Testing the Rotterdam Demand Model on the Japanese Expenditure Pattern*

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1. Introduction

There are several constraints on the demand equations which are derived from Micro-theory; that is, adding-up, homogeneity, symmetry, negativity, additivity, etc. Some of these constraints are introduced into the majority of complete demand systems without testing in order to decrease the number of parameters, to simplify their functional forms and to reduce the multicollinearity among income and prices. Although, in most demand systems, it is difficult to test the appropriateness of each of the applied constraints, we have two convenient systems by which each constraint in turn can be easily tested. These convenient demand systems are the Double Log System and the Rotterdam Model. However, in evaluating the results of testing by these systems, we must be careful that the rejection of a constraint does not necessarily imply the invalidity of Micro-theory, because we cannot judge whether the rejections are due to functional mis-specification, aggregation bias, or the general invalidity of Micro-theory. Furthermore, it is well known that the Double Log System and the Rotterdam Model have some drawbacks[1]. But it is very meaningful to test the constraints by these systems approximately. J. A. C. Brown and A. S. Deaton [1972] state the reasons why the test is important as follows.

"First if it turns out that investigations using aggregate data have produced results which are consistent with many of the postulates of the Micro-theory, then the problem of aggregation over consumers may be ignored with fewer misgivings than otherwise. Second, a knowledge of the weight of evidence for or against special aspects of the theory such as additivity or homogeneity will help when we come to assess those other models which have used such assumptions without submitting them to test."

This paper adopts the Rotterdam Model whose limitations are thought to be less restrictive. The following constraints are tested: (1) Homogeneity (2) Symmetry (3) Additivity, by making use of the Japanese "The Family Income and Expenditure Survey." In addition to these constraints, we test whether the constant term is useful, whether the substitution terms are important, and whether the demand system ——which depends upon only income and each commodity's own relative price——is meaningful.

Several studies on testing the constraints have been done on several countries other than Japan and on several commodity groups. A. P. Barten [1977] summarizes the previous studies as follows[2].

"Homogeneity passes more easily for small systems than for large ones, where it is usually firmly rejected. Symmetry passes more easily than homogeneity, but it also meets with various rejections. Negativity, given symmetry and homogeneity, does not seem to be very restrictive. Strong separability or additivity appears to be too limiting whenever its empirical validity is checked."

We will compare our Japanese results with those mentioned above in Section 4 of this paper.

We use the maximum likelihood estimators in estimating the parameters and the likelihood ratio test is used in testing each constraint in

* The author wishes to thank an anonymous referee for his valuable comments.

1) In the Double Log System, for instance, when income elasticities of all commodities are constant and the value share of luxury goods increases at a constant rate, the budget constraint will not be met. This model is consistent with the theory only when each income elasticity is unity. See R. P. Byron [1970] concerning in this system.

The defects of the Rotterdam Model are based on changing a differential equation into a difference equation. That is, there is a problem of non-integrability over all but infinitesimal time periods. See A. S. Deaton [1974] for a detail discussion about this point.

2) The Table II of A. P. Barten [1977] is very useful to obtain the information of the previous studies.

this paper.

2. The Rotterdam Model

Postulating that x_i and p_i are respectively the demand and price of the i–th commodity and y is the income or total expenditure, we can derive the following demand function by maximizing the well–defined utility function subject to the budget constraint.

$$x_i = x_i(y, p_1, \cdots\cdots, p_n). \tag{1}$$

From (1), we can derive

$$\partial x_i/\partial p_j = K_{ij} - \partial x_i/\partial y \cdot x_j, \tag{2}$$

where $K=\{K_{ij}\}$ indicates the substitution matrix. Differenciating equation (1) and using equation (2), we can obtain

$$\begin{aligned} dx_i &= \partial x_i/\partial y \cdot dy + \sum_{j=1}^{n} \partial x_i/p_j \cdot dp_j \\ &= \partial x_i/\partial y \{dy - \sum_{j=1}^{n} x_j dp_j\} + \sum_{k=1}^{n} K_{ik} dp_k. \end{aligned} \tag{3}$$

