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Menu Costs and Dynamic Duopoly

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Menu Costs and Dynamic Duopoly

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Abstract
Examining a state-dependent pricing model in the presence of menu costs and dynamic duopolistic interactions, this paper claims that the assumption regarding market structure is crucial for identifying the menu costs for price changes. Prices in a dynamic duopolistic market can be more rigid than those in more competitive markets, such as a monopolistic-competition market. Therefore, the estimates of menu costs under monopolistic competition are potentially biased upward due to the price rigidity from strategic interactions between dynamic duopolistic firms. By developing and estimating a dynamic discrete-choice model with duopoly to correct for this potential bias, this paper provides empirical evidence that dynamic strategic interactions, as well as menu costs, play an important role in explaining the observed degree of price rigidity in weekly retail prices.

Key Words: Menu Costs; Dynamic Discrete Choice Game; Retail Price.
JEL Classification Number: D43, L13, L81.

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1. Introduction

In this paper, I study a structural state-dependent pricing model with menu costs for price changes in which brands of retail products play a dynamic game of price competition. The model provides the claim of this paper: the estimates of menu costs identified under a maintained hypothesis of monopolistic competition could be biased upward due to the price rigidity generated from dynamic strategic interactions between two brands in a duopolistic market. Using scanner data collected from a large supermarket chain in the Chicago metropolitan area, I provide empirical evidence that not only menu costs but also dynamic strategic interactions play an important role in the high-frequency movements of weekly retail prices after correcting for potential bias. To the best of my knowledge, the bias in the estimates of menu costs due to dynamic strategic interactions in a duopolistic market has not been investigated thoroughly in the literature on state-dependent pricing.

Following past studies, this paper defines menu costs as any fixed adjustment costs a price setter has to pay when changing its price, regardless of the magnitude and direction of a price change. Several papers provide evidence that menu costs are empirically important in understanding retail price dynamics. Constructing direct measures of physical and labor costs in large supermarket chains in the United States, Lévy, Bergen, Dutta and Venable (1997) claim that menu costs play an important role in the price setting behavior of retail supermarkets. Estimating menu costs as structural parameters of single-agent dynamic discrete-choice models in monopolistically competitive markets, Slade (1998) and Aguirregabiria (1999) find that their estimates of menu costs are positive and statistically significant. More recently, using a dynamic oligopoly competition model, Nakamura and Zerom (2010) observe that menu costs are crucial for explaining price rigidity in the short run.

As is frequently observed in macroeconomics literature, monopolistic competition is the most commonly adopted market structure in past studies on price rigidity.\textsuperscript{1} This hypothesis of market structure, however, is problematic if the market under study is dominated by a small number of firms. In this case, duopolistic/oligopolistic competition may be a more appropriate market

\textsuperscript{1}The seminal paper that applies a monopolistic-competition model to aggregate price rigidity is Blanchard and Kiyotaki (1987).
structure for studying firms’ pricing behaviors. More importantly, if duopolistic/oligopolistic competition prevails in the market of investigation, the estimates of menu costs identified under the maintained assumption of monopolistic competition may be biased due to tighter strategic interactions among firms. For example, suppose that there are just two dominant firms in a market that compete in price. Although monopolistic-competition models create a degree of strategic complementarity among firms’ prices, each firm perceives its own market power to be so small that it recognizes the average price to be exogenous. In contrast, in a duopolistic market, firms explicitly take into account strategic interactions. Because this would lead to a stronger degree of strategic complementarity, firms may prefer less aggressive price competition. Due to their tighter strategic interactions, the equilibrium price of the market may be rigid to some extent regardless of the existence of menu costs. Within such markets with tighter strategic interactions among firms, the working hypothesis of monopolistic competition spuriously results in the overestimation of menu costs. This situation implies that in order to draw a precise inference on menu costs, it is essential to properly identify the market structure of a product under investigation and allow for dynamic duopolistic/oligopolistic interactions among the firms in the market.

Although a number of empirical papers study price rigidity using micro data, few investigate the relationship between the price rigidity of a product and its market structure, taking into account the effect of dynamic duopolistic/oligopolistic interactions. Slade (1999) estimates the thresholds of price changes as functions of strategic variables using a reduced-form statistical model. Assuming that firms follow a variant of the (s, S) policy, Slade observes that firms’ strategic interactions in a dynamic oligopolistic competition model exacerbate price rigidity. This observation suggests a potential upward bias of the estimates of menu costs, as previously discussed. In this paper, I go beyond the reduced-form model of Slade (1999) by developing a fully-structural dynamic discrete-choice model equipped with menu costs and dynamic duopolistic interactions. Because the effect of dynamic duopolistic interactions on equilibrium prices is captured by the strategies of the two firms in the model, the rigidity due to menu costs is separately inferred from that due to dynamic strategic interactions.

Carlton (1986), Cecchetti (1986), and Kashyap (1995) are among the empirical studies on price rigidity that use micro data. For more recent studies, see Nakamura and Steinsson (2008) and the references cited therein. For theoretical studies that deal with duopolistic/oligopolistic competitions in the presence of fixed adjustment costs, see Dutta and Rustichini (1995) and Lipman and Wang (2000). Unfortunately, it is not straightforward to construct econometric models from their theoretical implications.
interactions. Another important exception is Nakamura and Zerom (2010), who investigate the sources of the incompleteness of the pass-through of wholesale prices to retail prices observed within the coffee industry. They construct an empirical model under dynamic oligopolistic competition among manufacturers and identify the menu costs at the wholesale level. Their estimation indicates that though the menu costs are negligible, they are nevertheless important for explaining the price rigidity observed in the short run. Notice that the objective of this paper is different: I examine how an empirical inference about menu costs might be affected when the underlying market structure is misspecified.

By examining a small product market of graham crackers, I estimate menu costs under both monopolistic competition and dynamic duopoly. The former is the benchmark and the latter is the minimum extension of monopolistic competition with dynamic strategic interactions. It is worth noting that the main claim of this paper is not a theoretical consequence of dynamic-duopolistic competition; this is because in the estimation under dynamic duopoly, there is no restriction that would lead to price rigidity. Thus, the estimated menu costs can be either greater or smaller than that in the monopolistic-competition model. I find that the estimates of menu costs are statistically significant under the two market structures. The comparison between the estimation results from the two specifications supports the main claim of this paper: the dynamic strategic interactions between brands result in an upward bias of the estimates implied by the benchmark specification of monopolistic competition.

The next section describes the data used for analysis. Section 3 introduces the dynamic discrete-choice duopoly model. Section 4 describes the empirical strategy of this paper. Section 5 reports the empirical results, and Section 6 concludes.

## 2. Data

The data used in this paper are weekly scanner data collected across the branch stores of Dominick’s Finer Food (DFF, hereafter), the second largest supermarket chain in the Chicago metropolitan area during the sample period from September 1989 to May 1997.\(^3\) The data set contains information

\(^3\)The data set is publicly available online at the website of James M. Kilts Center, Graduate School of Business, University of Chicago. The website also provides links to papers that describe the pricing practice of DFF.
on actual transaction prices, quantities sold, indicators of promotions (simple price reductions and bonus-buys), and a variable called average acquisition cost (AAC, hereafter), which is a weighted average of the wholesale prices of inventory in each store, by stores and Universal Product Codes (barcodes).\(^4\) The products in the data set are priced on a weekly basis, which matches the sampling frequency of the data. The fact that the prices are actual transaction ones is ideal for studying price rigidity as the frequency and timing of price changes are the most important statistics in this study.

I choose standard graham crackers as the product to be analyzed for three reasons. First, only a small number of firms dominate the market. Second, across firms, there is only one similarly-sized package (15 or 16 ounces) for the product. Third, because a box of graham crackers is a minor product, I can avoid the possibility that pricing is affected by competition among retailers due to, for example, a loss-leader motivation. There are four brands in this market: two national brands (Keebler and Nabisco), one local brand (Sarelno), and one private brand (Dominick’s). The market share of the four brands is approximately 97 percent of the total sales of standard graham crackers. Note that DFF buys graham crackers directly from manufacturers.\(^5\) Further, note that prices are fairly uniform across stores; in other words, DFF does not adopt zone pricing, wherein stores are assigned to one of three categories: high-, mid-, or low-priced stores. The zone pricing strategy is typically used for products that sell in large volumes. In contrast, zone pricing is not adopted for products with small sales volumes such as graham crackers, probably because it is too costly for a retailer to tailor-make the prices of such goods. These facts suggest that manufacturers’ decisions are more likely to be reflected in retail prices, and the pass-through rate from the wholesale price to the retail store would be large.

Figure 1 plots the shelf prices of the four brands in a representative store, displaying the following important aspects of the data. First, the shelf prices discretely jump both upward and downward. Second, the prices stay at the same level for a certain period of time although temporary price reductions or “sales” are observed quite frequently. Third, the price levels vary over time for each brand. These patterns suggest that the pricing decisions can be decomposed into a discrete

\(^4\)For details on AAC, see Peltzman(2000).

\(^5\)The data set provides a code that indicates whether DFF buys a product directly from manufacturers or through wholesalers.
decision—whether or not to change the price—and a continuous decision—what level of price to set. Thus, it is important to incorporate the discrete decision into a model.

Figure 1 also reveals another important aspect of the data: the pricing patterns of the two national brands, Keebler and Nabisco, are similar to each other, but quite different from those of the other two brands. Observe that the prices of the two national brands move quite frequently around the higher levels for most of the sample period, while the prices of the other two brands move less frequently around the lower levels. Tables 1 and 2 provide further evidence to support this claim. Table 1 reports several summary statistics of the data across brands. The fourth column of the table shows the market shares in terms of revenue; the fifth column shows the means of the prices in U.S. dollars per ounce; and the sixth column shows the means of the quantities sold in ounces. Although the two national brands, Nabisco and Keebler, have very different market shares, their price levels are similar to each other. Table 2 summarizes the descriptive statistics related to the frequencies of price changes. The third column shows the frequencies of price changes in percentage terms; the fourth column shows the frequencies of downward price changes; the fifth column shows the frequencies of upward price changes; and the sixth column shows the average number of price changes per year. It is clear that the two national brands change their prices with similar frequencies: as high as 33 percent on average. The frequencies of downward and upward price changes of the two national brands are also close to each other, but those of the other two brands are, by comparison, much lower. These observations lead to an inference that Keebler and Nabisco are engaged in a dynamic competition that can be described by similar strategies, whereas the other brands are not.

As previously discussed, most of the downward price changes are temporary reductions, such as sales. As sales are conducted repeatedly, some consumers may feel that these follow some cycle. If so, taking into account such consumer behavior can impact the estimation of demand elasticity. One way to capture such behavior is to incorporate the information about the duration between sales. Using store-level data, Pesendorfer (2002) finds that the duration between sales is positively correlated with quantity sold. Hendel and Nevo (2003) show that the duration between promotions is important for deriving a reasonable inference about the relationship between sales and stockpiling behavior. From these findings in the literature, I exploit the indicator of promotional activity provided in the data set and its duration to capture the effect of stockpiling behavior.
The data set provides an indicator of in-store promotional activity, called a bonus-buy. A bonus-buy may be associated with an advertisement, an in-store display, or a promotion such as “buy-one-get-one-free.” Table 3 shows the frequency and mean duration of bonus-buy by brands. The percentage of weeks during which bonus-buy is in effect for Keebler and Nabisco are 28 and 21, respectively. The mean bonus-buy length is approximately two weeks for both brands. The problem with using this indicator is that it may overlap the period of a price reduction, and in such a case, if bonus-buy is included in demand estimation along with price, the bonus-buy may absorb a part of the price variation leading to a bias in demand elasticity.\(^6\) To examine the overlap of bonus-buy on price reduction, I decompose price into “regular” price and “sale” price. First, I look at the price of the two products at a representative store, store 73. I define regular price as the modal price over 5 weeks, and sale price as any price lower than the regular price. Out of the 763 weeks of observations, sale price is seen in 243 weeks. Out of these 243 weeks, bonus-buy is in effect for 177 weeks. In addition, bonus-buy is in effect with regular pricing for 21 weeks. Thus, bonus-buy and price reduction do not necessarily overlap. Later, I examine whether this degree of overlap biases the estimated parameter of demand elasticities.

