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Diamond-Rajan Bank Runs in a Production Economy*

Keiichiro Kobayashi

To analyze the macroeconomic consequences of a systemic bank run, we integrate the banking model à la Diamond and Rajan (2001a) into a simplified version of an infinite-horizon neoclassical growth model. The banking sector intermediates collateral-secured loans from households to entrepreneurs. Entrepreneurs also deposit their working capital in banks. A systemic bank run, which is a sunspot phenomenon in this model, results in a deep recession by causing a sudden shortage in working capital. We show that an increase in the probability of a systemic run can persistently lower output, consumption, labor, capital and asset prices, even if a systemic run does not actually occur. This implies that the slowdown in economic growth after a financial crisis may be caused by the increased fragility of the banking system or increased fears of the recurrence of a systemic run.
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1. Introduction

We experienced a severe systemic crisis in the global financial market in 2008-2009, along with vulnerable and slow economic recovery in the US and Europe thereafter. There are several ways to understand these events and formulate them into a formal economic model. For example, Gertler and Kiyotaki (2010) formulate the crisis as a large shock to capital depreciation in an economy where banks have a limited ability to commit; and Kurlat (2009) and Bigio (2010) model the crisis as a breakdown of the market for financial assets due to adverse selection à la Akerlof's (1970) lemon problem. In this paper we model the crisis as a systemic bank run, and we hypothesize that stagnant economic performance after a crisis is caused by widespread fears of the recurrence of a systemic bank run. To analyze the macroeconomic consequences of the financial crises, we integrate the banking model à la Diamond and Rajan (2001a) into a variant of the Kiyotaki-Moore (1997) model. The banking sector intermediates collateral-secured loans from households to entrepreneurs. Entrepreneurs also deposit working capital in banks. A systemic bank run, which is a sunspot phenomenon in this model, results in a deep recession by causing a shortage in working capital. We show in a version of our model where entrepreneurs accumulate capital that an increase in the probability of a systemic bank run causes a persistent recession and lowers asset prices, even if a systemic run does not actually occur. The contributions of this paper are as follows.

- We incorporate the Diamond-Rajan banks into an infinite-horizon business cycle model in an essential way: that is, we translate the "demand for liquidity" in the Diamond-Rajan models into a "demand for working capital for production" in the business cycle models.[1] A liquidity shortage in the banking models should represent disruptions in payment activities in various economic transactions; and frictions on payment are naturally modeled as financial constraints on working capital for wage payment and/or purchase of intermediate goods in the macroeconomic models.[2] The view we put forward in this paper is that a systemic bank run can cause *a sudden shortage of working capital that leads to severe declines in output.*

- Our model with capital accumulation implies that the fragility of the financial system, which is translated into an increased probability (θ) of another systemic bank run in the model, can be a fundamental cause of slow economic growth, which was observed in the US and Europe in 2008–2010, and in Japan in the 1990s.

Literature: This paper is related to dynamic stochastic general equilibrium (DSGE) models that analyze the financial crisis in 2008, such as Gertler and Karadi (2010) and Gertler and Kiyotaki (2010). These studies formalize the financial crisis as a large exogenous shock; e.g., productivity shocks or depreciation of capital stock.[3] Our model shows the theoretical possibility that a bank run triggered by a small shock may cause a large fluctuation in real variables. Gorton and Metrick (2012) conclude that an essential feature of the 2008 crisis was a run in the interbank repo market. Uhlig (2009) models the 2008 global financial crisis as a systemic bank run. He constructs a two-period model based on Diamond and Dybvig (1983), which is quite different from the Diamond-Rajan framework. Angeloni and Faia (2009) incorporate the Diamond-Rajan banks in a DSGE model as we do in this paper, but their model analyzes only idiosyncratic bank runs and does not discuss a systemic crisis. Ennis and Keister (2003) analyze the effects of bank runs on economic growth using a model that incorporates the Diamond-Dybvig banks into an overlapping generations model. Their results are quite similar to ours but their modeling method is quite different from our model: we consider the coordination failure through variations in factor prices, while Ennis and Keister do not.

The organization of the paper is as follows. In the next section, we present and analyze the basic model, in which land works as a factor of production and collateral for bank loans. In Section 3, we analyze the model with both land and capital accumulation. Section 4 provides concluding remarks.

2. The Model with Land

We describe financial contracts (demand deposits) in Appendix 1. Given that banks offer demand deposits, the model is described as follows.

2.1 The Environment

The economy is closed and time is discrete: $t=0, 1, 2, \cdots$. There are three agents: households; entrepreneurs; and banks.[4] The measures of these agents are normalized to one, respectively. Households and entrepreneurs live for infinite periods. Banks are one-period lived. At the end of period $t-1$, banks are born, accept deposits from households and entrepreneurs, and make loans to entrepreneurs. If there is no bank run in period t, they collect repayment of loans from the borrowing entrepreneurs at the end of period t, then payout depositors, and die. If a bank run occurs in period t, the banks just walk away from the market leaving the loan assets to the depositors, and die at the end of the period. Three objectives are traded:

- Land, a_t: Only entrepreneurs can own and operate land. Land is pledgeable as collateral for bank loans. Land is nondepletable and productive. The total supply of land is fixed: $a_t=1$, $\forall t$. Owner-entrepreneurs incur maintenance cost $\chi(a_t)$ in period t for holding land a_t.
- Labor, l_t: Only households can provide labor input to entrepreneurs. Labor supply l_t incurs disutility $\gamma(l_t)$ to households in period t.[5]

- Consumer goods, y_t: Only entrepreneurs can produce consumer goods from land and labor with the Cobb-Douglas technology.

$$y_t = Aa_{t-1}^{\alpha} l_t^{1-\alpha}.$$

Households' utility is

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\{c_t - h_t - \gamma(l_t)\}\right],$$

where c_t is consumption and h_t is disutility due to the following backyard production: we assume for simplicity of analysis that households and entrepreneurs can produce h_t units of consumer goods in their backyards, incurring h_t units of disutility contemporaneously. Entrepreneurs' utility is

$$E_0\left[\sum_{t=0}^{\infty}(\beta')^t\{c_t^E - h_t^E\}\right], \qquad (1)$$

where c_t^E is consumption and h_t^E is disutility due to backyard production. We assume that households' discount factor, β, is larger than that of entrepreneurs, β'. (Thus households are patient and entrepreneurs are impatient.)

$$0 < \beta' < \beta < 1.$$

A bank makes loan to an entrepreneur taking land as collateral. If the borrowing entrepreneur repudiates repayment in period t, the bank can seize $\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$, where a_{t-1} is the land that the borrowing entrepreneur owns, r_t^a is the return from the land, and q_t is the land price. Note that r_t^a and q_t may vary depending on the realization of the sunspot variable, s_t, which is defined below.[6)] If this loan is transferred to some other agent, the agent who acquires the loan can collect (at most) $\kappa\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$, where

$$0 < \kappa = z + (1-z)x < 1.$$

A borrowing entrepreneur cannot precommit to repay a debt. When an entrepreneur repudiates a debt, the collateral is (partially) seized by the creditor but there is no additional penalty for the repudiation. The value seized by the creditor is $\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$ if the creditor is the bank that originated the loan and $\kappa\{r_t^a(s_t) + q_t(s_t)\}a_{t-1}$ otherwise. We assume that there is a sunspot variable s_t, where

$$s_t = \begin{cases} e\,(\text{emergency}) & \text{with probability } \theta, \\ n\,(\text{normal time}) & \text{with probability } 1-\theta. \end{cases}$$

The variable s_t is revealed at the beginning of period t. Macroeconomic variables, e.g., $r_t^a(s_t)$, depend on s_t.

