### Quantifying Borrowing Constraints and Precautionary Savings

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#### Abstract

This paper quantifies the effects of precautionary savings. It demonstrates that Zeldes' estimate [12] of excess consumption growth for low asset holders is consistent with a dynamic general equilibrium model with uninsurable endowment shocks when borrowing is constrained at three months' worth of average wage income. I propose a Monte Carlo simulation of the stationary equilibrium as a method of indirectly testing the hypotheses of a no-borrowing specification and a natural debt limit specification. At the estimated borrowing constraint, an increase in endowment shocks within the range of empirical findings can cause a 1.6% increase in the savings rate and a 6.9% increase in capital.

JEL classification codes: E21, C68

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## 1 Introduction

This paper provides a numerical assessment of the importance of precautionary savings as an explanation of the excess consumption growth rate observed for consumers with low assets. In so doing, the paper also proposes a method of deriving a test distribution for the borrowing constraint that is inferred indirectly by an empirical estimate of the excess consumption growth.

The analysis is built upon Aiyagari's [1] dynamic general equilibrium model with uninsured endowment shocks and Zeldes' [12] empirical findings on individual consumption growth. Zeldes reports that those who hold assets of less than two months' income experience consumption growth rates that are 1.7% higher, on average, than those who hold higher assets. Zeldes interprets this excess growth as an effect of liquidity constraints. Other researchers such as Carroll [3], Deaton [5], and Kimball [9] have emphasized the importance of the precautionary motive of savings as an explanation for the excess growth. Empirically, the precautionary effect is often indistinguishable from the liquidity constraint effect (Browning and Lusardi [2]). It has also been recognized theoretically that the two effects work together in a dynamic consumption decision (Carroll and Kimball [4] and Huggett and Ospina [8]). Thus, this paper deals with the combined effect of liquidity constraints and precautionary savings.

The purpose of the paper is to quantify the contribution of the two effects and to identify the parameter range in which Zeldes' estimate of excess growth is consistent with Aiyagari's [1] model. The basic assumption is that the economy follows a stationary rational expectations equilibrium. I draw on Huggett's [7] and Aiyagari's method of numerically computing such stationary equilibria when the households can only partially insure against idiosyncratic endowment shocks via limited borrowing.

First, I estimate the borrowing constraint point b which is consistent with Zeldes' estimate of excess growth. The population mean of the excess growth rates can be computed at the stationary

equilibrium of the Aiyagari model. The result shows that the Zeldes' point estimate corresponds to the case when b is set at three months' worth of income. After this calibration exercise, I investigate the statistical robustness of the point estimate of b. In a Monte Carlo run, I randomly draw the model households and replicate Zeldes' estimator of the excess consumption growth rate for low asset holders. A test distribution for the estimator is constructed by using a large number of such Monte Carlo runs. I repeat the procedure for various values of b, and obtain 90% confidence intervals for bin which the simulated excess growth supports Zeldes' estimate. Although the p-values are close to significant, the interval shows that neither specification b = 0 (no borrowing) nor  $b = w\underline{l}/r$  (natural debt limit) can be rejected at a significance level of 5%.

The methodological contribution is that the stationary equilibrium distribution of the households' states is utilized to form a test distribution for the average excess growth rates. The excess growth estimate contains sampling errors, since the households' states are heterogeneous *within* each group of constrained or unconstrained households. The stationary equilibrium distribution accounts for the distribution of these sampling errors under the null hypothesis. This procedure extends the usual calibration method, which yields a parameter value so that a *population* statistic of interest is consistent with an empirical estimate. The extended procedure calibrates not only the point estimate but also the standard error of the estimate.

Using the estimated borrowing constraint point b, I examine the aggregate consequence of endowment shocks. I simulate the model with the calibrated b for the minimum and maximum estimates of the endowment shock variances in the empirical micro literature cited by Aiyagari. The results show that the capital level increases by 6.9% and the saving rate increases by 1.6% at the stationary equilibrium when the endowment risk is increased from the minimum to the maximum. A broad range of lower-middle asset holders contributes to the increase in aggregate assets. These results suggest that labor market fluctuations can have a significant effect on aggregate savings.

The remainder of the paper is organized as follows. Section 2 presents the model, and defines and computes the liquidity constraint and precautionary savings effects. Section 3 tests two specifications for b, and demonstrates that the estimate of b is robust to preference shifts. Section 4 quantifies the aggregate effects of a change in income risks. Section 5 concludes.

