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AGGLOMERATION, TAX, AND LOCAL PUBLIC GOODS*

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Abstract

In this paper, we explore the interaction between taxation and a local public good (LPG) to see how it impacts the spatial pattern in the framework of new economic geography (NEG). In the benchmark case of a pure LPG, the system displays a similar location pattern to the existing NEG taxation model, although the tax reduces the market size of manufactured goods. However, when we consider the inherent congestion of an LPG, we find a new agglomeration force due to the demand of the LPG and a new dispersion force due to its congestion. As a result of their interaction, the congestability is crucial in determining the spatial location pattern.

Keywords: tax, agglomeration, dispersion, congestible local public good
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I. *Introduction*

The relationship between factor mobility and the regional provision of a local public good (LPG) has long been a concern of economists. Previous classic literature can be classified into two categories. On the one hand, literature on tax competition demonstrates that tax competition leads to a “race to the bottom” and underprovision of LPGs among jurisdictions (see Wilson, 1999, for a complete survey). On the other hand, Tiebout (1956) and many important subsequent studies focus on the efficiency of LPG provision in a system of competing jurisdictions. They offer various combinations of LPG and taxation. In addition to the above, recently, a separate literature focuses on how tax competition impacts on industrial location by use of a general equilibrium framework called new economic geography (NEG). These studies rest heavily on the core-periphery (CP) model of Krugman (1991).

Interestingly, under the NEG framework, tax competition may generate different results from classic tax competition models due to the hump-shaped agglomeration rents. Theoretical studies along this line include Kind et al. (2000), Ludema and Wooton (2000), Andersson and Forslid (2003), Baldwin et al. (2003; Ch. 15, 16), Baldwin and Krugman (2004), Ottaviano and van Ypersele (2005) and Borck and Pflüger (2006). Most studies focus on the issue of “tax competition” to emphasis the tax differential outcome and its corresponding spatial pattern of mobile factor, resulting in a “race to the top” rather than a race to the bottom. Moreover, Ihara (2008) consider that LPG is input into production to analyze the provision efficiency of LPGs. Despite great progress in the issue of tax competition, the corresponding LPG issue is not sufficiently explored in the NEG framework.

Firstly, Ottaviano and van Ypersele (2005), Haufler and Wooton (2010), Kind et al. (2000), Ludema and Wooton (2000) assume that tax revenues are redistributed as income to workers instead of financing LPG provision. As a result, Ottaviano and van Ypersele (2005) show that asymmetric market size results in an incentive of taxation (resp. subsidization) to attract firms even in the absence of public good provision. Second, although Andersson and Forslid (2003) and Baldwin et al. (2003; Ch. 15, 16) include the LPG production in their models, they limit their studies to the case of a “pure” LPG without congestion. Furthermore, Andersson and Forslid (2003) and Baldwin et al. (2003; Ch. 15, 16) assume that the LPG is produced using the same composite of the agricultural good and the manufactured good as in consumers’ utility function. As a result, the market size of private sectors and the wage rate are independent of the tax rate. Therefore, the symmetric tax rate is neutral in the NEG framework. On the one hand, this assumption is successful to simplify their models to derive a tax game. However, it also makes the symmetric tax rate inessential for an agglomeration pattern. Furthermore, if the LPG production uses a different fraction of manufactured goods from that in the consumers’ utility function, some results of tax competition in previous papers may not hold. That is because their tax revenue is *linear* in the tax rate and they must use a specific functional form so that the government’s objective is *concave* in the tax rate to obtain an interior solution (Baldwin and Krugman, 2004; Borck and Pflüger, 2006).

The interaction between taxation and LPGs remains a salient issue in the NEG framework. For example, taxation may induce a chain reaction which successively impacts the market size of private consumption, wage rate, tax revenue, LPG provision and incentive of agglomeration (endogenous market size). Thus, the purpose of this paper is to add several distinctive features neglected in previous studies and to explore the chain reaction in the NEG framework.

To this end, as the first feature of this paper, we differentiate the input fraction of manufactured goods in the LPG production from the consumption fraction of manufactured goods in the private sector. As a result, the tax revenue becomes concave in tax rate.[1] This revenue is redistributed as income to workers as in Andersson and Forslid (2003)[2] and also has an impact on the market size of private goods. Some financial policies in the real world are actually based on the impact of tax on the private consumption size.[3] To prevent the model from being too complicated, we assume that the input of LPG production is unskilled labor only.[4] As a result, the model has full analytical results in the absence of tax competition[5] which complement the numerical examples in tax competition framework of Andersson and Forslid (2003).

Next, as our second strategic feature, we consider an economy of one state containing two regions. The central government initially chooses symmetric tax rates in two regions. More precisely, we follow Baldwin and Krugman (2004) and Andersson and Forslid (2003) by considering proportional income tax but have no tax game (competition). We also assume that tax revenue is only used to produce the LPG that is consumed by every resident in the same region. The above two features provide additional tension between the new centrifugal force from the reduced wage rate with respect to tax rate and a new centripetal force from LPG services. As a result, we find more and different agglomeration patterns under the regime of a symmetric tax rate. Notably, the second feature indicates that we are going to study LPGs rather than tax competition.

The third feature of this paper is that we consider not only a pure LPG but also a congestible LPG, which is neglected in Andersson and Forslid (2003). In the real world, all public services are characterized by some degree of congestion, such as the cases of social security service, transportation facilities, public utilities, courts, parks, and libraries. Their service quality should span the range between the polar cases of pure private and pure public goods. Previously, models with a congestible LPG can be seen in the literature of public economics and regional economics (Hochman, 1982; Edwards, 1990; Conley and Wooders (2001); Berliant et al., 2006) as well as in that of endogenous growth theory (Barro and Sala-i-Martin, 1992; Turnovsky, 1996; Eicher and Turnovsky, 2000). The congestion of an LPG is inherently related to the scale of the aggregate usage; it is particularly relevant in an economic-geography context that relative congestion and aggregate (absolute) congestion depend on the endogenous degree of spatial agglomeration. It is interesting to study how and how far the presence of usage congestion for an LPG affects the agglomeration pattern in an NEG framework.

This paper obtains the following results. First, in the benchmark case of a pure local public good (PLPG), we obtain a similar location pattern to previous NEG taxation models although

[1] The concave tax revenue is able to capture the essential trade-off of the government between a high tax revenue and a low tax rate. In contrast, in the studies of Andersson and Forslid (2003), Baldwin and Krugman, (2004) and Baldwin et al. (2003), the tax revenue is linear in tax rate.

[2] However, the possibility of income redistribution in Andersson and Forslid (2003) is comes from the different tax rates between skilled and unskilled workers.

[3] For example, tax refunds, the grant of a consumption coupon (voucher), and the expansion of domestic demand.

[4] This assumption enables us to have a simpler price index of the LPG than Andersson and Forslid (2003).

[5] The same tax rate between two regions is the second feature of this paper which is explained in the next paragraph.

the tax reduces the market size of manufactured goods. Meanwhile, we find that the interaction between the tax rate and the LPG taste is important in determining the value of the breakpoint.[6] Second, in the case of a congestible local public good (CLPG), the equilibria of both symmetric dispersion and full agglomeration could emerge from the autarky state (i.e., infinite transportation costs) due to the presence of congestion. In addition, the interaction between the tax rate and the usage congestion of the LPG determines the spatial structure of the ultimate state (i.e., zero transportation costs). This is because the economy scale and the corresponding LPG service may not counteract the endogenous congestion for agglomeration in a high tax rate. Third, using the fully analytically solvable framework, we show that the break point (resp. re-dispersion point[7]) increases (resp. decreases) in the tax rate and the LPG congestability. In particular, it is possible that the dispersion equilibrium is always stable for any level of trade freeness as long as the breakpoint and the re-dispersion point coincide or do not exist.

The rest of the article is organized as follows. Section II describes a basic two-region model that extends the footloose entrepreneur model of Forslid and Ottaviano (2003) by incorporating symmetric tax rates and congestible local public goods. Section III analyzes the long-run equilibrium of the benchmark case of pure local public goods. Section IV characterizes the structural change of the spatial pattern in the case of a general congestible local public good and describes how the location patterns vary with the tax rate, the LPG congestability, and other parameters. Section V presents our concluding remarks.

II. *The Model*

We adopt the footloose entrepreneur (henceforth, FE) model of Forslid and Ottaviano (2003), which is a solvable variant of Krugman (1991). Specifically, we consider an economic system with the following features: (1) There is one state containing two regions, denoted by 1 and 2. (2) Each region produces three kinds of goods, a homogenous agricultural good (A), a composite manufactured good (M) consisting of horizontally differentiated varieties (c), and a local public good (G). (3) The local public good (LPG) in a region is available only to residents in the same region, and we assume that there is "congestion," described by parameter $\alpha \in [0, 1)$. (4) The LPG is provided by the regional tax revenue, while each region levies a "proportional income tax" to every local resident. Two regions adopt the same tax rate (t), and there is no "tax game (competition)." (5) There are two factors of production in a private good market, skilled labor and unskilled labor. Skilled labor only works for the manufacturing sector and can migrate cost-free between the two regions, while unskilled labor works for either the manufacturing sector or the agricultural sector but is immobile between regions. (6) The agricultural good market is perfectly competitive. The agricultural good is freely traded between two regions, whose production employs unskilled labor only and is subject to constant returns to scale. (7) The manufactured good is subject to monopolistic competition with increasing returns to scale. A manufacturing firm employs both skilled and unskilled labor. (8) The

[6] The breakpoint is the critical value of trade freeness (which is inversely related to trade/transportation costs), above which the dispersion equilibrium is unstable.

[7] The re-dispersion point is the critical value of trade freeness, above which the dispersion equilibrium is stable again.

transport cost of the manufactured goods is modeled by the iceberg technology. (9) The total number of skilled labor is denoted by H, and that of unskilled labor, by L. The numbers of unskilled labor in the two regions are $L_1=L_2=L/2$. For simplicity, we choose the unit of population, such that the world population is 1: $H+L=1$.