From the budget constraint, we can obtain

$$\sum_{j=1}^{n} p_j dx_j = dy - \sum_{k=1}^{n} x_k dp_k.$$

Substituting this equation into equation (3) and multiplying both sides by p_i/y, we have

$$w_i d \ln x_i = b_i \sum_{k=1}^{n} w_k d \ln x_k + \sum_{j=1}^{n} S_{ij} d \ln p_j, \quad i=1, \cdots\cdots, n, \tag{4}$$

where $w_i = p_i x_i/y$, $b_i = p_i \partial x_i/\partial y$ and $S_{ij} = p_i K_{ij} p_j/y$. This system of demand equations is called the Rotterdam Model. In this system, each constraint is described as follows.

[1] Adding–up $\sum_i b_i = 1, \sum_i S_{ij} = 0,$

[2] Homogeneity $\sum_j S_{ij} = 0,$

[3] Symmetry $S_{ij} = S_{ji},$ (5)

[4] Negativity $S=\{S_{ij}\}$, negative semi–definite

[5] Additivity[3] $S_{ij} = \phi(\delta_{ij} b_i - b_i b_j),$

$$\delta_{ij} = \begin{cases} 1 \text{ when } i=j, \\ 0 \text{ when } i \neq j, \end{cases}$$

ϕ: any constant value.

Changing the differential equations (4) to the difference equations and introducting the disturbance terms, v_{it}, we have

$$\begin{aligned} \bar{w}_{it}\, \varDelta \ln x_{it} &= b_i \sum_{k=1}^{n} \bar{w}_{kt}\, \varDelta \ln x_{kt} \\ &\quad + \sum_{j=1}^{n} S_{ij}\, \varDelta \ln p_{jt} + v_{it}, \\ & i=1, \cdots\cdots, n-1. \end{aligned} \tag{6}$$

where $\varDelta \ln x_{it} = \ln x_{it} - \ln x_{it-1}$,

$\varDelta \ln p_{jt} = \ln p_{jt} - \ln p_{jt-1}$,

$\bar{w}_{it} = (w_{it} + w_{it-1})/2.$

Under the homogeneity restriction, equation (6) is written as

$$\begin{aligned} \bar{w}_{it}\, \varDelta \ln x_{it} &= b_i \sum_{k=1}^{n} \bar{w}_{kt}\, \varDelta \ln x_{kt} \\ &\quad + \sum_{j=1}^{n-1} S_{ij}\, \varDelta \ln (p_{jt}/p_{nt}) + v_{it}, \\ & i=1, \cdots\cdots, n-1. \end{aligned} \tag{7}$$

When additivity is imposed, the demand system is

$$\begin{aligned} \bar{w}_{it}\, \varDelta \ln x_{it} &= b_i \sum_{k=1}^{n} \bar{w}_{kt}\, \varDelta \ln x_{kt} \\ &\quad + \phi b_i \left\{ \varDelta \ln (p_{it}/p_{nt}) - \sum_{j=1}^{n-1} b_i\, \varDelta \ln (p_{jt}/p_{nt}) \right\} + v_{it}, \\ & i=1, \cdots\cdots, n-1. \end{aligned} \tag{8}$$

As other comparable models, we introduce the following three types. We adopt the model with constant terms as the first type. The constant terms a_i are thought to show the effects of factors other than prices and income, such as habit formations. The adding–up constraint leads to $\sum_{i=1}^{n} a_i = 0$. The second type is the model which does not have the substitution terms; that is, $S_{ij}=0$. This model tests the effects of price changes on demand. The last type is the model whose equations respectively depend upon its own relative price and is independent of the prices of other goods. This model is described as

$$\begin{aligned} \bar{w}_{it}\, \varDelta \ln x_{it} &= b_i \sum_{k=1}^{n} \bar{w}_{kt}\, \varDelta \ln x_{kt} \\ &\quad + S_{ii}\, \varDelta \ln (p_{it}/p_{nt}) + v_{it}, \\ & i=1, \cdots\cdots, n-1. \end{aligned} \tag{9}$$

This type of model can be found in some empirical studies, although these do not adopt the difference of the variables. The last two models fulfill the constraints of adding–up, homogeneity and symmetry.

