Theory of Demand for a Mutual Fund under Asymmetric Information*

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1. Introduction

The purpose of this paper is to provide a theoretical explanation for the 'raison d'être' of financial intermediaries limiting the analysis to the case of a mutual fund which is the simplest form of the financial intermediary.

As is well known, the essential role of financial intermediaries is to facilitate the flow of loanable funds from surplus spending units to deficit spending units. However, since it is possible for them to issue and trade securities directly in markets, they will utilize financial intermediaries only if they find themselves better off by doing so rather than trading only among them. Financial intermediaries can play their role, first, by issueing the totally different securities form those which deficit utits issue, namely securities which are more attractive to holders with respect to liquidity, default risk, terms of contracts and so on, secondly, by investing funds in more advantageous opportunities through their superior information, and thirdly, by economizing various costs associated with direct trades such as transactions costs and search costs.

A mutual fund is considered as the financial intermediary which specializes mainly in investing in common stocks. It does not lend money directly to borrowers (firms) unlike other financial intermediaries, but its role should remain the same. Individual investors will demand a mutual fund only when they become better off by holding it (in addition to original assets). However, a mutual fund is nothing but the linear combination of original assets. Why do individuals demand it instead of holding original assets and why do they regard a mutual fund as the different asset from the simple combination of original assets?

The traditional and intuitive answers to these questions have been stated as follows. (1) Taking into account the fact that there are minimum units for transactions of original risky assets, the individuals with only limited amount of money or the individuals with little ability to collect and analyze information about many kinds of securities can get the benefit of the diversification of portfolio by holding a mutual fund. (2) Individuals will expect the superior ability of the mutual fund manager to forecast the future returns on risky assets and they prefer holding a mutual fund to buying original assets by themselves.

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However, to the best of my knowledge, the rigorous analysis of these statements has not been made enough. For instance, as to (1), individuals may prefer selecting their own portfolios to holding a mutual fund seeking the higher expected returns at the expense of the benefit of portfolio diversifications, hence it will be necessary to clarify the conditions for the positive demand for a mutual fund. In order that the statement (2) make sense, we must define meaningfully the superior ability of the manager and the individuals' expectations about it[1]. The similar questions will arise even when we consider other types of financial intermediaries.

In this paper, we elaborate the role of a mutual fund along the line suggested by (2) above but with emphasis on the fact that individuals have also opportunities to buy original assets directly in markets as well as a mutual fund[2], and consider the conditions under which we can derive the nontrivial (positive at least for a sufficiently low commission rate) demand function for a mutual fund. It is not obvious, as is shown in section 2, because the demand for a mutual fund is always zero for a positive commission rate under idealized conditions if individuals make deterministic estimations concerning the mutual fund portfolio or they are informed of it in advance, no matter how good its performance may be.

As is suggested by Leland and Pyle [9], informational asymmetries between financial intermediaries and individuals may be a primary reason for the existence of financial intermediaries[3]. And also the empirical analyses on the performance of mutual funds

1) I shall mention so called "mutual fund theorem" of portfolio selection, relating to the function of mutual funds. This theory has its origin in the separation theorem of Tobin [21] and Lintner [10] and has extended by Black [2], Cass and Stiglitz [3] and Ross [17]. It finds the conditions either on the utility function or on the distribution function of returns on assets under which individuals can achieve the same utility level when they can trade only smaller number of mixed assets than original assets as when they are allowed to trade all original assets. According to this theory, the role of mutual funds is to reduce the number of assets on which individuals should focus when they make portfolio decisions without any loss of their utility levels. However, the implication of the reduction of the number of tradable assets is not clear as long as there already exist markets of original assets. The reduction of the number of assets does not necessarily save information costs or search costs since individuals have to analyze information about the returns on original assets as well as the portfolios of mutual funds in order to estimate the returns on mutual funds.

2) It is usually assumed, in the theoretical framework of the emprical analysis on the performance of mutual funds, that the portfolio manager of mutual funds seeks to maximize the utility function of the holders (their utility functions are all the same) without regard to their opportunities to buy assets directly in markets. Generally, this strategy will be different from those which are derived taking into account the possibility of their direct trades. The theory of the principal and agent relationship developed by Ross [16], Shavell [19] and so on deals with the cases where only one party, the agent, can get the outcome from his activity whereas the other party, the principal, has no choice but to 'enjoy' it. Although "Many economic arrangements which involve problems of risk sharing and incentives may be described in terms of the principal and agent relationship" (Shavell [19] p. 55), unfortunately we cannot apply this theory since individual investors as well as intermediaries are able to enter the financial markets and trade original assets.

3) They say "Traditional models of financial markets have difficulty explaining the existence of financial intermediaries, firms which hold one class of securities and sell securities of other types. If transactions costs are not present, ultimate lenders might just as well purchase the primary securities directly and avoid the costs which intermediation must involve. Transactions costs could explain intermediation, but their magnitude does not in many cases appear sufficient to be the sole cause. We suggest that information asymmetries may be a primary reason that intermediaries exist" ([9],

studied by Jensen [6], [7], Kon and Jen [8] and others seem to intend to check whether the mutual fund managers have superior ability to forecast the future returns on risky assets[4]. We shall formalize the superior ability of the manager by informational asymmetries between the manager and individuals. We assume the manager has the finer observation than individuals which are correlated with the future returns on risky assets.

In section 2, we propose the simple but basic model with a mutual fund, as a starting point of the analysis, to show that if the individual estimates the mutual fund portfolio deterministically or is informed of it in advance he has no incentive to demand a mutual fund even if the commission rate is zero. Section 3 deals with the portfolio selection behavior of the individual under informational asymmetries, where the mutual fund portfolio is regarded as random. Section 4 discusses the behavior of the manager and shows that, under certain contracts between the manager and the individual, if the manager's observation actually conveys information about the future returns on risky assets we can derive the nontrivial demand function for a mutual fund. Section 5 generalizes, to some extent, the assumptions made in sections 3 and 4. Section 6 presents a brief summary.

2. Fundamental Assumptions and the Demand for a Mutual Fund under the Deterministic Expectation on the Mutual Fund Portfolio

Let us consider the one period economy (time 0=present and time 1=future) where there are n original risky assets, one safe asset and one mutual fund company who issues only one type of mutual fund. We focus on the behavior of a representative individual in this economy. Let $\boldsymbol{R}=(R_1, \cdots\cdots, R_n)'$ be a vector of random future returns on original risky assets and $r_0 \geqq 1$ be the interest factor of a safe asset. Here the accounting units of risky assets are taken properly and a safe asset and a mutual fund are counted in Dollar terms.