Assumption 1 *The agents in this economy expect that $r_t^a(e)$ is much smaller than $r_t^a(n)$*

Due to this expectation, a systemic bank run occurs in equilibrium in the state where $s_t = e$. As a result of the systemic bank run, the expectation that $r_t^a = r_t^a(e)$ is smaller than $r_t^a(n)$ is justified in equilibrium. See Section 2.5 for details.

Working capital for wage payment: Entrepreneurs need to buy labor input from households in order to produce consumer goods. If entrepreneurs could commit to pay wages, they could have used the labor input just by promising to pay the wages afterwards and they could have actually paid wages in the form of the consumer goods after the production. We assume, however, the following assumption.

Assumption 2 *Worker-households cannot impose any penalty after production of consumer goods on entrepreneurs who break their promise to pay wages.*

This assumption prevents entrepreneurs from committing beforehand to pay wages to worker-households after production. Because of this lack of commitment, entrepreneurs must pay wages before production in the form of credible claims, which are the bank deposits or the collateral-secured loans to (other) entrepreneurs. At the end of period $t-1$, entrepreneurs choose to hold a certain amount of bank deposits for wage payment in period t. If a bank run does not occur at the beginning of period t, entrepreneurs pay wages in the form of bank deposits. If a bank run occurs, the banks walk away and the depositor-entrepreneurs are left with the loan assets that the banks originated at the end of period $t-1$. As the banks walk away, the value of the loan assets decreases to $\kappa\{r_t^a(e)+q_t(e)\}a_{t-1}$, which is the value that the depositors can recover after the bank run; and the depositor-entrepreneurs pay wages by transferring loan assets, the values of which are less than the original bank deposits, directly to the worker-households.

2.2 Timing of events

The timing of events during the representative period t is as follows.

- At the beginning of period t:
  Households carry over bank deposits, $(1+r_{t-1})d_{t-1}^H$, where d_{t-1}^H is the amount of deposits made at the end of period $t-1$ and r_{t-1} is the deposit rate from $t-1$ to t. Entrepreneurs carry over bank deposits, $(1+r_{t-1})d_{t-1}^E$, and land a_{t-1} as their assets, while a_{t-1} is pledged as collateral for bank loans b_{t-1} that they borrowed at the end of the previous period $t-1$. Banks carry over deposits $(1+r_{t-1})d_{t-1}$ as their liabilities, where $d_{t-1}=d_{t-1}^H+d_{t-1}^E$. The sunspot variable, $s_t\in\{n,e\}$, is revealed. If $s_t=n$, agents expect that $r_t^a=r_t^a(n)$ is large and there is no bank run. If $s_t=e$, agents expect that $r_t^a=r_t^a(e)$ is strictly smaller than $r_t^a(n)$. In this case, a systemic bank run occurs as an equilibrium outcome (see Section 2.5) and the banks walk away from the market leaving the loan assets to the depositors. As a result of the bank run, the value of loan assets becomes $\kappa\{r_t^a+q_t\}a_{t-1}$, which is the amount that depositors can recover from borrowers without the bank's help; and bank deposits become direct claims on the loans to entrepreneurs, the value of which are $\xi_t(1+r_{t-1})d_{t-1}^H$ for households and $\xi_t(1+r_{t-1})d_{t-1}^E$ for entrepreneurs, where $\xi_t(<1)$ is the recovery rate of bank deposits. Note that ξ_t is identical for all depositors due to Assumption 4 and that ξ_t is an equilibrium outcome.
- In the middle of period t:
  Households choose the labor supply l_t. They sell l_t to entrepreneurs at the wage rate w_t. If $s_t=n$, w_tl_t is paid by transferring bank deposits $(\{1+r_{t-1}\}d_{t-1}^E)$ from entrepreneurs to households. If $s_t=e$, w_tl_t is paid by transferring loan assets $(\xi_t\{1+r_{t-1}\}d_{t-1}^E)$ from entrepreneurs to households. Entrepreneurs produce consumer goods $(Aa_{t-1}^{\alpha}l_t(s_t)^{1-\alpha})$ from land a_{t-1} and labor $l_t(s_t)$.
- At the end of period t:
  The consumer goods market and the asset market open. Households choose consumption c_t and backyard production h_t. They withdraw bank deposits $(1+r_{t-1})(d_{t-1}^H+d_{t-1}^E)$ if $s_t=n$, or collect loans $\xi_t(1+r_{t-1})(d_{t-1}^H+d_{t-1}^E)$ from borrowing entrepreneurs directly if $s_t=e$.[7] They make new deposits d_t^H that they carry over to the next period. Entrepreneurs choose consumption c_t^E and backyard production h_t^E. They sell land a_{t-1} and repay $\{r_t^a+q_t\}a_{t-1}$ if $s_t=n$, or they repay a small part of bank loans $\kappa\{r_t^a+q_t\}a_{t-1}$ if $s_t=e$. Entrepreneurs make new deposits d_t^E, borrow new bank debts b_t, and buy land a_t that they carry over to the next period. Banks collect loans $(r_t^a+q_t)a_{t-1}$, payout deposits $(1+r_{t-1})d_{t-1}$, eat any remaining profit, and die if $s_t=n$. (If $s_t=e$, the banks simply die.) New banks are born and they accept deposits $d_t=d_t^H+d_t^E$ from households and entrepreneurs, and make loans b_t to entrepreneurs.

2.3 Optimization Problems

There is only one stochastic variable, $s_t\in\{n,e\}$, which is a sunspot variable.

Household:

$$\max_{c_t, h_t, d_t^H, l_t} E_0\left[\sum_{t=0}^{\infty}\beta^t\{c_t - h_t - \gamma(l_t)\}\right],$$

$$\text{s.t. } c_t(s_t) + d_t^H(s_t) = \xi_t(s_t)(1+r_{t-1})d_{t-1}^H + w_t(s_t)l_t(s_t) + h_t(s_t),$$

where $\xi_t(s_t)$ is the recovery rate of deposits, which is

$$\xi_t(s_t) = \begin{cases} 1 & \text{if a bank run does not occur,} \\ \xi_t(<1) & \text{if a bank run occurs,} \end{cases}$$

where the value of ξ_t is determined in equilibrium. It is shown in Section 2.5 that a bank run occurs if $s_t = e$ and does not occur if $s_t = n$. The first-order conditions (FOCs) for the household's problem imply

$$\gamma'(l_t(s_t)) = w_t(s_t), \tag{2}$$

$$1 = \beta\{(1-\theta)\xi_{t+1}(n) + \theta\xi_{t+1}(e)\}(1+r_t(s_t)). \tag{3}$$

Obviously, (3) implies that the interest rate does not depend on $s_t \in \{n, e\}$[8]:

$$r_t(n) = r_t(e) = r_t. \tag{4}$$

Entrepreneur: Given that $0<\beta'<\beta<1$, entrepreneurs solve the following problem.

$$\max_{c_t^E, h_t^E, d_t^E, l_t, a_t, b_t} E_0\left[\sum_{t=0}^{\infty}(\beta')^t\{c_t^E - h_t^E\}\right], \tag{5}$$

s.t.

