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Bundling Decision in Procurement Auctions with Risk-Averse Suppliers

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Abstract

In a two-stage procurement model, we compare two contracting schemes: bundling and unbundling. They differ in whether two sequential tasks of investment and service provision are bundled or not in the auction. We show that while unbundling causes underinvestment in cost reduction and bundling causes the *ex post* inefficiency of trade, each scheme imposes some forms of risk on the suppliers. The comparative statics results show that as the investment is more costly and/or a common cost component is more risky, bundling becomes less attractive to both the buyer and society.

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1 Introduction

Governments delegate the provision of a wide range of public infrastructures to the private sectors. These infrastructures consist of physical facilities (e.g. transportation, school, hospital) and public services (e.g. road maintenance, education, medical service). The government, which is responsible for the use of tax revenues, must find the efficient way of procurement.

An essential aspect of the infrastructure provision is that the process has a multi-stage feature. A public facility is designed, built, and operated for the purpose of delivering the associated service to people. The sequential process inevitably involves some issues concerning efficiency. The private company's investment in the design specifications or the building embodying innovative ideas can reduce costs in the subsequent stages. Thus, the public authority should induce the company to invest efficiently. On the other hand, the exact amount of cost is estimated only at the later stage, depending on exogenous factors such as the availability of resources or technological changes, and thus is uncertain at the outset. Then, for each task, the public authority must find a company which can perform the task at the least cost. Furthermore, the risk of construction or operation cost should be shared between the public authority and private company in the most efficient way.

As a way of procuring public infrastructures, public-private partnerships (PPPs) have become increasingly popular (Yescombe, 2007; OECD, 2008). One distinctive feature of PPPs is that a private party organized by some companies is responsible for performing many tasks such as design, construction, and operation under the long-term contract. On the other hand, in a way of traditional procurement, these sequential tasks are separated. In the case of construction, the scheme of design-build corresponds to PPPs, and that of design-bid-build corresponds to traditional procurement. Moreover, regardless of the scheme, the public procurement is often conducted through competitive bidding under the law (e.g. the Federal Acquisition Rules in the United States).

Our aim is to discover the factors affecting the optimal choice between *bundling* and *unbundling*. The two schemes differ in whether two sequential tasks of investment and service provision are bundled or not in the auction. We consider a two-stage model. A risk-neutral buyer must procure one unit of service (construction or operation of the facility) from one of risk-averse suppliers. Each supplier's production cost is determined by three elements: the (cost-reducing) investment, his private parameter and a common parameter. After the investment is made, each supplier's private parameter is privately known to him, and the common parameter is commonly known to all suppliers. It should be emphasized that no supplier initially knows these parameter values. Thus, there are risks of production costs. Each auction is held in a first-price format. A winner is awarded a fixed-price contract.

The main results show that the buyer and the society face the tradeoff which involves three factors: the *ex post* efficiency of trade, investment incentives, and risk sharing. First, unbundling allows the buyer to select the most efficient supplier, whereas bundling does not. Second, unbundling causes underinvestment relative to bundling. Thus, as the investment is more costly, bundling becomes relatively less attractive. Finally, each scheme imposes some forms of risk on the suppliers. The scheme of bundling exposes a winner to the risk of production cost. Although one may expect that the suppliers undertake less risks under unbundling, the scheme entails the risk associated with competition. On the other hand, the competition under unbundling can transfer the risk of common cost from the risk-averse suppliers to the risk-neutral buyer. Therefore, as the common cost is more risky, bundling becomes relatively less attractive.

A number of articles have addressed the issues of bundling decision in the context of PPPs. The main question is how each scheme of bundling and unbundling (possibly with an ownership structure of the facility) affects the suppliers' incentives for various kinds of investments. The literature can be classified into two categories according to the approach. The first one is the incomplete contract approach. In this strand, both the investment and resulting outcome (e.g. production cost) are assumed to be unverifiable. Hart (2003), who develops a leading model, shows that bundling provides stronger incentives for the cost-reducing investments than unbundling, at the expense of quality. In the experiment, Hoppe et al. (2011) find support for the theoretical prediction. For other studies, see Bennett and Iossa (2006), and Chen and Chiu (2010).

The second one is the complete contract approach. There are few studies which address risk-sharing issues, apart from Martimort and Pouyet (2008), and Li and Yu (2011); see also Schmitz (2005), Maskin and Tirole (2008), and Hoppe and Schmitz (2010). Martimort and Pouyet (2008) consider the quality-enhancing investment which reduces or increases the production cost, in an environment where contracts contingent on the production cost (and the quality level of the facility) are feasible. They show that the optimal bundling decision depends on the investment externality on the production cost. In contrast, Li and Yu (2011) explicitly consider an auction model. They examine how the bundling decision is affected by the externality of the first task and the intensity of competition (i.e. the number of suppliers). In the models of Martimort and Pouyet (2008) and Li and Yu (2011), the risks are shared between the buyer and supplier by writing performance contracts, as in standard moral-hazard models.

Although our model is close to that of Li and Yu (2011), there are two significant differences. First, we examine how the risks are shared in each scheme when the production cost is unverifiable. The point is that without relying on cost-sharing contracts, the risk of common cost (not private cost) can be effectively transferred to the buyer through competition under unbundling. Notice that all suppliers' (estimated) production costs are equally affected by the common parameter. Second, we introduce two kinds of risks of production cost (i.e. private and common parameters). In the model of Li and Yu (2011), there is no common cost parameter. With these parameters, it becomes clear what kind of risks in public procurement encourage the choice of each scheme. This has important policy implications. We will discuss the issue in Section 6.

The remainder of the paper is organized as follows. Section 2 presents the model. Sections 3 and 4 characterize the equilibrium under each scheme. Section 5 gives the comparative statics results which show when each scheme becomes more attractive to the buyer and society than the other. Section 6 concludes with a summary and discussion. All proofs are in the Appendix.

2 The model

Consider a buyer who must procure one unit of service from one of n suppliers. The buyer is risk-neutral. Each supplier $i \in N \equiv \{1, ..., n\}$ is risk-averse, and has a CARA utility function $u(\pi) = 1 - \exp(-r\pi)$, where $\pi \in \mathbb{R}$ is a profit from trade and $r > 0$ is his coefficient of absolute risk aversion.

A supplier i's (production) cost of service provision is given by $c(a, \theta_i, \omega)$, where $a \in \mathbb{R}_+$ is the investment level, $\theta_i \in [\underline{\theta}, \bar{\theta}]$ is the supplier i's private parameter, and $\omega \in [\underline{\omega}, \bar{\omega}]$ is a common parameter, which is identical across suppliers. We call the latter two variables the *risk of production cost*. We assume that $(\theta_1, ..., \theta_n, \omega)$ are independent random variables, and the cumulative distribution functions of θ_i and ω are respectively given by F and G, with $F' = f > 0$.[1] For convenience, let $\theta_{(n)} \equiv \min\{\theta_1, ..., \theta_n\}$, $F_{(n)}(\theta) \equiv 1 - (1 - F(\theta))^n$ and $f_{(n)}(\theta) \equiv n(1 - F(\theta))^{n-1} f(\theta)$ denote the lowest cost parameter, the cumulative distribution function and the probability density function of $\theta_{(n)}$, respectively. A supplier i incurs a cost $\psi(a)$ if he invests a. The buyer's valuation for the service is $v > 0$. Thus, all suppliers are *ex ante* symmetric. We make the following assumptions.

Assumption 1. $c : \mathbb{R}_+ \times [\underline{\theta}, \bar{\theta}] \times [\underline{\omega}, \bar{\omega}] \to \mathbb{R}_+$ is twice continuously differentiable in a, $\frac{\partial c}{\partial a} < 0$, $\frac{\partial^2 c}{\partial a^2} \geq 0$, continuous and increasing in θ_i.

Assumption 2. If $c(a, \theta_i, \omega) > c(a, \theta_i', \omega')$, then $-\frac{\partial c}{\partial a}(a, \theta_i, \omega) \geq -\frac{\partial c}{\partial a}(a, \theta_i', \omega')$.

Assumption 3. $\psi : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable, $\frac{d^2\psi}{da^2} > 0$, $\frac{d\psi}{da}(0) \leq 0$ and $\lim_{a\to\infty} \frac{d\psi}{da}(a) = \infty$.