### 2 The Model

I follow Aiyagari's [1] model of uninsured endowment shocks. The model is standard, as in Ljungqvist and Sargent [10]. Households maximize their utility over an infinite sequence of consumption with each period's utility exhibiting constant relative risk aversion:  $E\left[\sum_{t=0}^{\infty} \beta^t c_t^{1-\gamma}/(1-\gamma)\right]$ . A household is endowed with labor l, the logarithm of which follows an AR(1) process,  $\log l' = \phi \log l + \sigma \sqrt{1-\phi^2}\epsilon$ , where  $\epsilon$  follows the standard normal distribution independently over periods. There is no aggregate risk in the economy. Thus, there exists a stationary equilibrium where the interest rate is constant over the periods. Let r and w denote the real interest rate and the real wage. A household can lend and borrow, but its net assets cannot fall below the borrowing constraint point  $b \leq 0$ . Thus, the household is constrained by  $(1+r)a+wl \geq a'+c$  and  $a' \geq b$ . The second constraint is called a borrowing constraint or a liquidity constraint. Utility maximization given the prices yields the households' policy functions on consumption and the next period asset level: c(a, l; w, r) and a'(a, l; w, r).

A representative firm has a Cobb-Douglas production technology  $Y = AK^{\theta}L^{1-\theta}$ , where K, L, and Y are the aggregate capital, labor, and output, respectively. Capital depreciates at rate  $\delta$ . The stationary competitive prices are then equal to the marginal products:  $w = (1 - \theta)Y/L$  and  $r = \theta Y/K - \delta$ . Let f(a, l) denote the joint density function of assets and endowments across households. The conditional density of endowments p(l'; l) is determined by the AR(1) process. The stationary equilibrium is defined as the prices (w, r), the aggregate allocations (K, L), the policy function a'(a, l), and the distribution f(a, l), that solve the household's problem and satisfy the production function, the competitive price conditions, the market clearing conditions for factors K = $\int \int af(a, l) da \, dl$  and L = 1 (labor supply is normalized to one), and the stationarity condition  $f(a', l') = \int \int_{\{a:a'=a'(a,l)\}} f(a, l) p(l'; l) da \, dl$ .

The Euler equation must hold for an optimal consumption path. Define  $\lambda_1$  as a Lagrange multiplier for the liquidity constraint. Then the Euler equation is:  $\lambda_1 = c^{-\gamma} - \mathbb{E}[c'^{-\gamma} \mid a, l](1+r)\beta$ . Following Zeldes [12] and Dynan [6], the *liquidity constraint effect*  $\lambda$  and the *precautionary effect*  $\mu$  are defined as follows.

$$\lambda = \log\left(1 + \lambda_1 / (\operatorname{E}\left[c'^{-\gamma} \mid a, l\right](1+r)\beta)\right) / \gamma \tag{1}$$

$$\mu = \operatorname{Var}\left[\log\left(c'/c\right)|a,l\right]\gamma/2 \tag{2}$$

Then, by following Deaton [5], an approximation for the Euler equation is obtained:

$$\log\left(c'/c\right) = \log((1+r)\beta)/\gamma + \lambda + \mu + e.$$
(3)

The unbiasedness condition E[e|a, l] = 0 holds if c' follows a log-normal distribution. This is the case in this model if the elasticity of consumption to the current endowment,  $\partial \log c(a, l)/\partial \log l$ , is constant. The reason is that the random term in conditional consumption growth  $(\log(c'/c) | a, l)$  is concentrated in the second argument of the consumption policy function  $c' = c \left(a'(a, l), l^{\phi} e^{\sigma \sqrt{1-\phi^2}\epsilon}\right)$ , and that  $\epsilon$  is normally distributed. With this constant elasticity holding, the first three terms in the right hand side of (3) give an unbiased estimator for consumption growth.