We shall make the following assumptions throughout this paper.

(A. 1) All assets are completely divisible.

(A. 2) The mutual fund company does not issue the common stock and the future return on the mutual fund portfolio is distributed totally to its holders proportionately to their shares in its total supply. The commission revenue is distributed to individuals according to their predetermined shares in the ownership of the company.

(A. 3) Both the individual and the mutual fund manager are price takers for original assets. The commission rate of a mutual fund is determined by the mutual fund magager so as to equilibrate demand and supply of a mutual fund.

(A. 4) The asset markets are frictionless in the sense that there are no transactions costs, no tax and no search costs to get information except for the commis-

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pp. 382–383).

4) Their analyses are devoted to evaluating the portfolio performance not peculiar to mutual funds. To rationalize the use of the security market line as a bench mark for portfolio performance, Mayers and Rice [12] show that an individual with better information plots above the security market line. However, it is not clear that they recognize the differences between the criteria of portfolio selections of mutual funds and of individual investors.

sion of a mutual fund.

(A. 5) Short sales are admitted for original assets and their prices and the interest rate are the same in long and short sales. But, the short sale of a mutual fund is not admitted.

Some comments on these assumptions are in order. (A. 1), together with (A. 4), excludes the benefit of portfolio diversification by holding a mutual fund. Also (A. 4) excludes the benefit of economizing several costs. (A. 2) is important because if we assume alternatively that the future return on the mutual fund portfolio is divided into the dividend to the mutual fund holders and the dividend to the stockholders of the company, the impact of the existence of a mutual fund on the economy will crucially depend on the rule of division of the future return into two types of dividends unless in equilibrium every individual holds the equal proportion of these two outstanding assets, like the capital asset pricing model of Sharpe-Lintner-Mossin type. Although the violation of (A. 3) may cause the benefit of holding a mutual fund, it seems natural, in principle, to assume it. (A. 5) is made for the simplification of the analysis.

At time 0, the mutual fund manager determines the supply of a mutual fund $X_s>0$, and the demand for risky assets $\boldsymbol{z}=(z_1, \cdots\cdots, z_n)'$ and a safe asset z_0 which satisfy the budget constraint

$$X_s=z_0+\boldsymbol{p}'\boldsymbol{z}+qX_s \tag{1}$$

for given prices of risky assets $\boldsymbol{p}=(p_1, \cdots\cdots, p_n)'$, where q is the commission rate of a mutual fund $(q\geqq 0)$. We assume X_s is announced to the public. At time 1, the realized return on the portfolio $(z_0, \boldsymbol{z})$

$$Y^{n+1}=r_0z_0+\boldsymbol{R}'\boldsymbol{z} \tag{2}$$

is distributed as the dividend to the mutual fund holders. Thus the rate of return on a mutual fund is given by

$$r_{n+1}=Y^{n+1}/X_s=(r_0z_0+\boldsymbol{R}'\boldsymbol{z})/X_s \tag{3}$$

On the other hand, at time 0, the individual estimates the probability distribution of future returns on original risky assets and a mutual fund, and determines the portfolio so as to maximize his ex ante utility function of the future return

$$Y=r_0x_0+\boldsymbol{R}'\boldsymbol{x}+r_{n+1}x_{n+1} \tag{4}$$

subject to the budget constraint

$$\overline{W}=bqX_s+\bar{x}_0+\boldsymbol{p}'\bar{\boldsymbol{x}}=x_0+\boldsymbol{p}'\boldsymbol{x}+x_{n+1} \tag{5}$$

and the nonnegativity constraint $x_{n+1}\geqq 0$,

where x_0 : the individual's demand for a safe asset,

$\boldsymbol{x}=(x_1, \cdots\cdots, x_n)'$: the individual's demand vector for original risky assets,

x_{n+1} : the individual's demand for a mutual fund,

$\bar{x}_0$: the individual's initial holding of a safe asset, (given),

$\bar{\boldsymbol{x}}=(\bar{x}_1, \cdots\cdots, \bar{x}_n)'$: the individual's initial holding vector of risky assets, (given), and

b : the share of the individual in the ownership of the mutual fund company, (given).

In addition to the above assumptions, let us assume

(A. X) The individual estimates the portfolio of a mutual fund deterministically or the mutual fund manager announces it to the public in advance.

Denote the individual's estimation on the mutual fund portfolio as $(z_0{}^h, \boldsymbol{z}^h)$ such that the relation

$$X_s = z_0{}^h + \boldsymbol{p}'\boldsymbol{z}^h + qX_s \tag{6}$$

holds. Under this assumption, the individual will dissolve a mutual fund into the combination of original assets and estimate r_{n+1} based on his own estimation on original risky assets as long as his expectation is consistent. From his point of view, the rate of return on a mutual fund becomes

$$r_{n+1}{}^h = (r_0 z_0{}^h + \boldsymbol{R}'\boldsymbol{z}^h)/X_s \tag{7}$$

Using (6) and (7), we can rewrite (4) and (5) as

$$Y = r_0 s_0{}^h + \boldsymbol{R}'\boldsymbol{s}^h \quad \text{and} \tag{4'}$$

$$\overline{W} = s_0{}^h + \boldsymbol{p}'\boldsymbol{s}^h + aqX_s \tag{5'}$$

where $a = x_{n+1}/X_s$: the individual's demand for share in a mutual fund,

$$s_0{}^h = x_0 + az_0{}^h, \quad \text{and}$$

$$\boldsymbol{s}^h = \boldsymbol{x} + a\boldsymbol{z}^h. \tag{8}$$

$(s_0{}^h, \boldsymbol{s}^h)$ is explained as the individual's total demand for original assets since he is regarded as demanding $100a\%$ of the original assets in the mutual fund portfolio in addition to his own demand. Assuming that the individual's ex post utility is an increasing function of Y, his demand for a mutual fund will be zero if the commission rate q is positive, because he is only concerned with $(s_0{}^h, \boldsymbol{s}^h)$ and as he increases a he has the less amount of money to buy $(s_0{}^h, \boldsymbol{s}^h)$ in view of (4′) and (5′). The optimal value of $(s_0{}^h, \boldsymbol{s}^h)$ is determined by maximizing his ex ante utility function of Y subject to (5′) letting $a=0$. Therefore there is no market equilibrium with $X_s>0$ and $q>0$. If q is zero, the demand for a mutual fund is indeterminate and his own demand for original assets is determined by (8) in which $(s_0{}^h, \boldsymbol{s}^h)$ is the solution of the same problem. Thus we have

Propositon 1. Under the assumptions (A. 1)～(A. 5) and (A. X),

1. If the commission rate is positive, there is no demand for a mutual fund. Therefore in equilibrium the commission rate must be zero for $X_s>0$.
2. If the commission rate is equal to zero, the demand for a mutual fund is indeterminate and the individual's total demand for original assets remains constant and is equal to his demand for original assets with no demand for a mutual fund. Hence there is no incentive to demand a mutual fund.

