OPTIMAL DESIGN OF PROCUREMENT MECHANISMS

by
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Contents

Chapter 1

Introduction

Public procurement consists a substantial part of economic activity. The OECD (2012) reports that governments in OECD member countries spend on average 12% of their GDP on public procurement. A government procures a wide range of goods, works and services from private firms. Physical facilities such as railways, schools or hospitals and public services such as transportation, education or medical service have direct impacts on the society and the economy. There are potentially many ways to conduct the procurement: competitive bidding, negotiations, posted prices, and so on. A procurement authority must find an efficient procedure of procurement. In this thesis, we study the optimal design of procurement mechanisms in some economic environments.

A large economic literature has addressed the issue of the optimal design of procurement mechanisms. The book by Laffont and Tirole (1993) offers a comprehensive analysis of the issue and provides an excellent reference on related theoretical models. The key aspect of these studies is that suppliers have some private information to which a buyer has only a limited access. A procurement mechanism is then used to elicit suppliers' information about their types and determine the terms of trade. Although the buyer has full bargaining power to design a mechanism, suppliers with informational advantages can obtain positive information rents. The buyer then faces a tradeoff between *allocative efficiency* and *rent extraction*. On the one hand, the buyer aims to increase the gains from trade. On the other hand, the buyer aims to reduce suppliers' information rents. In order to justify the assumption that a planner dislikes monetary transfers to firms, Laffont and Tirole (1986) refer to an inefficient tax system.

The previous studies have characterized the buyer's *optimal (direct) mechanism* in many procurement environments. The optimal mechanism is defined as a mechanism that maximizes the buyer's expected payoff among those in which suppliers' truth-telling strategies form a Bayesian Nash equilibrium and any supplier obtains nonnegative expected payoff. The so-called "Revelation Principle" guarantees that the optimal direct mechanism yields to the buyer the maximum payoff among all Bayesian Nash equilibria in *any* procurement mechanism. The buyer's optimal direct mechanism is determined to balance the tradeoff between allocative efficiency and rent extraction. Under some regularity conditions, it is shown that, for any type profile, the buyer's optimal mechanism determines the terms of trade to maximize *virtual surplus* not social surplus. Here, the virtual surplus is defined as social surplus minus suppliers' information rents.

Given the characterization results, we have a more practical question: whether and under what conditions procurement mechanisms used in practice can implement the same equilibrium outcome as the optimal direct mechanism. By answering these questions, we can provide useful policy guidelines. In Chapter 2, we analyze a new method of procurement, *scoring auction*, recently introduced in the whole world. Also, there may be legal constraints which restrict the class of available procurement mechanisms. For instance, procurement authorities in the United States are bound to award contracts through competitive bidding under the Federal Acquisition Rules. In Chapter 3, we investigate whether the buyer should procure two kinds of tasks through a single auction or two sequential auctions. In the two chapters, we assume that the buyer has no private information in contrast with suppliers. In Chapter 4, we remove the strong assumption.

We will study three models in non-cooperative game theory:

1. Scoring auctions with multi-dimensional quality,
2. Sequential procurement auctions with risk-averse suppliers,
3. Bilateral trade in which a seller with private information selects a mechanism.

Optimal Design of Scoring Auctions with Multidimensional Quality

In Chapter 2, we study the optimal design of *scoring auctions*. In the procurement auction mechanism, suppliers offer not only price but also quality they promise to ensure in a project. Hence bids are multi-dimensional. Suppliers' bids are evaluated by a *scoring rule* so that a supplier with the highest score wins a contract. There are many examples of scoring rules adopted by state departments of transportation in the United States. For instance, the rule of "weighted criteria" puts a weight on each of price and quality attributes (e.g. delivery date, safety level) and gives a supplier's offer a weighted sum of subscores. We say that a scoring rule is optimal if a scoring auction with the rule implements the buyer's optimal direct mechanism. The purpose of this chapter is to construct an optimal scoring rule in an environment with various quality attributes.

We extend the model of Che (1993) by allowing for multi-dimensional quality. In an environment where quality is one-dimensional, Che derives a symmetric equilibrium bidding strategy in a game induced by the scoring auction, and constructs an optimal scoring rule. Although many scoring rules used in practice apply multiple quality criteria, there are no theoretical studies investigating how to construct an optimal scoring rule when quality is multi-dimensional. In the model of multi-dimensional quality, it will be anticipated that the interaction among quality attributes significantly affects the optimal form of scoring rules.

We consider two classes of scoring rules: a rule which is supermodular in quality and a rule which is additively separable in quality. The rule of "weighted criteria" is included in the latter class. First, we show that there exists an optimal scoring rule which is supermodular in quality if the virtual surplus is quasisupermodular in quality. These properties of supermodularity and quasisupermodularity represent the concept of complementarity between quality attributes. Second, we derive a necessary condition and a sufficient condition for the existence of an optimal scoring rule which is additively separable in some quality attributes. An example shows that an extension of Che (1993)'s scoring rule cannot implement the buyer's optimal direct mechanism. The scoring rule is additively separable in quality.

Sequential Procurement Auctions with Risk-Averse Suppliers[1]

In Chapter 3, we compare two procurement mechanisms, *bundling* and *unbundling*, in a two-stage auction model with risk-averse suppliers. They differ in whether two kinds of tasks of investment and production are procured through a single auction or two sequential auctions. For instance, in many construction projects, a procurement authority has the option of choosing between a bundling method called "design-build" and an unbundling method called "design-bid-build." Under the design-build method, a private party is responsible for performing both tasks of design and construction. Under the design-bid-build method, the two tasks are separated. The purpose of this chapter is to compare the performance of the two procurement mechanisms, and to investigate the buyer's choice problem of a mechanism.

Hart (2003) compares a bundling method with an unbundling method in an incomplete contract model. He shows that a bundling mechanism gives a supplier strong investment incentives. His model is so simple that the issues of suppliers' private information and production risks are not addressed. We formalize an auction model with these two elements. A risk-neutral buyer procures a public infrastructure from risk-averse suppliers. The investment stage is followed by the production stage. In a bundling mechanism, the buyer holds a single auction. In an unbundling mechanism, the buyer holds two sequential auctions.

[1]Published in *Journal of Economics*. The final publication is available at link.springer.com. Direct link: http://link.springer.com/article/10.1007%2Fs00712-013-0381-1

An important feature of our model is that there are two categories of production risks. The first category is an *aggregate risk*. The risk is common to all suppliers. The second one is an *idiosyncratic risk*. The risk is specific to each supplier. Each supplier's production cost is affected by the two risk factors as well as the cost-reducing investment. After the cost-reducing investment is made, the realized value of the aggregate risk is commonly known to all suppliers, and each supplier privately knows the realized value of the idiosyncratic risk to him.

The main result of this chapter is as follows. The choice of a bundling mechanism is optimal for the buyer if the aggregate risk is below certain threshold. The result may not hold true for the idiosyncratic risk. Key factors leading to the result are the differences of risk sharing, information rents, and investment incentives between the two mechanisms. In the bundling mechanism, all the risks are taken by a single supplier. The buyer has to pay a high risk premium to the supplier, while the mechanism implements the efficient investment. In contrast, the unbundling mechanism provides little incentives for the investment due to a moral hazard problem. The buyer must pay risk premia for the idiosyncratic risk, indirectly through the payment of information rents. The buyer pays no risk premium for the aggregate risk because the risk is transferred from the suppliers to the buyer through the second-stage auction. Therefore, a decrease in the aggregate risk is beneficial to the buyer and the society only in the bundling mechanism, unlike the idiosyncratic risk. A decrease in the idiosyncratic risk may reduce the information rents in the unbundling mechanism.

Informed Principal Problems in Bilateral Trading

In the above two chapters, we have assumed that the procurement authority has no private information. This assumption is too restrictive in many situations. For instance, the state Department of Transportation in the United States may possess superior information about the economic value of a new highway. Moreover, the department may have private information about political issues which affect a contractor's construction costs. In Chapter 4, we consider a bilateral trade environment where a seller with private information proposes a trading mechanism to a buyer with private information. The seller's selection of a mechanism may convey information about her type to the buyer. Although we assume that the seller has full bargaining power, we can exchange the roles. The purpose of this chapter is to prove the existence of a separating equilibrium, and to investigate allocative efficiency and distributional consequences in the equilibrium.

Our model belongs to the informed-principal literature. In the case of interdependent values, Maskin and Tirole (1992) characterize the set of mechanisms selected in equilibrium. Their analysis rests on a fundamental concept called an "RSW (Rothschild-Stiglitz-Wilson)" mechanism. They define it as a mechanism that maximizes the principal's interim payoff for each type within the class of mechanisms which are interim incentive compatible for the principal and both ex post incentive compatible and individually rational for the agent. They show that the set of mechanisms selected in equilibrium consists of incentive-feasible mechanisms which weakly dominate the RSW mechanism. After showing the result in the case where only the principal has private information, they extend it to the case where the agent also has private information. The extension, however, requires specific assumptions on the payoff functions.

We introduce a concept called an *LCS (least-cost-separating) mechanism*. The definition differs from the RSW mechanism in that the interim incentive compatibility constrains for the principal are replaced by the interim *upward* incentive compatibility constraints. Although the constraints are weakened in the LCS mechanism, the LCS and RSW mechanisms are equivalent in our model.

First, we show that there exists a separating equilibrium, and the seller's interim payoff vector in any separating equilibrium is uniquely determined by that in the LCS mechanism. A simple observation shows that the seller's interim payoff vector in the LCS mechanism is the minimum of the set of equilibrium payoff vectors. Therefore, our existence theorem implies the same characterization result as Maskin and Tirole (1992).

Next, we investigate allocative efficiency and distributional consequences in the LCS mechanism. We provide sufficient conditions on the seller's cost function under which the allocation rule is distorted upward or downward compared to the optimal mechanism when the seller's type is common knowledge. Accordingly, the buyer is weakly better off or worse off than in the optimal mechanism. This is in contrast to the case where the buyer with no private information cannot obtain any information rent in the LCS mechanism. The privacy of the seller's information weakens her monopolistic power. The effect may be socially desirable in our model.

Organization of This Thesis

The rest of the thesis is organized as follows. In Chapter 2, we study the optimal design of scoring auctions. We construct scoring rules which implement the optimal direct mechanism. In Chapter 3, we compare the performance of the bundling mechanism with that of the unbundling mechanism. We provide conditions on parameters under which each choice is optimal for the buyer. In Chapter 4, we address an informed principal problem in a bilateral trade environment. We prove the existence of a separating equilibrium, and investigate the efficiency properties of the LCS mechanism. Chapter 5 concludes.

Chapter 2

Optimal Design of Scoring Auctions with Multidimensional Quality

2.1 Introduction

Auction rules of public procurement have changed from one-dimensional bidding to multi-dimensional bidding. In contrast with the former traditional rule in which each supplier submits only a price-bid, the latter auction rule requires suppliers to offer not only price but also quality they promise to ensure in a project. For instance, in the EU, Article 53 of Directive 2004/18/EC specifies the "Most Economically Advantageous Tender." In the tendering procedure, procurement authorities award contracts based on various criteria such as price, technical merit, aesthetic characteristics, delivery date, and so on. The design of multi-dimensional auctions is a matter of great concern to the procurement authorities, reflecting the fact that public procurement accounts for about 16% of GDP in OECD member countries (OECD, 2008). The purpose of this chapter is to study the optimal design of multi-dimensional auctions in an environment with various quality attributes.

The essential element of the multi-dimensional auction is a scoring rule. The rule, which evaluates suppliers' offers and gives them scores, should be carefully designed because it considerably affects suppliers' decisions what offers to make. There are many examples of scoring rules adopted by state departments of transportation in the United States: "A+B bidding" (Arizona, etc.), "weighted criteria" (Delaware, Idaho, Massachusetts, Oregon, Utah, Virginia, etc.), "adjusted bid" (Arizona, Maine, Michigan, North Carolina, South Carolina, South Dakota, etc.), and so on. See Molenaar and Yakowenko (2007) for the detail. For instance, the rule of "weighted criteria" puts a weight on each of price and quality attributes (e.g. delivery date, safety level) and evaluates each attribute individually. A total score of each offer is a weighted sum of subscores and a supplier with the highest total score wins a contract.

In this chapter, we extend the model of Che (1993) by allowing for multi-dimensional quality. In the seminal work, Che shows that a *scoring auction* with a properly designed scoring rule implements the buyer's optimal mechanism. Although many scoring rules used in practice apply multiple quality criteria, there are no theoretical studies investigating whether a scoring auction can implement the optimal mechanism in an environment where quality is multi-dimensional. It is by no means trivial to answer the question and show what form of scoring rule succeeds in the implementation, at least from the "Revelation Principle." In the model of multi-dimensional quality, it will be anticipated that the interaction among quality attributes significantly affects the optimal form of scoring rule. These observations motivate our analysis.

A scoring rule announced by the buyer induces the following auction game. First, all suppliers' (one-dimensional) cost parameters are realized. Each supplier is privately informed about his own cost parameter. Second, each supplier simultaneously offers both price and quality. A single supplier wins if the score of his offer is the highest among suppliers and higher than the predetermined reserve score.[1] The winner's offer becomes a binding contract. We say that a scoring rule is optimal if a scoring auction with the rule implements the optimal mechanism.

We obtain two sets of results. First, we show that there exists an optimal scoring rule which is supermodular in quality if the so-called "virtual surplus" is quasisupermodular in quality. These properties of supermodularity and quasisupermodularity represent the concept of complementarity between quality attributes. They are specific to the model of multi-dimensional quality. In particular, it will be shown that the scoring rule should evaluate all quality attributes as a whole (not separately) to give a total score in a complementary way. Second, we derive a necessary condition and a sufficient condition for the existence of an optimal scoring rule which is additively separable in some quality attributes. Our results show that when the buyer establishes some sets of subcriteria to use the additively separable rule, each pair of quality attributes in the distinct sets should be complementary in terms of the production cost.

In a seminal article, Che (1993) shows that a scoring auction with a properly designed scoring rule implements the buyer's optimal mechanism characterized by Laffont and Tirole (1987), McAfee and McMillan (1987), and Riordan and Sappington (1987). Branco (1997) extends this result to an environment where each supplier's production cost has a common-cost component, so that his cost is correlated with the other suppliers' costs. Che and Branco assume that both quality and each supplier's type are one-dimensional.

In contrast, Asker and Cantillon (2008) consider a fully general environment. They allow that both elements are multi-dimensional. The main results of Asker and Cantillon are the characterization of equilibrium bidding behavior and the expected utility equivalence between some formats of scoring auction. They also show that the scoring

[1] The introduction of reserve score is also an extension of Che (1993).

auction outperforms some other mechanisms from the buyer's viewpoint. On the other hand, they have not investigated whether a scoring auction can implement the optimal mechanism. The likely reason is that it is extremely difficult to characterize the optimal mechanism when the supplier has multi-dimensional private information. Asker and Cantillon (2010), however, characterize the optimal mechanism in a specific environment. In the environment, quality is one-dimensional, each supplier's type consists of two parameters (fixed cost and marginal cost) and each parameter is a binary random variable. They show that the scoring auction yields a performance close to that of the optimal mechanism, taking a numerical simulation approach.

All of the above studies including the current one focus on "quasi-linear" scoring rules. Under the rule, a total score of supplier's offer is given by a quality score minus price. A typical example of quasi-linear rule is "weighted criteria." In a recent study, Hanazono et al. (2011) consider "price-quality ratio" scoring rules under which a total score is given by price divided by a quality score, and analyze the equilibrium bidding behavior.

In addition to these theoretical studies, there is some experimental evidence supporting the high performance of scoring auctions compared to that of traditional price-only auctions (Bichler, 2000; Chen-Ritzo et al., 2005).

Our analysis contributes to the literature on scoring auctions in two ways. First, we construct the optimal scoring rule in a new way. When quality is one-dimensional, the scoring rule constructed by Che (1993) and Branco (1997) provides suppliers with incentives to offer the appropriate quality level. On the other hand, our example shows that when quality is multi-dimensional, an extension of their scoring rule may induce unsuitable quality offers from the buyer's viewpoint. To resolve this problem, we construct the optimal scoring rule by applying the method of monotone comparative statics. The constructed rule satisfies supermodularity in quality to deter a suppliers' deviation from the desirable quality offer for the buyer. Also, as a by-product of using the monotone comparative statics method, we require no assumptions of concavity (or convexity) and differentiability in quality of the value and cost functions. Second, we provide a useful guide to designing scoring rules which are additively separable in some quality attributes. The result has important policy implications. Additively separable scoring rules (e.g. "weighted criteria") are widely adopted, and it must be easier for procurement authorities to administer those rules.

This chapter is organized as follows. Section 2.2 presents the model which generalizes that of Che (1993). Section 2.3 derives the equilibrium bidding strategy. Section 2.4 shows how to design a scoring rule to implement the optimal mechanism. Section 2.5 investigates how the buyer should classify quality attributes when using a scoring rule which is additively separable in the attributes. Section 2.6 concludes. All proofs are in the Appendix.

2.2 The Model

Consider a buyer who procures a single product from one of N suppliers. A *contract* between the buyer and supplier $i \in \{1, ..., N\}$ is denoted by $(p_i, \boldsymbol{q_i}) \in \mathbb{R}_+ \times \mathcal{Q}$. Under the contract, supplier i must deliver a product of *quality* $\boldsymbol{q_i} = (q_i^1, ..., q_i^M) \in \mathcal{Q} \subset \mathbb{R}^M$ in exchange for price $p_i \in \mathbb{R}_+$.[2] For each $m \in \{1, ..., M\}$, q_i^m represents a level of quality attribute. Supplier i's cost parameter is given by $\theta_i \in [\underline{\theta}, \bar{\theta}] \subset \mathbb{R}$. Suppliers' types $(\theta_1, ..., \theta_N)$ are random variables which are independent across suppliers. The cumulative distribution function of θ_i is given by F, with a density function f that is strictly positive everywhere. Each supplier has private information about his realized type respectively. The prior probability distribution is common knowledge.

Supplier i of type θ_i earns profits $p - c(\boldsymbol{q}, \theta_i)$ from a contract $(p, \boldsymbol{q})$. Here, $c(\boldsymbol{q}, \theta_i) > 0$ is his production cost. The buyer's utility from a contract $(p, \boldsymbol{q})$ with a supplier of type θ_i is a weighted sum of consumers' surplus and profits, i.e. $v(\boldsymbol{q}) - p + \alpha(p - c(\boldsymbol{q}, \theta_i))$. Here, $v(\boldsymbol{q})$ is the valuation for a product of quality $\boldsymbol{q}$, and $\alpha \in [0, 1]$ is a weight on profits. We make the following assumptions.

Assumption 2.1. $\mathcal{Q} = \times_{m=1}^{M} \mathcal{Q}^m$. Here, $\mathcal{Q}^m \subset \mathbb{R}$ is a closed interval or a finite set.

Assumption 2.2. v is continuous in $\boldsymbol{q}$.

Assumption 2.3. c is continuous in $(\boldsymbol{q}, \theta)$ and increasing in $\boldsymbol{q}$. c is differentiable and increasing in θ. $c_\theta := \partial c / \partial \theta$ is continuous in $(\boldsymbol{q}, \theta)$ and nondecreasing in θ.

Assumption 2.4. c has strictly increasing differences in $(\boldsymbol{q}, \theta)$, i.e. $c(\boldsymbol{q'}, \theta) - c(\boldsymbol{q}, \theta)$ is increasing in θ for each $\boldsymbol{q'} > \boldsymbol{q}$ in $\mathcal{Q}$. c_θ has increasing differences in $(\boldsymbol{q}, \theta)$, i.e. $c_\theta(\boldsymbol{q'}, \theta) - c_\theta(\boldsymbol{q}, \theta)$ is nondecreasing in θ for each $\boldsymbol{q'} > \boldsymbol{q}$ in $\mathcal{Q}$.

Assumption 2.5. $\frac{F}{f}$ is nondecreasing in θ.

We will apply the monotone comparative statics method. See Topkis (1998) for some notions. Assumption 2.1 ensures that $\mathcal{Q}$ is a compact lattice. Assumptions 2.4 and 2.5 ensure that the "virtual surplus" defined later has strictly decreasing differences in $(\boldsymbol{q}, \theta)$.

There is an auction rule (mechanism) that is available to the buyer: a *scoring auction*. We first define a *scoring rule* as a real-valued function $S : \mathbb{R}_+ \times \mathcal{Q} \to \mathbb{R}$. In the scoring auction, each supplier offers both price and quality. The scoring rule S assigns a score $S(p, \boldsymbol{q})$ to each offer $(p, \boldsymbol{q})$. With a *reserve score* normalized to zero, supplier i wins only if his score is nonnegative and the highest among suppliers.[3] We consider a *first-score* format. In the format, winner i is awarded a binding contract $(p_i, \boldsymbol{q_i})$ he offered in the auction. This format corresponds to the first-price sealed-bid format in standard auctions. We focus on a *quasi-linear* scoring rule. The rule takes a form of $S(p, \boldsymbol{q}) = s(\boldsymbol{q}) - p$. We

[2]Bold letters denote some vectors: $\boldsymbol{q} \geq \boldsymbol{\hat{q}}$ means $q^m \geq \hat{q}^m$ for each m; $\boldsymbol{q} > \boldsymbol{\hat{q}}$ means $\boldsymbol{q} \geq \boldsymbol{\hat{q}}$ and $\boldsymbol{q} \neq \boldsymbol{\hat{q}}$; $\boldsymbol{q} \gg \boldsymbol{\hat{q}}$ means $q^m > \hat{q}^m$ for each m.

[3]We assume that if there is a nonnegative tie score, then each supplier achieving the highest score wins with equal probability. All results hold for any other tie-breaking rule.

also call a function $s : \mathcal{Q} \to \mathbb{R}$ a scoring rule. We assume that s is upper semicontinuous in $\boldsymbol{q}$, and s has a cost parameter $\bar{\theta}^s \in (\underline{\theta}, \bar{\theta}]$ which satisfies $\max_{\boldsymbol{q}\in\mathcal{Q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \bar{\theta}^s)] = 0$.

A scoring rule s publicly announced by the buyer induces the following auction game. First, all suppliers' types $\boldsymbol{\theta} = (\theta_1, ..., \theta_N)$ are realized. Each supplier is privately informed about his own type respectively. Second, supplier i submits an offer $(p_i, \boldsymbol{q_i})$ as a sealed bid. Then, the game ends. If supplier i of type θ_i who offers $(p_i, \boldsymbol{q_i})$ wins, then he receives $p_i - c(\boldsymbol{q_i}, \theta_i)$. The other suppliers receive zero payoffs, and the buyer receives $v(\boldsymbol{q_i}) - p_i + \alpha(p_i - c(\boldsymbol{q_i}, \theta_i))$. If no supplier wins, then the buyer and all suppliers receive zero payoffs.

We consider a Bayesian Nash equilibrium in the auction game. In the following, we simply call it an equilibrium. With a slight abuse of notation, we denote supplier i's bidding strategy by $(p_i, \boldsymbol{q_i}) : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}_+ \times \mathcal{Q}$. We assume that no supplier uses weakly dominated strategies.

2.3 Equilibrium Bidding Strategy

In this section, we derive the equilibrium bidding strategy. The following lemma characterizes a symmetric equilibrium, where all suppliers use the same bidding strategy. Although our environment is substantially more general than Che (1993) and there are slight technical difficulties, we can apply his technique to prove the lemma.

Lemma 2.1. *The auction game induced by a scoring rule s has the following symmetric equilibrium $(p^*, \boldsymbol{q^*})$.*
(i) The bidding strategy is as follows. For any $\theta \in [\underline{\theta}, \bar{\theta}^s]$,

$$\boldsymbol{q}^*(\theta) \in \arg\max_{\boldsymbol{q}\in\mathcal{Q}} \; [s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)] \tag{2.1}$$

$$p^*(\theta) = c(\boldsymbol{q}^*(\theta), \theta) + \int_{\theta}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z) \left(\frac{1 - F(z)}{1 - F(\theta)}\right)^{N-1} dz. \tag{2.2}$$

For any $\theta \in (\bar{\theta}^s, \bar{\theta}]$, $(p^*(\theta), \boldsymbol{q}^*(\theta))$ is an arbitrary offer which satisfies $s(\boldsymbol{q}^*(\theta)) - p^*(\theta) < 0$.
(ii) A supplier of type θ_i wins only if $\theta_i = \min\{\theta_1, ..., \theta_N, \bar{\theta}^s\}$. For any $\theta \in [\underline{\theta}, \bar{\theta}^s]$, any offer $(p', \boldsymbol{q'})$ with $\boldsymbol{q'} \notin \arg\max_{\boldsymbol{q}\in\mathcal{Q}} \; [s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ is weakly dominated by $(p, \boldsymbol{q}^*(\theta))$ with $s(\boldsymbol{q}^*(\theta)) - p = s(\boldsymbol{q'}) - p'$.

It is worth emphasizing that any supplier who wins with positive probability chooses quality so as to maximize a quality score minus his production cost, as in (2.1). Actually, any offer which does not maximize $s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)$ is weakly dominated by the quality offer $\boldsymbol{q}^*(\theta)$ with some price offer. Given the optimal quality offer $\boldsymbol{q}^*(\theta)$, the optimal price offer $p^*(\theta)$ is determined by (2.2). The price offer is greater than the production cost.

Lemma 2.1 also implies that in equilibrium the most efficient supplier wins provided that his type is lower than $\bar{\theta}^s$. Let $\theta_{(N)} := \min\{\theta_1, ..., \theta_N\}$ be the lowest cost parameter among suppliers. We denote the cumulative distribution function of $\theta_{(N)}$ by $F_{(N)}(\cdot) := 1-(1-F(\cdot))^N$. Then, the buyer's expected utility from announcing a quasi-linear scoring rule s is

$$F_{(N)}(\bar{\theta}^s)E\left[v(\boldsymbol{q}^*(\theta_{(N)})) - p^*(\theta_{(N)}) + \alpha[p^*(\theta_{(N)}) - c(\boldsymbol{q}^*(\theta_{(N)}), \theta_{(N)})] \mid \theta_{(N)} \leq \bar{\theta}^s\right]$$
$$= \int_{\underline{\theta}}^{\bar{\theta}^s} \left[v(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta) - (1-\alpha)c_\theta(\boldsymbol{q}^*(\theta), \theta)\frac{F(\theta)}{f(\theta)}\right] dF_{(N)}(\theta),$$

where the equality follows from the substitution of $p^*(\theta_{(N)})$ and the interchange of the order of integration. We now define the *virtual surplus* as a function $\Phi := v - c - (1-\alpha)c_\theta \frac{F}{f}$. Its value $\Phi(\boldsymbol{q}, \theta) = v(\boldsymbol{q}) - c(\boldsymbol{q}, \theta) - (1-\alpha)c_\theta(\boldsymbol{q}, \theta)\frac{F(\theta)}{f(\theta)}$ times the density $f(\theta)$ is the social surplus generated by trading a product of quality $\boldsymbol{q}$ between the buyer and a supplier of type θ, minus the sum of information rents paid to the more efficient supplier than θ. Using this virtual surplus, the buyer's expected utility can be rewritten as

$$\int_{\underline{\theta}}^{\bar{\theta}^s} \Phi(\boldsymbol{q}^*(\theta), \theta) dF_{(N)}(\theta).$$

2.4 Optimal Scoring Rule

In this section, we study the implementation problem. The analysis proceeds in two steps. First, we characterize an optimal mechanism. We use the standard mechanism-design approach with the envelope theorem of Milgrom and Segal (2002). Second, we find the condition under which a scoring auction implements the optimal mechanism. We investigate how the scoring rule should be designed.

In a first step, we use the Revelation Principle to focus on "incentive compatible direct mechanisms." A *direct mechanism* is an $3N$-tuple of measurable functions $(P_i, \boldsymbol{Q_i}, X_i)_{i\in\{1,\dots,N\}}$. Here, $(P_i, \boldsymbol{Q_i}, X_i) : [\underline{\theta}, \bar{\theta}]^N \to \mathbb{R} \times \mathcal{Q} \times [0,1]$ for each i. Now, fix any profile of types $\hat{\boldsymbol{\theta}} = (\hat{\theta}_1, ..., \hat{\theta}_N)$ reported by suppliers. The payment schedule P_i specifies the expected payment $P_i(\hat{\boldsymbol{\theta}})$ from the buyer to supplier i. The quality schedule $\boldsymbol{Q_i}$ specifies quality $\boldsymbol{Q_i}(\hat{\boldsymbol{\theta}})$ supplier i must ensure when delivering the product. The function X_i specifies the trading probability $X_i(\hat{\boldsymbol{\theta}})$ between the buyer and supplier i. Since no trade is allowed, $\sum_{i=1}^N X_i(\hat{\boldsymbol{\theta}}) \leq 1$. A direct mechanism $(P_i^*, \boldsymbol{Q_i^*}, X_i^*)_{i\in\{1,\dots,N\}}$ is *optimal* (for the buyer) if it solves the following problem:

$$\max_{\rho=(P_i,\boldsymbol{Q_i},X_i)_{i\in\{1,\dots,N\}}} \sum_{i=1}^{N} E\left[X_i(\boldsymbol{\theta})v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - P_i(\boldsymbol{\theta}) + \alpha[P_i(\boldsymbol{\theta}) - X_i(\boldsymbol{\theta})c(\boldsymbol{Q_i}(\boldsymbol{\theta}),\theta_i)]\right]$$

$$\text{s.t. } \Pi_i^\rho(\theta_i \mid \theta_i) \geq \Pi_i^\rho(\hat{\theta}_i \mid \theta_i) \quad \text{for any } \theta_i, \hat{\theta}_i \in [\underline{\theta}, \bar{\theta}], i \in \{1, ...N\} \tag{2.3}$$

$$\Pi_i^\rho(\theta_i \mid \theta_i) \geq 0 \quad \text{for any } \theta_i \in [\underline{\theta}, \bar{\theta}], i \in \{1, ...N\} \tag{2.4}$$

Here, $\Pi_i^\rho(\hat{\theta}_i \mid \theta_i) := E_{\boldsymbol{\theta_{-i}}}[P_i(\hat{\theta}_i, \boldsymbol{\theta_{-i}}) - X_i(\hat{\theta}_i, \boldsymbol{\theta}_{-i})c(\boldsymbol{Q_i}(\hat{\theta}_i, \boldsymbol{\theta}_{-i}), \theta_i)]$. The first constraint is the interim incentive compatibility (IC) constraint for each supplier, and the second one is the interim individual rationality (IR) constraint for each supplier.

The next lemma characterizes an optimal mechanism. To explain the hypothesis of the lemma and the related notions, we introduce some definitions. Denote $\boldsymbol{q} \wedge \hat{\boldsymbol{q}} := (\min\{q^1, \hat{q}^1\}, ..., \min\{q^M, \hat{q}^M\})$ and $\boldsymbol{q} \vee \hat{\boldsymbol{q}} := (\max\{q^1, \hat{q}^1\}, ..., \max\{q^M, \hat{q}^M\})$. Here, $\boldsymbol{q}, \hat{\boldsymbol{q}} \in \mathcal{Q}$ implies $\boldsymbol{q}\wedge\hat{\boldsymbol{q}}, \boldsymbol{q}\vee\hat{\boldsymbol{q}} \in \mathcal{Q}$ because $\mathcal{Q}$ is a lattice. The virtual surplus Φ is *quasisupermodular* in quality if, for any $\boldsymbol{q}, \hat{\boldsymbol{q}} \in \mathcal{Q}$ and θ, $\Phi(\boldsymbol{q}\wedge\hat{\boldsymbol{q}}, \theta) \leq \Phi(\boldsymbol{q}, \theta)$ implies $\Phi(\hat{\boldsymbol{q}}, \theta) \leq \Phi(\boldsymbol{q}\vee\hat{\boldsymbol{q}}, \theta)$, and $\Phi(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta) < \Phi(\boldsymbol{q}, \theta)$ implies $\Phi(\hat{\boldsymbol{q}}, \theta) < \Phi(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta)$. The virtual surplus Φ is *supermodular* in quality if, for any $\boldsymbol{q}, \hat{\boldsymbol{q}} \in \mathcal{Q}$ and θ, $\Phi(\boldsymbol{q}, \theta) - \Phi(\boldsymbol{q}\wedge\hat{\boldsymbol{q}}, \theta) \leq \Phi(\boldsymbol{q}\vee\hat{\boldsymbol{q}}, \theta) - \Phi(\hat{\boldsymbol{q}}, \theta)$. We apply the same definition to a scoring rule s. It is easy to show that if Φ is supermodular in quality, then Φ is quasisupermodular in quality. Thus, quasisupermodularity is a weaker notion than supermodularity. These notions express the concept of complementarity between quality attributes.

Lemma 2.2. *Suppose that the virtual surplus Φ is quasisupermodular in quality. Then, the following direct mechanism $(P_i^*, \boldsymbol{Q^*}, X_i^*)_{i\in\{1,\dots,N\}}$ is optimal for the buyer:*

$$X_i^*(\boldsymbol{\theta}) := \begin{cases} 1 & \text{if } \theta_i < \min\{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N, \bar{\theta}^*\} \\ \frac{1}{\sharp\{j \mid \theta_j = \theta_i\}} & \text{if } \theta_i = \min\{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N, \bar{\theta}^*\} \\ 0 & \text{if } \theta_i > \min\{\theta_1, ..., \theta_{i-1}, \theta_{i+1}, ..., \theta_N, \bar{\theta}^*\} \end{cases} \tag{2.5}$$

$$\boldsymbol{Q^*}(\theta_i) \in \arg\max_{\boldsymbol{q}\in\mathcal{Q}} \Phi(\boldsymbol{q}, \theta_i) \quad \text{for any } i \tag{2.6}$$

$$P_i^*(\boldsymbol{\theta}) := X_i^*(\boldsymbol{\theta}) \left[c(\boldsymbol{Q^*}(\theta_i), \theta_i) + \int_{\theta_i}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q^*}(z), z) \left(\frac{1 - F(z)}{1 - F(\theta_i)}\right)^{N-1} dz\right] \tag{2.7}$$

Here, $\bar{\theta}^* \in [\underline{\theta}, \bar{\theta}]$ is a cost parameter such that $\Phi(\boldsymbol{Q^*}(\theta), \theta) \geq 0$ iff $\theta \in [\underline{\theta}, \bar{\theta}^*]$. In the mechanism, $\boldsymbol{Q^*}(\theta_i) \geq \boldsymbol{Q^*}(\theta_i')$ for any θ_i, θ_i' with $\theta_i < \theta_i'$.