3. Estimation Procedure and Likelihood Ratio Test

Under the adding–up constraint, the variance–convariance matrix of $V_t = [v_{1t}, \cdots\cdots, v_{nt}]'$,

3) When we impose the i–th constraint, the constraints of numbers smaller than i are imposed without notice.

Under the additivity constraint, the utility function is additive with respect to each commodity.

Ω, is singular; that is, let $l=[1, \cdots\cdots, 1]'$,

$$\Omega l=\{E(V_t V_t')l\}=E(V_t V_t' l)=0, \qquad (10)$$

so that one equation of n equations is redundant. Since it is well known that the result does not depend upon which equation is excluded, we exclude the n-th equation. Let $V_t^*=[v_{1t}, \cdots\cdots, v_{n-1t}]'$ and its variance-covariance matrix be Ω^*. When we assume that the components of V_t^* have a zero expectation value and are jointly normally distributed, the joint density of V_t is

$$f(V_t)=2\pi^{-(n-1)/2}|\Omega^*|^{-\frac{1}{2}}\exp(-V_t^{*\prime}\Omega^{*-1}V_t^*)/2. \qquad (11)$$

If the number of observations is T and we assume that $V_1^*, \cdots\cdots, V_T^*$ are mutually independent, the log-likelihood function is

$$L=-[T(n-1)\ln 2\pi+T\ln|\Omega^*| + \sum_t V_t^{*\prime}\Omega^{*-1}V_t^*]/2, \qquad (12)$$

where $E(v_{it}v_{js})\begin{cases}\neq 0 \text{ when } t=S,\\ =0 \text{ when } t\neq S.\end{cases}$

Since the maximum likelihood estimator of Ω^* is $A=\frac{1}{T}\sum_t V_t^* V_t^{*\prime}$, the log-likelihood function is

$$L_1=-T[(n-1)(1+\ln 2\pi)+\ln|A|]/2. \qquad (13)$$

The assumption that $V_1^*,\cdots\cdots,V_T^*$ are mutually independent corresponds to Zellner's seemingly uncorrelated assumption.

In the cases of the adding-up constraint (henceforth, we shall call this the no constraint case), the homogeneity constraint and no substitution term, the maximum likelihood estimators are consistent with the estimators of ordinary least squares which is applied to each equation separately. That is, we apply ordinary least squares to equation (6) in the case of no constraint and to equation (7) in the case of homogeneity.

Under the symmetry constraint, we apply iterative constrained generalized least squares to the equations of system (7). When we describe the symmetry constraint as

$$RS=0 \qquad (14)$$

where

$$S=[\beta_1, \cdots\cdots, \beta_{n-1}]',$$
$$\beta_i=[b_i S_{i1}, \cdots\cdots, S_{in-1}]',$$

we can write the estimator $\hat{S}$ of S as

$$\hat{S}=S^*-CR'(RCR')^{-1}RS^*, \qquad (15)$$

where

$$S^*=\begin{bmatrix}\hat{\beta}_1\\ \vdots\\ \hat{\beta}_{n-1}\end{bmatrix}=\begin{bmatrix}(X'X)^{-1}X'Y_1\\ \vdots\\ (X'X)^{-1}X'Y_{n-1}\end{bmatrix}$$