Remark In order to have a market equilibrium with $X_s>0$ and $q=0$, we must assume that $z_0{}^h = z_0$ and $\boldsymbol{z}^h = \boldsymbol{z}$ for all individuals, i. e., either their estimations on the mutual fund portfolio are correct or they are informed of it in advance. In this case, we can prove the equilibrium prices of original risky assets are the same as those of no mutual fund case provided that they do not change expectations about the future returns on risky assets when the mutual fund portfolio is announced.

Notice that Proposition 1 is free from the assumptions on the expectations about the future returns on original risky assets and on the portfolio selection behavior of the mutual fund manager. Thus even if the individual changes his personal beliefs about the distribution of future ruturns through the information of the announced portfolio of the mutual fund, Proposition 1 still remains true.

The implication of (A. X) is now obvious. Under this assumption, the individual does not regard a mutual fund as a different asset from the combination of original assets which he can buy in the markets by the same conditions as the mutual fund manager faces. Thus there is no benefit of buying a mutual fund instead of the same combination of original assets.

We have shown that, under (A. 1)$\sim$(A. 5) and (A. X), the demand for a mutual fund is always zero for the positive commission rate, no matter how good its performance may be. The empirical (and also theoretical) analysis on the performance of mutual funds will be meaningless unless we get out of the model discussed here. In this sense, we can think of this model as the starting point of the analysis on the role of a mutual fund.

In the subsequent sections, we shall abandon (A. X) and consider the informational asymmetries between the manager and the individual to seek the situation in which the individual becomes better off by holding a mutual fund.

3. Asymmetric Information and the Behavior of the Individual

As is noted in the introduction, we can find one of the main functions of a mutual fund in the manager's superior ability to forecast the future returns on risky assets. We shall formalize it in the following way.

(A. 6) The future returns on risky assets are correlated with some J signals $\boldsymbol{m}=(m_1, \cdots\cdots, m_J)'$ by the joint probability distribution function $F(\boldsymbol{R}, \boldsymbol{m})$. Both the manager and the individual know this prior distribution function.

(A. 7) The manager observes the realization of $\boldsymbol{m}$ in advance while the individual cannot. He is not informed of the value of $\boldsymbol{m}$ nor the mutual fund portfolio.

We will consider the more general case than that of (A. 7) in section 5.

The manager makes portfolio decision based on the posterior (conditional) probability distribution of $\boldsymbol{R}$ knowing the value of $\boldsymbol{m}$. Hence we can write the mutual fund portfolio of risky assets as $\boldsymbol{z}(\boldsymbol{m})$. (The derivation of this function from the behavior of the manager will be deferred to the next section.) Therefore from the individual's point of view, $\boldsymbol{z}(\boldsymbol{m})$ is considered as random.

Let us define the excess returns on original risky assets and a mutual fund as

$$\boldsymbol{e}=\boldsymbol{R}-r_0\boldsymbol{p} \tag{9}$$

$$e_{n+1}=r_{n+1}-r_0$$

and denote the joint distribution function of $\boldsymbol{e}$ and $\boldsymbol{m}$ as $H(\boldsymbol{e}, \boldsymbol{m})$ that is derived from $F(\boldsymbol{R}, \boldsymbol{m})$.

Using (9), we can rewrite the future return on the mutual fund portfolio as

$$Y^{n+1}=\boldsymbol{e}'\boldsymbol{z}+r_0(1-q)X_s \tag{10}$$

from (1) and (2). Thus we get

$$r_{n+1}=(\boldsymbol{e}'\boldsymbol{z})/X_s+r_0(1-q) \tag{11}$$

$$e_{n+1}=(\boldsymbol{e}'\boldsymbol{z}-r_0qX_s)/X_s \tag{12}$$

Similarly, we can rewrite the future return on the individual's portfolio $(x_0, \boldsymbol{x}, x_{n+1})$ as

$$Y=\boldsymbol{e}'\boldsymbol{x}+(\boldsymbol{e}'\boldsymbol{z}(\boldsymbol{m})-r_0qX_s)a+r_0\overline{W} \tag{13}$$

substituting (5) to (4) and applying (13), where $a=x_{n+1}/X_s$ as before.

As to the objective function of the individual, we assume

(A. 8) The individual maximizes the espected utility of the future return on his portfolio. The ex post utility function u is continuously twice differentiable, $u'>0$ and $u''<0$. Thus he is a risk averter.

Then the optimization problem of the individual is formalized as

$$\text{To } \underset{x,\, a}{\text{maximize}}\ E[u(Y)]=\int_{e,m} u(Y)\,dH(\boldsymbol{e}, \boldsymbol{m}) \qquad (14)$$

subject to $a \geqq 0$.

We obtain, as the necessary and sufficient conditions for this problem,

$$\frac{\partial E[u(Y)]}{\partial x_i}=E[u'(Y)e_i]=0 \qquad i=1, \cdots\cdots, n, \qquad (15)$$

$$\frac{\partial E[u(Y)]}{\partial a}=E[u'(Y)(\boldsymbol{e}'\boldsymbol{z}(\boldsymbol{m})-r_0 qX_s)]\leqq 0 \qquad (16)$$

$$\frac{\partial E[u(Y)]}{\partial a}a=0 \qquad (16^*)$$

We can prove the following proposition without any specific assumption on $\boldsymbol{z}(\boldsymbol{m})$.

Prosition 2. Given the assumptions (A. 1) through (A. 8), if $\boldsymbol{R}$ and $\boldsymbol{m}$, therefore $\boldsymbol{e}$ and $\boldsymbol{m}$, are mutually independent, the individual has no incentive to demand a mutual fund even if $q=0$ and does not demand it for $q>0$.

(Proof) We will show that $a=0$ and $\boldsymbol{x}=\hat{\boldsymbol{x}}$ which is defined as the solution of

$$E[u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})e_i]=0 \qquad i=1, \cdots\cdots, n \qquad (17)$$

is the optimal solution when $q=0$. If this is true, he does not demand a mutual fund for $q>0$ since his utility level clearly decreases by buying it.