1. Consumption

On the demand side, we follow the model of Andersson and Forslid (2003), in which skilled and unskilled labor have the same preferences described by a CES sub-utility over the manufacturing varieties nested in a Cobb-Douglas upper-tier function that includes consumption of A and G, while parameters $\mu, \gamma \in (0, 1)$ measure the tastes for M and G, respectively. Subsequently, i refers to either region 1 or region 2. For convenience, we also denote the other region by j. The consumption behavior is formally described as follows:

$$\begin{aligned} &\text{Max} \quad U_i=\frac{1}{\mu^{\mu}(1-\mu)^{1-\mu}} M_i^{\mu} A_i^{1-\mu} G_i^{\gamma} \\ &\text{s.t.} \quad \int_0^{n_i} p_{ii}(c_i)\, d_{ii}(c_i)\, \mathbf{d}c_i+\int_0^{n_j} p_{ji}(c_j)\, d_{ji}(c_j)\, \mathbf{d}c_j+p_{iA} A_i=(1-t)w_i, \end{aligned} \tag{1}$$

where $1/[\mu^{\mu}(1-\mu)^{1-\mu}]$ is a positive coefficient to ease the notation burden later, p_{iA} is the price of the agricultural good, and the manufacturing aggregate M is defined by the following CES form

$$M_i \equiv \left[\int_0^{n_i} d_{ii}(c_i)^{\frac{\sigma-1}{\sigma}}\, \mathbf{d}c_i+\int_0^{n_j} d_{ji}(c_j)^{\frac{\sigma-1}{\sigma}}\, \mathbf{d}c_j\right]^{\frac{\sigma}{\sigma-1}}, \quad 0<\mu, \gamma<1<\sigma$$

In the above, $d_{ji}(c_j)$ is the consumption of a manufacturing variety c_j produced in region j and sold in i, and $p_{ji}(c_j)$ denotes its consumer price. Notations $d_{ii}(c_i)$ and $p_{ii}(c_i)$ are defined similarly. n_i and n_j are the numbers of manufacturing varieties produced in region i and j, respectively. $\sigma \in (1, \infty)$ is the elasticity of substitution between any two varieties. In addition, because firms sell differentiated goods, each of them has some monopoly power facing an isoelastic demand function with elasticity σ. In addition, w_i is the wage rate of a consumer in region i. More precisely, we denote the wage rate of skilled (resp. unskilled) labor by w_{Hi} (resp. w_{Li}). Since each worker is levied a proportional income tax by rate $t \in (0, 1)$, the budget is limited to the disposable income $(1-t)w_i$.

Following Berliant et al. (2006), the LPG service consumed by an individual in region i is represented by

$$G_i \equiv \frac{g(T_i)}{\left(H_i+\frac{1-H}{2}\right)^{\alpha}} \quad ; \quad T_i=t\,Y_i \quad ; \quad Y_i=H_i w_{Hi}+\frac{1-H}{2} w_{Li}, \tag{2}$$

where g is the total output of the LPG, which is a function of the total tax revenue T_i, and Y_i denotes the aggregate regional income consisting of skilled and unskilled wages in region i. The denominator of G_i measures the "congestion" level of the LPG from two aspects, the endogenous number of users $H_i+(1-H)/2$ and the exogenous "congestability" parameter $\alpha\in[0, 1)$. Notably, $\alpha=0$ implies a pure LPG, whereas $\alpha\rightarrow 1$ implies a private good. Unlike Berliant et al. (2006), the number of LPG users is endogenously given rather than fixed.

Using the two-stage budgeting approach (Fujita et al., 1999, Ch. 4), we can obtain the demand functions in region i as follows

$$d_{ji}(c_j)=\frac{p_{ji}(c_j)^{-\sigma}}{\mathbb{P}_i^{1-\sigma}}\mu(1-t)w_i \quad ; \quad d_{ii}(c_i)=\frac{p_{ii}(c_i)^{-\sigma}}{\mathbb{P}_i^{1-\sigma}}\mu(1-t)w_i, \tag{3}$$

$$M_i=\mu\frac{(1-t)w_i}{\mathbb{P}_i} \quad ; \quad A_i=(1-\mu)\frac{(1-t)w_i}{p_{iA}}, \tag{4}$$

where $\mathbb{P}_i$ and $\mathbb{P}_j$ are the price indices of manufactured goods in region i and j, respectively. According to the ex-post symmetry among varieties, every variety c_i (resp. c_j) has the same price, so the price index in region i is simply expressed as:

$$\mathbb{P}_i=[n_i\, p_{ii}(c_i)^{1-\sigma}+n_j\, p_{ji}(c_j)^{1-\sigma}]^{\frac{1}{1-\sigma}}. \tag{5}$$

Substituting (2), (3) and (4) into (1), we obtain the indirect utility function of a skilled labor in region i:

$$V_i=\frac{(1-t)w_{Hi}}{(\mathbb{P}_i)^{\mu}(p_{iA})^{1-\mu}}\cdot\frac{g^{\gamma}(T_i)}{\left(H_i+\dfrac{1-H}{2}\right)^{\alpha\gamma}}. \tag{6}$$

An interior equilibrium $h=h^*$ is obtained when the utility differential is $\Delta V|_{h=h^*}=V_1-V_2=0$. The equilibrium is stable iff $\left.\frac{\partial\Delta}{\partial h}\right|_{h=h^*}<0$. Corner equilibrium $h=1$ is stable if $\Delta V|_{h=1}>0$ and corner equilibrium $h=0$ is stable if $\Delta V|_{h=0}<0$.

2. Production

Turning to the supply side, each region produces the agricultural good, the manufacturing goods, and the local public good. The following setup of the agricultural and the manufacturing sectors is standard.

(1) Agricultural sector

The homogenous agricultural good is produced under constant returns to scale, and the market is in perfect competition. Unskilled labor is the only input. The unit of the agricultural product is chosen such that one unskilled labor produces one unit of the agricultural good. These conditions imply that the wage of unskilled labor (w_{Li}) is equal to the price of the agricultural good: $w_{Li}=p_{iA}$. In addition, the agricultural good is freely traded between two regions so that its price is the same everywhere ($p_{iA}=p_{jA}$), which also implies interregional wage equalization ($w_{Li}=w_{Lj}$).[8] We choose the agricultural good as the numéraire. Then, we obtain $w_{Li}=p_{iA}=p_{jA}=w_{Lj}=1$.

[8] The agriculture sector needs to be big enough that no single region can specialize in manufacturing and the LPG (i.e. not produce any of good A).

(2) Local public good

As in the agricultural sector, we assume that the input in the LPG production is also unskilled labor only. Furthermore, the marginal labor requirement is assumed to be one for simplicity. Therefore, the amount $g(T_i)$ of the LPG is equal to $T_i/w_{Li}=T_i$. Then, the first part of (2) simply becomes

$$G_i \equiv \frac{T_i}{\left(H_i+\frac{1-H}{2}\right)^{\alpha}}. \tag{7}$$

(3) Manufacturing sector

Manufacturing firms employ both skilled and unskilled labor to produce differentiated goods subject to increasing returns to scale. Together with the assumption of costless differentiation, this ensures that each firm produces only one variety. In other words, there is a one-to-one relationship between firms and varieties. Each firm uses the same technology of production. Following the FE model, we assume that each firm employs one skilled labor as the fixed input. We further choose the unit of manufactured goods so that a marginal input of $(\sigma-1)/\sigma$ units of unskilled workers are required to produce one unit of a variety. The number of firms in a region is, therefore, equal to the number of skilled workers. A firm in region i sets its prices p_{ii} and p_{ij} to maximize its profit:

$$\underset{p_{ii},\, p_{ij}}{\text{Max}} \quad \pi_i(c_i)=p_{ii}(c_i)D_{ii}(c_i)+p_{ij}(c_i)D_{ij}(c_i)-\left\{w_{Hi}+\frac{\sigma-1}{\sigma}w_{Li}[D_{ii}(c_i)+\tau D_{ij}(c_i)]\right\}, \tag{8}$$

where $D_{ii}(c_i)$ and are the total demands for variety c_i in regions i and j, respectively. Here $\tau\, D_{ij}(c_i)$ represents the total supply to region j, inclusive of the Samuelson melting iceberg cost: for one unit of the manufactured good to reach the other region, $\tau \geq 1$ units must be shipped. Hence, under the market-clearing condition, $x_i(c_i)=D_{ii}(c_i)+\tau D_{ij}(c_i)$ holds for each variety c_i. Moreover, according to Equation (3), the total demand for each variety c_i in one region is determined as follows:

$$D_{ii}(c_i)=\frac{p_{ii}(c_i)^{-\sigma}}{\mathbb{P}_i^{1-\sigma}}\mu(1-t)Y_i \quad ; \quad D_{ij}(c_i)=\frac{p_{ij}(c_i)^{-\sigma}}{\mathbb{P}_j^{1-\sigma}}\mu(1-t)Y_j. \tag{9}$$

The first order conditions of (8) imply:

$$p_{ii}=p_{jj}=1 \quad ; \quad p_{ij}=p_{ji}=\tau, \tag{10}$$

so the manufactured price indices become:

$$\mathbb{P}_1=H^{\frac{1}{1-\sigma}}[h+\phi(1-h)]^{\frac{1}{1-\sigma}} \quad ; \quad \mathbb{P}_2=H^{\frac{1}{1-\sigma}}[1-h+\phi h]^{\frac{1}{1-\sigma}}, \tag{11}$$

where $\phi \equiv \tau^{1-\sigma} \in [0, 1]$ denotes the trade freeness, which is equal to one when the transportation cost $\tau-1$ is zero (free trade) and zero when the trade of manufacturing goods is impossible. Notation $h \equiv H_1/H \in [0, 1]$ is the share of skilled workers residing in region 1.

By the free-entry condition of firms, the operating profit earned by a typical firm is just sufficient to cover its fixed cost. We then obtain the nominal wages of skilled labor as

$$w_{H1}=\frac{\mu}{\sigma H}\left[\frac{(1-t)Y_1}{h+\phi(1-h)}+\frac{\phi(1-t)Y_2}{\phi(1-h)+h}\right] \quad ; \quad w_{H2}=\frac{\mu}{\sigma H}\left[\frac{(1-t)Y_2}{1-h+\phi h}+\frac{\phi(1-t)Y_1}{h+\phi(1-h)}\right]. \tag{12}$$

Finally, after plugging the aggregate income (2) into (12), we generate a system of two equations for w_{H1} and w_{H2}. Solving the equations obtains the equilibrium wages of skilled labor:

$$\begin{aligned} w_{H1}&=\frac{(1-H)(1-t)\mu\{\sigma[2\phi h+(1-h)(1+\phi^2)]-\mu(1-t)(1-h)(1-\phi^2)\}}{2H[\sigma-(1-t)\mu]\{h(1-h)[\sigma-(1-t)\mu]+\sigma\phi[1-2h(1-h)]+h(1-h)[\sigma+(1-t)\mu]\phi^2\}}, \\ w_{H2}&=\frac{(1-H)(1-t)\mu\{\sigma[2\phi(1-h)+h(1+\phi^2)]-\mu(1-t)h(1-\phi^2)\}}{2H[\sigma-(1-t)\mu]\{h(1-h)[\sigma-(1-t)\mu]+\sigma\phi[1-2h(1-h)]+h(1-h)[\sigma+(1-t)\mu]\phi^2\}}. \end{aligned} \tag{13}$$

It is noteworthy that Equation (13) is reduced to Equation (16) of Forslid and Ottaviano (2003) when $t=0$. In our model, the income tax reduces the consumption market of manufactured goods and then decreases the wage rate of skilled labor.