Finally, let us define unexplained growth  $\tilde{\mu}$  as the mean residual of the Euler equation regression:

the mean consumption growth less the fundamental growth  $\log((1+r)\beta)/\gamma$  and the liquidity effect  $\lambda$ , namely,

$$\tilde{\mu} = \mathbf{E} \left[ \log \left( c'/c \right) \mid a, l \right] - \log((1+r)\beta)/\gamma - \lambda(a, l).$$
(4)

Then we obtain:

$$\log\left(c'/c\right) = \log((1+r)\beta)/\gamma + \lambda + \tilde{\mu} + \tilde{e}$$
(5)

where  $E[\tilde{e}|a, l] = 0$  holds regardless of the form of the consumption function. The unexplained growth  $\tilde{\mu}$  coincides with the precautionary effect  $\mu$  when the elasticity of consumption to the endowment is constant. I call  $\lambda + \tilde{\mu}$  excess growth, since it represents the mean surplus consumption growth of consumers to that of consumers with maximum assets.

To summarize, the excess growth  $\lambda + \tilde{\mu}$  is decomposed into two factors: the liquidity constraint effect  $\lambda$  and the precautionary effect  $\mu$ . The precautionary effect  $\mu$  is an approximation of the unexplained growth  $\tilde{\mu}$ , that is,  $\lambda + \mu$  is an unbiased estimator of the excess growth  $\lambda + \tilde{\mu}$  if the consumption function c(a, l) has a constant elasticity with respect to l.

Next, I numerically calculate a stationary equilibrium when consumers cannot have a net debt position at all, i.e. b = 0. I employ Aiyagari's specification for parameters. The production and preference parameters are set at  $\beta = 0.96$ ,  $\gamma = 3$ , A = 1,  $\theta = 0.36$ , and  $\delta = 0.08$ . The parameters for the endowment shocks follow Aiyagari's benchmark case:  $\sigma = 0.4$  and  $\phi = 0.6$ . These values of  $\sigma$ and  $\phi$  fall on the upper bound of the empirical findings cited by Aiyagari. The endowment process is approximated by a five-state Markov transition matrix using the quadrature method of Tauchen and Hussey [11]. Then I numerically obtain the stationary equilibrium.

The left panel of Figure 1 plots the unexplained growth  $\tilde{\mu}$  along with the precautionary effect  $\mu(a, l) = \operatorname{Var}[\log(c'/c)|a, l]\gamma/2$  against asset levels normalized by the mean annual wage  $w \mathbb{E}[l]$  for three endowment states. The precautionary effect is large for low asset groups, and converges to the



Figure 1: Left: unexplained growth  $\tilde{\mu}$  and precautionary effect  $\mu$  (dotted). Right: probability distributions of consumption growth rates for various asset groups with mean endowments

minimal level of about 0.1% as assets increase. The precautionary effect  $\mu$  tracks the unexplained growth  $\tilde{\mu}$  well except for the lowest asset group with the worst labor shock. The divergence between  $\mu$ and  $\tilde{\mu}$  occurs exactly at the asset/labor states for which the consumers are liquidity-constrained. The unexplained growth  $\tilde{\mu}$  is constant in the liquidity-constrained region because the liquidity constraint effect ( $\lambda$ ) accounts for the additional excess growth. This significant effect of liquidity constraint occurs only for the consumer group with the lowest labor endowment and assets less than two months' worth of income. There is also an interesting discrepancy between  $\mu$  and  $\tilde{\mu}$  for the lowest endowment state with assets more than two months' worth and less than six months' worth of income. This discrepancy occurs because of the skewness of the consumption growth distribution in this region. The right panel of Figure 1 shows the growth rate distribution for households with the lowest endowment and various asset levels. The distribution is skewed to the left not only for the liquidity constrained asset level but also for the level above it (the middle distribution in the figure). Hence the log-normal approximation is not precise for these asset groups. It is well known that the liquidity constraint effect is dominated by the precautionary effect in aggregation at a stationary equilibrium of an economy with infinitely-living households, since few consumers are bound by the borrowing constraint. It is also known that Zeldes' estimator can pick up the precautionary savings effect [12, page 319]. The task of empirically disentangling the liquidity constraint and precautionary savings effects has been pursued elsewhere. In this paper I focus on the combined effect  $\lambda + \mu$  or the excess growth  $\lambda + \tilde{\mu}$  to match Zeldes' estimate.