$$\begin{aligned} q_t(s_t)a_t(s_t) + d_t^E(s_t) - b_t(s_t) + c_t^E(s_t) &= Aa_{t-1}^{\alpha}l_t(s_t)^{1-\alpha} + q_t(s_t)a_{t-1} - w_t(s_t)l_t(s_t) \\ &\quad + \xi_t(s_t)(1+r_{t-1})d_{t-1}^E - \kappa_t(s_t)\{r_t^a(s_t) \\ &\quad + q_t(s_t)a_{t-1} - \chi(a_t(s_t))\} + h_t^E(s_t), \end{aligned} \tag{6}$$

$$w_t(s_t)l_t(s_t) \le \xi_t(s_t)(1+r_{t-1})d_{t-1}^E, \tag{7}$$

$$b_t(s_t) \le B_t(a_t, s_t), \tag{8}$$

where $\kappa_t(s_t)$ is the recovery rate of bank loans, $r_t^a(s_t) = Aa_{t-1}^{\alpha}l_t(s_t)^{1-\alpha} - w_t(s_t)l_t(s_t)$ and in equilibrium

$$r_t^a(s_t) = \alpha A\left(\frac{l_t(s_t)}{a_{t-1}}\right)^{1-\alpha}, \tag{9}$$

and $B_t(a_t, s_t)$ is the debt capacity for the entrepreneur, which is determined as a solution to the bank's optimization problem. The recovery rate $\kappa_t(s_t)$ takes on the following values:

$$\kappa_t(s_t) = \begin{cases} 1 & \text{if a bank run does not occur,} \\ \kappa & \text{if a bank run occurs,} \end{cases}$$

It is shown in Section 2.5 that a bank run occurs if $s_t = e$ and does not occur if $s_t = n$.

Bank: Given the deposit rate r_t and the amount of collateral a_t of a borrowing entrepreneur, a bank maximizes the expected profit from a loan to an entrepreneur. Note that r_t is fixed at t and does not depend on the realization of s_{t+1}.

$$\begin{aligned} \max_{b_t, d_t} E_t\pi_{t+1}^B &= (1-\theta)\max\{[r_{t+1}^a(n) + q_{t+1}(n)]a_t - (1+r_t)d_t, 0\} \\ &\quad + \theta\max\{[r_{t+1}^a(e) + q_{t+1}(e)]a_t - (1+r_t)d_t, 0\}, \end{aligned} \tag{10}$$

subject to

$$b_t = d_t, \tag{11}$$

$$b_t \ge B_t(a_t, s_t), \tag{12}$$

where $B_t(a_t, s_t)$ is the lower limit of bank loans to a borrower who pledges a_t as collateral. The value of $B_t(a_t, s_t)$ is determined in equilibrium as a result of competition among banks. Competition among banks drives $E_t\pi_{t+1}^B$ to zero, which implies that $B_t(a_t, s_t)$ does not depend on s_t and

$$\begin{aligned} B_t(a_t, s_t) &= B_t(a_t) \\ &\equiv \frac{1}{1+r_t}\max\{r_{t+1}^a(n) + q_{t+1}(n), r_{t+1}^a(e) + q_{t+1}(e)\}. \end{aligned} \tag{13}$$

2.4 Equilibrium conditions

The recovery rate of deposits during a bank run, ξ_t, is determined by

$$\xi_t(s_t) = \min\left\{1, \kappa_t(s_t)\frac{\{r_t^a(s_t) + q_t(s_t)\}a_t}{(1+r_{t-1})d_{t-1}}\right\}, \tag{14}$$

where d_{t-1} is the total amount of deposits in a bank and a_t is the total amount of collateral assets for a bank. The market clearing conditions are

$$a_t(s_t) = 1, \tag{15}$$

$$c_t(s_t) - h_t(s_t) + c_t^E(s_t) - h_t^E(s_t) = Aa_{t-1}^{\alpha}l_t(s_t)^{1-\alpha}, \tag{16}$$

$$b_t(s_t) = d_t(s_t)\ (\equiv d_t^H(s_t) + d_t^E(s_t)). \tag{17}$$

2.5 Dynamics

Definition of a competitive equilibrium: *A competitive equilibrium is a set of prices, $\{r_t(s_t), \kappa_t(s_t), \xi_t(s_t), q_t(s_t), w_t(s_t), r_t^a(s_t)\}$, and quantities, $\{a_t(s_t), c_t(s_t), c_t^E(s_t), h_t(s_t), h_t^E(s_t), l_t(s_t)\}$, such that (i) given the prices, the quantities are the solution to the optimization problems of households, entrepreneurs, and banks; and (ii) the market clearing conditions are satisfied.*

To analyze the dynamics, we first check the FOCs for the entrepreneur's problem:

$$(1-\alpha)A\left(\frac{a_{t-1}}{l_t(s_t)}\right)^{\alpha} = w_t(s_t)\,(1+\mu_t(s_t)), \tag{18}$$

$$1 = \eta_t(s_t), \tag{19}$$

$$1 = \beta'[(1-\theta)\xi_{t+1}(n)\{1+\mu_{t+1}(n)\} + \theta\xi_{t+1}(e)\{1+\mu_{t+1}(e)\}](1+r_t), \tag{20}$$

$$\chi'(a_t(s_t)) + q_t(s_t) = (1-\kappa)\beta'\theta[r_{t+1}^a(e) + q_{t+1}(e)] + \eta_t(s_t)B_t'(a_t), \tag{21}$$

where $\mu_t(s_t)$ and $\eta_t(s_t)$ are the Lagrange multipliers for (7) and (8), respectively. Then, (19) implies that $\eta_t(n) = \eta_t(e) = 1$. Similarly, (21) and $a_t(s_t) = 1$ imply that the equilibrium asset price does not depend on the sunspot variable: $q_t(n) = q_t(e) \equiv q_t$. Summarizing the above arguments, we obtain the following lemma:

Lemma 1 *The variables $\{r_t, q_t, \eta_t\}$ do not depend on the realization of s_t.*

Since we assumed that $r_{t+1}^a(n) > r_{t+1}^a(e)$ in Assumption 1, this lemma and (13) implies that

$$B_t(a_t) = \frac{r_{t+1}^a(n) + q_{t+1}}{1+r_t}a_t. \tag{22}$$

Therefore, $d_t = b_t = B_t(a_t)$ and $B_t'(a_t) = \{r_{t+1}^a(n) + q_{t+1}\}/(1+r_t)$.