The quasisupermodularity of the virtual surplus, which is trivially satisfied if quality is one-dimensional, plays a key role in determining the property of the optimal quality schedule $\boldsymbol{Q^*}$. The proof shows that a necessary and sufficient condition for the IC constraints (2.3) is given by the two conditions. With these conditions, we can rewrite the buyer's maximization problem as follows:

$$\max_{(\boldsymbol{Q_i}, X_i)_{i\in\{1,\dots,N\}}} \sum_{i=1}^{N} E\left[X_i(\boldsymbol{\theta})\Phi(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i)\right]$$

$$\text{s.t.}\quad \text{For each } \theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}] \text{ with } \theta < \hat{\theta} \text{ and each } i \in \{1, ...N\},$$

$$\int_{\theta}^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), z)]dz \geq 0. \quad (2.8)$$

The condition (2.8), which we call the *monotonicity condition*, is automatically satisfied if both $\boldsymbol{Q_i}$ and X_i are nonincreasing in θ_i. The proof shows that the quasisupermodularity of the virtual surplus together with Assumptions 2.4 and 2.5 implies that a quality schedule $\boldsymbol{Q_i}$ which maximizes the virtual surplus $\Phi(\boldsymbol{q}, \theta_i)$ is nonincreasing in θ_i. The proof also shows that the maximized virtual surplus $\Phi(\boldsymbol{Q}^*(\theta_i), \theta_i)$ is nonincreasing in θ_i. As a result, the quality schedule $\boldsymbol{Q}^*$ and the trading probability X_i^* defined in Lemma 2.2 satisfy the monotonicity condition (2.8). The optimal mechanism prescribes that the more efficient a supplier is, the higher levels of *all* quality attributes he is required to achieve. On the other hand, without the quasisupermodularity, the optimal quality schedule $\boldsymbol{Q}^*$ may be *nonmonotonic* in θ_i. Even if Q^{m*} is increasing in θ_i for some m, the monotonicity condition (2.8) can be satisfied when $c_\theta(\boldsymbol{Q}(\theta_i), z)$ is decreasing in θ_i. The following example shows that this is the case.[4]

Example 2.1. Assume that $M = 2$, $\mathcal{Q}^1 = \mathcal{Q}^2 = \{1, 2\}$, $v(q^1, q^2) = 0$ if $q^1 = q^2 = 1$ and $v(q^1, q^2) = 100$ otherwise. Assume also that $c(q^1, q^2, \theta) = 6\theta q^1 + (3\theta + 9)q^2 - q^1 q^2$, θ is uniformly distributed on $[1, 3]$, and $\alpha = 0$. Then, one can show that $\Phi(q^1, q^2, \theta) = v(q^1, q^2) - (12\theta - 6)q^1 - (6\theta + 6)q^2 + q^1 q^2$, and Φ is *not* quasisupermodular in quality. Actually, Φ is submodular in quality, i.e. $-\Phi$ is supermodular in quality. The following levels of quality attributes maximizes $\Phi(q^1, q^2, \theta)$ for any θ:

$$\boldsymbol{Q}^*(\theta) = (Q^{1*}(\theta), Q^{2*}(\theta)) := \begin{cases} (2, 1) & \text{if } \theta \in [1, 2) \\ (1, 2) & \text{if } \theta \in [2, 3] \end{cases}$$

The quality schedule $\boldsymbol{Q}^*$ together with X_i^* defined in Lemma 2.2 and $\bar{\theta}^* = \bar{\theta} = 3$ satisfies the monotonicity condition (2.8) because $c_\theta(\boldsymbol{Q}^*(\theta), z) = 15$ if $\theta \in [1, 2)$ and $c_\theta(\boldsymbol{Q}^*(\theta), z) = 12$ if $\theta \in [2, 3]$. Thus, $c_\theta(\boldsymbol{Q}^*(\theta), z)$ is decreasing in θ. Therefore, the optimal quality schedule $\boldsymbol{Q}^*$ is *nonmonotonic* in θ because Q^{1*} is decreasing whereas Q^{2*} is increasing.

Remark 2.1. We now give primitive conditions which guarantee that the virtual surplus is quasisupermodular in quality. For simplicity, suppose that there are two quality attributes. We also assume that the quality space $\mathcal{Q}^m$ is a compact interval for each m, the functions v and c are twice differentiable, and $\underline{\theta} > 0$. We specify the

[4]This can occur even in a single-agent screening model. See Laffont and Martimort (2002).

valuation function by $v(q^1, q^2) = v^1(q^1) + \beta q^1 q^2 + v^2(q^2)$, and the cost function by $c(q^1, q^2, \theta) = (c^1(q^1) + \gamma q^1 q^2 + c^2(q^2))\theta$. Here, $\beta \in \mathbb{R}$ and $\gamma \in \mathbb{R}$ represent the degrees of complementarity between the two quality attributes in terms of the valuation and the production cost, respectively. The virtual surplus is then given by

$$\Phi(q^1, q^2, \theta) = \left[\beta - \left(\theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}\right)\gamma\right] q^1 q^2 + \sum_{m=1}^{2}\left[v^m(q^m) - \left(\theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}\right)c^m(q^m)\right].$$

It follows from Topkis (1998) that, for any θ, Φ is supermodular on $\mathcal{Q}$ if and only if the coefficient of the cross term is nonnegative. Hence, if both the valuation function and the cost function exhibit complementarity (i.e., $\beta \geq 0$ and $\gamma \leq 0$), then the virtual surplus is supermodular and thus quasisupermodular in quality. More generally, if $\beta \geq \left(\theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}\right)\gamma$ for any θ, then the virtual surplus is quasisupermodular in quality.

For some practical example of quality attributes, we can judge whether the cost (or valuation) function exhibits complementarity or not. For instance, assume that quality consists of the durability of the highway and the maintenance service level after delivery. In the example, it is natural for the cost function to exhibit complementarity. Furthermore, it might be possible to estimate the complementarity parameters β and γ for each procurement project by empirical analysis.[5]

In a second step, we study the implementation of the optimal mechanism via a scoring auction. We say that a scoring rule s is *optimal* or *implements the optimal mechanism* if the auction game induced by s has a Bayesian Nash equilibrium which yields the same outcome as $(P_i^*, \boldsymbol{Q}^*, X_i^*)_{i\in\{1,\ldots,N\}}$ for any realization of $\boldsymbol{\theta}$. Lemmas 2.1 and 2.2 imply that the buyer's goal is to construct a scoring rule s so that each supplier's equilibrium offer $\boldsymbol{q}^*(\theta)$ is equal to $\boldsymbol{Q}^*(\theta)$ for any $\theta \in [\underline{\theta}, \bar{\theta}^*]$ and the inefficient suppliers ($\theta \in (\bar{\theta}^*, \bar{\theta}]$) are excluded by the reserve score. The next proposition demonstrates how the scoring rule should be constructed under the same condition as Lemma 2.2.

Proposition 2.1. *Suppose that the virtual surplus Φ is quasisupermodular in quality. Then, there exists an optimal scoring rule s^* which is supermodular in quality.*

In the construction of the optimal scoring rule s^*, we fully utilize the monotonicity of the optimal quality schedule $\boldsymbol{Q}^*$. The monotonicity is guaranteed by the quasisupermodularity of the virtual surplus in quality. Comparing the two problems (2.1) and (2.6), we see that if a weight α on profits is equal to one so that the buyer does not care about information rents, then the scoring rule s which is equal to her valuation v succeeds in the implementation. In general, however, we must carefully devise the optimal scoring rule. Figure 2.1 shows how the rule s^* is constructed. Suppose that quality is two-dimensional.

[5] In the environment where quality is one-dimensional, Tsuruoka (2013) identifies and estimates the cost function by using a dataset of price-only auctions in Japan. Then, using the method of counterfactual simulations, he quantifies the welfare gain of switching from price-only auctions to scoring auctions.

In the left figure, the solid curve represents the qualities which the optimal quality schedule $\boldsymbol{Q}^*$ requires suppliers to ensure for any type $\theta \in [\underline{\theta}, \bar{\theta}^*]$. Lemma 2.2 implies that this curve is upward sloping due to the monotonicity of $\boldsymbol{Q}^*$. The rule s^* is constructed so that a score remains constant even if, starting from any point on the curve, the level of only one attribute increases, and a score falls to zero if the level of at least one attribute is lower than $Q^{m*}(\bar{\theta}^*)$. Thus, any supplier who wishes to win has no incentive to make a quality offer other than offers on the solid curve. In the right figure, the three solid curves below the dotted curve represent the production costs for some cost parameters. These production costs have the "single-crossing property" due to the monotonicity of $\boldsymbol{Q}^*$ with the assumption that the incremental cost is increasing in a cost parameter. Thus, there exists a "lower envelope" of the cost curves shifted up, which is described by the dotted curve. The envelope is given by the following function of $\hat{\theta}$:

$$c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z)dz.$$

This is used as a score of the quality offer $\boldsymbol{Q}^*(\hat{\theta})$ with $\hat{\theta} \in [\underline{\theta}, \bar{\theta}^*]$. The second term is a score added to the production cost $c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta})$ to facilitate the separation of types. Then, a supplier of type θ optimally makes the quality offer $\boldsymbol{Q}^*(\hat{\theta}) = \boldsymbol{Q}^*(\theta)$ so as to maximize a quality score $s^*(\boldsymbol{Q}^*(\hat{\theta}))$ minus his production cost $c(\boldsymbol{Q}^*(\hat{\theta}), \theta)$.

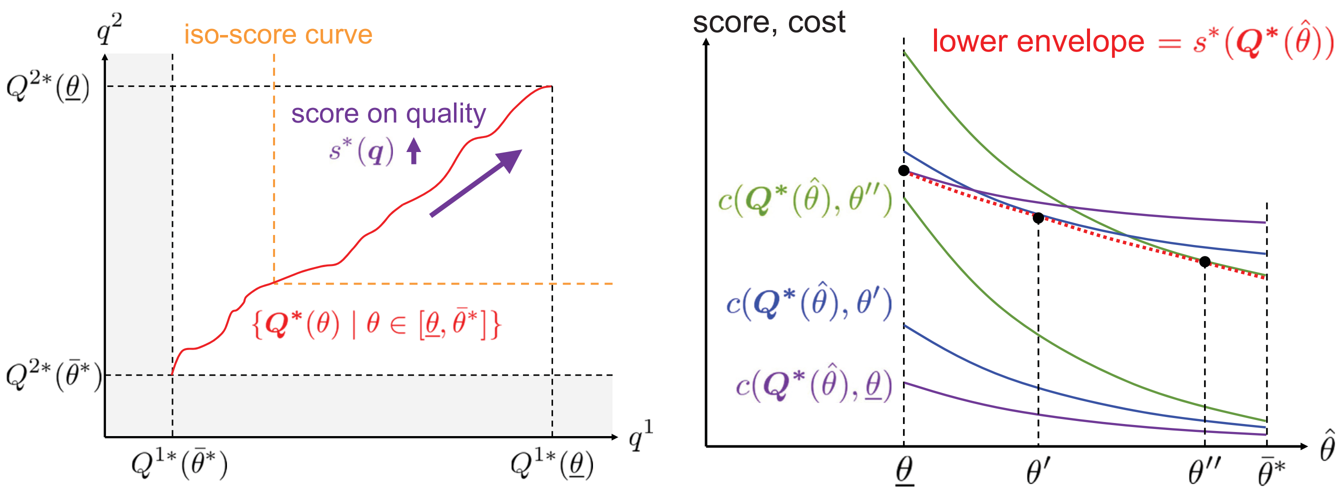

Figure 2.1: Construction of the optimal scoring rule

Proposition 2.1 states that the quasi-linear scoring rule s^*, which is supermodular in quality, implements the optimal mechanism. The supermodularity is derived from the Leontief-like shape of s^*. With the quasisupermodularity of the virtual surplus in quality, the buyer desires a more efficient supplier to achieve higher levels of *all* quality attributes. Then, a scoring rule which gives a score in a complementary way works well. On the other hand, without the quasisupermodularity, a supermodular scoring rule s may fail in the implementation. This problem arises from the nonmonotonicity of the optimal quality schedule, as shown in Example 2.1. The next example shows that this is the case.

Example 2.2. We make the same assumptions as Example 2.1. Suppose that a quasi-linear scoring rule s which is supermodular in quality implements the optimal mechanism. Because the optimal quality schedule satisfies $\boldsymbol{Q}^*(\theta) = (1, 2)$ for $\theta = 2$, Lemma 2.1 implies that $s(2, 2) - s(1, 2) \leq c(2, 2, \theta = 2) - c(1, 2, \theta = 2) = 10$. Now, $c(2, 1, \theta) - c(1, 1, \theta) = 6\theta - 1 > 10$ for any $\theta > 11/6$. Because s is supermodular in quality, $s(2, 1) - s(1, 1) \leq s(2, 2) - s(1, 2)$. Hence, $c(2, 1, \theta) - c(1, 1, \theta) > s(2, 1) - s(1, 1)$ for any $\theta > 11/6$, and thus a supplier of type $\theta \in (11/6, 2)$ never chooses $\boldsymbol{Q}^*(\theta) = (2, 1)$ in any equilibrium. This contradicts the hypothesis that s implements the optimal mechanism.

We construct the scoring rule s^* in a different way from Che (1993) (and Branco (1997)). We now show that a scoring rule *a la* Che (1993) which is extended to an environment where quality is multi-dimensional may fail in the implementation. Suppose first that quality is one-dimensional and a continuous variable, and the optimal quality schedule Q^* is decreasing in θ. Then, a scoring rule s constructed by Che is

$$s(q) = v(q) - (1 - \alpha) \int_0^q \frac{\partial^2 c}{\partial q \partial \theta}(y, (Q^*)^{-1}(y)) \frac{F((Q^*)^{-1}(y))}{f((Q^*)^{-1}(y))} dy.$$

Here, $(Q^*)^{-1}$ is the inverse function of Q^*. The rule underrewards quality relative to the valuation because the quality level which maximizes the social surplus $v(q) - c(q, \theta)$ is excessive from the buyer's viewpoint. With some differentiability assumptions and Inada conditions, the first-order condition of the problem $\max_q[s(q) - c(q, \theta)]$ is given by

$$\frac{dv}{dq}(q) - (1 - \alpha) \frac{\partial^2 c}{\partial q \partial \theta}(q, (Q^*)^{-1}(q)) \frac{F((Q^*)^{-1}(q))}{f((Q^*)^{-1}(q))} - \frac{\partial c}{\partial q}(q, \theta) = 0,$$

which is satisfied if $q = Q^*(\theta)$. Together with the assumptions in this chapter, the assumptions that v, $-c$ and $-c_\theta$ are concave in quality imply that s is also concave, as shown by Che (1993). Thus, $q = Q^*(\theta)$ is a global optimal solution to $\max_q[s(q) - c(q, \theta)]$.

Suppose now that quality is multi-dimensional. Consider the following scoring rule:

$$s(\boldsymbol{q}) = v(\boldsymbol{q}) - (1 - \alpha) \sum_{m=1}^{M} \int_0^{q^m} \frac{\partial^2 c}{\partial q^m \partial \theta}(y, \boldsymbol{Q}^{-\boldsymbol{m}*}(\theta^m(y)), \theta^m(y)) \frac{F(\theta^m(y))}{f(\theta^m(y))} dy. \tag{2.9}$$

Here, θ^m is the inverse function of Q^{m*}. With some differentiability assumptions and Inada conditions, the first-order conditions of $\max_{\boldsymbol{q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ are given by

$$\frac{\partial v}{\partial q^m}(\boldsymbol{q}) - (1 - \alpha) \frac{\partial^2 c}{\partial q^m \partial \theta}(q^m, \boldsymbol{Q}^{-\boldsymbol{m}*}(\theta^m(q^m)), \theta^m(q^m)) \frac{F(\theta^m(q^m))}{f(\theta^m(q^m))} - \frac{\partial c}{\partial q^m}(\boldsymbol{q}, \theta) = 0$$

for each m The conditions are satisfied if $\boldsymbol{q} = \boldsymbol{Q}^*(\theta)$. However, even if v, $-c$ and $-c_\theta$ are concave in quality, the scoring rule s defined by (2.9) may *not* be concave and cannot implement the optimal mechanism. The next example shows that this is the case.

Example 2.3. Assume that $M = 2$, $\mathcal{Q}^1 = \mathcal{Q}^2 = [0, 1]$, $v(\boldsymbol{q}) = q^1 + q^2 + 100$, $c(\boldsymbol{q}, \theta) = \theta((q^1)^2 + (q^2)^2) + q^1 q^2$, θ is uniformly distributed on $[1, 2]$, and $\alpha = 0$. Notice that because v is linear in $\boldsymbol{q}$ and both $-c$ and $-c_\theta$ are strictly concave in $\boldsymbol{q}$, Φ is strictly concave in $\boldsymbol{q}$. One can show that $\Phi(\boldsymbol{q}, \theta) = (q^1 + q^2) + 100 - (2\theta - 1)((q^1)^2 + (q^2)^2) - q^1 q^2$, $\boldsymbol{Q}^*(\theta) = (1/(4\theta - 1), 1/(4\theta - 1))$, and $\bar{\theta}^* = \bar{\theta} = 2$. The scoring rule s defined by (2.9) is

$$s(\boldsymbol{q}) = v(\boldsymbol{q}) - \sum_{m=1}^{2} \int_0^{q^m} 2y \left(\frac{1}{4y} - \frac{3}{4} \right) dy = \frac{1}{2}(q^1 + q^2) + \frac{3}{4}((q^1)^2 + (q^2)^2) + 100.$$

Then, for some θ, $s - c$ is *not* concave in $\boldsymbol{q}$. Hence, the quality offer $\boldsymbol{Q}^*(\theta) = (1/(4\theta - 1), 1/(4\theta - 1))$ may not be a global optimal solution to $\max_{\boldsymbol{q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$. Actually, for $\theta = 1$, $s(\boldsymbol{Q}^*(\theta)) - c(\boldsymbol{Q}^*(\theta), \theta) = 100 + 1/6 < 100 + 1/4 = s(1, 0) - c(1, 0, \theta)$.

2.5 Additively Separable Scoring Rule

In this section, we investigate whether a scoring rule which is additively separable in some quality attributes can implement the optimal mechanism.

The scoring rule s^* constructed in Proposition 2.1 is supermodular in quality. Then, a question arises: Can an additively separable scoring rule such as "weighted criteria" mentioned in the Introduction implement the optimal mechanism? To answer this question, we introduce a definition. We assume that the set of quality attributes $\{1, ..., M\}$ is partitioned into two nonempty subsets $\mathcal{M}^1$ and $\mathcal{M}^2$ of criteria.[6] A quasi-linear scoring rule s is *additively separable* if the rule takes a form of $s(\boldsymbol{q}) = s^1(\boldsymbol{q^1}) + s^2(\boldsymbol{q^2})$ for each $\boldsymbol{q} = (\boldsymbol{q^1}, \boldsymbol{q^2})$ with $\boldsymbol{q^1} \in \times_{m \in \mathcal{M}^1} \mathcal{Q}^m$ and $\boldsymbol{q^2} \in \times_{m \in \mathcal{M}^2} \mathcal{Q}^m$. In the next example, an additively separable scoring rule can *never* implement the optimal mechanism.

Example 2.4. Assume that $M = 2$, $\mathcal{Q}^1 = \mathcal{Q}^2 = \{1, 2\}$, $v(q^1, q^2) = 9(q^1 + q^2) + 100$, $c(q^1, q^2, \theta) = 3\theta(q^1 + q^2) + (3 - \theta) q^1 q^2$, θ is uniformly distributed on $[1, 3]$, and $\alpha = 0$. Then, one can show that $\Phi(q^1, q^2, \theta) = (12 - 6\theta)(q^1 + q^2) - (4 - 2\theta) q^1 q^2 + 100$, and Φ is quasisupermodular in quality. The optimal quality schedule is given by

$$\boldsymbol{Q}^*(\theta) = (Q^{1*}(\theta), Q^{2*}(\theta)) := \begin{cases} (2, 2) & \text{if } \theta \in [1, 2) \\ (1, 1) & \text{if } \theta \in [2, 3] \end{cases}$$

and $\bar{\theta}^* = \bar{\theta} = 3$. Suppose that a quasi-linear scoring rule s which is additively separable implements the optimal mechanism. Because $\boldsymbol{Q}^*(\theta) = (1, 1)$ for $\theta = 2$, Lemma 2.1 implies that $s(1, 2) - s(1, 1) \leq c(1, 2, \theta = 2) - c(1, 1, \theta = 2) = 7$. Now, $c(2, 2, \theta) - c(2, 1, \theta) = 6 + \theta > 7$ for any $\theta > 1$. Hence, $c(2, 2, \theta) - c(2, 1, \theta) > 7 \geq s(1, 2) - s(1, 1) = [s^1(1) -$

[6] Our analysis can be extended to the case of more than two subsets at the expense of notational complexity.

$s^1(1)] + [s^2(2) - s^2(1)] = [s^1(2) - s^1(2)] + [s^2(2) - s^2(1)] = s(2,2) - s(2,1)$ for any $\theta > 1$, and thus a supplier of type $\theta \in (1,2)$ never chooses $\boldsymbol{Q}^*(\theta) = (2,2)$ in any equilibrium. This contradicts the hypothesis that s implements the optimal mechanism.

We now provide a class of scoring rules which implement the optimal mechanism in this example. Consider a scoring rule given by a CES function $s(q^1, q^2) = 15(\frac{1}{2}(q^1)^\rho + \frac{1}{2}(q^2)^\rho)^{1/\rho}$, where $\rho \in \mathbb{R} \setminus \{0\}$ is a constant elasticity of substitution. When $\rho = 1$, the rule s is additively separable in quality. Hence, the above argument shows that the rule with $\rho = 1$ cannot implement the optimal mechanism. Since $s(2,2) - s(1,1) = 15$ for any ρ, any supplier with $\theta \in [1,2)$ has no incentive to deviate from $(q^1, q^2) = (2,2)$ to $(1,1)$, and any supplier with $\theta \in [2,3]$ has no incentive to deviate from $(q^1, q^2) = (1,1)$ to $(2,2)$. Finally, the scoring rule s converges to the Leontief function $15 \min\{q^1, q^2\}$ as $\rho \to -\infty$, so that if ρ is sufficiently small, then the rule s implements the optimal mechanism.

What is the cause of the failure of additively separable rules to implement the optimal mechanism? One can find the answer in the functional form of the production cost in the above example. For any type except $\theta = 3$, the incremental cost for each attribute is increasing in the level of the other attribute. That is, the two quality attributes are substitutable in terms of the production cost. Now, the set of types can be partitioned into two groups: From the buyer's point of view, any type in the efficient group $[1,2)$ should offer the greatest quality $(2,2)$, and any type in the inefficient group $[2,3]$ should offer the least quality $(1,1)$. Then, given any additively separable rule which deters the inefficient suppliers' deviations, an incentive for a supplier of type $\theta = 2$ to deviate from the quality offer $(1,1)$ to $(1,2)$ is weaker than that for a supplier of type $\theta \in (1,2)$ to deviate from the quality offer $(2,2)$ to $(2,1)$ because of the substitutability. This means that any additively separable rule causes deviations of some types in either group. As one would expect, if the production cost has decreasing differences in (q^1, q^2), then we can construct an additively separable scoring rule which is immune to the deviation.

We can obtain the more general results although the underlying structure is the same as Example 2.4. Here, the notion of increasing (decreasing) differences expresses the concept of complementarity (substitutability) between some quality attributes in terms of the production cost. The following proposition gives a necessary condition and a sufficient condition for the existence of an additively separable scoring rule which implements the optimal mechanism.

Proposition 2.2. *Suppose that the virtual surplus* Φ *is quasisupermodular. Then:*
(i) If c *has increasing differences in* $(\boldsymbol{q^1}, \boldsymbol{q^2})$ *and there exists* $(m, m') \in \mathcal{M}^1 \times \mathcal{M}^2$ *such that* c *has strictly increasing differences in* $(q^m, q^{m'})$, $Q^{m*}(\theta-) > Q^{m*}(\theta+)$ *and* $Q^{m'*}(\theta-) > Q^{m'*}(\theta+)$ *for some* $\theta \in (\underline{\theta}, \bar{\theta}^*)$, *then there is no additively separable scoring rule which implements the optimal mechanism.*
(ii) If c *has decreasing differences in* $(\boldsymbol{q^1}, \boldsymbol{q^2})$ *and* $\mathcal{Q}^m$ *is finite for each* m, *then there is*

an additively separable scoring rule which implements the optimal mechanism.

This proposition has several implications. First, the implementation possibility via an additively separable scoring rule heavily depends on whether the cost function has increasing differences or decreasing differences in quality attributes in the distinct sets of subcriteria, rather than the property of the value function. In particular, Example 2.4 shows that an additively separable scoring rule cannot implement the optimal mechanism even if the value function is additively separable in quality.

Second, the results of Proposition 2.2 provide a useful guide to designing additively separable scoring rules. Consider an example of the highway construction. Suppose that the quality attributes represent delivery date ($m = 1$), durability of the highway ($m = 2$), maintenance service after delivery ($m = 3$), respectively. Moreover, we consider the following plausible scenario: as the delivery date is earlier, it is more costly to increase the durability level; as the durability level is higher, it is less costly to increase the level of maintenance service; the incremental cost for the maintenance service is independent of the delivery date. That is, the production cost has strictly increasing differences in (q^1, q^2), decreasing differences in (q^2, q^3), and is additively separable in (q^1, q^3). Thus, the cost function has decreasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with $\mathcal{M}^1 = \{1, 2\}$ and $\mathcal{M}^2 = \{3\}$ whereas the cost function has increasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with $\mathcal{M}^1 = \{1\}$ and $\mathcal{M}^2 = \{2, 3\}$. Proposition 2.2 then implies that the buyer should classify quality attributes so that $s(\boldsymbol{q}) = s^1(q^1, q^2) + s^2(q^3)$ not $s(\boldsymbol{q}) = s^1(q^1) + s^2(q^2, q^3)$. When the buyer establishes some sets of subcriteria to use the additively separable rule, each pair of quality attributes in the distinct sets should be complementary in terms of the production cost.

2.6 Concluding Remarks

We have studied the optimal design of scoring auctions in an environment where quality is multi-dimensional. Our main result shows that if the virtual surplus is quasisupermodular in quality, then there exists an optimal scoring rule which is supermodular in quality. Thus, when the virtual surplus exhibits a kind of complementarity between quality attributes, a scoring rule which gives a quality score in a complementary way works well. This in turn implies that the buyer should carefully design scoring rules which are additively separable in some quality attributes.

One may wonder why suppliers should offer *all* quality attributes in a scoring auction. Alternatively, the buyer can require suppliers to offer only one quality attribute (with price). Then, with a scoring rule *a la* Che (1993), the most efficient supplier achieves the highest score, and the winner's type is revealed to the buyer. If the levels of the remaining quality attributes are properly specified based on the winner's type, then this modified auction mechanism can implement the optimal mechanism. The mechanism or procedure, however, requires the buyer to prespecify the levels of the remaining quality attributes for any cost parameter. In practice, it may be prohibitively costly for the buyer to do so. For instance, the buyer may not be able to specify aesthetic and functional characteristics of highway although suppliers with expertise can offer these characteristics. Therefore, when there are at least two such quality attributes, this study is of significance.

Finally, we should point out a potential limitation of the analysis. In our model, a more efficient supplier (i.e. a supplier with less cost parameter) has a superior technology of increasing the levels of *all* quality attributes. Thus, our model does not cover the following case: one type of supplier has a superior technology of increasing the level of one quality attribute to another type of supplier whereas the latter type has a superior technology of increasing the level of another quality attribute to the former type. To cover the case, we need to allow the supplier's type to be multi-dimensional. It is interesting and challenging to study the optimal design of scoring auctions in an environment where both quality and a supplier's type are multi-dimensional. This is left for future research.

2.7 Appendix

Proof of Lemma 2.1. (i) First, note that $\arg\max_{\boldsymbol{q}\in\mathcal{Q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ is nonempty for any θ because $\mathcal{Q}$ is compact and $s - c$ is upper semicontinuous in $\boldsymbol{q}$. (See, for example, Kolmogorov and Fomin (1975).) We now show that in equilibrium a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^s]$ never offers $(p', \boldsymbol{q'})$ such that $\boldsymbol{q'} \notin \arg\max_{\hat{\boldsymbol{q}}\in\mathcal{Q}}[s(\hat{\boldsymbol{q}}) - c(\hat{\boldsymbol{q}}, \theta)]$. Suppose, on the contrary, that a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^s]$ makes such an offer $(p', \boldsymbol{q'})$. Consider another offer $(p, \boldsymbol{q})$ such that $\boldsymbol{q} \in \arg\max_{\hat{\boldsymbol{q}}\in\mathcal{Q}}[s(\hat{\boldsymbol{q}}) - c(\hat{\boldsymbol{q}}, \theta)]$ and $s(\boldsymbol{q}) - p = s(\boldsymbol{q'}) - p'$. The score of $(p, \boldsymbol{q})$ is equal to that of $(p', \boldsymbol{q'})$, so that both offers yield the same winning probability given the other suppliers' strategies. The supplier's expected profit from $(p', \boldsymbol{q'})$ is not higher than his expected profit from $(p, \boldsymbol{q})$ because

$$
\begin{aligned}
&[p' - c(\boldsymbol{q'}, \theta)]Prob[win \mid S(p', \boldsymbol{q'})] \\
&\leq [p' - c(\boldsymbol{q'}, \theta) + (s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta) - (s(\boldsymbol{q'}) - c(\boldsymbol{q'}, \theta)))]Prob[win \mid S(p', \boldsymbol{q'})] \\
&= [p - c(\boldsymbol{q}, \theta)]Prob[win \mid S(p, \boldsymbol{q})],
\end{aligned}
$$

where the inequality follows from the hypothesis that $\boldsymbol{q'} \notin \arg\max_{\hat{\boldsymbol{q}}\in\mathcal{Q}}[s(\hat{\boldsymbol{q}}) - c(\hat{\boldsymbol{q}}, \theta)] \ni \boldsymbol{q}$, and the equality follows from the construction of $(p, \boldsymbol{q})$. The inequality is strict if $Prob[win \mid S(p, \boldsymbol{q})] > 0$, which occurs for some strategies of the other suppliers. This contradicts the assumption that no supplier uses weakly dominated strategies. Thus, the latter statement holds, and we can assume that a symmetric equilibrium bidding strategy $(p, \boldsymbol{q}^*)$ satisfies $\boldsymbol{q}^*(\theta) \in \arg\max_{\boldsymbol{q}\in\mathcal{Q}}[s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ for any $\theta \in [\underline{\theta}, \bar{\theta}^s]$.

Second, consider the following change of variable: $k(\theta) := s(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta)$ for any $\theta \in [\underline{\theta}, \bar{\theta}^s]$. Because c_θ is continuous and thus bounded on $[\underline{\theta}, \bar{\theta}]$, it follows from the integral form envelope theorem of Milgrom and Segal (2002) (see also Theorem 3.1 of Milgrom (2004)) that k is absolutely continuous, and is given by

$$k(\theta) = k(\underline{\theta}) - \int_{\underline{\theta}}^{\theta} c_\theta(\boldsymbol{q}^*(z), z)dz.$$

Finally, we show that the bidding strategy $(p^*, \boldsymbol{q}^*)$ in the lemma constitutes a symmetric equilibrium. For each $\theta \in [\underline{\theta}, \bar{\theta}^s]$, the score $s(\boldsymbol{q}^*(\theta)) - p^*(\theta)$ is given by

$$\begin{aligned}
s(\boldsymbol{q}^*(\theta)) - p^*(\theta) &= s(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta) - [p^*(\theta) - c(\boldsymbol{q}^*(\theta), \theta)] \\
&= k(\theta) - \int_{\theta}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta)}dz.
\end{aligned}$$

Note that $s(\boldsymbol{q}^*(\bar{\theta}^s)) - p^*(\bar{\theta}^s) = s(\boldsymbol{q}^*(\bar{\theta}^s)) - c(\boldsymbol{q}^*(\bar{\theta}^s), \bar{\theta}^s) = 0$ by assumption. Because k is continuous in $\theta \in [\underline{\theta}, \bar{\theta}^s]$, so is $s(\boldsymbol{q}^*(\theta)) - p^*(\theta)$. Moreover, the score $s(\boldsymbol{q}^*(\theta)) - p^*(\theta)$ is decreasing in $\theta \in [\underline{\theta}, \bar{\theta}^s]$ because for any $\theta, \theta' \in [\underline{\theta}, \bar{\theta}^s]$ with $\theta < \theta'$,

$$\begin{aligned}
&[s(\boldsymbol{q}^*(\theta)) - p^*(\theta)] - [s(\boldsymbol{q}^*(\theta')) - p^*(\theta')] \\
&= (k(\theta) - k(\theta')) - \left[\int_{\theta}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta)}dz - \int_{\theta'}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta')}dz\right] \\
&> (k(\theta) - k(\theta')) - \int_{\theta}^{\theta'} c_\theta(\boldsymbol{q}^*(z), z)\frac{1 - F_{(N-1)}(z)}{1 - F_{(N-1)}(\theta)}dz \\
&> (k(\theta) - k(\theta')) - \int_{\theta}^{\theta'} c_\theta(\boldsymbol{q}^*(z), z)dz \\
&= \int_{\theta}^{\theta'} \left[c_\theta(\boldsymbol{q}^*(z), z) - c_\theta(\boldsymbol{q}^*(z), z)\right] dz = 0.
\end{aligned}$$

When the other suppliers follow the strategy $(p^*, \boldsymbol{q}^*)$, the expected profit of a supplier of type $\theta \in [\underline{\theta}, \bar{\theta}^s]$ from offering $(p, \boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta)) - p = s(\boldsymbol{q}^*(\hat{\theta})) - p^*(\hat{\theta})$ for some $\hat{\theta} \in [\underline{\theta}, \bar{\theta}^s]$ can be written as

$$\begin{aligned}
&[p - c(\boldsymbol{q}^*(\theta), \theta)]Prob[win \mid S(p, \boldsymbol{q}^*(\theta))] \\
&= [s(\boldsymbol{q}^*(\theta)) - c(\boldsymbol{q}^*(\theta), \theta) - s(\boldsymbol{q}^*(\hat{\theta})) + p^*(\hat{\theta})]Prob[win \mid S(p^*(\hat{\theta}), \boldsymbol{q}^*(\hat{\theta}))] \\
&= [k(\theta) - s(\boldsymbol{q}^*(\hat{\theta})) + p^*(\hat{\theta})](1 - F_{(N-1)}(\hat{\theta})) \\
&= (k(\theta) - k(\hat{\theta}))(1 - F_{(N-1)}(\hat{\theta})) + \int_{\hat{\theta}}^{\bar{\theta}^s} c_\theta(\boldsymbol{q}^*(z), z)(1 - F_{(N-1)}(z))dz.
\end{aligned}$$

The second equality follows from the observation that the score $S(p^*(\hat{\theta}), \boldsymbol{q}^*(\hat{\theta})) = s(\boldsymbol{q}^*(\hat{\theta})) - p^*(\hat{\theta})$ is decreasing in $\hat{\theta}$. The supplier cannot obtain a higher expected profit by deviating from $(p^*(\theta), \boldsymbol{q}^*(\theta))$ to $(p, \boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta)) - p = s(\boldsymbol{q}^*(\hat{\theta})) - p^*(\hat{\theta})$ because the difference between the expected profits is given by

$$\begin{aligned}
&-(k(\theta)-k(\hat{\theta}))(1-F_{(N-1)}(\hat{\theta}))+\int_{\theta}^{\hat{\theta}} c_\theta(\boldsymbol{q}^*(z),z)(1-F_{(N-1)}(z))dz \\
&=\int_{\theta}^{\hat{\theta}}\left[-c_\theta(\boldsymbol{q}^*(z),z)(1-F_{(N-1)}(\hat{\theta}))+c_\theta(\boldsymbol{q}^*(z),z)(1-F_{(N-1)}(z))\right]dz \\
&=\int_{\theta}^{\hat{\theta}} c_\theta(\boldsymbol{q}^*(z),z)(F_{(N-1)}(\hat{\theta})-F_{(N-1)}(z))dz \geq 0.
\end{aligned}$$

It is easy to show that the supplier cannot obtain a higher expected profit by deviating from $(p^*(\theta),\boldsymbol{q}^*(\theta))$ to $(p,\boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta))-p\notin[0,s(\boldsymbol{q}^*(\underline{\theta}))-p^*(\underline{\theta})]$. Also, a supplier of type $\theta\in(\bar{\theta}^s,\bar{\theta}]$ obtains a negative expected profit if he offers $(p,\boldsymbol{q})$ such that $s(\boldsymbol{q})-p\geq 0$ whereas he obtains zero profit by offering $(p^*(\theta),\boldsymbol{q}^*(\theta))$ such that $s(\boldsymbol{q}^*(\theta))-p^*(\theta)<0$. This completes the proof of part (i).