$$C=[X'(\hat{A}^{-1}\otimes I)X]^{-1}=\hat{A}\otimes(X'X)^{-1}$$

$$X=\begin{bmatrix}\sum_{j=1}^n \bar{w}_{j1}\Delta\ln x_{j1} & \Delta\ln\left(\frac{p_{11}}{p_{n1}}\right) & \cdots & \Delta\ln\left(\frac{p_{n-11}}{p_{n1}}\right)\\ \vdots & \vdots & & \vdots\\ \sum_{j=1}^n \bar{w}_{jT}\Delta\ln x_{jT} & \Delta\ln\left(\frac{p_{1T}}{p_{nT}}\right) & \cdots & \Delta\ln\left(\frac{p_{n-1T}}{p_{nT}}\right)\end{bmatrix}$$

$$Y_i=\begin{bmatrix}\bar{w}_{i1}\ \Delta\ln\ x_{i1}\\ \vdots\\ \bar{w}_{iT}\ \Delta\ln\ x_{iT}\end{bmatrix}$$

Since $\hat{A}$ is an estimated value of variance-covariance matrix by residuals, we have to use a iterative procedure to obtain a maximum likelihood estimator. As the initial values, we use the estimates which are derived under the homogeneity constraint[4)].

In the case of the additivity constraint, when we define

$$B_{it}=b_i\left\{\Delta\ln(p_{it}/p_{nt})-\sum_{j=1}^{n-1}\Delta\ln(p_{jt}/p_{nt})\right\}$$

and describe equation (8) in vector form, we have

$$\begin{bmatrix}Y_1\\ \vdots\\ \vdots\\ Y_{n-1}\end{bmatrix}=\begin{bmatrix}X^* & 0 & \cdots & 0 & B_1\\ 0 & X^* & \cdots & 0 & B_2\\ \vdots & \vdots & & \vdots & \vdots\\ 0 & 0 & \cdots & X^* & B_{n-1}\end{bmatrix}\begin{bmatrix}b_1\\ \vdots\\ b_{n-1}\\ \phi\end{bmatrix}+\begin{bmatrix}v_1\\ \vdots\\ \vdots\\ v_{n-1}\end{bmatrix}$$

or

$$Y=\bar{X}\beta+V$$

where

$$X^*=\begin{bmatrix}\sum_{j=1}^n \bar{w}_{j1}\ \Delta\ln\ x_{j1}\\ \vdots\\ \sum_{j=1}^n \bar{w}_{jT}\ \Delta\ln\ x_{jT}\end{bmatrix}$$

$$B_i=[B_{i1}\cdots B_{iT}]'$$

Making use of the estimates which are derived under the symmetry constraint as the initial values, we can obtain the estimator of β as

$$\hat{\beta}=\{\bar{X}'(A^{-1}\otimes I)\bar{X}\}^{-1}\bar{X}'(A^{-1}\otimes I)Y$$
$$=\{A\otimes(\bar{X}'\bar{X})^{-1}\}A^{-1}\otimes\bar{X}'Y. \qquad (17)$$

By this estimate $\hat{\beta}$, we estimate the values of B_{it} and A. Then we estimate $\hat{\beta}$ by these new B_{it} and A. Until the convergence is attained, this iteration is continued[5)]. When we estimate

4) Under the symmetry constraint there are three estimation methods. Refer to A. P. Barten and E. Geyskens [1975] with respect to the other two methods.

5) This procedure is introduced in H. Theil [1971], but we use it in the iterative form.

the system which depends upon its own relative price, we also use the iterative GLS. All estimators under each constraint which are obtained by the above procedure are consistent with maximum likelihood estimators.