III. *The Benchmark Case: A Pure Local Public Good (PLPG)*

This section considers a pure local public good (PLPG): $\alpha=0$ in (2). Equilibrium is obtained by the utility indifference of skilled labor between two regions. According to the indirect utility function of (6), we express the utility differential of skilled labor between two regions as follows:

$$\Delta V\equiv V_i-V_j=\frac{(1-t)w_{Hi}}{(\mathbb{P}_i)^{\mu}}\cdot(G_i)^{\gamma}-\frac{(1-t)w_{Hj}}{(\mathbb{P}_j)^{\mu}}\cdot(G_j)^{\gamma}. \tag{14}$$

Substituting (7), (11), and (13) into (14), we analyze the long-run equilibria and location patterns of the whole system.

In the PLPG case, we first analyze how taxation affects the relative level of the wage rate between two regions. To do this, differentiating both $w_{iH}-w_{jH}$ and $w_{iH}/(\mathbb{P}_i)^{\mu}-w_{jH}/(\mathbb{P}_j)^{\mu}$ with respect to h and taking value at $h=1/2$, we obtain two thresholds of trade freeness ϕ_w^{Pure} and $\phi_\omega^{\text{Pure}}$, above which the larger region offers a higher nominal and real wage rate, respectively, where

$$\phi_w^{\text{Pure}}\equiv\frac{\sigma-(1-t)\mu}{\sigma+(1-t)\mu}\in(0,\,1) \quad ; \quad \phi_\omega^{\text{Pure}}\equiv\phi_w^{\text{Pure}}\cdot\frac{\sigma-1-\mu}{\sigma-1+\mu}\in(0,\,1).$$

It holds that $\phi_w^{\text{Pure}}>\phi_w^{\text{FE}}$, $\phi_\omega^{\text{Pure}}>\phi_\omega^{\text{FE}}$ and $\phi_w^{\text{Pure}}>\phi_\omega^{\text{Pure}}$. When $t=0$, ϕ_w^{Pure} and $\phi_\omega^{\text{Pure}}$ degenerate to the threshold values (18) and (26) in the FE model. Note that both ϕ_w^{Pure} and $\phi_\omega^{\text{Pure}}$ increase in t, showing a dispersion force of taxation. It is because tax reduces the local expenditure, or the market size. On the other hand, using (2), (7), and $\alpha=0$, it is clear that the services of PLPG form a new agglomeration force due to the following:[9]

[9] Note that

$$\frac{\partial}{\partial h}(G_i^{\text{Pure}}-G_j^{\text{Pure}})=\frac{t\mu\sigma\phi(1-H)(1-t)}{[\sigma-(1-t)\mu]\{h(1-h)[\sigma-(1-t)\mu]+\sigma\phi[1-2h(1-h)]+h(1-h)\phi^2[\sigma+(1-t)\mu]\}^2}\cdot Z,$$

$$\frac{\partial}{\partial h}(G_i^{\text{Pure}}-G_j^{\text{Pure}})=\frac{\partial}{\partial h}[t(Y_i-Y_j)]>0, \quad \forall\ t, \phi, h.$$

Obviously, the relative level of the PLPG service depends upon the relative level of regional incomes due to the symmetric tax rate regime. Residents in the larger region enjoy a better PLPG service than those in the smaller one for any tax rate and trade freeness; however, they may face a reduced wage rate by agglomeration when trade freeness is low. In particular, the relative real wage rate between two regions monotonically increases in h when $\phi>\phi_\omega^{\text{Pure}}$, which results from a standard study of the real wage rate differential between two regions (see Appendix A, Part 1). Summarizing the above results, in the PLPG case, $\Delta V>0$ is true for any tax rate $0<t<1$ when $h>1/2$ and $\phi>\phi_\omega^{\text{Pure}}$ hold.

Although income tax reduces the market size, the long-run configuration still undergoes a "pitchfork bifurcation," as in the FE model. Evidently, when ϕ is sufficiently large (before reaching $\phi_\omega^{\text{Pure}}$), the utility differential between two regions is monotonically increasing at agglomeration level h due to the additional agglomeration force of the PLPG.

As a result, full agglomeration is the unique stable configuration when ϕ is large enough, including the ultimate state $\phi=1$. This configuration of the ultimate state is different from that of the FE model because any spatial distribution is possible there when $\phi=1$. Intuitively, the PLPG specifies the final configuration to full agglomeration when the trade costs are sufficiently low, since the market (size/crowding) effect only forms a small impact on the wage rate. Finally, mobile workers choose to access the region with a higher level of PLPG services. Note that this result holds for any tax rate $t\in(0, 1)$ as long as the PLPG exists.

Next, we consider an important question whether the agglomeration force of PLPGs can be strong enough to change the stable equilibrium from dispersion to agglomeration for a very low trade freeness level, so that the dispersion equilibrium is stable forever (i.e., the system falls into a "dispersion black hole"). By the inspection in Appendix A (Part 2), when $\phi=0$, the symmetry distribution is a unique stable equilibrium if and only if $\sigma>1+\mu$; the full agglomeration is the unique stable equilibrium if and only if $\sigma<1+\mu$. They are shown in Figure 1, where solid lines are stable equilibria and heavy dashed lines are unstable ones.

The following analysis completes the discussion of Figure 1. On the one hand, the "sustain point" and "break point" further clarify the meaning of threshold $\sigma=1+\mu$ in the PLPG case. Since corner equilibrium $h=1$ is stable iff $\Delta V|_{\alpha=0,\,h=1}\geq 0$, the "sustain point" ϕ_S^{Pure} is determined by

$$2\phi\left[\frac{\sigma+(1-t)\mu}{\sigma-(1-t)\mu}\right]^{\tau}-\phi^{\frac{\mu}{\sigma-1}}\left\{1-(1-t)\frac{\mu}{\sigma}+\phi^2\left[1+(1-t)\frac{\mu}{\sigma}\right]\right\}=0. \tag{15}$$

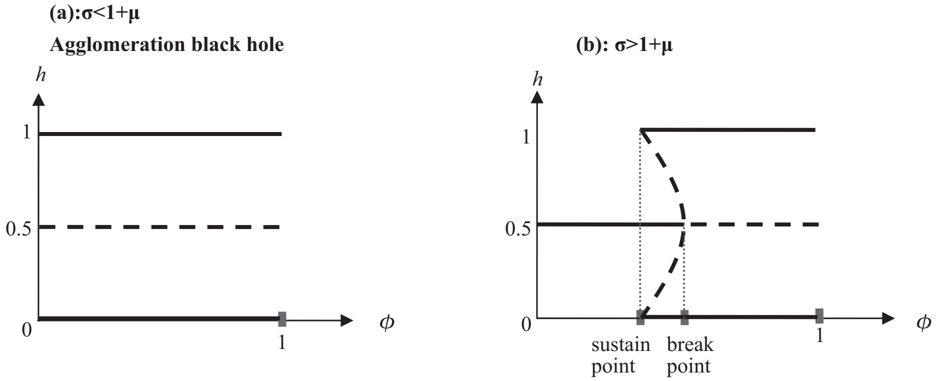
By inspection of the LHS of (15), full agglomeration is always a stable equilibrium when $\sigma<1+\mu$ (see Appendix B). On the other hand, a "break point" is a root in $[0, 1]$ of the following equation

$$\left.\frac{\partial\,\Delta V}{\partial\, h}\right|_{\alpha=0,\,h=\frac{1}{2}}=0, \tag{16}$$

where $Z=\sigma[1+\phi^2-2h(1-h)(\phi-1)^2]-\mu(1-t)(1-\phi^2)[1-2h(1-h)]$.

Therefore, $\frac{\partial}{\partial h}(G_i^{\text{Pure}}-G_j^{\text{Pure}})>0$ holds because $\frac{\partial}{\partial\phi}Z>0$ and $Z|_{\phi=0}=[1-(1-t)\mu][1-2h(1-h)]>0$.

FIG. 1. THRESHOLD $\sigma=1+\mu$ OF THE PLPG CASE



which can be rewritten as $a_1\phi^2+b_1\phi+c_1=0$, where

$$\begin{aligned} a_1&=[\sigma+(1-t)\mu](\sigma-1+\mu)>0,\\ b_1&=-2\{\mu(1-t)[2\gamma(\sigma-1)+\mu]+\sigma(\sigma-1)\}<0,\\ c_1&=[\sigma-(1-t)\mu](\sigma-1-\mu). \end{aligned} \tag{17}$$

According to (17), $a_1>0$ and $a_1+b_1+c_1=-4\gamma\mu(1-t)(\sigma-1)<0$ are true, then Equation (16) has only one root in $(-\infty, 1)$[10]. Therefore, the break point is max $\{0, \underline{\phi_B^{\text{Pure}}}\}$, where

$$\begin{aligned} \underline{\phi_B^{\text{Pure}}}&=\frac{-b_1-\sqrt{\Delta_1}}{2a_1},\\ \Delta_1&=(b_1)^2-4a_1c_1>0.^{11} \end{aligned} \tag{18}$$

According to (17) and (18), $\underline{\phi_B^{\text{Pure}}}<0$ if and only if c_1 is negative, which in turn is equivalent to $\sigma<1+\mu$. This reveals that there is no stable dispersion equilibrium when $\sigma<1+\mu$; meanwhile, the system falls into the so-called "black hole" which comes from the fact that full agglomeration equilibria exist in any trade freeness by inspection of the LHS of (15). Hence, the no-black-hole condition in the PLPG case is consistent with the FE model. On the other hand, $\underline{\phi_B^{\text{Pure}}}\in(0, 1)$ holds if $\sigma>1+\mu$.

To sum up, in the PLPG case, there are only two types of agglomeration patterns, $\underline{\phi_B^{\text{Pure}}}<0$ and $\underline{\phi_B^{\text{Pure}}}\in(0, 1)$, which display similar structures of the FE model.

Two remarks follow. First, the result that ultimate state ($\phi=1$) must be a full agglomeration is different from that in the FE model. This is because the service level of the PLPG increases in agglomeration level h. It turns out that the PLPG effect dominates the geographic configuration when trade costs are sufficiently small, since the market distribution is

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[10] Note that the other solution of $a_1\phi^2+b_1\phi+c_1=0$, $\dfrac{-b_1+\sqrt{\Delta_1}}{2a_1}$, is always larger than 1 because $a_1+b_1+c_1<0$.

[11] $\Delta_1=4\mu^2\sigma^2+4(1-t)\mu(\sigma-1)\{4(1-t)\mu\gamma[\mu+\gamma(\sigma-1)]+4\gamma\sigma(\sigma-1)+\mu[(1-t)(\sigma-1)+2\sigma]\}>0$.

no longer important for the market size and the market crowding effect (i.e., access to other firms and consumers is very easy). Note that this result does not depend upon the tax rate. Second, the no-black-hole condition of the PLPG case, $\sigma > 1 + \mu$, is also independent of the tax rate. It is clear that the only reason for the "black hole" in the PLPG case is the so-called "cost-of-living effect" (Forslid and Ottaviano, 2003; p.235), so the result is the same as that in the FE model. In other words, without the cost-of-living effect, the dispersion equilibrium is always stable in any tax rate and service level of PLPG when trade costs are sufficiently large. Proposition 1 summarizes those important features comparing the PLPG case with the FE model.