# 3 Quantifying the Borrowing Constraint

#### 3.1 Test for specifications on the borrowing constraint

In this section, I test specifications for the borrowing constraint point b by Monte Carlo simulations. I replicate Zeldes' estimation procedure for the excess growth of low asset consumers and derive a test distribution for the estimate. A state (a, l, l') is randomly drawn from a stationary distribution for low and high asset groups, and a corresponding consumption growth rate is calculated. By mimicking Zeldes' samples, 2731 samples of the consumption growth rates are drawn from the low asset group and 1583 from the high asset group, then the estimators such as average  $\lambda$  and  $\mu$  are computed. The splitting point for the low and high groups is set at two months' worth of mean annual wage income. I test two specifications: b = 0 and  $b = -\frac{l}{w}/r$ . The former specification, allowing no borrowing, is used by Zeldes as well as Aiyagari's [1] benchmark simulation. The latter specification is called a natural debt limit [1, 10] and implies a solvency constraint at the lowest labor income. Under these null hypotheses, the estimators.

Methodologically, the above procedure is an extension of the calibration procedure. In calibration,

the mean equilibrium excess growth rates are calculated by using the stationary distribution for various values of b, and a value of b is determined so that the equilibrium mean excess growth rate matches an empirical estimate. To derive a test distribution, sets of households are randomly drawn from the stationary distribution, and a sample average excess growth rate is calculated for each set. This testing procedure exploits the fact that the equilibrium model predicts both the stationary distribution and the policy function. In other words, the stationary equilibrium not only provides the *population* excess growth but also the *distribution* of excess growth. This method is particularly useful in our case where the underlying heterogeneity significantly contributes to the behavioral difference *within* the sample groups (the constrained and unconstrained groups). As seen in the previous section, the precautionary effect differs among the households in each group. Thus the distribution of the sampling errors in the estimate of the excess growth rate depends on the policy function and the stationary distribution of assets and endowments.

There are a number of disturbances for the estimated excess growth other than asset and endowment heterogeneity, such as measurement errors, preference shifts, and estimation errors of the other parameters. The *t*-statistic for the excess growth estimated by Zeldes is 1.63, which implies that the estimated excess growth suffers a standard error of about 0.017/1.63. It turns out in the following simulations that the asset and endowment heterogeneity only accounts for about one third of this estimated standard error. Therefore, I need to incorporate other sources of disturbances in the replication of Zeldes' estimation. I therefore add a disturbance term *s* to the estimated excess growth, where *s* is assumed to follow a normal distribution with mean zero and variance  $\sigma_s^2$ . To estimate  $\sigma_s$ , I first estimate the variance of the estimated excess growth from simulated data without measurement errors, and then set  $\sigma_s^2$  equal to Zeldes' estimated variance of  $(0.017/1.63)^2$  less the estimated variance without the measurement error. To obtain a test distribution with the measurement error, I



Figure 2: Distributions of estimated differences in liquidity constraint and precautionary effects between low and high asset holders for the no-borrowing specification (b = 0, left) and for the natural debt limit specification (b = -11.7, right). The vertical line shows Zeldes' point estimate.

numerically take the convolution of the histogram of the simulated excess growth rates and a normal distribution function with variance  $\sigma_s^2$ .

The left panel of Figure 2 shows the mean differences in  $\lambda$ ,  $\lambda + \mu$ , the excess growth  $\lambda + \tilde{\mu}$ , and the excess growth with measurement error  $\lambda + \tilde{\mu} + s$ , between low and high asset groups when b = 0. The difference in  $\lambda$  accounts for 0.26% of the higher growth rate of consumption on average. The difference in  $\mu$  accounts for 2.5%. The difference in excess growth  $\lambda + \tilde{\mu}$  is distributed around 2.8%. The distribution of the excess growth with measurement error provides a test distribution for the hypothesis b = 0. The *p*-value for the hypothesis b = 0 evaluated by the test distribution is 10.8%, failing to reject the hypothesis at a reasonable significance level. The hypothesis would be rejected if the estimate did not contain measurement errors, since the entire support of the distribution of the excess growth difference is above 1.7% (the vertical line).

The position of the histogram of  $\lambda + \mu$  matches the average excess growth  $\lambda + \tilde{\mu}$  reasonably well. This

is a convenient property, since both  $\lambda$  and  $\mu$  can be estimated only from the conditional consumption growth data with appropriate assumptions on parameters. Note, however, that the distribution of  $\lambda + \mu$  has a smaller variance than that of excess growth. This implies that the standard error of  $\lambda + \mu$ is conservative as an estimated standard error of the excess growth.