In testing the hypothesis that a certain constraint is meaningful, we use the log-likelihood ratio test; that is, -2 log λ (λ is the likelihood ratio). However, -2 log λ only asymptotically distributes as χ^2 - distribution. There is a possibility in the χ^2 test that rejects the important hypothesis, especially in small samples. In small samples, the true distribution has a fatter tail than the χ^2 - distribution. Accordingly, when we do not reject the hypothesis by the χ^2 test, we also do not reject it by the true distribution. But when we reject it, there remains some possibility that we will not reject it by the true distribution. However, with respect to the cases of linear hypotheses such as no constant, homogeneity and no substitution, we can test these hypotheses more precisely by making use of the approximate equation of T. W. Anderson [1958]. When we assume that N, p, q and q_1 are respectively the number of observations, the number of equations, the number of parameters in a equation and the number of parameters which are restricted by the hypothesis, the approximate equation is written as

$$
\begin{aligned}
P_r\left\{-2\frac{m}{N}\log\lambda\leq Z\right\} &= P_r\{\chi^2_{pq_1}\leq Z\} \\
&+\frac{\gamma_2}{m^2}(P_r\{\chi^2_{pq_1+4}\leq Z\}-P_r\{\chi^2_{pq_1}\leq Z\}) \\
&+\frac{1}{m^4}[\gamma_4(P_r\{\chi^2_{pq_1+8}\leq Z\}-P_r\{\chi^2_{pq_1}\leq Z\}) \\
&-\gamma_2^2(P_r\{\chi^2_{pq_1+4}\leq Z\}-P_r\{\chi^2_{pq_1}\leq Z\})] \\
&+R \qquad (18)
\end{aligned}
$$

where

$$
\begin{aligned}
m &= N-q+q_1-\frac{1}{2}(p+q_1+1) \\
\gamma_2 &= \frac{pq_1(p^2+q_1^2-5)}{48} \\
\gamma_4 &= \frac{\gamma_2^2}{2}+\frac{pq_1}{1920}[3p^4+3q_1^4+10p^2q_1^2 \\
&\quad -50(p^2+q_1^2)+159]
\end{aligned}
$$

If the first term of (18) is used, the error is in the order of N^{-2}; for the second, N^{-4}; and for the third, N^{-6}. As the observations become larger in equation (18), m/N approaches unity and the terms other than the first approach zero. Thus when we can use this correct test, we use it, and when we can not, we use the χ^2 test.

4. Results of Estimation

The data are derived from "Kakei–chōsa: 20 nen no hinmokubetsu shohi–keiretsu showa 26 nen∼46 nen" (Family income and expenditure survey: individual commodities' expenditure series for 20 years from 1951 to 1971) published by the Bureau of Statistics, the Office of the Prime Minister. The data on the family size are for cities with population of 50,000 or more and is derived from the "Annual report on the family income and expenditure survey." The observed period is 19 years from 1953 to 1971, and the commodities are divided into five groups, that is, (1) Food (2) Housing (3) Fuel and Light (4) Clothing (5) Miscellaneous. We exclude "other miscellaneous" and remittances from the miscellaneous group because it is difficult to obtain their exact price indices[6]. In the regression, real expendtiure for each commodity and total expenditure are divided by the family size. The estimated parameters under the no constraint case without constant terms, the homogeneity constraint, the symmetry constraint and the additivity constraint are respectively presented in Tables 1 to 4. In these tables, the values in the parentheses are standard errors or asymptotic standard errors, N_1 is the number of coefficients to be estimated and L_1 is the log-likelihood value corresponding to equation (13). In all cases in which iterative estimation procedures are adopted, we can obtain estimates with smooth convergences. In Table 5, the log-likelihood values and the number of coefficients to be estimated are collected. In Table 6, -2 log λ under each

6) We estimate each demand equation for the case in which "other miscellaneous" and remittances are included in the miscellaneous group. In this case the results of testing of each constraint are almost the same. However the hypothesis of no constant term is not rejected only by the correct test. In this estimation we find large positive values at S_{55}. For example, .2497 under the symmetry constraint and they are significant. By excluding "other miscellaneous" and remittances, these large positive values decrease. These large values are thought to depend upon the fact that the miscellaneous price indices do not correspond to its demands.

