Money Supply Uncertainty and Activist Stabilization Policy under Rational Expectations

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Economists of "new classical school" such as Sargent and Wallace [1975] have established the proposition that no anticipated stabilization policy exerts influences upon the real side of the macroeconomy. As an implied corollary to this proposition, they or Monetarism-2 support the Monetarism-1 prescription of constantly-growing-money-supply rule as the only recommendable government stabilization policy (Tobin [1980]). The purpose of this paper is to present a counter example to the direct linkage between the proposition and the corollary. More specifically, we present a simple macroeconomic model in which money is subject to inherent random exogenous shocks but they are possibly mitigated by active stabilization policies. We then show that, under some plausible situation, activist stabilization policy is more recommended than is the Monetarism-1 prescription, even though the anticipated-policy-ineffectiveness or Monetarism-2 proposition does hold true.

1. The Model

Consider the following simple macroeconomic model:

(1) $y_t=\alpha(p_t-E_{t-1}p_t),\ \alpha>0,$

(2) $y_t=\beta(m_t-p_t),\ \beta>0,$

where y_t=real output, p_t=the price level, and m_t=nominal money supply. These variables are in logarithmic form and are deviations from normal or trend levels so that the unconditional averages of them are all equal to zero. $E_{t-1}p_t$ denotes the rational expectation of p_t, conditional upon all the available information at period $t-1$.

Equation (1) is a standard Lucas type supply function whose crucial role to the Monetarism-2 proposition has been well known (e. g., Fischer [1980] and McCallum [1980]). Equation (2) is an aggregate demand function relating aggregate demand to real balances. For simplicity but without loss of generality, we do not introduce any additive exogenous random disturbances in (1) and (2).

Equating supply and demand, a direct manipulation of the model yields $E_{t-1}p_t=E_{t-1}m_t$, and

$$p_t-E_{t-1}p_t=\frac{\beta}{\alpha+\beta}(m_t-E_{t-1}m_t),$$

so that

(3) $$y_t=\frac{\alpha\beta}{\alpha+\beta}(m_t-E_{t-1}m_t).$$

Equation (3) establishes the alleged Monetarism-2 proposition that the behavior of real output is influenced only by that of unanticipated money supply. Therefore, in order to stabilize output fluctuations to the best extent, minimizing the fluctuations of unanticipated money supply is called for and for that matter the Monetarism-1 prescription is recommended. There is no flaw in this discussion insofar as money supply could be perfectly controlled.

If, however, money supply *per se* is subject to random disturbances and is under imprecise control of the authority, uncontrollable part of money supply need not be anticipated and it may exert influences upon the fluctuations of real output. More specifically, if the magnitude of uncontrollable part of money supply is made dependent upon policy rules (as has been pointed out by, for instance, Fellner [1980]) and if some active policy rule brings about smaller fluctuations of unanticipated money supply as a result, then activist stabilization policies, rather than the Monetarism-1 prescription, may in fact stabilize output fluctuations.

2. Uncertainties in Money Supply

In order to formally examine the above conjecture, consider the following money supply equation:

(4) $m_t=(\rho+\xi_t)m_{t-1}+(1+\eta_t)x_t+\varepsilon_t,$

where x_t denotes the magnitude of policy changes. It is assumed that the authority adopts the counter-cyclical feedback fine tuning for the policy changes x_t:

(5) $x_t=-\gamma y_{t-1}-\delta p_{t-1}.$

The parameters ρ, γ, and δ are nonegative constants; and ξ_t, η_t, and ε_t are mutually and serially independent random variables with zero means and variance σ_ξ^2, σ_η^2, and σ_ε^2, respectively. All the random variables are assumed to be independent of m_{t-1}, y_{t-1}, and p_{t-1}.

The first term of (4) captures the inherent dynamic process of money supply. It characterizes that money supply is a state variable, so that $\rho=1$ may be most natural although we also allow more general cases of $\rho<1$. Due, however, to a possibility that money supply is subject also to instantaneous and unsystematic shifts in the portfolios of banking and nonbanking sectors whose magnitudes may depend upon the previous balances, a random disturbance ξ_t is included in this term. The second term in (4) captures, as already noted, the changes in money supply through policy actions. Because the money supply changes initiated by policy intervention are often associated with uncontrollable and stochastic elements (such as that caused by the variability of money multiplier), we introduce a random variable in this term, too. The third term ε_t represents an additional random factor which is not captured by the first two terms.