We rewrite (17) as

$$\int_e u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})e_i dH_1(\boldsymbol{e})=0 \qquad i=1, \cdots\cdots, n \qquad (18)$$

where $dH_1(\boldsymbol{e})$ is the marginal probability density of $\boldsymbol{e}$. (17) or (18) is nothing but the condition (15) when $a=0$. By the independence assumption of $\boldsymbol{e}$ and $\boldsymbol{m}$, we can express the probability density of $(\boldsymbol{e}, \boldsymbol{m})$ as

$$dH(\boldsymbol{e}, \boldsymbol{m})=dH_1(\boldsymbol{e})\,dH_2(\boldsymbol{m})$$

Therefore, when $q=0$, at $a=0$ and $\boldsymbol{x}=\hat{\boldsymbol{x}}$, we have

$$\begin{aligned}\frac{\partial E[u(Y)]}{\partial a}&=\sum_{i=1}^{n}\int_{e,m} u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})e_i z_i(\boldsymbol{m})\,dH_1(\boldsymbol{e})\,dH_2(\boldsymbol{m})\\&=\sum_{i=1}^{n}\left[\int_e u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})e_i dH_1(\boldsymbol{e})\cdot\int_m z_i(\boldsymbol{m})\,dH_2(\boldsymbol{m})\right]\\&=0\end{aligned}$$

in view of (18). Hence the optimality conditions (15) and (16) are satisfied by $a=0$ and $\boldsymbol{x}=\hat{\boldsymbol{x}}$ when $q=0$. (Q. E. D.)

This proposition means that if the observation of the manager does not convey any information about the future returns on risky assets the individual has no incentive to demand a mutual fund even if the commission rate is zero and never demands it for the positive commission rate. Therefore the strictly finer observation of the manager is a necessary condition for the positive demand for a mutual fund.

4. The Behavior of the Mutual Fund Manager and the Demand for a Mutual Fund

Our next task is to derive the function $\boldsymbol{z}(\boldsymbol{m})$ from the portfolio selection behavior of the manager and information he announces to the individual. After treating this problem, we shall show, under some conditions, that if the manager's observation is actually effective, i. e., conveys some information about the distribution of $\boldsymbol{R}$, the individual demands a mutual fund at least for a sufficiently low commission rate.

We consider the situation where

(A. 9) $(\boldsymbol{R}, \boldsymbol{m})$ is distributed as the $(n+J)$ variate normal with mean $(\bar{\boldsymbol{R}}, \bar{\boldsymbol{m}})$ and covariance matrix $\boldsymbol{V}=\begin{bmatrix}\boldsymbol{P} & \boldsymbol{L}\\ \boldsymbol{L}' & \boldsymbol{T}\end{bmatrix}$ where $\boldsymbol{P}=E(\boldsymbol{R}-\bar{\boldsymbol{R}})(\boldsymbol{R}-\bar{\boldsymbol{R}})'$, $\boldsymbol{L}=E(\boldsymbol{R}-\bar{\boldsymbol{R}})(\boldsymbol{m}-\bar{\boldsymbol{m}})'$, and $\boldsymbol{T}=E(\boldsymbol{m}-\bar{\boldsymbol{m}})\cdot(\boldsymbol{m}-\bar{\boldsymbol{m}})'$. $\boldsymbol{V}$ is assumed to be positive difinite, (hence $\boldsymbol{P}$ and $\boldsymbol{T}$ are invertible.)

After the value of $\boldsymbol{m}$ is known, $\boldsymbol{e}$ is distributed as the n variate normal with mean

$$\bar{\boldsymbol{e}}_m(\boldsymbol{m})=\bar{\boldsymbol{e}}+\boldsymbol{L}\boldsymbol{T}^{-1}(\boldsymbol{m}-\bar{\boldsymbol{m}}) \tag{19}$$

and covariance matrix

$$\boldsymbol{P}_m=\boldsymbol{P}-\boldsymbol{L}\boldsymbol{T}^{-1}\boldsymbol{L}' \text{[5)]} \tag{20}$$

where $\bar{\boldsymbol{e}}=\bar{\boldsymbol{R}}-r_0\bar{\boldsymbol{p}}$.

As to the behavior of the manager, we shall assume, first,

(A. 10) The portfolio selection behavior of the manager is consistent with maximization of expected utility, as a risk averter, of either (i) the total return on the mutual fund portfolio Y^{n+1} or (ii) the rate of return r_{n+1}, when X_s and (qX_s) are arbitrarily fixed. The admissible form of his ex post utility function, say v, is restricted to those which are continuously twice differentiable, $v'>0$ and $v''<0$.

Notice that (A. 10) does not necessarily imply that the manager has his own utility function. Rather it should be considered as a part of the contracts between the manager and individuals[6)].

Under the normality assumption of (A. 9), Y^{n+1} and r_{n+1} (which are given by (10) and (11) respectively) are also distributed as normal. Therefore any expected utility function is expressed in terms of mean and standard deviation of Y^{n+1} or r_{n+1}. Denote

$$E[v(Y^{n+1})\,|\boldsymbol{m}]=U(E_m(Y^{n+1}), S_m(Y^{n+1})) \qquad \text{in case (i)} \quad \text{and}$$
$$E[v(r_{n+1})\,|\boldsymbol{m}]=U(E_m(r_{n+1}), S_m(r_{n+1})) \qquad \text{in case (ii)}$$

in which $E_m(\ )$ and $S_m(\ )$ stand for the conditional expected value and standard

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5) $\boldsymbol{P}_m$ is invertible since it is expressed as $\boldsymbol{P}_m=\boldsymbol{C}_1'\boldsymbol{V}\boldsymbol{C}_1$ with the $(n+J)\times n$ matrix $\boldsymbol{C}_1=\begin{bmatrix}\boldsymbol{I}\\ -\boldsymbol{T}^{-1}\boldsymbol{L}'\end{bmatrix}$ and $\boldsymbol{C}_1\boldsymbol{y}=0$ iff $\boldsymbol{y}=0$, where $\boldsymbol{I}$ is the identity matrix of order n.

6) (A. 10) can be restated as

(A. 10*) The manager chooses the efficient portfolio in the sense of mean-standard deviation approach based on his own estimation.