As a common problem in scanner data, some observations are missing when no purchase is made, when the product is out of stock, or when there are no data records.\(^7\) In particular, in the case of graham crackers, there are approximately 20 weeks for which no record is available for all brands in all stores. While it is possible to impute missing prices assuming no purchase activity and using prices in previous periods, such imputation can cause spurious price rigidity. Therefore, in this paper, I remove missing observations, including their lagged observations (i.e., list-wise deletion). As a result, I am left with unbalanced panel data for the two brands of 13,120 observations spread over 20 stores.\(^8\)

When necessary, prices and other nominal monetary values are deflated with a constant inflation rate.\(^9\) For the inflation rate, I use mean Consumer Price Index (CPI) for food obtained

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\(^6\)The data set contains another indicator of in-store promotion: a simple price reduction. This variable is not used in the analysis since there is no additional announcement effect on demand.

\(^7\)Other well-known scanner data such as A. C. Nielsen data also contain missing data in their original data. For the problem arising from missing data in the Nielsen data, see Erdem, Keane, and Sun (1999).

\(^8\)The stores chosen are store 12, 18, 44, 47, 53, 54, 56, 59, 73, 74, 80, 84, 98, 107, 111, 112, 116, 122, 124, and 131.

\(^9\)The constant inflation rate stems from the assumption of the model in this paper. From September 1989 to May 1997, the average weekly monthly rate is 0.2 percent. I convert it to the average weekly rate of 0.06 percent.
from the website of the Bureau of Labor Statistics (BLS).

To solve the profit maximization problem of each brand, I need a measure of marginal costs to produce graham crackers. I construct a measure of production costs by combining the information from a box of graham crackers, the Input-Output table, and the Producer Price Index (PPI). The main ingredients of graham crackers are wheat flour, whole grain wheat flour, sugar, and oil. According to the Input-Output table, in addition to these ingredients, cardboard for packaging, wage, and wholesale trade are major production factors in the cookies and crackers industry. Obtaining the PPI of these items, I combine them according to the ratios shown in the Input-Output table for the cookies and crackers industry. To derive the monetary value per unit, the AAC from the DFF data set is used as a proxy for the wholesale price at the starting period. By construction, the production costs explain approximately 35 percent of the price on average. The appendix discusses the details of the costs. The constructed series is monthly and in dollars, and is common to brands. Table 4 shows the summary statistics of the constructed costs. In particular, as shown in the third column, the standard deviation of the constructed costs is fairly reduced when it is deflated.

3. Model

This section introduces the structural model of the paper. I describe only the duopoly model in this section. The monopolistic-competition model is described in the appendix. The difference between the two models is whether a brand takes into account the impact of its own action on the rival’s reactions and future strategic interactions.

The model describes a dynamic competition between two brands to maximize their own inter-temporal profits from each store. Brands set wholesale prices for each store given the strategy of the other brand, and each store maximizes its joint profit from the products of the two brands. The main competition is the one between two brands within each store as stores are assumed to be local monopolists. Primary price setters are assumed to be brands while stores are allowed to set prices discretionally to some extent.

The following is a rough description of the timing of the game.
1. At the beginning of each period, two brands of graham crackers observe the following commonly observable state variables: the previous demand conditions and store prices of both brands, and a common marginal cost. In addition, each brand receives a private profitability shock.

2. Brands simultaneously set wholesale prices for stores given the other brand’s strategy, demand, and stores’ behavior. Brands also suggest the ranges of their profit-maximizing retail prices to the stores. Wholesale prices and suggested prices are not observable to the rival brand.

3. Demand shocks realize.

4. Observing wholesale prices, suggested ranges of retail prices, and demand shocks for the two brands, each store sets the retail prices of the two products as a local multi-product monopolist. If a store decides to change its shelf price following the suggestion made by a brand, the brand pays the menu costs. Otherwise, the menu costs are paid by the store.

5. Demand conditions realize (customers come to stores) and purchases are made.

6. At the end of each period, stores and brands receive their profits.

The model maintains several important assumptions. First, the main competition in the model is the one between brands. Previous works offer supportive evidence on the claim that the main price competitors in a narrowly defined category are brands, and not stores or chains. For example, analyzing the DFF data, Montgomery (1997) states that weekly deviations of prices from regular prices mainly reflect manufacturers’ competitive actions. Slade (1998) assumes brands as price setters with a passive retailer analyzing the brand competition in a saltine-cracker category. According to telephone interviews with supermarket-chain managers, she claims that the competition important in a category is the one among brands. Stores, instead, compete by overall-offerings of products and locations, and not on a product-by-product basis. Conducting interviews with DFF stores, Chintagunta, Dube, and Singh (2003) confirm Slade’s claim and assume that stores are local monopolists. The demand, nevertheless, may be affected by location or size of stores. These factors are controlled by store-fixed effects in the estimation.

Furthermore, the data show that the timings of price changes of products across different categories of a brand tend to be synchronized to a large extent. This observation also suggests that major price changes are determined...
Second, shelf prices are set by each store and not by the chain. This assumption on the pricing structure is based on data observation. The data show that pricing decisions at DFF are centralized to a certain extent but that stores exhibit some discretionary power in price setting. In the case of graham crackers, the retail prices of a graham cracker product from one brand are fairly uniform across stores, but the exact price levels and the timings of the price changes are not entirely same. The correlation of the timing of price changes across stores is approximately 0.8. In particular, sometimes, a few stores differ their prices by tiny amounts. This sort of pricing is likely to be done on a store basis, and not on a brand or chain basis. This fact suggests that while pricing decisions at the brand level are dominant for the price of graham crackers, stores have some discretionary power and it is reasonable to assume that a store sets its own price.

Third, brands sell products to stores, and not to a whole chain. According to Peltzman (2000), wholesale price is uniform across stores implying that it is the chain that negotiates with manufacturers. Peltzman (2000), however, states that manufacturers changed their promotion policy toward DFF during the sample period to prevent stores from exploiting geographical price differentials, thus implying that stores have a certain power in their negotiations with manufacturers.

As brands behave while taking demand and stores’ behavior as given, I start the description of my model with demand and stores’ behavior. A description of brand behavior then follows.

Suppose that store \( s \in \{1, \ldots, S\} \) sells the products of two brands \( i \in \{1, 2\} \). For simplicity, I assume a static linear demand function. Let \( q_{ist}, p_{ist}, r_{ pist}, \) and \( e_{ist} \) stand for the quantity, real store price, real store price of the rival brand, and demand shock of the product of brand \( i \) at store \( s \) in week \( t \), respectively. The coefficients on price and rival price are allowed to be asymmetric between brands. Defining a brand dummy variable that takes zero for brand 1 and one for brand 2 by \( br \), the asymmetricity of the brand’s price elasticity is expressed by including a cross term, \( p_{ist} \times br \). In the same manner, \( r_{ pist} \times br \) allows asymmetric cross-price elasticity. Demand shock \( e_{ist} \) is assumed to be mean-zero and decomposed into a store-brand specific component \( \xi_{ist} \), which at the brand level. For example, the timing of a price change for a package of saltine crackers and graham crackers is synchronized to some extent in a store. This observation suggests that it is ideal to model a brand as a multi-product manufacturer, but it is infeasible in the current exercise to model a large number of choices with different brands for many products.
may be correlated with price, and an idiosyncratic shock $\varepsilon_{ist}^d$: $e_{ist} = \xi_{ist} + \varepsilon_{ist}^d$. I define another variable, demand condition $d_{ist}$, to include other demand shifters. The demand condition includes, for example, an in-store promotion variable such as bonus-buy and the number of customers who visit store $s$ in week $t$ as a measure of the size of potential purchase. $d_{ist}$ will be discussed in detail in the section on demand estimation and the construction of state variables. The demand for a product of brand $i$ then is

$$q_{ist} = d_{ist} - b_0 p_{ist} + b_1 r p_{ist} + (b_2 p_{ist} + b_3 r p_{ist}) \times br + e_{ist}, \quad (1)$$

where $b_0 \geq 0$, $b_1 \geq 0$, and $b_1 < b_0$.

Store $s$ is a multi-product local monopolist who maximizes the joint profit generated by the two branded products each period. Given wholesale prices $(w_{1st}, w_{2st})$ and the realization of demand shocks $(e_{1st}, e_{2st})$, store $s$ sets real retail prices $(p_{1st}, p_{2st})$ and puts the products on its shelf. Part of demand conditions $(d_{1st}, d_{2st})$, such as customer count is yet to be realized. The stores form expectations with respect to its realization. The current period profit of store $s$ in week $t$ is

$$\pi_{st} = \sum_{i=1,2} (p_{ist} - w_{ist})q_{ist}. \quad (2)$$

Solving for $p_{1st}$ and $p_{2st}$ yields the following optimal retail prices:

$$p_{1st}^* = \lambda_{1}^{-1}[2(b_0 - b_2)\tilde{d}_{1st} + (2b_1 + b_3)\tilde{d}_{2st} + \lambda_2 w_{1st} - b_3(b_0 - b_2)w_{2st}] \quad (3)$$

and

$$p_{2st}^* = \lambda_{1}^{-1}[(2b_1 + b_3)\tilde{d}_{1st} + 2b_0\tilde{d}_{2st} - b_0b_3w_{1st} + \lambda_3 w_{2st}], \quad (4)$$

where $\tilde{d}_{ist} = E_{t}d_{ist} + e_{ist}$, $\lambda_1 = 4b_0(b_0 - b_2) - (2b_1 + b_3)^2$, $\lambda_2 = 2b_0(b_0 - b_2) - b_1(2b_1 + b_3)$, and $\lambda_3 = 2b_0(b_0 - b_2) - (b_1 + b_3)(2b_1 + b_3)$. $E_{t}d_{ist}$ is the conditional expectation with respect to the demand condition, which follows an exogenous first-order Markov process.

Given the decision rule of stores described above, brands compete with respect to wholesale prices, which are unobservable to the other brand, over infinite periods. In each period, brand $i$ observes the previous own and rival’s real retail prices, $p_{ist-1}$ and $rp_{ist-1}$, current real production costs $c_t$ that are common to both brands, and the previous demand conditions $d_{ist-1}$ for both brands. Brands observe the one-period lagged demand conditions as state variables because the
demand conditions are assumed to be realized during a week. Store-level demand shock \( \varepsilon_{ist} \) is not realized yet, and brands take the same expectations with respect to its realization. At the same time, each brand receives private information \( \varepsilon_{ist} \) that affects its profitability.

Observing the state variables, \((p_{1st-1}, p_{2st-1}, d_{1st-1}, d_{2st-1}, c_t, \varepsilon_{ist})\), brands simultaneously take their actions on real wholesale prices \( w_{ist} \), which are drawn from a continuous support, expecting that store \( s \) follows the decision rule of equations (3) and (4). At the same time, suppose that a brand suggests a retail price range from the \( L \) discretized bins. The suggested retail price range contains the \textit{ex-ante} optimal retail price level. Given each of the suggested price ranges, the optimal retail behavior reflected in equation (3) and (4) implies the corresponding range of wholesale price, \( w_{ist}^j, j \in \{1, \ldots, J+1\} \), where \( w_{ist}^1 \) is determined by \( p_{ist-1} \). Because the suggested price range always includes the \textit{ex-ante} optimal retail price level and because the optimal retail price perfectly reveals the underlying wholesale price through equations (3) and (4), choosing a suggested retail price range is equivalent to choosing the corresponding wholesale price range. This economizes the choice variable of brands and simplifies the brands’ decision problem. Below, I formalize the brand’s problem concentrating only on the suggested price range as the single relevant choice variable. Both wholesale price and suggested retail price are observable only to the store and the brand.

