

# Money in Kaldorian Cycle Theory\*

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## 1. Introduction

The purpose of this paper is to formulate a very simple macroeconomic model which purports to explain the existence of the persistent business cycle in a modern capitalist economy.

We can refer to Kaldor (1940) and Goodwin (1967) as the representatives of models of the 'endogenous' business cycle. They tried to explain the process of the business cycle using a nonlinear investment function (Kaldor) or a sort of the 'Phillips curve' (Goodwin) without resorting to exogenous factors outside an economy such as 'random shocks.' Almost every model of the endogenous business cycle which was formulated until now can be classified as a variant of either Kaldor-type or Goodwin-type or a mixture of these types.<sup>1)</sup>

Incidentally, those models are typically 'real models' of the business cycle, i. e., usually they neglect a 'monetary factor' or, even if the monetary factor is considered, it is only implicitly introduced into the models in spite of the fact that many of those models intend to reflect the vision of Keynes (1936) on the working of a modern capitalist economy in some senses.

On the other hand, we have the so-called 'Neoclassical monetary growth model' as a macrodynamic model which allows for the monetary factor explicitly.<sup>2)</sup> But, unfortunately, this model contradicts Keynes' vision since it negates the existence of investment functions which are independent of the saving functions of households and furthermore, more fatally, it *presupposes* the full employment of labour.<sup>3)</sup>

In this paper we introduce the monetary factor into Kaldorian model of the business cycle. Our model differs from the above mentioned models in the sense that the monetary factor as well as the real factors plays an important role as one of the causes of the business

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1) For example, we can refer to Chang and Smyth (1971) and Varian (1979) as the developments of the Kaldor-type model, and Desai (1973), Medio (1980), Pohjola (1981) and Velupillai (1979) as the developments of Goodwin-type model. Mino (1978) and Rose (1967) can be considered as the mixtures of these types.

2) With regard to the Neoclassical theory of monetary growth, see, for example, Burmeister and Dobell (1970) chap. 6 or Stein (1971) chap. 1.

3) It must be noted, however, for the sake of the fairness, that Goodwin (1967)'s original model also lacks the independent investment function and so his model does not necessarily conform to the Keynesian vision.

cycle, and the main motive powers of the business cycle in our model are the investment activities of entrepreneurs and the speculative behaviours of wealth holders. In this sense, the spirit of our model is essentially 'Keynesian.'

## 2. The model

### <Notation>

$Y \equiv$  real gross national income.  $K \equiv$  real capital stock.

$\delta \equiv$  rate of capital depreciation which is assumed to be constant ( $0 < \delta \leq 1$ ).

$M \equiv$  nominal money supply which is assumed to be constant.

$r \equiv$  rate of interest on bond.  $p \equiv$  general price level.

$I \equiv$  real gross investment demand.  $S \equiv$  real gross saving.

$s \equiv S/Y =$  average propensity to save which is assumed to be constant ( $0 < s < 1$ ).

### <System of equations>

Our model consists of the following system of equations.<sup>4)</sup>

$$\dot{Y} = \alpha [I(Y, K, r) - sY]; \quad \alpha > 0, I_Y > 0, I_K < 0, I_r < 0 \quad (1)$$

$$M/p = L(Y, r); \quad L_Y > 0, L_r < 0 \quad (2)$$

$$p = p(Y); \quad p_Y > 0 \quad (3)$$

$$\dot{K} = I(Y, K, r) - \delta K \quad (4)$$

Eq.(1) means that the real output fluctuates according to the discrepancy between the investment and the saving (excess demand in the good market), which is one of the fundamental hypotheses of 'Kaldorian' cycle theory.<sup>5)</sup>

Eq.(2) is the 'LM equation' which characterizes the equilibrium in the money market. Namely,  $L(Y, r)$  is the demand function for the real money balance and it is implicitly supposed that there exists the bond market behind the money market.