Proposition 1. *In the PLPG case,* (i) *the bifurcation diagrams have the same qualitative structure with the FE model;* (ii) *the no-black-hole condition is quantitatively identical with the FE model;* (iii) *full agglomeration is the only stable equilibrium in the ultimate state (ϕ=1).*

Evidently, this benchmark case also shows that mobile workers ignore the effect of the tax rate, which reduces the wage rate and/or the PLPG scale, when agglomeration takes place (Proposition 1, (iii)). However, we will show later (Section 4) that the above results may not hold in the general case of a congestible LPG. Proposition 1 (i) implies that qualitative results of PLPG case are "independent" of the tax rate. Nevertheless, "quantitative" results are different since the sustain and the break point formulas clearly depend on the tax rate.

The tax has an impact on the evolving equilibrium path with respect to trade freeness ϕ. To see the details, we compare the break point of our model with that of the FE model. Note that, in our framework, the income taxation (t) has a negative impact on the agglomeration force due to the reduced market. Besides, the preference (γ) of the PLPG forms an agglomeration force. As a result of their interaction, the break point ($\underline{\phi_B^{\text{Pure}}}$) may be either larger or smaller than the break point ϕ_B^{FE} of the FE model depending on parameters σ, μ, t and γ. We summarize the relationship between $\underline{\phi_B^{\text{Pure}}}$ and ϕ_B^{FE} in the following proposition.

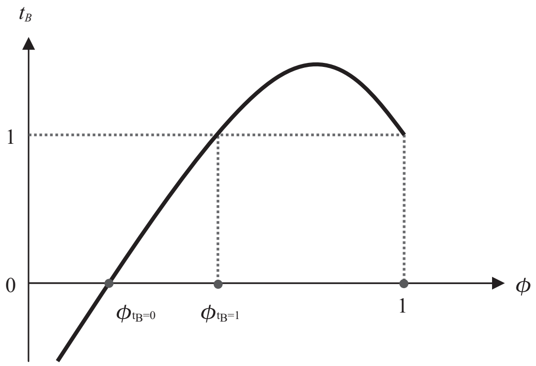
Proposition 2. (i) *When $\sigma > 1 + \mu$, both ϕ_B^{FE} and $\underline{\phi_B^{\text{Pure}}}$ belong to* (0, 1).
(ii) *For any $0 < \gamma < 1$ and $0 < \mu < 1$, $\underline{\phi_B^{\text{Pure}}} < \phi_B^{\text{FE}}$ holds iff $t < t_C$, where*

$$t_C \equiv \frac{\gamma(\sigma-1)(\sigma^2-\mu^2)}{\mu\sigma(2\sigma-1)+\gamma(\sigma-1)(\sigma^2-\mu^2)} \in (0,\ 1)$$

Proof: See Appendix B.

Proposition 2 complements Andersson and Forslid (2003) in the following two respects. Firstly, our analytical result shows that a low tax rate results in $\underline{\phi_B^{\text{Pure}}} < \phi_B^{\text{FE}}$, whereas a high tax rate results in $\underline{\phi_B^{\text{Pure}}} > \phi_B^{\text{FE}}$. Notation t_C is the threshold level of tax rate at which the agglomeration force of PLPG is just equal to its dispersion force of reducing the market size. Note that t_C increases in the LPG preference parameter γ. Specifically, for a higher γ, skilled workers are more likely to agglomerate in a region to enjoy the better LPG service, and save interregional trade costs, while tolerating a higher tax rate. Therefore, a higher tax rate induces a faster agglomeration with respect to trade freeness ϕ than the FE model. The novel results of Propositions 1 and 2 provide a policy insight: an applicable tax rate and its corresponding level of PLPG really enhance an agglomeration path in the FE framework, even the tax reduces the

FIG. 2. THE RELATIONSHIP BETWEEN t_B AND TRADE FREENESS ϕ



wage rate and the market size. In contrast, $\underline{\phi_B^{\mathrm{Pure}}}<\phi_B^{\mathrm{FE}}$ is always true in the numerical examples of Andersson and Forslid (2003). This is because their wage rate is independent of the tax rate. Then the tax revenue financing the LPG enhances the agglomeration force but there is no new counterpart of the dispersion force.

Secondly, their numerical example shows that skilled workers agglomerate in one region when the tax rate is high and disperse when the tax rate is low. We are able to calculate the "breakpoint in tax" in our model:

$$t_B=\frac{(1-\phi)^2[\mu^2+\sigma(\sigma-1)]-\mu[\phi^2-4\gamma\phi-1+2\sigma(1+2\gamma\phi-\phi^2)]}{\mu[\mu(1-\phi)^2+(\sigma-1)(\phi^2-4\gamma\phi-1)]}.$$

The above t_B depends on the trade freeness ϕ, whose relationship is illustrated in Figure 2, where

$$\phi_{t_B=0}=\frac{\mu^2+(2\gamma\mu+\sigma)(\sigma-1)-\sqrt{\mu\{\mu[(2\sigma-1)^2+4\gamma^2(\sigma-1)^2]+4\gamma(\sigma-1)[\mu^2+\sigma(\sigma-1)]\}}}{(\sigma+\mu)(\sigma-1+\mu)}$$

and

$$\phi_{t_B=1}=\frac{\sigma-1-\mu}{\sigma-1+\mu}.$$

Three facts are observed in Figure 2. (i) Any tax rate $0<t<1$ cannot induce an agglomeration pattern when $\phi<\phi_{t_B=0}$. This implies that the central government has an incentive to induce an agglomeration pattern by subsidization. (ii) Any tax rate $0<t<1$ induces an agglomeration pattern when $\phi>\phi_{t_B=1}$. This results from the fact that workers in the larger region always obtain a higher real wage and a better LPG service due to low trade costs ($\phi_{t_B=1}>\phi_{\omega}^{\mathrm{Pure}}$). In this phase, the central government has an incentive to impose tax on firms to maximize the welfare of local residents. (iii) When $\phi_{t_B=0}<\phi<\phi_{t_B=1}$, dispersion is an unstable (resp. a stable) if the tax rate is lower (resp. higher) than t_B, which is opposite to the pattern obtained in Figure 3 of Andersson and Forslid (2003). This result can be attributed to two negative effects of a high tax rate. On the one hand, a high tax rate reduces the market size so that the larger region may not provide a higher wage rate (the market size effect cannot prevail over the market crowded effect). On the other hand, the relative level of the LPG service in the larger region is decreasing in its tax rate.[12] Namely, the larger region loses the comparative advantage of LPG gradually.[13] Hence, when the trade freeness (ϕ) is intermediate, agglomeration is observed for low tax rate and it changes to dispersion for high tax rate.

IV. *A Congestible Local Public Good (CLPG)*

Almost all local public services are subject to some degree of *congestion*. This section investigates how the "usage congestion" of a congestible local public good (CLPG) impacts on the location patterns of firms. Interestingly, both the economy scale and the congestion of the LPG are related to the endogenous number of skilled workers. The tax rate becomes important even if it is symmetric between two regions, since it has an impact on the market size of the economy and the level of public good services.

In the following, we first examine the symmetric equilibrium by the concepts of break point and sustain point. We will see how the usage congestion of the LPG and the tax rate affect the values of the break point and the re-dispersion point. We then identify stable patterns when the trade costs are zero ($\phi=1$, the ultimate state) or low enough by use of the break point. We also examine the spatial configuration when the trade costs approach infinity ($\phi=0$, the autarky state).

1. The Break Point and the Re-dispersion Point

We first examine the stability of the symmetry equilibrium with respect to ϕ in the CLPG case. Equation

$$\left.\frac{\partial\,\Delta V}{\partial\, h}\right|_{h=\frac{1}{2}}=0$$

can be simplified as $a_2\phi^2+b_2\phi+c_2=0$ which may have two economically meaningful roots, where

$$\begin{aligned}
&a_2=[\sigma+(1-t)\mu][(\sigma-1+\mu)+\alpha H\gamma(\sigma-1)]>0,\\
&b_2=2\{(1-t)\mu[2\gamma(1-\sigma)-\mu]+\sigma H(1-\alpha\gamma)(1-\sigma)+\sigma(1-H)(1-\sigma)\}<0,\\
&c_2=[\sigma-(1-t)\mu][(\sigma-1-\mu)+\alpha H\gamma(\sigma-1)],\\
&\Delta_2=(b_2)^2-4a_2c_2.^{14}
\end{aligned}\tag{19}$$

The quadratic equation may have two economically meaningful roots

$$\underline{\phi_B^{\text{Cong}}}=\frac{-b_2-\sqrt{\Delta_2}}{2a_2}\quad;\quad\overline{\phi_B^{\text{Cong}}}=\frac{-b_2+\sqrt{\Delta_2}}{2a_2},\tag{20}$$

[12] $\frac{\partial}{\partial t}\left(\frac{G_1^{\text{Pure}}}{G_2^{\text{Pure}}}\right)=\frac{2\sigma\mu\phi(2h-1)[h(\phi-1)-\phi]}{[1+h(\phi-1)]\{\phi[\sigma+\mu(1-t)]+h[\sigma(1-\phi)-\mu(1-t)(1+\phi)]\}^2}<0,\quad\forall\ \phi,\ \frac{1}{2}<h<1.$

[13] On the contrary, in Andersson and Forslid (2003), since tax rate is independent of market size and wage rate, the relative level of LPG service between two regions is increasing in tax rate.

[14] $\Delta_2=\Delta_1-4\alpha\gamma(\sigma-1)H\{(1-t)^2\mu^2[2-\alpha\gamma(\sigma-1)H]+2t(1-t)\mu^2\sigma+4(1-t)\mu(\sigma-1)\sigma\gamma+4(\sigma-1)\sigma^2\}.$

which are the threshold values of the trade freeness for the dispersion equilibrium to become unstable and stable again. If Δ_2 is negative, then neither $\underline{\phi_B^{\mathrm{Cong}}}$ nor $\overline{\phi_B^{\mathrm{Cong}}}$ exists, and the symmetry is always stable. On the other hand, if Δ_2 is positive, it holds that $\underline{\phi_B^{\mathrm{Cong}}}<\overline{\phi_B^{\mathrm{Cong}}}$. When ϕ is lower than $\underline{\phi_B^{\mathrm{Cong}}}$ or larger than $\overline{\phi_B^{\mathrm{Cong}}}$, $h=1/2$ is stable; between them, the symmetry is unstable. Therefore, min $\{1, \max\{0, \underline{\phi_B^{\mathrm{Cong}}}\}\}$ is the "break point," and min $\{1, \max\{0, \overline{\phi_B^{\mathrm{Cong}}}\}\}$ is the "re-dispersion point."[15]

Obviously, when $\overline{\phi_B^{\mathrm{Cong}}}$ is larger than 1, there is no re-dispersion, and agglomeration occurs at $\phi=1$, which is similar to the PLPG case. On the other hand, when $\overline{\phi_B^{\mathrm{Cong}}}$ is smaller than 1, we have a re-dispersion process. Precisely, $\overline{\phi_B^{\mathrm{Cong}}}<1$ holds if and only if $a_2+b_2+c_2=4\gamma(\sigma-1)[\alpha H\sigma-(1-t)\mu]>0$ where $\lambda\equiv\alpha H\sigma-(1-t)\mu$. Thus, the condition for re-dispersion is written as

$$\alpha H>\frac{(1-t)\mu}{\sigma}\in(0,\ 1). \tag{21}$$

To examine (21), we calculate the partial derivatives with respect to all parameters:

$$\frac{\partial\lambda}{\partial t}>0,\quad \frac{\partial\lambda}{\partial\alpha}>0,\quad \frac{\partial\lambda}{\partial H}>0,\quad \frac{\partial\lambda}{\partial\sigma}>0,\quad \frac{\partial\lambda}{\partial\mu}<0. \tag{22}$$

Inequalities of (22) tell us that the re-dispersion point decreases in t, α, H and σ and increases in μ. Therefore, $\overline{\phi_B^{\mathrm{Cong}}}$ comes earlier for a larger t, α, H and σ and a lower μ.