(A. 10) and (A. 11) below are terms of contracts imposed on the manager. One may think that there is no incentive for the manager to fulfill (A. 10) so long as individuals do not know ex post the realized value of $\boldsymbol{m}$. As a matter of fact, there is no systematic mechanism in the actual world to check the behavior of the manager. But (A. 10) will be the minimum requirement for him to act with the best intereset of the mutual fund holders.

deviation for given value of $\boldsymbol{m}$ respectively. As is shown by Tobin [21], $v'>0$ and $v''<0$ imply $\frac{\partial U}{\partial E_m}>0$, $\frac{\partial U}{\partial S_m}<0$ and strict concavity of U in (E_m, S_m). Hence, we can solve the portfolio selection problem by the familiar method of mean-standard deviation approach to get the necessary and sufficient condition

$$\boldsymbol{z}(\boldsymbol{m})=k(\boldsymbol{m})\boldsymbol{P}_m^{-1}\bar{\boldsymbol{e}}_m(\boldsymbol{m}) \tag{21}$$

where $k(\boldsymbol{m})$ is a positive scalar given by

$$k(\boldsymbol{m})=-\left(\frac{\partial U}{\partial E_m}\bigg/\frac{\partial U}{\partial S_m}\right)\frac{S_m(Y^{n+1})}{2} \quad \text{in case (i)} \quad \text{and}$$

$$=-\left(\frac{\partial U}{\partial E_m}\bigg/\frac{\partial U}{\partial S_m}\right)\frac{S_m(r_{n+1})}{2}X_s \quad \text{in case (ii).}$$

Namely, as long as (A. 9) and (A. 10) hold, the portfolio of risky assets in a mutual fund is written in the form of (21), although $k(\boldsymbol{m})$ depends on the function v, arbitrarily fixed X_s and (qX_s), $\boldsymbol{P}_m$, $\bar{\boldsymbol{e}}_m(\boldsymbol{m})$, $\boldsymbol{p}$ and r_0. In order for the individual to estimate the function $\boldsymbol{z}(\boldsymbol{m})$, it is necessary to know the function $k(\boldsymbol{m})$, i. e., how the utility function of the manager v is like and how he estimates (qX_s) for different values of $\boldsymbol{m}$ with his different strategies. Furthermore, in order that the manager estimate (qX_s) he must know how individuals estimate $\boldsymbol{z}(\boldsymbol{m})$. These considerations suggest the transmission of information is indispensable. We assume

(A. 11) The manager informs the individual of the value of k.

This assumption has at least three implications. First, since we have

$$S_m(Y^{n+1})=k\sqrt{\bar{\boldsymbol{e}}_m(\boldsymbol{m})'\boldsymbol{P}_m^{-1}\bar{\boldsymbol{e}}_m(\boldsymbol{m})} \quad \text{in case (i)} \quad \text{and}$$

$$S_m(r_{n+1})=k\sqrt{\bar{\boldsymbol{e}}_m(\boldsymbol{m})'\boldsymbol{P}_m^{-1}\bar{\boldsymbol{e}}_m(\boldsymbol{m})}/X_s \quad \text{in case (ii),}$$

the greater value of k means the more riskiness of a mutual fund. Thus we can interpret k as a measure of riskiness of a mutual fund[7]. Secondly, in the actual economy it is very difficult or impossible to know the other person's utility function or transmit information about utility functions between persons. Under this assumption, we need not worry about this problem. Thirdly, the manager need not specify the form of v. From (13) and (14), we can observe that the demand for a mutual fund depends on (qX_s) and the function $\boldsymbol{z}(\boldsymbol{m})$ which is specified by k under (A. 11). Therefore the equilibrium value of (qX_s) in the market will depend on the value of k (if the market equilibrium exists). Hence the manager's estimation on the equilibrium commission revenue may be written as

$$(qX_s)^e=\phi(k) \tag{22}$$

As it is certain that the manager should act to maximize (qX_s) in this framework of the model, (22) is nothing but his objective function. He will choose the value of k that maximizes (22). This means k plays an essential role as a strategic variable of the manager instead of v[8].

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7) k is considered as the continuous form of the type of a mutual fund such as "balanced," "income," "income growth", "growth income" and "growth." Furthermore, it is linearly related to the "systematic risk" of the mutual fund portfolio β, since it can be written as

$$\beta=\frac{\text{Cov.}(r_{n+1}, r_M)}{S_m^2(r_M)}=k\cdot\frac{E_m(r_M)-r_0}{S_m^2(r_M)X_s}$$

where r_M is the rate of return on the market portfolio.

8) In the theory of the principal and agent relationship, it is assumed that both the principal and

After k is announced, the individual perceives the function $\boldsymbol{z}(\boldsymbol{m})$ as

$$\boldsymbol{z}(\boldsymbol{m})=k\boldsymbol{P}_m^{-1}\bar{\boldsymbol{e}}_m(\boldsymbol{m}) \tag{23}$$

which is rewritten, substituting (19), as

$$\boldsymbol{z}(\boldsymbol{m})=k(\boldsymbol{Bm}+\boldsymbol{c}) \tag{24}$$

where

$$\boldsymbol{B}=\boldsymbol{P}_m^{-1}\boldsymbol{L}\boldsymbol{T}^{-1} \tag{25}$$

$$\boldsymbol{c}=\boldsymbol{P}_m^{-1}(\bar{\boldsymbol{e}}-\boldsymbol{L}\boldsymbol{T}^{-1}\bar{\boldsymbol{m}}) \tag{26}$$

The individual may get some information about the distribution of $\boldsymbol{R}$ from the announced value of k. However, let us assume, for the moment

(A. 12) For the individual, the conditional distribution of $(\boldsymbol{R}, \boldsymbol{m})$ known k is the same as the prior distribution $F(\boldsymbol{R}, \boldsymbol{m})$.

In section 5, we shall relax this assumption to the case where the individual's perceived relation of k and m is a linear function[9].

Now, we can prove the following proposition.

Proposition 3. Suppose the assumptions (A. 1) through (A. 12) hold. If $\boldsymbol{L}$ is not a zero matrix, i. e., at least one of $\boldsymbol{m}$ is actually correlated with $\boldsymbol{R}$, the individual's demand for a mutual fund is strictly positive at least for a sufficiently low commission rate for any value of k.

(Proof) Substituting (24), we can rewrite the optimality condition (16) as

$$\frac{\partial E[u(Y)]}{\partial a}=E[u'(Y)(k\boldsymbol{e}'(\boldsymbol{Bm}+\boldsymbol{c})-r_0qX_s)]$$

$$=kE[u'(Y)\boldsymbol{e}'\boldsymbol{Bm}]-r_0qX_sE[u'(Y)]\leqq 0 \tag{16'}$$

since $kE[u'(Y)\boldsymbol{e}'\boldsymbol{c}]=0$ by (15).