Eq.(3) is the 'aggregate supply function' which expresses the positive correlation between the general price level and the real output level. Furthermore, in passing, it is implicitly supposed that real output and the employment correlate positively.

Eq.(4) is the 'capital accumulation equation' which means that the net investment contributes to the change of the capital stock.<sup>6)</sup>

Eq.(1) ~ Eq.(4) are sufficient to determine four endogenous variables ( $Y, K, r, p$ ). The remaining part of this paper is devoted to the analyses of the working of the model which has been formulated in this section.

## 3. System of the fundamental dynamical equations and its properties

Substituting eq.(3) into eq.(2) and solving with respect to  $r$ , we have

$$r = r(Y); \quad r_Y = - \left( \frac{Mp_Y}{p^2} + L_Y \right) / L_r > 0. \quad (5)$$

(+)

Substituting eq.(5) into eq.(1) and eq.(4), we can derive the following system of the

4) We assume that all relevant functions are continuously differentiable and  $I_Y, I_K$  etc. mean that  $\partial I / \partial Y, \partial I / \partial K$  etc.

5)  $I(Y, K, r)$  is the investment function which has both of the characters postulated by Kaldor (1940) and Keynes (1936).

6) In this model, it is assumed that the discrepancy between the demand and the output is absorbed through the changes of the inventories and hence always the intended demand realizes.

'fundamental dynamical equations' which characterizes the dynamics of our model economy.

$$\dot{Y} = \alpha [I(Y, K, r(Y)) - sY] \equiv I_1(Y, K) \quad (6a)$$

$$\dot{K} = I(Y, K, r(Y)) - \delta K \equiv I_2(Y, K) \quad (6b)$$

Now, we shall analyze the properties of this system.

### 3-1. The locus of $\dot{Y}=0$

First let us investigate the shape of the locus of  $\dot{Y}=0$ , i. e.,

$$I(Y, K, r(Y)) = sY. \quad (7)$$

Totally differentiating this equation, we have

$$(I_Y + I_{rr}r_Y - s)dY + I_K dK = 0. \quad (8)$$

Hence, we obtain

$$\begin{aligned} \left. \frac{dK}{dY} \right|_{\dot{Y}=0} &= - (I_Y + I_{rr}r_Y - s) / I_K \cong 0 \\ &\quad \begin{array}{cccc} & & (+) & (-)(+) & (-) \end{array} \\ \Leftrightarrow I_Y &\cong s - I_{rr}r_Y. \quad (9) \\ &\quad \quad \quad (-)(+) \end{aligned}$$

As for the magnitude of the sensitivity of the investment with respect to the change of  $Y$  (i. e.,  $I_Y$ ), we adopt Kaldor (1940)'s well-known hypothesis that it is small for extremely low levels and for extremely high levels of  $Y$ , and it is relatively large for 'normal' levels of  $Y$ .<sup>7)</sup> In what follows, more concretely we assume that

Ass. 1: *There exist the continuous functions  $Y_L(K)$  and  $Y_H(K)$  which satisfy the following properties for all  $K \geq 0$ .*

(i)  $0 < \underline{Y} < Y_L(K) < Y_H(K) < \infty$ .

(ii)  $I_Y < s - I_{rr}r_Y$  if  $Y \in (0, Y_L(K)) \cup (Y_H(K), \infty)$ , and  $I_Y > s - I_{rr}r_Y$  if  $Y \in (Y_L(K), Y_H(K))$ ; where  $\underline{Y}$  is a given constant.

Furthermore, as the 'boundary conditions' let us assume

Ass. 2: (i)  $I$  is bounded and  $I(0, K, r(0)) > 0$  for all  $K \geq 0$ .

(ii)  $I(Y, 0, r(Y)) > sY$  for all  $Y \in [0, Y_H(0)]$ .