Furthermore, we explore some general properties of the break point and the re-dispersion point in the CLPG case.

The following inequalities show the relationship with respect to the tax rate t, the LPG congestability α, and the LPG taste γ:

$$\frac{\partial\,\underline{\phi_B^{\mathrm{Cong}}}}{\partial\,t}>0,\quad \frac{\partial\,\underline{\phi_B^{\mathrm{Cong}}}}{\partial\,\alpha}>0,\quad \frac{\partial\,\overline{\phi_B^{\mathrm{Cong}}}}{\partial\,t}<0,\quad \frac{\partial\,\overline{\phi_B^{\mathrm{Cong}}}}{\partial\,\alpha}<0, \tag{23}$$

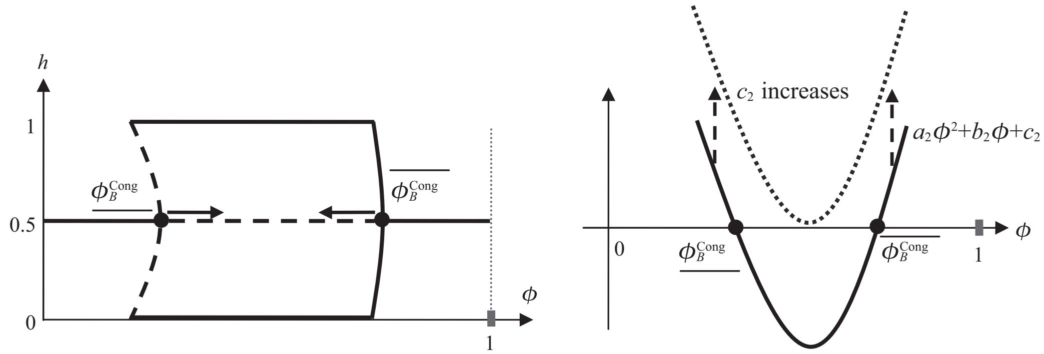
whose proof is relegated to Appendix C. As illustrated in the LHS panel of Figure 3, inequalities of (23) reveal that the stable symmetry equilibrium breaks later and the re-dispersion occurs earlier when the tax rate t and the congestability α are higher. Intuitively, taking account of the reduced market size and the LPG congestion, the agglomeration pattern is less likely to emerge and is more difficult to sustain.

Unfortunately, the impact of the taste parameter γ on $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ is indeterminate. The intuition is straightforward. For a larger taste γ, residents prefer more LPG service but dislike the increased congestion at the same time. The total effect is indeterminate.

The dispersion equilibrium is stable for any trade freeness as long as the break point and the re-dispersion point coincide or do not exist (i.e., $\Delta_2\leq 0$). The stable dispersion equilibrium never breaks in this situation. This happens when c_2 is large, as illustrated in the right-hand

[15] The analyses about the existence of the break point and the re-dispersion point are showed in Appendix F. More details are available from the authors upon request.

FIG. 3. MOVEMENT OF THE BREAK POINT AND THE RE-DISPERSION POINT



panel of Figure 3. As shown in (24) below, c_2 increases when the centripetal force is relatively weak (i.e. when μ is low and/or σ is high) and the centrifugal force is relatively strong (i.e., when t, α, γ, and H are high):

$$\frac{\partial c_2}{\partial t}>0,\ \frac{\partial c_2}{\partial \alpha}>0,\ \frac{\partial c_2}{\partial \gamma}>0,\ \frac{\partial c_2}{\partial H}>0,\ \frac{\partial c_2}{\partial \sigma}>0,\ \frac{\partial c_2}{\partial \mu}<0. \tag{24}$$

2. The Ultimate State ($\phi=1$)

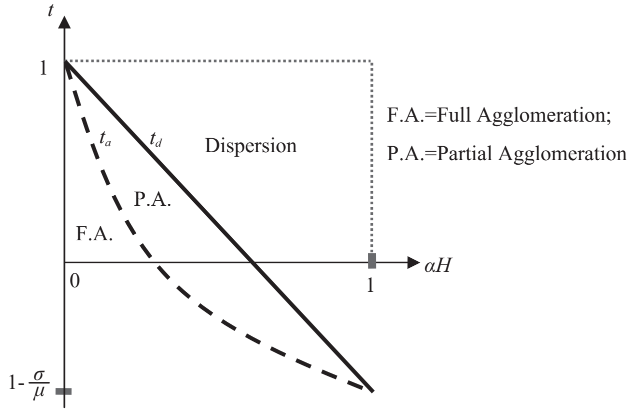
The following two equations (25) and (26) are used to examine whether the symmetry and the full agglomeration are stable:

$$\left.\frac{\partial \Delta V}{\partial h}\right|_{\phi=1,\ h=\frac{1}{2}}=\eta_1\cdot[(1-t)\mu-H\alpha\sigma], \tag{25}$$

$$\Delta V|_{\phi=1,\ h=1}=\eta_2\cdot\left\{(1+H)^{-\alpha\gamma}\left[\frac{\sigma+(1-t)\mu}{\sigma-(1-t)\mu}\right]^{\gamma}-(1-H)^{-\alpha\gamma}\right\}, \tag{26}$$

where positive coefficients η_1 and η_2 are listed in Appendix D. By (25), the condition for the symmetric equilibrium to be stable when $\phi=1$ is identified as the same as (21). The LHS of (21) αH measures a new dispersion force from the usage congestion of the LPG. If the congestion is large (i.e., high congestability of the LPG and/or a larger number of skilled workers), the symmetry distribution becomes a stable equilibrium when $\phi=1$. On the other hand, the RHS of (21) $(1-t)\mu/\sigma$ can be regarded as an index measuring the agglomeration force from the interaction between the reduced manufacturing market size $(1-t)\mu$ and monopoly power σ of the differentiated goods. If it is small, the location pattern changes from agglomeration to dispersion due to the LPG congestion. This means that, if the economy has high t, α, and σ or a low μ, the CLPG could not sustain a full agglomeration to the end ($\phi=1$). The intuition is straightforward: The new agglomeration force of the LPG is discounted by the LPG congestion. Furthermore, the agglomeration force is small for low trade costs. Comparatively, Proposition 1 says that full agglomeration is the unique stable equilibrium when $\phi=1$ in the PLPG case, so we know that the LPG congestion is critical for the location of

FIG. 4. ULTIMATE SPATIAL PATTERNS WITH RESPECT TO t_a AND t_d



economic activity.

To examine the role of the tax rate, we define two critical values of tax rate as follows:

$$t_a \equiv 1-\frac{\sigma}{\mu}\times\frac{(1+H)^{\alpha}-(1-H)^{\alpha}}{(1+H)^{\alpha}+(1-H)^{\alpha}}\in(-\infty,\ 1)\ \ ;\ \ t_d\equiv 1-\frac{\alpha H\sigma}{\mu}\in(-\infty,\ 1) \tag{27}$$

According to (25) and (26), when $\phi=1$, t_a is the threshold value of the tax rate below which the full agglomeration is stable, and t_d is the threshold value of the tax rate above which the symmetric equilibrium is stable. We can prove[16] that t_a is always smaller than t_d when $\alpha\in(0,\ 1)$.

Figure 4 illustrates how the ultimate spatial pattern is related to various parameters. The plane of $\alpha H\in(0,\ 1)$ and $t\in(0,\ 1)$ is divided into three parts by t_a and t_d. First, when αH is high and $t>t_d$, (21) is satisfied, and $\overline{\phi_B^{\mathrm{Cong}}}<1$. The ultimate spatial pattern is the symmetric dispersion. Second, when αH is intermediate and t lies between t_a and t_d, a partial agglomeration (an interior asymmetric equilibrium) is the ultimate spatial pattern. Third, when αH is low and $t<t_a$, the full agglomeration emerges. We can see that, if the economy is of a low monopoly power or consumers are not lovers of differentiated goods, the presence of "congestion" and "taxation" could easily transform the spatial pattern from agglomeration into dispersion for low trade costs.

The bifurcation of industrial location with respect to the tax rate is shown in Figure 5 and summarized in Proposition 3.

Proposition 3. *When $\phi=1$, the spatial pattern exhibits a pitchfork bifurcation with respect to t.*

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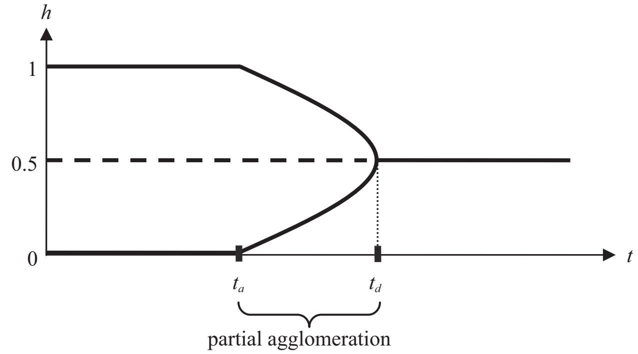
[16] Note that t_d is linear in α, while t_a is a convex function of α since

$$\frac{\partial^2 t_a}{\partial\alpha^2}=\frac{2\sigma(1-H^2)^{\alpha}[(1+H)^{\alpha}-(1-H)^{\alpha}]\{\ln[(1-H)/(1+H)]\}^2}{\mu[(1-H)^{\alpha}+(1+H)^{\alpha}]^3}>0.$$

Then, the conclusion immediately follows from $t_d|_{\alpha=0}=t_a|_{\alpha=0}=1$ and $t_d|_{\alpha=1}=t_a|_{\alpha=1}=1-(\sigma/\mu)H$.

FIG. 5. PITCHFORK BIFURCATION WITH RESPECT TO t WHEN $\phi=1$



Proof: See Appendix E.

Unlike the PLPG case, tax rate t does matter in the structure of spatial pattern due to the LPG congestion. Full agglomeration is sustainable only when congestion and tax rate are low. Otherwise, both partial agglomeration and symmetry are possible. It is worth stressing that partial agglomeration is also possible in the ultimate state. To the best of our knowledge, this phenomenon has not been reported elsewhere. Our result provides a credible explanation of the diverse landscape of economic geography in reality.