Table 1. No Constraint (without constant term)
$N_1=24$, $L_1=380.82$

	b_i	S_{ij} 1	2	3	4	5
1. Food	.2607 (.0565)	−.0538 (.0603)	.0776 (.0605)	−.0473 (.0503)	−.0173 (.0469)	.0336 (.0793)
2. Housing	.2541 (.0742)	.0420 (.0791)	−.0491 (.0793)	−.0495 (.0659)	−.0008 (.0616)	−.0469 (.1041)
3. Fuel and Light	.0328 (.0166)	.0008 (.0177)	−.0105 (.0177)	.0020 (.0147)	.0158 (.0137)	.0297 (.0232)
4. Clothing	.1404 (.0498)	−.0879 (.0531)	.0402 (.0532)	.1130 (.0443)	.0517 (.0413)	−.0234 (.0698)
5. Miscellaneous	.3121 (.0536)	.0989 (.0572)	−.0582 (.0573)	−.0182 (.0477)	−.0494 (.0455)	.0070 (.0752)

Table 2. Homogeneity $N_1=20$, $L_1=376.30$

	b_i	S_{ij} 1	2	3	4	5
1. Food	.2561 (.0343)	−.0565 (.0522)	.0796 (.0552)	−.0450 (.0435)	−.0144 (.0366)	.0363 (.0717)
2. Housing	.1879 (.0473)	.0025 (.0721)	−.0205 (.0762)	−.0165 (.0601)	.0405 (.0505)	−.0060 (.0990)
3. Fuel and Light	.0568 (.0114)	.0151 (.0174)	−.0208 (.0183)	−.0100 (.0145)	.0008 (.0122)	.0149 (.0238)
4. Clothing	.1997 (.0330)	−.0525 (.0503)	.0146 (.0531)	.0833 (.0418)	.0146 (.0352)	−.0600 (.0690)
5. Miscellaneous	.2994 (.0326)	.0913 (.0497)	−.0528 (.0525)	−.0118 (.0414)	−.0415 (.0348)	.0148 (.0682)

Table 3. Symmetry $N_1=14$, $L_1=372.55$

	b_i	S_{ij} 1	2	3	4	5
1. Food	.2919 (.0174)	−.0865 (.0303)	.0271 (.0217)	.0123 (.0095)	−.0290 (.0288)	.0761 (.0216)
2. Housing	.1850 (.0231)	.0271 (.0217)	.0090 (.0307)	−.0170 (.0086)	.0695 (.0207)	−.0886 (.0223)
3. Fuel and Light	.0539 (.0067)	.0123 (.0095)	−.0170 (.0086)	−.0166 (.0068)	.0076 (.0087)	.0136 (.0114)
4. Clothing	.1547 (.0182)	−.0290 (.0288)	.0695 (.0207)	.0076 (.0087)	.0276 (.0324)	−.0756 (.0214)
5. Miscellaneous	.3145 (.0167)	.0761 (.0216)	−.0886 (.0223)	.0136 (.0114)	−.0756 (.0214)	.0745 (.0289)

Table 4. Additivity
$N_1=5$, $L_1=349.86$

	b_i
1. Food	.2946 (.0168)
2. Housing	.2150 (.0240)
3. Fuel and Light	.0436 (.0060)
4. Clothing	.1495 (.0223)
5. Miscellaneous	.2973 (.0219)
ϕ	−.6489 (.0600)

Table 5. Log-likelihood Values

	L_1	N_1
No Constraint (with constant)	384.7649	28
No Constraint (without constant)	380.8178	24
Homogeneity	376.2960	20
Symmetry	372.5487	14
Additivity	349.8672	5
No Substitution	338.0982	4
Own Price	350.4299	8

hypothesis is presented and the values in parentheses are the χ^2 values of the 5% significant level.