Ass. 2(i) implies that even if  $Y=0$ , the gross investment is positive (although the net investment may be negative). Ass. 2(ii) says that since a boosting effect works on the investment at  $K=0$ , the investment exceeds the saving for the relevant region of  $Y$  at  $K=0$ .

Now, let us denote by  $K=h(Y)$  the function which expresses the locus of  $\dot{Y}=0$ . Then, under the above-mentioned assumptions, we can prove the following

#### PROPOSITION 1.

(i)  $h(Y)$  is continuously differentiable.

(ii) There exists  $h^{-1}(K) > 0$  for all  $K \geq 0$ .

(iii)  $h^{-1}(0)$  is unique, and  $h^{-1}(0) > h_H(0)$ .

(iv) There exist  $Y_i (i=1, 2, 3, 4)$  such that

$$0 < Y_1 \leq Y_2 < Y_3 \leq Y_4 < h^{-1}(0),$$

$$h'(Y) < 0 \text{ at } Y \in (0, Y_1) \cup (Y_4, h^{-1}(0)) \text{ and}$$

7) See Kaldor (1940) and Varian (1979) for the rationales of this hypothesis. The point is that the entrepreneurs become sluggish if they face the abnormal business conditions.

$h'(Y) > 0$  at  $Y \in (Y_2, Y_3)$ .

(Proof)

- (i) It is obvious from eq. (9).
- (ii) From Ass. 2(i), it follows that for all  $K \geq 0$ ,  $I(0, K, r(0)) > 0 = sY$  if  $Y = 0$ . On the other hand,  $I(Y, K, r(Y)) < sY$  for sufficiently large  $Y > 0$  since  $I$  is bounded (Ass. 2i). Hence, there exists  $\bar{Y} = h^{-1}(K) > 0$ , i. e.,  $\bar{Y} > 0$  such that  $I(\bar{Y}, K, r(\bar{Y})) = s\bar{Y}$  by continuity.
- (iii) From (ii), there exists  $Y^0 = h^{-1}(0) > 0$ . Furthermore, from Ass. 2(ii) it follows that  $Y^0 > Y_H(0)$ , i. e.,  $Y^0 \in (Y_H(0), \infty)$  and  $I(Y^0, 0, r(Y^0)) = sY^0$ .  $Y^0$  must be unique because of the following reason. Suppose that  $Y_H(0) < Y_1^0 < Y_2^0 < \infty$  and that  $I(Y_i^0, 0, r(Y_i^0)) = sY_i^0$  ( $i=1, 2$ ). In this case, there exists  $Y^{**} \in (Y_1^0, Y_2^0) \subset (Y_H(0), \infty)$  such that  $I_{Y^{**}} + I_{r^{**}} r_{Y^{**}} = s$  by virtue of the mean value theorem in differential calculus. But, this contradicts the definition of  $Y_H(0)$ .
- (iv) From Ass. 1 it follows that  $Y < Y_L(K)$  if  $Y < \bar{Y}$ , which implies that  $h'(Y) < 0$  for all  $Y \in (0, \bar{Y})$ . On the other hand, it follows from (iii) that  $Y^0 = h^{-1}(0) \in (Y_H(0), \infty)$ , which implies that  $h'(Y) < 0$  at the neighbourhood of  $h^{-1}(0)$  from the definition of  $Y_H(0)$ . Therefore, the locus  $H = \{(Y, K) \in R_+^2 | K = h(Y)\}$  must intersect the region  $Z = \{(Y, K) \in R_+^2 | Y_L(K) < Y < Y_H(K)\}$  by continuity. This assures that there exists an intermediate interval such that  $h'(Y) > 0$ .

(q. e. d.)