(ii) In the above equilibrium, the score $s(\boldsymbol{q}^*(\theta_i))-p^*(\theta_i)$ is decreasing in $\theta_i\in[\underline{\theta},\bar{\theta}^s]$ with $s(\boldsymbol{q}^*(\bar{\theta}^s))-p^*(\bar{\theta}^s)=0$, and is negative for any $\theta_i\in(\bar{\theta}^s,\bar{\theta}]$. Thus, a supplier of type θ_i wins only if $\theta_i=\min\{\theta_1,...,\theta_N,\bar{\theta}^s\}$. □

Proof of Lemma 2.2. (i) We show that a necessary and sufficient condition for the IC constraints (2.3) is given by the two conditions: *envelope condition* and *monotonicity condition*. We say that a direct mechanism ρ satisfies the envelope condition if for any i and θ,

$$\Pi_i^\rho(\theta)=\Pi_i^\rho(\bar{\theta})+\int_{\theta}^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z,\boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(z,\boldsymbol{\theta_{-i}}),z)]dz.$$

Here, $\Pi_i^\rho(\theta):=\Pi_i^\rho(\theta\mid\theta)$. We say that a direct mechanism ρ satisfies the monotonicity condition if for any i, θ and $\hat{\theta}$ with $\hat{\theta}>\theta$,

$$\int_{\theta}^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z,\boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(z,\boldsymbol{\theta_{-i}}),z)-X_i(\hat{\theta},\boldsymbol{\theta_{-i}})c_\theta(\boldsymbol{Q_i}(\hat{\theta},\boldsymbol{\theta_{-i}}),z)]dz\geq 0.$$

Because c has (strictly) increasing differences in $(\boldsymbol{q},\theta)$, it must hold that for any $\boldsymbol{q},\boldsymbol{q'}\in\mathcal{Q}$ with $\boldsymbol{q}\leq\boldsymbol{q'}$ and any $z,z'\in[\underline{\theta},\bar{\theta}]$ with $z\leq z'$, $c(\boldsymbol{q'},z)-c(\boldsymbol{q},z)\leq c(\boldsymbol{q'},z')-c(\boldsymbol{q},z')$, and thus $c_\theta(\boldsymbol{q'},z)\geq c_\theta(\boldsymbol{q},z)>0$. Therefore, if both $X_i(\theta_i,\boldsymbol{\theta_{-i}})$ and $\boldsymbol{Q_i}(\theta_i,\boldsymbol{\theta_{-i}})$ are nonincreasing in θ_i for each $\boldsymbol{\theta_{-i}}$, then the monotonicity condition is automatically satisfied.

First, we prove sufficiency. Suppose that supplier i's IC constraint is not satisfied. Then, there exist θ and $\hat{\theta}$ such that $\Pi_i^\rho(\hat{\theta}\mid\theta)>\Pi_i^\rho(\theta)$. Hence, $E_{\boldsymbol{\theta_{-i}}}[X_i(\hat{\theta},\boldsymbol{\theta_{-i}})c(\boldsymbol{Q_i}(\hat{\theta},\boldsymbol{\theta_{-i}}),\hat{\theta})-X_i(\hat{\theta},\boldsymbol{\theta_{-i}})c(\boldsymbol{Q_i}(\hat{\theta},\boldsymbol{\theta_{-i}}),\theta)]>\Pi_i^\rho(\theta)-\Pi_i^\rho(\hat{\theta})$ by definition of Π_i^ρ. Rewriting the left-hand side as the definite integral and applying the envelope condition to the right-hand side, we obtain

$$\int_{\theta}^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), z)] dz > \int_{\theta}^{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz.$$

This contradicts the monotonicity condition.

Next, we prove necessity. Using the integral form envelope theorem of Milgrom and Segal (2002), the IC constraints (2.3) imply that for any i and θ, $\Pi_i^\rho(\theta) = \max_{\hat{\theta}} E_{\boldsymbol{\theta_{-i}}}[P_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \theta)]$ is given by

$$\Pi_i^\rho(\theta) = \Pi_i^\rho(\bar{\theta}) - \int_{\theta}^{\bar{\theta}} \frac{\partial \Pi_i^\rho}{\partial \theta}(z \mid z) dz = \Pi_i^\rho(\bar{\theta}) + \int_{\theta}^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz.$$

We thus obtain the envelope condition. Also, the IC constraints (2.3) imply that for any i, θ and $\hat{\theta}$, $E_{\boldsymbol{\theta_{-i}}}[X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \hat{\theta}) - X_i(\hat{\theta}, \boldsymbol{\theta_{-i}}) c(\boldsymbol{Q_i}(\hat{\theta}, \boldsymbol{\theta_{-i}}), \theta)] \leq \Pi_i^\rho(\theta) - \Pi_i^\rho(\hat{\theta})$. Rewriting the left-hand side as the definite integral and applying the envelope condition to the right-hand side, we obtain the monotonicity condition.

(ii) We solve the optimization problem. The IC constraints (2.3) imply that for each i, $\Pi_i^\rho(\theta)$ is nonincreasing in θ because $\Pi_i^\rho(\theta) \geq \Pi_i^\rho(\theta' \mid \theta) \geq \Pi_i^\rho(\theta')$ for each $\theta < \theta'$. The second inequality follows from the assumption that c is increasing in θ_i. Hence, the IR constraints (2.4) are replaced by $\Pi_i^\rho(\bar{\theta}) \geq 0$ for each i. Using the result (i), the IC constraints (2.3) are replaced by the envelope and monotonicity conditions. By the envelope condition and the interchange of the order of integration, $E[\Pi_i^\rho(\theta_i)]$ is given by

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \Pi_i^\rho(\theta_i) f(\theta_i) d\theta_i &= \Pi_i^\rho(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta_i}^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] dz f(\theta_i) d\theta_i \\
&= \Pi_i^\rho(\bar{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} E_{\boldsymbol{\theta_{-i}}}[X_i(z, \boldsymbol{\theta_{-i}}) c_\theta(\boldsymbol{Q_i}(z, \boldsymbol{\theta_{-i}}), z)] \frac{F(z)}{f(z)} f(z) dz.
\end{aligned}$$

Hence, the buyer's objective function is rewritten as

$$\begin{aligned}
&\sum_{i=1}^N E\left[X_i(\boldsymbol{\theta}) v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - P_i(\boldsymbol{\theta}) + \alpha \Pi_i^\rho(\theta_i)\right] \\
&= \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta})[v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - c(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i)] - (1-\alpha)\Pi_i^\rho(\theta_i)\right] \\
&= \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta})\left[v(\boldsymbol{Q_i}(\boldsymbol{\theta})) - c(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i) - (1-\alpha) c_\theta(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i) \frac{F(\theta_i)}{f(\theta_i)}\right] - (1-\alpha)\Pi_i^\rho(\bar{\theta})\right] \\
&= \sum_{i=1}^N E\left[X_i(\boldsymbol{\theta}) \Phi(\boldsymbol{Q_i}(\boldsymbol{\theta}), \theta_i) - (1-\alpha)\Pi_i^\rho(\bar{\theta})\right].
\end{aligned}$$

Note that $\arg\max_{\boldsymbol{q} \in \mathcal{Q}} \Phi(\boldsymbol{q}, \theta_i)$ is nonempty for any θ_i because $\mathcal{Q}$ is compact and Φ is continuous in $\boldsymbol{q}$. The above objective function is maximized when $\Pi_i^\rho(\bar{\theta}) = 0$, and $\boldsymbol{Q_i}(\boldsymbol{\theta})$ and $X_i(\boldsymbol{\theta})$ are respectively given by $\boldsymbol{Q}^*(\theta_i)$ and $X_i^*(\boldsymbol{\theta})$ in the lemma. This is because $\boldsymbol{Q}^*(\theta_i)$ maximizes $\Phi(\boldsymbol{q}, \theta_i)$ and the maximized value $\Phi(\boldsymbol{Q}^*(\theta_i), \theta_i)$ is decreasing in θ_i. The latter fact follows from $\Phi(\boldsymbol{Q}^*(\theta), \theta) \geq \Phi(\boldsymbol{Q}^*(\theta'), \theta) > \Phi(\boldsymbol{Q}^*(\theta'), \theta')$ for any $\theta < \theta'$. The second inequality follows from the assumptions that c is increasing in θ_i, and both c_θ and F/f are increasing in θ_i.

Finally, we show that the direct mechanism $\rho^* = (P_i^*, \boldsymbol{Q_i^*}, X_i^*)_{i \in \{1,\dots,N\}}$ satisfies the ignored monotonicity condition. Now, Φ is quasisupermodular in $\boldsymbol{q}$ by hypothesis, and has strictly increasing differences in $(\boldsymbol{q}, -\theta_i)$ from Assumptions 2.4 and 2.5. It then follows from Theorem 4' of Milgrom and Shannon (1994) that $\boldsymbol{Q}^*(\theta_i) \geq \boldsymbol{Q}^*(\theta_i')$ for any $\theta_i < \theta_i'$. Also, X_i^* is decreasing in θ_i. These facts imply that ρ^* satisfies the monotonicity condition. □

Proof of Proposition 2.1. (i) We first show that there exists a quasi-linear scoring rule which implements the optimal mechanism. From Lemmas 2.1 and 2.2, it suffices to show that there exists an upper semicontinuous function $s^* : \mathcal{Q} \to \mathbb{R}$ which satisfies $\boldsymbol{Q}^*(\theta) \in \arg\max_{\boldsymbol{q} \in \mathcal{Q}}[s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$ for any $\theta \in [\underline{\theta}, \bar{\theta}^*]$ and $s^*(\boldsymbol{Q}^*(\bar{\theta}^*)) - c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*) = 0$. Lemma 2.2 implies that Q^{m*} is nonincreasing in θ. Hence, $Q^{m*} : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}$ can have no more than countably many points of discontinuity. Let $\{\theta^1, \theta^2, \dots, \theta^l, \dots\}$ be the discontinuous points of $\boldsymbol{Q}^*$ in $[\underline{\theta}, \bar{\theta}^*]$. For each l with $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$, there exist some m such that $Q^{m*}(\theta^l+) := \lim_{\theta \to \theta^l+} Q^{m*}(\theta) \leq Q^{m*}(\theta^l) \leq \lim_{\theta \to \theta^l-} Q^{m*}(\theta) =: Q^{m*}(\theta^l-)$ with either or both inequalities being strict.

First, we define a function $\sigma : [\underline{\theta}, \bar{\theta}^*] \to \mathbb{R}$ as follows: For any $\theta \in [\underline{\theta}, \bar{\theta}^*]$,

$$\sigma(\theta) = c(\boldsymbol{Q}^*(\theta), \theta) + \int_\theta^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz.$$

Note that if $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta')$, and thus $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta'')$ for any $\theta'' \in [\theta, \theta']$, then $\sigma(\theta) = \sigma(\theta')$. Using the function σ, we construct s^* in the following way. (a) If $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\bar{\theta}^*)$, then let $s^*(\boldsymbol{q}) = 0$. (b) If there exists $\theta' \in (\underline{\theta}, \bar{\theta}^*) \setminus \{\theta^1, \theta^2, \dots, \theta^l, \dots\}$ or $\theta' \in \{\underline{\theta}, \bar{\theta}^*\}$ such that $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta')$ and there exists no $\theta < \theta'$ which satisfies $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta) > \boldsymbol{Q}^*(\theta')$, then let $s^*(\boldsymbol{q}) = \sigma(\theta')$.[7] (c) If there exists l such that $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$, $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta^l+)$ and $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\theta^l)$, then let $s^*(\boldsymbol{q}) = \sigma(\theta^l+)$. (d) If there exists l such that $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$, $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta^l)$ and $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\theta^l-)$, then let $s^*(\boldsymbol{q}) = \sigma(\theta^l)$. (e) If there exists l such that $\theta^l \in (\underline{\theta}, \bar{\theta}^*)$ and there exists no $\theta < \theta^l$ which satisfies $\boldsymbol{q} \geq \boldsymbol{Q}^*(\theta) > \boldsymbol{Q}^*(\theta^l-)$, then let $s^*(\boldsymbol{q}) = \sigma(\theta^l-)$.

Second, we show that $s^*(\boldsymbol{Q}^*(\bar{\theta}^*)) - c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*) = 0$ and s^* is upper semicontinuous on $\mathcal{Q}$. The former is trivial because $s^*(\boldsymbol{Q}^*(\bar{\theta}^*)) = \sigma(\bar{\theta}^*) = c(\boldsymbol{Q}^*(\bar{\theta}^*), \bar{\theta}^*)$. By construction, the function σ is continuous in $[\underline{\theta}, \bar{\theta}^*] \setminus \{\theta^1, \theta^2, \dots, \theta^l, \dots\}$. Also, $\sigma(\theta) = \sigma(\theta')$ for any

[7] For some $\boldsymbol{q}$, there may exist another $\theta (\neq \theta')$ which satisfies the condition. However, because it then follows that $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta')$, the equality $s^*(\boldsymbol{q}) = \sigma(\theta) = \sigma(\theta')$ holds.

$\theta, \theta' \in [\underline{\theta}, \bar{\theta}^*]$ with $\theta < \theta'$ if $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta')$ and thus $\boldsymbol{Q}^*(\theta) = \boldsymbol{Q}^*(\theta'') = \boldsymbol{Q}^*(\theta')$ for any $\theta'' \in [\theta, \theta']$. Hence, for any $\boldsymbol{q} \in \mathcal{Q}$ and $\epsilon > 0$, there exists a neighborhood of $\boldsymbol{q}$ in which $s^*(\boldsymbol{q'}) \leq s^*(\boldsymbol{q}) + \epsilon$. This means that the function s^* is upper semicontinuous on $\mathcal{Q}$.

Finally, we show that $\boldsymbol{Q}^*(\theta) \in \arg\max_{\boldsymbol{q} \in \mathcal{Q}}[s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)]$. By construction of s^*, $\boldsymbol{q} \notin \{\boldsymbol{Q}^*(\theta) \mid \theta \in [\underline{\theta}, \bar{\theta}^*]\} \cup \{\boldsymbol{Q}^*(\theta^l+) \mid l = 1, 2, ...\} \cup \{\boldsymbol{Q}^*(\theta^l-) \mid l = 1, 2, ...\}$ cannot maximize $s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta)$ for any $\theta \in [\underline{\theta}, \bar{\theta}^*]$ because $s^*(\boldsymbol{Q}^*(\theta)) - c(\boldsymbol{Q}^*(\theta), \theta) \geq 0$ whereas $s^*(\boldsymbol{q}) - c(\boldsymbol{q}, \theta) < 0$ for any $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\bar{\theta}^*)$, and c is increasing in $\boldsymbol{q}$. Hence, it suffices to show that for any $\theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}^*]$, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\hat{\theta}) - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)$, and for each l, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l+) - c(\boldsymbol{Q}^*(\theta^l+), \theta)$, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l-) - c(\boldsymbol{Q}^*(\theta^l-), \theta)$. First, for any $\theta \in [\underline{\theta}, \bar{\theta}^*]$, it is shown that $\hat{\theta} = \theta$ maximizes $\sigma(\hat{\theta}) - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)$ because for any $\hat{\theta} \neq \theta$,

$$
\begin{aligned}
&[\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta)] - [\sigma(\hat{\theta}) - c(\boldsymbol{Q}^*(\hat{\theta}), \theta)] \\
&= \int_{\theta}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz - \left[c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta}) + \int_{\hat{\theta}}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz - c(\boldsymbol{Q}^*(\hat{\theta}), \theta) \right] \\
&= \int_{\theta}^{\hat{\theta}} [c_\theta(\boldsymbol{Q}^*(z), z) - c_\theta(\boldsymbol{Q}^*(\hat{\theta}), z)] dz \geq 0.
\end{aligned}
$$

The inequality holds because Lemma 2.2 implies that $\boldsymbol{Q}^*(z) \geq \boldsymbol{Q}^*(\hat{\theta})$ for each $z < \hat{\theta}$, and the assumption that c has (strictly) increasing differences in $(\boldsymbol{q}, \theta)$ implies that $c_\theta(\boldsymbol{Q}^*(z), z) \geq c_\theta(\boldsymbol{Q}^*(\hat{\theta}), z)$. Second, for any $\theta \in [\underline{\theta}, \bar{\theta}^*]$ and l, $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l+) - c(\boldsymbol{Q}^*(\theta^l+), \theta)$ because

$$
\begin{aligned}
&[\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta)] - [\sigma(\theta^l+) - c(\boldsymbol{Q}^*(\theta^l+), \theta)] \\
&= \int_{\theta}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz - \left[\lim_{\hat{\theta} \to \theta^l+} c(\boldsymbol{Q}^*(\hat{\theta}), \hat{\theta}) + \lim_{\hat{\theta} \to \theta^l+} \int_{\hat{\theta}}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz - c(\boldsymbol{Q}^*(\theta^l+), \theta) \right] \\
&= \int_{\theta}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz - \left[c(\boldsymbol{Q}^*(\theta^l+), \theta^l) + \int_{\theta^l}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q}^*(z), z) dz - c(\boldsymbol{Q}^*(\theta^l+), \theta) \right] \\
&= \int_{\theta}^{\theta^l} [c_\theta(\boldsymbol{Q}^*(z), z) - c_\theta(\boldsymbol{Q}^*(\theta^l+), z)] dz \geq 0.
\end{aligned}
$$

The first equality follows from the construction of σ. The second equality follows from the continuity of c in $(\boldsymbol{q}, \theta)$. The last inequality holds because Lemma 2.2 and Assumption 2.3 imply that $c_\theta(\boldsymbol{Q}^*(z), z) \geq c_\theta(\boldsymbol{Q}^*(\theta^l+), z)$ for any $z \leq \theta^l$. Similarly, we can show that $\sigma(\theta) - c(\boldsymbol{Q}^*(\theta), \theta) \geq \sigma(\theta^l-) - c(\boldsymbol{Q}^*(\theta^l-), \theta)$.

(ii) We now claim that the function s^* is supermodular in quality. First, we show that s^* is nondecreasing in $\boldsymbol{q}$. Note that σ is nonincreasing in θ because for any θ, θ' with

$\theta < \theta'$,

$$\begin{aligned}
\sigma(\theta) - \sigma(\theta') &= c(\boldsymbol{Q^*}(\theta), \theta) + \int_{\theta}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q^*}(z), z)dz - c(\boldsymbol{Q^*}(\theta'), \theta') - \int_{\theta'}^{\bar{\theta}^*} c_\theta(\boldsymbol{Q^*}(z), z)dz \\
&\geq c(\boldsymbol{Q^*}(\theta'), \theta) - c(\boldsymbol{Q^*}(\theta'), \theta') + \int_{\theta}^{\theta'} c_\theta(\boldsymbol{Q^*}(z), z)dz \\
&= \int_{\theta}^{\theta'} [c_\theta(\boldsymbol{Q^*}(z), z) - c_\theta(\boldsymbol{Q^*}(\theta'), z)]dz \geq 0.
\end{aligned}$$

Take any $\boldsymbol{q}, \boldsymbol{q'} \in \mathcal{Q}$ with $\boldsymbol{q'} > \boldsymbol{q}$. If $\boldsymbol{q} \not\geq \boldsymbol{Q}^*(\bar{\theta}^*)$, then $s^*(\boldsymbol{q'}) \geq s^*(\boldsymbol{q}) = 0$. If $\boldsymbol{q} \geq \boldsymbol{Q}^*(\bar{\theta}^*)$ and $\boldsymbol{q'} \not\geq \boldsymbol{Q}^*(\underline{\theta})$, then $s^*(\boldsymbol{q'}) \geq s^*(\boldsymbol{q})$ because there exist θ and θ' with $\theta \geq \theta'$ such that $s^*(\boldsymbol{q})$ is equal to $\sigma(\theta-)$, $\sigma(\theta)$ or $\sigma(\theta+)$ and $s^*(\boldsymbol{q'})$ is equal to $\sigma(\theta'-)$, $\sigma(\theta')$ or $\sigma(\theta'+)$. Here, $s^*(\boldsymbol{q}) = \sigma(\theta'-)$ implies that $s^*(\boldsymbol{q'}) \geq \sigma(\theta'-)$, and $s^*(\boldsymbol{q}) = \sigma(\theta')$ implies that $s^*(\boldsymbol{q'}) \geq \sigma(\theta')$. If $\boldsymbol{q'} \geq \boldsymbol{Q}^*(\underline{\theta})$, then $s^*(\boldsymbol{q'}) = \sigma(\underline{\theta}) \geq s^*(\boldsymbol{q})$.

Second, we show that the function s^* is supermodular in $\boldsymbol{q}$. Fix any unordered pair $\boldsymbol{q}$ and $\hat{\boldsymbol{q}}$ in $\mathcal{Q}$, so that $q^m > \hat{q}^m$ and $q^{\hat{m}} < \hat{q}^{\hat{m}}$ for some $m, \hat{m}$. Because s^* is nondecreasing in $\boldsymbol{q}$, $s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) \geq s^*(\hat{\boldsymbol{q}})$ and $s^*(\boldsymbol{q}) \geq s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. Thus, if $s^*(\boldsymbol{q}) = s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$, then $s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s^*(\hat{\boldsymbol{q}}) \geq 0 = s^*(\boldsymbol{q}) - s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. We claim that if $s^*(\boldsymbol{q}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$, then $s^*(\hat{\boldsymbol{q}}) = s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. Suppose, on the contrary, that $s^*(\boldsymbol{q}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$ and $s^*(\hat{\boldsymbol{q}}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. Then, by construction of s^*, there exist $m, \hat{m}, \theta, \hat{\theta}$ such that $q^m \geq Q^{m*}(\theta) > \min\{q^m, \hat{q}^m\}$ with $q^{\hat{m}} \geq Q^{\hat{m}*}(\theta)$, and $\hat{q}^{\hat{m}} \geq Q^{\hat{m}*}(\hat{\theta}) > \min\{q^{\hat{m}}, \hat{q}^{\hat{m}}\}$ with $\hat{q}^m \geq Q^{m*}(\hat{\theta})$. This contradicts the result that $\boldsymbol{Q^*}$ is nondecreasing in θ because $Q^{m*}(\theta) > Q^{m*}(\hat{\theta})$ and $Q^{\hat{m}*}(\theta) < Q^{\hat{m}*}(\hat{\theta})$, so that $\boldsymbol{Q^*}(\theta)$ and $\boldsymbol{Q^*}(\hat{\theta})$ are unordered. Thus, if $s^*(\boldsymbol{q}) > s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$, then $s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s^*(\hat{\boldsymbol{q}}) = s^*(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) \geq s^*(\boldsymbol{q}) - s^*(\boldsymbol{q} \wedge \hat{\boldsymbol{q}})$. This completes the proof. □

Proof of Proposition 2.2. (i) Suppose that $Q^{m*}(\theta-) > Q^{m*}(\theta+)$ and $Q^{m'*}(\theta-) > Q^{m'*}(\theta+)$ with $(m, m') \in \mathcal{M}^1 \times \mathcal{M}^2$ and $\theta \in (\underline{\theta}, \bar{\theta}^*)$. Then, $(Q^{m*}(\theta-))_{m \in \mathcal{M}^1} > (Q^{m*}(\theta+))_{m \in \mathcal{M}^1}$ and $(Q^{m*}(\theta-))_{m \in \mathcal{M}^2} > (Q^{m*}(\theta+))_{m \in \mathcal{M}^2}$. Because c has increasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ and strictly increasing differences in $(q^m, q^{m'})$, we obtain $c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{Q}}, \theta) > c(\boldsymbol{Q}, \theta) - c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta)$, where $\boldsymbol{Q} = ((Q^{m*}(\theta+))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta-))_{m \in \mathcal{M}^2})$ and $\hat{\boldsymbol{Q}} = ((Q^{m*}(\theta-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta+))_{m \in \mathcal{M}^2})$, so that $\boldsymbol{Q} \vee \hat{\boldsymbol{Q}} = \boldsymbol{Q^*}(\theta-)$ and $\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}} = \boldsymbol{Q^*}(\theta+)$. Let ϵ be a positive real number such that $2\epsilon < [c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{Q}}, \theta)] - [c(\boldsymbol{Q}, \theta) - c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta)]$. Fix any θ', θ'' with $\theta'' < \theta < \theta'$ such that $[c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta) - c(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta')] - [c(\boldsymbol{Q}, \theta) - c(\boldsymbol{q}, \theta')] < \epsilon$ and $[c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta'')] - [c(\hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{q}}, \theta'')] < \epsilon$, where $\boldsymbol{q} = ((Q^{m*}(\theta'))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta''))_{m \in \mathcal{M}^2})$ and $\hat{\boldsymbol{q}} = ((Q^{m*}(\theta''))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta'))_{m \in \mathcal{M}^2})$, so that $\boldsymbol{q} \vee \hat{\boldsymbol{q}} = \boldsymbol{Q^*}(\theta'')$ and $\boldsymbol{q} \wedge \hat{\boldsymbol{q}} = \boldsymbol{Q^*}(\theta')$. The existence of such θ' and θ'' is guaranteed by the continuity of c in $(\boldsymbol{q}, \theta)$.

Suppose that a scoring rule s implements the optimal mechanism. Then, it follows from Lemma 2.1 that the inequality $s(\boldsymbol{q}) - c(\boldsymbol{q}, \theta') \leq s(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) - c(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta')$ must hold because $\boldsymbol{q} \wedge \hat{\boldsymbol{q}} = \boldsymbol{Q^*}(\theta')$ whereas $\boldsymbol{q} \neq \boldsymbol{Q^*}(\theta')$. Hence, we obtain

$$
\begin{aligned}
s(\boldsymbol{q}) - s(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) &\leq c(\boldsymbol{q}, \theta') - c(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}, \theta') \\
&< c(\boldsymbol{Q}, \theta) - c(\boldsymbol{Q} \wedge \hat{\boldsymbol{Q}}, \theta) + \epsilon \\
&< c(\boldsymbol{Q} \vee \hat{\boldsymbol{Q}}, \theta) - c(\hat{\boldsymbol{Q}}, \theta) - \epsilon \\
&< c(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta'') - c(\hat{\boldsymbol{q}}, \theta'').
\end{aligned}
$$

Now, if the scoring rule s is additively separable, then $s(\boldsymbol{q}) - s(\boldsymbol{q} \wedge \hat{\boldsymbol{q}}) = s^2(\boldsymbol{q^2}) - s^2(\hat{\boldsymbol{q}}^{\boldsymbol{2}}) = s(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s(\hat{\boldsymbol{q}})$ because $\boldsymbol{q^1} = \boldsymbol{q^1} \wedge \hat{\boldsymbol{q}}^{\boldsymbol{1}} = (Q^{m*}(\theta'))_{m \in \mathcal{M}^1}$ and $\hat{\boldsymbol{q}}^{\boldsymbol{1}} = \boldsymbol{q^1} \vee \hat{\boldsymbol{q}}^{\boldsymbol{1}} = (Q^{m*}(\theta''))_{m \in \mathcal{M}^1}$. Then, $s(\boldsymbol{q} \vee \hat{\boldsymbol{q}}) - s(\hat{\boldsymbol{q}}) < c(\boldsymbol{q} \vee \hat{\boldsymbol{q}}, \theta'') - c(\hat{\boldsymbol{q}}, \theta'')$, and thus a supplier of type θ'' never chooses $\boldsymbol{q} \vee \hat{\boldsymbol{q}} = \boldsymbol{Q^*}(\theta'')$ from Lemma 2.1. Therefore, there is no additively separable scoring rule which implements the optimal mechanism.

(ii) It follows from Lemma 2.1 that Q^{m*} is nonincreasing in θ. Because $\mathcal{Q}^m$ is finite, $Q^{m*} : [\underline{\theta}, \bar{\theta}] \to \mathbb{R}$ can have no more than finitely many points of discontinuity. Suppose, without loss of generality, that the discontinuous points of $\boldsymbol{Q^*}$ in $[\underline{\theta}, \bar{\theta}^*]$ are indexed in decreasing order, i.e. $\theta^1 > \theta^2 > ... > \theta^L$. Note that $\boldsymbol{Q^*}(\theta^{l+1}+) = \boldsymbol{Q^*}(\theta) = \boldsymbol{Q^*}(\theta^l-)$ for any $\theta \in (\theta^{l+1}, \theta^l)$, and $Q^{m*}(\theta^l-) \geq Q^{m*}(\theta^l+)$. Assume for simplicity that $\bar{\theta} > \theta^1$ and $\theta^L > \underline{\theta}$. Without this assumption, the proof proceeds with some notational complexity.

First, we define functions $\sigma^1 : [\underline{\theta}, \bar{\theta}^*] \to \mathbb{R}$ and $\sigma^2 : [\underline{\theta}, \bar{\theta}^*] \to \mathbb{R}$: For any $\theta \in [\underline{\theta}, \bar{\theta}^*]$ such that $\theta \in (\theta^{l+1}, \theta^l]$ for some l or $\theta \in [\underline{\theta}, \theta^l]$ for $l = L$,

$$
\begin{aligned}
\sigma^1(\theta) = \frac{1}{2} c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*) &+ \sum_{r=1}^{l-1} [c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r) - c(\boldsymbol{Q^*}(\theta^r+), \theta^r)] \\
&+ [c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^l+))_{m \in \mathcal{M}^2}, \theta^l) - c((Q^{m*}(\theta^l+))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^l+))_{m \in \mathcal{M}^2}, \theta^l)], \\
\sigma^2(\theta) = \frac{1}{2} c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*) &+ \sum_{r=1}^{l-1} [c(\boldsymbol{Q^*}(\theta^r-), \theta^r) - c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r)] \\
&+ [c((Q^{m*}(\theta^l-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta^l) - c((Q^{m*}(\theta^l-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^l+))_{m \in \mathcal{M}^2}, \theta^l)],
\end{aligned}
$$

and for any $\theta \in (\theta^1, \bar{\theta}^*]$, $\sigma^1(\theta) = \sigma^2(\theta) = \frac{1}{2} c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*)$. Using these functions, we construct a function $s^a : \times_{m \in \mathcal{M}^a} \mathcal{Q}^m \to \mathbb{R}$ for each $a = 1, 2$ in the following way. (a) If $\boldsymbol{q^a} \ngeq (Q^{m*}(\bar{\theta}^*))_{m \in \mathcal{M}^a}$, then let $s^a(\boldsymbol{q^a}) = -\max_{\boldsymbol{q} \in \mathcal{Q}} c(\boldsymbol{q}, \bar{\theta})$. (b) If there exists $\theta' \in [\underline{\theta}, \bar{\theta}^*]$ such that $\boldsymbol{q^a} \geq (Q^{m*}(\theta'))_{m \in \mathcal{M}^a}$ and there exists no $\theta < \theta'$ which satisfies $\boldsymbol{q^a} \geq (Q^{m*}(\theta))_{m \in \mathcal{M}^a} > (Q^{m*}(\theta'))_{m \in \mathcal{M}^a}$, then let $s^a(\boldsymbol{q^a}) = \sigma^a(\theta')$. It is easy to show that $s^1((Q^{m*}(\bar{\theta}^*))_{m \in \mathcal{M}^1}) + s^2((Q^{m*}(\bar{\theta}^*))_{m \in \mathcal{M}^2}) - c(\boldsymbol{Q^*}(\bar{\theta}^*), \bar{\theta}^*) = 0$. Moreover, the function $\underline{s} + \bar{s}$ is trivially upper semicontinuous on $\mathcal{Q}$ because $\mathcal{Q}$ is finite.

Next, we show that $\boldsymbol{Q^*}(\theta) \in \arg\max_{\boldsymbol{q} \in \mathcal{Q}} [s^1(\boldsymbol{q^1}) + s^2(\boldsymbol{q^2}) - c(\boldsymbol{q}, \theta)]$. By construction of s^1 and s^2, $\boldsymbol{q} \notin \{(Q^{m*}(\theta))_{m \in \mathcal{M}^1} \mid \theta \in [\underline{\theta}, \bar{\theta}^*]\} \times \{(Q^{m*}(\theta))_{m \in \mathcal{M}^2} \mid \theta \in [\underline{\theta}, \bar{\theta}^*]\}$ cannot be the maximizer. Hence, it suffices to show that for any $\theta, \theta', \theta'' \in [\underline{\theta}, \bar{\theta}^*]$, $\sigma^1(\theta) + \sigma^2(\theta) -$

$c(\boldsymbol{Q^*}(\theta), \theta) \geq \sigma^1(\theta') + \sigma^2(\theta'') - c((Q^{m*}(\theta'))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta''))_{m \in \mathcal{M}^2}, \theta)$. Suppose first that $\theta \in (\theta^{l+1}, \theta^l)$ and $\theta' \in (\theta^{l'+1}, \theta^{l'})$ with $l \geq l'$. Then, we obtain

$$
\begin{aligned}
&\sigma^1(\theta) - \sigma^1(\theta') \\
&= \sum_{r=l'+1}^{l} [c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r) - c(\boldsymbol{Q^*}(\theta^r+), \theta^r)] \\
&\geq \sum_{r=l'+1}^{l} [c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta^r) - c((Q^{m*}(\theta^r+))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta^r)] \\
&\geq \sum_{r=l'+1}^{l} [c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta) - c((Q^{m*}(\theta^r+))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta)] \\
&= c(\boldsymbol{Q^*}(\theta), \theta) - c((Q^{m*}(\theta'))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta).
\end{aligned}
$$

The first equality follows from the observation that $Q^{m*}(\theta^l-) = Q^{m*}(\theta)$ and $Q^{m*}(\theta^{l'}-) = Q^{m*}(\theta')$. The first inequality follows from the hypothesis that c has decreasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with the observation that $Q^{m*}(\theta^r+) \leq Q^{m*}(\theta)$ for each $r \in \{l'+1, ..., l\}$. The second inequality follows from the assumption that c has (strictly) increasing differences in $(\boldsymbol{q}, \theta)$. The last equality follows from the observation that $Q^{m*}(\theta^r-) = Q^{m*}(\theta^{r+1}+)$, $Q^{m*}(\theta^l-) = Q^{m*}(\theta)$ and $Q^{m*}(\theta^{l'+1}+) = Q^{m*}(\theta')$. Suppose next that $\theta'' \in (\theta^{l''+1}, \theta^{l''})$ with $l \leq l''$. Then, the same argument as above yields

$$
\begin{aligned}
&\sigma^2(\theta) - \sigma^2(\theta'') \\
&= -\sum_{r=l+1}^{l''} [c(\boldsymbol{Q^*}(\theta^r-), \theta^r) - c((Q^{m*}(\theta^r-))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r)] \\
&\geq -\sum_{r=l+1}^{l''} [c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r-))_{m \in \mathcal{M}^2}, \theta^r) - c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta^r)] \\
&\geq -\sum_{r=l+1}^{l''} [c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r-))_{m \in \mathcal{M}^2}, \theta) - c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta^r+))_{m \in \mathcal{M}^2}, \theta)] \\
&= -[c((Q^{m*}(\theta))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta''))_{m \in \mathcal{M}^2}, \theta) - c(\boldsymbol{Q^*}(\theta), \theta)] \\
&\geq -[c((Q^{m*}(\theta'))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta''))_{m \in \mathcal{M}^2}, \theta) - c((Q^{m*}(\theta'))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta))_{m \in \mathcal{M}^2}, \theta)].
\end{aligned}
$$

The last inequality follows from the hypothesis that c has decreasing differences in $(\boldsymbol{q^1}, \boldsymbol{q^2})$ with the observation that $Q^{m*}(\theta') \leq Q^{m*}(\theta)$. Therefore, we obtain

$$
[\sigma^1(\theta) - \sigma^1(\theta')] + [\sigma^2(\theta) - \sigma^2(\theta'')] \geq c(\boldsymbol{Q^*}(\theta), \theta) - c((Q^{m*}(\theta'))_{m \in \mathcal{M}^1}, (Q^{m*}(\theta''))_{m \in \mathcal{M}^2}, \theta),
$$

for any $\theta, \theta', \theta'' \in [\underline{\theta}, \bar{\theta}^*]$ such that $\theta \in (\theta^{l+1}, \theta^l)$, $\theta' \in (\theta^{l'+1}, \theta^{l'})$ and $\theta'' \in (\theta^{l''+1}, \theta^{l''})$ with $l'' \geq l \geq l'$. A similar argument applies to the other combinations of θ, θ' and θ''. $\square$

Chapter 3

Sequential Procurement Auctions with Risk-Averse Suppliers

3.1 Introduction

Governments delegate the provision of a wide range of public infrastructure to the private sector. Such infrastructure consists of physical facilities (e.g., railways, schools, hospitals) and public services (e.g., transportation, education, medical services). An essential aspect of infrastructure provision is that it is a multi-stage process. A public facility is designed, built, and operated over a long period of time as it delivers services to the public. Inevitably, an infrastructure project involves many kinds of uncertainty. Risk sharing between public authorities and private contractors is thus a key factor for a successful project. The government must find an efficient procedure for procurement. The purpose of this chapter is to compare the performance of two procurement mechanisms, *bundling* and *unbundling*, and to investigate the choice problem of a mechanism.