The offer of a wholesale price may cause a change in the nominal retail price; this incurs menu costs. The relationship between real price \( p_{ist} \) and nominal price \( P_{ist} \) is given by a one-to-one correspondence, \( \log(p_{ist}) = \log(P_{ist}) - \rho t \), where \( \rho > 0 \) is a constant inflation rate. I assume that if a resulting retail price change is “large” and the change is in accordance with a store’s \textit{ex-ante} optimal retail price, the brand pays menu costs. If the price change is “small’ and not expected \textit{ex-ante}, the store pays menu costs.

I first define large and small price changes. Consider a discretization of the support of real price into \( L \) mutually exclusive discrete elements, \( p_{ist} \in \{(p_1, \bar{p}_1), (p_2, \bar{p}_2), \ldots, (p_L, \bar{p}_L)\} \). I define a large price change as the one across different bins: \( p_{ist} \neq p_{ist-1} \) and \( P_{ist} \neq P_{ist-1} \). A small price change is the one within a bin: \( P_{ist} \neq P_{ist-1} \) and \( p_{ist} = p_{ist-1} \).\(^{11}\) A large price change corresponds

\(^{11}\)To see an implication of the assumption on the data sample, I discretize the actual real prices into five segments so that each segment is visited with approximately equal probability. Nominal price changes occur 36 percent of the time in the whole sample. Among these nominal price changes, 25 percent are associated with changes across the
to a relatively significant price change such as the offering or terminating of a large discount while a small one is a store-specific price change of a tiny amount.

Second, I define when and by whom menu costs are paid. Suppose that the store makes a large retail price change. If the \textit{ex-post} optimal price level is the same as the \textit{ex-ante} optimal price level, the brand pays menu costs: $\gamma > 0$. The brand also expects that depending on the realization of the demand shock and rival’s wholesale price, the \textit{ex-post} optimal retail price may deviate from the \textit{ex-ante} optimal retail price level. I assume, even in this case, that the brand pays menu costs if the \textit{ex-post} optimal retail price is within the suggested retail price range that contains the \textit{ex-ante} optimal retail price level. At the same time, the store may change its retail price by its discretion reflecting changes in the retail environment captured by the demand shock. I assume that the brand is not responsible for paying menu costs with respect to such a small price change.\footnote{This model does not describe menu costs paid by stores. Modeling and estimating such costs requires dynamic models for both retailers and brands, which is beyond the scope of this paper.}I further assume that large price changes reflect brands’ decisions while small price changes reflect stores’ decisions. This is an identification assumption. The suggested price range and wholesale price are both unobservable to a researcher and the other brand, and thus, it is impossible to identify who initiated a large price change for each observation. I impose an identification assumption that a large price change is due to the suggestion made by brands. In addition, the structure of menu costs reduces a store’s incentive to conduct a large price change by its own discretion.\footnote{I further assume that large price changes reflect brands’ decisions while small price changes reflect stores’ decisions. This is an identification assumption. The suggested price range and wholesale price are both unobservable to a researcher and the other brand, and thus, it is impossible to identify who initiated a large price change for each observation. I impose an identification assumption that a large price change is due to the suggestion made by brands. In addition, the structure of menu costs reduces a store’s incentive to conduct a large price change by its own discretion.}

This structure assumes that the main price setters are brands, but allows retailers to exhibit some power to affect prices accounting for various conditions in the stores. A smaller number of $L$ allows stores to use greater discretion.

Private information $\varepsilon^j_{ist}$ is drawn randomly from a set of $J \equiv L+1$ alternatives: $\{\varepsilon^1_{ist}, \ldots, \varepsilon^J_{ist}\}$. The first element $\varepsilon^1_{ist}$ corresponds to the case of no price change: $p_{ist} = p_{ist-1}$; the second $\varepsilon^2_{ist}$, the case of a price change to $(p_1, \bar{p}_1)$: $p_{ist} \in (p_1, \bar{p}_1)$ and $p_{ist} \neq p_{ist-1}$; and the third $\varepsilon^3_{ist}$, the case of a price change to $(p_2, \bar{p}_2)$: $p_{ist} \in (p_2, \bar{p}_2)$ and $p_{ist} \neq p_{ist-1}$, and so on. This private shock explains the gap between the retail price predicted by the model and the observed price for each state. An interpretation of private shock would be an unobservable idiosyncratic component of the price adjustment costs. Under such an interpretation, the adjustment costs consist of a component discretized bins in the space of real prices. The rest of the nominal price changes are categorized into small price changes that do not accompany changes across the bins in the space of real prices.
common to brands, stores, and price level—menu costs \( \gamma \)—and an idiosyncratic component.\(^{14}\)

Let \( x_{st} = \{p_{1st-1}, p_{2st-1}, d_{1st-1}, d_{2st-1}, c_t, br\} \) denote the vector stacking the common-knowledge state variables observable to the brands, store, and a researcher. The demand conditions and production costs follow independent stationary first-order Markov processes with transition probability matrices independent of the actions taken by the brands. Private information, which is observable to only brand \( i \), \( \varepsilon_{ist} \) is assumed to be i.i.d. with a known density function, \( g(\varepsilon_{ist}) \), common across actions, brands, and periods of time. The choice variable of brands, suggested price range \( p_{ist} \), is observable only to brand \( i \) and store \( s \).\(^{15}\) When brands set their suggested prices, each brand forms an expectation with respect to the suggested price of the other brand conditional on the commonly observable state variables.

Under the above simplification, given the rival’s choice, the one-period profit of brand \( i \) in store \( s \) in week \( t \) conditional on choosing a discrete alternative \( j \) is

\[
\Pi_{ist}^j(x_{st}) = (w_{ist}^j - c_t)E_t[q_{ist}] + \varepsilon_{ist}^j - \gamma I(p_{ist} \neq p_{ist-1})I(P_{ist} \neq P_{ist-1}),
\]

where \( w_{ist}^j \) is the wholesale price range associated with alternative \( j \), \( E_t \) stands for the conditional expectation operator on the realization of \( d_{ist} \), which is conditional on the current realization of state variable \( x_{st} \). The one-period profit for brand \( i \) depends on the action its rival takes given own wholesale price. A brand maximizes its expected discounted sums of future profits by taking into account the strategy of its rival and the evolutions of demand conditions and production costs. The objective function of brand \( i \) in store \( s \) at period \( t \) is

\[
E\left\{ \sum_{m=t}^{\infty} \beta^{m-t} \Pi_{is}^m(x_{sm}) \mid x_{st}, \varepsilon_t \right\},
\]

where \( \beta \in (0, 1) \) is the discount factor, and \( E\{ \cdot \mid x_{st}, \varepsilon_t \} \) is the conditional expectation operator on the payoff-relevant state variables in store \( s \) at period \( t \). As the time horizon is infinite and the problem has a stationary Markov structure, I assume a Markov-stationary environment. I drop the

\(^{14}\)This interpretation is a mixture of existing models with menu costs such as Slade (1998) and Aguirregabiria (1999), who specify menu costs as a fixed parameter, and macroeconomic studies such as Dotsey, King, and Wolman (1999) and Nakamura and Zerom (2010), who specify menu costs as a random shock. While I keep the term of menu costs for the constant adjustment costs, it is reasonable that there exists an idiosyncratic shock. Sources of such shocks may be temporary changes in information gathering and processing costs, labor costs, and display costs.

\(^{15}\)Again, the range of wholesale price is perfectly related to the suggested price range.
time and store subscript from all the variables adopting the notations of \( x = x_{st} \) and \( x' = x_{st+1} \) for any variable \( x \). I investigate only the Markov-perfect equilibrium in which brands follow symmetric pure-Markov strategies with imperfect information.

Let \( \sigma = \{\sigma_1, \sigma_2\} \) denote a set of arbitrary strategies of the two brands, where \( \sigma_i \) defines a mapping from the state space of \( (x, \varepsilon_i) \) into the action space. Denote the one-period profit without private information conditional on choosing \( j \) by \( \pi_i^\sigma(x, j) \). Let \( V_i^\sigma(x) \) express the value of brand \( i \) when both brands follow strategy \( \sigma \) and the state is \( x \). Furthermore, let \( f(x'|x, j) \) represent the transition probability of the observable state variables conditional on the action of choosing alternative \( j \). When private information is integrated out, the corresponding Bellman equation is

\[
V_i^\sigma(x) = \int \max_{j \in J} \left\{ \pi_i^\sigma(x, j) + \varepsilon_i^j + \beta \sum_{x'} f(x'|x, j)V_i^\sigma(x') \right\} g_i(\varepsilon_i)d\varepsilon_i,
\]

where \( \Pi_i^\sigma(x, j) \) is the profit defined by common-knowledge state variables \( x \) conditional on brand \( i \) choosing alternative \( j \) given that the rival brand follows strategy \( \sigma_2 \). Then, the conditional choice probability—or the best-response probability—for brand \( i \) is to choose alternative \( j \) given the strategy of the other brand that is associated with a set of Markov strategies \( \sigma \), can be written as

\[
Pr_i(j|x) = \int I\{j = \arg \max_{j \in J} \left\{ \pi_i^\sigma(x, j) + \varepsilon_i^j + \beta \sum_{x'} f(x'|x, j)V_i^\sigma(x') \right\} \} g_i(\varepsilon_i)d\varepsilon_i.
\]

Aguirregabiria and Mira (2007) show that a Markov-perfect equilibrium, associated with equilibrium strategy \( \{\sigma_1^*, \sigma_2^*\} \) is characterized as a set of probability functions \( \{Pr_1(x), Pr_2(x)\} \) that solve the coupled-fixed-point problem presented by equations (7) and (8) in its probability space. The representation in the probability space is used to describe the likelihood function for estimation.\(^{16}\)

As noted previously, the monopolistic-competition model is described in the appendix. The important difference between the monopolistic-competition model and the duopoly model in this paper is that following Slade (1998) and Aguirregabiria (1999), I treat the evolution of \( r_{p_{ist}} \) as exogenous in the econometric model. An interpretation of this treatment would be that a brand takes into account its rival’s price but treats the effect of its own decision through the rival’s reaction in the future as trivial. In other words, the observed outcomes are simply those of the static Bayesian-Nash equilibrium. In this sense, the monopolistic-competition model studied in the

\(^{16}\)For the representation in the probability space, see the appendix.
previous papers lacks dynamic strategic interactions.\(^{17}\)

Note that in the duopoly model, no detailed structure to introduce price rigidity due to dynamic strategic interactions, such as collusion, is imposed. Therefore, the estimates of menu-cost parameters under the assumption of a dynamic duopoly can be either smaller or greater than those under the assumption of monopolistic competition. The strategy of this paper is to see whether the data reveal this bias.

### 4. Empirical strategy

This section describes the empirical implementation of the model. I first estimate demand equation. Second, the state variables are constructed, and their transition probability matrices are estimated. Third, wholesale price ranges are constructed. Finally, the menu costs parameter is estimated. I describe the details below in order.

#### 4.1. Demand estimation

Demand equation (1) is common to the duopoly model and the monopolistic-competition model. In this section, I discuss only the endogeneity problem in demand estimation, and leave the detailed description of the estimation to the next section.

Demand error term \(e_{ist}\) is assumed to include the unobserved store-brand term that affects demand and possibly correlates with price variables. Having included a brand dummy variable and time dummies, \(\xi_{ist}\) may include unobserved promotional activity (Nevo and Hatzitaskos 2006) and weekly in-store valuation affected by shelf space and display (Chintagunta, Dube, and Singh 2003). To control for these endogeneities, I need an effective promotional variable or instruments that are correlated with price but uncorrelated with the weekly store-brand demand error term. First, I include a promotional variable, that is, a bonus-buy indicator provided by the data set. Second, I use AAC as instrumental variables for the price. The correlation between the retail price and AAC is 0.73 in my sample. Chintagunta, Dube, and Singh (2003) use a measure of wholesale cost created from AAC and its lags as instruments. Having controlled for display and feature,

\(^{17}\)These two papers, however, feature other aspects of the models that are absent from this paper. Slade (1998) incorporates consumer goodwill accumulated from price reductions into her model. Aguirregabiria (1999) finds a crucial role of the inventory held by retail stores in the pricing behaviors of retail products.
they argue that the wholesale price, which is uniform across stores, is independent of current store-brand demand. Nevo and Hatzitaskos (2006), who study both category and product demand over a chain, use AAC as the instrument of price in one of their estimations.\textsuperscript{18} They note the potential endogeneity of AAC, since regarding it as a wholesale price, it may be correlated with unobserved promotion captured in the error term. They, however, also note that AAC does not denote the current wholesale prices but the weighted average of past and current wholesale prices, and thus they conclude that the problem will be less serious. I also assume that the rival and Salerno and Dominick’s prices are endogenous, and use the corresponding AAC and their lags as instruments.

