Hitotsubashi University
Graduate School of Economics

Discussion Paper No.2013-02

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March 2013

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March 6, 2013

Abstract

When encountering a set of alternatives displayed in the form of a list, the decision maker usually determines a particular alternative, after which she *stops* checking the remaining ones, and *chooses* an alternative from those observed so far. We present a framework in which both decision problems are explicitly modeled, and axiomatically characterize a stop-and-choose rule which unifies position-biased successive choice and satisficing choice.

JEL Classification Numbers: D00, D11, D83
Keywords: choice function, list, satisficing choice, stopping decision, successive choice

*We would like to thank Salvador Barberà, Walter Bossert, Dipjyoti Majumdar, Paola Manzini, Marco Mariotti, Jordi Massó, Debasis Mishra, Ariel Rubinstein, and Arunava Sen for their helpful comments. Saptarshi Mukherjee gratefully acknowledges the support of FEDER and of the Spanish Ministry of Science and Innovation through grant "Consolidated Group-C" ECO2008-04756.

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1 Introduction

Classical choice theory discusses the act of choosing one or more alternatives from a given set of alternatives. A close observation on our regular choice problems indicates that often the choice behavior is affected by the structure with which the alternatives appear to the decision maker. In particular, there are many situations where the decision problem takes the form of choice from a *list*. Common examples are for instance buying decisions from an e-commerce website where the products are displayed sequentially, or selecting a combo-menu from a list of menus in some fast-food center. For such kind of problems, Rubinstein and Salant (2006) introduce a very intuitive axiom on choice functions from lists as to show that it is exactly the structure of the list that helps the decision maker select an alternative from a given set of alternatives. It is important to note here that the crucial assumption in the mentioned work is that the decision maker observes *all* alternatives from the given list.

However, in many circumstances an individual might not want to continue the search till the end of the sequence. This for instance might not be feasible due to a decision maker's limited cognitive capacity or just due to the specific way in which the alternatives are presented to her (cf. Diehl and Zauberman, 2005). In such a case, the individual faces two different decision problems: she has to determine a particular alternative, after which she *stops* checking the remaining ones, and to *choose* an alternative from those observed so far. Rubinstein and Salant (2006) fully characterize the *selection* rule from the lists, while ignoring the *search* component of the decision. Our aim in this paper is to unify the problems of choice and search when the decision maker faces a list. For this, we introduce in Section 2 the notion of a *stop-and-choose rule* as to analyze the effect of the list structure on both types of decision problems. Interestingly, we identify two different aspects of the search behavior when the decision maker reaches the end of a list: (i) she observes the last alternative and decides to stop - here we say that the stopping decision is *endogenous* to the decision maker; (ii) the decision maker observes the last alternative and would like to search further, but cannot because the list ends - here we say that the stopping decision is *exogenous* to her. As an example, let us consider the situation in which one is looking at the menu in a restaurant and finds nothing satisfactory enough in the list. It may happen that the decision maker turns a page of the menu as to search for other dishes, but finds nothing more because that was the last page. In other words, the search stops exogenously in such a case. An observer perceiving this type of behavior has then some evidence about the process leading the individual to her decision (cf. Eliaz and Rubinstein, 2012) and can clearly distinguish between exogenous and endogenous stop.

The axioms we discuss in Section 3 take care of both types of search behavior, and incorporate various notions of consistency in the decision maker's choice. We show then in Section 4 that these axioms, being independent, characterize a special stop-and-choose rule that unifies *successive choice* and *satisficing choice* as introduced in Salant (2003) and Simon (1955), respectively. This unification is in terms of the generation of a complete binary relation over the set of alternatives as well as of a partition of this set into two subsets such that the rule uses either position-biased successive choice (capturing the influence of the list's structure in a more subtile manner) or applies a satisficing choice procedure depending on whether the corresponding list contains alternatives only from one of these subsets or from both of them.