Substituting out p_{t-1} from (5) by making use of (2), we have

$$(6)\qquad x_t=-\delta m_{t-1}-\left(\gamma-\frac{\delta}{\beta}\right)y_{t-1},$$

so that (4) is rewritten as

$$(7)\qquad m_t=[\rho+\xi_t-\delta(1+\eta_t)]m_{t-1} -\left(\gamma-\frac{\delta}{\beta}\right)(1+\eta_t)y_{t-1}+\varepsilon_t.$$

Then, taking the conditional expectations of (7) based upon all the information available at period $t-1$, we obtain the anticipated money supply as

$$(8)\qquad E_{t-1}m_t=(\rho-\delta)m_{t-1}-\left(\gamma-\frac{\delta}{\beta}\right)y_{t-1},$$

and thereby the unanticipated money supply as

$$(9)\qquad m_t-E_{t-1}m_t=(\xi_t-\delta\eta_t)m_{t-1} -\left(\gamma-\frac{\delta}{\beta}\right)\eta_t y_{t-1}+\varepsilon_t.$$

Equation (9) clearly indicates that the behavior of unanticipated money supply is dependent upon policy rules which are represented by the parameters γ and δ.

3. The Case for Activist Stabilization Policy

The substitution of (9) into (3) yields

$$(10)\qquad Jy_t=(\xi_t-\delta\eta_t)m_{t-1} -\left(\gamma-\frac{\delta}{\beta}\right)\eta_t y_{t-1}+\varepsilon_t,$$

where

$$J=\frac{1}{\alpha}+\frac{1}{\beta}.$$

Also, from (7) and (10) or, alternatively, substituting (8) into (3), we have

$$(11)\qquad m_t-Jy_t=(\rho-\delta)m_{t-1}-\left(\gamma-\frac{\delta}{\beta}\right)y_{t-1}.$$

Apparently, not all of (7), (10), and (11) are independent of each other and any combination of two from three equations composes a system of independent bivariate stochastic difference equations. For this system with reasonable assumptions upon the values of structural and policy parameters, there exists a stochastic stationary state only to which we confine our analysis below.

By assumption, after multiplying (10) by y_{t-1} and by m_{t-1} and taking the unconditional expectations, we can easily obtain, respectively, $E(yy_{-1})=0$ and $E(ym_{-1})=0$, where $E(yy_{-1})$, for instance, denotes the unconditional expected value of $y_t y_{t-1}$ in the stochastic stationary state. Then, after multiplying (11) by y_t and utilizing the above relations, we have

$$(12)\qquad E(ym)=JE(y^2).$$

Also, squaring both sides of (11) and substituting (12) will yield

$$(13)\qquad E(m^2)=\frac{1}{1-(\rho-\delta)^2}\left[J^2-2J(\rho-\delta)\left(\gamma-\frac{\delta}{\beta}\right)+\left(\gamma-\frac{\delta}{\beta}\right)^2\right]E(y^2).$$

But, from (10):

$$J^2E(y^2)=(\sigma_\xi^2+\delta^2\sigma_\eta^2)E(m^2) +2\delta\left(\gamma-\frac{\delta}{\beta}\right)\sigma_\eta^2E(ym) +\left(\gamma-\frac{\delta}{\beta}\right)^2\sigma_\eta^2E(y^2)+\sigma_\varepsilon^2,$$

so that, substituting (12) and (13) into the above gives

$$(14)\qquad E(y^2)=\frac{\sigma_\varepsilon^2}{H},$$

where

$$(15)\qquad H=J^2-\frac{1}{1-(\rho-\delta)^2}\left[\left\{J^2-2J(\rho-\delta)\left(\gamma-\frac{\delta}{\beta}\right)+\left(\gamma-\frac{\delta}{\beta}\right)^2\right\}\sigma_\xi^2 +\left\{J^2\delta^2-2J(\rho^2-\rho\delta-1)\delta\left(\gamma-\frac{\delta}{\beta}\right) +(1+2\rho\delta-\rho^2)\left(\gamma-\frac{\delta}{\beta}\right)^2\right\}\sigma_\eta^2\right].$$

From (14) the fluctuations of output can be stabilized to the best extent by the policy rule which maximizes the expression for H. Since, however, the expression for H is so complicated that it is not fruitful to further pursue the general optimal policy rule. Instead we obtain

$$\left.\frac{\partial H}{\partial \gamma}\right|_{\substack{\gamma=0\\ \delta=0}}=2J\frac{\rho}{1-\rho^2}\sigma_\xi^{\ 2},$$

and

$$\left.\frac{\partial H}{\partial \delta}\right|_{\substack{\gamma=0\\ \delta=0}}=2J\frac{(\alpha\rho^2+\beta)\rho}{\alpha\beta(1-\rho^2)^2}\sigma_\xi^{\ 2}.$$

Therefore, unless either $\sigma_\xi^{\ 2}=0$ and/or $\rho=0$, the activist stabilization policies ($\gamma>0$ and/or $\delta>0$) perform better than doing nothing. In other words, the Monetarism-1 prescription is optimum only when either $\sigma_\xi^{\ 2}=0$ and/or $\rho=0$.