One problem in the data set is that prices show fairly small variations across stores. The timings of price changes synchronize across stores for approximately 80 percent of the period. This lack of cross-sectional variations in prices may be problematic in estimating pricing behaviors because using the observations from all the stores results in spuriously small standard errors of the estimates of menu costs without much difference in their values.\textsuperscript{19} Therefore, in the exercise below, I provide the results from the five stores that have the fewest missing observations. The number of observations is now 3694.

\textbf{4.2. State variables} From the estimated demand equation, I construct demand condition $d_{ist}$, computed from the estimated coefficients on $cc$, $sdp$, $bonus$, $duration$, and $duration\ bonus\ 2$, store and time dummy variables, outlier, and a constant in demand equation. The state variables consist of $x_i = \{p_1, p_2, d_1, d_2, c, br\}$ in the duopoly model and $xs_i = \{p, rp, d, rd, c, br\}$ in the monopolistic-competition model.

State space is discretized according to a uniform grid in the space of the empirical probability distribution of each variable. I apply the same state space to all the price variables: $p_1$ and $p_2$ for the duopoly model and $p$ and $rp$ for the monopolistic-competition model. In addition, $d_1$ and $d_2$ are also discretized so that they have the same support. This is to ensure that the estimation results do not depend on the difference in state space construction. Therefore, the potential difference in the estimates of menu costs parameter $\gamma$ between the duopoly model and the monopolistic-competition model is solely due to the specification regarding the interactions between the brands.

\textsuperscript{18}The corresponding estimation result is shown in their appendix. They use the result from OLS to derive their main result.

\textsuperscript{19}I owe this point to the helpful comments from the seminar participants at Queen’s University.
The transition probabilities of the demand condition and rival price are estimated following the method by Tauchen (1986). This method generates more smooth transition processes than the alternative method such as counting the number of the samples that fall into each cell of the discretized state space. To evaluate the representative value in each cell of state space, I use the middle point of the range of each cell.

4.3. Wholesale price

As described in the model, a suggested price range corresponds to a discretized bin of observed retail price. In the empirical implementation, the suggested price ranges are evaluated at their middle values. The corresponding wholesale price range is backed out, thereby exploiting the optimal retail behavior.\footnote{Using AAC is another way to measure the wholesale price. However, I do not directly exploit this variable since (1) AAC need not be the same as the wholesale price if stores hold inventory, and (2) the literature does not agree with the validity of this variable as a measure of wholesale price (Peltzman 2000). The first problem is more serious for a storable good such as graham crackers.}

Solving equations (3) and (4) for $w_{1st}$ and $w_{2st}$, wholesale price range $w_{ist}$ is expressed as a function of suggested price $p_{ist}$ as follows:

$$w_{1st} = [\lambda_2 \lambda_3 + b_0 b_3 (b_0 - b_2)]^{-1} \{\lambda_1 \lambda_3 p_{1st} + b_3 (b_0 - b_2) \lambda_1 p_{2st} - (b_0 - b_2) [2 \lambda_3 + (2 b_1 + b_3) b_3] \tilde{d}_{1st} - [(2 b_1 + b_3) \lambda_3 + 2 b_0 b_3 (b_0 - b_2)] \tilde{d}_{2st} \}$$

and

$$w_{2st} = [\lambda_2 \lambda_3 + b_0 b_3 (b_0 - b_2)]^{-1} \{\lambda_1 \lambda_2 p_{2st} + b_3 b_0 \lambda_1 p_{1st} - [2 \lambda_2 - (2 b_1 + b_3) b_3] b_0 \tilde{d}_{2st} - [(2 b_1 + b_3) \lambda_2 - 2 b_0 b_3 (b_0 - b_2)] \tilde{d}_{1st} \}.$$

Given the derived wholesale price range evaluated at its mid-value, the profit is evaluated at its middle value as well.

4.4. Estimation of menu costs

To estimate menu costs parameter $\gamma$, I exploit the nested pseudo-likelihood (NPL) estimator developed by Aguirregabiria and Mira (2002, 2007). The advantage of using the NPL estimator over a full-solution method is computational because I do not need to solve a dynamic-programming problem for each iteration of the maximum-likelihood estimation of the structural parameters of the model. Moreover, the method is useful in the current application
since it allows me to estimate all the parameters in demand equation and the transition process separately from the dynamic one, which is a menu-cost parameter in this paper. The value function is recovered from data by exploiting the infinite-horizon Markov-stationary structure of the model. I leave the details of the estimation procedure to the appendix.

5. Empirical results

This section describes the empirical implementation and results of this paper. The demand equation and the transition processes of exogenous state variables are estimated separately from the menu costs parameter. I first describe the estimation results of demand equation; second, I state the discretization of state variables; third, I state the construction of wholesale price; fourth, I report the results of the estimated menu costs; and finally, I report the results of the simulation exercise to examine the property of price rigidities implied by the estimated results and the model in this paper.

5.1. Demand estimation results

Table 5 shows the results of the demand estimations. I provide the results of 7 specifications: 3 OLS and 4 IV estimations. In all the specifications, the dependent variable is the quantity sold standardized by 10 oz. The independent variables common to all the specifications are own price (\(price\)), rival price (\(rp\)), the weighted average of the prices of non-national brands (Dominick’s and Sarelno) with weight being the total quantity sold in the sample period (\(sdp\)), a brand dummy variable that takes one for Nabisco and zero for Keebler (\(br\)), the customer count (\(cc\)), the store dummy variables, the time dummies for month and year, and the dummy variable to control for outliers.\(^{21}\) The customer count, which is the average number of customers per day who visit the corresponding store within a week, is used to control for the time-varying size of potential purchasers.\(^{22}\) The independent variables appearing in some of the specifications are a cross term of \(p\) and \(br\), a cross-term of \(rp\) and \(br\), a dummy variable of bonus-buys, the duration since the end of the last bonus-buy, and the duration within a period of consecutive bonus-buys. All of the monetary variables are per 10 oz. and deflated by the CPI of food in the U.S.

\(^{21}\)The dummy variable to control for outliers takes one when the quantity sold exceeds 5000 oz. Such events occur 2.84 percent of the times.

\(^{22}\)The unit of customer count is 1,000.
The first column shows the names of the variables. The second to the last columns show the results of the different specifications. OLS 1 includes the following variables: \textit{price}, \textit{rp}, \textit{sdp}, \textit{cc}, \textit{br}, and constant. The store-fixed effects, time dummies, and a dummy variable to control for outliers are also included but their coefficients are not shown. The signs of the coefficients are as expected. The own demand elasticity evaluated at mean is -2.8. The own elasticities evaluated at brand-specific means are -4.34 for Keebler and -2.04 for Nabisco. Elasticity, which is calculated as \( \frac{\partial q_{it}}{\partial p_{it}} \frac{\bar{q}_i}{\bar{p}_i} \), where \( \bar{q}_i \) and \( \bar{p}_i \) are the means of price and quantity of brand \( i \), respectively, is greater for Keebler because \( \frac{\bar{q}_i}{\bar{p}_i} \) is much smaller for Keebler. The cross-elasticities are calculated as \( \frac{\partial q_{it}}{\partial p_{-it}} \frac{\bar{q}_i}{\bar{p}_{-i}} \), where \( i \in \{1, 2\} \) and \( -i \in \{2, 1\} \). The cross-price elasticity of Keebler’s demand with respect to Nabisco’s price is 0.64 while that of Nabisco with respect to Keebler’s price is 0.34. The fourth to the fifth columns (OLS2) show the estimated coefficients of the specification allowing asymmetric coefficients on own price and rival price across brands. Although the coefficients on asymmetry are statistically significant, the brand-specific elasticities are similar to those calculated in OLS1.

Specification OLS3 includes the following variables: \textit{bonus}, which is the dummy variable that takes one when a bonus-buy takes place and zero otherwise; \textit{bonus duration}, which is the number of weeks elapsed since the end of the last bonus-buy; and \textit{bonus duration 2}, which is the number of weeks elapsed since the beginning of the bonus-buy.\textsuperscript{23} The coefficient on \textit{bonus} shows a positive effect, as expected. The coefficient on \textit{bonus duration} is negative but not statistically significant. Sometimes, a bonus-buy takes place for consecutive multiple periods. If most consumers buy products during the first week of the bonus-buy, the demand for the second week may decline. To capture such dynamics, I include the variable \textit{bonus duration 2}. This variable takes one at the second week of the bonus-buy, two at the third week, and so on. The estimated coefficient on \textit{bonus duration 2} is negative showing that continuing the bonus-buy does not increase demand as much as in the first week. Importantly, in OLS3, the estimated coefficients on \textit{price} and the other price variables are not significantly affected by including the variables of bonus-buy. The estimated coefficient on \textit{price} is slightly lower than that of OLS2, but \textit{bonus} does not significantly absorb the price variation. This is expected because bonus-buy is not necessarily associated with price reduction. The estimated elasticities for both brands evaluated at the brand-specific means are -3.99 and -2.37. The cross-price elasticities are 0.92 for Keebler’s demand and 0.22 for Nabisco’s.

\textsuperscript{23}I divide variables \textit{bonus duration} and \textit{bonus duration 2} by 10.
Columns eight through last display the results of the IV estimations. IV1 shows the estimated values of coefficients with AAC, lagged AAC, rival AAC, lagged rival AAC, and the AAC of Salerno and Dominick’s as instruments treating price, \( rp \), and \( sdp \) as endogenous variables. Compared to OLS1, the size of own price coefficient increases in absolute value. IV2 includes \( br \times price \) and \( br \times rp \) with additional instruments of the cross-term of AAC and \( br \), and the cross-term of rival price and \( br \). While the sizes of own and rival price coefficients do not change much between IV1 and IV2, the coefficient on \( br \times price \) is now insignificant. Allowing asymmetry in the coefficients on rival price, the coefficient on \( rp \) increases while its magnitude is almost same as that on \( rp \times br \). IV3 includes \textit{bonus}, \textit{duration}, and \textit{bonus duration 2}, which are assumed to be exogenous. The properties of the estimated coefficients are similar to those in OLS3 except that the cross term on own price is insignificant. In addition, the signs of the cross-price elasticities of Nabisco in IV2 and IV3 are not right, though their values are very small. IV4 treats the bonus-related variables as endogenous. The mean-elasticities are approximately -3.4, and the brand-specific elasticities are approximately -5.3 for Keebler and -2.3 for Nabisco. The cross-price elasticity of Keebler’s demand is 1.56 while that of Nabisco is 0.06, showing a strong asymmetry. The result shows that Keebler’s demand is sensitive to Nabisco’s prices while Nabisco’s demand is not. The over-identification test by J-statistics is not rejected in all estimations, thus demonstrating empirical support for the validity of instruments.

The results of demand estimations indicate that own-price elasticity is approximately -2.5 in OLS and -3.5 in the IV estimations when using store-level AAC and its lags as instruments. Cross-price elasticities under OLS and IV are different: asymmetry is much stronger in the IV estimations. Although the main claim of this paper regarding the relative size of menu costs between the monopolistic-competition model and the duopoly model will not be affected by the size of demand elasticity, the size of the point estimate of menu costs will not be immune. I try the estimation of menu costs using results from both OLS and IV.