In many construction projects, a public authority has the option of choosing between a bundling method called "design-build" and an unbundling method called "design-bid-build." Under the design-build method, a private party called a design-builder, which may be organized by a group of companies, is responsible for performing tasks of both design and construction.[1] Similarly, contractual arrangements called "public-private partnerships (PPPs)" are characterized by bundled tasks. Under the design-bid-build method, the two tasks are separated. Architects draw the initial design, and then bidders are invited to submit bids for the construction. While unbundling methods have been employed traditionally in public procurement, there is a recent trend towards bundling methods. There is an ongoing debate about the two methods. It is often said that the bundling method transfers risks from public authorities to private parties so that the private parties are motivated to invest more. However, the risk transfer may become excessive and,

[1] Engineering News-Record (ENR) annually reports the top 100 design-build firms (ENR, 2012).

as a result, the public authority must pay a higher risk premium in exchange for shifting the burden of risk to private parties. See Gibson et al. (2007) and Iossa et al. (2007) for more about these methods and detailed arguments in the debate. It remains an open question, theoretically and practically, which of the bundling and unbundling methods is most desirable for a public authority and for society.

We analyze the procurement problem in a two-stage auction model with risks. A risk-neutral buyer procures a public infrastructure project from risk-averse suppliers. The investment stage is followed by the production stage. In a bundling mechanism, the buyer holds a single auction. In an unbundling mechanism, the buyer holds two sequential auctions. Each auction adopts a first-price format. A distinctive feature of our model is that it has two categories of production risks. The first category is an *aggregate risk*. The risk is common to all suppliers. The second one is an *idiosyncratic risk*. The risk is specific to each supplier. Each supplier's production cost is affected by the two risk factors as well as by the cost-reducing investment. After the cost-reducing investment is made, the realized value of the aggregate risk is commonly known to all suppliers, and each supplier privately knows the realized value of the idiosyncratic risk to him.

Yescombe (2007) provides many examples of aggregate risk and idiosyncratic risk in construction projects. He classifies risks into several categories related to sites, construction, and completion. Site risks including ground conditions are typical examples of aggregate risk. If the geology of a site turns out not to be as expected, then piling is required for foundations. Design risks in the category of completion are other examples of aggregate risk. If there is a flaw in the design drawings, then all builders are damaged. Market uncertainty such as fluctuations in material prices, political uncertainty, and climate conditions are other examples of aggregate risk. Many uncertain elements in economic, financial, labor and organizational conditions of construction companies constitute idiosyncratic risk to them. For instance, a construction company may be uncertain of the technical ability of its hired subcontractors. If the subcontractors are not able to do the construction work properly, then the constructor must bear additional costs.

The main result of this chapter is as follows. The bundling mechanism is optimal for the buyer and also socially desirable if the aggregate risk is below certain thresholds. The result may not hold true for the idiosyncratic risk. Key factors leading to the result are the differences in risk sharing, information rents, and investment incentives between the bundling and unbundling mechanisms. In the bundling mechanism, all the risks are assumed by a single supplier, the winner of the auction. The buyer has to pay a high risk premium to the winner, while the mechanism implements the efficient investment. In contrast, the unbundling mechanism provides little incentive for the investment due to a moral hazard problem. The buyer must pay risk premia for the idiosyncratic risk indirectly through the payment of information rents. The buyer pays no risk premium for the aggregate risk because the risk is transferred from the suppliers to the buyer through the second-stage auction. Therefore, a decrease in the aggregate risk is beneficial to the buyer and society only in the bundling mechanism, unlike the idiosyncratic risk. A decrease in the idiosyncratic risk reduces the information rents in the unbundling mechanism if the idiosyncratic risk is exponentially distributed.

Based on the result above, we can provide the following policy recommendation on the choice of procurement mechanisms: A public authority should not lump all possible risks together, but classify them into aggregate risks and idiosyncratic risks. A low aggregate risk for a project is a strong reason to choose a bundling method from the viewpoints of both the public authority and society. On the contrary, when a project's idiosyncratic risk is low, it may be recommended for the authority to choose an unbundling method rather than a bundling method.

Kay (2009) reports a real case of a construction project that is relevant for our result. During 1996–2004, a new metro system called Tren Urbano was constructed in Puerto Rico. The 10.7-mile metro line connects the cities of San Juan, Guaynabo, and Bayamón. The Puerto Rico Highway and Transit Authority (PRHTA) split the civil work into seven sections, and employed design-build methods (i.e., bundling) in all sections. Prior to requesting proposals for the design-build contract, the PRHTA investigated geotechnical conditions significantly. The research outcome was shared by prospective bidders, and it reduced aggregate risks to them.

This chapter is related to several previous studies in the literature which also considered the choice problem of bundling and unbundling mechanisms. Hart (2003) presents an incomplete contract model and shows that a bundling mechanism gives a supplier strong incentives for socially productive and unproductive investments. His result is supported experimentally by Hoppe et al. (2013). Li and Yu (2011) present an auction model similar to ours, and examine how the optimality of a bundling mechanism is affected by the competition among suppliers. Martimort and Pouyet (2008) show that the buyer's optimal choice is a bundling mechanism if a quality-enhancing effort in the first stage reduces production costs. In a second-price auction model, Grimm (2007) shows that the buyer always prefers a bundling mechanism with subcontracting to an unbundling mechanism in the case of risk-neutral suppliers. Unlike all works mentioned above, this chapter focuses on risk sharing between the buyer and suppliers in the choice of a procurement mechanism, and shows that aggregate risk and idiosyncratic risk have different effects on the choice of mechanism. Finally, our result is closely related to the "yardstick competition effect" studied by Auriol and Laffont (1992). As in their regulation model, the unbundling mechanism in our model has the yardstick competition effect in the sense that information on aggregate risk is revealed to the buyer through the second-stage auction. Owing to the information revelation, the unbundling mechanism becomes beneficial to the buyer.

This chapter is organized as follows. Section 2 presents a two-stage auction model.

Sections 3 and 4 characterize an equilibrium of the model under each mechanism. Section 5 gives the main results. Section 6 discusses an extension of the model. Section 7 is the conclusion. All proofs are given in the Appendix.

3.2 The Model

Consider a buyer who must procure one unit of public infrastructure (a physical facility or public service) from one of n suppliers. The buyer is risk-neutral. Each supplier $i \in N := \{1, ..., n\}$ is risk-averse, and has a CARA utility function $u(\pi) := 1 - \exp(-r\pi)$. Here, $\pi \in \mathbb{R}$ is a profit from trade, and $r > 0$ is the coefficient of absolute risk aversion.

The procurement process involves two tasks: investment and production. First, one of the suppliers invests in design. The supplier incurs a cost $\psi(a)$ if he makes an investment $a \in [0, \bar{a}]$. Second, one of the suppliers develops the infrastructure based on the design. Supplier i's production cost $c(a, \theta_i, \omega)$ depends on three elements. The investment a is common to all suppliers. Namely, it is a "public good." Here, $\theta_i \in [\underline{\theta}, \bar{\theta}]$ is supplier i's private parameter,[2] and $\omega \in [\underline{\omega}, \bar{\omega}]$ is a common parameter among all suppliers. The buyer's valuation for the infrastructure is $v > 0$.

We assume that $(\theta_1, ..., \theta_n, \omega)$ are independent random variables, and the θ_i's are identically distributed. In the following, we call the random variable θ_i *idiosyncratic risk* to supplier i and the random variable ω *aggregate risk* common to all suppliers. The cumulative distribution functions of θ_i and ω are, respectively, given by F and G, with $F' = f > 0$.[3] The distribution F is parameterized by $\kappa > 0$ so that $F(\cdot; \kappa)$ second-order stochastically dominates $F(\cdot; \kappa')$ if $\kappa' > \kappa$.[4] Similarly, the distribution G is parameterized by $\lambda > 0$ so that $G(\cdot; \lambda)$ second-order stochastically dominates $G(\cdot; \lambda')$ if $\lambda' > \lambda$. We set $E[\theta_i] = E[\omega] = 0$ for normalization. As the parameter κ (resp. λ) converges to zero, the distribution $F(\cdot; \kappa)$ (resp. $G(\cdot; \lambda)$) converges in law to a degenerate distribution which takes zero with probability one.[5] Let $\theta_{(n)} := \min\{\theta_1, ..., \theta_n\}$ denote the lowest value of n cost parameters. The cumulative distribution function of $\theta_{(n)}$ is given by $F_{(n)}(\theta) := 1 - (1 - F(\theta))^n$, and the probability density function of $\theta_{(n)}$ is given by $f_{(n)}(\theta) := n(1 - F(\theta))^{n-1} f(\theta)$. All suppliers are ex ante symmetric.

We assume for simplicity that the cost function c is linear in θ_i, ω, a as

$$c(a, \theta_i, \omega) = \theta_i + \omega - \delta a. \tag{3.1}$$

[2] We allow the case that $\bar{\theta} = \infty$. In this case, we also assume that $E[\exp(r\theta_i)] < \infty$, which guarantees that an associated risk premium is finite. Here, $E[\cdot]$ is the expectation operator.

[3] We write $f > 0$ if $f(\theta_i) > 0$ for any $\theta_i \in [\underline{\theta}, \bar{\theta}]$. We use the same notation for other functions.

[4] We allow $\underline{\theta}$ and $\bar{\theta}$ to depend on κ. We also assume that if $\bar{\theta} = \infty$ for some κ, then there exists $m > 0$ such that $f(\theta; \kappa') \leq f(\theta; \kappa)$ for any $\theta \geq m$ and any κ' with $\kappa' < \kappa$. See Hanoch and Levy (1969) for the definition of second-order stochastic dominance.

[5] That is, $F(\theta; \kappa) \to 0$ as $\kappa \to 0$ for any $\theta < 0$, and $F(\theta; \kappa) \to 1$ as $\kappa \to 0$ for any $\theta > 0$.

Here, $\delta > 0$ is the marginal benefit of investment. The investment reduces all suppliers' production costs to the same degree. The positive externality of investment arises in a situation where the public facility is constructed based on the same design, or the public service is provided using the same facility. We make the following assumption about the investment cost function.

Assumption 3.1. $\psi : [0, \bar{a}] \to \mathbb{R}_+$ is twice continuously differentiable, ψ is increasing in a, $\frac{d\psi}{da}(0) = 0$, $\frac{d^2\psi}{da^2} > 0$, and $\lim_{a \to \bar{a}} \frac{d\psi}{da}(a) = \infty$.

There are two kinds of procurement mechanisms: *bundling* and *unbundling*. In a bundling mechanism, the buyer bundles two sequential tasks of investment and production, and awards a contract for both tasks to a single supplier via an auction. In an unbundling mechanism, the buyer separates those tasks, and sequentially awards a contract for each task via an auction. Each auction adopts a first-price sealed-bid format. The winner is a supplier who bids the lowest price.[6]

The game proceeds as follows. At date 0, the buyer chooses either a bundling or an unbundling mechanism. At date 1, each supplier $i \in N$ simultaneously submits a bid $p_i^1 \in \mathbb{R}$ in the *first-stage auction*. At date 2, a winner in the first-stage auction chooses an investment level $a \in [0, \bar{a}]$. At date 3, $(\theta_1, ..., \theta_n, \omega)$ are realized. In the case of a bundling mechanism, the game ends. At date 4, which occurs only in the case of an unbundling mechanism, each supplier $j \in N$ submits a bid $p_j^2 \in \mathbb{R}$ in the *second-stage auction*. The winner performs the task.

The player payoffs are defined as follows. When the game ends at date 3 under a bundling mechanism, the winner obtains $u(p_i^1 - \psi(a) - c(a, \theta_i, \omega))$, while the other suppliers obtain $u(0) = 0$, and the buyer obtains $v - p_i^1$. When the game ends at date 4 under an unbundling mechanism, each supplier i obtains $u(\pi_i^1 + \pi_i^2)$, and the buyer obtains $v - (p_j^1 + p_k^2)$. Here, (p_j^1, p_k^2) are winning prices in the auctions, and (π_i^1, π_i^2) are supplier i's profits in the auctions. If i wins in the first stage, then $\pi_i^1 = p_i^1 - \psi(a)$. If i wins in the second stage, then $\pi_i^2 = p_i^2 - c(a, \theta_i, \omega)$. If i wins in both stages, then i receives $\pi_i^1 + \pi_i^2$. If i loses in the first (second) stage, then $\pi_i^1 = 0$ ($\pi_i^2 = 0$).

The information structure of the game is as follows. The buyer's bundling decision is commonly known to all players. The realized values of θ_i and $c(a, \theta_i, \omega)$ become supplier i's private information, and that of ω becomes common knowledge among all suppliers. No supplier can observe the other suppliers' decisions. The winner's investment level a can be estimated perfectly by all suppliers at date 3 through the one-to-one correspondence between the production cost $c(a, \theta_i, \omega)$ and a, given (θ_i, ω).

Every player's (pure) strategy is defined in a standard way. The buyer's strategy is the choice of a mechanism. Each supplier's strategy is represented by a triple (p^1, a, p^2). A bidding strategy p^1 in the first-stage auction is a function from {bundling, unbundling}

[6] We assume that if there is a tie, then the supplier submitting the lowest bid wins with equal probability. All results hold for any other tie-breaking rule.

to $\mathbb{R}$, a : {bundling, unbundling} $\to [0, \bar{a}]$ is a choice of investment level conditional on winning, and $p^2 : [0, \bar{a}] \times [\underline{\theta}, \bar{\theta}] \times [\underline{\omega}, \bar{\omega}] \to \mathbb{R}$ is a bidding strategy in the second-stage auction.[7] We consider a symmetric perfect Bayesian equilibrium in which all suppliers use the same strategies. In the following, we simply call it an equilibrium. Since $(\theta_1, ..., \theta_n, \omega)$ are independent, no supplier updates his belief about the other suppliers' types in equilibrium.

The *social welfare* is defined as the buyer's expected utility plus the sum of each supplier's certainty equivalent which gives the same utility as his expected utility.[8] The investment level $\tilde{a}$ is *efficient* if it minimizes a total expected cost $\psi(a) + E[\theta_{(n)} + \omega] - \delta a$.

Finally, we remark that the investment level a and the realized values of $(\theta_1, ..., \theta_n, \omega)$ are unverifiable in the model. If these values are verifiable, then the buyer can initially offer a contract in which prices are contingent on the investment level and the cost parameters without any incentive constraints. In this verifiable case, the buyer can implement the ex ante efficient outcome in a straightforward manner. The buyer pays the investment cost $\psi(\tilde{a})$ to an arbitrary supplier for the efficient investment $\tilde{a}$, and pays the production cost $\theta_{(n)} + \omega - \delta\tilde{a}$ to a supplier with the lowest private parameter $\theta_{(n)}$ for the production. The buyer then obtains the *first-best utility* $v - \{\psi(\tilde{a}) + E[\theta_{(n)} + \omega] - \delta\tilde{a}\}$.

3.3 A Bundling Mechanism

In this section, we characterize an equilibrium when the buyer chooses a bundling mechanism. Applying backward induction, we analyze suppliers' equilibrium bidding strategies at date 1 and the winner's optimal investment.

By investing a, a winner in the auction obtains the expected utility $E[u(p_i^1 - \psi(a) - c(a, \theta_i, \omega))]$. The certainty equivalent is $p_i^1 - \psi(a) + \delta a - \rho^*$. Here, the risk premium is

$$\rho^* = \frac{1}{r} \ln E[\exp(r\theta_i)] + \frac{1}{r} \ln E[\exp(r\omega)]. \tag{3.2}$$

Notice that the investment a has no effect on the risk premium when a supplier has a CARA utility function.

The next proposition characterizes the equilibrium under bundling.

Proposition 3.1. *Let $\tilde{a}$ be the efficient investment level and ρ^* be the risk premium in (3.2). In a bundling mechanism, the unique symmetric equilibrium is characterized as follows. Every supplier submits the same bid $\psi(\tilde{a}) - \delta\tilde{a} + \rho^*$. A winner chooses the efficient investment level $\tilde{a}$.*

[7] It will be shown that the first-stage bid and the identity of the first-stage winner have no effect on either the investment level or the second-stage bid in equilibrium.

[8] For any random variable π, the certainty equivalent $E[\pi] - \rho$ with the risk premium ρ is determined by $u(E[\pi] - \rho) = E[u(\pi)]$. The risk premium is calculated as $\rho = E[\pi] + \frac{1}{r} \ln E[\exp(-r\pi)]$.

This proposition shows the following properties of a bundling mechanism. First, the efficient level $\tilde{a}$ of investment can be attained in equilibrium. The contractual obligation under bundling motivates a winner to make an efficient investment. This result provides partial support for the OECD (2008) policy favoring risk be transferred to the party best able to carry it. Second, the buyer exploits the social welfare W^*. That is, her expected utility EU_B^* is given by

$$EU_B^* = W^* = v - \{\psi(\tilde{a}) - \delta\tilde{a} + \rho^*\}. \tag{3.3}$$

This result stems from the fact that all suppliers are ex ante symmetric and have no private information. The first-stage auction has the same structure as the Bertrand competition game with complete information. No supplier can earn a positive rent in the auction. Third, the bundling leads to two types of costs for both the buyer and society. The first one is the risk premium for the production risk. Every supplier adds the risk premium to his bid. The second one is the opportunity cost of not being able to switch to a more efficient supplier. The buyer cannot change suppliers under a bundling mechanism. If the parameters κ and λ of the risks converge to zero, then the buyer can obtain the first-best utility. In the limit, the risk premium ρ^* converges to zero and the "sampling effect" asymptotically disappears (i.e., $E[\theta_{(n)}] \to E[\theta_i]$ as $\kappa \to 0$). This will be formally shown in the proof of Theorem 3.1.

3.4 An Unbundling Mechanism

In this section, we characterize an equilibrium when the buyer chooses an unbundling mechanism. Similarly to Section 3, we analyze suppliers' equilibrium bidding strategies at dates 1 and 4 and the winner's optimal investment, applying backward induction.

Since the second stage of the model is the first-price auction with symmetric risk-averse bidders, a symmetric equilibrium of the second stage is given by the work of Holt (1980) as follows.

Lemma 3.1. *In an unbundling mechanism, given an investment level a and a common parameter ω, the following bidding strategy $\mathrm{p}^2(a, \cdot, \omega)$ constitutes a symmetric equilibrium in the second-stage auction:*

$$p^2(a, \theta_i, \omega) = b(\theta_i) + \omega - \delta a. \tag{3.4}$$

Here, $b(\theta_i) := -\frac{1}{r} \ln E_{\theta_{(n-1)}}[\exp(-r\theta_{(n-1)}) \mid \theta_{(n-1)} > \theta_i]$.

The lemma has the following four implications to our analysis. (1) The bidding strategy is significantly affected by the information structure of the game. It can be seen that $b(\theta_i) > \theta_i$ for every $\theta_i < \bar{\theta}$. This property is due to the so-called "bid shading" by supplier i who has private information about his idiosyncratic risk θ_i. On the other hand, the aggregate risk ω is commonly known to all suppliers, and thus the second term in the equilibrium bid is exactly equal to ω. This bidding behavior of suppliers has an interesting effect in that, although the aggregate risk ω is unverifiable, the information rent with respect to ω is zero as if the information on ω was revealed to the buyer through competition. This effect is similar to the "yardstick competition effect" studied by Auriol and Laffont (1992). Since suppliers are risk-averse in our model, the effect has an additional consequence for risk sharing. The aggregate risk is transferred from risk-averse suppliers to the risk-neutral buyer, whereby the winner's profit $p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega) = b(\theta_i) - \theta_i$ no longer depends on ω. The effect improves the efficiency of risk sharing, which is a crucial advantage of the unbundling mechanism over the bundling mechanism. These results hold for a general class of cost functions which have no cross-term between θ_i and ω. (2) The equilibrium bidding strategy $p^2(\cdot)$ is independent of the identity of the winner in the first-stage auction. Since the CARA utility function has no wealth effect, the winner's bidding behavior is independent of p_i^1 and his sunk cost $\psi(a)$ of investment. (3) The most efficient supplier wins because the equilibrium bidding strategy $p^2(\cdot)$ is increasing in θ_i. (4) The effect of the investment on the second-stage bidding strategy is given by

$$\frac{\partial p^2}{\partial a}(a, \theta_i, \omega) = \frac{\partial c}{\partial a}(a, \theta_i, \omega) = -\delta. \tag{3.5}$$

Each supplier's bid in the second stage is decreasing in the first-stage winner's investment, and the marginal effect is equal to that of cost reduction. This is because the winner's cost-reducing investment reduces all suppliers' production costs to the same degree, and thus induces aggressive bidding by all of them.

By investing a, a winner in the first-stage auction obtains the expected utility $E[(1 - F_{(n-1)}(\theta_i))u(p_i^1 - \psi(a) + p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega)) + F_{(n-1)}(\theta_i)u(p_i^1 - \psi(a))]$. The first term in the expectation corresponds to the event whereby the supplier also wins the second-stage auction; the second term corresponds to the event that he loses. The certainty equivalent is given by

$$p_i^1 - \psi(a) + E[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)] - \rho^{**}, \tag{3.6}$$

where the risk premium ρ^{**} is

$$\begin{aligned} \rho^{**} = &\frac{1}{r} \ln E[(1 - F_{(n-1)}(\theta_i)) \exp(-r(b(\theta_i) - \theta_i)) + F_{(n-1)}(\theta_i)] \\ &+ E[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)]. \end{aligned} \tag{3.7}$$

The winner makes no investment in equilibrium because he cannot obtain any benefit from investment.

The next proposition characterizes an equilibrium under unbundling.

Proposition 3.2. *Let ρ^{**} be the risk premium in (3.7). In an unbundling mechanism, there exists an equilibrium characterized as follows. (i) All suppliers employ the bidding strategy $p^2(\cdot)$ with $b(\cdot)$ in Lemma 3.1 in the second-stage auction. A winner in the first-stage auction chooses the investment level* 0*. Every supplier submits the same bid $\psi(0)$ in the first-stage auction. (ii) The buyer's expected utility, EU_B^{**}, supplier i's expected utility, EU_i^{**}, and the social welfare, W^{**}, are respectively given by*

$$\begin{aligned} EU_B^{**} &= v - \left\{\psi(0) + E[\theta_{(n)}] + nE[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)]\right\}, \\ EU_i^{**} &= u(E[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)] - \rho^{**}), \\ W^{**} &= v - \left\{\psi(0) + E[\theta_{(n)}] + n\rho^{**}\right\}. \end{aligned}$$

This proposition shows the following properties of an unbundling mechanism. First, the equilibrium investment level is lower than the efficient level $\tilde{a}$. In fact, a winner has no incentive to invest due to a moral hazard problem.

Second, in contrast to a bundling mechanism, the buyer's utility is less than the social welfare. This is because suppliers have private information about their private cost parameters in the second-stage auction. Every supplier can earn information rents. Notice that even if a supplier loses in the first stage, he has an option to participate in the second-stage auction. Hence, every supplier's reservation utility in the first-stage auction is endogenously determined by the expected utility in the second-stage auction.

Third, an unbundling mechanism causes two types of costs to the buyer and society. The first one is the risk premium. Each supplier bears a fraction of the risk of the production cost because each wins with positive probability in the second-stage auction. Moreover, each supplier must bear another risk associated with competition. Participation in the second-stage auction means each supplier faces a risky outcome (i.e., winning or losing, and a winning bid price). The buyer must pay the premium indirectly because the expected information rent $E[(1-F_{(n-1)}(\theta_i))(b(\theta_i)-\theta_i)]$ is greater than the risk premium ρ^{**}. The second one is an efficiency loss from underinvestment.

3.5 Bundling versus Unbundling

This section presents the main results. By analyzing the equilibrium outcomes in Sections 3 and 4, we investigate how changes in the risk parameters λ, κ and the marginal benefit δ of investment affect the buyer's choice of mechanism and the social welfare. We say that the bundling (resp., unbundling) mechanism is *socially desirable* if $W^* > W^{**}$ (resp., $W^* < W^{**}$). To choose a mechanism, the buyer must take into account three factors:

risk premia, information rents, and investment incentives.

The following theorem shows how the change of the aggregate risk affects the buyer's optimal choice and the social welfare.

Theorem 3.1. *Assume that $\lambda \in (0,1)$ represents the level of aggregate risk. (i) Fixing all parameters other than λ, there exist two thresholds $\underline{\lambda}, \bar{\lambda} \in [0,1]$ with $\underline{\lambda} \leq \bar{\lambda}$ such that the buyer's optimal choice is the bundling mechanism iff $\lambda < \bar{\lambda}$, and it is socially desirable iff $\lambda < \underline{\lambda}$. (ii) $\underline{\lambda} > 0$ in the limit as $\kappa \to 0$. $\bar{\lambda} < 1$ in the limit as $\delta \to 0$ and $n \to \infty$.*

The main result implies that the bundling mechanism is optimal for the buyer and also socially desirable if the aggregate risk is below certain thresholds. The intuition is simple. As explained after Lemma 3.1, no supplier bears the aggregate risk in the unbundling mechanism because the risk is transferred to the risk-neutral buyer. In the bundling mechanism, a winner bears all the risk. Therefore, a decrease in the aggregate risk is beneficial to both the buyer and society only with the bundling mechanism.

The theorem also states that the thresholds $\underline{\lambda}$ and $\bar{\lambda}$ are bounded away from the endpoints of $[0,1]$ for some values of the parameters κ, δ and n. On the one hand, the bundling mechanism implements the efficient investment, but requires the buyer to pay a risk premium for all the risks. The disadvantage in terms of the premium for the idiosyncratic risk vanishes as $\kappa \to 0$ so that the risk is negligible. Then, the bundling mechanism becomes optimal for the buyer and socially desirable for sufficiently small λ. On the other hand, the unbundling mechanism causes underinvestment, and requires the buyer to pay information rents. These disadvantages vanish as $\delta \to 0$ and $n \to \infty$ so that the underinvestment problem is less serious and the market is competitive. Then, the unbundling mechanism becomes optimal for the buyer and socially desirable for sufficiently large λ. Figure 1 illustrates the result. We can see that the bundling mechanism chosen by the buyer is not socially desirable for $\lambda \in (\underline{\lambda}, \bar{\lambda})$. This happens because the buyer cannot exploit the social welfare in the unbundling mechanism.

Next, we examine the effect of a change in the idiosyncratic risk. One may expect that a decrease in the idiosyncratic risk also encourages the buyer to choose the bundling mechanism. Interestingly, this is not always the case. Although a decrease in the idiosyncratic risk reduces a risk premium under bundling, it may reduce the suppliers' information rents under unbundling by changing the competition structure. An example is provided in the following.

Example 3.1. Assume that the idiosyncratic risks θ_i's are exponentially distributed on $[-\kappa, \infty)$ with $F(\theta_i;\kappa) = 1 - \exp(-\theta_i/\kappa - 1)$ for each $\theta_i \geq -\kappa$. For the distribution, $E[\theta_i] = 0$ for each $\kappa > 0$, and $F(\cdot;\kappa)$ second-order stochastically dominates $F(\cdot;\kappa')$ if $\kappa' > \kappa$. The distribution also satisfies the assumption in footnote 4. Then, each supplier's

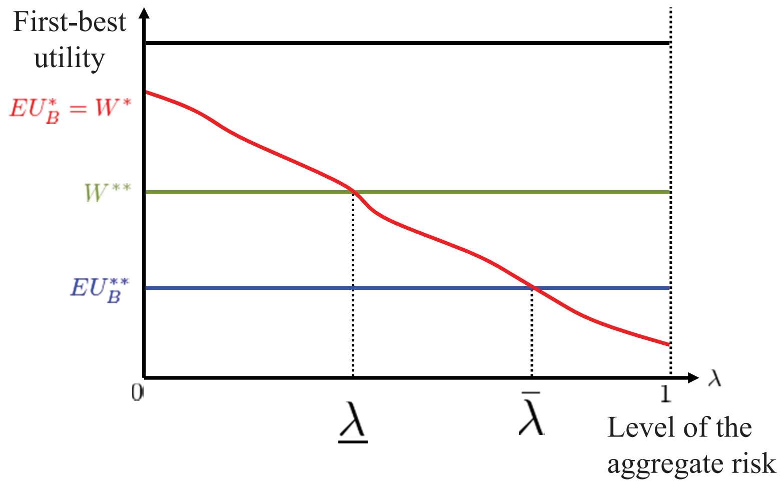

Figure 3.1: Illustration of Theorem 3.1

expected information rent is

$$E[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)] = \frac{1}{nr} \left(\ln(\kappa r + n - 1) - \ln(n-1)\right).$$

Obviously, the rent is increasing in κ. Assuming that $\kappa r \in (0,1)$, a calculation shows that the risk premium ρ^* in the bundling mechanism is higher than the risk premia $n\rho^{**}$ in the unbundling mechanism. The calculations are given in the proof of Example 3.1 in the Appendix.

Moreover, assume that ω is distributed on $\{-2, 0, 2\}$ with equal probability, $v = 10$, $r = 1$, $\kappa \in (0,1)$, $\delta = 2$, $n = 2$ and $\psi(a) = a^2$. Then, the buyer's expected utility under bundling is

$$EU_B^* = v - \{\tilde{a}^2 - 2\tilde{a} + \rho^*\} = 11 + \kappa + \ln(1-\kappa) - \ln\left(\frac{\exp(-2) + 1 + \exp(2)}{3}\right).$$

The buyer's expected utility under unbundling is

$$EU_B^{**} = v - \{E[\theta_{(2)}] + 2E[(1 - F_{(1)}(\theta_i))(b(\theta_i) - \theta_i)]\} = 10 + \frac{\kappa}{2} - \ln(1+\kappa).$$

Figure 2 depicts the buyer's expected utilities under bundling and unbundling. In this example, one can see that a decrease in the idiosyncratic risk κ changes the buyer's optimal choice from bundling to unbundling.

The next proposition examines the effect of the marginal benefit δ of investment on the mechanism choice.

Proposition 3.3. *There exist two thresholds $\underline{\delta}, \bar{\delta} \geq 0$ with $\underline{\delta} \leq \bar{\delta}$ such that the buyer's optimal choice is the bundling mechanism iff $\delta > \underline{\delta}$, and it is socially desirable iff $\delta > \bar{\delta}$.*

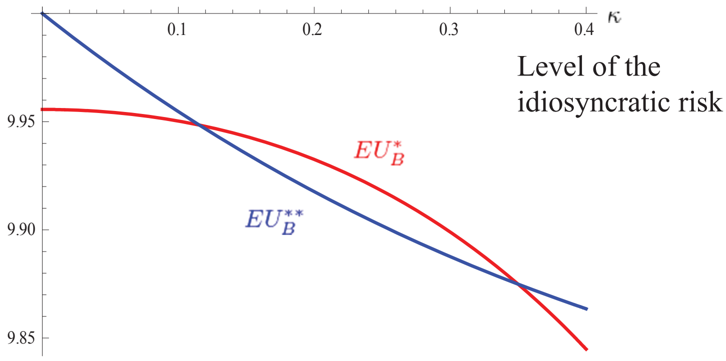

Figure 3.2: Illustration of Example 3.1.

Proposition 3.3 implies that the bundling mechanism is optimal and socially desirable when the marginal benefit of investment is sufficiently high. It follows from Lemma 3.1 that the second-stage winner's profit $p^2(a, \theta_i, \omega) - c(a, \theta_i, \omega)$ does not depend on the marginal benefit δ. The cost-reduction effect is exactly offset by the price-reduction effect. On the other hand, an increase in the marginal benefit provides a winner under bundling with stronger investment incentives. Figure 3 illustrates this result.

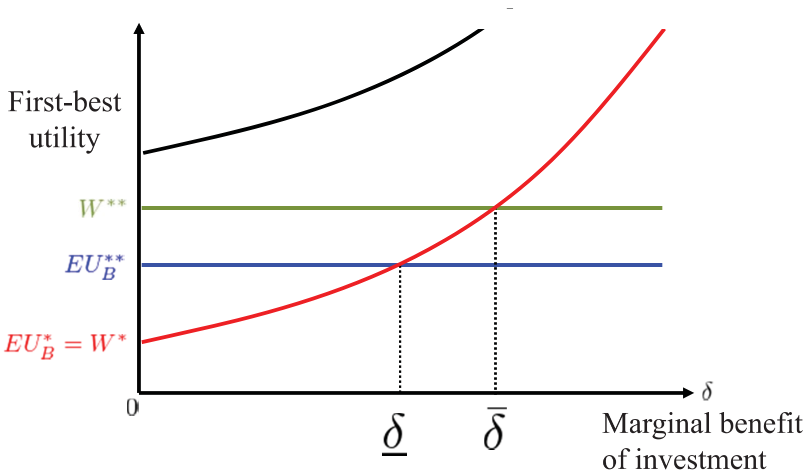

Figure 3.3: Illustration of Proposition 3.3.