Assumption 1 says that the investment reduces each supplier's production cost. Assumption 2, which means that a supplier with higher production cost can enjoy a greater benefit of *ex ante* investment in a weak sense, will play a key role in the analysis to yield clear-cut results. For instance, Assumptions 1 and 2 are satisfied by $c(a, \theta_i, \omega) = (\theta_i + \omega)/\bar{c}(a)$ or $c(a, \theta_i, \omega) = \theta_i + \omega - \bar{c}(a)$ with an appropriate function $\bar{c}(\cdot)$. Assumption 3 allows for the possibility that the investment cost is decreasing when the investment levels are low. This occurs if, for example, $\psi(a) = \bar{\psi}(a) - a$ is a total cost of investment; the term $-a$ means that the investment also reduces the total cost of the task.

There are two feasible contracting schemes: *bundling* and *unbundling*. Under bundling, the buyer bundles the two sequential tasks of investment and service provision, and awards a contract for both tasks to a single supplier via an auction. Under unbundling, the buyer

[1] We say that $f > 0$ if $f(\theta_i) > 0$ for all θ_i. We use the same notation for other functions.

separates those tasks, and sequentially awards a contract for each task via an auction. Each auction is held in a first-price sealed-bid format; a supplier $j \in N$ wins only if his bid $p_j \in \mathbb{R}_+$ is the lowest among $(p_1, ..., p_n)$.[2] The winner j must perform each task in exchange for a fixed payment p_j.

The game proceeds as follows. At date 0, the buyer chooses either bundling or unbundling. At date 1, each supplier $i \in N$ (simultaneously and independently) submits a bid $p_i^1 \in \mathbb{R}_+$ in the *first-stage auction*. At date 2, a winner j in the first-stage auction chooses an investment level $a \in \mathbb{R}_+$. At date 3, $(\theta_1, ..., \theta_n, \omega)$ are realized; in the scheme of bundling, the game then ends. At date 4, only in the scheme of unbundling, each supplier $i \in N$ submits a bid $p_i^2 \in \mathbb{R}_+$ in the *second-stage auction*; the game then ends.

The player payoffs are defined as follows. When the game ends at date 3 under bundling, a winner j with his bid p_j^1 obtains $u(p_j^1 - \psi(a) - c(a, \theta_j, \omega))$, the other suppliers obtain $u(0) = 0$, and the buyer obtains $v - p_j^1$. When the game ends at date 4 under unbundling, a winner j in the first-stage auction obtains $u(p_j^1 - \psi(a) - \pi_j)$, the other suppliers obtain $u(\pi_i)$, and the buyer obtains $v - (p_j^1 + p_k^2)$; (p_j^1, p_k^2) are winning prices in the auctions, and π_i is the supplier $i(\in N)$'s profit in the second-stage auction, where $\pi_i = p_i^2 - c(a, \theta_i, \omega)$ if i wins $(i = k)$ and $\pi_i = 0$ if i loses $(i \neq k)$.

The information structure is as follows. The buyer's choice of scheme and the identity of the winner in the first-stage auction become common knowledge among all players. The realized values of θ_i and $c(a, \theta_i, \omega)$ become the supplier i's private information, and that of ω becomes common knowledge among all suppliers. No supplier can observe the other suppliers' decisions. However, the winner's investment level a is commonly known to suppliers at date 3 because the assumption $\partial c/\partial a < 0$ implies that each supplier

[2] We assume that if there is a tie, then each supplier submitting the lowest bid wins with equal probability. All results hold for any other tie-breaking rule.

exactly knows the level a by his production cost and realized cost parameters.

Each player's (pure) strategy is defined in a standard way. The buyer's strategy is a choice of contracting scheme. Each supplier i's strategy is a triple (p_i^1, a_i, p_i^2), where $p_i^1 : \{\text{bundling}, \text{unbundling}\} \to \mathbb{R}_+$ is a bidding strategy in the first-stage auction, $a_i : \{\text{bundling}, \text{unbundling}\} \times \mathbb{R}_+ \to \mathbb{R}_+$ is a choice of investment level conditional on winning, and $p_i^2 : \mathbb{R}_+ \times N \times \mathbb{R}_+ \times [\underline{\theta}, \bar{\theta}] \times [\underline{\omega}, \bar{\omega}] \to \mathbb{R}_+$ ($\ni p_i^2(p_i^1, j, a, \theta_i, \omega)$ where j is a winner in the first-stage auction) is a bidding strategy in the second-stage auction. In the following sections, we explore the (pure strategy) perfect Bayesian equilibrium of the game. Since $(\theta_1, ..., \theta_n, \omega)$ are independent, no supplier updates his belief about the other suppliers' types in equilibrium; we also assume that this is the case in any off-equilibrium path.

We assume that the investment level a and the realized value of ω are unverifiable. If the buyer can initially offer a contract in which prices are contingent on the investment level a and the realized values of $(\theta_1, ..., \theta_n, \omega)$, then the efficient outcome is realized: (i) The buyer induces an arbitrary supplier i to choose the *efficient investment level* $\tilde{a}$, which minimizes expected total cost $\psi(a) + E[c(a, \theta_{(n)}, \omega)]$.[3] (ii) Given the realized values of $(\theta_1, ..., \theta_n, \omega)$, the buyer pays $c(\tilde{a}, \theta_{(n)}, \omega)$ to a supplier with the lowest private parameter $\theta_{(n)}$, who provides the service. The buyer then obtains the *first-best (expected) utility* $v - \{\psi(\tilde{a}) + E[c(\tilde{a}, \theta_{(n)}, \omega)]\}$.

3 Bundling

In this section, we characterize the equilibrium in the subgame after the buyer has chosen bundling. We apply backward induction.

By investing a, a winner i in the first-stage auction obtains the expected utility

[3] In this paper, $E[\cdot]$ represents the expectation operator of random variables.

$E[u(p_i^1 - \psi(a) - c(a, \theta_i, \omega))]$. The certainty equivalent which gives the same utility to the winner i is given by

$$p_i^1 - \psi(a) - E[c(a, \theta_i, \omega)] - \rho^*(a), \tag{1}$$

where $\rho^*(a) = \frac{1}{r} \ln E[\exp(rc(a, \theta_i, \omega))] - E[c(a, \theta_i, \omega)] > 0$ is his risk premium. We first examine the effect of the investment on the risk premium.

Lemma 1. $\frac{d\rho^*}{da}(a) \leq 0$ *for all* a.

The lemma states that a winner's risk premium is nonincreasing in his investment. This result depends on Assumption 2. With this assumption, the winner's investment has an effect to decrease the riskiness of production cost, and thus to decrease his risk premium. We will use this lemma to compare the equilibrium investment level with the efficient level.

The next proposition characterizes the equilibrium under bundling. Now, the *social welfare* in equilibrium is defined as the buyer's expected utility plus the sum of each supplier's certainty equivalent profit which gives the same utility as the equilibrium expected utility to the supplier.

Proposition 1. *Under bundling, any equilibrium is characterized as follows.*

(i) At least two suppliers submit the same bid $p^{1*} = \psi(a^*) + E[c(a^*, \theta_i, \omega)] + \rho^*(a^*)$, *and the other suppliers submit bids higher than* p^{1*}. *A winner* i *always chooses the investment level* a^* *determined by*

$$\frac{d\psi}{da}(a^*) = -E\left[\frac{\partial c}{\partial a}(a^*, \theta_i, \omega)\right] - \frac{d\rho^*}{da}(a^*). \tag{2}$$

(ii) $a^* \geq \tilde{a}$.

(iii) The buyer's utility EU_B^*, each supplier i's expected utility EU_i^*, and the social welfare W^* are respectively given by

$$EU_B^* = v - \{\psi(a^*) + E[c(a^*, \theta_i, \omega)] + \rho^*(a^*)\},$$

$$EU_i^* = 0,$$

$$W^* = v - \{\psi(a^*) + E[c(a^*, \theta_i, \omega)] + \rho^*(a^*)\}.$$

This proposition identifies some properties of bundling. First, the equilibrium investment level a^* is higher than the efficient level $\tilde{a}$. As in the first-order condition (2), a winner chooses the investment level to equate the marginal effects. Lemma 1 then implies that with the positive effect on the risk premium, the winner has an additional incentive to invest. Moreover, in contrast to the efficient outcome, the winner must provide the service even if his production cost is extremely high. Assumption 2 then implies that this contractual obligation increases the winner's incentive. Notice that overinvestment by the winner is efficient provided that he bears all the risk of production cost. This fact partially supports the argument of OECD (2008) that risk must be transferred to the party best able to carry it.