Our work fits into the strand of literature on sequential choice procedures or choices with frames (cf. Mandler et al., 2011; Salant, 2011; Salant and Rubinstein, 2008; Yildiz, 2012), and it is related also to works dealing with the analysis of search problems. For instance, Weitzman (1979) considers a situation in which the decision maker has several boxes to open and each box has a reward that is a random variable. The decision maker can then choose to open the boxes in *any* order and if she stops, the pay-off would be the maximum value found so far. Weitzman (1979) characterizes then the optimal decision strategy in this model. More recently, Masatlioglu and Nakajima (2012) focus on choice problems when the search path depends on an initial and externally observable reference point, while Caplin and Dean (2011) characterize two types of search behavior (alternative-based and reservation-based), and consider a rich data set as to test their models. Let us however stress the fact that the mentioned papers do not address the issue of *how* the sequence of alternatives the decision maker is facing *affects* her search decisions. Finally, the closest to our work is the paper by Guney (2010), in which she characterizes a choice process where the decision maker selects an alternative from a list by performing an iterative search that utilizes her "mental constraint sets" and the order of alternatives in the list. In general, one can interpret a mental constrained set attached to a given alternative x as the set of alternatives the decision maker looks for after visiting x. Thus, the search process characterized by Guney (2010) does not necessarily involve observation of all alternatives in a list. Additionally, the stopping behavior in our model, apart from being explicitly modeled, depends only on the alternatives (and their order) in the corresponding lists.

2 Decision problem

Let X be a set of alternatives. A *list* ℓ is a finite sequence of alternatives drawn from X. We assume that each alternative from X appears only once in a list, and denote the set of all possible lists from X by Λ. For $\ell \in \Lambda$, $X(\ell)$

is the set of alternatives appearing in ℓ, while the *length* $le(\ell)$ of ℓ is defined by $le(\ell) := |X(\ell)|$.

As already discussed in the Introduction, when confronted with a choice problem from a list $\ell \in \Lambda$, an individual makes in fact two decisions. The first one is to determine a particular alternative $x \in X(\ell)$ such that, having observed x in ℓ, she stops checking the remaining alternatives displayed in ℓ. The second decision is about choosing an alternative from the series of alternatives that she has observed in the list before x. In other words, a *stop-and-choose rule* (sc-rule) C assigns to each $\ell \in \Lambda$ an ordered pair $\langle x, k\rangle \in X(\ell) \times (\{\emptyset\} \cup \{1, \ldots, le(\ell)\})$ such that $x \in \cup_{k'=1}^{k} \{\ell_{k'}\}$ if $k \neq \emptyset$, and $x \in X(\ell)$ if $k = \emptyset$.

The above formulation of a sc-rule allows to differentiate between the two situations regarding an individual's decision on "when to stop" as elaborated in the Introduction. Suppose first that $C(\ell) = \langle x, k\rangle$ with $k \in \{1, \ldots, le(\ell)\}$ for some $\ell \in \Lambda$. This corresponds to a situation of *endogenous* stop, that is, the individual has decided to stop the search by herself and, if $k < le(\ell)$, to not encounter the rest of the list; of course, $k = le(\ell)$ is also possible. On the other hand, we interpret $C(\ell) = \langle x, k\rangle$ with $k = \emptyset$ as a case of *exogenous* stop, where the decision maker wants to continue her search even after observing the last alternative in ℓ, but cannot because the list ends. Moreover, as an endogenous stopping decision indicates the fact that the decision maker does not want to encounter the remaining alternatives from the list, we assume in what follows that, for any two lists $\ell, \ell' \in \Lambda$ with $le(\ell') > le(\ell)$ and $\ell'_i = \ell_i$ for all $i \in \{1, \ldots, le(\ell)\}$, we have that $C(\ell) = \langle x, k\rangle$ with $k \neq \emptyset$ implies $C(\ell') = \langle x, k\rangle$.

There are many decision procedures that fit well into the framework described above. The common features of the examples we have chosen and provide next, are as follows. First, each procedure is parametrized by a complete binary relation (not necessarily a strict one). Second, this binary relation allows to explicitly partition the set of alternatives into two sets such that the decision maker stops her search after observing an alternative from one of these sets for first time in the list. Finally, in each of the examples, when the decision maker stops her search the alternative being lastly observed is also the chosen one.

Example 1 (Goods and bads: I) The decision maker partitions the set X into a set of good alternatives (X_1) and a set of bad ones (X_2). She continues her search until she observes a good alternative from the list and chooses it. If she does not encounter any good alternative, then the last alternative in the list is chosen. Thus, for $\ell \in \Lambda$, $C(\ell) = \langle \ell_k, k \rangle$ if $\ell_i \in X_2$ for all $i \in \{1, \ldots, k-1\}$ and $\ell_k \in X_1$, and $C(\ell) = \left\langle \ell_{le(\ell)}, \emptyset \right\rangle$, otherwise.