Remark 3.1. The following comment may be helpful to understand our results further. The buyer could implement the efficient investment by an unbundling mechanism with a more sophisticated contract than the first-price auction. The mechanism is as follows. The buyer holds the first-price auction at date 1 as in the unbundling mechanism. A winner of the auction chooses an investment level at date 2. After all values of risks θ_i and ω are realized at date 3, all suppliers submit reports on the investment level. The second auction for production is held at date 4. The winner is chosen by the same rule as in the original unbundling mechanism, that is, the first-price auction. In addition to this, all suppliers incur infinite penalties if there are different reports. Even if all suppliers make the same reports, the first-stage winner incurs an infinite penalty if that report is not equal to the efficient investment level $\tilde{a}$. It can be shown that there exists an equilibrium under the sophisticated contract such that the first-stage winner chooses the efficient investment level, and every supplier truthfully reports it. Thus, the sophisticated contract solves the moral hazard problem regarding investment in the unbundling mechanism. The social welfare under the sophisticated contract is given by

$$v - \left\{\psi(\tilde{a}) - \delta\tilde{a} + E[\theta_{(n)}] + n\rho^{**}\right\}. \tag{3.8}$$

Example 3.1 shows that $\rho^* > n\rho^{**}$ if the idiosyncratic risks θ_i's are exponentially distributed. Then, the unbundling mechanism with the sophisticated contract *always* yields a higher social welfare than the bundling mechanism.

However, even if the sophisticated contract above is implementable, the buyer still faces a tradeoff between risk premia and information rents in the mechanism choice. The buyer pays the risk premium ρ^* under bundling, and does the information rents $nE[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)]$ under unbundling. When the idiosyncratic risks θ_i's are exponentially distributed as in Example 3.1, the buyer's optimal choice is a bundling mechanism for sufficiently small λ, κ and n. The proof is given in the Appendix. Therefore, we can conclude that the optimality of a bundling mechanism still holds true for some values of the parameters even when the sophisticated contract is implementable.

Finally, we consider the issue of ex post efficiency of contract allocation. The contractual flexibility of the unbundling mechanism allows the buyer to select the most efficient supplier, whereas the contractual rigidity of the bundling mechanism does not. The latter is a harmful effect of the buyer's commitment not to switch suppliers under bundling. Thus, with respect to the ex post efficiency of contract allocation, the unbundling mechanism is superior to the bundling mechanism. We discuss further the issue in the next section.

3.6 A Bundling Mechanism with Subcontracting

In this section, we discuss how our main results in Section 5 can be generalized to a situation where subcontracting is possible under a bundling mechanism.

A crucial difference between bundling and unbundling is that the buyer commits at the outset to the production contract in the bundling mechanism. In particular, the buyer has an incentive to change suppliers under bundling if the contract allocation is ex post inefficient. Suppose, for example, that the buyer can change suppliers at date 4 by the first-price auction. In this case, the equilibrium outcome of bundling is equivalent to that of unbundling. The difference between the two mechanisms disappears.

Instead, we consider another scenario in which the buyer allows a first-stage winner to hire a subcontractor. We assume that, at date 4, a winner under bundling selects a subcontractor by the first-price auction with a reserve price being equal to the winner's production cost.[9] We call the modified mechanism *bundling with subcontracting*, and the auction held by the first-stage winner a *subcontracting auction*. For simplicity of analysis, we consider the case that all $n-1$ suppliers participate in the subcontracting auction.[10]

The following lemma characterizes a symmetric equilibrium in the subcontracting auction. Since the lemma can be proved in the same way as Lemma 3.1, we have omitted the proof in this text.

Lemma 3.2. *In a bundling mechanism with subcontracting, given the investment level a, a common parameter ω and the reserve price $c(a,\theta,\omega)$, the following bidding strategy $p(a,\cdot,\omega,\theta)$ constitutes a symmetric equilibrium in the subcontracting auction: For any $\theta_i \in [\underline{\theta},\theta]$,*

$$p(a,\theta_i,\omega,\theta) = b(\theta_i,\theta) + \omega - \delta a,$$

where $b(\theta_i,\theta) := -\frac{1}{r}\ln E_{\theta_{(n-2)}}[\max\{\exp(-r\theta_{(n-2)}), \exp(-r\theta)\} \mid \theta_{(n-2)} > \theta_i]$. For any $\theta_i \in (\theta,\bar{\theta}]$, $p(a,\theta_i,\omega,\theta)$ is any bid higher than $c(a,\theta,\omega)$.

Lemma 3.2 implies that the contract allocation is ex post efficient in the bundling mechanism with subcontracting because the bidding strategy $p(\cdot)$ increases in each supplier's private parameter and the reserve price is given by the first-stage winner's production cost. Notice that the first-stage winner bears the aggregate risk ω in any state. The fact that the payment $p(a,\theta_i,\omega,\theta)$ is a subcontracting cost to the first-stage winner, not the buyer, is crucial to our result.

By investing a, a winner in the first-stage auction obtains the expected utility $E[(1-F_{(n-1)}(\theta_i))u(p_i^1-\psi(a)-c(a,\theta_i,\omega))+F_{(n-1)}(\theta_i)u(p_i^1-\psi(a)-p(a,\theta_{(n-1)},\omega,\theta_i))]$. The first term in the expected utility corresponds to the event that no subcontracting occurs, and the second term corresponds to the event that subcontracting does occur. The certainty equivalent is

$$p_i^1 - \psi(a) - E\left[(1-F_{(n-1)}(\theta_i))\theta_i + \int_{\underline{\theta}}^{\theta_i} b(s,\theta_i) f_{(n-1)}(s)ds\right] + \delta a - \rho_w. \tag{3.9}$$

[9]Under this assumption, all suppliers know the first-stage winner's private parameter from the announced reserve price with the investment level and the realized value of the aggregate risk.

[10]We implicitly assume that $n \geq 3$. However, even if $n = 2$, all results are still valid, assuming that a first-stage winner offers a price equal to his production cost to the other supplier.

The winner's risk premium is given by

$$\rho_w = \frac{1}{r} \ln E\left[(1 - F_{(n-1)}(\theta_i)) \exp(r\theta_i) + \int_{\underline{\theta}}^{\theta_i} \exp(rb(s,\theta_i)) f_{(n-1)}(s) ds\right]$$
$$- E\left[(1 - F_{(n-1)}(\theta_i))\theta_i + \int_{\underline{\theta}}^{\theta_i} b(s,\theta_i) f_{(n-1)}(s) ds\right] + \frac{1}{r}\ln E[\exp(r\omega)].$$

As in the case of unbundling, even if a supplier loses in the first stage, he has an outside option to participate in the subcontracting auction. A first-stage loser wins the subcontracting auction if and only if he has the lowest private parameter among all the suppliers. Thus, his expected profit is $E\left[F(\theta_j)E_{\theta_i}[(1 - F_{(n-2)}(\theta_i))(p(a,\theta_i,\omega,\theta_j) - c(a,\theta_i,\omega)) \mid \theta_i < \theta_j]\right]$, and his risk premium is

$$\rho_l = E\left[F(\theta_j)E_{\theta_i}[(1 - F_{(n-2)}(\theta_i))(b(\theta_i,\theta_j) - \theta_i) \mid \theta_i < \theta_j]\right]$$
$$+ \frac{1}{r}\ln E\big[F(\theta_j)E_{\theta_i}[(1 - F_{(n-2)}(\theta_i)) \exp(-r(b(\theta_i,\theta_j) - \theta_i)) + F_{(n-2)}(\theta_i) \mid \theta_i < \theta_j] + (1 - F(\theta_j))\big].$$

Next, we characterize an equilibrium under bundling with subcontracting.

Lemma 3.3. *In a bundling mechanism with subcontracting, the following equilibrium exists: (i) Every supplier submits the same bid*

$$\psi(\tilde{a}) - \delta\tilde{a} + E\left[\frac{n}{n-1}\int_{\underline{\theta}}^{\theta_i} b(s,\theta_i) f_{(n-1)}(s) ds\right] + \rho_w - \rho_l$$

in the first-stage auction. The winner chooses the efficient investment level $\tilde{a}$. The other $n-1$ suppliers employ the bidding strategy $p(\cdot)$ with $b(\cdot,\cdot)$ given in Lemma 3.2 in the subcontracting auction. (ii) The buyer's expected utility, EU_B^s, supplier i's expected utility, EU_i^s, and the social welfare, W^s, are respectively given by the following:

$$EU_B^s = v - \left\{\psi(\tilde{a}) - \delta\tilde{a} + E\left[\frac{n}{n-1}\int_{\underline{\theta}}^{\theta_i} b(s,\theta_i) f_{(n-1)}(s) ds\right] + \rho_w - \rho_l\right\},$$
$$EU_i^s = u\left(E\left[F(\theta_j)E_{\theta_i}[(1 - F_{(n-2)}(\theta_i))(b(\theta_i,\theta_j) - \theta_i) \mid \theta_i < \theta_j]\right] - \rho_l\right),$$
$$W^s = v - \left\{\psi(\tilde{a}) - \delta\tilde{a} + E[\theta_{(n)}] + \rho_w + (n-1)\rho_l\right\}.$$

As previously indicated, this lemma shows the properties of the bundling mechanism with subcontracting. As in the case of unbundling, each supplier obtains positive utility from his information rent. The allocation of the production contract is ex post efficient. An efficient investment level can be attained in equilibrium. The suppliers must bear some risks of production and subcontracting.

Finally, the following proposition shows that essentially the same results as Theorem 3.1 and Proposition 3.3 hold true even if subcontracting is possible in a bundling mechanism. The reasoning behind the result is that a first-stage winner who is responsible for the production as a prime contractor always bears the aggregate risk but has strong investment incentives. In the proof, we also show that each supplier's expected utility EU_i^s in the bundling mechanism with subcontracting is equal to that in the unbundling mechanism. This implies that $EU_B^s > EU_B^{**}$ if and only if $W^s > W^{**}$. Thus, we can obtain a single threshold $\hat{\lambda}$ with respect to the aggregate risk and a single threshold $\hat{\delta}$ with respect to the marginal benefit of investment.

Proposition 3.4. *(i) Assume that $\lambda \in (0,1)$ represents the level of aggregate risk. Then, there exists a threshold $\hat{\lambda} \in [0,1]$ such that $EU_B^s > EU_B^{**}$ iff $\lambda < \hat{\lambda}$, and $W^s > W^{**}$ iff $\lambda < \hat{\lambda}$. Also, $\hat{\lambda} > 0$ in the limit as $\kappa \to 0$; $\hat{\lambda} < 1$ in the limit as $\delta \to 0$ and $n \to \infty$. (ii) There exists a threshold $\hat{\delta} \geq 0$ such that $EU_B^s > EU_B^{**}$ iff $\delta > \hat{\delta}$, and $W^s > W^{**}$ iff $\delta > \hat{\delta}$.*

3.7 Concluding Remarks

We have compared the performance of a bundling method with that of an unbundling method in an auction model with two risk factors, aggregate risk and idiosyncratic risk. Through the classification of risks in infrastructure projects, we provide a new perspective to the debate regarding the choice between the two methods. In particular, our findings show that each risk factor has a different effect on a public authority's optimal choice. As recognized by many practitioners, a public authority must pay a high risk premium in exchange for the burden of risks on a private party in the bundling method. As a result, the associated low aggregate risk is a strong reason for the public authority to choose the bundling method. A decrease in the idiosyncratic risk may reduce information rents for private parties and thus may encourage the public authority to choose the unbundling method.

Appendix

Proof of Proposition 3.1. First, the efficient investment level $\tilde{a}$ minimizes $\psi(a) + E[\theta_{(n)} + \omega] - \delta a$; thus, the level is uniquely determined by the first-order condition

$$\frac{d\psi}{da}(\tilde{a}) = \delta. \tag{3.10}$$

In the bundling mechanism, a winner chooses an investment level to maximize the certainty equivalent $p_i^1 - \psi(a) + \delta a - \rho^*$. Since the risk premium ρ^* does not depend on a, the equilibrium investment level is the same as the efficient level $\tilde{a}$. Next, the auction is equivalent to the Bertrand competition among symmetric suppliers. Thus, in any symmetric equilibrium, all suppliers submit the same bid $\psi(\tilde{a}) - \delta\tilde{a} + \rho^*$. □

Proof of Lemma 3.1. We must show that for any a and ω, a supplier with θ cannot gain by deviating from a bid $p^2(a, \theta, \omega)$ when the other suppliers follow the strategy $p^2(\cdot)$. Note that $p^2(\cdot)$ is increasing and continuous in θ.

First, we can assume without loss of generality that no supplier submits a bid $p \notin [p^2(a, \underline{\theta}, \omega), p^2(a, \bar{\theta}, \omega)]$. If a supplier bids $p > p^2(a, \bar{\theta}, \omega)$, then he loses with probability one. By bidding $p^2(a, \bar{\theta}, \omega)$, he can obtain the same utility. If a supplier bids $p < p^2(a, \underline{\theta}, \omega)$, then he wins with probability one. By bidding $p^2(a, \underline{\theta}, \omega)$, he can win with probability one and obtain higher utility.

Second, we show that it is optimal for a supplier with θ to bid $p = p^2(a, \theta, \omega)$. Consider the following two cases: The supplier wins or loses in the first stage. If the supplier loses in the first-stage auction, then his expected utility from bidding $p^2(a, \hat{\theta}, \omega)$ is

$$
\begin{aligned}
&(1 - F_{(n-1)}(\hat{\theta}))u(p^2(a, \hat{\theta}, \omega) - c(a, \theta, \omega)) + F_{(n-1)}(\hat{\theta})u(0) \\
&= (1 - F_{(n-1)}(\hat{\theta})) \int_{\hat{\theta}}^{\bar{\theta}} [1 - \exp(-r(s - \theta))] \frac{f_{(n-1)}(s)}{1 - F_{(n-1)}(\hat{\theta})} ds \\
&= \int_{\hat{\theta}}^{\bar{\theta}} u(s - \theta) f_{(n-1)}(s) ds.
\end{aligned}
$$

The equalities follow from the definitions of $u(\cdot)$ and $p^2(\cdot)$. The difference between the expected utility from bidding $p^2(a, \theta, \omega)$ and that from bidding $p^2(a, \hat{\theta}, \omega) \neq p^2(a, \theta, \omega)$ is

$$
\int_{\theta}^{\hat{\theta}} u(s - \theta) f_{(n-1)}(s) ds > 0.
$$

In contrast, if the supplier wins with a bid p^1 in the first-stage auction, then his expected utility from bidding $p^2(a, \hat{\theta}, \omega)$ is given by

$$
\begin{aligned}
&(1 - F_{(n-1)}(\hat{\theta}))u(p^1 - \psi(a) + p^2(a, \hat{\theta}, \omega) - c(a, \theta, \omega)) + F_{(n-1)}(\hat{\theta})u(p^1 - \psi(a)) \\
&= u(p^1 - \psi(a)) + \exp(-r(p^1 - \psi(a))) \left\{ \int_{\hat{\theta}}^{\bar{\theta}} u(s - \theta) f_{(n-1)}(s) ds \right\}.
\end{aligned}
$$

The equality follows from some additional calculations. The difference between the expected utility from bidding $p^2(a, \theta, \omega)$ and that from bidding $p^2(a, \hat{\theta}, \omega) \neq p^2(a, \theta, \omega)$ is

$$
\exp(-r(p^1 - \psi(a))) \left\{ \int_{\theta}^{\hat{\theta}} u(s - \theta) f_{(n-1)}(s) ds \right\} > 0.
$$

Therefore, it is optimal for a supplier with θ to bid $p = p^2(a, \theta, \omega)$ regardless of whether he wins or loses in the first stage. □

Proof of Proposition 3.2. (i) First, Lemma 3.1 implies that $p^2(\cdot)$ constitutes a symmetric equilibrium in the second-stage auction. Second, a winner in the first-stage auction chooses an investment level to maximize the certainty equivalent (3.6). Since the certainty equivalent decreases in a, the winner optimally chooses $a = 0$. Third, in the first-stage auction, every supplier submits a bid which makes him indifferent between winning and losing, in any symmetric equilibrium. Even if a supplier loses the first-stage auction, he can obtain the positive expected utility in the second-stage auction. The certainty equivalent is given by $E[(1-F_{(n-1)}(\theta_i))(b(\theta_i)-\theta_i)] - \rho^{**}$. Hence, all suppliers submit the same bid $\psi(0)$ in the first-stage auction.

(ii) Since a supplier with $\theta_{(n)}$ wins the second-stage auction in equilibrium, the buyer's expected utility is $EU_B^{**} = v - \psi(0) - E[p^2(0, \theta_{(n)}, \omega)]$. The expected payment in the second-stage auction $E[p^2(0, \theta_{(n)}, \omega)]$ can be rewritten as

$$
\begin{aligned}
E[b(\theta_{(n)})] + E[\omega] &= \int_{\underline{\theta}}^{\bar{\theta}} b(s) f_{(n)}(s) ds + E[\theta_{(n)}] - \int_{\underline{\theta}}^{\bar{\theta}} s f_{(n)}(s) ds \\
&= E[\theta_{(n)}] + \int_{\underline{\theta}}^{\bar{\theta}} [b(s) - s] \cdot n(1 - F_{(n-1)}(s)) f(s) ds \\
&= E[\theta_{(n)}] + nE[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)].
\end{aligned}
$$

The second equality follows from the definition of $f_{(n)}(\cdot)$. Thus, $EU_B^{**} = v - \big\{\psi(0) + E[\theta_{(n)}] + nE[(1-F_{(n-1)}(\theta_i))(b(\theta_i)-\theta_i)]\big\}$. Since every supplier wins the first-stage auction with equal probability, his equilibrium expected utility is given by $EU_i^{**} = u(E[(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)] - \rho^{**})$. Finally, the social welfare is given by $W^{**} = EU_B^{**} + n \cdot u^{-1}(EU_i^{**}) = v - \big\{\psi(0) + E[\theta_{(n)}] + n\rho^{**}\big\}$. □

Proof of Theorem 3.1. (i) It follows from the first-order condition (3.10) that $\tilde{a}$ does not depend on λ. Since $G(\cdot;\lambda)$ second-order stochastically dominates $G(\cdot;\lambda')$ if $\lambda' > \lambda$, the risk premium ρ^* under bundling increases in λ. Thus, both EU_B^* and W^* decrease in λ. On the other hand, under unbundling, the second-stage winner's profit $b(\theta_i) - \theta_i$ does not depend on λ; therefore, the risk premium ρ^{**} is invariant to λ. Thus, both EU_B^{**} and W^{**} are invariant to λ. Proposition 3.2 (ii) also implies that $EU_B^{**} < W^{**}$.

Then, we have thresholds $\underline{\lambda}$ and $\bar{\lambda}$ with the properties in part (i) of the theorem. If $EU_B^* > EU_B^{**}$ (resp. $W^* > W^{**}$) for each $\lambda \in (0,1)$, then we set $\bar{\lambda} = 1$ (resp. $\underline{\lambda} = 1$). If $EU_B^* < EU_B^{**}$ (resp. $W^* < W^{**}$) for each $\lambda \in (0,1)$, then we set $\bar{\lambda} = 0$ (resp. $\underline{\lambda} = 0$).

(ii) First, we claim that $\underline{\lambda}$ is greater than zero in the limit as $\kappa \to 0$. We show that as $\kappa \to 0$ and $\lambda \to 0$, the difference between $W^*(= EU_B^*)$ and the first-best utility $v - \{\psi(\tilde{a}) + E[\theta_{(n)}] - \delta\tilde{a}\}$ converges to zero. According to the definition of the convergence in law, for any $\theta > 0$, $F(\theta;\kappa) \to 1$ as $\kappa \to 0$. Also, for any κ, $\lim_{m\to\infty} \int_m^\infty \exp(r\theta) dF(\theta;\kappa) = 0$. This implies that, for any $\varepsilon > 0$, there exist positive numbers δ, m with $0 < \delta < m$

and $\bar{\kappa} > 0$ such that for any $\kappa < \bar{\kappa}$, the following inequalities remain true:[11]

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} \exp(r\theta)dF(\theta;\kappa) &= \int_{-\infty}^{\delta} \exp(r\theta)dF(\theta;\kappa) + \int_{\delta}^{m} \exp(r\theta)dF(\theta;\kappa) + \int_{m}^{\infty} \exp(r\theta)dF(\theta;\kappa) \\ &< \exp(r\delta) + \exp(rm)[F(m;\kappa) - F(\delta;\kappa)] + \int_{m}^{\infty} \exp(r\theta)f(\theta;\kappa)d\theta \\ &< \left(1 + \frac{\varepsilon}{3}\right) + \frac{\varepsilon}{3} + \frac{\varepsilon}{3} = 1 + \varepsilon.\end{aligned}$$

Since $E[\exp(r\theta_i)] > \exp(rE[\theta_i]) = 1$ always remains true, $\ln E[\exp(r\theta_i)] \to 0$ as $\kappa \to 0$. Similarly, $\ln E[\exp(r\omega)] \to 0$ as $\lambda \to 0$. Hence, the risk premium ρ^* converges to zero as $\kappa, \lambda \to 0$. Also, according to the definition of the convergence in law, for any $\theta < 0$, $F(\theta;\kappa) \to 0$ as $\kappa \to 0$. Hence, a similar argument shows that $E[\theta_{(n)}] \to 0$ as $\kappa \to 0$. On the other hand, the difference between W^{**} and the first-best utility is strictly smaller than zero even in the limit as $\kappa, \lambda \to 0$. This is because $0 < \tilde{a}$. Therefore, $\underline{\lambda} > 0$ in the limit as $\kappa \to 0$.

Next, we claim that $\bar{\lambda}$ is smaller than one in the limit as $n \to \infty$ and $\delta \to 0$. Using integration by parts, we transform the bidding function $b(\cdot)$ in Lemma 3.1 into

$$\begin{aligned} b(\theta_i) &= -\frac{1}{r}\ln\left\{-\left[\exp(-rs)\frac{1 - F_{(n-1)}(s)}{1 - F_{(n-1)}(\theta_i)}\right]_{\theta_i}^{\bar{\theta}} - r\int_{\theta_i}^{\bar{\theta}} \exp(-rs)\frac{1 - F_{(n-1)}(s)}{1 - F_{(n-1)}(\theta_i)}ds\right\} \\ &= -\frac{1}{r}\ln\left\{\exp(-r\theta_i) - r\int_{\theta_i}^{\bar{\theta}} \exp(-rs)\left(\frac{1 - F(s)}{1 - F(\theta_i)}\right)^{n-1} ds\right\}. \end{aligned} \tag{3.11}$$

Lebesgue's dominated convergence theorem implies that as the number n of suppliers goes to infinity, the term $b(\theta_i)$ converges to θ_i for each θ_i. Based on this fact, we can easily verify that $E[n(1 - F_{(n-1)}(\theta_i))(b(\theta_i) - \theta_i)]$ converges to zero as $n \to \infty$. Moreover, $E[\theta_{(n)}]$ converges to $\underline{\theta}$ as $n \to \infty$. Thus, EU_B^{**} converges to $v - \{\psi(0) + \underline{\theta}\}$ as $n \to \infty$. In contrast, EU_B^{*} is invariant to n. It follows from the first-order condition (3.10) that $\tilde{a} \to 0$ as $\delta \to 0$. Therefore, $\bar{\lambda} = 0$ for sufficiently large n and small δ. This establishes the claim. □

Proof of Example 3.1. Assume that $F(\theta_i;\kappa) = 1 - \exp(-\theta_i/\kappa - 1)$ for each $\theta_i \geq -\kappa$. From the distribution, the bidding function $b(\cdot)$ in the second-stage auction is given as follows:

$$\begin{aligned} b(\theta_i) &= -\frac{1}{r}\ln\int_{\theta_i}^{\infty} \exp(-rs)\frac{n-1}{\kappa}\exp\left(-(n-1)\left(\frac{s}{\kappa}+1\right)\right)\exp\left((n-1)\left(\frac{\theta_i}{\kappa}+1\right)\right)ds \\ &= \theta_i + \frac{1}{r}\left(\ln(\kappa r + n - 1) - \ln(n-1)\right). \end{aligned}$$

[11]Even if $\bar{\theta} = \infty$, we can find such number m using the assumption detailed in footnote 4.

Therefore, a simple calculation shows that

$$E[(1-F_{(n-1)}(\theta_i))(b(\theta_i)-\theta_i)] = \frac{1}{nr}\left(\ln(\kappa r+n-1)-\ln(n-1)\right),$$

which increases in κ. The equality follows from $E[(1-F_{(n-1)}(\theta_i))] = 1/n$.

We next show that $\rho^* > n\rho^{**}$. From the distribution of θ_i, ρ^* is given as follows:

$$\begin{aligned}\rho^* &= \frac{1}{r}\ln\int_{-\kappa}^{\infty}\exp(rs)\frac{1}{\kappa}\exp\left(-\frac{s}{\kappa}-1\right)ds + \frac{1}{r}\ln E[\exp(r\omega)] \\ &= -\frac{1}{r}\ln(1-\kappa r)-\kappa+\frac{1}{r}\ln E[\exp(r\omega)].\end{aligned}$$

This is well-defined because $\kappa r \in (0,1)$. We can compute $n\rho^{**}$ as follows:

$$\begin{aligned}n\rho^{**} &= \frac{n}{r}\ln E\left[(1-F_{(n-1)}(\theta_i))\frac{n-1}{\kappa r+n-1}+F_{(n-1)}(\theta_i)\right] \\ &\quad + nE[(1-F_{(n-1)}(\theta_i))]\frac{1}{r}(\ln(\kappa r+n-1)-\ln(n-1)) \\ &= \frac{n-1}{r}\left[\ln(n-1)-\ln(\kappa r+n-1)\right]+\frac{n}{r}\left[\ln(\kappa r+n)-\ln(n)\right].\end{aligned}$$

It is easy to show that $\rho^* \to \frac{1}{r}\ln E[\exp(r\omega)] > 0$ and $n\rho^{**} \to 0$ as $\kappa \to 0$. Also, a simple calculation shows that

$$\frac{\partial\rho^*}{\partial\kappa} = \frac{\kappa r}{1-\kappa r} > \frac{\kappa r}{(\kappa r+n)(\kappa r+n-1)} = \frac{\partial(n\rho^{**})}{\partial\kappa}$$

for each κ, r with $\kappa r \in (0,1)$ and $n \geq 2$. Therefore, it holds that $\rho^* > n\rho^{**}$. □

Proof of Remark 3.1. We claim that if the idiosyncratic risks θ_i's are exponentially distributed as in Example 3.1, then for sufficiently small λ, κ and n, the buyer's expected utility EU_B^* in the bundling mechanism is greater than the following expected utility in the unbundling mechanism with the sophisticated contract; i.e.,

$$\begin{aligned}&v-\left\{\psi(\tilde{a})-\delta\tilde{a}+E[\theta_{(n)}]+nE[(1-F_{(n-1)}(\theta_i))(b(\theta_i)-\theta_i)]\right\} \\ &= v-\left\{\psi(\tilde{a})-\delta\tilde{a}-\left(\frac{n-1}{n}\right)\kappa+\frac{1}{r}\left(\ln(\kappa r+n-1)-\ln(n-1)\right)\right\}.\end{aligned}\tag{3.12}$$

The difference between the expected utility (3.12) and the first-best utility converges to zero as $\kappa \to 0$. For the expected utility (3.12), the derivative with respect to κ is given by

$$\frac{n-1}{n}-\frac{1}{\kappa r+n-1}.\tag{3.13}$$

As shown in the proof of Theorem 3.1, the difference between the buyer's expected utility

EU_B^* in the bundling mechanism and the first-best utility converges to zero as $\lambda \to 0$ and $\kappa \to 0$. The derivative with respect to κ is

$$\frac{\partial EU_B^*}{\partial \kappa} = -\frac{\partial \rho^*}{\partial \kappa} = -\frac{\kappa r}{1-\kappa r}. \tag{3.14}$$

The derivative (3.14) is greater than (3.13) when κ is close to zero and $n = 2$. Therefore, EU_B^* is greater than the expected utility (3.12) for sufficiently small λ, κ and $n = 2$. $\square$

Proof of Proposition 3.3. First, using the envelope theorem, we can show that $\frac{\partial EU_B^*}{\partial \delta} = \frac{\partial W^*}{\partial \delta} = \tilde{a}$. Next, the assumption of additively separable production cost implies that neither the second-stage winner's profit $b(\theta_i) - \theta_i$ nor the risk premium ρ^{**} under unbundling depends on δ. Hence, $\frac{\partial EU_B^{**}}{\partial \delta} = \frac{\partial W^{**}}{\partial \delta} = 0$. Since $\tilde{a} > 0$, $\frac{\partial EU_B^*}{\partial \delta} = \frac{\partial W^*}{\partial \delta} = \tilde{a} > 0 = \frac{\partial EU_B^{**}}{\partial \delta} = \frac{\partial W^{**}}{\partial \delta}$. Proposition 3.2 (ii) also implies that $EU_B^{**} < W^{**}$.

Then, we have thresholds $\underline{\delta}$ and $\bar{\delta}$ with the properties in the proposition. If $EU_B^* > EU_B^{**}$ (resp. $W^* > W^{**}$) for any $\delta > 0$, then we set $\underline{\delta} = 0$ (resp. $\bar{\delta} = 0$). Also, $\underline{\delta}, \bar{\delta} < \infty$ because $W_B^* > W_B^{**}$ and $EU_B^* > EU_B^{**}$ for some sufficiently large δ. This completes the proof. $\square$

Proof of Lemma 3.3. (i) First, Lemma 3.2 implies that $p(\cdot)$ is an equilibrium bidding strategy in the subcontracting auction. Second, a first-stage winner chooses an investment level to maximize the certainty equivalent (3.9). Differentiating the certainty equivalent yields the first-order condition (3.10). Hence, the winner chooses the efficient investment level. Third, in the first-stage auction, every supplier submits a bid which makes him indifferent between winning and losing, in any symmetric equilibrium. Even if supplier i loses the first-stage auction, he can obtain the positive expected utility in the subcontracting auction. The certainty equivalent is given by

$$E\left[F(\theta_j)E_{\theta_i}[(1 - F_{(n-2)}(\theta_i))(b(\theta_i, \theta_j) - \theta_i) \mid \theta_i < \theta_j]\right] - \rho_l.$$

Hence, all suppliers submit the following bid:

$$\begin{aligned}
&\psi(\tilde{a}) + E\left[(1 - F_{(n-1)}(\theta_i))\theta_i + \int_{\underline{\theta}}^{\theta_i} b(s, \theta_i) f_{(n-1)}(s) ds + \omega - \delta\tilde{a}\right] + \rho_w \\
&\quad + E\left[\int_{\underline{\theta}}^{\theta_i} (1 - F_{(n-2)}(s))(b(s, \theta_i) - s) f(s) ds\right] - \rho_l \\
&= \psi(\tilde{a}) - \delta\tilde{a} + E\left[\frac{n}{n-1}\int_{\underline{\theta}}^{\theta_i} b(s, \theta_i) f_{(n-1)}(s) ds\right] + \rho_w - \rho_l.
\end{aligned}$$

The equality follows from the interchange of the integrals.

(ii) It is obvious that EU_B^s and EU_i^s are the same as in the proposition. The social welfare $W^s = EU_B^s + n \cdot u^{-1}(EU_i^s)$ can be rewritten as

$$
\begin{aligned}
& v - \left\{ \psi(\tilde{a}) - \delta \tilde{a} + E\left[\frac{n}{n-1} \int_{\underline{\theta}}^{\theta_i} b(s,\theta_i) f_{(n-1)}(s) ds \right] + \rho_w - \rho_l \right\} \\
& \quad + nE\left[\int_{\underline{\theta}}^{\theta_i} (1 - F_{(n-2)}(s))(b(s,\theta_i) - s) f(s) ds \right] - n\rho_l \\
& = v - \left\{ \psi(\tilde{a}) - \delta \tilde{a} + E[\theta_{(n)}] + \rho_w + (n-1)\rho_l \right\}.
\end{aligned}
$$

□

Proof of Proposition 3.4. As a preliminary, we prove that $EU_i^s = EU_i^{**}$; thus, $W^s - EU_B^s = W^{**} - EU_B^{**}$. From Proposition 3.2 (ii) and equation (3.11), the supplier's expected utility EU_i^{**} in the unbundling mechanism can be rewritten as

$$
\begin{aligned}
& 1 - E[(1 - F_{(n-1)}(\theta_i)) \exp(-r(b(\theta_i) - \theta_i)) + F_{(n-1)}(\theta_i)] \\
& = 1 - \int_{\underline{\theta}}^{\bar{\theta}} \left\{ (1 - F_{(n-1)}(\theta_i)) \left[1 - r \int_{\theta_i}^{\bar{\theta}} \frac{\exp(-rs)}{\exp(-r\theta_i)} \left(\frac{1 - F(s)}{1 - F(\theta_i)} \right)^{n-1} ds \right] + F_{(n-1)}(\theta_i) \right\} f(\theta_i) d\theta_i \\
& = r \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta_i}^{\bar{\theta}} \frac{\exp(-rs)}{\exp(-r\theta_i)} (1 - F(s))^{n-1} f(\theta_i) ds d\theta_i. \qquad (3.15)
\end{aligned}
$$

Using integration by parts, we transform the bidding function $b(\cdot,\cdot)$ in Lemma 3.2 into

$$
\begin{aligned}
b(\theta_i, \theta) &= -\frac{1}{r} \ln \left\{ \int_{\theta_i}^{\theta} \exp(-rs) \frac{f_{(n-2)}(s)}{1 - F_{(n-2)}(\theta_i)} ds + \int_{\theta}^{\bar{\theta}} \exp(-r\theta) \frac{f_{(n-2)}(s)}{1 - F_{(n-2)}(\theta_i)} ds \right\} \\
&= -\frac{1}{r} \ln \left\{ \int_{\theta_i}^{\theta} \exp(-rs) \frac{f_{(n-2)}(s)}{1 - F_{(n-2)}(\theta_i)} ds + \exp(-r\theta) \left(\frac{1 - F(\theta)}{1 - F(\theta_i)} \right)^{n-2} \right\} \\
&= -\frac{1}{r} \ln \left\{ \exp(-r\theta_i) - r \int_{\theta_i}^{\theta} \exp(-rs) \left(\frac{1 - F(s)}{1 - F(\theta_i)} \right)^{n-2} ds \right\}. \qquad (3.16)
\end{aligned}
$$

From Lemma 3.3 (ii) and equation (3.16), the supplier's expected utility EU_i^s in the bundling mechanism with subcontracting can be rewritten as

$$
\begin{aligned}
& 1 - E\big[F(\theta_j) E_{\theta_i}[(1 - F_{(n-2)}(\theta_i)) \exp(-r(b(\theta_i,\theta_j) - \theta_i)) + F_{(n-2)}(\theta_i) \mid \theta_i < \theta_j] + (1 - F(\theta_j))\big] \\
& = \frac{1}{2} - \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta_j} \left\{ (1 - F_{(n-2)}(\theta_i)) \left[1 - r \int_{\theta_i}^{\theta_j} \frac{\exp(-rs)}{\exp(-r\theta_i)} \left(\frac{1 - F(s)}{1 - F(\theta_i)} \right)^{n-2} ds \right] + F_{(n-2)}(\theta_i) \right\} \\
& \qquad f(\theta_i) d\theta_i f(\theta_j) d\theta_j \\
& = r \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta_j} \int_{\theta_i}^{\theta_j} \frac{\exp(-rs)}{\exp(-r\theta_i)} (1 - F(s))^{n-2} f(\theta_i) f(\theta_j) ds d\theta_i d\theta_j \\
& = r \int_{\underline{\theta}}^{\bar{\theta}} \int_{\theta_i}^{\bar{\theta}} \frac{\exp(-rs)}{\exp(-r\theta_i)} (1 - F(s))^{n-1} f(\theta_i) ds d\theta_i. \qquad (3.17)
\end{aligned}
$$

The first two equalities follows from $E[1 - F(\theta_j)] = 1/2$, and the third equality follows from Fubini's theorem. Hence, it follows from (3.15) and (3.17) that $EU_i^{**} = EU_i^s$.