5.2. \textit{State variables} Table 6 shows the means and standard deviations of the state variables before discretization. The third column reports that price has a moderate degree of variance, demand condition has a relatively large variance, and production costs vary little. I discretize the state variables in vector \( x_i \) as follows. In the main exercise, the size of state space for each model is
1800; that is, $np = 5$, $nd = 6$, $nc = 1$, and $nbr = 2$, where $np$, $nd$, $nc$, and $nbr$ are the number of grids for price, demand condition, cost, and brand dummy, respectively. I set the lower and upper bounds of state space to the 5 percent and 95 percent tiles of the samples. The number of grids of each variable is relatively small compared to the recent applications of dynamic discrete choice models.\footnote{For example, the size of state space in Collard-Wexler (2010) who focuses on the U.S. concrete industry is 1.4 million. In contrast, studies such as Slade (1998) and Aguirregabiria (1999), whose results are used for comparison, use a smaller state space.} This size of discretization is, however, appropriate in the current application because the range of the choice variable, real price, is small. The 10 percent quartile of real price is 2.08 per box and the 90 percent quartile is 2.46 per box. Thus, dividing it into 5 grids creates small bins. The last variable in the vector of state variables, $br$, is a fixed state variable that takes one for Nabisco ($i = 2$) and zero for Keebler ($i = 1$). In addition, the coarseness of state space does not affect the estimated size of menu costs. Trying estimations with various sizes of state space, I find no systematic relationship between the coarseness of state space and the estimated size of menu costs in the following exercise.

5.3. Wholesale price Table 7 shows the mean value of derived wholesale prices and the frequency of wholesale price changes. On average, both brands change their wholesale prices 26 percent of time, with Keebler making changes slightly more frequently.\footnote{When recovered wholesale price exceeds retail price, I scale down the directly recovered wholesale price so that wholesale price is equivalent to the mean of AAC, although this is an ad-hoc way to construct wholesale price.}

5.4. Estimation of menu costs Table 8 presents the results of the structural estimation of $\gamma$ for both the duopoly model and the monopolistic-competition model using the result of IV4 in the demand estimation. The size of the estimate of $\gamma$ is 4.53 for the monopolistic-competition model and 1.96 for the duopoly model. While the two estimates are statistically significant at the 1 percent level, the duopoly model results in a higher likelihood, which means a better fit to the data. Estimated $\gamma$ in the duopoly model is much smaller than that in the monopolistic-competition model. From the difference in estimated $\gamma$ between the two models, this upward bias can be inferred to be due to the specification of the monopolistic-competition model.

The above result depends on the specification of a demand equation and a specific size of
state space. To demonstrate the robustness of the above result, I first estimate the duopoly model by different specifications of demand equation and then by different sizes of state space. Table 9 shows the results across different specifications of demand estimation. The second to the fifth columns show the estimated menu costs under the assumption of the duopoly model using the results from all the specifications. Although the results using the IV estimations are slightly higher than those using OLS, the difference among the results is small. Thus, the result is robust with respect to which demand estimation result is employed. Second, Table 10 shows the estimates by different levels of state space coarseness. The rows indicate the number of grids of demand condition \( nd \), and the columns take the number of grids of price \( np \). For example, \( nd = 2 \) and \( np = 2 \) means that the price and demand condition are divided into two grids for each. This implies that the size of state space is 32. As stated in the section on state space, there is no systematic relationship between the size of state space and the estimated size of the menu costs. On average, the size of menu costs is approximately 1.85, which is close to the estimate in Table 7.

Table 11 compares the results of this paper with those of previous studies. Due to the specific structure of this model, the estimated menu costs may not be directly comparable to the ones in the previous studies. Nevertheless, it will be valuable to examine what factor can contribute to the differences and similarities in the results. The first row of the table shows the result of the duopoly model. Its point estimate of the menu costs parameter, 1.96, is greater than the result obtained by Aguirregabiria (1999), 1.45, and the result obtained by Lévy et al. (1997), 0.52, while it is smaller than the result obtained in Slade (1998).\(^{26}\) It is not surprising that the estimate of this paper is greater than the direct measure of menu costs calculated by Lévy et al. (1997), 0.52, because my estimate captures any costs associated with price changes, whereas the reported number by Lévy et al. (1997) includes only the physical and labor costs of price changes.

The size of menu costs with respect to the percentage of revenue is 18 percent in this paper. While this number is much greater than those reported in previous studies, it is closest to the estimate obtained by Slade (1998), which is fairly large in the previous studies. Note that

\(^{26}\)The result of Aguirregabiria (1999), 1.45, is calculated from the reported values of asymmetric menu costs using reported shares in revenue as weights from Table 6. He also reports the results of the specification with symmetric menu costs, whose estimated results are also close to this value (for example, 1.12 in specification 2 in Table 5). Slade (1998) does not report the estimate of menu costs as a percentage of revenue. Revenue is calculated as the weighted average across brands using the information provided in her paper.
Aguirregabiria (1999) estimates menu costs using various products, while Slade (1998) examines a single product, as do I. This difference implies that menu costs might be relatively uniform across products in retail stores, and that the large estimate of menu costs as a percentage of revenue that this paper observes might simply reflect the small revenues generated by graham crackers.

The bottom row of Table 11 shows the estimated value of menu costs from a recent study by Nakamura and Zerom (2010) who use a dynamic oligopolistic model. Their estimate of menu costs as a percentage of revenue is much smaller than the one I obtain.\footnote{Their estimate of the absolute magnitude of menu costs is not comparable because their menu costs are for price changes within the entire U.S. market.} One reason may be that when they estimate menu costs at the level of wholesale markets, their menu costs may not include an important part of price changes at retail markets, such as the costs to print and deliver price tags. Another reason may be the difference in the specification of the market structure between this study and theirs. As this paper assumes a duopolistic model abstracting potential strategic interactions with the other two brands, the estimate of menu costs in this study may still be biased upward.

Although the estimated size of menu costs in this paper is from a single product, it is informative to compare the size of menu costs with that calibrated commonly in past studies in macroeconomics. For example, under a general equilibrium model with monopolistic competitions and menu costs, Blanchard and Kiyotaki (1987) calculate that menu costs amounting to 0.08 percent of total revenue suffices to prevent firms from adjusting their prices. The subsequent studies in macroeconomics require a size of 0.5-0.7 percent of total revenue to fit the models to selected sample moments and to affect aggregate price dynamics (e.g., Golosov and Lucas 2007). The empirical results from grocery stores, as studied herein, show that the estimated size of menu costs is large enough to have significant effects on aggregate price adjustments. Therefore, I conclude that menu costs have significant implications for price adjustment behaviors economically and statistically.

5.5. \textit{Price rigidity and state space} The above estimation result has shown that not only menu costs but also dynamic duopolistic interactions play an important role in explaining the price rigidities observed in the data. Menu costs comprise an exogenous source for price rigidity while strategic interactions create price rigidity endogenously. The overall price rigidity implied by the model under
particular menu costs is expressed by the equilibrium conditional choice probabilities of no price change. To examine the properties of price rigidity due to strategic interactions, I next examine the properties of the conditional choice probability of no price changes by conducting a simulation exercise.

The main results of this exercise are as follows. First, overall price rigidity is stronger in the duopoly model than in the monopolistic-competition model. Second, own-price elasticity and cross-price elasticities are crucial in determining how price rigidity relates to own and rival prices in state space. As strategic complementarity becomes stronger, price tends to be more rigid in response to the higher past price levels of both brands. Third, dynamics also play an important role for strategic complementarity to impact price rigidity. Taking into account future reactions leads to more complex reactions to rival’s state variables, as compared to in a myopic model under the assumption of duopolistic competition. I discuss these three results in detail below.

Figures 2(a) to 2(d) show the contour plots of the predicted choice probabilities of no price changes in the monopolistic-competition model and the duopoly model assuming that the menu costs are set to be 1.96, which is the result of the duopoly model in Table 7. The results are based on the estimation using IV4 with the number of grids being np = 5 and nd = 6. Figure 2(a) shows the predicted choice probabilities of no price change in Keebler in the duopoly model; Figure 2(b), of Keebler in the monopolistic-competition model; Figure 2(c), of Nabisco in the duopoly model; and Figure 2(d), of Nabisco in the monopolistic-competition model. The horizontal axis shows own past price in state space, \( p_{t-1} \), and the vertical axis shows \( rp_{t-1} \). In other words, 1 on the horizontal axis corresponds to the lowest previous price level: \( p^1 \) for \( p_{t-1} \), and so on. Thus, the figures can tell us how price rigidity due to strategies varies over previous own price level, previous rival price level, and previous relative price. The discrepancy between the previous prices of the two brands is zero on a 45-degree line. The choice probabilities for the duopoly model are those estimated in section 4.4 while those for the monopolistic-competition model are simulated. The choice probabilities are shown on the curves of the contour plots. The darker an area, the higher is price rigidity.\(^{29}\)

\[^{28}\] The figures are drawn using contours in MATLAB. All probabilities between the discretized prices are approximated.

\[^{29}\] Predicted choice probabilities are constructed conditional on the demand conditions and the production costs, whose transitions are exogenous. I took the averages of the predicted choice probabilities over the demand conditions.
The figures highlight three important aspects of the estimated conditional choice probabilities. First, the difference between the monopolistic-competition model and the duopoly model is apparent: the monopolistic-competition model predicts lower probabilities of no price changes than the duopoly model for both brands: 0.11 vs. 0.39 for Keebler and 0.39 vs. 0.60 for Nabisco, on average. As noted previously, the duopoly model in this paper specifies no deep theoretical structure to generate a higher price rigidity in the presence of dynamic duopolistic interactions. However, since the only difference between the monopolistic competition and the duopoly model is how strategy is formed, this result suggests that price rigidity in the duopoly model is generated from tighter interactions between the two national brands. Such a strong strategic interaction observed in the duopoly model is the primary source for upward bias in the estimates of menu costs if the underlying data-generating process is specified as the monopolistic-competition model. Second, the table highlights the asymmetry between the two brands. Both in the monopolistic-competition model and the duopoly model, Keebler tends to change its price more frequently than Nabisco. This property is consistent with the observed data. Third, price rigidity is highly responsive to own state. Price rigidity dramatically increases as own state becomes lower, and this tendency is more strong in the duopoly model.

The question is how the strategic interactions in the duopoly model lead to more price rigidities as compared to the monopolistic-competition model. For example, as stated in the introduction, Slade (1999) suggests that price rigidities will be stronger as previous price level is higher due to strategic complementarity. If dynamic duopolistic competition exacerbates such strategic complementarity, it can be the source of stronger price rigidity in the duopoly model. Such observation is, however, not seen in the previous figures. To see the effect of strategic complementarity, I examine the changes in price rigidities as the coefficient on rival price varies. The key parameters in this exercise are the coefficients on \( rp \) and \( p \). Therefore, I first examine how the coefficient on \( p \) affects the degree of price rigidity.

Figures s3(a) to 3(f) show the contour plots of the predicted choice probabilities under the different sizes of the coefficient on \( p \). In this exercise, I keep the degree of asymmetricity between the brands low: the coefficients on \( price \times br \), \( rp \times br \), and \( br \) are set to be 1. The menu costs are set to be 2.0. To highlight the impact of own price coefficient, the coefficient on \( rp \) is set to be 1, and cost for each pair of \((p_{1t-1}, p_{2t-1})\).
with which strategic complementarity is fairly weak. The figures on the top show the relationship between price rigidity and state variables of prices when the coefficient on $p$ is as large as -30, implying a relatively high own demand elasticity on average. This value is close to the one in the main result. With such a high demand elasticity, the brands tend to price their products at the lower level. The relationship reverses as the own price coefficient becomes smaller. When the size of the own price coefficient is -20, price rigidity becomes higher as own state is higher, indicating that the brands are exploiting less elastic demand.