Example 2 (Goods and bads: II) We change Example 1 as follows: if the decision maker does not observe any good alternative till the list ends, then she chooses the first alternative in the list; that is, for $\ell \in \Lambda$, we have $C(\ell) = \langle \ell_1, \emptyset \rangle$ in this case. Apparently, this example might appear to be not significantly different from the earlier one, but as we will see later, the difference between them is crucial in terms of the properties of the search behavior that we consider.

Example 3 (Limited memory capacity) The alternatives one may choose from are again categorized into "good" or "bad". The decision maker can remember only the last k^* alternatives in the series of alternatives seen so far. She then stops if the last k^* remembered alternatives are good and chooses the last of them. If this never happens, the last alternative in the list is chosen. In other words, for $\ell \in \Lambda$, we have $C(\ell) = \langle \ell_k, k \rangle$ if $\ell_i \in X_1$ for all $i \in \{k - k^* + 1, k - k^* + 2, \ldots, k\}$ and $\ell_{k-k^*} \in X_2$, and $C(\ell) = \left\langle \ell_{le(\ell)}, \emptyset \right\rangle$, otherwise.

Example 4 (Satisficing choice (Simon, 1955)) Suppose that the decision maker has a strict linear order P over X and a satisfactory threshold alternative x^*. Given a list of alternatives, she stops and chooses the first element in the list that is not inferior to x^*; if there is no such alternative, she chooses the best element in the list according to P. Hence, for $\ell \in \Lambda$, $C(\ell) = \langle \ell_k, k \rangle$ if $x^* P \ell_i$ for all $i \in \{1, \dots, k-1\}$ and $\ell_k P x^*$ (or $\ell_k = x^*$), and $C(\ell) = \langle y, \emptyset \rangle$ with $y P \ell_i$ for all $\ell_i \in X(\ell)$, $\ell_i \neq y$, otherwise.

Example 5 (Successive choice (Salant, 2003)) The successive choice rule can be defined in terms of a strict linear order P over X and works as follows. For $\ell \in \Lambda$, the decision maker stores first ℓ_1 in a "register" and at stage t of the computation, $t \in \{1, \dots, le(\ell) - 1\}$, she replaces the register value y with ℓ_{t+1} if $\ell_{t+1} P y$. When the list ends, the alternative in the register, say $x(\ell)$, is chosen. In this case we have $C(\ell) = \langle x(\ell), \emptyset \rangle$ for all $\ell \in \Lambda$.

3 Axioms

Let us consider a list ℓ and an alternative x not belonging to it, and suppose that the decision maker exogenously stops for each of the lists ℓ and (x), that is, she would like to continue her search but has to stop because the lists terminate. Our first axiom incorporates an 'additivity of continuation' idea and naturally recommends the continuation of the search when x is added after the last alternative from ℓ. Note that no restriction on the choice behavior is imposed in such a case.

Axiom 1 For $\ell \in \Lambda$ and $x \in X \setminus X(\ell)$, $C(\ell) = \langle y, \emptyset \rangle$ and $C(x) = \langle x, \emptyset \rangle$ imply $C(\ell, x) = \langle z, \emptyset \rangle$.

The second axiom requires that if the decision maker decides to stop after observing x in the list containing only that alternative (and thus, by definition, to choose x), then from any other list she should either stop immediately after observing x (and choose x) or she must have decided to stop before x. Notice that $C(x) = \langle x, 1 \rangle$ implicitly contains the fact that the decision maker does not even want to know whether the list continues or terminates, and in that sense one can say that there is no alternative in X letting the decision maker continue after observing x. We can then interpret our second requirement as a consistency condition in the sense that if the decision maker continues her search after encountering all alternatives before x, then she must recall the fact that no alternative after x makes her continue (and she has chosen x) and thus, she must stop after x and choose x.

Axiom 2 If $C(x) = \langle x, 1 \rangle$, then for any $\ell \in \Lambda$ with $\ell_n = x$ we have either $C(\ell) = \langle y, m \rangle$ for some $m \in \{1, \ldots, n-1\}$ or $C(\ell) = \langle x, n \rangle$.

Our third axiom resembles the weak axiom of revealed preference from the standard theory of choice, and it starts with a situation in which the decision maker exogenously stops and selects an alternative x from a list ℓ. We require then the decision maker to be consistent with her decision (and continue selecting x) also in the following two situations: (i) she should never choose any alternative (say y) that appears later than x in the list ℓ over x in any list comprising only x and y, and (ii) her choice decision should be independent of the way, in which alternatives after x in ℓ are arranged in any list ℓ' coinciding with ℓ up to x.