Condition for a bank run: The above results imply that when $s_t = n$, the value of bank assets is $\{r_t^a(n) + q_t\}a_{t-1}$ and the value of bank liabilities (i.e., deposits) is $(1+r_{t-1})d_{t-1} = (1+r_{t-1})B_{t-1}(a_{t-1}) = \{r_t^a(n) + q_t\}a_{t-1}$. Therefore, a bank is solvent when $s_t = n$, and a bank run does not occur in this state. On the other hand, when $s_t = e$, the value of bank assets is $\{r_t^a(e) + q_t\}a_{t-1}$ and the value of bank liabilities is $\{r_t^a(n) + q_t\}a_{t-1}$. Assumption 1 implies that a bank is insolvent in this state, and therefore a bank run occurs when $s_t = e$. The recovery rate of bank loans is thus given by

$$\kappa_t(s_t) = \begin{cases} 1 & \text{if } s_t = n, \\ \kappa & \text{if } s_t = e, \end{cases}$$

and the recovery rate of bank deposits is given by

$$\xi_t(s_t) = \begin{cases} 1 & \text{if } s_t = n, \\ \xi_t & \text{if } s_t = e, \end{cases}$$

where

$$\xi_t = \kappa\frac{r_t^a(e) + q_t}{r_t^a(n) + q_t}. \tag{23}$$

The liquidity constraint (7) implies that

$$\xi_t w_t(n) l_t(n) = w_t(e) l_t(e). \tag{24}$$

The equilibrium path is determined as a sequence $\{a_t, r_t, q_t, w_t(s_t), r_t^a(s_t), l_t(s_t), \xi_t, \mu_t(s_t)\}_{t=0}^{\infty}$,[9] which satisfies (2), (9), (15), (18), (23), (24), and

$$1 = \beta\{1-\theta+\xi_{t+1}\theta\}(1+r_t),$$

$$1 = \beta'[(1-\theta)1+\mu_{t+1}(n) + \theta\xi_{t+1}\{1+\mu_{t+1}(e)\}](1+r_t),$$

$$\chi'(1) + q_t = (1-\kappa)\beta'\theta[r_{t+1}^a(e) + q_{t+1}] + \frac{r_{t+1}^a(n) + q_{t+1}}{1+r_t}.$$

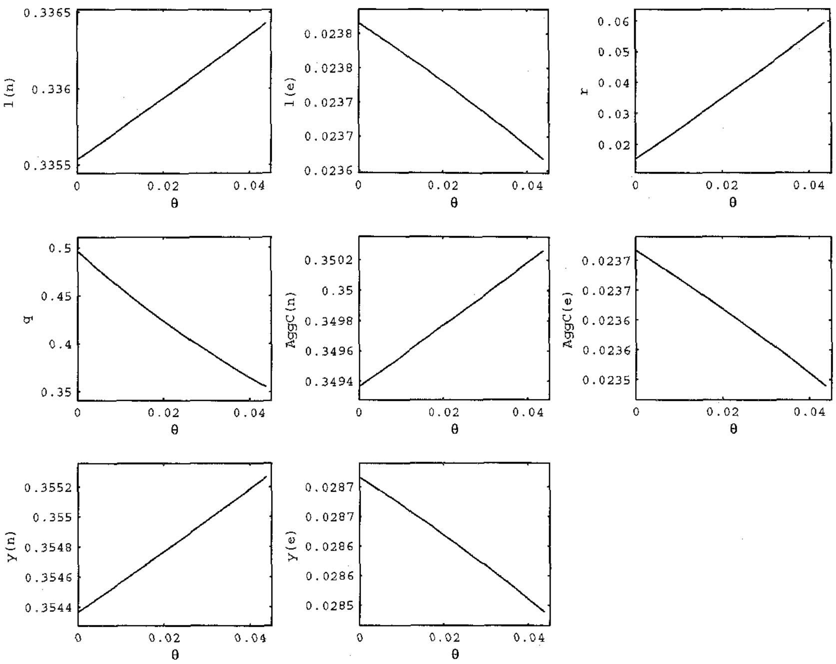
The equilibrium path must also satisfy the transversality condition:

$$\lim_{t\to\infty}(\beta')^t q_t = 0.$$

2.6 Stationary Equilibrium

Since the state variable in this model is a_{t-1} and it is time-invariant, there exists an equilibrium path, along which prices and quantities are all time-invariant. We focus on this stationary equilibrium.[10] In the station-

Figure 1. Stationary Equilibrium in the Model with Land



Parameters: $A=1$, $\alpha=0.3$, $\beta=0.985$, $\beta'=0.9\times\beta$, $\delta=0.04$, $\sigma=1$, $\psi=0.6$, $\phi=0.01$, $\kappa=0.05$.
Note: $AggC(s)=c(s)+c^E(s)-h(s)-h^E(s)$.

ary equilibrium, macroeconomic variables are time-invariant and depend only on the realization of $s_t\in\{n,e\}$. Given θ, the stationary equilibrium is specified by the set of variables $\{r,q,w(n),w(e),r^a(n),r^a(e),l(n),l(e),\xi,\mu(n),\mu(e)\}$, which solves the following system of equations.

$$1=\beta\{1-(1-\xi)\theta\}(1+r), \tag{25}$$

$$1=\beta'[(1-\theta)\{1+\mu(n)\}+\theta\xi\{1+\mu(e)\}](1+r), \tag{26}$$

$$(1-\alpha)Al(n)^{-\alpha}=w(n)\{1+\mu(n)\}, \tag{27}$$

$$(1-\alpha)Al(e)^{-\alpha}=w(e)\{1+\mu(e)\}, \tag{28}$$

$$\xi=\kappa\frac{r^a(e)+q}{r^a(n)+q}, \tag{29}$$

$$\chi'(1)+q=(1-\kappa)\beta'\theta[r^a(e)+q]+\frac{r^a(n)+q}{1+r}, \tag{30}$$

$$r^a(n)=\alpha Al(n)^{1-\alpha}, \tag{31}$$

$$r^a(e)=\alpha Al(e)^{1-\alpha}, \tag{32}$$

$$\xi w(n)l(n)=w(e)l(e), \tag{33}$$

$$\gamma'(l(n))=w(n), \tag{34}$$

$$\gamma'(l(e))=w(e). \tag{35}$$

We show the existence of a steady-state equilibrium by solving the system of equations (25)-(35) numerically. For numerical calculation, we specify the functional forms as follows:

$$\gamma(l_t)=-\psi\frac{(1-l_t)^\sigma}{1-\sigma},$$

$$\chi(a_t)=\frac{\phi}{2}a_t^2.$$

Figure 1 plots the variables corresponding to each value of θ. It confirms that output, labor and consumption are smaller when $s_t=e$ than when $s_t=n$. The economy falls into severe recession when a bank run occurs ($s_t=e$) because of a shortage of liquidity for wage payment. A counterintuitive feature of Figure 1 is that when $s_t=n$, output, labor and consumption are slightly increasing in θ.[11] It is easily confirmed, however, that the expected values of labor and output, $E_tl_{t+1}=(1-\theta)l(n)+\theta l(e)$ and $E_ty_{t+1}=(1-\theta)y(n)+\theta y(e)$, are both decreasing in θ in the example shown in Figure 1.[12] We can interpret z as representing the severity of financial friction: it is easily confirmed that as z decreases the discrepancies between the variables in state $s_t=n$ and in state $s_t=e$ increases.[13]

As we show in the next section, these variables when $s_t=n$ decrease in θ in the modified model in which entrepreneurs accumulate capital stocks.