There are two reasons for the fact that full agglomeration loses stability for a high trade freeness. First, the tax revenue reduces the market size of manufactured goods, and it then has an impact on the wage rate of skilled labor (w_H) and the level of the regional aggregated income (Y). Since the tax rate is symmetric between two regions, skilled labor does not care about the tax rate directly. However, the relative level of interaction between the wage rate and the regional income ($w_H \times Y^{\gamma}$) depends on the tax rate (t). Second, the LPG congestion provides another new centrifugal force in the system. When trade freeness increases sufficiently, the centripetal force from pecuniary externalities associated with the manufactured market is no longer strong, since access to another region's market is very easy. Meanwhile, once the benefit of a higher regional income fails to defeat the costs of congestion, skilled labor would rather choose "dispersion" to avoid the crowding region.

3. The Autarky State ($\phi=0$)

This section focuses on the autarky case. We check the stability of $h=1/2$ and $h=1$ by the following equations:

$$\left.\frac{\partial \Delta V}{\partial h}\right|_{\phi=0,\, h=\frac{1}{2}} = \eta_3 \cdot [\mu-(\sigma-1)(1+\alpha H\gamma)], \tag{28}$$

$$\Delta V|_{\phi=0} = \eta_4 \cdot \left\{ \frac{h^{\frac{1+\mu-\sigma}{\sigma-1}}}{(2Hh+1-H)^{\alpha\gamma}} - \frac{(1-h)^{\frac{1+\mu-\sigma}{\sigma-1}}}{[2H(1-h)+1-H]^{\alpha\gamma}} \right\}, \tag{29}$$

with positive coefficients η_3 and η_4 listed in Appendix D. Obviously, the stability of $h=1/2$ and

$h=1$ is independent of the tax rate in the autarky case.

According to (28), the condition for the symmetric equilibrium to be stable when $\phi=0$ is identified as:

$$\alpha H\gamma > \frac{1+\mu-\sigma}{\sigma-1}. \tag{30}$$

Equation (30) implies that the value of $\alpha H\gamma$ determines the spatial pattern in the autarky state when $\sigma<1+\mu$. The no-black-hole condition $\sigma>1+\mu$ of the PLPG case does not hold, since the new centrifugal force from congestion works. Note that this distinctive feature does not depend upon the tax rate because market size of the manufacturing sector is inessential in the autarky state.

Representing the preference of the LPG, parameter γ also measures the congestion degree of the LPG to the utility level. If consumers care more about the LPG (i.e., a higher γ), the LPG congestion is more sensitive (i.e., $\alpha\gamma$ is high). Hence, $\alpha H\gamma$ represents the strength of the new centrifugal force.

To have a deeper view, we examine $\underline{\phi_B^{\text{Cong}}}$ and $\overline{\phi_B^{\text{Cong}}}$ of the CLPG case by (20). The sign of term c_2 is indefinite. When $\alpha H\gamma<(1+\mu-\sigma)/(\sigma-1)$, c_2 is negative, and Δ_2 is positive. Meanwhile, $\underline{\phi_B^{\text{Cong}}}$ is negative, and $\overline{\phi_B^{\text{Cong}}}$ is either between 0~1 or larger than 1 depending on the tax rate. On the other hand, when $\alpha H\gamma>(1+\mu-\sigma)/(\sigma-1)$, then $c_2>0$ holds, and both $\underline{\phi_B^{\text{Cong}}}$ and $\overline{\phi_B^{\text{Cong}}}$ are positive if they exist. Therefore, we know that, when condition (30) is met, $\underline{\phi_B^{\text{Cong}}}$ might be positive, and the symmetry equilibrium is stable in an economy of high trade costs.

To be precise, we define two critical values, σ_S and σ_B, as follows

$$\sigma_B \equiv 1+\frac{\mu}{\alpha H\gamma+1} \quad ; \quad \sigma_S \equiv 1+\mu. \tag{31}$$

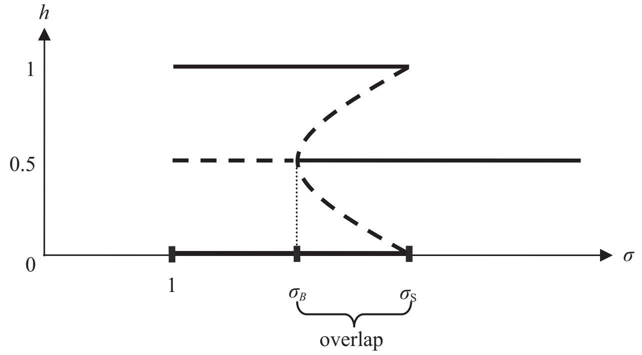
According to (28) and (29), σ_B is the threshold value above which the symmetric equilibrium is stable, and σ_S is the threshold value below which the full agglomeration is stable. Obviously, $\sigma_B<\sigma_S$ holds. When σ lies between the two thresholds, we observe a new overlap of stable full agglomeration and dispersion when ϕ is small. This observation is new since such an overlap only occurs at intermediate level of ϕ from dispersion to agglomeration in the existing literature. Indeed, this new overlap does not appear in PLPG case (note that $\sigma_S=\sigma_B$ when $\alpha=0$).

The bifurcation of industrial location with respect to σ is shown in Figure 6 and summarized in Proposition 4.

Proposition 4. *When the trade costs (interregional transportation costs) are sufficiently large, the initial spatial pattern exhibits a tomahawk bifurcation with respect to σ.*

Proof: See Appendix E.

FIG. 6. TOMAHAWK BIFURCATION OF h WITH RESPECT TO σ FOR $\phi=0$



V. *Concluding Remarks*

This paper presents a simple, analytically solvable, new economic geography model to analyze the effects of LPGs on the industrial location. We add new features to the existing literature in several respects. First, a proportional income tax is imposed on both skilled and unskilled workers to finance the LPG in two regions with symmetric tax rates. Second, the tax revenue reduces the disposable income level of workers and the market size of private goods. Third, the LPG is "congestible" depending on the endogenous scale of local user/resident and an exogenous congestability parameter. In our framework, the benchmark case of PLPG displays a similar location pattern to the previous NEG taxation model, although the tax reduces the market size of manufactured goods. The interaction between the tax rate and the LPG taste makes the appearance of the break point different from that in the FE model. Furthermore, in the CLPG case, both the symmetry and the full agglomeration equilibria could emerge from the autarky state due to the presence of congestability. In addition, the interaction between the tax rate and the LPG congestion determines the spatial pattern of the ultimate state (transportation costs are zero). This is because the economy scale and the corresponding LPG service may not counteract the endogenous congestion for an agglomeration in a high tax rate. Finally, using this fully analytic framework, we have shown that the break point (re-dispersion point) increases (decreases) in tax rate t and the LPG congestability. In particular, it is possible that the dispersion equilibrium is always stable for any level of trade freeness as long as the break point and the re-dispersion point coincide or do not exist. This distinctive feature also stems from the fact of inherent congestability of the LPG. Thereby, we are able to derive a richer menu of normative results.

Decentralization of governance is a societal trend nowadays. In most developed countries, local governments are able to assign tax rates by themselves. Therefore, it is important to explore the endogenous tax rates and tax competition for future research.

Appendix A

Part 1: The relative real wage rate between two regions monotonically increases in h when $\phi > \phi_{\omega}^{\text{Pure}}$. This is due to the following standard study of the real wage rate differential between two regions:

$$\left.\frac{w_{iH}}{(\mathbb{P}_i)^{\mu}} - \frac{w_{jH}}{(\mathbb{P}_j)^{\mu}}\right|_{h=\frac{1}{2}} = 0, \quad \forall \phi,\ t;$$

$$\left.\frac{\partial}{\partial h}\left[\frac{w_{iH}}{(\mathbb{P}_i)^{\mu}} - \frac{w_{jH}}{(\mathbb{P}_j)^{\mu}}\right]\right|_{h=\frac{1}{2},\ \phi=\phi_{\omega}^{\text{Pure}}} = 0, \quad \left.\frac{\partial^2}{\partial h\, \partial \phi}\left[\frac{w_{iH}}{(\mathbb{P}_i)^{\mu}} - \frac{w_{jH}}{(\mathbb{P}_j)^{\mu}}\right]\right|_{h=\frac{1}{2},\ \phi=\phi_{\omega}^{\text{Pure}}} > 0, \quad \forall t;$$

$$\left.\frac{\partial^2}{\partial h^2}\left[\frac{w_{iH}}{(\mathbb{P}_i)^{\mu}} - \frac{w_{jH}}{(\mathbb{P}_j)^{\mu}}\right]\right|_{h=\frac{1}{2},\ \phi=\phi_{\omega}^{\text{Pure}}} = 0, \quad \left.\frac{\partial^3}{\partial h^3}\left[\frac{w_{iH}}{(\mathbb{P}_i)^{\mu}} - \frac{w_{jH}}{(\mathbb{P}_j)^{\mu}}\right]\right|_{h=\frac{1}{2},\ \phi=\phi_{\omega}^{\text{Pure}}} > 0, \quad \forall t.$$

Part 2: We examine the following equations checking the stability of the dispersion ($h=1/2$) and the full agglomeration ($h=1$ or 0) equilibria at $\phi=0$, respectively:

$$\left.\frac{\partial \Delta V}{\partial h}\right|_{\alpha=0,\ \phi=0,\ h=\frac{1}{2}} = \frac{2^{\frac{\mu(\gamma-2)}{\sigma-1}}(1-t)^2\mu}{H^{\frac{\sigma-1-\mu}{\sigma-1}}\sigma(\sigma-1)t} \cdot \left[\frac{(1-H)t\sigma}{\sigma-(1-t)\mu}\right]^{1+\gamma} \cdot (1+\mu-\sigma), \tag{A1}$$

$$\Delta V|_{\alpha=0,\ \phi=0} = \frac{\mu(1-t)^2}{H^{\frac{\sigma-1-\mu}{\sigma-1}}\sigma t} \cdot \left\{\frac{(1-H)t\sigma}{2[\sigma-(1-t)\mu]}\right\}^{1+\gamma} \cdot \left[h^{\frac{\mu+1-\sigma}{\sigma-1}} - (1-h)^{\frac{\mu+1-\sigma}{\sigma-1}}\right]. \tag{A2}$$

The RHS of (A1) is positive so that the symmetry equilibrium is unstable if and only if $\sigma < 1+\mu$. On the other hand, when $\sigma < 1+\mu$, the RHS of (A2) is positive for $h > 1/2$ so that the full agglomeration is stable. If $\sigma > 1+\mu$, as long as $h > 1/2$, the RHS of (A2) is negative so that the full agglomeration is unstable.