We shall show $(a=0,\ \boldsymbol{x}=\hat{\boldsymbol{x}})$ does not satisfy this condition for $q=0$. (Notice that if $a=0$, $\boldsymbol{x}$ must be $\hat{\boldsymbol{x}}$ from (15).) This means the optimal value of a is strictly positive for a sufficiently low value of q by continuity. First, we write

$$dH(\boldsymbol{e}, \boldsymbol{m})=dH_2(\boldsymbol{m}|\boldsymbol{e})\,dH_1(\boldsymbol{e}).$$

When $q=0$, at $(a=0,\ \boldsymbol{x}=\hat{\boldsymbol{x}})$,

$$\frac{\partial E[u(Y)]}{\partial a}=kE[u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})\boldsymbol{e}'\boldsymbol{Bm}]$$

$$=k\int_{\boldsymbol{e},\boldsymbol{m}}u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})\boldsymbol{e}'\boldsymbol{Bm}\,dH(\boldsymbol{e}, \boldsymbol{m})$$

$$=k\int_{\boldsymbol{e}}u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})\boldsymbol{e}'\boldsymbol{B}\int_{\boldsymbol{m}}\boldsymbol{m}\,dH_2(\boldsymbol{m}|\boldsymbol{e})\,dH_1(\boldsymbol{e})$$

$$=k\int_{\boldsymbol{e}}u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})\boldsymbol{e}'\boldsymbol{B}\bar{\boldsymbol{m}}(\boldsymbol{e})\,dH_1(\boldsymbol{e}) \tag{27}$$

where $\bar{\boldsymbol{m}}(\boldsymbol{e})=\int_{\boldsymbol{m}}\boldsymbol{m}\,dH_2(\boldsymbol{m}|\boldsymbol{e})$: the conditional expected value of $\boldsymbol{m}$ given $\boldsymbol{e}$.

$\bar{\boldsymbol{m}}(\boldsymbol{e})$ is explicitly expressed, by the normality assumption (A. 9), as

$$\bar{\boldsymbol{m}}(\boldsymbol{e})=\bar{\boldsymbol{m}}+\boldsymbol{L}'\boldsymbol{P}^{-1}(\boldsymbol{e}-\bar{\boldsymbol{e}})$$

---

the agent have their own utility functions. But if the agent is the commission maximizer, his criterion function should be derived from maximization of the commission revenue. For instance, even when all principals have the same utility function, the agent may not behave as if he has that utility function in order to maximize his revenue. We are free from this type of the problem.

9) If k and/or the market price $\boldsymbol{p}$ are sufficient statistics of the signal $\boldsymbol{m}$, no one will demand a mutual fund for positive values of q.

$$=\boldsymbol{L}'\boldsymbol{P}^{-1}\boldsymbol{e}+\boldsymbol{g} \tag{28}$$

where $\boldsymbol{g}=\bar{\boldsymbol{m}}-\boldsymbol{L}'\boldsymbol{P}^{-1}\bar{\boldsymbol{e}}$.

Substitution of (28) into (27) leads

$$\frac{\partial Eu(Y)}{\partial a}=k\int_{\boldsymbol{e}} u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})\boldsymbol{e}'\boldsymbol{B}\boldsymbol{L}'\boldsymbol{P}^{-1}\boldsymbol{e}dH_1(\boldsymbol{e})$$

$$+k\int_{\boldsymbol{e}} u'(\boldsymbol{e}'\hat{\boldsymbol{x}}+r_0\overline{W})\boldsymbol{e}'\boldsymbol{B}\boldsymbol{g}dH_1(\boldsymbol{e}) \tag{29}$$

The second term of the RHS vanishes from the definition of $\hat{\boldsymbol{x}}$ in view of (18). We claim that an $n\times n$ matrix $\boldsymbol{A}=\boldsymbol{B}\boldsymbol{L}'\boldsymbol{P}^{-1}$ is *strictly positive semidefinite*, by which we mean (i) $\boldsymbol{A}$ is positive semidefinite and (ii) the set of n dimensional vectors $\{\boldsymbol{e}|\boldsymbol{e}'\boldsymbol{A}\boldsymbol{e}=0\}$ is limited to the strictly lower dimensional linear subspace than n. If this is true, (29) is proved to be strictly positive, and therefore $a=0$, $\boldsymbol{x}=\hat{\boldsymbol{x}}$ is not the optimal solution. This means the optimal value of a is strictly positive.

Since $\boldsymbol{A}$ is rewritten as

$$\begin{aligned}\boldsymbol{A}=\boldsymbol{B}\boldsymbol{L}'\boldsymbol{P}^{-1}&=\boldsymbol{P}_m^{-1}\boldsymbol{L}\boldsymbol{T}^{-1}\boldsymbol{L}'\boldsymbol{P}^{-1} &&\text{(from (25))}\\ &=\boldsymbol{P}_m^{-1}(\boldsymbol{P}-\boldsymbol{P}_m)\boldsymbol{P}^{-1} &&\text{(from (20))}\\ &=\boldsymbol{P}_m^{-1}-\boldsymbol{P}^{-1},\end{aligned}$$

it suffices to show $\boldsymbol{P}_m^{-1}-\boldsymbol{P}^{-1}$ is strictly positive semidefinite. Let the $(n+j)\times n$ matrix $\boldsymbol{C}=\begin{bmatrix}-\boldsymbol{P}^{-1}\boldsymbol{P}_m+\boldsymbol{I}\\ \boldsymbol{T}^{-1}\boldsymbol{L}'\end{bmatrix}$ where $\boldsymbol{I}$ is the identity matrix of order n. We can verify that, using (20),

$$\boldsymbol{C}'\boldsymbol{V}\boldsymbol{C}=\boldsymbol{P}_m-\boldsymbol{P}_m\boldsymbol{P}^{-1}\boldsymbol{P}_m=\boldsymbol{P}_m(\boldsymbol{P}_m^{-1}-\boldsymbol{P}^{-1})\boldsymbol{P}_m.$$

As $\boldsymbol{V}$ is positive definite, $\boldsymbol{P}_m^{-1}-\boldsymbol{P}^{-1}$ is clearly positive semidefinite, and it is strictly positive semidefinite if a set of n dimensional vector $\{\boldsymbol{y}|\boldsymbol{C}\boldsymbol{y}=0\}$ is limited to the lower dimentional linear subspace than n. $\boldsymbol{C}\boldsymbol{y}=0$ means

$$(-\boldsymbol{P}^{-1}\boldsymbol{P}_m+\boldsymbol{I})\boldsymbol{y}=0 \quad \text{and} \tag{30}$$

$$\boldsymbol{T}^{-1}\boldsymbol{L}'\boldsymbol{y}=0 \tag{31}$$

Premultiplying $\boldsymbol{P}$ and using (20), (30) becomes

$$\boldsymbol{L}\boldsymbol{T}^{-1}\boldsymbol{L}'\boldsymbol{y}=0 \tag{30'}$$

and premultiplying $\boldsymbol{T}$ to (31),

$$\boldsymbol{L}'\boldsymbol{y}=0 \tag{31'}$$

Let $\rho=$rank of $\boldsymbol{L}$. If $\boldsymbol{L}$ is not a zero matrix, $\rho\geqq 1$.