Second, the buyer's utility is the same level as the social welfare. This is because each supplier has no private information in the first-stage auction, and thus he cannot earn any rent.

Finally, the scheme of bundling involves two types of costs to both the buyer and society. The first one is the risk premium for the risk of production cost. Each supplier adds the risk premium to his bid. The second one is an efficiency loss from forgoing an opportunity to switch to a more efficient supplier. This is because the buyer commits not to switch suppliers under bundling.

4 Unbundling

In this section, we characterize an equilibrium in the subgame after the buyer has chosen unbundling. We apply backward induction.

The following lemma characterizes a symmetric equilibrium in the second-stage auction, in which all suppliers use the same bidding strategy. This result is based on Holt (1980).

Lemma 2. *Under unbundling, given the investment level a and the realized value of ω, the following bidding strategy $p^2(a, \cdot, \omega)$ and the suppliers' belief F constitute a symmetric equilibrium in the second-stage auction:*

$$p^2(a, \theta_i, \omega) = -\frac{1}{r} \ln E_{\theta_{(n-1)}}[\exp(-rc(a, \theta_{(n-1)}, \omega)) \mid \theta_{(n-1)} > \theta_i]. \tag{3}$$

In the following analysis, we focus on this symmetric equilibrium in the second-stage auction. This lemma has three implications. First, this bidding strategy is significantly affected by the information structure of the game. We can easily show that $p^2(a, \theta_i, \omega) > c(a, \theta_i, \omega)$. This is due to the "shading behavior" of the supplier i, who has private information about his private parameter θ_i. On the other hand, a common parameter ω is common knowledge among all suppliers, so that there is no room for shading. This fact will become clearer in Proposition 4. Second, the equilibrium bidding strategy $p^2(\cdot)$ is independent of the identity of the winner j in the first-stage auction. This is because the winner j's CARA utility function guarantees the absence of wealth effect, and thus his bidding behavior is independent of p_j^1 and his sunk investment cost $\psi(a)$. Third, the most efficient supplier wins because the equilibrium bidding strategy $p^2(\cdot)$ is increasing in θ_i.

We now examine the effect of the investment on the bidding strategy in the second-stage auction.

Lemma 3. $\frac{\partial p^2}{\partial a}(a, \theta_i, \omega) \leq \frac{\partial c}{\partial a}(a, \theta_i, \omega) < 0$ *for all* a, θ_i *and* ω.

The lemma states that each supplier's bid in the second-stage auction is decreasing in a winner's investment in the first-stage. This is because the winner's cost-reducing investment reduces all suppliers' production costs, and thus induces aggressive bidding by all of them. Moreover, the marginal effect is larger than that of cost reduction. As is well known in first-price auction models, a supplier i with θ_i computes his equilibrium bid by estimating the second-lowest production cost conditional on his parameter θ_i being the lowest; this fact is shown in (3). In short, the supplier i cares about more inefficient suppliers' production costs. Then, the statement $\partial p^2/\partial a \leq \partial c/\partial a$ follows from both the assumption that c is increasing in θ_i and Assumption 2 because suppliers with higher parameters than θ_i can enjoy greater benefits of cost-reducing investment than the supplier i.

By investing a, a winner i in the first-stage auction obtains the expected utility $E[(1 - F_{(n-1)}(\theta_i))u(p_i^1 - \psi(a) + p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega)) + F_{(n-1)}(\theta_i)u(p_i^1 - \psi(a))]$; the first term in the expectation corresponds to the case where the supplier i also wins the second-stage auction, and the second term corresponds to the case where he loses. The certainty equivalent which gives the same utility to the winner i in the first-stage auction is given by

$$p_i^1 - \psi(a) + E[(1 - F_{(n-1)}(\theta_i))(p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega))] - \rho^{**}(a), \tag{4}$$

where

$$\rho^{**}(a) = \frac{1}{r} \ln E[(1 - F_{(n-1)}(\theta_i)) \exp(-r(p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega))) + F_{(n-1)}(\theta_i)]$$
$$+ E[(1 - F_{(n-1)}(\theta_i))(p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega))]$$
$$> 0 \tag{5}$$

is his risk premium.

The next proposition characterizes an equilibrium under unbundling.

Proposition 2. *Under unbundling, there exists an equilibrium characterized as follows.*
(i) At least two suppliers submit the same bid $p^{1**} = \psi(a^{**})$ *in the first-stage auction, and the other suppliers submit bids higher than* p^{1**}. *A winner* i *always chooses the investment level* a^{**} *which satisfies*

$$\frac{d\psi}{da}(a^{**}) \geq \frac{E\left[\frac{\partial \pi}{\partial a}(a^{**}, \theta_i, \omega)(1 - F_{(n-1)}(\theta_i)) \exp(-r\pi(a^{**}, \theta_i, \omega))\right]}{E[(1 - F_{(n-1)}(\theta_i)) \exp(-r\pi(a^{**}, \theta_i, \omega)) + F_{(n-1)}(\theta_i)]}, \tag{6}$$

where $\pi(a, \theta_i, \omega) = p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega)$. *All suppliers follow the bidding strategy* $p^2(\cdot)$ *in the second-stage auction defined in Lemma 2.*
(ii) $\tilde{a} > a^{**}$.
(iii) The buyer's expected utility EU_B^{**}, *each supplier* i*'s expected utility* EU_i^{**}, *and the social welfare* W^{**} *are respectively given by*

$$EU_B^{**} = v - \left\{\psi(a^{**}) + E[c(a^{**}, \theta_{(n)}, \omega)] + nE[(1 - F_{(n-1)}(\theta_i))\pi(a^{**}, \theta_i, \omega)]\right\},$$
$$EU_i^{**} = u(E[(1 - F_{(n-1)}(\theta_i))\pi(a^{**}, \theta_i, \omega)] - \rho^{**}(a^{**})),$$
$$W^{**} = v - \left\{\psi(a^{**}) + E[c(a^{**}, \theta_{(n)}, \omega)] + n\rho^{**}(a^{**})\right\}.$$

This proposition identifies some properties of unbundling. First, the equilibrium investment level a^{**} is lower that the efficient level $\tilde{a}$. Lemma 3 implies that the right-hand side of (6) cannot be positive because the marginal effect of price reduction dominates that of cost reduction. Thus, the winner has no incentive to invest unless the investment has a considerable cost-reduction effect in the first stage (i.e. $d\psi(a)/da$ is sufficiently negative for some investment levels).

Second, in contrast to bundling, the buyer's utility is less than the social welfare. This is because each supplier acquires private information about his private cost parameter in the second-stage auction, and thus he can earn information rents. Notice that even if a supplier loses in the first-stage auction, he can participate in the second-stage auction. Hence, each supplier's reservation utility in the first-stage auction is endogenously determined by the expected utility in the second-stage auction.

Finally, the scheme of unbundling involves two types of costs to the buyer and society. The first one is the risk premium. Each supplier bears a fraction of the risk of production cost because he wins with positive probability in the second-stage auction. Moreover, each supplier must bear the risk associated with competition; the participation in the second-stage auction entails the risk of auction outcome (i.e. winning or losing, and winning price). The buyer must pay the premium indirectly because the expected information rent $E[(1-F_{(n-1)}(\theta_i))\pi(a^{**},\theta_i,\omega)]$ is greater than the risk premium $\rho^{**}(a^{**})$. The second one is an efficiency loss from underinvestment. Actually, a winner in the first-stage auction does not internalize the positive externality on the other suppliers.

5 Bundling versus unbundling

This section presents the main results. We compare the performance of bundling with that of unbundling, by analyzing each equilibrium in Sections 3 and 4. We say that bundling (unbundling) is *socially desirable* if $W^* > W^{**}$ ($W^* < W^{**}$). It is important to emphasize that the buyer and the society face the tradeoff which involves three factors:

the *ex post* efficiency of trade, investment incentives, and risk sharing.