Appendix B

Here, we prove that the LHS of (15) is always positive for any when $\sigma < 1+\mu$. First, we rewrite the LHS of (15) as $\phi^{\mu/(\sigma-1)} \cdot f$, where

$$f \equiv 2\phi^{\frac{\sigma-1-\mu}{\sigma-1}}\left[\frac{\sigma+(1-t)\mu}{\sigma-(1-t)\mu}\right]^{\gamma} - \left\{1-(1-t)\frac{\mu}{\sigma} + \phi^2\left[1+(1-t)\frac{\mu}{\sigma}\right]\right\}.$$

We immediately have the following

(i) $\dfrac{\partial f}{\partial \phi} = \dfrac{2(\sigma-1-\mu)}{\sigma-1}\phi^{\frac{-\mu}{\sigma-1}}\left[\dfrac{\sigma+(1-t)\mu}{\sigma-(1-t)\mu}\right]^{\gamma} - 2\phi\left[1+(1-t)\dfrac{\mu}{\sigma}\right] < 0$ when $\sigma < 1+\mu$,

(ii) $f|_{\phi=1} = 2\left\{\left[\dfrac{\sigma+(1-t)\mu}{\sigma-(1-t)\mu}\right]^{\gamma} - 1\right\} > 0.$

By putting (i), (ii), and $\phi^{\mu/(\sigma-1)} > 0$ together, we know that the full agglomeration equilibria are always stable for any $\phi \in (0, 1)$ when $\sigma < 1+\mu$. ■

Proof of Proposition 2: (i) When $\sigma > 1+\mu$, ϕ_B^{FE} falls in (0, 1) evidently. Furthermore, $c_1 > 0$ and $a_1+b_1+c_1 < 0$ hold. The conclusion of $\underline{\phi_B^{\text{Pure}}} \in (0,1)$ is derived from the following facts: (ia) $\underline{\phi_B^{\text{Pure}}}$ is the smaller root of $a_1\phi^2+b_1\phi+c_1=0$; (ib) $a_1\phi^2+b_1\phi+c_1>0$ when $\phi=0$; (ic) $a_1\phi^2+b_1\phi+c_1<0$ when $\phi=1$; (id) $a_1>0$.

(ii) We first calculate the derivative of ϕ_B^{FE} with respect to t:

$$\frac{\partial \underline{\phi_B^{\text{Pure}}}}{\partial t}=\frac{-\underline{\phi_B^{\text{Pure}}}}{2a_1\underline{\phi_B^{\text{Pure}}}+b_1}\left(\underline{\phi_B^{\text{Pure}}}\frac{\partial a_1}{\partial t}+\frac{\partial b_1}{\partial t}+\frac{1}{\underline{\phi_B^{\text{Pure}}}}\frac{\partial c_1}{\partial t}\right) \text{ by use of } a_1(\underline{\phi_B^{\text{Pure}}})^2+b_1\underline{\phi_B^{\text{Pure}}}+c_1=0.$$

Furthermore, we have $\dfrac{-\underline{\phi_B^{\text{Pure}}}}{2a_1\underline{\phi_B^{\text{Pure}}}+b_1}=\dfrac{-\underline{\phi_B^{\text{Pure}}}}{-\sqrt{(b_1)^2-4a_1c_1}}>0$, and

$$\underline{\phi_B^{\text{Pure}}}\frac{\partial a_1}{\partial t}+\frac{\partial b_1}{\partial t}+\frac{1}{\underline{\phi_B^{\text{Pure}}}}\frac{\partial c_1}{\partial t}=\frac{\mu}{\underline{\phi_B^{\text{Pure}}}}\{(\sigma-1)[1+4\gamma\underline{\phi_B^{\text{Pure}}}-(\underline{\phi_B^{\text{Pure}}})^2]-\mu(1-\underline{\phi_B^{\text{Pure}}})^2\}>0$$

for $\sigma>1+\mu$. Therefore, $\underline{\phi_B^{\text{Pure}}}$ increases in t when $\sigma>1+\mu$. On the other hand, ϕ_B^{FE} is independent of t, and t_C is obtained by solving $\underline{\phi_B^{\text{Pure}}}-\phi_B^{\text{FE}}=0$. Then $\underline{\phi_B^{\text{Pure}}}>\phi_B^{\text{FE}}$ holds iff $t>t_C$. Evidently, $t_C\in(0, 1)$ for any $0<\gamma<1$, $\sigma>1+\mu$, $0<\mu<1$. ■

Appendix C

Here, we prove expression (23):

$$\frac{\partial\, \underline{\phi_B^{\text{Cong}}}}{\partial\, t}>0,\quad \frac{\partial\, \underline{\phi_B^{\text{Cong}}}}{\partial\, \alpha}>0,\quad \frac{\partial\, \overline{\phi_B^{\text{Cong}}}}{\partial\, t}<0,\quad \frac{\partial\, \overline{\phi_B^{\text{Cong}}}}{\partial\, \alpha}<0 \text{ when } \underline{\phi_B^{\text{Cong}}} \text{ and } \overline{\phi_B^{\text{Cong}}}\in(0, 1).$$

As shown in (20) and (19), $\underline{\phi_B^{\text{Cong}}}$ and $\overline{\phi_B^{\text{Cong}}}$ are two roots of equation $a_2\phi^2+b_2\phi+c_2=0$. Both of them are positive when $c_2>0$. Differentiating this equation with respect to t and α gives

$$\frac{\partial\phi}{\partial\theta}=-\frac{\phi}{2a_2\phi+b_2}\left(\phi\frac{\partial a_2}{\partial\theta}+\frac{\partial b_2}{\partial\theta}+\frac{1}{\phi}\frac{\partial c_2}{\partial\theta}\right),\ \phi=\{\underline{\phi_B^{\text{Cong}}}, \overline{\phi_B^{\text{Cong}}}\},\ \theta=\{t, \alpha\}.$$

(i) At $\phi=\underline{\phi_B^{\text{Cong}}}\in(0, 1)$, we have

$$-\frac{\phi}{2a_2\phi+b_2}=\frac{1}{\sqrt{\Delta_2}}\underline{\phi_B^{\text{Cong}}}>0,$$

$$\phi\frac{\partial a_2}{\partial t}+\frac{\partial b_2}{\partial t}+\frac{1}{\phi}\frac{\partial c_2}{\partial t}=\frac{\mu}{\underline{\phi_B^{\text{Cong}}}}\left\{\frac{c_2[1-(\underline{\phi_B^{\text{Cong}}})^2]}{\sigma-(1-t)\mu}+2\mu\underline{\phi_B^{\text{Cong}}}(1-\underline{\phi_B^{\text{Cong}}})+4\gamma\underline{\phi_B^{\text{Cong}}}(\sigma-1)\right\}>0, \tag{C1}$$

$$\phi\frac{\partial a_2}{\partial\alpha}+\frac{\partial b_2}{\partial\alpha}+\frac{1}{\phi}\frac{\partial c_2}{\partial\alpha}=\frac{H\gamma}{\underline{\phi_B^{\text{Cong}}}}(\sigma-1)(1+\underline{\phi_B^{\text{Cong}}})[\sigma(1+\underline{\phi_B^{\text{Cong}}})-\mu(1-t)(1-\underline{\phi_B^{\text{Cong}}})]>0 \tag{C2}$$

(ii) At $\phi=\overline{\phi_B^{\text{Cong}}}\in(0, 1)$, we have

$$-\frac{\phi}{2a_2\phi+b_2}=-\frac{1}{\sqrt{\Delta_2}}\overline{\phi_B^{\text{Cong}}}<0.$$

On the other hand, the positiveness of (C1) and (C2) holds similarly. Therefore, we obtain the 3rd and 4th inequalities of (23). ■

Appendix D

$$\eta_1\equiv\frac{2^{(\alpha-1)\gamma+2}(1-H)^{1+\gamma}(1-t)^2t^\gamma\gamma\mu\sigma^{\gamma-1}}{H^{\frac{\sigma-1-\mu}{\sigma-1}}[\sigma-(1-t)\mu]^{1+\gamma}}>0,\ \eta_2\equiv\frac{2^{(\alpha-1)\gamma}(1-H)^{1+\gamma}(1-t)^2t^\gamma\mu}{H^{\frac{\sigma-1-\mu}{\sigma-1}}[\sigma-(1-t)\mu]}>0.$$

$$\eta_3 \equiv \frac{2^{(\alpha-1)\gamma+2-\mu/(\sigma-1)}(1-H)^{1+\gamma}(1-t)^2\mu(t\sigma)^\gamma}{H^{\frac{\sigma-1-\mu}{\sigma-1}}(\sigma-1)[\sigma-(1-t)\mu]^{1+\gamma}}>0,\ \eta_4 \equiv \frac{2^{\alpha\gamma-(1+\gamma)}(1-H)^{1+\gamma}\mu(1-t)^2(t\sigma)^\gamma}{H^{\frac{\sigma-1-\mu}{\sigma-1}}[\sigma-(1-t)\mu]^{\gamma+1}}>0.$$

Appendix E

Proof of Proposition 3: Note that

$\Delta V \equiv V_i - V_j = \left[\frac{(1-t)w_{Hi}}{(\mathbb{P}_i)^\mu}\cdot(G_i)^\gamma\right] - \left[\frac{(1-t)w_{Hj}}{(\mathbb{P}_j)^\mu}\cdot(G_j)^\gamma\right]$ and $\Delta V|_{\phi=1} = \eta_5\cdot\Omega_1$,

where $\eta_5 \equiv \frac{2^{-\gamma}(1-H)^{1+\gamma}(1-t)^2 t^\gamma \mu}{H^{\frac{\sigma-1-\mu}{\sigma-1}}[\sigma-(1-t)\mu]^{1+\gamma}}$ is a positive bundling parameter and

$$\Omega_1 \equiv \left\{\frac{\sigma-(1-2h)(1-t)\mu}{\left[Hh+\frac{1-H}{2}\right]^\alpha}\right\}^\gamma - \left\{\frac{\sigma+(1-2h)(1-t)\mu}{\left[H(1-h)+\frac{1-H}{2}\right]^\alpha}\right\}^\gamma.$$

Therefore, Ω_1 can be used to establish the bifurcation properties of $\Delta V|_{\phi=1}$ with respect to t:

$$\Omega_1\Big|_{\phi=1,\ h=\frac{1}{2}} = 0, \quad \forall t \tag{E1}$$

$$\frac{\partial\Omega_1}{\partial h}\Big|_{\phi=1,\ h=\frac{1}{2},\ t=t_d} = 0, \text{ where } t_d = 1-\frac{\alpha H\sigma}{\mu} \text{ by Equation (27)} \tag{E2}$$

$$\frac{\partial^2\Omega_1}{\partial h\,\partial t}\Big|_{\phi=1,\ h=\frac{1}{2},\ t=t_d} = \frac{-2^{2+\alpha\gamma}\gamma\mu}{\sigma^{1-\gamma}}<0 \tag{E3}$$

$$\frac{\partial^2\Omega_1}{\partial h^2}\Big|_{\phi=1,\ h=\frac{1}{2},\ t=t_d} = 0 \tag{E4}$$

$$\frac{\partial^3\Omega_1}{\partial h^3}\Big|_{\phi=1,\ h=\frac{1}{2},\ t=t_d} = 2^{5+\alpha\gamma}H^3\sigma^\gamma\gamma\alpha(\alpha^2-1)<0 \tag{E5}$$

Equation (E1) means that $h=1/2$ is always a stable equilibrium. Equations (E2) and (E3) show that, as t decreases from a large value, $h=1/2$ turns from being stable to unstable as soon as t falls below t_d. Equations (E4) and (E5) imply that, as soon as $h=1/2$ changes its stability, two asymmetric *stable* equilibria appear in its neighborhood.