As is easily seen,

$$\{\boldsymbol{y}|\boldsymbol{C}\boldsymbol{y}=0\}=\{\boldsymbol{y}|(30')\ \text{and}\ (31')\}=\{\boldsymbol{y}|(31')\}=\{\boldsymbol{y}|\boldsymbol{L}'\boldsymbol{y}=0\}$$

which is the $(n-\rho)$ dimensional linear subspace whose dimension is strictly less than n. This completes the proof. (Q. E. D.)

5. Relaxation of (A. 7) and (A. 12)

We have assumed that the individual cannot observe any signal in the assumption (A. 7). In this section, we shall relax this assumption to some extent and it will enable us to relax the assumption (A. 12). Let us assume, instead of (A. 7),

(A. 7*) Before the market opens, the manager can observe the realization of $\boldsymbol{m}$, while the individual can only get the observation of $\boldsymbol{w}=\boldsymbol{D}\boldsymbol{m}$ where $\boldsymbol{w}=$

$(w_1, \cdots\cdots w_K)$, $K<J$ and $\boldsymbol{D}$ is a $K\times J$ matrix with rank K. The individual is informed neither of the realized value of $\boldsymbol{m}$ nor of the mutual fund portfolio.

This assumption includes the cases where the individual can only observe a subset of $\boldsymbol{m}$ or he can only observe some linear aggregates or $\boldsymbol{m}$.

Consider the $J\times J$ invertible matrix

$$\boldsymbol{\Gamma}=\begin{bmatrix}\boldsymbol{Q}\\ \boldsymbol{D}\end{bmatrix}$$

where $\boldsymbol{Q}$ is a $(J-K)\times J$ matrix such that $|\Gamma|\neq 0$. Such a matrix $\boldsymbol{Q}$ always exists. (To see this, partition the matrix $\boldsymbol{D}$ as $(\boldsymbol{D}_1\boldsymbol{D}_2)$ so that $\boldsymbol{D}_1$ is a $K\times(J-K)$ matrix and $\boldsymbol{D}_2$ is an invertible $K\times K$ matrix, which is always possible by renaming signals if necessary, and let $\boldsymbol{Q}=(\boldsymbol{I}_{J-K}\ 0)$ where $\boldsymbol{I}_{J-K}$ is the identity matrix of order $J-K$.) We can reduce the case of (A. 7*) to that of (A. 7) by transforming $\boldsymbol{m}$ with the matrix $\boldsymbol{\Gamma}$

$$\begin{bmatrix}\eta_Q\\ \boldsymbol{w}\end{bmatrix}=\boldsymbol{\Gamma}_m=\begin{bmatrix}\boldsymbol{Qm}\\ \boldsymbol{Dm}\end{bmatrix} \tag{32}$$

and regarding the conditional distribution of $(\boldsymbol{e}, \eta_Q)$ given $\boldsymbol{w}$ as the prior distribution of $(\boldsymbol{e}, \boldsymbol{m})$ in the previous discussions.

Let $G_m(\boldsymbol{e}|\boldsymbol{m})$ and $G_w(\boldsymbol{e}|\boldsymbol{w})$ be the conditional distributions of $\boldsymbol{e}$ given $\boldsymbol{m}$ and $\boldsymbol{w}$ respectively. If $G_w(\boldsymbol{e}|\boldsymbol{Dm})$ is equal to $G_m(\boldsymbol{e}|\boldsymbol{m})$ for all $\boldsymbol{m}$, the additional information of $\boldsymbol{m}$ to $\boldsymbol{w}$ has no value to estimate the distribution of $\boldsymbol{e}$ (or $\boldsymbol{R}$) and the observation of $\boldsymbol{w}$ is as effective as the observation of $\boldsymbol{m}$. Therefore, in this case, we cannot say the manager has the superior observation to forecast $\boldsymbol{e}$. On the contrary, if the additional information of $\boldsymbol{m}$ changes the individual's perceived distribution of $\boldsymbol{e}$, we can say the manager's observation is finer than the individual's observation. Formally,

Definition 1. We say the observatin of $\boldsymbol{w}$ is as effective as the observation of $\boldsymbol{m}$ and express as $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$ iff

$$G_w(\boldsymbol{e}|\boldsymbol{Dm})=G_m(\boldsymbol{e}|\boldsymbol{m}) \qquad \text{for all } \boldsymbol{m} \tag{33}$$

i. e., $\boldsymbol{w}$ is a sufficient statistic of $\boldsymbol{m}$ for estimating the distribution of $\boldsymbol{e}$.

Definition 2. We say the observation of $\boldsymbol{m}$ is finer than the observation of $\boldsymbol{w}$ and express as $\{\boldsymbol{m}\}\succ\{\boldsymbol{w}\}$ iff $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$ does not hold.

Lemma. The following three statements are all equivalent.

(i) $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$,

(ii) For *any* $\boldsymbol{Q}$, $\boldsymbol{e}$ and η_Q are conditionally independent for any given value of $\boldsymbol{w}$, i. e., the conditional probability density of $(\boldsymbol{e}, \eta_Q)$ can be written as

$$dG_1(\boldsymbol{e}, \eta_Q|\boldsymbol{w})=dG_w(\boldsymbol{e}|\boldsymbol{w})\,dG_2(\eta_Q|\boldsymbol{w}) \qquad \text{for all } \boldsymbol{w} \tag{34}$$

(iii) For *some* $\boldsymbol{Q}$, (34) holds,

where $dG_2(\eta_Q|\boldsymbol{w})$ is the conditional probalibity density of η_Q given $\boldsymbol{w}$.

(Proof) We can write

$$\begin{aligned} dG_1(\boldsymbol{e}, \eta_Q|\boldsymbol{w}) &= dG_m(\boldsymbol{e}|\eta_Q, \boldsymbol{w})\,dG_2(\eta_Q|\boldsymbol{w})\\ &= dG_m(\boldsymbol{e}|\boldsymbol{m})\,dG_2(\eta_Q|\boldsymbol{w}) \end{aligned} \tag{35}$$

for all $\boldsymbol{w}$ and $\boldsymbol{m}$ such that $\boldsymbol{w}=\boldsymbol{Dm}$.