First, we consider the issue of *ex post* efficiency of trade. The contractual flexibility of unbundling allows the buyer to select the most efficient supplier, whereas the contractual rigidity of bundling does not. The latter is a harmful effect of the buyer's commitment not to switch suppliers under bundling. Thus, with respect to the *ex post* efficiency of trade, unbundling is superior to bundling. However, notice that the buyer is obliged to pay the information rents to the suppliers, and thus cannot extract the full surplus under unbundling.

Second, we consider the issue of investment incentives.

Corollary 1. $a^* \geq \tilde{a} > a^{**}$. *If* $\frac{d\psi}{da}(0) = 0$, *then* $a^{**} = 0$.

This corollary is an immediate consequence of Propositions 1 and 2. In the scheme of bundling, a winner is responsible for providing the service in any state. Consequently, the contractual obligation affords strong investment incentives to the winner. On the other hand, in the scheme of unbundling, a winner in the first-stage auction has little incentive to invest. This is because the investment induces the suppliers to bid aggressively, and thus intensifies price competition in the second-stage auction. Without the cost-reducing effect in the first stage, the winner makes no investment.

The next proposition provides the comparative statics with respect to the investment cost.

Proposition 3. *Assume that* $\psi(a) = (k+1)\bar{\psi}(a)$, *where* $k \geq 0$ *is a parameter of the investment cost, and the function* ψ *satisfies Assumption 3 with* $\bar{\psi}(0) = d\bar{\psi}(0)/da = 0$. *Then, there exist thresholds* $\underline{k}$ *and* $\bar{k}$ *such that (i) the buyer chooses bundling in equilibrium for all* $k < \bar{k}$, *and unbundling for all* $k > \bar{k}$, *(ii) the socially desirable scheme is bundling for all* $k < \underline{k}$, *and unbundling for all* $k > \underline{k}$, *and (iii)* $\underline{k} \leq \bar{k}$.

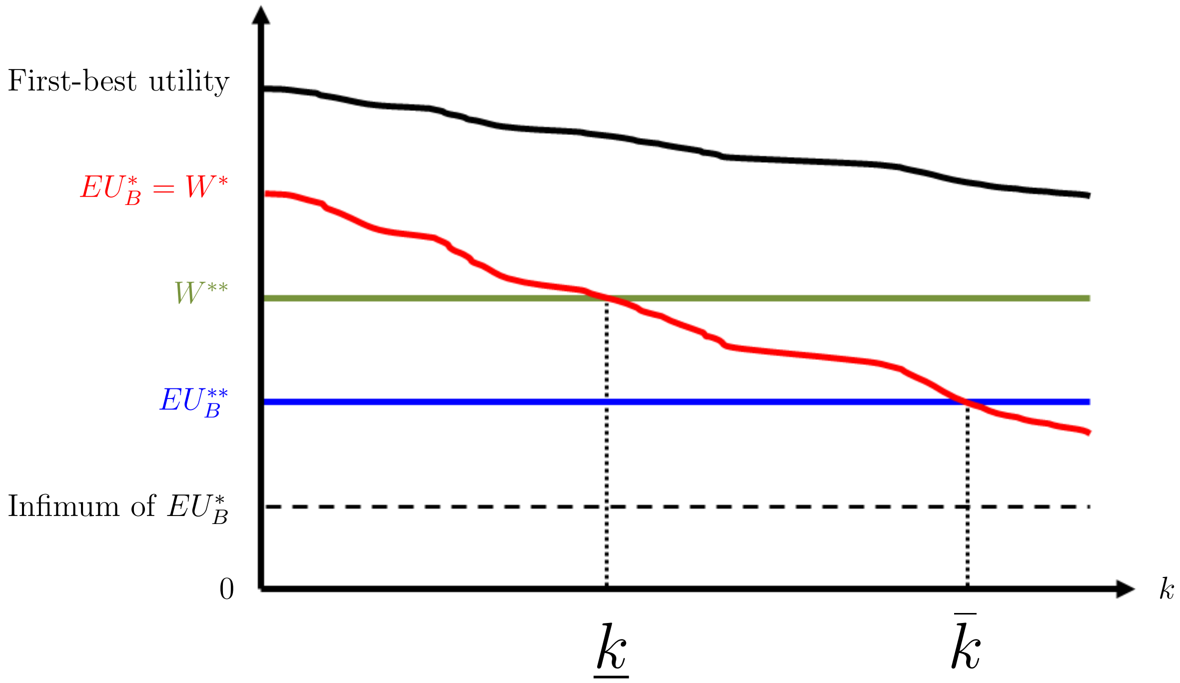

Figure 1: Illustration of Proposition 3.

The intuition of Proposition 3 is simple. In the scheme of unbundling, a winner in the first-stage auction has no incentive to invest because of the assumption $d\bar{\psi}(0)/da = 0$. Both the buyer's utility and social welfare under unbundling are thus independent of whether the investment is costly or not. On the other hand, in the scheme of bundling, a winner chooses the positive investment level. Therefore, as the investment is more costly (i.e. k becomes greater), the scheme of bundling becomes less attractive to both the buyer and society. Figure 1 illustrates this result.

Finally, we examine the issue of risk sharing. As analyzed in Sections 3 and 4, each supplier must bear some forms of risk in either scheme. Under bundling, a winner undertakes all the risk of production cost, which entails the risk premium $\rho^*(a^*)$. Under unbundling, the suppliers bear the risk associated with competition in the second-stage auction (and a fraction of the risk of production cost), which entails the risk premia $n\rho^{**}(a^{**})$. Unfortunately, we cannot say which one is greater than the other. However, we can obtain a noteworthy result by specifying the production cost function as follows.

Suppose that a common cost parameter is additively separable from the other terms, that is, $c(a, \theta_i, \omega) = \bar{c}(a, \theta_i) + \omega$. Notice that the equilibrium bidding strategy in the second-stage auction is then given by

$$p^2(a, \theta_i, \omega) = -\frac{1}{r} \ln E_{\theta_{(n-1)}}[\exp(-r\bar{c}(a, \theta_{(n-1)})) \mid \theta_{(n-1)} > \theta_i] + \omega.$$

The cost of common component ω is thus compensated by the buyer through competition regardless of its realized value. Then, in the scheme of unbundling, the risk of common cost component is effectively transferred from the suppliers to the buyer. With this specification, the next proposition provides the comparative statics with respect to the riskiness of common cost component.

Proposition 4. *Assume that $c(a, \theta_i, \omega) = \bar{c}(a, \theta_i) + \omega$, and an increase in $\beta \in [0, \hat{\beta}]$ is a mean-preserving spread of $G(\omega; \beta)$. Then, there exist thresholds $\underline{\beta}$ and $\bar{\beta}$ such that (i) the buyer chooses bundling in equilibrium for all $\beta < \bar{\beta}$, and unbundling for all $\beta > \bar{\beta}$, (ii) the socially desirable scheme is bundling for all $\beta < \underline{\beta}$, and unbundling for all $\beta > \underline{\beta}$, and (iii) $\underline{\beta} \leq \bar{\beta}$.*

Proposition 4 establishes the main result. As explained above, in the scheme of unbundling, no supplier undertakes the risk of common cost component, which is transferred to the risk-neutral buyer. Both the buyer's utility and social welfare under unbundling are thus independent of the riskiness of common cost component. On the other hand, in the scheme of bundling, a winner must undertake all the risk of production cost. Therefore, as the common cost component is more risky in the sense of Rothschild and Stiglitz (1970) (i.e. β becomes greater), the scheme of bundling becomes less attractive to both the buyer and society. Figure 2 illustrates this result.