Moreover, by the explanation after Equation (27), we know that full agglomeration is sustained from t_a to zero and t_a is always smaller than t_d. Summing up, $\Delta V|_{\phi=1}$ exhibits a "supercritical" pitchfork bifurcation as in Figure 5. ■

Proof of Proposition 4: Note that $\Delta V|_{\phi=0} = \eta_4\cdot\Omega_0$, where η_4 defined in Appendix D is positive and

$$\Omega_0 \equiv h^{\frac{1+\mu-\sigma}{\sigma-1}}\cdot\left(\frac{2}{2Hh+1-H}\right)^{\alpha\gamma} - (1-h)^{\frac{1+\mu-\sigma}{\sigma-1}}\cdot\left[\frac{2}{2H(1-h)+1-H}\right]^{\alpha\gamma}.$$

Then, Ω_0 can be used to establish the bifurcation properties of $\Delta V|_{\phi=0}$ with respect to σ:

$$\Omega_0\Big|_{\phi=0,\ h=\frac{1}{2}} = 0 \quad \forall\sigma \tag{E6}$$

$$\frac{\partial\Omega_0}{\partial h}\Big|_{\phi=0,\ h=\frac{1}{2},\ \sigma=\sigma_B} = 0, \text{ where } \sigma_B \equiv 1+\frac{\mu}{1+H\alpha\gamma} \text{ by Equation (31)} \tag{E7}$$

$$\left.\frac{\partial^2 \Omega_0}{\partial h\, \partial \sigma}\right|_{\phi=0,\, h=\frac{1}{2},\, \sigma=\sigma_B} = \frac{-2^{2+\alpha\gamma}(1+H\alpha\gamma)^2}{2^{1+H\alpha\gamma}\mu} < 0 \tag{E8}$$

$$\left.\frac{\partial^2 \Omega_0}{\partial h^2}\right|_{\phi=0,\, h=\frac{1}{2},\, \sigma=\sigma_B} = 0 \tag{E9}$$

$$\left.\frac{\partial^3 \Omega_0}{\partial h^3}\right|_{\phi=0,\, h=\frac{1}{2},\, \sigma=\sigma_B} = \frac{2^{5+\alpha\gamma}(1-H^2)H\alpha\gamma}{2^{1+H\alpha\gamma}} > 0 \tag{E10}$$

Equation (E6) reveals that $h=1/2$ is always an equilibrium. Equations (E7) and (E8) show that, as σ decreases from a large value, $h=1/2$ turns from being stable to unstable as soon as σ falls below σ_B. Equations (E9) and (E10) express that, as soon as $h=1/2$ changes its stability, two asymmetric *unstable* equilibria appear in its neighborhood.

Moreover, by the explanation after Equation (31), we know that full agglomeration is sustained from $\sigma_S=1+\mu$ to 1 and σ_S is always larger than σ_B. Summing up, $\Delta V|_{\phi=0}$ exhibits a "subcritical" pitchfork bifurcation with respect to σ as in Figure 6. ■

Appendix F: Existence of the Break Point and the Re-dispersion Point

In Section IV.1, the break point and the re-dispersion point are defined when Δ_2 is positive so that $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ exist. Further analysis of Δ_2 yields the following results.

Result 1: Both $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ exist for any tax rate $t \in (0, 1)$ if $\alpha H < \mathfrak{f}_1 \equiv \dfrac{\mu^2}{4\gamma(\sigma-1)^2}$.

Result 2: Neither $\underline{\phi_B^{\mathrm{Cong}}}$ nor $\overline{\phi_B^{\mathrm{Cong}}}$ exists for any tax rate $t \in (0, 1)$ if

$$\alpha H > \mathfrak{f}_2 \equiv \frac{2\sigma(\sigma+\gamma\mu)}{\gamma\mu^2} + \frac{1}{\gamma(\sigma-1)} - \frac{2}{\gamma\mu^2}\sqrt{\frac{(\sigma^2-\mu^2)(\sigma+\gamma\mu)[\mu^2+(\sigma-1)(\sigma+\gamma\mu)]}{\sigma-1}}.$$

Result 3: Both $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ exist iff $t < t_\Delta$ when $\mathfrak{f}_1 < \alpha H < \mathfrak{f}_2$, where

$$t_\Delta \equiv \frac{\mu\dfrac{2\sigma-1}{\sigma-1} + \gamma\left\{2\sigma - \alpha H\sigma\left[2\gamma + \dfrac{\mu}{\sigma-1}\right] + \mu\left[\alpha H(2+\alpha H\gamma) + 4\left(\gamma + \dfrac{\mu}{\sigma-1}\right)\right]\right\} - \sqrt{\Lambda}}{\mu\left\{1 + \gamma\left[\alpha H(2+\alpha H\gamma) + 4\left(\gamma + \dfrac{\mu}{\sigma-1}\right)\right]\right\}},$$

and $\Lambda \equiv 4\sigma^2\gamma\left[(1+\alpha H\gamma)^2 - \left(\dfrac{\mu}{\sigma-1}\right)^2\right]\left[\alpha H + \gamma + \dfrac{\mu}{\sigma-1}\right]$.

Then, we prove the above results. Note that both $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ exist iff Δ_2 is nonnegative. We rewrite Δ_2 as a function of tax rate t: $\Delta_2 = 8\sigma^2(\sigma-1)^2(a_3 t^2 + b_3 t + c_3)$, where

$$a_3 = \frac{1}{2}\left(\frac{\mu}{\sigma}\right)^2\left\{1 + \gamma\left[2\alpha H + (\alpha H)^2\gamma + 4\gamma + \frac{4\mu}{\sigma-1}\right]\right\} > 0,$$

$$b_3 = -\frac{\mu}{\sigma}\left\{\frac{\mu}{\sigma-1} + \frac{\mu}{\sigma} + \gamma\left[2 - \alpha H\left(2\gamma + \frac{\mu}{\sigma-1}\right) + \left(2\alpha H + (\alpha H)^2\gamma + 4\gamma + \frac{4\mu}{\sigma-1}\right)\frac{\mu}{\sigma}\right]\right\} < 0,$$

$$c_3 = \frac{1}{2}\left\{\left(\frac{\mu}{\sigma-1}\right)^2 + (\alpha H)^2\gamma^2\left(\frac{\mu}{\sigma}\right)^2 + \frac{2\mu^2}{\sigma(\sigma-1)}\left(1 + \frac{2\gamma\mu}{\sigma}\right)\right.$$

$$+\frac{\mu}{\sigma}\left(4\gamma+\frac{\mu}{\sigma}+\frac{4\gamma^2\mu}{\sigma}\right)-2\alpha H\gamma\left[2+\frac{2\gamma\mu}{\sigma}+\frac{\mu^2}{\sigma(\sigma-1)}-\left(\frac{\mu}{\sigma}\right)^2\right]\Bigg\}.$$

I. If $\alpha H\leq(1+\mu-\sigma)/[\gamma(\sigma-1)]$, then $c_2\leq 0$ and $\Delta_2>0$ always hold according to (19). Then, both $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ exist. On the other hand, if $\alpha H>(1+\mu-\sigma)/\gamma(\sigma-1)$, then we have two roots of equation $a_3t^2+b_3t+c_3=0$ .The larger root $(-b_3+\sqrt{(b_3)^2-4a_3c_3})/(2a_3)$ is always larger than 1 since $2a_3+b_3<0<\sqrt{(b_3)^2-4a_3c_3}$ holds. Therefore, the smaller root

$$\frac{-b_3-\sqrt{(b_3)^2-4a_3c_3}}{2a_3}=t_\Delta$$

is the only meaningful solution for an economic system. Clearly, $t_\Delta>1$ holds if and only if

$$a_3+b_3+c_3=\frac{1}{2}\left[\left(\frac{\mu}{\sigma-1}\right)^2-4\alpha H\gamma\right]>0.$$

Hence, both $\underline{\phi_B^{\mathrm{Cong}}}$ and $\overline{\phi_B^{\mathrm{Cong}}}$ exist when $\dfrac{1+\mu-\sigma}{\gamma(\sigma-1)}<\alpha H<\dfrac{\mu^2}{4\gamma(\sigma-1)^2}\equiv\mathfrak{f}_1$.

II. Neither $\underline{\phi_B^{\mathrm{Cong}}}$ nor $\overline{\phi_B^{\mathrm{Cong}}}$ exists if $t_\Delta<0$, which holds if and only if $c_3<0$. Rewriting c_3 as a function of αH, we obtain $c_3=a_4(\alpha H)^2+b_4\alpha H+c_4$, where

$$a_4=\frac{1}{2}\left(\frac{\gamma\mu}{\sigma}\right)^2>0,$$
$$b_4=-\left\{\frac{2\gamma^2\mu}{\sigma}+\gamma\left[2+\frac{\mu^2}{\sigma(\sigma-1)}-\left(\frac{\mu}{\sigma}\right)^2\right]\right\}<0,$$
$$c_4=\frac{1}{2}\left\{\left(\frac{\mu}{\sigma-1}\right)^2+\frac{2\mu^2}{\sigma(\sigma-1)}\left(1+\frac{2\gamma\mu}{\sigma}\right)+\frac{\mu}{\sigma}\left[4\gamma+\frac{\mu}{\sigma}(1+4\gamma^2)\right]\right\}>0.$$

There are two possible roots of equation $a_4(\alpha H)^2+b_4\alpha H+c_4=0$ with respect to αH. The larger root $(-b_4+\sqrt{(b_4)^2-4a_4c_4})/(2a_4)$ is always larger than 1 since $2a_4+b_4<0<\sqrt{(b_4)^2-4a_4c_4}$. Therefore, the smaller root $(-b_4-\sqrt{(b_4)^2-4a_4c_4})/(2a_4)$ is the only possible solution of the economic system. Then, $t_\Delta<0$ if and only if

$$\alpha H>\frac{-b_4-\sqrt{(b_4)^2-4a_4c_4}}{2a_4}\equiv\mathfrak{f}_2$$
$$=\frac{2\sigma(\sigma+\gamma\mu)}{\gamma\mu^2}+\frac{1}{\gamma(\sigma-1)}-\frac{2}{\gamma\mu^2}\sqrt{\frac{(\sigma^2-\mu^2)(\sigma+\gamma\mu)[\mu^2+(\sigma-1)(\sigma+\gamma\mu)]}{\sigma-1}}.$$

In addition, we obtain the result 2.

III. When $\mathfrak{f}_1<\alpha H<\mathfrak{f}_2$, $\Delta_2>0$ holds if and only if $t<t_\Delta$, and we obtain the result 3.

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