First we test the hypothesis about the constant term. From the theoretical point of view, the existence of the constant term is not acceptable. This test investigates whether factors other than income and prices influence expenditure. By the χ^2 test, the value of -2 log λ (7.89) is less than the χ^2 values (9.49) so that the hypothesis of no constant term is not rejected. This fact means that there is a possibility that using only income and prices we can explain the demand. Our result is different from those of A. S. Deaton [1974] and A. P. Barten [1969]. A. S. Deaton [1974] used the U. K. data from 1900 to 1970 and A. P. Barten [1969] used the Netherlands data from 1922 to 1961. One possible interpretation of this difference is that it is due to the differences in the length of observed periods. As we lengthen the data preiod, factors other than income and prices, such as habit formation may become more important. However, since there are very few studies concerning the testing of the constant terms, we need more results to verify this hypothesis. According to this result of testing, when we estimate the coefficients under the other constraints, we exclude the constant terms.

According to Table 6, the results of the other tests show that we cannot reject the three hypotheses, i. e., homogeneity, joint test of homogeneity and symmetry and symmetry under the assumption of homogeneity, but we can reject additivity, no substitution and own relative price system. Even if we use the correct test of equation (18), we

Table 6. Log-likelihood Ratio Test Statistics

Null \ Alternative	No Constraint (with constant)	No Constraint (without constant)	Homogeneity	Symmetry
No Constraint (without constant)	7.89(9.49)			
Homogeneity		9.04 (9.49)		
Symmetry		16.54(18.31)	7.50(12.59)	
Additivity		61.90(30.14)	52.86(25.00)	45.36(16.92)
No Substitution		85.44(31.41)	76.40(26.30)	68.90(21.03)
Own Price		60.78(26.30)	51.73(21.03)	44.24(12.59)

reject the constraint of no substitution term. Comparing our results with the previous studies, we can say the following. Our results show, too, that additivity is an extremely strong constraint. Symmetry and homogeneity easily pass the χ^2 test in our five commodity groups. The rejection of the no substitution and own relative price systems show the importance of the substitution terms and we may say that the prices of the other commodities, as well as the own price, have important effects on the expenditures in this system. The characteristic roots which are calculated by the estimates under symmetry——that is, Table 3——are -0.1164, -0.0299, -0.0090, and 0.0888. These values do not satisfy the negativity condition. If the characteristic roots are all negative, we can say that the negativity constraint passes without testing, but if the characteristic roots are not all negative, we have to estimate the demand system under the negativity constraint. However, we do not attempt this in this paper[7].

Since the S_{ij} term shows the reaction of the i–th commodity's compensated demand corresponding to the change of the j–th commodity's price, by dividing S_{ij} by the i–th commodity's value share we can obtain the compensated price elasticity e_{ij}^*, i. e., $e_{ij}^*=S_{ij}/w_i$. We can also obtain the total expenditure elasticity e_i by $e_i=b_i/w_i$. The value shares of 1965 are as follows: Food .4613; housing .1083, fuel and light .0516, clothing .1380, miscellaneous .2408. The price elasticities, which are based on 1965, can be easily calculated by the estimated parameters of Table 3 and these value shares. The total expenditure elasticity of each commodity under the symmetry constraint is .6328 for food, 1.7082 for housing, 1.045 for fuel and light, 1.1217 for clothing, 1.3061 for miscellaneous.

7) See A. P. Barten and E. Geyskens [1975] with respect to estimation procedure under the negativity constraint.

5. Concluding Remarks

We approximately test some constraints of the demand theory by making use of the Rotterdam Model and the Japanese expenditure data of the five commodity groups from 1953 to 1971. We adopt the maximum likelihood estimators in each regression and apply the correct test in the cases of linear hypotheses and use the χ^2 test in the other cases. As the results of testing, we obtain the following findings. There is a possibility that the factors other than prices and income are not meaningful in the demand system of such short period as 19 years. We cannot reject the homogeneity and symmetry constraints. However, the additivity is shown to be extremely strong. This agrees with the results of previous studies. Finally, the effects of each commodity's prices are very important in explaining the expenditure patterns.

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