Axiom 3 If $C(\ell) = \langle x, \emptyset \rangle$ for $\ell \in \Lambda$ with $x = \ell_k$ for some $k \in \{1, \ldots, le(\ell)\}$, then $C(x, y) = \langle x, \emptyset \rangle$ for any $y \in \left\{\ell_{k+1}, \ldots, \ell_{le(\ell)}\right\}$. Moreover, we have $C(\ell_1, \ldots, \ell_k, \ell') = \langle x, \emptyset \rangle$ for any $\ell' \in \Lambda$ with $X(\ell') \subseteq \left\{\ell_{k+1}, \ldots, \ell_{le(\ell)}\right\}$.

Finally, we consider again a situation of exogenous stop and interpret $C(x, y) = \langle x, \emptyset \rangle$ as a preference for x over y if the decision maker has to stop after y. Then it is natural to require that this revealed preference is kept when y is attached as the last alternative to a list, from which x has also been the selected alternative.

Axiom 4 If $C(\ell) = C(\ell') = \langle x, \emptyset \rangle$ for some $\ell, \ell' \in \Lambda$ with $\ell = (x, y)$, then $C(\ell', y) = \langle x, \emptyset \rangle$.

In order to show the independence of these four axioms, let us consider the following examples of sc-rules each of which satisfies three of the axioms but violates the fourth one.

Example 6 Let C be such that for all $\ell \in \Lambda$, $C(\ell) = \langle \ell_1, \emptyset \rangle$ if $le(\ell) = 1$ and $C(\ell) = \langle \ell_1, 2 \rangle$, otherwise. This rule clearly violates axiom 1, while satisfying all other axioms.

Example 7 Consider the sc-rule C defined as follows: for all $\ell \in \Lambda$, $C(\ell) = \langle \ell_1, 1 \rangle$ if $le(\ell) = 1$ and $C(\ell) = \langle \ell_2, 2 \rangle$, otherwise. It is easy to see that the rule satisfies axioms 1, 3, and 4. However, it violates axiom 2.

Example 8 Fix $x^* \in X$ and define C as follows: for all $\ell \in \Lambda$, $C(\ell) = \langle \ell_1, \emptyset \rangle$ if $X(\ell) \subseteq X \setminus \{x^*\}$ with $le(\ell) \neq 2$; $C(\ell) = \langle x^*, k \rangle$ with $x^* = \ell_k$ if $x^* \in X(\ell)$; and $C(\ell) = \langle \ell_2, \emptyset \rangle$, otherwise. This rule satisfies axioms 1, 2, and 4. It clearly violates axiom 3 since $C(a, b, c) = \langle a, \emptyset \rangle$ and $C(a, b) = \langle b, \emptyset \rangle$.

Example 9 Fix $x^* \in X$ and let C be defined as follows: for all $\ell \in \Lambda$, $C(\ell) = \langle \ell_1, \emptyset \rangle$ if $le(\ell) < 3$ and $x^* \notin X(\ell)$; $C(\ell) = \langle \ell_3, \emptyset \rangle$ if $le(\ell) \geq 3$ and $x^* \notin X(\ell)$; and $C(\ell) = \langle x^*, k \rangle$ with $\ell_k = x^*$, otherwise. This rule satisfies our first three axioms. Notice however that we have $C(a, b) = \langle a, \emptyset \rangle = C(a, c)$ and $C(a, b, c) = \langle c, \emptyset \rangle$ in violation of axiom 4.

4 Characterization

As already mentioned in the Introduction, the decision procedure we characterize unifies position-biased successive choice and satisficing choice. In order to define this procedure, we first provide a generalization of the successive choice function (called position-biased successive choice) used in Rubinstein and Salant (2006). This generalization is parametrized by a complete binary relation R (with P and I being its asymmetric and symmetric part, respectively) allowed to contain indifferences and by a labeling rule (indicator function) $\delta : X \times X \rightarrow \{1, 2\}$. Since the main assumption in Rubinstein and Salant (2006) is that the decision maker observes all alternatives appearing in a list, we present the position-biased successive choice as a special sc-rule, where for each list the decision maker would like to continue her search after the list ends.

