

5) If $k=0$, (8) is reduced to

$$w_t - w_f = -\frac{7}{72b} [r - (a-c)]^2$$

This can not be positive for any value of r .

$$\begin{aligned} 6) \quad \Delta x &= x_t - x_f = \frac{2}{3(4-k^2)b} [(3-k)(a-c) \\ &\quad + kr] - \frac{1}{(4-k^2)b} [(2-k)(a-c) + kr] \\ &= \frac{k}{3(4-k^2)b} (a-c-r) \end{aligned}$$

$$\begin{aligned} 7) \quad \Delta y &= y_t - y_f = \frac{1}{3(4-k^2)b} [(4-3k)(a-c) \\ &\quad - 4r] - \frac{1}{(4-k^2)b} [(2-k)(a-c) - 2r] \\ &= -\frac{2}{3(4-k^2)b} (a-c-r) \end{aligned}$$

$$\begin{aligned} 8) \quad \bar{w}_t &= w_t + ry_t = \frac{1}{18(4-k^2)^2 b} [(18k^3 - 21k^2 \\ &\quad - 120k + 188)(a-c)^2 - (18k^3 - 24k^2 \\ &\quad - 120k + 160)r(a-c) + (80 - 12k^2)r^2] \\ &\quad + \frac{r}{3(4-k^2)b} [(4-3k)(a-c) - 4r] \\ &= \frac{1}{18(4-k^2)^2 b} [(18k^3 - 21k^2 - 120k + 188) \\ &\quad (a-c)^2 + (48k - 64)r(a-c) + (12k^2 \\ &\quad - 16)r^2] \end{aligned}$$

$$\begin{aligned} 9) \quad \bar{w}_f &= w_f + ry_f = \frac{1}{2(4-k^2)^2 b} [2(2-k)^2(3+k) \\ &\quad (a-c)^2 - 2(2-k)^2(3+k)r(a-c) \\ &\quad + (12-k^2)r^2] + \frac{r}{(4-k^2)b} [(2-k) \\ &\quad (a-c) - 2r] = \frac{1}{2(4-k^2)^2 b} [2(2-k)^2 \\ &\quad (3+k)(a-c)^2 - 2(2-k)^2 r(a-c) \\ &\quad + (3k^2 - 4)r^2] \end{aligned}$$

$$\begin{aligned} 10) \quad \bar{w}_t - \bar{w}_f &= -\frac{1}{18(4-k^2)^2 b} [(3k^2 - 24k + 28) \\ &\quad (a-c)^2 - (18k^2 - 24k + 8)r(a-c) \\ &\quad + (15k^2 - 20)r^2] = -\frac{1}{18(4-k^2)^2 b} \end{aligned}$$

$$[r - (a - c)][5(4 - 3k^2)r + (3k^2 - 24k + 28)(a - c)]$$

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