The right panel of Figure 2 shows the same plots for b = -11.7, which corresponds to the natural debt limit  $-w\underline{l}/r$ . Note that the natural debt limit is determined endogenously by the stationary equilibrium prices, and the simulations show that the prices w and r and the natural debt limit are consistently aligned when b = -11.7. As seen in the graph, the simulated distributions of the liquidity constraint and the precautionary effects are generally below 1.7%. The *p*-value for the hypothesis b = -11.7 evaluated by the test distribution with measurement error is 5.7%, however, again failing to reject the hypothesis at a significance level of 5%.

The Monte Carlo simulations could not reject either specification b = 0 or  $b = -w\underline{l}/r$ . However, the *p*-values suggest that the specification b = 0 is too strict and  $b = -w\underline{l}/r$  too slack. The next section will show that a *b* exists between the two extremes such that Zeldes' estimate is consistent under the Aiyagari model with varying parameter values.

### 3.2 Confidence interval of the borrowing constraint point

In this section the same Monte Carlo simulations are executed for various values of the borrowing constraint, b, and other parameters. Figure 3 shows the means and 90% confidence intervals of the difference in excess growth between the low and high asset groups. The left panels show the cases where  $\gamma = 2, 3, 4$ . The right panels show the cases where  $\beta = 0.94, 0.96, 0.98$ . The benchmark case,  $\gamma = 3$  and  $\beta = 0.96$ , is shown in the middle row.

The solid lines show the mean growth difference between low and high asset groups for various



Figure 3: Mean (solid line) and 90% confidence interval (dashed lines) of growth difference as functions of the borrowing constraint point b. The mean is calculated by using the stationary distribution. The interval is obtained by 500 Monte Carlo runs. The dotted lines show the confidence intervals if there were no measurement errors.

values of b. The plot shows that the mean difference in excess growth is increasing in b. This is because as b increases the average asset (or debt) of the low asset group becomes closer to the constraint point, and hence the average effects of liquidity constraints and precautionary savings become larger. The plot shows that a 1.7% excess growth difference corresponds to b = -0.25 in the benchmark parameter set. Thus a borrowing constraint at net debt worth three months' wages yields excess growth compatible with Zeldes' estimate.

The dashed lines show confidence intervals of b. The confidence interval is obtained from 500 iterations of a Monte Carlo run which simulates Zeldes' estimation procedure with a fixed b. The 500 estimates of the consumption growth difference are transformed to a cumulative distribution, and numerically convoluted with a normal distribution with mean zero and variance  $\sigma_s^2$ . Then the 5-th percentile and 95-th percentile are taken from the resulting distribution as the boundaries of the confidence interval of the excess growth difference for the particular b. I define the 90% confidence interval of b as the region that contains Zeldes' point estimate of the growth difference (1.7%) within the confidence intervals of the excess growth. This conversion of the confidence interval from the excess growth to the borrowing constraint is justified by the increasing excess growth difference with respect to b. The simulations show, however, that the confidence interval does not put a restriction on the parameter range  $b \leq 0$ . This is due to a large measurement error taken into account in the test distribution. The dotted lines in Figure 3 show the 90% confidence interval if there were no measurement error in the estimate. The variation of the estimated b accounted for in these intervals only reflects the consumer heterogeneity of assets and endowments. For the benchmark parameter values, this interval is (-0.5, -0.2), which is a range of net debts worth from two to six months' wages.

The point estimate and the confidence interval are fairly robust to parameter specifications. The panels in the top and bottom rows in Figure 3 show the confidence intervals for various  $\gamma$  and  $\beta$ . For

the case without measurement errors, all the intervals are contained in the range (-0.7, -0.1). Hence for various parameter settings, Zeldes' estimate is consistent with an Aiyagari model with a mild liquidity constraint less than one year's worth of income. Note that b decreases in  $\gamma$  and increases in  $\beta$ . This result is natural, since the precautionary effect strengthens when consumers are more risk averse or less patient.

### 4 Aggregate Effects of Income Risk

In this section, I quantify the effects of income risks on aggregate savings and capital accumulation. I fix the borrowing constraint point at b = -0.25, for which the excess growth is consistent with Zeldes' estimate. In the benchmark case, the standard deviation of innovation in log endowments is set at  $\sigma = 0.4$ . The equilibrium interest rate r is 3.53%, and the average savings rate is 28.2%. Naturally, the interest rate is greater than in Aiyagari's benchmark case (2.78%) where b was set to zero, yet it is still significantly smaller than in Aiyagari's full insurance case (4.17%). Hence, precautionary savings decrease the equilibrium interest rate significantly when the borrowing constraint is set to be compatible with Zeldes' estimate.