Definition 1 Let R be a complete binary relation over X and δ be an indicator function. A sc-rule $Su_{R,\delta}$ is a *position-biased successive choice rule* if for all $\ell \in \Lambda$, $Su_{R,\delta}(\ell) = \langle x, \emptyset \rangle$ with $x \in X(\ell)$ being determined according to the following procedure. At the beginning of the procedure the decision maker stores ℓ_1 in a "register" and at stage t of the computation, $t \in \{1, \ldots, le(\ell) - 1\}$, she replaces the register value y with ℓ_{t+1} if (i) $\ell_{t+1} I y$ and $\delta(y, \ell_{t+1}) = 2$; or (ii) $\ell_{t+1} P y$. When the list ends the DM has x in the register.

In other words, if $\ell_{t+1} I y$ holds at some stage t, then the indicator function δ uses the ordering of y and ℓ_{t+1} in the list to break the tie and to determine the alternative to be registered at the end of stage t. Thus, this rule captures the influence of the way in which the alternatives are displayed on the decision-making in a more subtle manner.

We show in what follows that the axioms presented above allow us (i) to define a complete binary relation R over X and an indicator function δ, and (ii) to partition the set of alternatives into two sets (X_1 and X_2) such that, on lists containing alternatives only from X_2, the decision procedure uses $Su_{R,\delta}$, while on lists containing alternatives from both X_1 and X_2, the procedure considers any alternative in X_1 as being superior to any alternative from X_2 and selects the alternative from X_1 which appears firstly in the list. In that sense, the divide-and-choose rule we characterize below unifies both position-biased successive choice and satisficing choice. An axiomatic characterization of the position-biased successive choice on its own can then be derived as a side result.

We show in Lemma 1 how axioms 1 and 2 help us suitably construct R and partition X. Moreover, the joint application of axioms 1, 2, and 4 care for a consistent choice under exogenous stop when the last alternative from a list has been deleted (Lemma 2). The interplay of all axioms gives us then the characterization of a divide-and-choose rule (Theorem 1), while the combination of axioms 3 and 4 allow us to provide a connection between any (suitably restricted) sc-rule satisfying the axioms and the position-biased successive choice rule (Corollary 1).

Lemma 1 *If a sc-rule C satisfies axioms 1 and 2, then there exist a complete binary relation R over X and a partition (X_1, X_2) of X such that xPy for $x \in X_1$ and $y \in X_2$.*

Proof. Let C be as above and consider the choice from the lists $\ell' = (a, b)$ and $\ell'' = (b, a)$ for $a, b \in X$. By definition, the outcomes $C(\ell') = \langle b, 1\rangle$ and $C(\ell'') = \langle a, 1\rangle$ are excluded. Let us now suppose that $C(\ell') = \langle a, 2\rangle$ and show that this leads to contradictions. Consider first the outcomes from the lists containing only a and b, respectively. By the definition of a sc-rule, $C(a) = \langle a, 1\rangle$ implies $C(\ell') = \langle a, 1\rangle$, a contradiction. Thus, we should have $C(a) = \langle a, \emptyset\rangle$. On the other hand, $C(b) = \langle b, 1\rangle$ leads by axiom 2 to either $C(\ell') = \langle a, 1\rangle$ or $C(\ell') = \langle b, 2\rangle$, and we have again a contradiction. We conclude then that $C(b) = \langle b, \emptyset\rangle$ should hold. Notice finally that $C(a) = \langle a, \emptyset\rangle$ and $C(b) = \langle b, \emptyset\rangle$ imply by axiom 1 that $C(\ell') = \langle c, \emptyset\rangle$ for some $c \in \{a, b\}$ in contradiction to $C(\ell') = \langle a, 2\rangle$. We conclude then that $C(\ell') \neq \langle a, 2\rangle$. The same type of reasoning gives us $C(\ell'') \neq \langle b, 2\rangle$.

Consider now the combination of outcomes $C(\ell') = \langle b, 2\rangle$ and $C(\ell'') = \langle a, 2\rangle$. If $C(a) = \langle a, 1\rangle$ or $C(b) = \langle b, 1\rangle$, then, by the definition of a sc-rule, one must have $C(\ell') = \langle a, 1\rangle$ or $C(\ell'') = \langle b, 1\rangle$, a contradiction. Thus, $C(a) = \langle a, \emptyset\rangle$ and $C(b) = \langle b, \emptyset\rangle$ should hold. By axiom 1, $C(a, b) = \langle c, \emptyset\rangle$ for some $c \in \{a, b\}$, a contradiction. Hence, having both $C(\ell') = \langle b, 2\rangle$ and $C(\ell'') = \langle a, 2\rangle$ is not possible. A similar argument rules out the following combinations of outcomes: $C(\ell') = \langle a, \emptyset\rangle$ and $C(\ell'') = \langle a, 2\rangle$; $C(\ell') = \langle b, \emptyset\rangle$ and $C(\ell'') = \langle a, 2\rangle$; $C(\ell') = \langle b, 2\rangle$ and either $C(\ell'') = \langle b, \emptyset\rangle$ or $C(\ell'') = \langle a, \emptyset\rangle$.