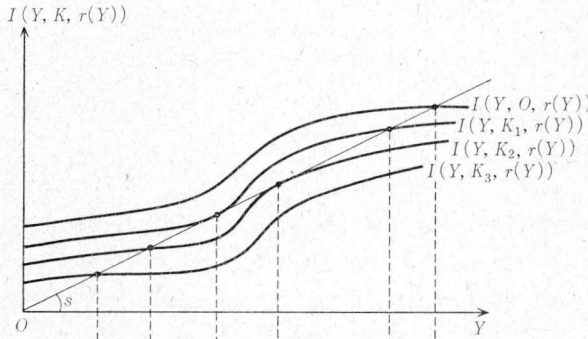


Fig. 1

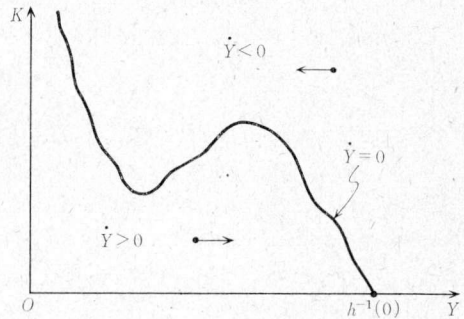


Fig. 2

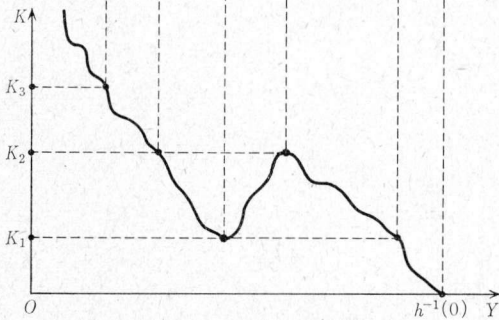


Fig. 3

As for the intuitive illustration of this proposition, see Fig. 1. In passing, it must be noted that if the position of  $(Y, K)$  is above (below) the locus of  $\dot{Y}=0$ , then  $\dot{Y} < 0$  ( $\dot{Y} > 0$ ) since eq. (6a) implies that  $\partial \dot{Y} / \partial K = \alpha I_K < 0$ . This is illustrated by Fig. 2.

**3-2. The locus of  $\dot{K}=0$**

Next, let us consider the locus of  $\dot{K}=0$ , i. e.,

$$I(Y, K, r(Y)) = \delta K. \tag{10}$$

Totally differentiating this equation, we obtain

$$\left. \frac{dK}{dY} \right|_{\dot{K}=0} = \frac{(I_Y + I_r r_Y)}{(+)(-)(+)} \frac{(\delta - I_K)}{(-)}. \tag{11}$$

Henceforth we denote by  $K=q(Y)$  the function which expresses the locus of  $\dot{K}=0$ . It follows from Proposition 1 (iv) and eq. (9) that in the intermediate range of  $Y$ ,  $I_Y + I_r r_Y > s > 0$ , and so  $q'(Y) > 0$ , while in the other range of  $Y$ ,  $q'(Y) \leq 0$  may occur. Furthermore, we have the following

PROPOSITION 2.  $q(0) > 0$ .

(Proof)

Let us define  $\Phi(K) \equiv I(0, K, r(0)) - \delta K$ . From Ass. 2 (i) we have  $\Phi(0) = I(0, 0, r(0)) > 0$ . On the other hand, we see that  $\Phi'(K) = I_K - \delta < 0$  and  $\lim_{K \rightarrow +\infty} \Phi(K) = -\infty$  since  $I$  is finite (Ass. 2 (i)). Hence, there exists unique  $K^0 > 0$  which satisfies  $\Phi(K^0) = 0$ . This proves that  $q(0) > 0$ .

(q. e. d.)

In passing, from eq. (6b) we obtain

$$\partial \dot{K} / \partial K = I_K - \delta < 0. \tag{12}$$

Therefore, if the position of  $(Y, K)$  is above (below) the locus of  $\dot{K}=0$ , then  $\dot{K} < 0$  ( $\dot{K} > 0$ ). This is illustrated by Fig. 3.