3. The Model with Land and Capital

In this section we modify our basic model such that entrepreneurs accumulate capital stocks, k_t, in each period. Consumer goods are produced from land, capital and labor by the following Cobb-Douglas technology:

$$y_t = Aa_{t-1}^{\nu}k_{t-1}^{\alpha}l_t^{1-\alpha-\nu}.$$

Entrepreneurs can transform consumer goods to capital on a one-to-one basis, and vice versa. Capital k_{t-1} depreciates to $(1-\delta)k_{t-1}$ at the end of period t. We assume that k_t is not pledgeable as collateral when entrepreneurs borrow in period t and that the only pledeable asset is land. If a borrower repudiates the repayment of debt in period t, banks can collect $\{r_t^a(s_t)+q_t\}a_{t-1}$, where

$$r_t^a(s_t) = \nu A\left(\frac{k_{t-1}}{a_{t-1}}\right)^{\alpha}\left(\frac{l_t(s_t)}{a_{t-1}}\right)^{1-\alpha-\nu},$$

and depositors can collect $\kappa\{r_t^a(s_t)+q_t\}a_{t-1}$. Entrepreneur's optimization problem in the modified model is

$$\max_{c_t^E, h_t^E, d_t^E, k_t, l_t, a_t, b_t} E_0\left[\sum_{t=0}^{\infty}(\beta')^t\{c_t^F-h_t^F\}\right],$$

s.t.

$$\begin{aligned}
&q_t(s_t)a_t(s_t)+k_t(s_t)+d_t^E(s_t)-b_t(s_t)+c_t^E(s_t)\\
&\quad = Aa_{t-1}^{\nu}k_{t-1}^{\alpha}l_t(s_t)^{1-\alpha-\nu}+q_t(s_t)a_{t-1}\\
&\qquad +(1-\delta)k_{t-1}-w_t(s_t)l_t(s_t)\\
&\qquad +\xi_t(s_t)(1+r_{t-1})d_{t-1}^E\\
&\qquad -\kappa(s_t)\{r_t^a(s_t)+q_t(s_t)\}\\
&\qquad -\chi(a_t(s_t))+h_t^E(s_t),\\
&\qquad w_t(s_t)l_t(s_t) \le \xi_t(s_t)(1+r_{t-1})d_{t-1}^E,\\
&\qquad b_t(s_t) \le B_t(a_t).
\end{aligned}$$

The resource constraint for consumer goods is

$$\begin{aligned}
&c_t(s_t)+c^E(s_t)+k_t(s_t)\\
&\quad = Aa_{t-1}^{\nu}k_{t-1}^{\alpha}l_t(s_t)^{1-\alpha-\nu}+h_t(s_t)+h_t^E(s_t)\\
&\qquad +(1-\delta)k_{t-1}.
\end{aligned}$$

The FOC with respect to k_t is

$$\begin{aligned}
1 = \beta'\{&(1-\theta)\alpha Aa_t^{\nu}k_t^{\alpha-1}l_{t+1}(n)^{1-\alpha-\nu}\\
&+\theta\alpha Aa_t^{\nu}k_t^{\alpha-1}l_{t+1}(e)^{1-\alpha-\nu}+1-\delta\},
\end{aligned} \tag{36}$$

which, together with $a_t=1$, implies that k_t does not depend on the realization of s_t.

Stationary Equilibrium: Equation (36) implies that there exists a stationary equilibrium in which k_t is time-invariant. Capital stock can be time-invariant along the equilibrium path because backyard production is available for households and entrepreneurs and this technology enables them to make any amount of investment. Therefore, the amount of investment at every period is chosen such that capital stock is kept at a steady-state level. The stationary equilibrium is specified by the set of variables $\{r, q, w(n), w(e), r^a(n), r^a(e), k, l(n), l(e), \xi, \mu(n), \mu(e)\}$, which solves the following system of equations:

$$1 = \beta\{1-(1-\xi)\theta\}(1+r),$$

$$\begin{aligned}
1 = \beta'\{&(1-\theta)\alpha Ak^{\alpha-1}l(n)^{1-\alpha-\nu}\\
&+\theta\alpha Ak^{\alpha-1}l(e)^{1-\alpha-\nu}+1-\delta\},
\end{aligned} \tag{37}$$

$$\begin{aligned}
1 = \beta'[&(1-\theta)\{1+\mu(n)\}\\
&+\theta\xi\{1+\mu(e)\}](1+r),
\end{aligned}$$

$$\begin{aligned}
&(1-\alpha-\nu)Ak^{\alpha}l(n)^{-\alpha-\nu}\\
&\quad = w(n)\{1+\mu(n)\},
\end{aligned} \tag{38}$$

$$(1-\alpha-\nu)Ak^{\alpha}l(e)^{-\alpha-\nu} = w(e)\{1+\mu(e)\}, \tag{39}$$

$$\xi = \kappa\frac{r^a(e)+q}{r^a(n)+q},$$

$$\chi'(1)+q = (1-\kappa)\beta'\theta[r^a(e)+q] + \frac{r^a(n)+q}{1+r},$$

$$r^a(n) = \nu Ak^{\alpha}l(n)^{1-\alpha-\nu},$$

$$r^a(e) = \nu Ak^{\alpha}l(e)^{1-\alpha-\nu},$$

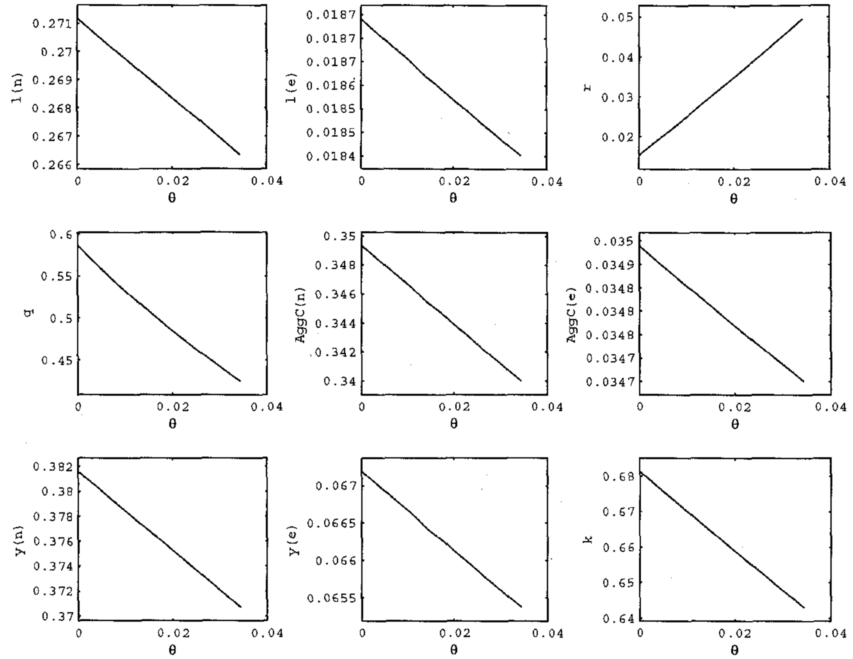
$$\xi w(n)l(n) = w(e)l(e),$$

$$\gamma'(l(n)) = w(n),$$

$$\gamma'(l(e)) = w(e).$$

We solve this system of equations numerically and show the result in Figure 2. The functional forms of $\gamma(l_t)$ and $\chi(a_t)$ are the same as those in the previous section. As Figure 2 shows, output, labor and consumption when $s_t=n$ are all decreasing in θ. Capital stock and the land price are also decreasing in θ. Al though this result may depend on the parameter values, we are confident that this result qualitatively holds for a standard range of parameters. We compare the FOCs in the two models at $\theta=0$ to see why k and l are decreasing in θ. In the basic model without capital, differentiation of (27) with respect to θ implies

Figure 2. Stationary Equilibrium in the Model with Land and Capital



Parameters: $A=1$, $\alpha=0.3$, $\beta=0.985$, $\beta'=0.9\times\beta$, $\delta=0.04$, $\sigma=1$, $\phi=0.6$, $\phi=0.01$, $\kappa=0.05$, $\nu=0.05$.
Note: $AggC(s)=c(s)+c^E(s)-h(s)-h^E(s)$.