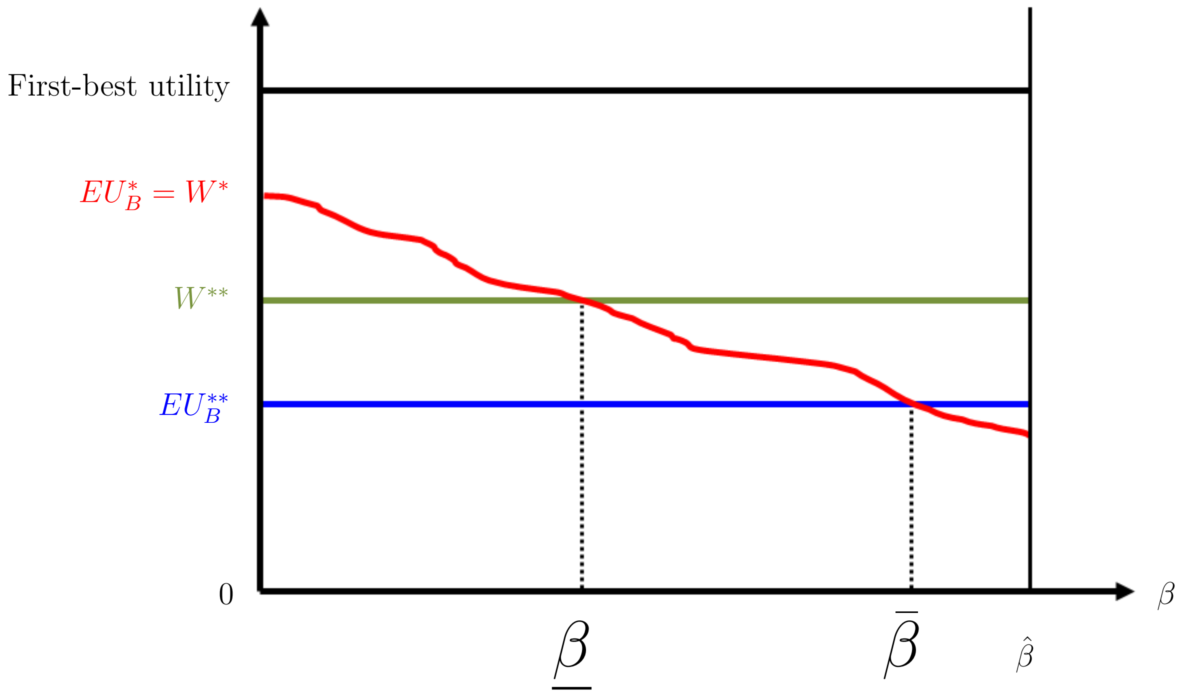

Figure 2: Illustration of Proposition 4.

6 Concluding remarks

We have explored what factors affect the buyer's optimal choice between bundling and unbundling. The buyer (and the society) faces the tradeoff which involves three factors: the *ex post* efficiency of trade, investment incentives, and risk sharing. The comparative statics results show that as the investment is more costly and/or a common cost component is more risky, bundling becomes relatively less attractive to both the buyer and society. The interesting effect of unbundling on risk sharing is that although the common cost parameter is unverifiable, this information is revealed through competition, so that the risk can be transferred from the suppliers to the buyer. This effect improves risk sharing.

It is now possible to answer the question what kind of risks encourage the choice of each scheme; Yescombe (2007) classifies project risks into some categories (p. 246). First, the increase in the riskiness of common cost parameter will encourage the choice of unbundling. We can give the following examples of the risk of common cost: the uncertainty about a project scope, the uncertainty about ground conditions, and rapid technological changes. Yescombe (2007) argues that those projects where technology is changing rapidly are not suitable for PPPs (p. 27). Actually, the United Kingdom abandoned the uses of PPPs for IT projects. Second, the increase in the riskiness of private cost parameter may or may not encourage the choice of unbundling. We can give the following examples of the risk of private cost: the uncertainty about the availability of a company's human and material resources. As we have shown, the scheme of unbundling entails the risk associated with competition, which is affected by the riskiness of private parameter. Thus, the effect of the riskiness on the public authority's choice is ambiguous.

We make some final remarks. First, we discuss the assumption that a winner in the first stage auction can participate in the second stage auction under unbundling. This setting is different from other studies on PPPs. However, even if the winner is excluded from the second stage auction, all our results are essentially unchanged. In this case, it is clear that the winner has no incentive to invest unless the cost-reducing effect in the first stage is positive.

Second, we discuss the issue of the buyer's commitment. The crucial difference between bundling and unbundling is the buyer's commitment. In particular, there is room for Pareto improvement under bundling. Suppose now that under bundling, the buyer can switch suppliers at date 4 by the first-price auction. Then, the equilibrium of bundling is equivalent to that of unbundling. There may be another scenario. Suppose that under bundling, the buyer allows a winner in the first-stage auction to hire a subcontractor in the second stage. When the winner selects the subcontractor by the first-price auction, the winner will undertake all the risk of common cost. Thus, our main result (Proposition

4) will still hold.

Finally, we examine how cost-sharing contracts affect the result. Suppose now that the production cost becomes verifiable after the service is provided. Then, some cost-sharing contracts are feasible. Under bundling, the buyer faces a standard tradeoff in moral hazard models. If the buyer compensates all the production cost, then a winner makes no investment. If the winner bears all the production cost, then he bears all the risk of production cost. Thus, the buyer optimally chooses the intermediate sharing level so that the winner always bears a fraction of common cost. On the other hand, in the scheme of unbundling, the risk of common cost can be transferred to the buyer, as in Proposition 4. We leave this analysis for future research.

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Appendix

Proof of Lemma 1. Differentiating ρ^* with respect to a yields

$$\begin{aligned}
\frac{d\rho^*}{da}(a) &= \frac{E\left[\frac{\partial c}{\partial a}(a,\theta_i,\omega)\cdot \exp(rc(a,\theta_i,\omega))\right]}{E[\exp(rc(a,\theta_i,\omega))]} - E\left[\frac{\partial c}{\partial a}(a,\theta_i,\omega)\right] \\
&= -\frac{\mathrm{Cov}\left(-\frac{\partial c}{\partial a}(a,\theta_i,\omega), \exp(rc(a,\theta_i,\omega))\right)}{E[\exp(rc(a,\theta_i,\omega))]} \\
&\leq 0;
\end{aligned}$$

the second equality follows from the formula for the covariance, and the inequality follows from the fact that the covariance between two positively covarying variables is nonnegative, together with Assumption 2. □

Proof of Proposition 1. (i) A winner i chooses an investment level to maximize his expected utility $E[u(p_i^1 - \psi(a) - c(a,\theta_i,\omega))]$, which is equal to the utility level $u(p_i^1 - \psi(a) - E[c(a,\theta_i,\omega)] - \rho^*(a))$ from the certainty equivalent (1). Note that the expected utility $E[u(p_i^1 - \psi(a) - c(a,\theta_i,\omega))]$ is strictly concave in a because the second derivative is given by $E[u''(\pi)\cdot(\partial\pi/\partial a)^2 + u'(\pi)\cdot \partial^2\pi/\partial a^2] < 0$, where $\pi = p_i^1 - \psi(a) - c(a,\theta_i,\omega)$; thus, $u(p_i^1 - \psi(a) - E[c(a,\theta_i,\omega)] - \rho^*(a))$ is also strictly concave in a. Hence, the equilibrium investment level a^* is uniquely determined by the first-order condition

$$\frac{d\psi}{da}(a^*) = -E\left[\frac{\partial c}{\partial a}(a^*,\theta_i,\omega)\right] - \frac{d\rho^*}{da}(a^*).$$

If a supplier i who submits a bid p_i^1 wins the first-stage auction, then he can obtain the expected utility $E[u(p_i^1 - \psi(a^*) - c(a^*,\theta_i,\omega))]$ in equilibrium. The certainty equivalent is $p_i^1 - \psi(a^*) - E[c(a^*,\theta_i,\omega)] - \rho^*(a^*)$. The first-stage auction is equivalent to the game of Bertrand competition among symmetric suppliers. Hence, in any equilibrium, there exist at least two suppliers who submit the same bid $p^{1*} = \psi(a^*) + E[c(a^*,\theta,\omega)] + \rho^*(a^*)$, which makes each supplier indifferent between winning and losing, and the other suppliers submit bids higher than p^{1*}.

(ii) First, the necessary and sufficient first-order condition for the efficient investment level $\tilde{a} = \arg\min\{\psi(a) + E[c(a, \theta_{(n)}, \omega)]\}$ is given by

$$\frac{d\psi}{da}(\tilde{a}) = -E\left[\frac{\partial c}{\partial a}(\tilde{a}, \theta_{(n)}, \omega)\right]. \tag{7}$$

It follows from the assumption $\frac{d\psi}{da}(0) \leq 0$ that $\tilde{a} > 0$.