**3-3. The process of the business cycle**

Now we are in a position to fully investigate the working of the model combining Fig. 2 and Fig. 3.

First, let us define an 'equilibrium' as the state  $(Y^*, K^*)$  in which  $K^* = h(Y^*) = q(Y^*)$ . As for the equilibrium point, in particular, we assume that

Ass. 3: *There exists a unique equilibrium point  $(Y^*, K^*) > (0, 0)$  such that*

$$h'(Y^*) < q'(Y^*) \tag{13}$$

*in the 'intermediate' range of  $Y$ .*

The situation under Ass. 3 is illustrated by Fig. 4.

Now, we can calculate the Jacobian matrix  $J^*$  of the system (6) which is evaluated at the equilibrium point  $(Y^*, K^*)$  as follows:

$$J^* \equiv \begin{bmatrix} \alpha(I_{Y^*} + I_r^* r_{Y^*} - s) & \alpha I_{K^*} \\ I_{Y^*} + I_r^* r_{Y^*} & I_{K^*} - \delta \end{bmatrix} \tag{14}$$

Then, from the equations (9), (11), (12), (14) and Ass. 3 we obtain

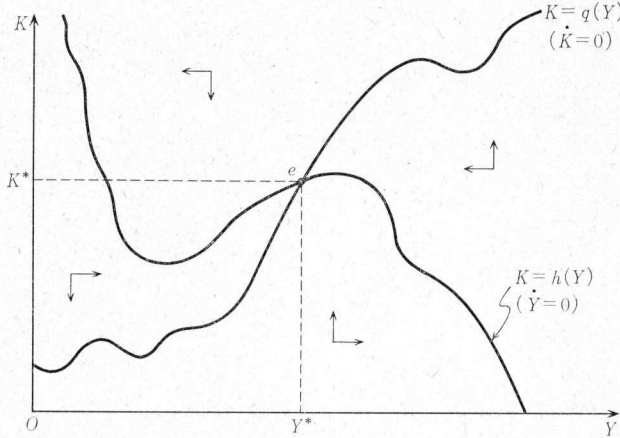
$$\det J^* = \alpha \{ (I_{Y^*} + I_r^* r_{Y^*} - s)(I_{K^*} - \delta) - (I_{Y^*} + I_r^* r_{Y^*}) I_{K^*} \}$$

$$= \alpha I_{K^*} (I_{K^*} - \delta) \{q'(Y^*) - h'(Y^*)\} > 0. \tag{15}$$

(-) (-)

On the other hand, we have

$$\text{trace } J^* = \underbrace{\alpha (I_{Y^*} + I_r^* r_{Y^*} - s)}_{(+)} + \underbrace{I_{K^*} - \delta}_{(-)}. \tag{16}$$

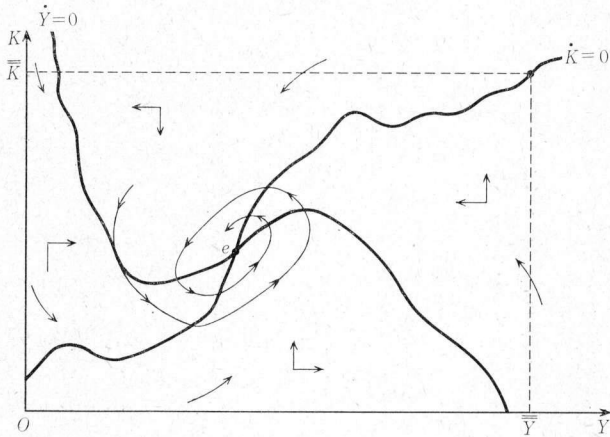


If  $\alpha (I_{Y^*} + I_r^* r_{Y^*} - s) < -I_{K^*} + \delta$ , then  $\text{trace } J^* < 0$  and in this case the equilibrium point becomes *locally stable*. On the other hand, if  $\alpha (I_{Y^*} + I_r^* r_{Y^*} - s) > -I_{K^*} + \delta$ , then  $\text{trace } J^* > 0$  and  $(Y^*, K^*)$  becomes *locally totally unstable*. The former case is illustrated by Fig. 5.<sup>8)</sup>