$$\left[\gamma''(l(n))\frac{\beta}{\beta'}+(1-\alpha)\alpha Al(n)^{-1-\alpha}\right]\frac{dl(n)}{d\theta}=-\gamma'(l(n))\frac{d\mu(n)}{d\theta},$$

while in the model with capital, differentiation of (38) implies

$$\left[\gamma''(l(n))\frac{\beta}{\beta'}+(1-\alpha-\nu)(\alpha+\nu)Ak^{\alpha}l(n)^{-1-\alpha-\nu}\right]\frac{dl(n)}{d\theta}=-\gamma'(l(n))\frac{d\mu(n)}{d\theta}+(1-\alpha-\nu)\alpha Ak^{\alpha-1}l(n)^{-\alpha-\nu}\frac{dk}{d\theta}. \quad (40)$$

In both models it is easily shown that $\frac{d\mu(n)}{d\theta}<0$ at $\theta=0$ (See Appendix for the proof). Therefore, $\frac{dl(n)}{d\theta}>0$ at $\theta=0$ in the model without capital. The intuition is as follows: if θ increases, entrepreneurs increase d^E to hold liquidity in case of a bank run; the increase in d^E loosens the liquidity constraint on wage payment when $s_t=n$; and as a result of loosening of the constraint on wage payment labor input increases when $s_t=n$. In the model with capital, if the sign of $\frac{dk}{d\theta}$ is negative and its absolute value is sufficiently large, $\frac{dl(n)}{d\theta}$ is negative at $\theta=0$. The intuition is as follows: if θ increases, the loosening of the constraint on wage payment has the effect of increasing $l(n)$, while the decrease in k lowers the marginal product of labor and has the effect to decreasing $l(n)$; therefore, if k decreases to a sufficient extent in response to an increase in θ, the negative effect overwhelms and $\frac{dl(n)}{d\theta}$ becomes negative. On the other hand, a decrease in $l(n)$ directly reduces the (expected) marginal product of capital (MPK), and the decrease in MPK leads to a decrease in k in equilibrium. This relationship is demonstrated as follows: Differentiating (37) with respect to θ, we obtain

$$(1-\alpha)\frac{l(n)^{1-\alpha-\nu}}{k}\frac{dk}{d\theta}=-l(n)^{1-\alpha-\nu}+l(e)^{1-\alpha-\nu}+(1-\alpha-\nu)l(n)^{-\alpha-\nu}\frac{dl(n)}{d\theta}, \quad (41)$$

which implies that if $\frac{dl(n)}{d\theta}<0$ then $\frac{dk}{d\theta}<0$, because $l(n)>l(e)$. We can derive the following lemma from equations (40) and (41).

Lemma 2 *The necessary and sufficient con-*

dition for $\frac{dl(n)}{d\theta}<0$ *and* $\frac{dk}{d\theta}<0$ *at* $\theta=0$ *in the model with land and capital is that the parameter values are set such that the following inequality holds at* $\theta=0$:

$$l(e) < \{(1-\alpha)\xi+\alpha\}^{\frac{1}{1-\alpha-\nu}} l(n). \quad (42)$$

See Appendix for the proof. A sufficient condition for $\frac{dl(n)}{d\theta}<0$ and $\frac{dk}{d\theta}<0$ at $\theta=0$ is given by setting $\xi=0$ in (42):

$$l(e) < \alpha^{\frac{1}{1-\alpha-\nu}} l(n).$$

For example, for $\alpha=0.3$ and $\nu=0.05$, this condition is $l(e)<0.156\times l(n)$, which implies that κ (and $l(e)$) should be considerably small to make k and l decreasing in θ at $\theta=0$.

The exogenous parameter θ in this model can be considered to represent the fragility of the financial system, such as the deterioration in the capital ratio and/or the increase in the ratio of nonperforming assets on the balance sheets of financial institutions. It may be interpreted as representing a loss of confidence in the market or increased fears of the recurrence of a systemic bank run. The result of our simulation implies that economic activities shrink and asset prices decrease as financial fragility (θ) increases, even if a systemic run does not actually occur ($s_t=n$). The mechanism in our model whereby an increase in θ exacerbates macroeconomic variables when $s_t=n$ may explain the slowdown in economic growth observed after episodes of systemic crisis.

4. Conclusion

We present a model of systemic runs on a fragile banking system à la Diamond and Rajan (2000, 2001a, 2001b, 2005) in an infinite-horizon production economy, where the borrowers are subject to collateral constraints à la Kiyotaki and Moore (1997). The mechanism of the bank runs in our model is as follows:

- When the sunspot variable turns out to be bad ($s_t=e$), all agents expect that the value of bank assets becomes $\{r_t^a(e)+q_t\}a_t$, while that of bank liabilities is $(1+r_{t-1})d_{t-1}=\{r_t^a(n)+q_t\}a_t$, where $r_t^a(e)<r_t^a(n)$.
- Since depositors expect that the bank cannot payout the full amount of deposits, depositors initiate a run on the bank.
- In a bank run, depositors (households and entrepreneurs) can withdraw only part of their deposits.
- Since entrepreneurs must pay wages with withdrawn deposits, a bank run reduces the amount of funds available to entrepreneurs to pay wages.
- The decrease in wage payment reduces aggregate labor input. The decrease in labor input lowers the marginal product of land and thus decreases the return on land from $r_t^a(n)$ to $r_t^a(e)$.
- In this way, the prophecy that $r_t^a=r_t^a(e)$ is self-fulfilled.

A bank run leads the economy into a severe recession due to a shortage of liquidity for paying wages. A bank run in our model is not caused by a coordination failure among depositors in one bank as in Diamond and Dybvig (1983), but is caused by an economy-wide coordination failure, which changes the market value of collateral assets. The key is that we translate the demand for liquidity in the Diamond-Rajan models into the demand for working capital, i.e., borrowing for factor payments, in the business cycle models. The modified model with capital accumulation shows that an increase in financial fragility (θ) causes a shrinkage of the economic activities and a decline in asset prices even if a systemic bank run does not actually occur. This indicates that the typical slowdown in economic growth and stagnant asset prices after a financial crisis may be caused by the increase in financial fragility or raised fears of the recurrence of another systemic crisis.

In this paper we did not analyze liquidity provision by the government in the form of government bonds or fiat currency. The possibility of public provision of liquidity would produce rich policy implications for crisis management in the wake of systemic bank runs. This will be a topic for future research.

Appendix 1: Financial Contracts

In this appendix we describe financial contracts (demand deposits) between banks and depositors, which we embed in the model of the text.

There are three agents in this economy: households, entrepreneurs, and banks. In this ssection, we consider a one-period financial contract between these agents. Banks raise funds from depositors (i.e., households and entrepreneurs) and lend the funds to entrepreneurs at the end of each period $t-1$. Entrepreneurs borrow from banks and also deposit working capital in banks. The bank loan and deposits are paid off at the end of the next period t. We assume the following assumptions for a bank, a depositor (a household or an entrepreneur), and a borrowing entrepreneur.