Second, the necessary and sufficient first-order condition for the equilibrium investment level a^* under bundling is given by (2) in Proposition 1. The right-hand side of (2) satisfies the following inequalities:

$$-E\left[\frac{\partial c}{\partial a}(a^*, \theta_i, \omega)\right] - \frac{d\rho^*}{da}(a^*) \geq -E\left[\frac{\partial c}{\partial a}(a^*, \theta_i, \omega)\right] \geq -E\left[\frac{\partial c}{\partial a}(a^*, \theta_{(n)}, \omega)\right]; \tag{8}$$

the first inequality follows from Lemma 1, and the second inequality follows from the fact that the distribution function F of θ_i first-order stochastically dominates that of $\theta_{(n)}$, together with Assumptions 1 and 2 (see, for example, Appendix B of Krishna (2009)). It follows from (2), (7), (8) and the convexity of ψ that $a^* \geq \tilde{a}$.

(iii) The buyer's equilibrium utility is given by

$$EU_B^* = v - p^{1*} = v - \{\psi(a^*) + E[c(a^*, \theta, \omega)] + \rho^*(a^*)\}.$$

Since each supplier i who submits p^{1*} wins with equal probability under the tie-breaking rule, his equilibrium expected utility is given by

$$EU_i^* = \frac{1}{m}u(p^{1*} - \psi(a^*) - E[c(a^*, \theta_i, \omega)] - \rho^*(a^*)) + \frac{m-1}{m}u(0) = 0,$$

where $m \in \{2, ..., n\}$ is the number of suppliers who submit p^{1*}. Each supplier who submits a higher bid than p^{1*} receives zero payoff. Finally, the social welfare is given by

$$W^* = EU_B^* + n \cdot u^{-1}(EU_i^*) = v - \{\psi(a^*) + E[c(a^*, \theta, \omega)] + \rho^*(a^*)\}.$$

$\square$

Proof of Lemma 2. We must show that for all a and ω, a supplier i with θ cannot gain by deviating from a bid $p^2(a, \theta, \omega)$ when the other suppliers follow the strategy $p^2(\cdot)$. Note that $p^2(\cdot)$ is increasing and continuous in θ.

First, we can assume without loss of generality that no supplier submits a bid $p \notin [p^2(a, \underline{\theta}, \omega), p^2(a, \bar{\theta}, \omega)]$. If a supplier i bids $p > p^2(a, \bar{\theta}, \omega)$, then he loses with probability one; by bidding $p^2(a, \bar{\theta}, \omega)$, he can obtain the same utility. If a supplier i bids $p < p^2(a, \underline{\theta}, \omega)$, then he wins with probability one; by bidding $p^2(a, \underline{\theta}, \omega)$, he can win with probability one and obtain higher utility.

Second, we show that it is optimal for the supplier i with θ to bid $p = p^2(a, \theta, \omega)$. There are two cases to consider: The supplier i wins or loses in the first-stage auction. If the supplier i is different from the winner in the first-stage auction, then his expected utility from bidding $p^2(a, \hat{\theta}, \omega)$ is given by

$$
\begin{aligned}
&(1 - F_{(n-1)}(\hat{\theta}))u(p^2(a, \hat{\theta}, \omega) - c(a, \theta, \omega)) + F_{(n-1)}(\hat{\theta})u(0) \\
&= (1 - F_{(n-1)}(\hat{\theta})) \left[1 - \exp(rc(a, \theta, \omega)) \int_{\hat{\theta}}^{\bar{\theta}} \exp(-rc(a, s, \omega)) \frac{f_{(n-1)}(s)}{1 - F_{(n-1)}(\hat{\theta})} ds \right] \\
&= (1 - F_{(n-1)}(\hat{\theta})) \int_{\hat{\theta}}^{\bar{\theta}} \left[1 - \exp(-r(c(a, s, \omega) - c(a, \theta, \omega))) \right] \frac{f_{(n-1)}(s)}{1 - F_{(n-1)}(\hat{\theta})} ds \\
&= \int_{\hat{\theta}}^{\bar{\theta}} u(c(a, s, \omega) - c(a, \theta, \omega)) f_{(n-1)}(s) ds;
\end{aligned}
$$

the first equality is obtained by substituting $p^2(a, \hat{\theta}, \omega)$, and the other equalities follow from simple calculations. Thus, the difference between the expected utility from bidding $p^2(a, \theta, \omega)$ and that from bidding $p^2(a, \hat{\theta}, \omega) \neq p^2(a, \theta, \omega)$ is given by

$$
\int_{\theta}^{\hat{\theta}} u(c(a, s, \omega) - c(a, \theta, \omega)) f_{(n-1)}(s) ds > 0;
$$

the inequality follows from the assumption that $c(a, \theta_i, \omega)$ is increasing in θ_i.

If the supplier i is the winner with a winning price p_i^1 in the first-stage auction, then his expected utility from bidding $p^2(a,\hat{\theta},\omega)$ is given by

$$\begin{aligned}
&(1-F_{(n-1)}(\hat{\theta}))u(p_i^1-\psi(a)+p^2(a,\hat{\theta},\omega)-c(a,\theta,\omega))+F_{(n-1)}(\hat{\theta})u(p_i^1-\psi(a))\\
&=1-\exp(-r(p_i^1-\psi(a)))\left\{(1-F_{(n-1)}(\hat{\theta}))\exp(-r(p^2(a,\hat{\theta},\omega)-c(a,\theta,\omega)))+F_{(n-1)}(\hat{\theta})\right\}\\
&=1+\exp(-r(p_i^1-\psi(a)))\left\{(1-F_{(n-1)}(\hat{\theta}))\left[1-\exp(-r(p^2(a,\hat{\theta},\omega)-c(a,\theta,\omega)))\right]-1\right\}\\
&=1+\exp(-r(p_i^1-\psi(a)))\left\{\int_{\hat{\theta}}^{\bar{\theta}}u(c(a,s,\omega)-c(a,\theta,\omega))f_{(n-1)}(s)ds-1\right\};
\end{aligned}$$

the third equality is obtained in the same way as above, and the other equalities follow from simple calculations. Thus, the difference between the expected utility from bidding $p^2(a,\theta,\omega)$ and that from bidding $p^2(a,\hat{\theta},\omega)\neq p^2(a,\theta,\omega)$ is given by

$$\exp(-r(p_i^1-\psi(a)))\left\{\int_{\theta}^{\hat{\theta}}u(c(a,s,\omega)-c(a,\theta,\omega))f_{(n-1)}(s)ds\right\}>0.$$

Therefore, it is optimal for the supplier i with θ to bid $p=p^2(a,\theta,\omega)$, no matter whether he wins or loses in the first-stage auction. □

Proof of Lemma 3. Differentiating p^2 with respect to a yields

$$\begin{aligned}
\frac{\partial p^2}{\partial a}(a,\theta_i,\omega)&=\frac{E_{\theta_{(n-1)}}\left[\frac{\partial c}{\partial a}(a,\theta_{(n-1)},\omega)\cdot\exp(-rc(a,\theta_{(n-1)},\omega))\mid\theta_{(n-1)}>\theta_i\right]}{E_{\theta_{(n-1)}}\left[\exp(-rc(a,\theta_{(n-1)},\omega))\mid\theta_{(n-1)}>\theta_i\right]}\\
&\leq\frac{\frac{\partial c}{\partial a}(a,\theta_i,\omega)\cdot E_{\theta_{(n-1)}}\left[\exp(-rc(a,\theta_{(n-1)},\omega))\mid\theta_{(n-1)}>\theta_i\right]}{E_{\theta_{(n-1)}}\left[\exp(-rc(a,\theta_{(n-1)},\omega))\mid\theta_{(n-1)}>\theta_i\right]}\\
&=\frac{\partial c}{\partial a}(a,\theta_i,\omega)<0;
\end{aligned}$$

the first inequality follows from both the assumption that c is increasing in θ_i and Assumption 2. □

Proof of Proposition 2. We show that the subgame has the equilibrium characterized in the proposition, by backward induction.

(i) Lemma 2 implies that $p^2(\cdot)$ can be an equilibrium bidding strategy in the second-stage auction.