As for the combination $C(\ell') = \langle a, \emptyset\rangle$ and $C(\ell'') = \langle b, 1\rangle$, note that $C(\ell'') = \langle b, 1\rangle$ implies by the definition of a sc-rule $C(b) = \langle b, 1\rangle$ which, by axiom 2, excludes $C(\ell') = \langle a, \emptyset\rangle$, a contradiction. The same type of reasoning rules out the combination of $C(\ell') = \langle a, 1\rangle$ and either $C(\ell'') = \langle b, \emptyset\rangle$ or $C(\ell'') = \langle a, \emptyset\rangle$.

Based on the other possible outcomes, for every $a, b \in X$ we define:

(1) $a \sim_1 b$ if $C(\ell') = \langle a, \emptyset\rangle$ and $C(\ell'') = \langle b, \emptyset\rangle$,

(2) $a \sim_2 b$ if $C(\ell') = \langle b, \emptyset\rangle$ and $C(\ell'') = \langle a, \emptyset\rangle$,

(3) $a \sim_3 b$ if $C(\ell') = \langle a, 1\rangle$ and $C(\ell'') = \langle b, 1\rangle$,

(4) $a \succ_1 b$ if $C(\ell') = \langle a, \emptyset\rangle$ and $C(\ell'') = \langle a, \emptyset\rangle$,

(5) $a \succ_2 b$ if $C(\ell') = \langle a, 1\rangle$ and $C(\ell'') = \langle a, 2\rangle$.

Generate then the binary relation R as follows: for all $a, b \in X$, aRb if and only if either of the above five possibilities holds. In view of the above application of axioms 1 and 2, R is complete. The asymmetric and symmetric part of R are denoted by P and I, respectively, and these are derived from R in the usual way.

Define now the partition (X_1, X_2) of X in the following way:

$$X_1 = \{x \in X : x \succ_2 y \text{ or } x \sim_3 y \text{ for some } y \in X\}, \; X_2 = X \setminus X_1.$$

Notice that if $x \succ_2 y$ or $x \sim_3 y$ holds *for some* $y \in X$, then we have $x \succ_2 y$ or $x \sim_3 y$ *for all* $y \in X$. As to see it, recall that either of $x \succ_2 y$ and $x \sim_3 y$ requires $C(x, y) = \langle x, 1 \rangle$ and hence, by the definition of a sc-rule, $C(x, z) = \langle x, 1 \rangle$ should hold for any $z \in X$.

Take now $x \in X_1$ and $y \in X_2$. If $x \sim_3 y$, then the symmetry of $\sim_3$ implies $y \in X_1$, a contradiction. Thus, the only remaining possibility is to have $x \succ_2 y$. Hence, we have xRy but not yRx, i.e., xPy should hold. ■

Lemma 2 *If a sc-rule C satisfies axioms 1, 2, and 4, then $C(\ell) = \left\langle \ell_{le(\ell)}, \emptyset \right\rangle$ for some $\ell \in \Lambda$ and $C(\ell') = \langle x, \emptyset \rangle$ for $\ell' = \ell \setminus \left\{ \ell_{le(\ell)} \right\}$ imply $C(x, \ell_{le(\ell)}) = \left\langle \ell_{le(\ell)}, \emptyset \right\rangle$.*