Assumption 3 *An entrepreneur pledges her own land, a_{t-1}, as collateral when she borrows from other agents at the end of period $t-1$. If the lender is a bank, the bank has a relation-specific loan-collection skill that enables it to seize $\{r_t^a+q_t\}a_{t-1}$ units of consumer goods from the borrower at the end of period t, where r_t^a is the return from the land and q_t is the land price. If the lender is a household or another entrepreneur, the lender can seize $z\{r_t^a+q_t\}a_{t-1}$ with $0<z<1$. The bank's loan-collection skills are relation-specific in that only the bank that originated the loan can collect $\{r_t^a+q_t\}a_{t-1}$ from the borrower, while the other banks can collect only $z\{r_t^a+q_t\}a_{t-1}$. The borrowing entrepreneur cannot commit to repay a predetermined amount to the lender and can walk away without any penalty except for seizure of the above-mentioned amounts. The bank has no funds to lend and needs to borrow from depositors (households and entrepreneurs) in order to lend funds to entrepreneurs. The banks cannot commit to use their relation-specific skills on behalf of their depositors and banks can walk away from depositors in the middle of period t without any penalty, leaving the loan assets to depositors. When a bank walks away, depositors (households and entrepreneurs) become the collective owner of the bank loans to borrowing entrepreneurs.*

Banks are the sole lenders to entrepreneurs: Under this assumption, a borrowing entrepreneur cannot commit to repay a prespecified amount but can pledge collateral, a_{t-1}, for the debt. Therefore, an entrepreneur is subject to a collateral constraint. We assume and justify later in the general equilibrium model that a collateral constraint is binding in equilibrium. Given that a collateral constraint is binding, an entrepreneur wants to borrow as much as possible. Banks can offer $R_t^{-1}E_{t-1}[\{r_t^a+q_t\}a_{t-1}]$ to lend to an entrepreneur who has a_{t-1}, while the other agents (households and other entrepreneurs) can offer $R_t^{-1}zE_{t-1}[\{r_t^a+q_t\}a_{t-1}]$ to the same borrower, where R_t is the gross rate of return. Given that a borrower's collateral constraint binds, a borrower always chooses to borrow from banks, not from other agents. Therefore, as long as banks can raise funds, they become the sole lenders in this economy as a result of lending competition among banks and other agents (households and entrepreneurs).

Banks cannot raise funds without issuing demandable debt: Since banks have no funds to lend at the end of period $t-1$, they need to raise funds from households and entrepreneurs. It is shown as follows, however, that it is impossible for a bank to raise funds unless it issues demandable debt. Suppose that a bank raises debt B, which is not demandable, from households and entrepreneurs and the bank lends it to an entrepreneur. Suppose also that the bank can collect C from the borrower using relation-specific loan-collection skills. We assume that

$$\frac{C}{B} \geq 1+r_t^m > \{z+(1-z)x\}\frac{C}{B}, \quad (43)$$

where $x\,(0<x<1)$ is the parameter that represent depositors' bargaining power (see below) and r_t^m is the risk-free rate of interest.[14] We assume that x is sufficiently small. By Assumption 3, bank depositors (households and entrepreneurs) can collect at most zC if bank walks away without collecting on the loan and they recover the loan by themselves. Since the bank cannot precommit to use relation-specific loan collection skills on behalf of depositors, it is *ex post* rational for the bank after making the loan to initiate the following renegotiation with depositors: the bank offers to pay $\{z+(1-z)x\}C\,(<C)$ to depositors and says that it will walk away leaving the loan assets to depositors if they do not accept this offer. Since the bank can collect C and depositors can collect zC, this offer means splitting surplus $(1-z)C$ between the bank and depositors, according to Nash bargaining between them, while depositors' bargaining power is x and the bank's bargaining power is $1-x$. Depositors have no other choice than to accept the bank's offer $\{z+(1-z)x\}C$, because the bank's relation-specific skills are necessary to generate surplus $(1-z)C$. Anticipating that the bank will initiate renegotiation in the middle of period t, households and entrepreneurs do not deposits their funds in the bank at the end of period $t-1$, because (43) implies that the return on bank deposits $\{z+(1-z)x\}\frac{C}{B}$ is lower than risk-free interest rate $1+r_t^m$. By issuing demand deposits, banks can credibly commit to use their loan-collection skills on behalf of depositors to successfully raise funds.

Demand deposits as a commitment device: The demand deposit contract is a contract that gives a depositor who deposits funds at the end of period $t-1$ the unilateral right to withdraw a predetermined amount, C, at anytime in period t. The demand deposit contract has the following features:

- One bank issues demand deposits to many depositors simultaneously.
- If a depositor withdraws at the end of period t, the bank pays the depositor C units of consumer goods.
- If a depositor withdraws in period t before the consumer goods are produced, the bank gives C/κ units of loan assets to the withdrawer as long as the bank assets remain, where κ is the recovery rate for the depositor if the depositor directly recovers the loan from the borrower. κ is defined as $\kappa=z+(1-z)x$ and therefore $0<\kappa<1$. Note that depositors obtain κC, not zC, from the borrowing entrepreneur. This is shown in the bargaining process described in the proof of Lemma 3.
- If many depositors withdraw before production of the consumer goods and the bank runs out of loan assets, the remaining depositors get nothing. This is the *first-come, first-served principle.*

Demand deposits make banks credibly commit to pay the promised amount of deposits and not to renegotiate the payment down. This is because if a bank tries to renegotiate with depositors they immediately initiate a run on the bank and the bank ends up getting zero as a result of the bank run. We show this result by a similar argument as Diamond and Rajan (2001a, 2001b).

Lemma 3 *If a bank tries to renegotiate with depositors, they all initiate a run on the bank immediately and the bank ends up getting zero as a result of the bank run.*

(Proof) If a bank initiates renegotiation with depositors to reduce payment, the dominant strategy for depositors is to unilaterally withdraw the predetermined amount of deposits. (See pp. 309–313 of Diamond and Rajan 2001a.) When a bank run occurs, the ownership of the bank loan is transferred to depositors. The depositors who successfully withdrew become a collective owner of the bank loan. Depositors can collect $z\{r_t^a + q_t\}a_{t-1}$ by themselves if they directly collect the loan from a borrowing entrepreneur, while the bank can collect $\{r_t^a + q_t\}a_{t-1}$. After the bank run, depositors collectively decide whether they directly collect the loan from the entrepreneur or they rehire the original bank to collect the loan on behalf of the depositors. It is easily shown as follows that depositors decide not to hire the bank and that the bank can collect zero rent.

- Suppose that depositors hire the original bank. The bank takes $\{r_t^a + q_t\}a_{t-1}$ from an entrepreneur. Since the surplus $(1-z)\{r_t^a + q_t\}a_{t-1}$ must be divided between the bank and the depositors according to Nash bargaining, the bank offers the depositors payment of $\{z+(1-z)x\}\{r_t^a + q_t\}a_{t-1}$.
- In order to prevent depositors from hiring the bank, the borrowing entrepreneur offers to pay $\{z+(1-z)x\}\{r_t^a + q_t\}a_{t-1}+\varepsilon$ directly to depositors, where $\varepsilon(>0)$ is an infinitesimally small amount. If this offer is accepted by depositors the entrepreneur pays $\{z+(1-z)x\}\{r_t^a + q_t\}a_{t-1}+\varepsilon$ to depositors, while if the bank is hired by depositors the entrepreneur must pay $\{r_t^a + q_t\}a_{t-1}$. Obviously the entrepreneur is better off by preventing depositors from hiring the bank.
- Depositors accept the entrepreneur's offer and never hire the bank again after the bank run.[15)]

Therefore, the bank can get no rent after the bank run. (End of Proof)

The demand deposit contract enables banks to credibly commit to use their human capital on behalf of depositors, and it enables banks to act as an intermediary between households and entrepreneurs. Meanwhile, demand deposits make the banking system susceptible to a systemic bank run, because depositors initiate a run on the banks and withdraw deposits unilaterally in response to an adverse macroeconomic shock or a sunspot shock as we see in the following sections.