(i) It follows from the first-order condition (3.10) and the definition of the first-stage loser's risk premium ρ_l that neither $\tilde{a}$ nor ρ_l depends on the parameter λ of the aggregate risk ω. According to the definition of the second-order stochastic dominance, the first-stage winner's risk premium ρ_w increases in λ. Hence, both EU_B^s and W^s decrease in λ. Now, the above preliminary result implies that $EU_B^s > EU_B^{**}$ iff $W^s > W^{**}$. Therefore, we have a threshold $\hat{\lambda}$ such that $EU_B^s > EU_B^{**}$ iff $\lambda < \hat{\lambda}$, and $W^s > W^{**}$ iff $\lambda < \hat{\lambda}$.

Now, we claim that $\hat{\lambda} > 0$ in the limit as $\kappa \to 0$. From equation (3.16), it is easy to see that $b(\theta_i, \theta)$ increases in θ_i and $b(\theta, \theta) = \theta$ for each θ. Using this fact, a similar argument to that for the proof of Theorem 3.1 shows that as $\kappa \to 0$ and $\lambda \to 0$, the difference between W^s and the first-best utility converges to zero. The difference between W^{**} and the first-best utility is strictly smaller than zero, even in the limit, because $0 < \tilde{a}$. Hence, it holds that $\hat{\lambda} > 0$ in the limit as $\kappa \to 0$.

Next, we claim that $\hat{\lambda} < 1$ in the limit as $\delta \to 0$ and $n \to \infty$. From equation (3.16), Lebesgue's dominated convergence theorem implies that as the number of suppliers, n, goes to infinity, the term $b(\theta_i, \theta)$ converges to θ_i for any θ_i and θ with $\theta_i \leq \theta$. In the limit as $n \to \infty$, subcontracting occurs with probability one, and the subcontracting cost is $\underline{\theta} + \omega - \delta\tilde{a}$. Hence, as $n \to \infty$, $\rho_l \to 0$ and $\rho_w \to \frac{1}{r} \ln E[\exp(r\omega)]$, so that EU_B^s converges to $v - \{\psi(\tilde{a}) - \delta\tilde{a} + \underline{\theta} + \frac{1}{r} \ln E[\exp(r\omega)]\}$. The proof of Theorem 3.1 shows that EU_B^{**} converges to $v - \{\psi(0) + \underline{\theta}\}$ as $n \to \infty$. Also, it follows from the first-order condition (3.10) that $\tilde{a} \to 0$ as $\delta \to 0$. Since $\frac{1}{r} \ln E[\exp(r\omega)] > 0$, it holds that $\hat{\lambda} = 0$ for sufficiently large n and small δ. Thus, this establishes the claim.

(ii) The envelope theorem implies that $\frac{\partial EU_B^s}{\partial \delta} = \frac{\partial W^s}{\partial \delta} = \tilde{a}$. It follows from the proof of Proposition 3.3 that $\frac{\partial EU_B^{**}}{\partial \delta} = \frac{\partial W^{**}}{\partial \delta} = 0$. Therefore, we can obtain the threshold $\hat{\delta}$ in the same way as part (i) of this proposition. $\square$

Chapter 4

Informed Principal Problems in Bilateral Trading

4.1 Introduction

In this chapter, we study an informed principal problem in a bilateral trade environment. A privately informed seller trades with a privately informed buyer. The two traders' private information directly affects their ex post payoffs in both ways. There are multiple units of goods to be traded. The strategic situation is modeled as a noncooperative game, called the *mechanism-selection game*. The seller has full bargaining power to design a trading mechanism. The model thus has both aspects of signaling and (monopolistic) screening.[1] The purpose of this chapter is to prove the existence of a separating equilibrium and investigate how the seller's "signaling activity" through the choice of mechanism affects allocative efficiency and distributional consequences.

Since the seminal work of Myerson and Satterthwaite (1983), a number of studies have discovered the efficiency properties of various mechanisms in bilateral trade environments. The central questions are whether efficient trade between two informed traders is possible or not, and which mechanism is optimal for a particular party. Most previous studies have implicitly assumed that the trading mechanism is designed by a third party (e.g., social planner, broker) with no private information.[2] In contrast, we assume that the seller with private information designs a trading mechanism with the same level of generality as previous studies.[3] Then, our model belongs to the informed-principal literature developed by Myerson (1983) and Maskin and Tirole (1990, 1992).

We analyze the following mechanism-selection game. At the outset, both the seller and buyer have private information about payoff-relevant types. Their types are inde-

[1]Riley (2001) provides an excellent survey of the literature on screening and signaling.

[2]Two exceptions are Yilankaya (1999) and Mylovanov and Tröger (2012b). They analyze the case of private values, defined shortly.

[3]Conversely, we can presume that the buyer designs a trading mechanism.

pendently distributed. Next, the seller announces a direct mechanism. The mechanism induces a game in which each player reports her/his own type. The buyer can also opt out. Given the reported types, the mechanism determines the (randomized) quantity to be traded and a payment.

There are two cases to be distinguished. The first one is the case of private values. In this case, the seller's information does not directly affect the buyer's payoff. As Maskin and Tirole (1990) have shown in the case of quasi-linear payoff functions, we can verify that the mechanism-selection game has a unique equilibrium outcome.[4] The seller's interim payoff is the same as in the optimal screening mechanism when her type is commonly known. The second one is the case of interdependent values.[5] In this case, the seller's information directly affects the buyer's payoff. Some type of seller may no longer select the optimal screening mechanism to avoid being mistaken for the other types and prevent an unfavorable outcome. This observation raises an issue which mechanism the seller selects in equilibrium.

Our focus is on the case of interdependent values. In particular, we assume that the so-called virtual valuation is monotonic in the seller's type. First, we prove the existence of a separating equilibrium in the mechanism-selection game. In the equilibrium, the buyer's posterior belief about the seller's type is reasonable in some sense. In any separating equilibrium, the seller's interim payoff vector is uniquely determined by the vector in the *LCS (least-cost-separating) mechanism*. The mechanism is defined so that it yields to any type of seller the maximum interim payoff among mechanisms which satisfy the interim upward incentive compatibility constraints for the seller and is both ex post incentive compatible and individually rational for the buyer. Next, we investigate the efficiency properties of the LCS mechanism in comparison with the seller's optimal screening mechanism when her type is commonly known. It is obvious that the seller is weakly worse off due to the signaling cost. We provide sufficient conditions on the seller's cost function under which the allocation rule is distorted upward or downward. Accordingly, the buyer's ex post payoff is weakly higher or lower than in the optimal screening mechanism.

Myerson (1983) develops a general theory of how an informed principal should design a mechanism to interact with informed agents. Myerson discovers the "Inscrutability Principle." The theorem states that any equilibrium outcome of the mechanism-selection game arises from some pooling equilibrium. The principle is so general that it holds true in our model. However, in order for the seller to obtain a strictly higher payoff than in the LCS mechanism, some pooling *must* occur in equilibrium.

In the case of interdependent values, Maskin and Tirole (1992) characterize the set of

[4] Example 2 in Mylovanov and Tröger (2012a) shows that this may not be the case in a partnership dissolution model à la Cramton et al. (1987). In some equilibrium of the example, the principal benefits from the privacy of her information.

[5] Maskin and Tirole (1992) call it the case of common values.

mechanisms selected in equilibrium. They show that the set consists of incentive-feasible mechanisms which weakly dominate the "RSW (Rothschild-Stiglitz-Wilson)" mechanism. Their analysis mainly focuses on the case in which only the principal has private information. By making relatively strong assumptions, they show that the characterization result holds true in a model with an informed agent. A contribution of this chapter is to generalize their result in many aspects. Since the LCS and RSW mechanisms are equivalent in our model, our existence theorem of separating equilibrium implies the same characterization result as Maskin and Tirole. Moreover, we investigate the properties of the agent's payoffs in the separating equilibrium as well as the principal's payoff. In Section 4.5, we will discuss the relation between the LCS mechanism and other solution concepts developed in the literature on informed principal problems.

The following studies analyze games in which a seller offers a price in trading environments with interdependent values. In all models, a seller as well as buyers has private information, and there is a single object to be traded. Shneyerov and Xu (2013) consider a bilateral trade environment. They provide a sufficient condition that ensures existence and uniqueness of a separating equilibrium. Jullien and Mariotti (2006) and Cai et al. (2007) consider auction environments. A seller offers a reserve price to buyers. The seller can signal her type through the proposal of prices. In the above three studies, they exogenously restrict the class of available mechanisms to one-dimensional prices. In contrast, we endogenously show that a mechanism in which the seller has no opportunity to choose actions can be selected in a separating equilibrium. Therefore, our result provides a justification for assuming that a seller with monopolistic power selects a (non-linear) pricing mechanism for screening purposes, which is often observed in practice, even when she has some private information.

This chapter is organized as follows. Section 4.2 presents the model. Section 4.3 provides some preliminary observations. Section 4.4 proves the existence of a separating equilibrium and investigates the allocations of goods and the players' payoffs in the separating equilibrium. Section 4.5 discusses the results. Section 4.6 concludes. All proofs are in the Appendix.

4.2 The Model

Consider a seller (principal) who sells some indivisible goods to a buyer (agent). The seller's type is denoted by $s \in S := \{1, ..., \bar{s}\}$. The buyer's type is denoted by $t \in T := \{1, ..., \bar{t}\}$.[6] We denote by $q \in \{0, 1, ..., \bar{q}\}$ the quantity of goods to be traded. Let $c : \{0, 1, ..., \bar{q}\} \times S \times T \to \mathbb{R}_+$ be a cost function,[7] and $v : \{0, 1, ..., \bar{q}\} \times S \times T \to \mathbb{R}_+$ be a

[6]A bilateral-trading model in which each player's type is binary is analyzed by Matsuo (1989).

[7]If we interpret the environment as a pure exchange economy, then $c(q, s, t)$ is an opportunity cost. If we interpret the environment as a production economy, then $c(q, s, t)$ is a production cost.

valuation function. They are normalized so that $c(q,s,t) \equiv 0$ and $v(q,s,t) \equiv 0$ for $q = 0$. We denote by Δ the set of all lotteries over $\{0, 1, ..., \bar{q}\}$. With a slight abuse of notation, for any lottery $Q \in \Delta$, we let $c(Q,s,t) := \sum_q Q_q c(q,s,t)$ and $v(Q,s,t) := \sum_q Q_q v(q,s,t)$ be the expected cost and the expected valuation respectively.[8] Let $\geq_{FSD}$ be the partial order on Δ such that $Q' \geq_{FSD} Q$ if Q' first-order stochastically dominates Q. Let $M \in \mathbb{R}$ be a monetary transfer from the buyer to the seller. We call $(Q, M) \in \Delta \times \mathbb{R}$ an *outcome*. Given any type profile (s,t) and outcome (Q,M), the seller's ex post payoff is $M - c(Q,s,t)$, and the buyer's ex post payoff is $v(Q,s,t) - M$.

The types (s,t) are random variables which are independent across players. The prior distribution of s is given by $(p^s)_{s \in S}$ with $\sum_{s \in S} p^s = 1$ and $p^s > 0$ for each s. The prior distribution of t is given by $(p_t)_{t \in T}$ with $\sum_{t \in T} p_t = 1$ and $p_t > 0$ for each t. Let $P(t) := \sum_{t'=1}^{t} p_{t'}$ be the cumulative distribution function of t. The prior distributions are common knowledge.

For instance, in a used-car market studied by Akerlof (1970), a seller's type s represents quality of her car, and a buyer's type t represents his preference parameter for the car. In a labor market studied by Spence (1973), a manager's type s represents her productivity, and an employer's type t represents his capacity to pay. In a monopolistic market studied in the literature on non-linear pricing, a monopolist's type s represents how innovative a new product is, and a retailer's type t represents local demands for the product. In each case, the seller's type variable s can directly affect the buyer's ex post payoff.

As in standard screening models, we define the *virtual valuation*[9] by

$$\psi(q,s,t) := v(q,s,t) - \frac{1 - P(t)}{p_t} \left[v(q,s,t+1) - v(q,s,t) \right].$$

We may extend the function ψ to $\Delta \times S \times T$ so that $\psi(Q,s,t) := \sum_q Q_q \psi(q,s,t)$ for any $Q \in \Delta$. Throughout the chapter, we maintain the following assumptions.

Assumption 4.1.
(i) (single-crossing condition in screening) v has strictly increasing differences in (q,t).[10]
(ii) ψ has strictly increasing differences in (q,t).
(iii) c has decreasing differences in (q,t).

Notice that Assumption 4.1 (i) together with $v(0,s,t) \equiv 0$ implies that v is increasing in t for any $q > 0$. Assumptions 4.1 (ii) and (iii) imply that the virtual surplus $\psi(q,s,t) - c(q,s,t)$ has strictly increasing differences in (q,t). For instance, Assumptions 4.1 (i) and

[8] Here, $Q_q \in [0,1]$ is a probability with which the seller supplies q units of goods. So, $\sum_{q=0}^{\bar{q}} Q_q = 1$.

[9] For any q and s, let $v(q,s,\bar{t}+1)$ be a real value.

[10] Let f be a real-valued function on $\{0, ..., \bar{q}\} \times S \times T$. We say that f has strictly increasing (resp., decreasing) differences in (q,t) if $f(q,s,t) - f(q-1,s,t)$ is increasing (resp., decreasing) in t for any $q > 0$ and $s \in S$. We say that f has increasing (resp., decreasing) differences in (q,t) if the above difference is nondecreasing (resp., nonincreasing) in t. The conditions with respect to (q,s) and (s,t) are analogous.

(ii) are satisfied if $v(q, s, t) = (s + t)q$ and $(1 - P(t))/p_t$ is nonincreasing in t.

The seller has full bargaining power to design a trading mechanism. In our framework, a *(direct) mechanism* $G := (Q, M)$ consists of a pair of functions. Here, $Q : S \times T \to \Delta$ is an *allocation rule*, and $M : S \times T \to \mathbb{R}$ is a *payment rule*. We thus allow a mechanism to be stochastic. Let $\mathcal{G} \subset [0, 1]^{\bar{s}\bar{t}\bar{q}} \times \mathbb{R}^{\bar{s}\bar{t}}$ be the set of all mechanisms. The restriction to direct mechanisms is justified by the "Revelation Principle." The statement and proof are presented in Proposition 4.7 in the Appendix.

The *mechanism-selection game* proceeds as follows. First, a type profile (s, t) of players is realized according to the prior distributions. Each player is privately informed about her/his own type. Second, the seller announces a mechanism $(Q, M) \in \mathcal{G}$. Third, the seller makes some report $\hat{s} \in S$, and, simultaneously, the buyer either makes some report $\hat{t} \in T$ or opts out. If the buyer opts out, then each player obtains zero payoff. If not, then the game ends with the outcome $(Q(\hat{s}, \hat{t}), M(\hat{s}, \hat{t}))$. Each player obtains her/his ex post payoff associated with the outcome and the realized types.

A (behavior) strategy for each player is defined in a standard manner. The seller's strategy consists of a (possibly randomized) mechanism-selection plan and a function $\sigma : S \times \mathcal{G} \to \Delta(S)$.[11] Here, $\sigma(\hat{s} \mid s, G)$ is a probability of reporting $\hat{s} \in S$ in a mechanism G given her type s. The buyer's strategy is a function $\tau : T \times \mathcal{G} \to \Delta(T \cup \{out\})$. Here, $\tau(\hat{t} \mid t, G)$ is a probability of reporting $\hat{t} \in T$ in a mechanism G or opting out for $\hat{t} = out$, given his type t.

The buyer's *posterior belief* about the seller's type is denoted by $\rho(\cdot \mid G) \in \Delta(S)$ for any mechanism $G \in \mathcal{G}$ announced by the seller. We assume that the posterior belief does not depend on the buyer's own type t because (s, t) are stochastically independent.

We now define some kinds of incentive compatibility (IC) conditions on mechanisms. To this end, we define the players' payoffs associated with a mechanism $G = (Q, M)$ and a posterior belief $\rho(\cdot \mid G)$ as follows. We define

$$\Pi(\hat{s} \mid s) := \sum_{t \in T} p_t \left[M(\hat{s}, t) - c\left(Q(\hat{s}, t), s, t\right) \right] \tag{4.1}$$

as the seller's interim payoff when she reports $\hat{s}$ given her own type s,

$$U(\hat{t} \mid s, t) := v(Q(s, \hat{t}), s, t) - M(s, \hat{t}) \tag{4.2}$$

as the buyer's ex post payoff when he reports $\hat{t}$ given the realized type profile (s, t), and

$$U(\hat{t} \mid t) := \sum_{s \in S} \rho(s \mid G) U(\hat{t} \mid s, t) \tag{4.3}$$

as the buyer's interim payoff when he reports $\hat{t}$ given his own type t. In each definition,

[11] For any countable set X, let $\Delta(X)$ be the set of all probability distributions on the set X.

the other player is supposed to report her/his type truthfully.

Definition 4.1. Let $G \in \mathcal{G}$ be a mechanism, $\rho(\cdot \mid G) \in \Delta(S)$ be a posterior belief, and $\Pi(\hat{s} \mid s), U(\hat{t} \mid s, t), U(\hat{t} \mid t)$ be the associated payoff functions defined by (4.1), (4.2) and (4.3).
(i) The mechanism G is *interim IC for the seller* if

$$\Pi(s \mid s) \geq \Pi(\hat{s} \mid s) \quad \text{for any } s, \hat{s} \in S. \tag{4.4}$$

(ii) The mechanism G with $\rho(\cdot \mid G)$ is *interim IC for the buyer* if

$$U(t \mid t) \geq U(\hat{t} \mid t) \quad \text{for any } t, \hat{t} \in T. \tag{4.5}$$

(iii) The mechanism G is *ex post IC for the buyer* if

$$U(t \mid s, t) \geq U(\hat{t} \mid s, t) \quad \text{for any } t, \hat{t} \in T, s \in S. \tag{4.6}$$

Next, we define some kinds of individually rationality (IR) conditions on mechanisms.

Definition 4.2. Let $G \in \mathcal{G}$ be a mechanism, $\rho(\cdot \mid G) \in \Delta(S)$ be a posterior belief, and $U(\hat{t} \mid s, t), U(\hat{t} \mid t)$ be the associated payoff functions defined by (4.2) and (4.3).
(i) The mechanism G with $\rho(\cdot \mid G)$ is *interim IR for the buyer* if $U(t \mid t) \geq 0$ for any t.
(ii) The mechanism G is *ex post IR for the buyer* if $U(t \mid s, t) \geq 0$ for any s and t.

We have some remarks about the IC and IR conditions. A mechanism G with the buyer's posterior belief $(\rho(s \mid G))_{s \in S}$ (and the seller's belief $(p_t)_{t \in T}$) induces a finite incomplete information game. We may simply call it a *continuation game* induced by G with ρ. For any mechanism G with ρ which is interim IC for both players and interim IR for the buyer, truth-telling forms a Bayesian equilibrium in the continuation game. Given the same mechanism G, however, this need not be the case with another posterior belief $\rho'(\cdot \mid G)$. We say that a mechanism G is *incentive feasible* if G is interim IC for the seller and G with the prior belief $(p^s)_{s \in S}$ is both interim IC and IR for the buyer.

The solution concept applied to the mechanism-selection game is perfect Bayesian equilibrium. In the following, we simply call it an equilibrium. For some mechanism $G \in \mathcal{G}$, there may be multiple Bayesian equilibria in the continuation game. Following Maskin and Tirole (1992), we suppose that players can coordinate over these Bayesian equilibria by means of some public randomizing device such as a coin flip. The device is formulated as $(\mathbb{N}, \pi)$, where $\mathbb{N}$ is the set of natural numbers and $\pi \in \Delta(\mathbb{N})$. Then, the players' strategies (σ, τ) are extended so that $\sigma : S \times \mathbb{N} \times \mathcal{G} \to \Delta(S)$ and $\tau : T \times \mathbb{N} \times \mathcal{G} \to \Delta(T \cup \{out\})$. Formally, a profile $((G^{s*})_{s \in S}, \sigma^*, \tau^*, \rho)$ of strategies and a belief system is a perfect Bayesian equilibrium if the profile satisfies the following three conditions. (i) Take any mechanism $G \in \mathcal{G}$. Then, there exists a public randomizing device $\pi^G \in \Delta(\mathbb{N})$

such that for any public signal $\omega \in \mathbb{N}$ drawn according to π^G, the strategy profile $(\sigma^*(\cdot \mid \cdot, \omega, G), \tau^*(\cdot \mid \cdot, \omega, G))$ forms a Bayesian equilibrium in the continuation game induced by G with ρ. (ii) Any type of seller has no incentive to deviate from the mechanism G^{s*} to another mechanism $G \neq G^{s*}$. That is, defining the seller's interim payoff function Π^G in a mechanism $G = (Q, M)$ by

$$\Pi^G(s) := \sum_{t \in T} p_t \sum_{\hat{s} \in S} \sum_{\hat{t} \in T \cup \{out\}} \sum_{\omega \in \mathbb{N}} \pi^G(\omega) \sigma^*(\hat{s} \mid s, \omega, G) \tau^*(\hat{t} \mid t, \omega, G) \left[M(\hat{s}, \hat{t}) - c(Q(\hat{s}, \hat{t}), s, t)\right], \tag{4.7}$$

it must hold that $\Pi^{G^{s*}}(s) \geq \Pi^G(s)$ for any $s \in S$ and $G \in \mathcal{G}$.[12] (iii) The belief system ρ is consistent in the sense that, for any mechanism G such that $G = G^{s*}$ for some $s \in S$,

$$\rho(s \mid G) = \frac{p^s}{\sum_{s' : G^{s'*} = G} p^{s'}} \text{ if } G = G^{s*}, \qquad \rho(s \mid G) = 0 \text{ if } G \neq G^{s*}.$$

The buyer may infer from the seller's selection of a mechanism G that the seller cannot be a particular type s. We focus on equilibria where each player adopts a truth-telling strategy in any mechanism G which is interim IC for the seller and both ex post IC and IR for the buyer.[13]

The ex post efficient allocation maximizes social surplus for any type profile (s, t):

$$\tilde{Q}(s, t) \in \arg\max_{Q \in \Delta} \left[v(Q, s, t) - c\left(Q, s, t\right)\right]. \tag{4.8}$$

We call such a function $\tilde{Q} : S \times T \to \Delta$ an *efficient allocation rule.*

Finally, we introduce the concept of "domination."

Definition 4.3. A mechanism (Q, M) *weakly dominates* another mechanism (Q', M') if

$$\sum_{t \in T} p_t \left[M(s, t) - c(Q(s, t), s, t)\right] \geq \sum_{t \in T} p_t \left[M'(s, t) - c(Q'(s, t), s, t)\right] \tag{4.9}$$

for each $s \in S$. A mechanism (Q, M) *dominates* another mechanism (Q', M') if the above inequalities are satisfied for each $s \in S$, with strict inequality for some $s \in S$. A mechanism (Q, M) is *undominated* if it is incentive feasible and there exists no incentive-feasible mechanism which dominates (Q, M).

It is worth mentioning that the concept is defined in reference to the seller's interim payoffs only. Thus, even if a mechanism G dominates another mechanism G', some type of buyer with some posterior belief may be strictly worse off by a change from G' to G.

[12] Here, $(Q(\hat{s}, out), M(\hat{s}, out)) = (0, 0)$ is a no-trade outcome. Note that $c(0, s, t) \equiv 0$, $v(0, s, t) \equiv 0$.

[13] The buyer's truth-telling strategy is defined by τ such that $\tau(t \mid t, \omega, G) = 1$ for any $t \in T$ and $\omega \in \mathbb{N}$. The seller's truth-telling strategy is defined in the same way.

4.3 Preliminary Results

This section provides some preliminary observations.

First, we provide sufficient conditions for the buyer's IC conditions. The inequalities (4.10) and (4.11) are called the *interim* and *ex post local downward ICs* for the buyer, respectively.

$$\sum_{s\in S}\rho(s\mid G)U(t\mid s,t)\geq\sum_{s\in S}\rho(s\mid G)U(t-1\mid s,t)\quad\text{for any } t>1. \tag{4.10}$$

$$U(t\mid s,t)\geq U(t-1\mid s,t)\quad\text{for any } t>1\ \text{ and } s\in S. \tag{4.11}$$

The inequalities (4.12) and (4.13) are called the *imterim* and *ex post local upward ICs* for the buyer, respectively.

$$\sum_{s\in S}\rho(s\mid G)U(t\mid s,t)\geq\sum_{s\in S}\rho(s\mid G)U(t+1\mid s,t)\quad\text{for any } t<\bar{t}. \tag{4.12}$$

$$U(t\mid s,t)\geq U(t+1\mid s,t)\quad\text{for any } t<\bar{t}\ \text{ and } s\in S. \tag{4.13}$$

The inequalities (4.14) are called the *monotonicity conditions*.

$$Q(s,t')\geq_{FSD}Q(s,t)\quad\text{for any } t'>t\ \text{ and } s. \tag{4.14}$$

Since we allow an allocation rule to be stochastic, the monotonicity conditions are in general stronger than necessary.

Lemma 4.1. *(i) A mechanism (Q,M) with a posterior belief ρ is interim IC for the buyer if the interim payoff function $U(\cdot)$ defined by (4.3) satisfies both the interim local downward and upward ICs for the buyer, and the allocation rule $Q(\cdot)$ satisfies the monotonicity conditions.*
(ii) A mechanism (Q,M) is ex post IC for the buyer if the ex post payoff function $U(\cdot)$ defined by (4.2) satisfies both the ex post local downward and upward ICs for the buyer, and the allocation rule $Q(\cdot)$ satisfies the monotonicity conditions.

The following lemma provides a version of the "revenue equivalence."

Lemma 4.2. *Let $G=(Q,M)$ be a mechanism, and ρ be the buyer's posterior belief. Suppose that (Q,M) with ρ satisfies the interim local downward ICs for the buyer with equality, and $\sum_{s\in S}\rho(s\mid G)[v(Q(s,1),s,1)-M(s,1)]=K$ for some $K\in\mathbb{R}$. Then, it must hold that*

$$\sum_{s\in S}\rho(s\mid G)\sum_{t\in T}p_tM(s,t)=\sum_{s\in S}\rho(s\mid G)\sum_{t\in T}p_t\psi(Q(s,t),s,t)-K. \tag{4.15}$$

Consider now a hypothetical situation in which the seller's type s is common knowledge. The buyer's type t is, however, still his private information. We say that a mechanism $\bar{G} = (\bar{Q}, \bar{M})$ is an *optimal screening mechanism* if, for each s, the mechanism is a solution to the following problem:

$$\begin{aligned} \max_{(Q,M)\in\mathcal{G}} \quad & \sum_{t\in T} p_t \left[M(s,t) - c\left(Q(s,t), s, t\right)\right] \\ \text{s.t.} \quad & (Q,M) \;\text{ with } \rho(s \mid (Q,M)) = 1 \text{ is interim IC for the buyer.} \\ & (Q,M) \;\text{ with } \rho(s \mid (Q,M)) = 1 \text{ is interim IR for the buyer.} \end{aligned} \tag{4.16}$$

It is shown that the following mechanism $(\bar{Q}, \bar{M})$ is optimal:

$$\bar{Q}(s,t) \in \arg\max_{Q\in\Delta} \left[\psi(Q,s,t) - c\left(Q,s,t\right)\right], \tag{4.17}$$

$$\bar{M}(s,t) := v\left(\bar{Q}(s,t), s, t\right) - \sum_{t'=1}^{t-1} \left[v\left(\bar{Q}(s,t'), s, t'+1\right) - v\left(\bar{Q}(s,t'), s, t'\right)\right]. \tag{4.18}$$

The buyer's optimal allocation maximizes the virtual surplus for any realization of the types. We assume that the maximization problem (4.17) has a unique deterministic solution for any type profile (s,t). In fact, the assumption is satisfied generically in the space of functions (ψ, c). The maximum expected payoff is given by

$$\bar{\Pi}(s) := \sum_{t\in T} p_t \left[\psi\left(\bar{Q}(s,t), s, t\right) - c\left(\bar{Q}(s,t), s, t\right)\right]. \tag{4.19}$$

The next proposition summarizes these results.

Proposition 4.1. *The mechanism $(\bar{Q}, \bar{M})$ given by (4.17) and (4.18) is the optimal screening mechanism.*

The next proposition states that the optimal allocation rule $\bar{Q}$ inevitably entails undersupply of goods, compared with the efficient rule $\tilde{Q}$.

Proposition 4.2 (Undersupply problem)**.** *Let $\bar{Q}$ be the optimal allocation rule and $\tilde{Q}$ be the efficient allocation rule. Then, for each s, $\bar{Q}(s,t) \leq_{FSD} \tilde{Q}(s,t)$ for any $t < \bar{t}$ and $\bar{Q}(s,\bar{t}) = \tilde{Q}(s,\bar{t})$.*

The undersupply problem is caused by the seller's incentive to reduce the buyer's information rents. However, the allocation $\bar{Q}(s,\bar{t})$ for the highest type $\bar{t}$ is ex post efficient. That is, there is "no distortion at the top."

In the remaining sections, we maintain the following assumptions.

Assumption 4.2.
(i) The virtual valuation $\psi(q,s,t)$ is increasing in s for each $q > 0$.

(ii) (single-crossing condition in signaling) c has strictly decreasing or strictly increasing differences in (q, s).
(iii) $c(q, s, t) - c(q-1, s, t)$ has decreasing differences in (s, t) for each $q > 0$.
(iv) $\sum_{t \in T} p_t[\psi(\bar{q}, \bar{s}, t) - c(\bar{q}, s, t)] \leq 0$ for each s.

By Assumption 4.2 (i), the optimal screening mechanism may no longer be interim IC for the seller. Some type of seller has an incentive to pretend to be higher type, in order to "screw" money from the buyer. Then, the seller with higher type has an incentive to reveal her type by selecting a mechanism different from the optimal screening mechanism. By analogy with Spence's model, Assumption 4.2 (i) can be described as a situation where a "wage" acceptable to an "employer" when facing a "high-productivity worker" is higher than an acceptable wage when facing a "low-productivity worker."

Assumption 4.2 (ii) is a single-crossing condition in signaling. By analogy with Spence's model, the former condition of Assumption 4.2 (ii) can be described as a situation where the marginal cost of raising an "education level" for the high-productivity worker is lower than that for the low-productivity worker. We also cover the opposite case. This is in contrast to Maskin and Tirole (1992) because they focus on the one-sided case. See Assumption S in their article.

Assumption 4.2 (iii) makes the seller's signaling activity worthwhile in a monotonic way with respect to the buyer's types. This assumption together with Assumptions 4.1 (ii) and (iii) is used to make the monotonicity conditions (4.14) nonbinding in some maximization problems defined later.[14] Finally, Assumption 4.2 (iv) means that the "full production" $\bar{q}$ is too costly for each type of seller.

Finally, the following proposition demonstrates the "Inscrutability Principle" discovered by Myerson (1983).

Proposition 4.3 (Inscrutability Principle)**.** *For any equilibrium $(\gamma, \sigma, \tau, \rho)$ such that $\gamma(\cdot \mid s)$ is a probability measure on $\mathcal{G}$ for each $s \in S$, there exists a pooling equilibrium $(G^*, \sigma^*, \tau^*, \rho^*)$ which is outcome-equivalent to the original equilibrium.*

The principle states that any equilibrium outcome of the mechanism-selection game arises from some pooling equilibrium. Myerson (1983) also shows that, in a more general model than ours, the mechanism-selection game has an equilibrium. The game thus has a pooling equilibrium. The Inscrutability Principle, however, tells us nothing about whether the mechanism-selection game has a *separating equilibrium.* This issue is examined in the next section.

[14]In the proof of Proposition 4.1, we have already used Assumptions 4.1 (ii) and (iii) to make the monotonicity condition (4.14) nonbinding in the problem of the optimal screening mechanism.

4.4 Separating Equilibrium

In this section, we show the existence of a separating equilibrium. Each type of seller reveals her type through the selection of a mechanism. In any separating equilibrium, the seller's interim payoff vector is uniquely determined by the vector in a particular mechanism. The existence theorem is used to characterize the set of mechanisms selected in equilibrium. We then show how the privacy of the seller's information affects the allocations and the players' payoffs in the separating equilibrium.

We define a mechanism which plays a crucial role in our analysis. Now, the following inequalities are called the *interim upward* and *downward ICs* for the seller, respectively:

$$\Pi(s \mid s) \geq \Pi(\hat{s} \mid s) \quad \text{for any } s, \hat{s} \in S \text{ with } s < \hat{s}. \tag{4.20}$$

$$\Pi(s \mid s) \geq \Pi(\hat{s} \mid s) \quad \text{for any } s, \hat{s} \in S \text{ with } s > \hat{s}. \tag{4.21}$$

Definition 4.4. A mechanism (Q^*, M^*) is an *LCS (least-cost-separating) mechanism* if it is a solution to the following maximization problem for each $s \in S$:

$$\max_{(Q,M)\in\mathcal{G}} \quad \Pi(s \mid s) = \sum_{t\in T} p_t \left[M(s,t) - c\left(Q(s,t), s, t\right)\right] \tag{4.22}$$

$$\begin{aligned} \text{s.t.} \quad & (Q, M) \text{ satisfies the interim upward ICs (4.20) for the seller.} \\ & (Q, M) \text{ is ex post IC for the buyer.} \\ & (Q, M) \text{ is ex post IR for the buyer.} \end{aligned}$$

The LCS mechanism is required to be both ex post IC and IR for the buyer. Moreover, Lemma 4.5 in the Appendix shows that the LCS mechanism is interim IC for the seller because it also satisfies the interim downward ICs (4.21). Hence, a simple observation is that, for each type of seller, the maximum payoff in the problem (4.22) is a lower bound of the set of equilibrium payoffs in the mechanism-selection game. If any type of seller obtains a strictly lower payoff, then she can profitably deviate to the LCS mechanism, regardless of the buyer's posterior belief. Indeed, whatever the seller's type, the buyer is willing to report truthfully in the LCS mechanism.

Remark 4.1. Maskin and Tirole (1992) define the LCS (least-cost-separating) allocation in the case where only the principal has private information.[15] Since the agent has no private information, the associated maximization problems impose no IC constraint for the agent. In addition, a difference from the LCS mechanism is that the upward ICs for the principal is weakened to the local upward ones in the LCS allocation.

It is worth pointing out that the LCS mechanism is closely related to the "RSW (Rothschild-Stiglitz-Wilson)" mechanism defined by Maskin and Tirole (1992). The

[15]The allocation in their model is defined as a direct mechanism in our model.

RSW mechanism is defined as a mechanism which is a solution to the problem (4.22) for each $s \in S$ with an additional constraint that (Q, M) satisfies the interim downward ICs (4.21) for the seller. Using the RSW mechanism, Maskin and Tirole (1992) characterize the set of mechanisms selected in equilibrium. They show that the set of equilibrium mechanisms consists of incentive-feasible mechanisms that weakly dominate the RSW mechanism, provided that the RSW mechanism is "interim efficient" for some strictly positive beliefs about the principal's type. In our framework, the LCS and RSW mechanisms are equivalent. This equivalence may not hold in other environments.