Now, let us consider the following nonempty compact set.

$$\Delta \equiv \{(Y, K) \in R_+^2 \mid Y \leq \bar{Y}, K \leq \bar{K} \equiv q(\bar{Y})\} \tag{17}$$

where  $\bar{Y}$  is the sufficiently large positive constant. It is easily seen that the locus in the system (6) which starts outside  $\Delta$  necessarily enters into  $\Delta$  sooner or later, and as long as it starts inside  $\Delta$ , it is throughout within  $\Delta$ . On the other hand, if the equilibrium point is *locally totally unstable* (the latter case), the system (6) never converges to the equilibrium point  $e = (Y^*, K^*)$  as long as it starts from any point but the equilibrium point. These situations are enough to apply the limit cycle theorem due to Poincaré and Bendixon.<sup>9)</sup>



8) The solutions of the characteristic equation  $|\lambda I - J^*| = 0$  are given by

$$\lambda_1, \lambda_2 = \frac{\text{trace } J^* \pm \sqrt{(\text{trace } J^*)^2 - 4 \det J^*}}{2}$$

From this we see that (i) if  $\text{trace } J^* < 0$  and  $\det J^* > 0$ , then the real parts of both roots become negative, and (ii) if  $\text{trace } J^* > 0$  and  $\det J^* > 0$ , then the real parts of both roots become positive. Note that in the latter case the equilibrium point does not become the saddle point but becomes *locally totally unstable*.

9) Poincaré-Bendixon theorem can be stated as follows (as for the proof, see Hirsch and Smale (1974) chap. 11): "A nonempty compact limit set of a continuously differentiable dynamical system in the plane, which contains no equilibrium point, is a closed orbit."

There are numerous examples of the applications of this theorem to the macrodynamic economics, in particular, the theory of the business cycle. See, for example, Chang and Smyth (1971), Medio (1980), Mino (1978), Rose (1967), Schinasi (1982), Varian (1979), et al.

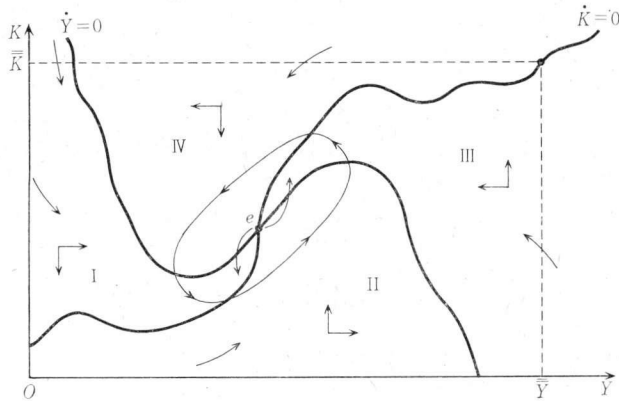


Fig. 6 (trace  $J^* > 0$ )