Since $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$ is equivalent to

$$dG_w(\boldsymbol{e}|\boldsymbol{w})=dG_m(\boldsymbol{e}|\boldsymbol{m}) \qquad \text{for all } \boldsymbol{w} \text{ and } \boldsymbol{m} \text{ such that } \boldsymbol{w}=\boldsymbol{Dm} \tag{36}$$

if $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$, substitution of (36) into (35) gives (34) for any $\boldsymbol{Q}$. Thus (i) means (ii). Obviously (ii) means (iii). Further, if (33) holds for some $\boldsymbol{Q}$, comparison of (34) and (35) leads (36). Thus $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$. (Q. E. D.)

Immediately we have, as the converse of this lemma,

Corollary. The following three statements are all equivalent.

(i) $\{\boldsymbol{m}\}\succ\{\boldsymbol{w}\}$,

(ii) (34) does not hold for *some* $\boldsymbol{Q}$,

(iii) (34) does not hold for *any* $\boldsymbol{Q}$.

Therefore we can check whether $\{\boldsymbol{m}\}\sim\{\boldsymbol{w}\}$ or $\{\boldsymbol{m}\}\succ\{\boldsymbol{w}\}$ by calculating $dG_1(\boldsymbol{e}, \boldsymbol{\eta}_Q|\boldsymbol{w})$ for some $\boldsymbol{Q}$ such that $|\boldsymbol{\Gamma}|=0$.

From the lemma, we can generalize Proposition 2 to the case of (A. 7*) as

*Proposition 2**. Suppose (A. 1) ~ (A. 6), (A. 7*) and (A. 8) hold. If $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$, the individual has no incentive to demand a mutual fund even if $q=0$ and does not demand it for $q>0$.

That is, if the individual's observation is as effective as the manager's observation, he has no incentive to demand a mutual fund even when the commission rate is zero and never demands it for the positive commission rate.

Next, we consider the case where $(\boldsymbol{e}, \boldsymbol{m})$ is distributed as normal. In this case, the conditional distribution of $(\boldsymbol{e}, \boldsymbol{\eta}_Q)$ given $\boldsymbol{w}$ is the $n+(J-K)$ variate normal. Hence, $\boldsymbol{e}$ and η_Q are mutually independent if and only if their covariance matrix is a zero matrix. This means, together with the corollary,

*Proposition 3**. Suppose (A. 1) ~ (A. 6), (A. 7*) and (A. 8) ~ (A. 12) hold. If $\{\boldsymbol{m}\}\succ\{\boldsymbol{w}\}$, i. e., the manager's observation is finer than his observation, the individual actually demand a mutual fund at least for a sufficiently low commission rate for any value of $k>0$.

Now we can relax the assumption (A. 12) as follows. Let us assume

(A. 12*) The individual perceives that the announced value of k conveys information about the value of m by the linear function

$$k=\sum_{j=1}^{J}\alpha_j m_j+\beta$$

where $\alpha=(\alpha_1, \cdots\cdots\alpha_J)$ and β are some constants and the rank of the $K+1\times J$ matrix $\hat{\boldsymbol{D}}=\begin{bmatrix}\boldsymbol{D}\\ \alpha\end{bmatrix}$ is $K+1$ and $K+1<J$.

From Proposition 3* we immediately obtain

*Proposition 3***. Suppose (A. 1) ~ (A. 6), (A. 7*), (A. 8) ~ (A. 11) and (A. 12*) hold. If the manager's observation is finer than his observation, the individual actually demand a mutual fund at least for a sufficiently low commission rate for any positive value of k.

Finally we will derive the condition for $\{\boldsymbol{m}\}\succ\{\boldsymbol{w}\}$.

Proposition 4. Under the assumtions (A. 7*) and (A. 9),

$$\{\boldsymbol{m}\}\succ\{\boldsymbol{w}\} \quad \text{iff} \quad \boldsymbol{IM}\neq 0$$

where

$$\boldsymbol{M}=\boldsymbol{T}^{-1}-\boldsymbol{D}'(\boldsymbol{DTD}')^{-1}\boldsymbol{D} \tag{37}$$

(Proof) We directly compare two conditional distributions, $G_w(\boldsymbol{e}|\boldsymbol{Dm})$ and $G_m(\boldsymbol{e}|\boldsymbol{m})$.

The conditional distribution of $\boldsymbol{e}$ given $\boldsymbol{w}$ is the n variate normal with mean

$$\bar{\boldsymbol{e}}_w(\boldsymbol{w})=\bar{\boldsymbol{e}}+\boldsymbol{LD}'(\boldsymbol{DTD}')^{-1}(\boldsymbol{w}-\boldsymbol{D}\bar{\boldsymbol{m}}) \tag{38}$$

and covariance matrix

$$\boldsymbol{P}_w=\boldsymbol{P}-\boldsymbol{LD}'(\boldsymbol{DTD}')^{-1}\boldsymbol{DL}'. \tag{39}$$

Thus we have, from (19) and (38)

$$\bar{\boldsymbol{e}}_m(\boldsymbol{m})-\bar{\boldsymbol{e}}_w(\boldsymbol{Dm})=\boldsymbol{LM}(\boldsymbol{m}-\bar{\boldsymbol{m}}) \tag{40}$$

and from (20) and (39)

$$\boldsymbol{P}_m-\boldsymbol{P}_w=-\boldsymbol{LML}'. \tag{41}$$

Therefore $\bar{\boldsymbol{e}}_m(\boldsymbol{m})=\bar{\boldsymbol{e}}_w(\boldsymbol{Dm})$ for all $\boldsymbol{m}$ iff $\boldsymbol{LM}=0$, and if $\boldsymbol{LM}=0$, $\boldsymbol{P}_m=\boldsymbol{P}_w$ holds. This means $\{\boldsymbol{w}\}\sim\{\boldsymbol{m}\}$ iff $\boldsymbol{LM}=0$. Thus we get the conclusion. (Q. E. D.)

6. Summary

We have shown in this paper that

1. If the individual estimates the mutual fund portfolio deterministically or he is informed of it in advance, he has no incentive to demand a mutual fund even if the commission rate is zero and he does not demand it for the positive commission rate, no matter how good the performance of the mutual fund may be.
2. If the individual's observation is as effective as the manager's observatin to estimate the future returns on risky assets, there is no incentive for him to demand a mutual fund even when the commission rate is zero and he does not demand it for the positive commission rate.
3. If the manager's observation is actually finer than the individual's observation, he will demand a mutual fund at least for a sufficiently low commission rate.

We can conclude that the information transmitted from the manager to individuals and the manager's finer observation are important factors in discussing the role of a mutual fund and deriving the demand function for it.

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