Our first main result is that the mechanism-selection game has a separating equilibrium. In the equilibrium, each type of seller reveals the type through her choice of mechanism. Moreover, the buyer's posterior belief is "reasonable" in the following sense. Let $BE(\rho, G)$ be the set of Bayesian equilibria in the continuation game induced by a mechanism $G = (Q, M)$ with a posterior belief $(\rho(s \mid G))_{s \in S}$. For any subset S' of S, define $BE(S', G) := \cup_{\rho \in \Delta(S')} BE(\rho, G)$.[16] Denote by $(\Pi^{G^{s*}}(s))_{s \in S}$ the seller's interim payoff vector in a candidate equilibrium. Then, define for any $G \in \mathcal{G}$ the set $S(G)$ as follows:

$$S(G) := \{s \in S \mid \text{ condition (4.24) holds}\}. \tag{4.23}$$

$$\Pi^{G^{s*}}(s) > \sup_{(\sigma,\tau) \in BE(S,G)} \sum_{t \in T} p_t \sum_{\hat{s} \in S} \sum_{\hat{t} \in T \cup \{out\}} \sigma(\hat{s} \mid s)\tau(\hat{t} \mid t) \left[M(\hat{s}, \hat{t}) - c(Q(\hat{s}, \hat{t}), s, t)\right]. \tag{4.24}$$

That is, $S(G)$ is the set of the seller's types for which the selection of the mechanism G is "dominated by the equilibrium payoff." Using the set $S(G)$, we define the criterion of reasonableness. It is easy to verify that the criterion in the following definition is stronger than the "intuitive criterion" by Cho and Kreps (1987). See also Mas-Collel et al. (1995).

Definition 4.5. Let $((G^{s*})_{s \in S}, \sigma^*, \tau^*, \rho)$ be an equilibrium of the mechanism-selection game. Let $(\Pi^{G^{s*}}(s))_{s \in S}$ be the seller's equilibrium payoff vector defined by (4.7), and define the set $S(G)$ by (4.23) for any mechanism G. Then, the belief system ρ is *reasonable* if $\rho(s \mid G) = 0$ for any $G \in \mathcal{G}$ with $S(G) \neq \varnothing$ and $s \in S(G)$.

Theorem 4.1. *The mechanism-selection game has a separating equilibrium. In the equilibrium, the belief system ρ is reasonable.*

Theorem 4.1 is proved in the following way. First, Lemma 4.7 in the Appendix shows

[16]Define the set $\Delta(S')$ by $\Delta(S') := \{\rho \in \Delta(S) \mid \rho(s) = 0 \text{ for any } s \notin S'\}$.

that any LCS mechanism is a solution to the following problem.

$$\max_{(Q,M)\in\mathcal{G}} \sum_{s\in S}\sum_{t\in T} p_t\left[M(s,t) - c\left(Q(s,t),s,t\right)\right] \tag{4.25}$$

$$\begin{aligned}\text{s.t.}\quad & (Q,M) \text{ satisfies the interim upward ICs (4.20) for the seller.}\\ & (Q,M) \text{ satisfies the ex post local downward ICs (4.11) for the buyer.}\\ & v(Q(s,1),s,1) - M(s,1) \geq 0 \quad \text{for any } s\in S.\end{aligned}$$

Thus, the LCS mechanism is not dominated by any other mechanism which satisfies the constraints in the problem (4.25). Lemmas 4.4, 4.6 and 4.7 in the Appendix imply that the LCS mechanism satisfies the ex post local downward ICs for the buyer and the constraints $v(Q(s,1),s,1) - M(s,1) \geq 0$ for any s in the problem (4.25) with equality. Using this fact, we make a weak assumption that the Lagrange multipliers associated with those constraints are strictly positive.

Next, Theorem 4.1 is proved using the following two propositions. The proof of Proposition 4.5 shows that, for any s, the type-s seller selects $G^{s*} := (Q^*(s,\cdot), M^*(s,\cdot))$ where (Q^*, M^*) is an LCS mechanism, and thus G^{s*} no longer depends on the seller's report $\hat{s}$.

Proposition 4.4. *Let (Q^*, M^*) be an LCS mechanism and λ be a vector of Lagrange multipliers such that $((Q^*, M^*), \lambda)$ satisfies the Kuhn-Tucker conditions for the problem (4.25). Let $(\lambda^{s,\hat{s}})_{s\in S,\hat{s}>s}$ be components of λ associated with the seller's interim upward ICs. Define the buyer's belief $(\rho(s))_{s\in S}$ by $\rho(s) := (1+\sum_{\hat{s}>s}\lambda^{s,\hat{s}} - \sum_{\hat{s}<s}\lambda^{\hat{s},s})/\bar{s}$. Then, $\rho(s) > 0$ for any s, and the LCS mechanism (Q^*, M^*) is not dominated by any other mechanism which is interim IC for both players and interim IR for the buyer with respect to ρ.*

Proposition 4.5. *Suppose that there exists a belief $(\rho'(s))_{s\in S}$ of the buyer such that $\rho'(s) > 0$ for any s and the LCS mechanism is not dominated by any other mechanism which is interim IC for both players and interim IR for the buyer with respect to ρ'. Then, the mechanism-selection game has a separating equilibrium $((G^{s*})_{s\in S}, \sigma^*, \tau^*, \rho)$. In the equilibrium, the belief system ρ is reasonable in the sense of Definition 4.5.*

The next theorem together with Theorem 4.1 implies that the seller's payoff vector in the LCS mechanism is the minimum of the set of equilibrium payoff vectors. This means that the set of equilibrium mechanisms consists of incentive-feasible mechanisms that weakly dominate the LCS mechanism. In order for the seller to obtain a strictly higher payoff than in the LCS mechanism, some pooling *must* occur in equilibrium. Since the LCS and RSW mechanisms are equivalent, we obtain the same characterization result as Maskin and Tirole (1992) in our environment.

Theorem 4.2. *In any separating equilibrium of the mechanism-selection game, the seller's interim payoff vector is uniquely determined by that in the LCS mechanism.*

The following theorem provides sufficient conditions under which the privacy of the seller's information mitigates or worsens the undersupply problem.

Theorem 4.3. *Let (Q^*, M^*) be an LCS mechanism and $\bar{Q}$ be the optimal allocation rule. Then: $Q^*(s,\cdot) = \bar{Q}(s,\cdot)$ for $s = 1$, and the following inequalities hold for any $s > 1$.*
(i) If the marginal cost is decreasing in s, then $Q^*(s,t) \geq_{FSD} \bar{Q}(s,t)$ for each t.
(ii) If the marginal cost is increasing in s, then $\bar{Q}(s,t) \geq_{FSD} Q^*(s,t)$ for each t.

In order to obtain some intuition, we consider the economic implications of the conditions on the cost function. We now interpret the seller as a firm and her type as the level of product innovation (high or low). When a high-level innovation occurs, the quality of the product improves, so that the virtual valuation becomes higher. We then consider two situations. The first one is a situation where the innovation also improves the production process. Then, the high-type seller can effectively convey the fact that the high-level innovation occurs by offering a contract with which large supplies are expected. The second one is a situation where the innovation raises the marginal cost because the seller must produce high-quality products. In this situation, the high-type seller can effectively convey the fact by offering a contract with which limited supplies are expected.

Since the problem (4.22) is more constrained than the problem (4.16) for each s, the seller's interim payoff in any separating equilibrium is weakly lower than that in the optimal screening mechanism. That is, the seller is weakly worse off due to the signaling cost, except for the lowest type. The next theorem shows how the privacy of the seller's information changes the buyer's ex post payoffs. The result has policy implications for antitrust laws. In case (i) of the theorem, a regulator who oversees the seller (e.g., monopolistic firm) and aims at maximizing consumer surplus should *not* disclose the seller's information to the buyer.

Theorem 4.4. *Fix any separating equilibrium, and any realization of the players' types.*
(i) If the marginal cost is decreasing in s, then the buyer's ex post payoff is weakly higher than that in the optimal screening mechanism.
(ii) If the marginal cost is increasing in s, then the buyer's ex post payoff is weakly lower than that in the optimal screening mechanism.

Remark 4.2. In a mechanism-selection game where the buyer, instead of the seller, designs a trading mechanism, we can obtain similar results to those in Sections 4.3 and 4.4 by changing Assumptions 4.1 and 4.2. Let $\phi(q,s,t) := c(q,s,t) + \frac{1-\sum_{s'=1}^{s} p^{s'}}{p^s}[c(q,s,t) - c(q,s+1,t)]$ be the "virtual cost." Then, Assumptions 4.1 and 4.2 should be replaced with the following assumptions.
Assumption 4.1'

(i) c has strictly decreasing differences in (q, s).
(ii) ϕ has strictly decreasing differences in (q, s).
(iii) v has increasing differences in (q, s).
Assumption 4.2'
(i) ϕ is decreasing in t for each $q > 0$.
(ii) v has strictly decreasing or strictly increasing differences in (q, t).
(iii) $v(q, s, t) - v(q-1, s, t)$ has increasing differences in (s, t) for each $q > 0$.
(iv) $\sum_{s \in S} p^s[v(\bar{q}, s, t) - \phi(\bar{q}, s, \bar{t})] \leq 0$ for each t.

4.5 Discussion

We discuss several implications of our results. First, we explain the significance of our results in the literature on screening and signaling. The LCS allocation defined by Maskin and Tirole (1992) had been of crucial importance for the theory of information economics. In a competitive screening model of an insurance market, Rothschild and Stiglitz (1976) show that there exists at most one market equilibrium, and the equilibrium outcome is determined by the LCS allocation. In a signaling model of a labor market, Spence (1973) shows that there exist multiple market equilibria. Cho and Kreps (1987) establish a number of equilibrium refinement criteria, and illustrate that a noncooperative game of the Spence signaling model has a unique equilibrium outcome, which is determined by the LCS allocation. Based on these previous studies, Maskin and Tirole (1992) formulate an informed-principal model, and show that, under some conditions, there exists an equilibrium in which the outcome is determined by the LCS allocation. While Maskin and Tirole provide a general and thorough analysis in the case where only the principal has private information, they make relatively strong assumptions to show that their main result holds true in the case where the agent also has private information. They then conjecture that the analysis carries over to much more general environments. Our results provide an affirmative answer to the conjecture. Moreover, Theorems 4.3 and 4.4 in this chapter show the efficiency properties of the LCS mechanism in comparison with the optimal screening mechanism. These theorems help us to better understand the interaction between screening and signaling.

Second, we discuss the relation between the LCS mechanism and other solution concepts developed in the literature. Myerson (1983) proposes many solution concepts (strong solution, core mechanism, neutral optimum) in informed principal problems. In our model, the strong solution is defined as an undominated mechanism which is interim IC for the seller and ex post IC and IR for the buyer. A drawback of the concept is that it does not always exist. In a model with a principal and an agent, a strong solution exists if and only if the RSW mechanism is undominated. Myerson then introduces two weaker concepts, the core mechanism and the neutral optimum, and proves the existence.

Mylovanov and Tröger (2012a) define the concept of a strong neologism proof allocation in a private-values environment which is more general than Maskin and Tirole (1990). Among the above mechanisms, the core mechanism is shown to be the logically weakest one. Since the core mechanism is by definition undominated, so are the other mechanisms. Balestrieri and Izmalkov (2012) construct a Hotelling model where an informed seller trades with an informed buyer. They characterize the optimal selling mechanism, which maximizes the seller's ex ante expected payoff among incentive-feasible mechanisms. Balkenborg and Makrisz (2013) define the concept of an assured mechanism in an interdependent-values environment where only the principal has private information. They show that if either the principal's type is binary or the primitives of the model satisfy a condition which ensures that there is no bunching in the assured allocation, then the assured allocation is a neutral optimum and thus undominated.

Contrary to all the above mechanisms, the LCS mechanism may be dominated by another incentive-feasible mechanism. Even then, however, the LCS mechanism can be interim incentive efficient in the sense of Holmström and Myerson (1983). The reason is that a change from the LCS mechanism may reduce the interim payoff for some type of buyer, as noted at the end of Section 4.2. Then, the selection of the LCS mechanism is admitted based on the normative criterion. This is in stark contrast with the result that the agent with no private information obtains no rent in the LCS allocation, and thus the LCS allocation is *never* interim incentive efficient provided that it is dominated by another incentive-feasible mechanism; see Proposition 2 of Maskin and Tirole (1992).

Finally, we consider the implications for the bilateral-trading problem studied by Hagerty and Rogerson (1987). They argue that a trading mechanism be "robust" in the sense that it should be ex post IC and IR for both traders. Their argument is based on the idea that a trading institution such as a stock exchange is used by a variety of traders over a long period of time, and thus a social planner should design a robust mechanism to changes in the information structure of the market. They show that, in a private-values environment with an indivisible good, posted-price mechanisms are essentially the only mechanisms which satisfy the two requirements. Our Theorems 4.1 and 4.2 imply that a mechanism which is ex post IC and IR for the buyer can be endogenously chosen by the seller in equilibrium. In many cases, however, the LCS mechanism is neither ex post IC nor ex post IR for the seller herself. For instance, assume that $\bar{q} = 1$, $T = S = \{1, 2\}$, $p_1 = p_2 = 1/2$, $v(q, s, t) = 4(s+t)q$ and $c(q, s, t) = (17-2s)q$. The prior distribution of s is arbitrary. Then, the LCS mechanism (Q^*, M^*) is given by $Q^*_1(1, t) \equiv 0$, $M^*(1, t) \equiv 0$, $Q^*_1(2, 1) = 1/7$, $M^*(2, 1) = 12/7$, $Q^*_1(2, 2) = 1$ and $M^*(2, 2) = 108/7$. Truth-telling strategies yield the negative ex post payoff $-1/7$ to the seller when $(s, t) = (2, 1)$, and the seller can be better off by reporting $\hat{s} = 1$ when $(s, t) = (2, 1)$ and $\hat{s} = 2$ when $(s, t) = (1, 2)$. Therefore, our result suggests that it might be costly for a trader with power of the mechanism selection to restrict a class of mechanisms to those which are ex post IC and IR for *both* traders.

4.6 Concluding Remarks

In this chapter, we have proved that, in a bilateral trade environment, the mechanism-selection game has a separating equilibrium with a reasonable posterior belief. It is shown that the seller's interim payoff vector in any separating equilibrium is uniquely determined by that in the LCS (lease-cost-separating) mechanism. We have also investigated allocative efficiency and the distributional consequences of the LCS mechanism.

Our existence theorem implies that the set of mechanisms selected in equilibrium consists of incentive-feasible mechanisms which weakly dominate the LCS mechanism. Therefore, unless the LCS mechanism is undominated, there exist multiple equilibria in the mechanism-selection game. In the case where only the principal has private information, Maskin and Tirole (1992) apply the equilibrium refinement of Cho and Kreps (1987). They show that, under some conditions, the RSW allocation is a unique mechanism which passes the Cho-Kreps criterion. It is an important topic for future research to investigate whether some equilibrium refinements can rule out equilibria other than the separating equilibrium in our model.

Appendix

We first prove the following lemma to use the monotone comparative statics. See Topkis (1998) for the related definitions and conditions.

Lemma 4.3. *Let f be a real-valued function on $\{0, ..., \bar{q}\} \times S \times T$, and $f(Q, s, t) := \sum_q Q_q f(q, s, t)$ for any $Q \in \Delta$. (i) The partially ordered set $(\Delta, \geq_{FSD})$ is a lattice. (ii) If $f(q, s, t)$ has increasing (resp. strictly increasing) differences in (q, s), then $f(Q, s, t)$ has increasing (resp. strictly increasing) differences in (Q, s) on $\Delta \times S$. If $f(q, s, t)$ has increasing (resp. strictly increasing) differences in (q, t), then $f(Q, s, t)$ has increasing (resp. strictly increasing) differences in (Q, t) on $\Delta \times T$. (iii) The function $f(\cdot, s, t)$ is both supermodular and submodular on Δ.*

Proof of Lemma 4.3. (i) Take any two cumulative distribution functions $Q, Q' \in \Delta$. It is easy to show that the distribution function $\min\{Q, Q'\} \in \Delta$ is a least upper bound of $\{Q, Q'\}$ and the distribution function $\max\{Q, Q'\} \in \Delta$ is a greatest lower bound of $\{Q, Q'\}$. Hence, Δ is a lattice. (ii) Take any $Q, Q' \in \Delta$ with $Q' >_{FSD} Q$, $s, s' \in S$ with $s' > s$ and $t \in T$. Suppose that $f(q, s, t)$ has strictly increasing differences in (q, s). Then, we can show by induction on $\bar{q}$ that

$$\sum_{q=0}^{\bar{q}} Q'_q \left[f(q, s', t) - f(q, s, t)\right] > \sum_{q=0}^{\bar{q}} Q_q \left[f(q, s', t) - f(q, s, t)\right].$$

This implies that $f(Q', s', t) - f(Q, s', t) > f(Q', s, t) - f(Q, s, t)$. The proof for the other case is analogous. (iii) Take any $Q, Q' \in \Delta$, $s \in S$ and $t \in T$. Let $Q \vee Q'$ and $Q \wedge Q'$ be the cumulative distribution functions defined by $Q \vee Q' := \min\{Q, Q'\}$ and $Q \wedge Q' := \max\{Q, Q'\}$. It is easy to show that the reduced lottery $\frac{1}{2} \circ (Q \vee Q') + \frac{1}{2} \circ (Q \wedge Q')$ is equal to $\frac{1}{2} \circ Q + \frac{1}{2} \circ Q'$. It follows that

$$\frac{1}{2} f(Q \vee Q', s, t) + \frac{1}{2} f(Q \wedge Q', s, t) = \frac{1}{2} f(Q, s, t) + \frac{1}{2} f(Q', s, t).$$

This implies that $f(\cdot, s, t)$ is both supermodular and submodular on Δ. □

Proof of Lemma 4.1. (i) The proof is by induction on $\bar{t}$. The proof is obvious for $\bar{t} = 2$. Now assume that the statement is true for $\bar{t} = k$ with $k \geq 2$. We must show that the interim payoff function $U(\cdot)$ defined by (4.3) satisfies the following inequalities:

$$\sum_{s \in S} \rho(s \mid G) U(k+1 \mid s, k+1) \geq \sum_{s \in S} \rho(s \mid G) U(t \mid s, k+1) \quad \text{for each } t \leq k. \tag{4.26}$$

$$\sum_{s \in S} \rho(s \mid G) U(t \mid s, t) \geq \sum_{s \in S} \rho(s \mid G) U(k+1 \mid s, t) \quad \text{for each } t \leq k. \tag{4.27}$$

The inequality (4.26) is satisfied for $t = k$ because the buyer's interim local downward IC for $\bar{t} = k + 1$ implies that

$$\sum_{s \in S} \rho(s \mid G) \left[v(Q(s, k+1), s, k+1) - v(Q(s, k), s, k+1)\right] \geq \sum_{s \in S} \rho(s \mid G) \left[M(s, k+1) - M(s, k)\right].$$

Take any $t \leq k - 1$. By the induction hypothesis, the following inequality is satisfied:

$$\sum_{s \in S} \rho(s \mid G) \left[v(Q(s, k), s, k) - v(Q(s, t), s, k)\right] \geq \sum_{s \in S} \rho(s \mid G) \left[M(s, k) - M(s, t)\right].$$

Summing up the above two inequalities, we obtain the following inequality:

$$\sum_{s \in S} \rho(s \mid G) \left[v(Q(s, k+1), s, k+1) - v(Q(s, k), s, k+1) + v(Q(s, k), s, k) - v(Q(s, t), s, k)\right]$$
$$\geq \sum_{s \in S} \rho(s \mid G) \left[M(s, k+1) - M(s, t)\right]. \tag{4.28}$$

Since $Q(s, k) \geq_{FSD} Q(s, t)$ for any s by the monotonicity condition and v has (strictly) increasing differences in (q, t), it follows from Lemma 4.3 (ii) that

$$\sum_{s\in S}\rho(s\mid G)\left\{[v(Q(s,k),s,k+1)-v(Q(s,t),s,k+1)]-[v(Q(s,k),s,k)-v(Q(s,t),s,k)]\right\}\geq 0.$$

The above two inequalities imply that

$$\sum_{s\in S}\rho(s\mid G)\left[v(Q(s,k+1),s,k+1)-v(Q(s,t),s,k+1)\right]\geq\sum_{s\in S}\rho(s\mid G)\left[M(s,k+1)-M(s,t)\right].$$

Therefore, the inequality (4.26) is satisfied for each $t \leq k$. The proof for the inequality (4.27) is analogous.

(ii) From part (i), both the ex post local downward and upward ICs for the buyer together with the monotonicity conditions imply that, for any $s \in S$ and posterior belief ρ such that $\rho(s \mid G) = 1$, the mechanism G with $\rho(\cdot \mid G)$ is interim IC for the buyer. This means that the mechanism G is ex post IC for the buyer. □

Proof of Lemma 4.2. Since (Q, M) with ρ satisfies the inequalities (4.10) with equality for each $t > 1$, we can obtain

$$\sum_{s\in S}\rho(s\mid G)\left[v(Q(s,t),s,t)-M(s,t)\right]=\sum_{s\in S}\rho(s\mid G)\sum_{t'=1}^{t-1}\left[v\left(Q(s,t'),s,t'+1\right)-v\left(Q(s,t'),s,t'\right)\right]+K$$

for each $t \in T$. Taking the expectation with respect to t, it is shown by an interchange of summations that

$$\sum_{s\in S}\rho(s\mid G)\sum_{t\in T}p_t M(s,t)=\sum_{s\in S}\rho(s\mid G)\sum_{t\in T}p_t\psi(Q(s,t),s,t)-K.$$

□

Proof of Proposition 4.1. Fix any $s \in S$, We first claim that the mechanism $(\bar{Q}, \bar{M})$ given by (4.17) and (4.18) is a solution to the following problem:

$$\max_{(Q,M)\in\mathcal{G}}\ \sum_{t\in T}p_t\left[M(s,t)-c\left(Q(s,t),s,t\right)\right]$$
$$\text{s.t.}\quad U(t\mid s,t)\geq U(t-1\mid s,t)\ \text{ for any } t>1,\ \text{ and } U(1\mid s,1)\geq 0.$$

It is easy to verify that any solution to the above problem must satisfy all the constraints with equality. By construction, the mechanism $(\bar{Q}, \bar{M})$ satisfies this condition. Letting $\rho(s \mid (\bar{Q}, \bar{M})) = 1$ and $K = 0$, it follows from Lemma 4.2 that $\sum_{t\in T} p_t\bar{M}(s,t) = \sum_{t\in T} p_t\psi(\bar{Q}(s,t),s,t)$. Substituting the expected payment into the objective function shows that the mechanism $(\bar{Q}, \bar{M})$ is a solution to the above problem.

Since the above problem is less constrained than the problem (4.16), it remains to show that $(\bar{Q}, \bar{M})$ satisfies all the constraints in the latter problem. It follows from Theorem 2.8.4 of Topkis (1998) together with Lemma 4.3 that the allocation rule $\bar{Q}$ given by (4.17) satisfies the monotonicity condition. Then, since $(\bar{Q}, \bar{M})$ with $\rho(s \mid (\bar{Q}, \bar{M})) = 1$ satisfies the interim local downward ICs with equality, Assumption 4.1 (i) with Lemma 4.3 (ii) implies that it also satisfies the interim local upward ICs for the buyer. Therefore, Lemma 4.1 implies that the mechanism $(\bar{Q}, \bar{M})$ with $\rho(s \mid (\bar{Q}, \bar{M})) = 1$ is interim IC for the buyer. Then, $U(t \mid s, t) \geq U(1 \mid s, t) \geq U(1 \mid s, 1) = 0$ for any $t > 1$, where the second inequality follows from the monotonicity of v in t. Thus, the mechanism $(\bar{Q}, \bar{M})$ with $\rho(s \mid (\bar{Q}, \bar{M})) = 1$ is interim IR for the buyer. This completes the proof. □

Proof of Proposition 4.2. Fix any $s \in S$ and $t < \bar{t}$. Let f be a real-valued function such that, for $Q \in \Delta$ and $\alpha \in \{0, 1\}$, $f(Q, \alpha) := v(Q, s, t) - c(Q, s, t) - (1-\alpha)\frac{1-P(t)}{p_t}[v(Q, s, t+1) - v(Q, s, t)]$. Then, $f(Q, 1)$ is the objective function of (4.8), and $f(Q, 0)$ is that of (4.17). Take any Q, Q' with $Q' >_{FSD} Q$. Lemma 4.3 (iii) implies that f is supermodular on Δ. Lemma 4.3 (ii) also implies that v has strictly increasing differences in (Q, t). Hence, if $f(Q', 0) \geq f(Q, 0)$, then $f(Q', 1) > f(Q, 1)$. This implies that the function f satisfies the strict single crossing property in (Q, α) in the sense of Milgrom and Shannon (1994). Therefore, Theorem 4' in their article implies that $\bar{Q}(s, t) \leq_{FSD} \tilde{Q}(s, t)$.

The latter statement is trivial because we assume that the maximization problem (4.17) has a unique deterministic solution for $t = \bar{t}$. □

Proof of Proposition 4.3. We first construct a candidate equilibrium $(G^*, \sigma^*, \tau^*, \rho^*)$ in which every type of seller selects the same mechanism G^*. Define the mechanism $G^* := (Q^*, M^*) \in \mathcal{G}$ to be

$$Q^*_q(s,t) := \int_{\mathcal{G}} \sum_{\hat{s} \in S} \sum_{\hat{t} \in T \cup \{out\}} \sum_{\omega \in \mathbb{N}} \pi^G(\omega)\sigma(\hat{s} \mid s, \omega, G)\tau(\hat{t} \mid t, \omega, G) Q_q(\hat{s}, \hat{t})\gamma(dG \mid s),$$

$$M^*(s,t) := \int_{\mathcal{G}} \sum_{\hat{s} \in S} \sum_{\hat{t} \in T \cup \{out\}} \sum_{\omega \in \mathbb{N}} \pi^G(\omega)\sigma(\hat{s} \mid s, \omega, G)\tau(\hat{t} \mid t, \omega, G) M(\hat{s}, \hat{t})\gamma(dG \mid s).$$

Here, for any $G \in \mathcal{G}$, π^G is a public randomizing device associated with the original equilibrium. Define the buyer's posterior belief ρ^* as follows: $\rho^*(s \mid G^*) := p^s$ for any $s \in S$, and $\rho^*(\cdot \mid G) := \rho(\cdot \mid G)$ for any $G \in \mathcal{G}$ with $G \neq G^*$. If the mechanism G^* is selected, then both players adopt truth-telling strategies so that $\sigma^*(s \mid s, \omega, G^*) := 1$ and $\tau^*(t \mid t, \omega, G^*) := 1$ for any $s \in S$, $t \in T$ and $\omega \in \mathbb{N}$. If any other mechanism $G \neq G^*$ is selected, then both players adopt the original strategies so that $\sigma^*(\cdot, G) := \sigma(\cdot, G)$ and $\tau^*(\cdot, G) := \tau(\cdot, G)$. For any $G \in \mathcal{G}$, we use the original public randomizing device π^G.

By construction, the mechanism G^* with the posterior belief $\rho^*(\cdot \mid G^*)$ is interim IC for both players and interim IR for the buyer. Since for any $G \neq G^*$, the original strategy profile $(\sigma(\cdot, G), \tau(\cdot, G))$ forms a Bayesian equilibrium in a continuation game induced by

G with $\rho^*(\cdot \mid G)$, so does $(\sigma^*(\cdot, G), \tau^*(\cdot, G))$. The linearity of the expected utility function in probabilities implies that, by selecting the mechanism G^*, any type of seller obtains the same interim payoff as in the original equilibrium $(\gamma, \sigma, \tau, \rho)$. So, the seller has no incentive to deviate to any other mechanism $G \neq G^*$. By construction, the belief system ρ^* is consistent. Finally, the construction of G^* implies that the pooling equilibrium $(G^*, \sigma^*, \tau^*, \rho^*)$ is outcome-equivalent to the original equilibrium $(\gamma, \sigma, \tau, \rho)$. □

Proposition 4.6. *Let (Q^*, M^*) be an LCS mechanism, and $(\bar{Q}, \bar{M})$ be the optimal screening mechanism. Then, $Q^*(1, t) = \bar{Q}(1, t)$ and $M^*(1, t) = \bar{M}(1, t)$ for any $t \in T$.*

Proof of Proposition 4.6. First, we claim that the maximum payoff of the problem (4.22) for $s = 1$ is bounded by the maximum payoff $\bar{\Pi}(1)$ when the seller's type is common knowledge. This is because the maximization problem of the seller's optimal screening mechanism is less constrained than the problem (4.22).

Next, define a mechanism (Q, M) so that $(Q(1, t), M(1, t)) := (\bar{Q}(1, t), \bar{M}(1, t))$ and $(Q(s, t), M(s, t)) := (\bar{q}, 0)$ for any s, t with $s > 1$. By construction, the mechanism (Q, M) satisfies the interim upward ICs for the seller and both ex post IC and IR for the buyer. Hence, the payoff $\bar{\Pi}(1)$ can be achieved in the problem (4.22) for $s = 1$. By assumption, the problem (4.17) has a unique solution. By definition, the LCS mechanism (Q^*, M^*) is a solution to the problem (4.22) for $s = 1$. Thus, (Q^*, M^*) must satisfy $Q^*(1, t) = \bar{Q}(1, t)$ and $M^*(1, t) = \bar{M}(1, t)$. This completes the proof. □

Lemma 4.4. *For any $s \in S$, let (Q, M) be a solution to the following problem:*

$$\max_{(Q,M)\in\mathcal{G}} \sum_{t\in T} p_t \left[M(s, t) - c\left(Q(s, t), s, t\right)\right] \tag{4.29}$$
$$\text{s.t.} \quad (Q, M) \text{ satisfies the interim upward ICs for the seller.}$$
$$(Q, M) \text{ satisfies the ex post local downward ICs (4.11) for the buyer.}$$
$$v(Q(s, 1), s, 1) - M(s, 1) \geq 0 \ \text{ for each } s \in S.$$

Then, the mechanism (Q, M) satisfies the buyer's ex post local downward ICs (4.11) for s with equality, and $v(Q(s, 1), s, 1) - M(s, 1) = 0$.

Proof of Lemma 4.4. Take any $s \in S$. Let (Q, M) be a solution to (4.29). By way of contradiction, suppose that $v(Q(s, t), s, t) - M(s, t) > v(Q(s, t-1), s, t) - M(s, t-1)$ for some $t \in T$ with $t \geq 2$, or $v(Q(s, 1), s, 1) - M(s, 1) > 0$. In the following, we construct a new mechanism (Q', M') which leads to a contradiction. There are two cases to be considered.

(i) First, assume that $c(q, s, t)$ has strictly decreasing differences in (q, s). Then, the following inequality holds for any allocation rule Q' such that $Q'(s, t) \geq_{FSD} Q(s, t)$ with strict inequality for some t:

$$\sum_{t\in T} p_t \left[c(Q'(s,t),s,t) - c(Q(s,t),s,t)\right] < \sum_{t\in T} p_t \left[c(Q'(s,t),s',t) - c(Q(s,t),s',t)\right]$$

for any $s' < s$. Thus, by continuity of the payoff functions on Δ, there exists $(Q'(s,t), M'(s,t))_{t\in T}$ such that $Q'(s,t) \geq_{FSD} Q(s,t)$ with strict inequality for some t,

$$\sum_{t\in T} p_t \left[c(Q'(s,t),s,t) - c(Q(s,t),s,t))\right] < \sum_{t\in T} p_t \left[M'(s,t) - M(s,t)\right] \tag{4.30}$$

$$\sum_{t\in T} p_t \left[c(Q'(s,t),s',t) - c(Q(s,t),s',t)\right] \geq \sum_{t\in T} p_t \left[M'(s,t) - M(s,t))\right] \tag{4.31}$$

for any $s' < s$, and $(Q'(s,t), M'(s,t))_{t\in T}$ satisfies both the ex post local downward ICs for the buyer and the constraint $v(Q'(s,1),s,1) - M'(s,1) \geq 0$. Then, fix such $(Q'(s,t), M'(s,t))_{t\in T}$. For any $s' < s$, define $(Q'(s',\cdot), M'(s',\cdot)) := (Q(s',\cdot), M(s',\cdot))$. For any $s' > s$, define $(Q'(s',\cdot), M'(s',\cdot)) := (\bar{q}, 0)$. By the construction of the new mechanism (Q', M'), $(Q'(s,\cdot), M'(s,\cdot))$ satisfies both the ex post local downward ICs for the buyer and the constraint $v(Q'(s',1),s',1) - M'(s',1) \geq 0$ for any $s' \in S$. Now, the new mechanism (Q', M') satisfies the inequality (4.31) for any $s' < s$. Thus, since the original mechanism (Q, M) satisfies the interim upward ICs for the seller, so does the new mechanism (Q', M').

However, the inequality (4.30) means that the type-s seller is strictly better off by selecting the new mechanism (Q', M'). This contradicts the hypothesis that the original mechanism (Q, M) is a solution to the problem (4.29) for s.

(ii) Assume that $c(q,s,t)$ has strictly increasing differences in (q,s). In a similar way to case (i), we can obtain a contradiction. □

Lemma 4.5. *Suppose that a mechanism (Q, M) is a solution to the problem (4.29) for each $s \in S$. Then, the mechanism (Q, M) satisfies the interim downward ICs for the seller, and thus it is interim IC for the seller.*

Proof of Lemma 4.5. Take any mechanism (Q, M) which is a solution to the problem (4.29) for each $s \in S$. It follows from Lemmas 4.2 and 4.4 that, for each s, the expected payment $\sum_{t\in T} p_t M(s,t)$ is equal to $\sum_{t\in T} p_t \psi(Q(s,t),s,t)$. Now, the proof proceeds by induction on s: For each $s \in S$, we claim that

$$\sum_{t\in T} p_t \left[\psi(Q(s,t),s,t) - c(Q(s,t),s,t))\right] \geq \sum_{t\in T} p_t \left[\psi(Q(\hat{s},t),\hat{s},t) - c(Q(\hat{s},t),s,t))\right] \tag{4.32}$$

for any $\hat{s} < s$. For $s = 1$, the claim is vacuously true. Suppose now that, for any $s \leq k$ with $k \geq 1$, $(Q(s,\cdot), M(s,\cdot))$ satisfies the above inequalities. Take $s := k+1$. By way of contradiction, suppose that

$$\sum_{t \in T} p_t \left[\psi(Q(s,t), s, t) - c(Q(s,t), s, t))\right] < \sum_{t \in T} p_t \left[\psi(Q(\hat{s},t), \hat{s}, t) - c(Q(\hat{s},t), s, t))\right] \quad (4.33)$$

for some $\hat{s} < s$. Fix such $\hat{s}$. Then, it must be the case that, for some t, the lottery $Q(\hat{s}, t)$ assigns positive probability to some $q > 0$. Also, since (Q, M) is a solution to the problem (4.29) for the type $\hat{s}$, Assumption 4.2 (iv) guarantees that, for some t, the lottery $Q(\hat{s}, t)$ assigns positive probability to some $q < \bar{q}$. In the following, we construct a new mechanism (Q', M') which leads to a contradiction. There are two cases to be considered.