The savings rate as a function of (a, l) is plotted in the left panel of Figure 4. It is clear that the low asset holders increase their savings rates as the prospect of the liquidity constraint binding in the future becomes higher. Note that the savings rates vary more across the endowment states than across the asset levels. For the asset holders with two to six years' worth of wages, more than a half of consumption is financed by dissaving when the worst endowment shock hits (the saving rate is -0.6). For consumers with assets less than two years' worth of wages facing the worst shock, dissaving is reduced because of the current and future borrowing constraints. This corresponds to the large  $\mu$  for



Figure 4: Saving rates when  $\sigma = 0.4$  (left) and  $\sigma = 0.2$  (right)

these states.

Now let us see how the savings behavior changes as the riskiness of the endowment process  $\sigma$  varies. I take  $\sigma = 0.2$ , which is the lower bound among the empirical findings cited by Aiyagari, and compare its steady state to the benchmark case  $\sigma = 0.4$  which is about the upper bound. The quadrature approximation requires the endowment state vector to be altered for different  $\sigma$ , but the percentile of each state in the stationary distribution is kept unchanged. The right panel of Figure 4 shows the savings functions for  $\sigma = 0.2$ . It is clear that the dispersion of the savings rates across the endowment states is much decreased for the smaller  $\sigma$ . The savings pattern across asset levels is not changed very much. These results imply that a smaller  $\sigma$  causes a smaller variance in consumption growth and thus smaller precautionary savings for a wide range of assets.

The aggregate equilibria for different  $\sigma$  are reported in Table 1. The results are consistent with precautionary savings models. The riskier the income process is, the larger are the steady state capital accumulation and the savings rate. The interest rate becomes lower as the endowment becomes riskier since the precautionary motives generate a larger supply of savings. Finally, Figure 5 shows the

σ	K	w	r~(%)	saving rate $(\%)$
0.2	5.54	1.19	4.04	26.6
0.4	5.92	1.21	3.53	28.2

Table 1: Aggregate equilibria for different endowment shocks



Figure 5: Asset distributions

stationary distribution of assets for different endowment shocks. The asset level is normalized by the mean annual wage of the case  $\sigma = 0.4$  so that the two distributions compare in the same units. The plot shows that increased asset holding occurs across a wide range of asset levels. The consumers up to the 90-th percentile shift their asset position up, and the magnitude of the shift is largest in the lower middle asset holders. The change in aggregate savings can be attributed to the change in savings of consumers in this range. This implies that the precautionary motive can be quantitatively an important factor to transmit the effect of a structural change in the labor market to the behavior of consumers with a wide range of assets. Finally, note that the stationary asset distribution exhibits a typical caveat of this kind of models: too much of the asset is concentrated on the middle. The prices and policy functions can be different in a model which generates more realistic asset distributions.

### 5 Conclusion

In this paper, I quantify the effects of precautionary savings on the consumption growth rate difference between low and high asset holders, as well as on aggregate savings, in the framework of a stationary dynamic general equilibrium model.

I propose a method for deriving a test distribution for the borrowing constraint point indirectly from an empirical estimate of the excess growth rate of consumption. The test distribution is generated by Monte Carlo simulations at a stationary equilibrium under various borrowing constraint points. The *p*-values for the null hypotheses b = 0 (no-borrowing) and  $b = -w\underline{l}/r$  (natural debt limit) are computed as 10.8% and 5.7%, respectively. One third of the standard error of the estimated excess growth accrues to the heterogeneity of assets and endowments *within* the groups of the liquidity constrained and unconstrained. A mild borrowing constraint at three months' worth of wage income proves consistent with Zeldes' point estimate. Using the same Monte Carlo simulations, I form 90% confidence intervals of b under various parameter sets. The point estimate of b is shown to be robust to changes in parameter values.

Under the borrowing constraint consistent with Zeldes' estimate, aggregate savings is affected considerably by the riskiness of the labor endowment process. When the riskiness is changed from the minimum to the maximum in the range of empirical findings, the aggregate savings rate increases by 1.6% points and the aggregate capital increases by 6.9% at the stationary equilibrium. The stationary asset distribution is shifted upward in a wide range of the lower middle asset holders. This suggests that a persistent change in labor market fluctuations affects the aggregate consumption propensity in a quantitatively significant manner.

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