Figures 4(a) to 4(f) show the relationship between price rigidity and strategic complementarity, which is captured by the coefficient on $r_p$. The larger size of the coefficient implies greater strategic complementarity. These results are from an exercise similar to that in Figure 3, but now, the coefficient on $r_p$ is changed keeping the coefficient on $p$ at -30. The other coefficients are the same as in Figure 3. The sizes of the coefficient on $r_p$ are 1, 10, and 15 for the top, middle, and bottom figures. Comparing the three figures under the assumption of duopoly competition highlights how price rigidity can vary in response to previous rival price depending on the size of strategic complementarity. The comparison shows that first, as strategic complementarity gets stronger, price rigidity at a higher level of own state increases. Second, price rigidities become more responsive to rival state as strategic complementarity becomes stronger. In Figure 4(e), the area with the highest price rigidities is the one with the highest prices, both own and rival. This is along the intuition of Slade (1999), as discussed above. Third, as price complementarity becomes stronger, brands are more likely to change prices as the discrepancy between own and rival prices in state space increases. The probability of no price change is the lowest at the top-left and the bottom-right of the figures where the discrepancy between prices is the highest. This uncovers the strong tendency to try to catch-up with the rival in an environment with high strategic complementarities. Thus, brands become more sensitive to relative price as strategic complementarities become greater. Finally, comparing the figures in the duopoly and the monopolistic-competition model makes it clear that the choice probabilities of the duopoly model are more responsive to past rival’s price. This comparison shows that strategic complementarity is more likely to lead to price rigidity under dynamic duopolistic interactions than under the monopolistic-competition model.

The final question is how dynamics play a role in the above result. Figure 5 compares price rigidity in the static and dynamic models. Figures 5(a) and 5(b) show the same plots as the
bottom plots in Figure 4. In these plots, the size of the discount factor is set to be 0.99. Figures 5(c) and 5(d) are the contour plots of the choice probabilities when the discount factor is set to be 0 keeping the other conditions the same as in Figures 4(e) and 4(f). Comparing Figures 5(a) and 5(c) reveal how the presence of dynamics is important for strategic interactions to impact price rigidities. In Figure 5(a), choice probabilities vary much along the different states of rival prices. This implies that own current action also influences rival’s future actions, and each brand takes into account such dynamic interactions. In contrast, such interactions are almost absent in Figure 5(c), where brands act myopically. The figures also shows the contrast between the duopoly model and the monopolistic-competition model. Figures 5(b) and 5(d), which show the predicted choice probabilities in the monopolistic-competition model with $\beta = 0.99$ and $\beta = 0$, indicate that the monopolistic-competition model lacks clear reactions to rival price.

The results of the simulation have shown that dynamic strategic interactions could induce a significant degree of price rigidity. This result implies an important message of this paper: not only menu costs but also dynamic strategic interactions among brands are important for explaining the observed degree of price rigidity.

6. Conclusion

This paper studies weekly price movements of a typical product sold in retail stores, graham crackers. As is commonly observed in retail price data, the price movements of the product are well characterized by frequent discrete jumps. To explain the discreteness of price changes, I employ a dynamic discrete-choice model with menu costs as the hypothesized data-generating process. Because the market of graham crackers is dominated by only a few brands and the pricing behaviors of the two national brands are similar to each other, I further take into account duopolistic interactions between the two national brands to examine the possible effects of dynamic strategic interactions on the discrete behavior of prices. I estimate this dynamic discrete-choice model with duopolistic competition by exploiting a recent development in the estimation of dynamic discrete choice games, the NPL estimator. The results show that menu costs are important both statistically and economically. In addition, I claim that adopting a monopolistic-competition model for explaining price data could lead to a possible bias in the estimate of menu costs. If dynamic
strategic interactions among firms affect the pricing behavior in the sample, the estimated menu costs in a monopolistic-competition model are biased upwards because strategic interactions in a duopolistic competition potentially create price rigidity. The results show that the estimate of menu costs under a dynamic-duopoly market is smaller than and significantly different from that under monopolistic competition. This finding means that dynamic-duopoly competitions explain some part of price rigidity, which is captured only by menu costs unless a researcher incorporates dynamic-duopoly interactions in the data. Thus, at least in the sample examined in this paper, I conclude that dynamic strategic interactions could be a crucial source of price rigidity and the assumption about market structure is important to identify menu costs.

A caveat should be mentioned on the whole exercise of this paper. As mentioned before, this paper does not specify any theoretical structure in strategic interactions between brands that leads to price rigidity a priori. An extension of this paper will be to incorporate a structure that can more explicitly cause price rigidity due to dynamic strategic interactions such as an implicit collusion. Rotemberg and Saloner (1990), Rotemberg and Woodford (1992), and Athey, Bagwell, and Sanchirico (2004) show theoretically that strategies with rigid prices can be supported as results of a collusion between duopolistic and oligopolistic environments. Using the entry-and-exit model, Fershtman and Pakes (2000) numerically analyze a dynamic game allowing collusion. I leave developing a dynamic pricing model by incorporating the implications of these studies to future research.
Appendix

1 Construction of production costs

This section explains the construction of production costs. According to the package of graham crackers, the main ingredients are enriched flour (wheat flour and fortification ingredients such as iron), whole grain wheat flour, sugar, oil, salt, corn syrup, baking soda, cornstarch, and artificial flavor. In addition, according to the Input-Output table, wage and paper also make up for a significant portion of costs. In the 1992 benchmark for the cookies and crackers industry (industry number 141802), the top components in production costs are the value added (35 percent), compensation to employees (21 percent), paperboard containers and boxes (4.9 percent), flour and other grain mill products (4.3 percent), wholesale trade (3.3 percent), sugar (2.8 percent), and edible fats and oil (2.8 percent). These components account for about 70 percent of the output value.

I use monthly PPI of these main components to create a measure of production costs combining the information of wholesale price.\(^{30}\) First, the PPI of the matching major components in the Input-Output table are collected from the BLS web-site. These are the PPI of wheat flour (series id: WPU02120301), fats and oils (WPU027), sugar (WPU0253), wholesale trade (CEU4142000035), and paper boxes and containers (WPU091503). I also obtained the wage index of hourly earnings of non-durable manufacturers (CEU3200000008) as a measure of wage. Since the PPI of whole grain flour was not available, that of plain flour was used instead.

Having obtained these PPI values, I create a measure of costs in the following steps.

1. Normalize each series by the values of September 1989, which is the first month in the sample period, so that the values in the starting period are all 1.

2. Calculate the average wholesale price of a graham cracker in the starting period. I use the average of AAC per box of three brands, except the private brand, for September 1989 and October 1989, from the DFF data set. I omit the private brand since its AAC is very low, and may not include the margin in the same manner as the other brands. The AAC of October 1989 is included because of the small sample number in September 1989.

3. Set the cost of flour in September 1989 as 4.3 percent of the above average wholesale price, and calculate the dollar value, which yields $ 0.07. Calculate the costs of the other variables in the same

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\(^{30}\)To create a measure of production costs, it is ideal to obtain the wholesale prices of the main ingredients, and use their utilization ratios to create a box of graham crackers. However, wholesale prices per pound are available for only wheat flour and cane sugar, and brands’ recipes for graham cracker are not obtainable.
manner.

4. Calculate the costs from October 1989 and thereafter by adopting the growth of the PPI series. For example, the cost of flour in October 1989 ($0.07) is the cost in September 1989 ($0.07) times the PPI of flour in the same period (0.99).

5. Take the sum of the costs of wheat flour, fats, sugar, wholesale trade, and wage. The average of the implied costs from September 1989 to the end period May 1998 is $0.71 per box.

2 Monopolistic-competition model

This section describes the monopolistic-competition model with menu costs. The difference from the duopoly model in the main text is that as in the monopolistic-competition models in Slade (1998) and Aguirregabiria (1999), a brand regards the evolution of \( r_p t \) as exogenous. The problem of retail stores is the same as in the duopoly model. In the problem of brands, linear demand is the same as in the duopolistic model but comes without a brand-specific subscript:

\[
q_{st} = d_{st} - b_0 p_{st} + b_1 r_p_{st} + (b_2 p_{st} + b_3 r_p_{st}) \times br + e_{st}. \tag{A1-1}
\]

The one-period profit at period \( t \) conditional on choosing alternative \( j \) is defined as

\[
\Pi^j_{st}(x_{st}) = (w^j_{st} - c_t)E_t[q_{st}] + e^j_{st} - \gamma I(p_{st} \neq p_{st-1})I(P_{st} \neq P_{st-1}), \tag{A1-2}
\]

where \( E_t \) stands for the conditional expectation operator on the realization of \( d_{ist} \) conditional on the current realization of state variable \( x_{st} \). A brand takes into account \( r_p t \) but regards its evolution as exogenous.

The timing of the game is as described in the duopoly model. First, a brand observes state variables \((p_{t-1}, r_p t_{-1}, d_{t-1}, r d_{t-1}, c_t, br)\). The assumptions about the evolution of demand condition and production costs are the same as before. The brand also receives private information \( \varepsilon_t \) that affects its profitability.

Private information consists of \( J = L + 1 \) randomly drawn unobserved profit components, which distribute \( i.i.d. \) across time and alternatives. Then, the brand chooses whether or not to suggest a price change, and chooses the wholesale price. Considering a stationary Markov environment, I denote state space as \( \{xs, \varepsilon\} = \{p, rp, d, rd, c, br, \varepsilon\} \).

Let \( \Pi \) be the expected one-period profit conditional on choosing alternative \( j \) and \( xs \), and let \( V(xs') \) be the value with private information being integrated out. Given state \( xs \) and private information \( \varepsilon \), the Bellman equation conditional on choosing \( j \) after integrating out private information is

\[
V^j(xs) = \int \max_{j' \in J} \Pi(xs, j) + e^j + \beta \sum_{xs} f(xs'|xs, j)V(xs')g(\varepsilon)d\varepsilon, \tag{A1-3}
\]
where \( \Pi(xs, j) \) is the profit defined by a set of state variables \( ks \) conditional on player \( i \) choosing alternative \( j \). The conditional choice probability to choose alternative \( j \) is

\[
Pr(j|xs) = \int I\{\max_{j \in J}\{\Pi(xs, j) + \varepsilon_j + \beta \sum_{xs' \in xs} f(xs'|xs, j)V(xs')\}\}g(\varepsilon)d\varepsilon.
\]  

(A1-4)

The right-hand side of equation (A1-3) defines a contraction mapping in the space of the integrated value functions. There exists a unique value function \( V_i \) that solves functional equation (A1-3).

3 Estimation procedure

This paper exploits the NPL algorithm developed by Aguirregabiria and Mira (2002, 2007). Below, I describe some details of the estimation procedure in this paper. The estimation consists of the following steps.

1. Estimation of demand equation
2. Construction of state space:
   - construction of demand conditions using the result of demand equation
   - discretization of state variables
3. Estimation of initial choice probabilities
4. Estimation of the laws of the evolutions of state variables
5. Estimation of dynamic parameter

Steps 1 to 3 are discussed in the main text. Now, I talk about steps 4 and 5. In actual practice, I modified the procedure described in Aguirregabiria (2001): “A Gauss program for the estimation of discrete choice dynamic programming models using a nested pseudo likelihood algorithm.”

Step 4: The estimation of the transition probability matrices of state variables I construct the transition probability matrices for \( f(d_{ist}|d_{ist-1}) \) as follows. The construction of transition probability matrices of \( d, rd, \) and \( rp \) are analogous. The transition probability matrix for \( rp \) is used in the monopolistic-competition model. For example, the stochastic process of the demand condition for brand \( i \) is specified as follows:

\[
d_{ist} = \delta_{d0} + \delta_{d1}d_{ist-1} + \epsilon^d_{ist},
\]  

(A3-1)

where \( d_{ist} \) and \( d_{ist-1} \) are continuous demand conditions; \( \delta_{d0} \) and \( \delta_{d1} \) are the coefficients; and \( \epsilon^d_{ist} \) follows an i.i.d. distribution function \( f_{\epsilon^d_{ist}} \). The process of rival price is specified in an analogous manner. The coefficients of the above process are estimated using OLS. Then, using the Kernel density estimation, I derive the distribution of residual non-parametrically. I construct the transition probability matrix counting the frequency of realization of each pair of \( d_{ist} \) and \( d_{ist-1} \).
Step 5: Estimation of menu costs parameter

According to Aguirregabiria (2001) and Aguirregabiria and Mira (2007), I derive an alternative presentation of value functions and conditional choice probabilities, which are used in the pseudo-likelihood estimation of the menu costs parameter.