It is interesting to note that the value of $\sigma_\eta^{\ 2}$ and $\sigma_\varepsilon^{\ 2}$ are not essential to the above conclusion, although the optimum degree of activism does generally depend upon the value of $\sigma_\eta^{\ 2}$. In order to see this, assume for simplicity that $\delta\equiv 0$. Then, we have $\frac{\partial H}{\partial \gamma}=0$ with

$$\gamma=J\frac{\rho\sigma_\xi^{\ 2}}{\sigma_\xi^{\ 2}+(1-\rho^2)\sigma_\eta^{\ 2}},$$

so that the optimal γ is the greater the greater is $\sigma_\xi^{\ 2}$ and the smaller is $\sigma_\eta^{\ 2}$(provided that $\rho>0$).

When $\rho=0$, (15) reduces to

$$H=J^2-\frac{1}{1-\delta^2}\left[\left\{J^2+2J\delta\left(\gamma-\frac{\delta}{\beta}\right)+\left(\gamma-\frac{\delta}{\beta}\right)^2\right\}\sigma_\xi^{\ 2}+\left\{J^2\delta^2+2J\delta\left(\gamma-\frac{\delta}{\beta}\right)+\left(\gamma-\frac{\delta}{\beta}\right)^2\right\}\sigma_\eta^{\ 2}\right],$$

so that H is maximized by

$$(16)\qquad \gamma=\frac{\delta}{\beta}-J\delta,$$

for which H further reduces to

$$H=J^2(1-\sigma_\xi^{\ 2}),$$

irrespective of the value of δ. Therefore, insofar as γ is optimally set in accordance with δ by (16) δ can be arbitrary. This implies in particular that the Monetarism-1 prescription $\gamma=0$ and $\delta=0$ is as good as any other active feedback fine tuning policies.

Yet another insight can be obtained when ξ_t and η_t are dependent. Suppose, as an extreme case, that $\xi_t=\rho\eta_t$ implying that ξ_t and η_t are perfectly correlated. Then, (7) becomes

$$m_t=(1+\eta_t)\left[(\rho-\delta)m_{t-1}-\left(\gamma-\frac{\delta}{\beta}\right)y_{t-1}\right]+\varepsilon_t.$$

Therefore, the fluctuations of unanticipated money supply and thereby those of output can be reduced to their bounded minima, which are unavoidable by the radom term ε_t, if only the policy rule is chosen to be $\gamma=\frac{\rho}{\beta}$ and $\delta=\rho$. And this is true irrespective of the inherent random disturbances of ξ_t and η_t.

4. Interpretation and Concluding Remarks

The foregoing analysis has been rather mechanical. What has been an essential mechanism, however, is simple. Equation (2) indicates that changes in nominal money supply are absorbed into contemporaneous changes in real output and/or changes in the price level. This implies that there are correlations between the actual money supply and real output and the price level. In the mean time, the fluctuations of unanticipated money supply at period t depend upon the previous balances of money supply insofar as the random factor ξ_t is present in (4). Therefore, by the introduction of the appropriate feedback fine tuning based upon the realizations of real output and the price level in the previous period, the resultant money supply base at period $t-1$, over which the fluctuations of unanticipated money supply at period t are amplified, can be reduced. When ξ_t is absent the above stabilizing channel disappears and the activist feedback fine tuning only adds destabilizing fluctuations of unanticipated money supply. Therefore, the Monetarism-1 prescription is preferred in this case.

When $\rho=0$ or when the current money supply is on average independent of the previous money supply, money supply follows essentially a static, rather than a dynamic, process in the absence of policy interventions. Then, it is natural that the Monetarism-1 prescription again becomes an optimal choice. Note, however, that it is but one of the optimal policies because any other appropriate fine tuning policies can attain the same goal insofar as (16) is satisfied. Moreover, it should be pointed out that the Monetarism-1 prescription becomes generally inferior to the activist fine tuning policies once demand and/or supply equation for output contains autocorrelated terms, $\rho=0$ notwithstanding.

Note also that, although the essential problem is the stabilization of unanticipated component of money supply *pes se* under the

Monetarism-2 proposition, the feedback fine tuning should generally look at everything. In other words, the stability of unanticipated money supply should not be pursued by looking at the behavior of money supply alone. This can be easily verified by the following exercise. Suppose that the feedback fine tuning is pursued by looking only at money supply, then we can analyze this case by setting $\gamma=\frac{\delta}{\beta}$ in (6). However, as is clear for instance from (16), this choice is generally not optimal.

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