A winner i in the first-stage auction chooses an investment level to maximize his expected utility

$$E[(1 - F_{(n-1)}(\theta_i))u(p_i^1 - \psi(a) + p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega)) + F_{(n-1)}(\theta_i)u(p_i^1 - \psi(a))],$$

which is equal to the utility level

$$u(p_i^1 - \psi(a) + E[(1 - F_{(n-1)}(\theta_i))(p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega))] - \rho^{**}(a))$$

from the certainty equivalent (4). Differentiating the certainty equivalent (4) yields

$$E\left[(1 - F_{(n-1)}(\theta_i))\left(\frac{\partial p^2}{\partial a}(a, \theta_i, \omega) - \frac{\partial c}{\partial a}(a, \theta_i, \omega)\right)\right] - \frac{d\rho^{**}}{da}(a) - \frac{d\psi}{da}(a).$$

If this first-order derivative is greater than zero at $a = 0$, then there is an interior solution a^{**} determined by (6) with equality. Otherwise, either a corner solution $a^{**} = 0$ or an interior solution exists.

If a supplier i who submits a bid p_i^1 wins the first-stage auction, then he can obtain the expected utility $E[(1 - F_{(n-1)}(\theta_i))u(p_i^1 - \psi(a^{**}) + p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega)) + F_{(n-1)}(\theta_i)u(p_i^1 - \psi(a^{**}))]$ in equilibrium. The certainty equivalent is $p_i^1 - \psi(a^{**}) + E[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))] - \rho^{**}(a^{**})$. The first-stage auction is equivalent to the game of Bertrand competition among symmetric suppliers. However, note that even if the supplier i loses the first-stage auction, he can obtain the expected utility

$$E[(1 - F_{(n-1)}(\theta_i))u(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega)) + F_{(n-1)}(\theta_i)u(0)]$$

in the second-stage auction; the certainty equivalent is given by

$$E[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))] - \rho^{**}(a^{**}).$$

Hence, in any equilibrium, there exist at least two suppliers who submit the same bid $p^{1**} = \psi(a^{**})$, which makes each supplier indifferent between winning and losing, and the other suppliers submit bids higher than p^{1**}.

(ii) We show that $\tilde{a} > a^{**}$. If there exists a corner solution $a^{**} = 0$, then $\tilde{a} > a^{**} = 0$. If there exists an interior solution $a^{**} > 0$, then a^{**} is determined by (6) with equality in Proposition 2. The first-order condition can be rewritten as

$$\begin{aligned} \frac{d\psi}{da}(a) &= \frac{E\left[\left(\frac{\partial p^2}{\partial a}(a,\theta,\omega) - \frac{\partial c}{\partial a}(a,\theta,\omega)\right)(1 - F_{(n-1)}(\theta))\exp(-r\pi(a,\theta,\omega))\right]}{E\left[(1 - F_{(n-1)}(\theta))\exp(-r\pi(a,\theta,\omega)) + F_{(n-1)}(\theta)\right]} \\ &\leq 0; \end{aligned} \tag{9}$$

the inequality follows from Lemma 3. It then follows from (7) and (9) with the convexity of ψ that $\tilde{a} > a^{**}$.

(iii) Since in equilibrium the supplier with $\theta_{(n)}$ wins the second-stage auction, the buyer's expected utility is $EU_B^{**} = v - p^{1**} - E[p^2(a^{**}, \theta_{(n)}, \omega)]$. It follows from (i) that $p^{1**} = \psi(a^{**})$. The expected payment in the second-stage auction is given by

$E_\omega[E_{\theta_{(n)}}[p^2(a^{**}, \theta_{(n)}, \omega)]]$. Now, $E_{\theta_{(n)}}[p^2(a^{**}, \theta_{(n)}, \omega)]$ can be rewritten as

$$\begin{aligned}
&E_{\theta_{(n)}}[p^2(a^{**}, \theta_{(n)}, \omega)] \\
&= E_{\theta_{(n)}}\left[-\frac{1}{r}\ln E_{\theta_{(n-1)}}[\exp(-rc(a^{**}, \theta_{(n-1)}, \omega)) \mid \theta_{(n-1)} > \theta_{(n)}]\right] \\
&= E_{\theta_{(n)}}[c(a^{**}, \theta_{(n)}, \omega)] - \int_{\underline{\theta}}^{\bar{\theta}} c(a^{**}, s, \omega) f_{(n)}(s) ds \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} -\frac{1}{r}\ln E_{\theta_{(n-1)}}[\exp(-rc(a^{**}, \theta_{(n-1)}, \omega)) \mid \theta_{(n-1)} > s] f_{(n)}(s) ds \\
&= E_{\theta_{(n)}}[c(a^{**}, \theta_{(n)}, \omega)] \\
&\quad + \int_{\underline{\theta}}^{\bar{\theta}} -\frac{1}{r}\ln E_{\theta_{(n-1)}}[\exp(-r(c(a^{**}, \theta_{(n-1)}, \omega) - c(a^{**}, s, \omega))) \mid \theta_{(n-1)} > s] f_{(n)}(s) ds \\
&= E_{\theta_{(n)}}[c(a^{**}, \theta_{(n)}, \omega)] \\
&\quad - \int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{r}\ln E_{\theta_{(n-1)}}[\exp(-r(c(a^{**}, \theta_{(n-1)}, \omega) - c(a^{**}, s, \omega))) \mid \theta_{(n-1)} > s] n(1 - F_{(n-1)}(s)) f(s) ds \\
&= E_{\theta_{(n)}}[c(a^{**}, \theta_{(n)}, \omega)] + nE_{\theta_i}[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))];
\end{aligned}$$

the first and fifth equalities follow from the definition of $p^2(\cdot)$ in Lemma 2, and the fourth equality follows from the definition of $f_{(n)}$. Thus, EU_B^{**} is given by

$$\begin{aligned}
EU_B^{**} = v - \big\{&\psi(a^{**}) + E[c(a^{**}, \theta_{(n)}, \omega)] \\
&+ nE[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))]\big\}.
\end{aligned}$$

Since each supplier i who submits p^{1**} wins the first-stage auction with equal probability under the tie-breaking rule, his equilibrium expected utility is given by

$$\begin{aligned}
EU_i^{**} &= \frac{1}{m}u(p^{1**} - \psi(a^{**}) + E[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))] - \rho^{**}(a^{**})) \\
&\quad + \frac{m-1}{m}u(E[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))] - \rho^{**}(a^{**})) \\
&= u(E[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - c(a^{**}, \theta_i, \omega))] - \rho^{**}(a^{**})),
\end{aligned}$$

where $m \in \{2, ..., n\}$ is the number of suppliers who submit p^{1**}. Each supplier who submits a higher bid than p^{1**} also obtains the same level of expected utility.

Finally, the social welfare is given by

$$W^{**} = EU_B^* + n \cdot u^{-1}(EU_i^{**}) = v - \left\{\psi(a^{**}) + E[c(a^{**}, \theta_{(n)}, \omega)] + n\rho^{**}(a^{**})\right\}.$$

$\square$

Proof of Proposition 3. The proof consists of four steps: We show that (a) EU_B^* is continuous and decreasing in k, (b) both EU_B^{**} and W^{**} are independent of k, (c) $EU_B^* = W^*$ for all k, $\lim_{k\to\infty} EU_B^* < EU_B^{**} < W^{**}$, and (d) the statements in the proposition follow from (a)-(c).

(a) Since the equilibrium investment level a^* under bundling depends on k, we denote $a^* = a^*(k)$ and $EU_B^* = EU_B^*(k)$. Now, the buyer's equilibrium utility under bundling is

$$EU_B^*(k) = v - \left\{(k+1)\bar{\psi}(a^*(k)) + E[c(a^*(k), \theta_i, \omega)] + \rho^*(a^*(k))\right\}.$$

The assumption that both $c(a, \theta_i, \omega)$ and $\bar{\psi}(a)$ are twice continuously differentiable in a implies that $a^*(k)$ is differentiable, and thus continuous in k. Thus, EU_B^* is continuous in k. Using the envelope theorem, differentiating $EU_B^*(k)$ with respect to k yields

$$\frac{dEU_B^*}{dk}(k) = -\bar{\psi}(a^*(k)) < 0.$$

Hence, $EU_B^*(k)$ is decreasing in k.

(b) It follows from Corollary 1 that $a^{**} = 0$. Since the investment cost is $\psi(0) = (k+1)\bar{\psi}(0) = 0$ in equilibrium, both EU_B^{**} and W^{**} are independent of k.