Let $P^*$ be a matrix of equilibrium probabilities, which are best-response probabilities, and $V^*_i$ be the corresponding value functions of brand $i$. Using $P^*$ and $V^*_i$, I can rewrite the Bellman equation (7) as

$$V^*_i(x) = \sum_{j \in J} P^*_i(j | x) [\Pi^*_i(j, x) + \epsilon^*_i(j)] + \beta \sum_{x' \in X} f^*_i(x' | x) V^*_i(x'), \quad (A3-2)$$

where $f^*_i(x' | x)$ is the transition probability induced by $P^*$, and $\epsilon^*_i(j)$ is the expectation of $\varepsilon^*_i$ conditional on $x$.$^{31}$ In vector form, equation (A3-2) is

$$V^*_i = \sum_{j \in J} P^*_i(j)[\Pi^*_i(j) + \epsilon^*_i(j)] + \beta \sum_{x' \in X} F^*_i V^*_i, \quad (A3-3)$$

where $V^*_i$, $P^*_i(j)$, $\Pi^*_i(j)$, and $\epsilon^*_i(j)$ are the vectors of corresponding elements in equation (A3-2) with dimension $M$, which is the size of state space. $F^*_i$ is a matrix of the transition probabilities of $f^*_i(x' | x)$.

Under condition $\beta < 1$, the value function given $P^*$ can be obtained as a solution of the following linear equation:

$$(I - \beta F^*_i) V^*_i = \sum_{j \in J} P^*_i(j)[\Pi^*_i(j) + \epsilon^*_i(j)], \quad (A3-4)$$

where $I$ is an identity matrix with dimension $M$. Denote the mapping for the solution of equation (A3-4) as $\Gamma_i(x; P^*)$. For an arbitrary set of probabilities $P$, the mapping operator $\Gamma_i(x; P)$ gives the values for brand $i$ when all the brands behave according to $P$. Note that this mapping is constructed given the conditional choice probabilities of brand $i$ as well as those of its rival brand. Using mapping $\Gamma$, instead of $V^*_i$ in equation (A3-4), I define a mapping $\Psi_i$ to calculate the expected value for brand $i$ to choose action $a_i$ for $P$:

$$\Psi_i(j | x) = \int I\{j = \arg \max_{j \in J} \Pi^*_i(j, x) + \epsilon^*_i + \beta \sum_{x'} f(x' | x, j) \Gamma_i P^*_i(x')\} g_i(\varepsilon) d\varepsilon_i, \quad (A3-5)$$

I use the two mappings, $\Gamma_i(x; P)$ and $\Psi_i(j | x)$, to estimate menu costs $\gamma$.

Next, the pseudo-likelihood function to estimate menu costs is derived. For convenience, define the following notations. The expected price of a competing brand under its conditional choice probability $P$ is $P^*_{-i} = \sum_{j \neq i} P(j \neq i | x) p_{-i}$ for given $x_t$. Given the estimated coefficients of demand equation and constructed demand conditions $\hat{d}_{ist}$, I set up the expected one-period profit associated with action $a$ as

$$\hat{\Pi}^*_i(j, x_t) = (\hat{w}^*_i - c_{ist})(\hat{d}_{ist} - \hat{b}_0 p^*_i + \hat{b}_1 P^*_{-i}) + (\hat{b}_2 p_{ist} + \hat{b}_3 P^*_{-ist}) \times br - \gamma I\{j = 1\}. \quad (A3-6)$$

For exposition, denote $\hat{\Pi}^*_i(j, x_t) = \hat{\Pi}^*_{ist} \theta$, where $\varepsilon^*_i = \{(\hat{w}^*_i - c_{ist})(\hat{d}_{ist} - \hat{b}_0 p^*_i + \hat{b}_1 P^*_{-i}) + (\hat{b}_2 p_{ist} + \hat{b}_3 P^*_{-ist}) \times br) -\gamma I\{j = 1\}\}$ and $\theta = \{1, \gamma\}$. Let $F^*$ be the transition probability matrix representing all the transi-

$^{31}$That is, $f^*_i(x' | x) = \sum_{j, j \neq i} P^*_i(j | x) P^*_{-i}(j \neq i | x) f(x' | x, j, j \neq i)$. 

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tion processes of state variables $x$ under conditional choice probabilities $P$, and $\epsilon^p_j(j)$ be a vector of the expectation of $\varepsilon^j$ conditional on $x$.\textsuperscript{32}

The empirical counterparts of the value functions and the best-response probabilities are derived according to the mapping expression by Aguirregabiria and Mira (2007). Let $\Gamma_i(P)$ denote the mapping operator of the value function in vector form given conditional choice probabilities $P$, and $\Psi(j | x)$ be the operator representing the best-response probabilities given $\Gamma_i(P)$. $\Gamma_i(P)$ can be written as $\Gamma_i(P) = Z_i^P \theta + \tau_i^P$, where $Z_i^P = (I - \beta F^P)^{-1} \sum_j P_i^*(j) \Pi_i(j)$ and $\tau_i^P = (I - \beta F^P)^{-1} \sum_{j} \epsilon_i^P(j)$, and where the value of discount factor is assumed to be known a priori and fixed at 0.99. Assume that private information follows an i.i.d. Type I Extreme Value distribution. Then, $\epsilon_i^P(j) = \text{Euler’s constant} - \ln(P_i^j)$, where Euler’s constant is about 0.577. The mapping of best-response probabilities $\Psi_i$ given $P$ is

$$\Psi_i(j) = \frac{\exp\{z_{ist}^j \theta + \beta F^j(Z_i^P \theta + \tau_i^P)\}}{\sum_j \exp\{z_{ist}^j \theta + \beta F^j(Z_i^P \theta + \tau_i^P)\}}. \quad (A3-7)$$

I construct a pseudo-likelihood function to estimate $\theta$ treating the conditional choice probability as nuisance parameters. Let $P^o$ and $\theta^o$ denote the true conditional choice probabilities and menu costs. Given true conditional choice probabilities $P^o$, the corresponding pseudo-log-likelihood function is

$$\sum_{i=1}^2 \sum_{s=1}^S \sum_{t=1}^{\infty} \sum_{j \in J} I\{j_{ist} = j\} \ln \Psi_i(j \mid x_{st}; P^o, \theta^o), \quad (A3-8)$$

where $\Psi_i(j \mid x; P^o, \theta^o)$ shows the dependence of $\Psi$ on conditional choice probabilities $P^o$ and menu costs $\theta^o$. The NPL estimator is obtained by the following procedure. I conduct the pseudo-maximum likelihood estimation of $\theta$ given a vector of the initial values of conditional choice probabilities, $P_0$, and then obtain the updated $\hat{P}_1$ according to mapping $\Psi$. I iterate this procedure for $K \geq 1$ stages. In the estimation, the $K$-stage pseudo-log-likelihood is constructed as:

$$\sum_{i=1}^2 \sum_{s=1}^S \sum_{t=1}^{T} \sum_{j \in J} I\{j_{ist} = j\} \ln \Psi_i(j \mid x_{st}; \hat{P}_{K-1}, \theta). \quad (A3-9)$$

Letting $\hat{\theta}_K$ denote the structural parameter that maximizes equation (A3-9) in the $K$-th stage, I can obtain the $K$-stage estimator of conditional choice probabilities:

$$\hat{P}_K = \Psi(\hat{P}_{K-1}; \hat{\theta}_K). \quad (A3-10)$$

Under standard regularity conditions, the parameter is consistent and asymptotically normal. Moreover, the estimator gains efficiency by repeating for $K > 1$ stages as compared to the estimator without iterations in terms of $K$. In practice, I conduct the estimation for stage $K$ until $\hat{P}_K = \hat{P}_{K-1}$, or equivalently, $\hat{\theta}_K = \hat{\theta}_{K-1}$.

\textsuperscript{32}F^P = \sum_{j_i} \sum_{j_{i-1}} P(j_i) * P(j_{i-1}) * (F^p_i \otimes F^p_i \otimes F^p_i \otimes F^c_i \otimes F^c_i \otimes F^c_i)$, where $*$ represents the element-by-element product, $\otimes$ represents the Kronecker product, and $F^p_i$ represents the matrix of transition probability $f^p_i$. 

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is obtained. The estimates converge fairly quickly (within 20 iterations). Note that the conditional expected profit except the menu costs parameter consists of the product of conditional expected demand and price-cost margin in U.S. dollars. Since the parameter of menu costs has the same unit as the conditional expected profit as specified in $\theta$, the estimated $\gamma$ is interpreted in the same unit.
References


Nevo, Aviv, and Konstantinos Hatzitaskos (2006) ‘Why Does the Average Price Paid Fall during High Demand Periods?’ Northwestern University, *manuscript*


Figure 1: Prices of the Four Brands of Standard Graham Crackers
Figure 2: Choice Probabilities of No Price Change at $\gamma = 1.96$

(a) Keebler duopoly
(b) Keebler MC
(c) Nabisco duopoly
(d) Nabisco MC

MC stands for the monopolistic-competition model.
Figure 3: Price Rigidity and Own Demand Elasticity

1. MC stands for the monopolistic-competition model.
2. The coefficient on rival price is set to be 1.
Figure 4: Price Rigidity and Strategic Complementarity

1. MC stands for the monopolistic-competition model.
2. The coefficient on own price is set to be -30.
Figure 5: Comparison with the Model with Myopic Agents

(a) duopoly, beta=0.99
(b) MC, beta=0.99
(c) duopoly, beta=0
(d) MC, beta=0

1. MC stands for the monopolistic-competition model.
2. The coefficient on own price is -30.
3. The coefficient on rival price is 15.
Table 1: Summary statistics of brands

<table>
<thead>
<tr>
<th>Brand</th>
<th>nob</th>
<th>Size of a box</th>
<th>Market share</th>
<th>Mean price</th>
<th>Mean quantity sold</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>unit</td>
<td>oz</td>
<td>%</td>
<td>$/oz</td>
<td>oz</td>
</tr>
<tr>
<td>Keebler</td>
<td>7333</td>
<td>15</td>
<td>18.0</td>
<td>0.17</td>
<td>118</td>
</tr>
<tr>
<td>Nabisco</td>
<td>7485</td>
<td>16</td>
<td>34.6</td>
<td>0.16</td>
<td>234</td>
</tr>
<tr>
<td>Sarelno</td>
<td>7418</td>
<td>16</td>
<td>16.9</td>
<td>0.15</td>
<td>126</td>
</tr>
<tr>
<td>Dominick’s</td>
<td>7340</td>
<td>16</td>
<td>30.4</td>
<td>0.12</td>
<td>280</td>
</tr>
</tbody>
</table>

1. Market shares are those of revenue.
2. Prices are nominal.
3. The observations are those with a positive purchase. The statistics are calculated before list-wise deletion for the estimation.

Table 2: Frequency of nominal price changes

<table>
<thead>
<tr>
<th>Brand</th>
<th>nob</th>
<th>Price Changes</th>
<th>Downward</th>
<th>Upward</th>
<th>Yearly change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>unit</td>
<td>%</td>
<td>%</td>
<td>%</td>
<td>times per year</td>
</tr>
<tr>
<td>Keebler</td>
<td>7057</td>
<td>31.8</td>
<td>16.2</td>
<td>15.6</td>
<td>16.5</td>
</tr>
<tr>
<td>Nabisco</td>
<td>7330</td>
<td>33.9</td>
<td>17</td>
<td>15.6</td>
<td>17.6</td>
</tr>
<tr>
<td>Sarelno</td>
<td>7193</td>
<td>22.5</td>
<td>11.4</td>
<td>11.1</td>
<td>11.7</td>
</tr>
<tr>
<td>Dominick’s</td>
<td>7142</td>
<td>26.8</td>
<td>14.1</td>
<td>12.8</td>
<td>13.9</td>
</tr>
<tr>
<td>average</td>
<td></td>
<td>32.85</td>
<td>16.6</td>
<td>15.6</td>
<td>17.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Nabisco &amp; Keebler)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>average</td>
<td></td>
<td>24.65</td>
<td>12.75</td>
<td>11.95</td>
<td>12.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(Sarelno &amp; Dominick’s)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Yearly change is the average number of price changes per year (52 weeks).
2. The observations are those with a positive purchase and a lagged value. The statistics are calculated before list-wise deletion for the estimation.