Proof. Let $\ell, \ell' \in \Lambda$ be such that $\ell = (\ell', le(\ell))$, $C(\ell) = \langle y, \emptyset \rangle$ with $y = \ell_{le(\ell)}$, and $C(\ell') = \langle x, \emptyset \rangle$. We have to show that $C(x, y) = \langle y, \emptyset \rangle$. Notice first that if $C(x) = \langle x, 1 \rangle$ or $C(y) = \langle y, 1 \rangle$, then axiom 2 requires the decision maker to stop after observing x in ℓ' (and choose x) and after observing y in ℓ (and choose y) in contradiction to $C(\ell') = \langle x, \emptyset \rangle$ and $C(\ell) = \langle y, \emptyset \rangle$, respectively. We conclude that $C(x) = \langle x, \emptyset \rangle$ and $C(y) = \langle y, \emptyset \rangle$ should hold which, by axiom 1, leads to $C(x, y) = \langle z, \emptyset \rangle$ for some $z \in \{x, y\}$. Finally, the possibility of $z = x$ is ruled out by the fact that $\ell = (\ell', y)$ and $C(x, y) = C(\ell') = \langle x, \emptyset \rangle$ imply by axiom 4 that $C(\ell) = \langle x, \emptyset \rangle$, a contradiction to $C(\ell) = \langle y, \emptyset \rangle$. Hence, $C(x, y) = \langle y, \emptyset \rangle$ should hold. ■

The proof of Lemma 2 becomes even shorter if the sc-rule C is required to be *exogenous* (i.e., it is just a choice function from lists in the terminology of Rubinstein and Salant (2006)), that is, if for all $\ell \in \Lambda$, $C(\ell) = \langle x, \emptyset \rangle$ for some $x \in X(\ell)$. Notice that in this case axioms 1 and 2 are vacuously satisfied and thus, we have the following simple observation.

Observation 1 If an exogenous sc-rule satisfies axiom 4, then Lemma 2 still holds.

Let us finally provide the formal definition of a divide-and-choose rule and show how the joint application of axiom 3 and the above two lemmas shapes its characterization.

Definition 4 Let R be a complete binary relation over X and δ be an indicator function. A sc-rule is a *divide-and-choose rule* if there exists a partition (X_1, X_2) of X with $x_1 P x_2$ for $x_1 \in X_1$ and $x_2 \in X_2$ such that for all $\ell \in \Lambda$ the following holds: (i) $C(\ell) = Su_{R,\delta}(\ell)$ if $X(\ell) \subseteq X_2$, and (ii) $C(\ell) = \langle x, k \rangle$ with $x = \ell_k$ for some $k \in \{1, \ldots, le(\ell)\}$ if $\ell_i \in X_2$ for $i \in \{1, \ldots, k-1\}$ and $\ell_k \in X_1$.

Theorem 1 *A sc-rule satisfies axioms 1, 2, 3, and 4 if and only if it is a divide-and-choose rule.*

Proof. Let C be a sc-rule satisfying axioms 1, 2, 3, and 4, and let the binary relation R and the partition (X_1, X_2) of X be as defined in the proof of Lemma 1. Moreover, let $\delta : X \times X \to \{1, 2\}$ be an indicator function such that for all $a, b \in X$, $\delta(a, b) = \delta(b, a) = 1$ if $a \sim_1 b$ and $\delta(a, b) = \delta(b, a) = 2$ if $a \sim_2 b$. Take $\ell \in \Lambda$ and note that two cases are possible: (i) $X(\ell) \cap X_1 \neq \emptyset$ and (ii) $X(\ell) \cap X_1 = \emptyset$.

(i) $X(\ell) \cap X_1 \neq \emptyset$. We have to show that $C(\ell) = \langle x, k \rangle$ if $\ell_k = x \in X_1$ and $\ell_i \in X_2$ for all $i \in \{1, \ldots, k-1\}$. In view of Lemma 1, $x \in X(\ell) \cap X_1$ implies $C(x, y) = \langle x, 1 \rangle$ for all $y \in X$ and thus, by the definition of a sc-rule and axiom 2, $C(\ell) = \langle z, \emptyset \rangle$ for some $z \in X(\ell)$ cannot happen. If $X(\ell) \cap X_2 = \emptyset$, then $C(\ell) = \langle x, 1 \rangle$ immediately follows from $x \in X_1$ (that is, from $C(x, z) = \langle x, 1 \rangle$ for some $z \in X$) and the definition of a sc-rule. Suppose now that $X(\ell) \cap X_2 \neq \emptyset$. It follows from the definition of a sc-rule and axiom 2 that $C(\ell) = \langle y, j \rangle$ for some $j > k$ cannot happen. As $C(a) = \langle a, \emptyset \rangle$ holds for all $a \in X_2$ (otherwise, $C(a) = \langle a, 1 \rangle$ would imply by axiom 2 that $C(a, b) = \langle a, 1 \rangle$ for all $b \in X \setminus \{a\}$ in contradiction to $a \in X_2$), we have by the repeated application of axiom 1 that $j < k$ cannot happen, either. Thus, the only possibility is to have $j = k$ which implies $y = x$ by $C(x) = \langle x, 1\rangle$ and axiom 2. We conclude that $C(\ell) = \langle x, k\rangle$ with $x = \ell_k$ should hold.