(c) First, Proposition 1 states that $EU_B^*(k) = W^*(k)$ for all k. Second, it follows from the first-order condition (2) that

$$\lim_{k\to\infty} a^*(k) = 0.$$

This implies that the infimum of $EU_B^*(k)$ is given by

$$\begin{aligned}\lim_{k\to\infty} EU_B^*(k) &= v - \{E[c(0,\theta_i,\omega)] + \rho^*(0)\} \\ &= v - \left\{\frac{1}{r}\ln E[\exp(rc(0,\theta_i,\omega))]\right\};\end{aligned}$$

the second equality follows from the definition of ρ^*. The bracketed term satisfies the following inequalities:

$$\begin{aligned}\frac{1}{r}\ln E[\exp(rc(0,\theta_i,\omega))] &> E[c(0,\theta_i,\omega)] \\ &> E[c(0,\theta_{(n)},\omega)] \\ &= E\left[-\frac{1}{r}\ln\exp(-rc(0,\theta_{(n)},\omega))E_{\theta_{(n-1)}}[1 \mid \theta_{(n-1)} > \theta_{(n)}]\right] \\ &> E\left[-\frac{1}{r}\ln E_{\theta_{(n-1)}}[\exp(-rc(0,\theta_{(n-1)},\omega)) \mid \theta_{(n-1)} > \theta_{(n)}]\right] \\ &= E[p^2(0,\theta_{(n)},\omega)];\end{aligned}$$

the first inequality follows by Jensen's inequality, the second inequality follows from the fact that the distribution function F of θ_i first-order stochastically dominates that of $\theta_{(n)}$, together with Assumptions 1 and 2 (see, for example, Appendix B of Krishna (2009)), and the last equality follows from the definition of $p^2(\cdot)$. On the other hand, the buyer's equilibrium expected utility under unbundling is

$$\begin{aligned}EU_B^{**} &= v - \left\{p^{1**} + E[p^2(a^{**},\theta_{(n)},\omega)]\right\} \\ &= v - \left\{(k+1)\bar{\psi}(a^{**}) + E[p^2(a^{**},\theta_{(n)},\omega)]\right\} \\ &= v - E[p^2(0,\theta_{(n)},\omega)];\end{aligned}$$

the third equality follows from $\bar{\psi}(a^{**}) = \bar{\psi}(0) = 0$. Therefore, we obtain that

$$\lim_{k\to\infty} EU_B^*(k) < EU_B^{**}.$$

Third, it follows from Proposition 2 that $W^{**} - EU_B^{**} = n \cdot u^{-1}(EU_i^{**}) > 0$. We thus obtain $EU_B^{**} < W^{**}$.

(d) There are three cases to consider.

First, suppose that $W^{**} < EU_B^*(0)$. Then, it follows from (a)-(c) that there exist thresholds $\underline{k}$ and $\bar{k}$ such that the statements (i) and (ii) in the proposition hold, with $0 < \underline{k} < \bar{k} < \infty$.

Second, suppose that $EU_B^{**} < EU_B^*(0) \leq W^{**}$. Then, it follows from (a)-(c) that there exist thresholds $\underline{k}$ and $\bar{k}$ such that the statements (i) and (ii) in the proposition hold, with $0 = \underline{k} < \bar{k} < \infty$.

Last, suppose that $EU_B^*(0) < EU_B^{**}$. Then, it follows from (a)-(c) that $EU_B^*(k) < EU_B^{**}$ and $W^*(k) < W^{**}$ for all k. By setting $\underline{k} = \bar{k} = 0$, the statements (i) and (ii) in the proposition hold. $\square$

Proof of Proposition 4. The proof proceeds in the same way as Proposition 3, and consists of four steps: We show that (a) EU_B^* is decreasing in β, (b) both EU_B^{**} and W^{**} are independent of β, (c) $EU_B^* = W^*$ for all β, $EU_B^{**} < W^{**}$ and (d) the statements in the proposition follow from (a)-(c).

(a) Since the risk premium $\rho^*(a^*)$ under bundling depends on β, we denote $\rho^*(a^*) = \rho^*(a^*;\beta)$ and $EU_B^* = EU_B^*(\beta)$. Now, the buyer's equilibrium utility under bundling is

$$EU_B^*(\beta) = v - \left\{ \psi(a^*) + E[\bar{c}(a^*, \theta_i)] + \int_{\underline{\omega}}^{\bar{\omega}} \omega dG(\omega;\beta) + \rho^*(a^*;\beta) \right\}.$$

Since an increase in β represents a mean-preserving spread of G, the expected value of ω is independent of β, but the risk premium $\rho^*(a^*;\beta)$ is increasing in β. Hence, $EU_B^*(\beta)$ is decreasing in β.

(b) First, the buyer's equilibrium utility under unbundling is

$$
\begin{aligned}
&EU_B^{**} \\
&= v - \Bigg\{ \psi(a^{**}) + E[\bar{c}(a^{**}, \theta_{(n)})] + \int_{\underline{\omega}}^{\bar{\omega}} \omega dG(\omega; \beta) \\
&+ nE[(1 - F_{(n-1)}(\theta_i))(p^2(a^{**}, \theta_i, \omega) - (\bar{c}(a^{**}, \theta_i) + \omega))] \Bigg\} \\
&= v - \Bigg\{ \psi(a^{**}) + E[\bar{c}(a^{**}, \theta_{(n)})] + \int_{\underline{\omega}}^{\bar{\omega}} \omega dG(\omega; \beta) \\
&+ nE\left[(1 - F_{(n-1)}(\theta_i)) \left(-\frac{1}{r} \ln E_{\theta_{(n-1)}}[\exp(-r\bar{c}(a^{**}, \theta_{(n-1)})) \mid \theta_{(n-1)} > \theta_i] - \bar{c}(a^{**}, \theta_i) \right) \right] \Bigg\};
\end{aligned}
$$

note that under the assumption $c(a, \theta_i, \omega) = \bar{c}(a, \theta_i) + \omega$, the bidding strategy $p^2(a, \theta_i, \omega)$ is now given by

$$
p^2(a, \theta_i, \omega) = -\frac{1}{r} \ln E_{\theta_{(n-1)}}[\exp(-r\bar{c}(a, \theta_{(n-1)})) \mid \theta_{(n-1)} > \theta_i] + \omega. \tag{10}
$$

Since an increase in β represents a mean-preserving spread of G, EU_B^{**} is independent of β.

Second, it follows from (5) and (10) with the assumption $c(a, \theta_i, \omega) = \bar{c}(a, \theta_i) + \omega$ that the risk premium $\rho^{**}(a^{**})$ under unbundling is independent of β; note that a^{**} is independent of β because the first-order condition (9) does not include ω. Hence, W^{**} is also independent of β.

(c) Proposition 1 states that $EU_B^*(\beta) = W^*(\beta)$ for all β. In the same way as (a), we can show that $EU_B^{**} < W^{**}$.

(d) There are four cases to consider.

First, suppose that $W^{**} \leq EU_B^*(\hat{\beta})$. Then, it follows from (a)-(c) that $EU_B^*(\beta) > EU_B^{**}$ and $W^*(\beta) \geq W^{**}$ for all β. By setting $\underline{\beta} = \bar{\beta} = \hat{\beta}$, the statements (i) and (ii) in the proposition hold.

Second, suppose that $EU_B^{**} \leq EU_B^*(\hat{\beta}) < W^{**}$. Then, it follows from (a)-(c) that there exist thresholds $\underline{\beta}$ and $\bar{\beta}$ such that the statements (i) and (ii) in the proposition hold, with $0 \leq \underline{\beta} < \bar{\beta} = \hat{\beta}$.

Third, suppose that $EU_B^*(\hat{\beta}) < EU_B^{**} < EU_B^*(0)$. Then, it follows from (a)-(c) that there exist thresholds $\underline{\beta}$ and $\bar{\beta}$ such that the statements (i) and (ii) in the proposition hold, with $0 \leq \underline{\beta} < \bar{\beta} < \hat{\beta}$.

Last, suppose that $EU_B^*(0) \leq EU_B^{**}$. Then, it follows from (a)-(c) that $EU_B^*(\beta) \leq EU_B^{**}$ and $W^*(\beta) < W^{**}$ for all β. By setting $\underline{\beta} = \bar{\beta} = 0$, the statements (i) and (ii) in the proposition hold. $\square$