Table 3: Summary statistics of bonus

<table>
<thead>
<tr>
<th>Brand</th>
<th>nob</th>
<th>frequency</th>
<th>mean length</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>unit</td>
<td>%</td>
<td>week</td>
</tr>
<tr>
<td>Keebler</td>
<td>7579</td>
<td>28</td>
<td>2.3</td>
</tr>
<tr>
<td>Nabisco</td>
<td>7579</td>
<td>21</td>
<td>2.1</td>
</tr>
</tbody>
</table>

1. The observations are for all the weeks in the sample.

Table 4: Summary statistics of cost ($U.S. per oz)

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>std.dev</th>
<th>min</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>cost (nominal)</td>
<td>0.04</td>
<td>0.003</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>cost (real)</td>
<td>0.04</td>
<td>0.001</td>
<td>0.04</td>
<td>0.041</td>
</tr>
</tbody>
</table>
Table 5: Demand Estimation Results: OLS and IV with outliers

<table>
<thead>
<tr>
<th>Variable</th>
<th>OLS1</th>
<th>OLS2</th>
<th>OLS3</th>
<th>IV1</th>
<th>IV2</th>
<th>IV3</th>
<th>IV4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>-32.43 (1.03)</td>
<td>-29.23 (1.62)</td>
<td>-29.82 (1.39)</td>
<td>-39.02 (2.16)</td>
<td>-38.94 (2.70)</td>
<td>-39.72 (3.18)</td>
<td>-39.49 (6.44)</td>
</tr>
<tr>
<td>$r p$</td>
<td>5.78 (1.00)</td>
<td>8.36 (1.82)</td>
<td>8.36 (1.49)</td>
<td>2.97 (1.58)</td>
<td>13.87 (2.43)</td>
<td>14.05 (2.41)</td>
<td>14.12 (3.41)</td>
</tr>
<tr>
<td>$s dp$</td>
<td>0.33 (1.42)</td>
<td>0.38 (1.74)</td>
<td>0.43 (1.42)</td>
<td>18.18 (7.17)</td>
<td>15.05 (7.33)</td>
<td>13.72 (7.67)</td>
<td>5.77 (10.33)</td>
</tr>
<tr>
<td>$price \times br$</td>
<td>-7.52 (2.41)</td>
<td>-7.84 (2.01)</td>
<td>2.32 (4.96)</td>
<td>1.91 (4.69)</td>
<td>7.39 (7.17)</td>
<td>7.39 (7.17)</td>
<td>7.39 (7.17)</td>
</tr>
<tr>
<td>$r p \times br$</td>
<td>-5.21 (2.41)</td>
<td>-4.59 (1.97)</td>
<td>-14.95 (3.32)</td>
<td>-15.47 (3.30)</td>
<td>-13.06 (5.45)</td>
<td>-13.06 (5.45)</td>
<td>-13.06 (5.45)</td>
</tr>
<tr>
<td>$cc$</td>
<td>6.79 (0.24)</td>
<td>25.71 (0.91)</td>
<td>25.16 (3.88)</td>
<td>6.62 (0.26)</td>
<td>25.27 (9.54)</td>
<td>26.61 (8.88)</td>
<td>14.39 (17.96)</td>
</tr>
<tr>
<td>$bonus$</td>
<td>1.68 (0.47)</td>
<td>1.51 (0.74)</td>
<td>1.54 (4.25)</td>
<td>-0.16 (0.09)</td>
<td>0.73 (1.11)</td>
<td>-0.16 (0.09)</td>
<td>0.73 (1.11)</td>
</tr>
<tr>
<td>$bonus\ duration$</td>
<td>-0.043 (0.10)</td>
<td>-0.16 (0.09)</td>
<td>0.73 (1.11)</td>
<td>-0.16 (0.09)</td>
<td>0.73 (1.11)</td>
<td>-0.16 (0.09)</td>
<td>0.73 (1.11)</td>
</tr>
<tr>
<td>$bonus\ duration 2$</td>
<td>-10.29 (1.72)</td>
<td>-11.18 (1.95)</td>
<td>-6.08 (14.13)</td>
<td>-11.18 (1.95)</td>
<td>-6.08 (14.13)</td>
<td>-11.18 (1.95)</td>
<td>-6.08 (14.13)</td>
</tr>
<tr>
<td>$constant$</td>
<td>33.35 (3.02)</td>
<td>24.16 (3.55)</td>
<td>25.82 (3.65)</td>
<td>25.82 (3.65)</td>
<td>15.65 (9.63)</td>
<td>20.09 (13.59)</td>
<td>26.85 (23.86)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>N</th>
<th>14024</th>
<th>14024</th>
<th>14024</th>
<th>13120</th>
<th>13120</th>
<th>13120</th>
<th>12308</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R^2$</td>
<td>0.57</td>
<td>0.57</td>
<td>0.57</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
<td>0.56</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>J-stat (p-value)</th>
<th>3.91 (0.27)</th>
<th>2.67 (0.45)</th>
<th>1.80 (0.61)</th>
<th>6.08 (0.11)</th>
</tr>
</thead>
<tbody>
<tr>
<td>elasticity (average)</td>
<td>-2.80</td>
<td>-2.52</td>
<td>-2.57</td>
<td>-3.37</td>
</tr>
<tr>
<td>elasticity (Keebler)</td>
<td>-4.34</td>
<td>-3.91</td>
<td>-3.99</td>
<td>-5.22</td>
</tr>
<tr>
<td>elasticity (Nabisco)</td>
<td>-2.04</td>
<td>-2.31</td>
<td>-2.37</td>
<td>-2.45</td>
</tr>
<tr>
<td>cross-price elasticity (Keebler)</td>
<td>0.64</td>
<td>0.92</td>
<td>0.92</td>
<td>0.33</td>
</tr>
<tr>
<td>cross-price elasticity (Nabisco)</td>
<td>0.34</td>
<td>0.18</td>
<td>0.22</td>
<td>0.17</td>
</tr>
<tr>
<td>mean demand condition</td>
<td>53.30</td>
<td>44.23</td>
<td>46.00</td>
<td>46.81</td>
</tr>
</tbody>
</table>

1. The regressions include a dummy variable for outliers, store-dummy, and time-dummy variables (month and year).
2. Price variables are real-valued. The unit is the U.S. dollars per 10 ounces.
3. The dependant variable is log of quantity sold in 10 ounces.
4. The endogenous variables in IV1 - IV4 are $price$, $r p$, $s dp$, $price \times br$, and $r p \times br$. IV4 additionally treats $bonus$, $bonus \ duration$, $bonus \ duration 2$ as endogenous variables. The excluded instruments are AAC, rival AAC, AAC of Salerno and Dominick's, the lagged variables of these three AAC, the cross term of AAC and brand dummy, the cross term of rival AAC and brand dummy. IV4 includes the second order lagged AAC variables as the excluded instruments as well as the variables used for IV1-IV4.
5. Standard errors are in parenthesis. They are heteroscedasticity and auto-correlation robust.
6. Cross-price elasticity (Keebler) is the elasticity of Keebler's quantity demanded with respect to Nabisco's price. Similarly, cross-price elasticity (Nabisco) is the elasticity of Nabisco's quantity demanded with respect to Keebler's price.
Table 6: Summary statistics of state variables

<table>
<thead>
<tr>
<th></th>
<th>nob</th>
<th>mean</th>
<th>std.dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>price (per 10 ounces)</td>
<td>3678</td>
<td>1.49</td>
<td>0.12</td>
</tr>
<tr>
<td>demand condition (IV4)</td>
<td>3694</td>
<td>52.65</td>
<td>4.89</td>
</tr>
<tr>
<td>cost (per 10 oz)</td>
<td>3678</td>
<td>0.39</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 7: Mean statistics of the Wholesale Price

<table>
<thead>
<tr>
<th></th>
<th>Keebler</th>
<th>Nabisco</th>
</tr>
</thead>
<tbody>
<tr>
<td>wholesale price ($U.S. per 10 ounces, deflated)</td>
<td>1.07</td>
<td>1.10</td>
</tr>
<tr>
<td>frequency of large price change (%)</td>
<td>27</td>
<td>25</td>
</tr>
<tr>
<td>nob</td>
<td>1839</td>
<td>1839</td>
</tr>
</tbody>
</table>

Table 8: Estimated menu costs

<table>
<thead>
<tr>
<th></th>
<th>monopolistic competition</th>
<th>duopoly</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\gamma} )</td>
<td>4.53</td>
<td>1.96</td>
</tr>
<tr>
<td>s.e.</td>
<td>(0.10)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>log-likelihood</td>
<td>-945</td>
<td>-503</td>
</tr>
<tr>
<td>nob</td>
<td>3528</td>
<td>3528</td>
</tr>
</tbody>
</table>

1. The estimated results are based on the results of IV 4.
The size of the state space is 1800 (np=5, nd=6, nc=1, nbr=2).
2. The standard errors are inside parenthesis and
based on 5000 non-parametric bootstrapping re-samples.

Table 9: Estimated menu costs by different results of the demand estimation

<table>
<thead>
<tr>
<th></th>
<th>OLS1</th>
<th>OLS2</th>
<th>OLS3</th>
<th>IV1</th>
<th>IV2</th>
<th>IV3</th>
<th>IV4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\gamma} )</td>
<td>1.71</td>
<td>1.64</td>
<td>1.73</td>
<td>1.71</td>
<td>1.66</td>
<td>1.96</td>
<td>1.96</td>
</tr>
<tr>
<td>s.e.</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.05)</td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.06)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>log-likelihood</td>
<td>-478</td>
<td>-449</td>
<td>-433</td>
<td>-723</td>
<td>-677</td>
<td>-506</td>
<td>-503</td>
</tr>
<tr>
<td>nob</td>
<td>3678</td>
<td>3678</td>
<td>3678</td>
<td>3528</td>
<td>3528</td>
<td>3528</td>
<td>3528</td>
</tr>
</tbody>
</table>

1. The size of the state space is 1800.
2. The standard errors are inside parenthesis and
based on 5000 non-parametric bootstrapping re-samples.
Table 10: Estimated menu costs and the numbers of grids

<table>
<thead>
<tr>
<th>nd \ np</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.37</td>
<td>2.40</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>1.62</td>
<td>1.82</td>
<td>2.50</td>
<td>1.78</td>
</tr>
<tr>
<td>4</td>
<td>1.73</td>
<td>1.54</td>
<td>1.99</td>
<td>2.43</td>
</tr>
<tr>
<td>5</td>
<td>1.80</td>
<td>1.38</td>
<td>1.88</td>
<td>2.12</td>
</tr>
<tr>
<td>6</td>
<td>1.83</td>
<td>1.27</td>
<td>1.81</td>
<td>1.96</td>
</tr>
</tbody>
</table>

1. The estimated results are those of the duopoly model. The specification of the demand model is IV4.
2. ‘np’ stands for the number of grids of price and rp. ’nd’ stands for the number of grids of the demand condition, d.
3. No convergence was achieved for -.

Table 11: Comparison of estimated menu costs with previous studies

<table>
<thead>
<tr>
<th></th>
<th>size</th>
<th>% in revenues</th>
</tr>
</thead>
<tbody>
<tr>
<td>this study: \hat{\gamma}</td>
<td>1.96</td>
<td>18</td>
</tr>
<tr>
<td>Levy et al.</td>
<td>0.52</td>
<td>0.7</td>
</tr>
<tr>
<td>Slade</td>
<td>2.55</td>
<td>5.11 §</td>
</tr>
<tr>
<td>Aguirregabiria</td>
<td>1.45 †</td>
<td>0.7</td>
</tr>
<tr>
<td>Nakamura and Zerom</td>
<td>7000</td>
<td>0.23</td>
</tr>
</tbody>
</table>

1. §The value is calculated from Table 1A and VB as the share-weighted average.
2. †The value is calculated from Table 6. The reported value in this table is the result from the specification allowing for asymmetric menu costs. The result without asymmetry is close to this value.