(ii) $X(\ell) \cap X_1 = \emptyset$. By $C(a) = \langle a, \emptyset\rangle$ for all $a \in X_2$, we have again by the repeated application of axiom 1 that $C(\ell) = \langle x, \emptyset\rangle$ should hold for some $x \in X(\ell)$. In order to prove that $C(\ell) = Su_{R,\delta}(\ell)$, let us consider the list $\ell' = \ell \setminus \left\{\ell_{q+1}, \ldots, \ell_{le(\ell)}\right\}$ for some $q \in \{1, \ldots, le(\ell)\}$ and an alternative $x \in X(\ell) \setminus X(\ell')$, and show the following claim.

Claim 1 $C(\ell') = Su_{R,\delta}(\ell')$ *implies* $C(\ell', x) = Su_{R,\delta}(\ell', x)$.

Proof. Let $C(\ell') = Su_{R,\delta}(\ell') = \langle y, \emptyset\rangle$ for some $y \in X(\ell')$. By definition, $Su_{R,\delta}(\ell', x)$ is either $\langle x, \emptyset\rangle$ or $\langle y, \emptyset\rangle$. We consider these two possibilities sequentially.

(i) $Su_{R,\delta}(\ell', x) = \langle y, \emptyset\rangle = C(\ell')$. Suppose that $C(\ell', x) = \langle z, \emptyset\rangle$ with $z \neq y$. If $z \in X(\ell')$, then axiom 3 requires $C(\ell') = \langle z, \emptyset\rangle$, and we have a contradiction since $z \neq y$. Consider next the case of $C(\ell', x) = \langle x, \emptyset\rangle$. By Lemma 2, $C(x, y) = \langle x, \emptyset\rangle$. Hence, either $x \succ_1 y$ (and thus, xPy) or $x \sim_1 y$ (implying xIy and $\delta(x, y) = 1$). Either of these two possibilities calls for $Su_{R,\delta}(\ell', x) = \langle x, \emptyset\rangle$, a contradiction. We conclude then that $C(\ell', x) = \langle y, \emptyset\rangle = Su_{R,\delta}(\ell', x)$ should hold.

(ii) $Su_{R,\delta}(\ell', x) = \langle x, \emptyset\rangle$. If $C(\ell', x) = \langle z, \emptyset\rangle$ with $z \neq y$ and $z \in X(\ell')$, we have the same type of contradiction as above. Suppose now that $C(\ell', x) = \langle y, \emptyset\rangle$. By axiom 3, $C(y, x) = \langle y, \emptyset\rangle$. Hence, either $y \succ_1 x$ (and thus, yPx) or $x \sim_2 y$ (implying xIy and $\delta(x, y) = 2$). Either of these two possibilities requires $Su_{R,\delta}(\ell', x) = \langle y, \emptyset\rangle$, a contradiction. Hence, we have $C(\ell', x) = \langle x, \emptyset\rangle = Su_{R,\delta}(\ell', x)$. ■

Finally, since $C(\ell^*) = Su_{R,\delta}(\ell^*)$ with $\ell^* = (\ell_1)$ trivially holds, we can repeatedly use Claim 1 by adding the alternatives from $X(\ell) \setminus X(\ell')$ in the corresponding order as to conclude that $C(\ell) = Su_{R,\delta}(\ell)$ should hold. ■

In view of the above result, one can immediately provide a characterization of the position-biased successive choice rule by just noticing that it is an exogenous sc-rule.

Corollary 1 *An exogenous sc-rule satisfies axioms 3 and 4 if and only if it is a position-biased successive choice rule.*

Proof. Let C be a sc-rule as above. For every $a, b \in X$ define $a \sim_1 b$, $a \sim_2 b$, and $a \succ_1 b$ as in the proof of Lemma 1. Let the complete binary relation R and the indicator function δ be defined as in the proof of Lemma 1 and Theorem 1, respectively. Notice that, in view of Observation 1, we are allowed to use Lemma 2 and conclude that the proof of the corollary directly follows from part (ii) of the proof of Theorem 1. ■

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