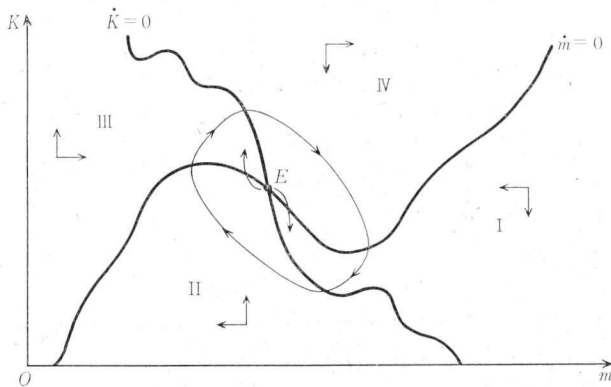


Fig. 7

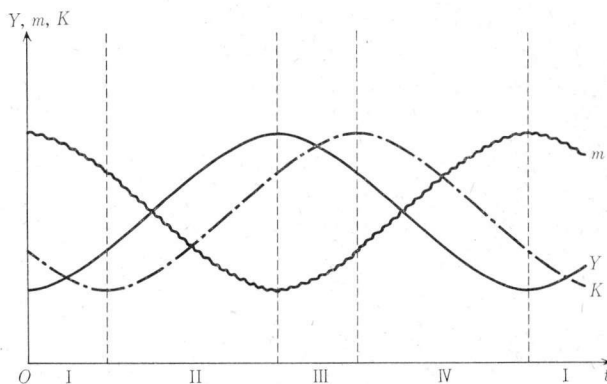


Fig. 8

Hence, we have the following theorem under the assumptions Ass. 1 ~ Ass. 3.

**THEOREM.**

- (i) If  $\alpha(I_{Y^*} + I_r^* r_{Y^*} - s) < -I_{K^*} + \delta$ , then, the equilibrium point is locally stable.
- (ii) If  $\alpha(I_{Y^*} + I_r^* r_{Y^*} - s) > -I_{K^*} + \delta$ , then, there exists at least one closed orbit inside  $\Delta$ , and the path described by the system (6) which starts from any point of  $(Y, K) \in R_+^2$  but the equilibrium point necessarily converges to one of the closed orbits (limit cycles).

The case of Theorem (ii) is illustrated by Fig. 6.

In passing, the real money balance ( $m$ ) is written as follows:

$$\begin{aligned}
 m &\equiv M/p(Y) = m(Y); \\
 m'(Y) &= -Mp'(Y)/p^2 < 0
 \end{aligned}
 \tag{18}$$

Therefore, we can derive from Fig. 6 the phase diagram in the  $m - K$  plane as Fig. 7.

**3-4. Verbal interpretation of the cycle**

Before closing this section, let us try to give a verbal interpretation of the process of the cycle in our model. The trough of the business condition lies on the boundary between the phase IV and the phase I of Fig. 6 or Fig. 7 where the real national income ( $Y$ )

arrives at the minimum and the real capital stock ( $K$ ) is relatively small, while the real money balance ( $m$ ) arrives at the maximum because of the low price level. In this case, the rate of unemployment is very high because of the lack of the effective demand. Yet the decreasing capital stock due to the negative net investment as well as the low interest rate

due to the high real money balance stimulates the investment expenditures for the productive capital, and so the business condition recovers. This is the situation of the phase I.

The phase II is the phase of the 'accelerated accumulation' where 'the investment induces the investment.' This process is the process of the absorption of the unemployed labour force, and in this process the price level increases and so the real money balance decreases.

But, this process of the 'accelerated accumulation' ends before long because of the following reasons. First, the increasing capital stock has a depressing effect on the investment expenditure (the 'Kaldorian' effect). Second, the decreasing real money balance also suppresses the investment through the rise in the interest rate caused by the financial deficit. The upper turning point of the real national income exists on the boundary between the phase II and the phase III.

The phase IV is the phase of the 'accelerated decumulation.' In this phase the rate of unemployment increases and the general price level decreases. But, sooner or later, this process is reversed since the falls in the capital stock and the rate of interest stimulate the investment expenditures. In this way, the alternations of the 'booms' and the 'slumps' are repeated periodically (see Fig. 8).

Note that in our model the rate of interest plays an important role to anchor the economy around the stationary state, i. e., it defends the economic system from the infinite divergency so that it works as a sort of the 'stabilizer.'