Simplification of the first-come, first-served principle: We make the following assumption to simplify analysis of the equilibrium. The first-come, first-served (FCFS) principle is essential in Diamond and Rajan (2001a) to derive the result that unconditional withdrawal is the dominant strategy for depositors when a bank initiates renegotiation. The FCFS principle divides depositors into two groups, i.e., the successful withdrawer and the unsuccessful withdrawer, when a bank run occurs. The heterogeneity of depositors complicates the analysis. To avoid complication, we adopt Allen and Gale's (1998) simplifying assumption for the depositors' payoff in the bank run.

Assumption 4 *All depositors divide up bank assets on a pro rata basis when a bank run occurs. Therefore, a depositor who has the right to withdraw D_i can seize ξD_i during the bank run, where $\xi\ (0 \leq \xi < 1)$ is determined as an equilibrium outcome and identical for all depositors in the bank.*

The pro rata payoff is realized if, for example, the depositors in one bank form a fair insurance contingent on the bank run.[16)] Through this assumption, we can analyze macroeconomic variables assuming that households and entrepreneurs are identical, respectively, after the bank run. Although we assume this simplifying assumption, which is not rigorously consistent with Lemma 3, we are confident that the following analysis of the general equilibrium model does not change qualitatively even without Assumption 4.

Appendix 2

Proof of $\frac{d\mu(n)}{d\theta}<0$ at $\theta=0$: We consider the basic model presented in Section 2. (The same arguments hold for the other models.)

Equation (25) implies that $1+r=1/\beta$ and $\beta\frac{dr}{d\theta}=1-\xi$ at $\theta=0$. Equation (26) implies that $1+\mu(n)=\beta/\beta'$ at $\theta=0$. Together with these results, differentiation of (26) with respect to θ at $\theta=0$ implies

$$\begin{aligned}\frac{d\mu(n)}{d\theta} &= \frac{\beta\xi}{\beta'}\left[1-\frac{\beta'}{\beta}(1+\mu(e))\right] \\ &= \frac{\beta\xi}{\beta'}\left[1-\frac{1+\mu(e)}{1+\mu(n)}\right]. \qquad (44)\end{aligned}$$

Equations (33), (34), and (35) imply that $l(e)<l(n)$. Equations (27) and (28) imply that $1+\mu$ is a decreasing function of l. Therefore, $\frac{1+\mu(e)}{1+\mu(n)}>1$, since $l(e)<l(n)$. This result and equation (44) imply that $\frac{d\mu(n)}{d\theta}<0$.

Proof of Lemma 2: Equations (40) and (41) imply

$$\begin{aligned}&\left[\gamma'(l(n))\frac{\beta}{\beta'}+\frac{(1-\alpha-\nu)^{\nu}}{1-\alpha}Ak^{\alpha}l(n)^{-1-\alpha-\nu}\right]\frac{dl(n)}{d\theta} \\ &= -\gamma'(l(n))\frac{d\mu}{d\theta}-\{l(n)^{1-\alpha-\nu} \\ &\quad -l(e)^{1-\alpha-\nu}\}\frac{(1-\alpha-\nu)\alpha}{1-\alpha}\frac{Ak^{\alpha}}{l(n)}. \qquad (45)\end{aligned}$$

The right-hand side of (45) is rewritten as follows using (44), (38) and (39):

$$\frac{(1-\alpha-\nu)\alpha}{1-\alpha}\frac{Ak^{\alpha}}{l(n)}[l(e)^{1-\alpha-\nu}-\{(1-\alpha)\xi+\alpha\}l(n)^{1-\alpha-\nu}]. \qquad (46)$$

Therefore, $\frac{dl(n)}{d\theta}<0$ at $\theta=0$ if and only if (42) holds. Equation (41) implies that $\frac{dk}{d\theta}<0$ if $\frac{dl(n)}{d\theta}<0$.

(Institute of Economic Research, Hitotsubashi University)

Notes

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1) Our interpretation of liquidity as a working capital in production is similar to that in Holmstrom and Tirole (1997, 1998). The Holmstrom-Tirole models do not address a banking crisis or a bank run, which is the focus of the present paper.

2) It is well known that if working capital is subject to borrowing constraints, financial frictions that tighten the constraints amplify the economic downturn. See, for example, Jermann and Quadrini (2007), Kobayashi and Nutahara (2008), Kobayashi, Nakajima, and Inaba (2010), and Mendoza (2010).

3) Cole and Ohanian (1999) and Kehoe and Prescott (2002) argue that large and persistent economic downturns (great depressions) are caused by large and persistent TFP shocks.

4) Banks in this model represents various financial institutions that take short-term debt and make long-term loans. Households in this model also represent various depositors and lenders of short-term loans in reality, which include lending financial institutions in the interbank markets.

5) Labor in this model represents not only labor literally, but also various inputs of production, such as intermediate goods or materials.

6) In this paper we focus on the stationary equilibrium path in which macroeconomic variables, e. g., r_t^a and q_t, are time-invariant and depend only on the realization of s_t. Nevertheless, we put the subscript t on the variables in the following analysis in order to distinguish the variables at date t from those at other dates. Therefore, we denote macroeconomic variable x in period t as $x_t(s_t)$.

7) Note that in the middle of period t households obtain $(1+r_{t-1})d_{t-1}^E$ units of bank deposits if $s_t=n$ or the same amount of loans originated by banks if $s_t=e$, as wage payment. Loans with face value $(1+r_{t-1})d_{t-1}^E$ have a market value of $\xi_t(1+r_{t-1})d_{t-1}^E$ for households.

8) Note that $\xi_{t+1}(s_{t+1})\in\{\xi_{t+1}(n),\xi_{t+1}(e)\}$ does not depend on the realization of s_t.

9) The remaining variables $\{c_t(s_t), c_t^E(s_t), h_t(s_t), h_t^E(s_t)\}$ are determined by the resource constraints and the non-negativity constraints of respective variables.

10) The macroeconomic variables in the equilibrium path may vary over time if the initial value of q_t is

different from its value in the stationary equilibrium. We do not consider these cases in this paper.

11) It is analytically proven that $l(n)$ is increasing in θ at $\theta=0$. See Section 3 for the details.

12) I would like to thank Reiko Aoki for pointing out this fact.

13) I would like to thank Ryo Kambayashi for pointing this out.

14) The risk-free rate is determined in general equilibrium. Although we do not explicitly consider it in the following sections, we can introduce the market for real government bonds in our model and define the risk-free rate as the interest rate for government bonds.

15) This result does not depend on the protocol of Nash bargaining between the bank and depositors, which may take place after the bank run. Given any division of the surplus between the bank and depositors, depositors and borrowing entrepreneurs would be better off by dividing the bank's surplus between themselves. Therefore, an entrepreneur can always make an offer to the depositors that lead them to decide not to hire the original bank.

16) The insurance contract should be such that a depositor is eligible for insurance only if he does not make concessions to the bank and try to withdraw his deposit unilaterally.

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