#### 4. Concluding remarks

The main idea of our model is essentially according to 'Kaldorian' cycle theory. Namely, in the 'extreme' levels of income the entrepreneurs become sluggish and so the sensitivity of the investment (with respect to the change of the national income) becomes small, which implies that the 'centripetal force' is exerted at the 'extreme' levels of income, and in the intermediate levels of income the sensitivity of the investment becomes relatively large, so that the 'centrifugal force' is exerted at the intermediate levels of income. Like Kaldor (1940), in our model the process of the endogenous business cycle has been explained on the bases of the reciprocal actions of these counteracting forces.

The significance of our model lies in the fact that in our model, unlike Kaldor (1940), the money market as well as the good market is explicitly incorporated.

In passing, it may be noted that our formulation is rather *ad hoc*, i. e., "the hypotheses of the models are neither derived from microeconomic model of maximizing behavior, nor are they subjected to serious empirical testing" (Varian (1979) p. 14). This feature of the *ad hocness* is not peculiar to our model but is common to a great many of the existing macroeconomic models, those of the so-called 'macro rationalists' are *not* the exceptions. It goes without saying, however, that "the hypotheses (of our model) are not without plausibility" (Varian (1979) p. 14) and hence such an attempt as this paper may be of some interests as a first step towards the more realistic theoretical investigation of the 'business cycle phenomena' in so far as the orthodox 'microeconomic' theory never even aims at ex-

10) Obviously, we must complicate our basic model by introducing the public sector, the international trade and so on to explain the process of the business cycle in the modern capitalist economy more realistically. But, this is beyond the scope of this paper. Moreover, our model is the 'pure cycle model' which abstracts from the economic growth. In passing, for the analysis of the government's monetary stabilization policy in a somewhat different framework of the 'cyclical growth,' see Asada (1984).



plaining such phenomena.<sup>10)</sup>

(Seijo University and Komazawa University)

### <Appendix: Introduction of the growth factor>

In the text of the paper, the economic growth was abstracted from for the simplicity of the exposition. Obviously, we can introduce the growth factor into the model, but at the cost of the analytical simplicity. In this appendix, an example of such reformulation is presented.

Let us re-interpret the variables such as  $Y$ ,  $K$ ,  $M/p$  and  $I$  as the *per capita* real variables  $y = Y/N$ ,  $k = K/N$ ,  $m = M/(pN)$  and  $i = I/N$ , where  $N$  is the labour supply which grows exponentially at the 'natural rate'  $n$ . Then, we reformulate the system of equations (1) ~ (4) as follows.

$$\dot{y} = \alpha[i(y, k, r - \pi^e) - sy]; \alpha > 0, i_y > 0, i_k < 0, i_{r - \pi^e} < 0 \quad (\text{A } 1)$$

$$m = \phi(y, r, \pi^e); \phi_y > 0, \phi_r < 0, \phi_{\pi^e} \leq 0 \quad (\text{A } 2)$$

$$\pi = f(y, \pi^e); f_y > 0, f_{\pi^e} > 0 \quad (\text{A } 3)$$

$$\dot{k} = i(y, k, r - \pi^e) - (n + \delta)k \quad (\text{A } 4)$$

where  $\pi \equiv \dot{p}/p$  is the rate of price inflation and  $\pi^e \equiv \hat{p}^e/p^e$  is the 'expected' rate of inflation. Eq. (A3) is the general form of the 'long run' supply function or the 'expectation-augmented' Phillips curve. If we assume that the nominal money stock  $M$  grows at the exogenously given rate  $\mu$ , we have

$$\dot{m}/m = \mu - n - \pi. \quad (\text{A } 5)$$

Since this system has six endogenous variables ( $y$ ,  $k$ ,  $m$ ,  $r$ ,  $\pi$  and  $\pi^e$ ), there remains 'one degree of freedom' in this system. We can close the model by introducing *any* type of the expectation hypothesis concerning the rate of inflation. This reformulated model, however, requires dynamical system in *at least* three-dimensional phase space. This makes it difficult to analyze the 'global' nature of the system although the 'local' nature can be analyzed even in this case.

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