(i) First, assume that $c(q, s, t)$ has strictly increasing differences in (q, s). Since ψ is increasing in s by Assumption 4.2 (i), the intermediate value theorem implies that there exist allocations $(Q'(s,t))_{t \in T}$ such that $Q'(s,t) \leq_{FSD} Q(\hat{s}, t)$ with strict inequality for some t, and

$$\sum_{t \in T} p_t \left[c(Q'(s,t), s, t) - c(Q(\hat{s},t), s, t)\right] = \sum_{t \in T} p_t \left[\psi(Q'(s,t), s, t) - \psi(Q(\hat{s},t), \hat{s}, t)\right]. \quad (4.34)$$

Since $c(Q, s, t)$ has strictly increasing differences in (Q, s), the above equality implies the following inequality:

$$\sum_{t \in T} p_t \left[c(Q'(s,t), s', t) - c(Q(\hat{s},t), s', t)\right] \geq \sum_{t \in T} p_t \left[\psi(Q'(s,t), s, t) - \psi(Q(\hat{s},t), \hat{s}, t)\right] \quad (4.35)$$

for any $s' < s$. Now, define the payment rule M as in (4.18). For any $s' < s$, define (Q', M') to be $(Q'(s', \cdot), M'(s', \cdot)) := (Q(s', \cdot), M(s', \cdot))$. For any $s' > s$, define (Q', M') to be $(Q'(s', \cdot), M'(s', \cdot)) := (\bar{q}, 0)$. By the construction of the new mechanism (Q', M'), $(Q'(s, \cdot), M'(s, \cdot))$ satisfies both the ex post local downward ICs for the buyer and the constraint $v(Q'(s', 1), s', 1) - M'(s', 1) \geq 0$ for any $s' \in S$. Now, the new mechanism (Q', M') satisfies the inequality (4.35) for any $s' < s$. Moreover, the induction hypothesis implies that the original mechanism (Q, M) satisfies the following inequalities

$$\sum_{t \in T} p_t \left[M(s', t) - c(Q(s', t), s', t)\right] \geq \sum_{t \in T} p_t \left[M(\hat{s}, t) - c(Q(\hat{s}, t), s', t)\right]$$

for any $s', \hat{s} \in S$ with $s' < s$ and $\hat{s} < s$. Hence, the new mechanism (Q', M') also satisfies the above inequality with $\hat{s} = s$ for any $s' < s$. Therefore, the new mechanism is feasible in the problem (4.29).

However, the inequality (4.33) with the equality (4.34) means that the type-s seller is strictly better off by selecting the new mechanism (Q', M'). This contradicts the hypothesis that the original mechanism (Q, M) is a solution to the problem (4.29) for s.

(ii) Second, assume that $c(q,s,t)$ has strictly decreasing differences in (q,s). In a similar way to case (i), we can obtain a contradiction. $\square$

Lemma 4.6. *A mechanism (Q,M) is an LCS mechanism if and only if it is a solution to the problem (4.29) for each $s \in S$.*

Proof of Lemma 4.6. Fix any $s \in S$, and take any solution (Q,M) to the problem (4.29) for s. First, we claim that the mechanism (Q,M) satisfies the monotonicity condition (4.14) for s. Define the Lagrangian function L as follows:

$$
\begin{aligned}
L(Q,M,\lambda) := & \sum_{t\in T} p_t \left[M(s,t) - c(Q(s,t),s,t)\right] \\
& + \sum_{s\in S}\sum_{t\geq 2} \lambda_t^s \left[v(Q(s,t),s,t) - M(s,t) - v(Q(s,t-1),s,t) + M(s,t-1)\right] \\
& + \sum_{s\in S} \lambda_1^s \left[v(Q(s,1),s,1) - M(s,1)\right] \\
& + \sum_{s\in S}\sum_{\hat{s}>s} \lambda^{s,\hat{s}} \sum_{t\in T} p_t \left[M(s,t) - c(Q(s,t),s,t) - M(\hat{s},t) + c(Q(\hat{s},t),s,t)\right] \\
& + \sum_{s\in S}\sum_{t\in T} \bar{\lambda}_t^s \left[1 - \sum_{q=0}^{\bar{q}} Q_q(s,t)\right],
\end{aligned}
$$

where $\lambda = (\lambda_t^s, \lambda^{s,\hat{s}}, \bar{\lambda}_t^s)_{s\in S, \hat{s}>s, t\in T} \in \mathbb{R}_+^{\bar{s}\bar{t}+\bar{s}(\bar{s}-1)/2} \times \mathbb{R}^{\bar{s}\bar{t}}$. Since (Q,M) is a solution to the problem (4.29), there exist Lagrange multipliers λ such that a triple $((Q,M),\lambda)$ satisfies the Kuhn-Tucker conditions. Notice that since the problem (4.29) is a linear programming problem which satisfies the Slater constraint qualification, the Kuhn-Tucker conditions are necessary and sufficient for the solution. Fix any such λ. Then, the first-order condition $\frac{\partial L}{\partial M(s,\bar{t})}(Q,M,\lambda) = 0$ implies that

$$
p_{\bar{t}} \left(1 + \sum_{\hat{s}>s} \lambda^{s,\hat{s}} - \sum_{\hat{s}<s} \lambda^{\hat{s},s})\right) = \lambda_{\bar{t}}^s.
$$

Let $\alpha := 1 + \sum_{\hat{s}>s} \lambda^{s,\hat{s}} - \sum_{\hat{s}<s} \lambda^{\hat{s},s}$. Since $\lambda_{\bar{t}}^s \geq 0$, so is α. Lemma 4.4 shows that (Q,M) satisfies the buyer's ex post local downward ICs (4.11) for s with equality and $v(Q(s,1),s,1) - M(s,1) = 0$. Thus, Lemma 4.2 implies that the expected payment is given by $\sum_{t\in T} p_t M(s,t) = \sum_{t\in T} p_t \psi(Q(s,t),s,t)$. By substituting the expected payment into the Lagrangian function L, we can show that $Q(s,\cdot)$ must be a solution to the following maximization problem:

$$
\max_{Q(s,\cdot)\in\Delta^{\bar{t}}} \sum_{t\in T} p_t \left\{ \alpha \left[\psi(Q(s,t),s,t) - c(Q(s,t),s,t)\right] + \sum_{\hat{s}<s} \lambda^{\hat{s},s} \left[c(Q(s,t),\hat{s},t) - c(Q(s,t),s,t)\right] \right\}.
$$

Thus, for each $t \in T$, the allocation $Q(s,t)$ is a solution to

$$\max_{Q' \in \Delta} \alpha \left[\psi(Q', s, t) - c(Q', s, t)\right] + \sum_{\hat{s} < s} \lambda^{\hat{s},s} \left[c(Q', \hat{s}, t) - c(Q', s, t)\right].$$

It follows from Lemma 4.3 that $\psi(Q,s,t)$ has strictly increasing differences in (Q,t), $c(Q,s,t)$ has decreasing differences in (Q,t), and ψ and c are both supermodular and submodular on Δ. Moreover, for any $\hat{s} < s$, $c(Q,\hat{s},t) - c(Q,s,t)$ has increasing differences in (Q,t) by Assumption 4.2 (iii). If $\alpha > 0$, then Theorem 2.8.4 of Topkis (1998) implies that $Q(s,t) \geq_{FSD} Q(s,t')$ for each $t > t'$. If $\alpha = 0$, then $\lambda^{\hat{s},s} > 0$ for some $\hat{s} < s$. Then, $Q(s,t)$ assigns probability 1 to either $q = 0$ for each t or $q = \bar{q}$ for each t because $c(Q,s)$ has either strictly increasing differences or strictly decreasing differences in (Q,s). Thus, the allocation rule Q satisfies the monotonicity condition (4.14) for s.

Second, we claim that the solution (Q,M) to the problem (4.29) for s is both ex post IC and ex post IR for the buyer with respect to s. The above arguments show that (Q,M) satisfies the buyer's ex post local downward ICs (4.11) for s with equality and the monotonicity condition (4.14) for s. Thus, since $v(Q,s,t)$ has strictly increasing differences in (Q,t),

$$v(Q(s,t+1),s,t) - v(Q(s,t),s,t) \leq v(Q(s,t+1),s,t+1) - v(Q(s,t),s,t+1) = M(s,t+1) - M(s,t)$$

for each $t \leq \bar{t} - 1$. Hence, (Q,M) satisfies the ex post local upward ICs (4.13) for s. Lemma 4.1 (i) then implies that the mechanism (Q,M) is ex post IC for the buyer for s. Also, (Q,M) is ex post IR for the buyer for s because v is increasing in t, (Q,M) is ex post IC for the buyer for s and $v(Q(s,0),s,0) - M(s,0) = 0$.

Finally, we prove the statement in the lemma. The proof of the "only-if-part" is as follows. By way of contradiction, suppose that an LCS mechanism is not a solution to the problem (4.29) for some $s' \in S$. Fix such s'. By definition, the LCS mechanism is feasible in the problem (4.29). So, there exists another feasible mechanism (Q', M') in which the buyer with s' obtains strictly higher payoff. Then, the above claim implies that (Q', M') is both ex post IC and IR for the buyer with respect to s'. In a similar way to Lemma 4.5, we can construct a new mechanism (Q'', M'') with $(Q''(s',\cdot), M''(s',\cdot)) := (Q'(s',\cdot), M'(s',\cdot))$ which satisfies the interim upward ICs for the seller and is both ex post IC and IR for the buyer. This is a contradiction because the LCS mechanism is a solution to the problem (4.29) for s'. The proof of the "if-part" is as follows. By the above claim, if a mechanism is a solution to the problem (4.29) for each s, then it is both ex post IC and IR for the buyer. The mechanism also satisfies the interim upward ICs for the seller. Thus, it must be an LCS mechanism. □

Lemma 4.7. *Any LCS mechanism (Q^*, M^*) is a solution to the problem (4.25).*

Proof of Lemma 4.7. Take any LCS mechanism (Q^*, M^*). It follows from Lemma 4.6 that

(Q^*, M^*) is a solution to the problem (4.29) for each $s \in S$. By way of contradiction, suppose that (Q^*, M^*) is not a solution to the problem (4.25). Then, there exists another mechanism (Q, M) which satisfies the interim upward ICs for the seller, and the ex post local downward ICs for the buyer and the constraint $v(Q(s,t), s, t) - M(s,t) \geq 0$ for any s and

$$\sum_{t \in T} p_t \left[M(s,t) - c\left(Q(s,t), s\right)\right] > \sum_{t \in T} p_t \left[M^*(s,t) - c\left(Q^*(s,t), s\right)\right] \tag{4.36}$$

for some $s \in S$. If the inequality is also satisfied in a weak sense for any $s' \neq s$, then we have a contradiction. So, we assume that the reverse inequality holds for some $s' \neq s$. Let S' be the set of the seller's types for which the reverse inequality holds in a weak sense. Hence, $s' \in S'$. We construct a mechanism (Q', M') as follows: $(Q'(s, \cdot), M'(s, \cdot)) := (Q(s, \cdot), M(s, \cdot))$ for any $s \in S \setminus S'$ and $(Q'(s, \cdot), M'(s, \cdot)) := (Q^*(s, \cdot), M^*(s, \cdot))$ for any $s \in S'$. By construction, for any $s \in S \setminus S'$ and $s' \in S'$,

$$\sum_{t \in T} p_t \left[M'(s,t) - c\left(Q'(s,t), s, t\right)\right] > \sum_{t \in T} p_t \left[M^*(s,t) - c\left(Q^*(s,t), s, t\right)\right] \geq \sum_{t \in T} p_t \left[M^*(\hat{s},t) - c\left(Q^*(\hat{s},t), s, t\right)\right]$$
$$\sum_{t \in T} p_t \left[M'(s',t) - c\left(Q'(s',t), s', t\right)\right] > \sum_{t \in T} p_t \left[M(s',t) - c\left(Q(s',t), s', t\right)\right] \geq \sum_{t \in T} p_t \left[M(\hat{s}',t) - c\left(Q(\hat{s}',t), s', t\right)\right]$$

for any $\hat{s} > s$ and $\hat{s}' > s'$. The two weak inequalities follow from the hypotheses that the original mechanisms (Q^*, M^*) and (Q, M) satisfy the interim upward ICs for the seller. Since (Q^*, M^*) and (Q, M) satisfy the ex post local downward ICs for the buyer and the constraint $v(Q(s,t), s, t) - M(s,t) \geq 0$ for any s, so does (Q', M'). This contradicts the fact that (Q^*, M^*) is a solution to the problem (4.29) for s which satisfies the inequality (4.36). $\square$

Proof of Proposition 4.4. Take any LCS mechanism (Q^*, M^*). For the problem (4.25), define the Lagrangian function L as in Lemma 4.6. Let $\lambda^* := (\lambda_t^{s*}, \lambda^{s,\hat{s}*}, \bar{\lambda}_t^{s*})_{s \in S, \hat{s} > s, t \in T}$ be a vector of Lagrange multipliers such that $((Q^*, M^*), \lambda^*)$ satisfies the Kuhn-Tucker conditions. The first-order conditions $\frac{\partial L}{\partial M(s,t)}(Q^*, M^*, \lambda^*) = 0$ imply that

$$\lambda_t^{s*} = \sum_{t'=t}^{\bar{t}} p_{t'} \left(1 + \sum_{\hat{s} > s} \lambda^{s,\hat{s}} - \sum_{\hat{s} < s} \lambda^{\hat{s},s}\right)$$

for each s and t. Summing over s, it is shown that $\sum_{s \in S} \lambda_t^{s*} = \bar{s} \sum_{t'=t}^{\bar{t}} p_{t'}$ for each t. We now define a belief ρ so that $\rho(s) := \lambda_t^{s*} / \sum_{s' \in S} \lambda_t^{s'*} = (1 + \sum_{\hat{s} > s} \lambda^{s,\hat{s}} - \sum_{\hat{s} < s} \lambda^{\hat{s},s}) / \bar{s}$. Since Lemmas 4.4 and 4.6 imply that $\lambda_t^{s*} > 0$, we obtain $\rho(s) > 0$ for any s. Let $\lambda_t^* := \sum_{s \in S} \lambda_t^{s*}$. By construction, $\lambda_t^* \rho(s) = \lambda_t^{s*}$ for each s and t.

Now, consider the following maximization problem:

$$\max_{(Q,M)\in\mathcal{G}} \sum_{s\in S}\sum_{t\in T} p_t\left[M(s,t) - c\left(Q(s,t),s,t\right)\right] \tag{4.37}$$

$$\begin{aligned}\text{s.t.}\quad & (Q,M) \text{ satisfies the interim upward ICs for the seller.}\\ & (Q,M) \text{ with } \rho \text{ satisfies the interim local downward ICs (4.10) for the buyer.}\\ & \sum_{s\in S}\rho(s)\left[v(Q(s,1),s,1) - M(s,1)\right] \geq 0.\end{aligned}$$

Define the Lagrangian function L' as follows:

$$\begin{aligned}
&L'(Q,M,\lambda)\\
&:= \sum_{s\in S}\sum_{t\in T} p_t\left[M(s,t) - c(Q(s,t),s,t)\right]\\
&\quad + \sum_{t\geq 2}\lambda_t\sum_{s\in S}\rho(s)\left[v(Q(s,t),s,t) - M(s,t) - v(Q(s,t-1),s,t) + M(s,t-1)\right]\\
&\quad + \lambda_1\sum_{s\in S}\rho(s)\left[v(Q(s,1),s,1) - M(s,1)\right]\\
&\quad + \sum_{s\in S}\sum_{\hat{s}>s}\lambda^{s,\hat{s}}\sum_{t\in T}p_t\left[M(s,t) - c(Q(s,t),s,t) - M(\hat{s},t) + c(Q(\hat{s},t),s,t)\right]\\
&\quad + \sum_{s\in S}\sum_{t\in T}\bar{\lambda}^s_t\left[1 - \sum_{q=0}^{\bar{q}}Q_q(s,t)\right],
\end{aligned}$$

where $\lambda = (\lambda_t, \lambda^{s,\hat{s}}, \bar{\lambda}^s_t)_{s\in S,\hat{s}>s,t\in T} \in \mathbb{R}_+^{\bar{t}+\bar{s}(\bar{s}-1)/2}\times\mathbb{R}^{\bar{s}\bar{t}}$. By the construction of ρ and λ^*_t, the mechanism (Q^*,M^*) with the multipliers $(\lambda^*_t, \lambda^{s,\hat{s}*}, \bar{\lambda}^{s*}_t)_{s\in S,\hat{s}>s,t\in T}$ satisfies the Kuhn-Tucker conditions. Hence, (Q^*,M^*) is a solution to the problem (4.37). This implies that the LCS mechanism (Q^*,M^*) is not dominated by any other mechanism which is interim IC for both players and interim IR for the buyer with respect to ρ. $\square$

Proof of Proposition 4.5. Let (Q^*,M^*) be an LCS mechanism. For each $s\in S$, define a mechanism by $G^{s*} := (Q^*(s,\hat{t}), M^*(s,\hat{t}))_{\hat{t}\in T}$ so that the mechanism G^{s*} no longer depends on the seller's report $\hat{s}$. We assume without loss of generality that $G^{s*}\neq G^{s*}$ for any s,s' with $s\neq s'$.[17] For any s, denote the type-s seller's interim payoff in the mechanism G^{s*} by $\Pi^*(s) := \sum_{t\in T}p_t[M^*(s,t) - c(Q^*(s,t),s,t)]$.

We claim that there exists a separating equilibrium in which, for any s, the type-s seller selects the mechanism G^{s*}. Since the LCS mechanism is both ex post IC and IR for the buyer, given any s, it is sequentially rational for the buyer with a posterior belief $\rho(s\mid G^{s*}) = 1$ to report his type truthfully in the mechanism G^{s*}. Then, the type-s seller has no incentive to deviate to the mechanism $G^{s'*}$ of another type $s'\neq s$ because the

[17] If not, then we redefine $(G^{s*})_{s\in S}$ so that $(Q^{s*}(\hat{s},\cdot), M^{s*}(\hat{s},\cdot)) := (Q^*(s,\cdot), M^*(s,\cdot))$ if $\hat{s}=s$ and $(Q^{s*}(\hat{s},\cdot), M^{s*}(\hat{s},\cdot)) := (0,-s)$ if $\hat{s}\neq s$.

LCS mechanism is interim IC for the seller by Lemmas 4.5 and 4.6. Hence, to prove the claim, it suffices to show that, for any mechanism G different from $(G^{s*})_{s\in S}$, there exists some posterior belief $(\rho(s \mid G))_{s\in S}$ and some randomizing device π such that the seller's equilibrium payoff $\Pi^G(s)$ defined by (4.7) in the continuation game is weakly lower than $\Pi^*(s)$ for any s.

In the following, fix any mechanism $G = (Q, M)$ with $G \neq G^{s*}$ for each $s \in S$. Lemma 4.7 implies that there exists a vector of Lagrange multipliers λ such that $((Q^*, M^*), \lambda)$ satisfies the Kuhn-Tucker conditions for the problem (4.25). Let $(\lambda^{s,\hat{s}})_{s\in S,\hat{s}>s}$ be components of λ associated with the interim upward ICs for the seller. For the interim payoff vector $(\Pi^*(s))_{s\in S}$, let $S(G)$ be the set of the seller's types defined by (4.23). We can assume $S(G) \neq S$ because if $S(G) = S$, then any type of seller has no incentive to deviate to G, no matter what equilibrium is played in the continuation game.

For any belief $\rho \in \Delta(S \setminus S(G))$ about the seller's type, let $F(\rho) \subset \mathbb{R}^{\bar{s}}$ be the set of the seller's equilibrium payoff vectors in the continuation game induced by the mechanism G with the belief ρ. Since the set of outcomes is finite in the mechanism G, both $F(\rho)$ and $\cup_{\rho\in\Delta(S\setminus S(G))}F(\rho)$ are nonempty and bounded. Also, since we permit public randomizing devices, the set $F(\rho)$ is convex for any ρ. Let $\mathcal{F}$ be a compact and convex set containing $\cup_{\rho\in\Delta(S\setminus S(G))}F(\rho)$. For any $s \in S$, $\Pi(\cdot) \in \mathbb{R}^{\bar{s}}$ and $\delta \in [0, 1]$, define a correspondence Φ^s_δ by $\Phi^s_\delta(\Pi(s)) := \{\tilde{r} \mid \tilde{r} \in \arg\max_{r\in[\delta,1]} [r\Pi(s) + (1 - r)\Pi^*(s)]\}$ for $s \notin S(G)$ and $\Phi^s_\delta(\Pi(s)) := \{0\}$ for $s \in S(G)$, and let $\Phi_\delta(\Pi) := \Phi^1_\delta(\Pi(1)) \times \cdots \times \Phi^{\bar{s}}_\delta(\Pi(\bar{s}))$. Obviously, the correspondences Φ^s_δ and Φ_δ are convex-valued. Now, take any belief $\rho' \in \Delta(S)$ with $\rho'(s) > 0$ for any s. For any $s \in S, t \in T$ and $r \in [0, 1]^{\bar{s}}$ with $r \neq (0, ..., 0)$, define a function ϕ^s by

$$\phi^s(r) := \frac{r(s)\rho'(s)}{\sum_{s'\in S} r(s')\rho'(s')},$$

which is well-defined because $\rho'(s') > 0$ for each s'. Let $\phi(r) := (\phi^s(r))_{s\in S}$.

By the above arguments, the following self-correspondence

$$(F, \Phi_\delta, \phi) : \Delta(S \setminus S(G)) \times \mathcal{F} \times \left([\delta, 1]^{S\setminus S(G)} \times \{0\}^{S(G)}\right) \rightrightarrows \mathcal{F} \times \left([\delta, 1]^{S\setminus S(G)} \times \{0\}^{S(G)}\right) \times \Delta(S \setminus S(G))$$

is convex-valued. Also, the correspondence has a closed graph. The domain of the correspondence is nonempty, compact and convex. Then, the Kakutani fixed point theorem implies that the correspondence (F, Φ_δ, ϕ) has a fixed point $(\Pi_\delta, r_\delta, \rho_\delta) \in F(\rho_\delta) \times \Phi_\delta(\Pi_\delta) \times \phi(r_\delta)$ for any $\delta \in (0, 1]$. Let $(\Pi, r, \rho) \in \mathcal{F} \times [0, 1]^{\bar{s}} \times \Delta(S \setminus S(G))$ be a subsequential limit of the sequence $(\Pi_{\frac{1}{k}}, r_{\frac{1}{k}}, \rho_{\frac{1}{k}})_{k=1}^{\infty}$, the existence of which is guaranteed by the compactness of $\mathcal{F} \times [0, 1]^{\bar{s}} \times \Delta(S \setminus S(G))$. Since the correspondence F has a closed graph, $\Pi \in F(\rho)$. Hence, for some randomizing device π, for any $\omega \in \mathbb{N}$, the continuation game induced by the mechanism G with the belief $(\rho(s \mid G))_{s\in S}$ has a Bayesian equilibrium

$(\sigma^*(\cdot,\omega,G),\tau^*(\cdot,\omega,G))$ such that $\Pi(s)=\Pi^G(s)$ for any $s\in S$, where

$$\Pi^G(s) := \sum_{t\in T} p_t \sum_{\hat{s}\in S} \sum_{\hat{t}\in T\cup\{out\}} \sum_{\omega\in\mathbb{N}} \pi(\omega)\sigma^*(\hat{s}\mid s,\omega,G)\tau^*(\hat{t}\mid t,\omega,G)\left[M(\hat{s},\hat{t})-c(Q(\hat{s},\hat{t}),s,t)\right].$$

Fix such a strategy profile $(\sigma^*(\cdot,G),\tau^*(\cdot,G))$, using the Axiom of Choice. Also, for any s, let $(\sigma^*(\cdot,G^{s*}),\tau^*(\cdot,G^{s*}))$ be truth-telling strategies and $\rho(s\mid G^{s*})=1$.

Finally, we claim that the profile $((G^{s*})_{s\in S},\sigma^*,\tau^*,\rho)$ defined above is a separating equilibrium which has the properties in the theorem. Since $r_\delta(s)=0$ and $\phi^s(r_\delta)=0$ for any $s\in S(G)$ and $\delta\in(0,1]$, we obtain $\rho(s\mid G)=0$ if $s\in S(G)$. Now, we show that $\Pi^G(s)\le\Pi^*(s)$ for any $s\in S$. By way of contradiction, suppose not. Then, there exists $s\in S(G)$ such that $\Pi^G(s)>\Pi^*(s)$. We now construct a new mechanism (Q',M') as follows:

$$Q'(s,t) := r(s)\left(\sum_{\hat{s}\in S}\sum_{\hat{t}\in T\cup\{out\}}\sum_{\omega\in\mathbb{N}}\pi(\omega)\sigma^*(\hat{s}\mid s,\omega,G)\tau^*(\hat{t}\mid t,\omega,G)Q(\hat{s},\hat{t})\right)+(1-r(s))Q^*(s,t),$$

$$M'(s,t) := r(s)\left(\sum_{\hat{s}\in S}\sum_{\hat{t}\in T\cup\{out\}}\sum_{\omega\in\mathbb{N}}\pi(\omega)\sigma^*(\hat{s}\mid s,\omega,G)\tau^*(\hat{t}\mid t,\omega,G)M(\hat{s},\hat{t})\right)+(1-r(s))M^*(s,t).$$

Since the correspondence Φ_δ with $\delta=0$ has a closed graph, we obtain $r(\cdot)\in\Phi_0(\Pi(\cdot))$. Thus, $r(s)=0$ if $\Pi^G(s)<\Pi^*(s)$ and $r(s)=1$ if $\Pi^G(s)>\Pi^*(s)$. By the hypothesis, the latter case occurs for some s. Hence, (Q',M') dominates (Q^*,M^*). By definition, the LCS mechanism (Q^*,M^*) is both ex post IC and IR for the buyer, and interim IC for the seller by Lemma 4.5. Moreover, for any $\omega\in\mathbb{N}$, $(\sigma^*(\cdot,\omega,G),\tau^*(\cdot,\omega,G))$ is a Bayesian equilibrium in the continuation game induced by G with ρ. Here, $\rho(s\mid G)=r(s)\rho'(s)/\sum_{s'\in S}r(s')\rho'(s')$ for each s because $\rho(\cdot)=\phi(r(\cdot))$ by the continuity of ϕ on $[0,1]^{\bar{s}}\setminus\{0,...,0\}$. Therefore, it must be the case that the mechanism (Q',M') with ρ' is interim IC for both players, and interim IR for the buyer. This contradicts the hypothesis in the proposition. Thus, $((G^{s*})_{s\in S},\sigma^*,\tau^*,\rho)$ is a separating equilibrium which has the properties in the theorem. □

Proof of Theorem 4.2. Fix any $s\in S$. Take any separating equilibrium of the mechanism-selection game, and let $\Pi(s)$ be the seller's interim payoff in the equilibrium. Let $\Pi^*(s)$ be the seller's interim payoff in the LCS mechanism. Since $\Pi^*(s)$ is a lower bound of the set of equilibrium payoffs by the definition of the LCS mechanism, we obtain $\Pi^*(s)\le\Pi(s)$. We claim that $\Pi^*(s)\ge\Pi(s)$. By way of contradiction, suppose that $\Pi^*(s)<\Pi(s)$. This contradicts the fact that the LCS mechanism is a solution to the problem (4.22) for s. Therefore, $\Pi^*(s)=\Pi(s)$ as desired. □

Proof of Theorem 4.3. Proposition 4.6 implies that $Q^*(s,\cdot)=\bar{Q}(s,\cdot)$ for $s=1$. Now, let

λ be a vector of Lagrange multipliers such that $((Q^*, M^*), \lambda)$ satisfies the Kuhn-Tucker conditions for the problem (4.25). Let $(\lambda^{s,\hat{s}})_{s\in S, \hat{s}>s}$ be components of λ associated with the interim upward ICs for the seller. Fix any $s \in S$ and $t \in T$. In a similar way to the proof of Lemma 4.6, it is shown that $Q^*(s,t)$ is a solution to the following problem:

$$\max_{Q\in\Delta} \left(1 + \sum_{\hat{s}>s} \lambda^{s,\hat{s}} - \sum_{\hat{s}<s} \lambda^{\hat{s},s}\right) [\psi(Q,s,t) - c(Q,s,t)] + \sum_{\hat{s}<s} \lambda^{\hat{s},s} \left[c(Q,\hat{s},t) - c(Q,s,t)\right].$$

By Lemmas 4.4 and 4.6, we obtain $1 + \sum_{\hat{s}>s} \lambda^{s,\hat{s}} - \sum_{\hat{s}<s} \lambda^{\hat{s},s} > 0$.

(i) Take any Q, Q' with $Q' >_{FSD} Q$. Suppose that the marginal cost is decreasing in s. Then, $c(Q',\hat{s},t) - c(Q',s,t) > c(Q,\hat{s},t) - c(Q,s,t)$ for any $\hat{s} < s$. Assume first that $\lambda^{\hat{s},s} = 0$ for any $\hat{s} < s$. Since the problem $\max_q[\psi(q,s,t) - c(q,s,t)]$ has a unique solution by assumption, we obtain $Q^*(s,t) = \bar{Q}(s,t)$. Assume next that $\lambda^{\hat{s},s} > 0$ for some $\hat{s} < s$. If $\psi(Q',s,t) - c(Q',s,t) \geq \psi(Q,s,t) - c(Q,s,t)$, then

$$\begin{aligned}
&\left(1 + \sum_{\hat{s}>s} \lambda^{s,\hat{s}} - \sum_{\hat{s}<s} \lambda^{\hat{s},s}\right) [\psi(Q',s,t) - c(Q',s,t)] + \sum_{\hat{s}<s} \lambda^{\hat{s},s} \left[c(Q',\hat{s},t) - c(Q',s,t)\right] \\
> &\left(1 + \sum_{\hat{s}>s} \lambda^{s,\hat{s}} - \sum_{\hat{s}<s} \lambda^{\hat{s},s}\right) [\psi(Q,s,t) - c(Q,s,t)] + \sum_{\hat{s}<s} \lambda^{\hat{s},s} \left[c(Q,\hat{s},t) - c(Q,s,t)\right].
\end{aligned}$$

This implies that the objective function satisfies the strict single crossing property in $(Q, \sum_{\hat{s}<s} \lambda^{\hat{s},s})$ in the sense of Milgrom and Shannon (1994). Therefore, Theorem 4' in their article implies that $Q^*(s,t) \geq_{FSD} \bar{Q}(s,t)$. (ii) The proof is similar to that of (i). $\square$

Proof of Theorem 4.4. It follows from Proposition 4.6 and Theorem 4.2 that the statements are trivially satisfied for $s = 1$. So, fix any $s \geq 2$ and $t \in T$. Take any LCS mechanism (Q^*, M^*). Let $\bar{Q}$ be the optimal allocation rule.

(i) Assume that the marginal cost is decreasing in s. Lemmas 4.2 and 4.4 imply that in the mechanism (Q^*, M^*), the type-t buyer's ex post payoff is

$$\sum_{t'=2}^{t} \left[v\left(Q^*(s,t'-1), s, t'\right) - v\left(Q^*(s,t'-1), s, t'-1\right)\right].$$

Since the marginal cost is decreasing in s, Theorem 4.3 implies that $Q^*(s,t') \geq_{FSD} \bar{Q}(s,t')$ for each t'. Hence, Lemma 4.3 (ii) implies that

$$\begin{aligned}
&\sum_{t'=2}^{t} \left[v\left(Q^*(s,t'-1), s, t'\right) - v\left(Q^*(s,t'-1), s, t'-1\right)\right] \\
\geq &\sum_{t'=2}^{t} \left[v\left(\bar{Q}(s,t'-1), s, t'\right) - v\left(\bar{Q}(s,t'-1), s, t'-1\right)\right].
\end{aligned}$$

Therefore, the buyer's ex post payoff is weakly higher than that in the optimal screening mechanism. (ii) The proof is similar to that of (i). $\square$

The following proposition presents the statement and proof of the "Revelation Principle." We call $\bar{G} := ((A, B), (\bar{Q}, \bar{M}))$ a *generalized mechanism*, where A, B are nonempty action sets of the seller and the buyer, $\bar{Q} : A \times B \to \Delta$ is an allocation rule and $\bar{M} : A \times B \to \mathbb{R}$ is a payment rule. We assume that $\bar{Q}_0(a, out) \equiv 1$ and $\bar{M}(a, out) \equiv 0$. For any generalized mechanism $\bar{G}$, let $\rho(\cdot \mid \bar{G}) \in \Delta(S)$ be the buyer's posterior belief about the seller's type. For any $s \in S$ and $t \in T$, define $\alpha(\cdot \mid s)$ and $\beta(\cdot \mid t)$ to be probability measures on A and $B \cup \{out\}$ endowed with some sigma-fields, respectively.

Proposition 4.7 (Revelation Principle)**.** *Let $\bar{G}$ be a generalized mechanism, and $\rho(\cdot \mid \bar{G})$ be the buyer's posterior belief. Denote by (α, β) a Bayesian equilibrium in a continuation game induced by $\bar{G}$ with $\rho(\cdot \mid \bar{G})$. Then, there exists a direct mechanism $G \in \mathcal{G}$ which has the following two properties. (i) The mechanism G with $\rho(\cdot \mid \bar{G})$ is interim IC for both players, and interim IR for the buyer. (ii) The outcomes of the truth-telling strategies in G are equivalent to those of (α, β).*

Proof of Proposition 4.7. We construct a direct mechanism $G := (Q, M)$ as follows:

$$\begin{aligned} Q_q(s,t) &:= \int_A \int_{B \cup \{out\}} \bar{Q}_q(a, b)\alpha(da \mid s)\beta(db \mid t), \\ M(s,t) &:= \int_A \int_{B \cup \{out\}} \bar{M}(a, b)\alpha(da \mid s)\beta(db \mid t). \end{aligned}$$

In the continuation game induced by G with $\rho(\cdot \mid \bar{G})$, the buyer has no incentive to tell a lie when the seller truthfully reports her type, because for each $t, \hat{t} \in T$,

$$\begin{aligned} &\sum_{s \in S} \rho(s \mid \bar{G}) \left[v(Q(s,t), s, t) - M(s,t)\right] \\ &= \sum_{s \in S} \rho(s \mid \bar{G}) \int_A \int_{B \cup \{out\}} \left[v(\bar{Q}(a,b), s, t) - \bar{M}(a,b)\right] \alpha(da \mid s)\beta(db \mid t) \\ &\geq \sum_{s \in S} \rho(s \mid \bar{G}) \int_A \int_{B \cup \{out\}} \left[v(\bar{Q}(a,b), s, t) - \bar{M}(a,b)\right] \alpha(da \mid s)\beta(db \mid \hat{t}) \\ &= \sum_{s \in S} \rho(s \mid \bar{G}) \left[v(Q(s,\hat{t}), s, t) - M(s,\hat{t})\right]. \end{aligned}$$

The inequality follows from the hypothesis that (α, β) is a Bayesian equilibrium in the continuation game induced by $\bar{G}$ with $\rho(\cdot \mid \bar{G})$. The hypothesis also implies that the buyer has no incentive to opt out. In a similar way, we can show that the seller has no incentive to tell a lie when the buyer truthfully reports his type. Finally, the construction of G proves part (ii). This completes the proof. $\square$

Chapter 5

Conclusion

In the previous chapters, we have analyzed three game-theoretic models.

In Chapter 2, we have studied the optimal design of scoring auctions in an environment with various quality attributes. We show that if the virtual surplus is quasisupermodular in quality, then there exists an optimal scoring rule which is supermodular in quality. As an optimal scoring rule, we have constructed a Leontief-like function. An example shows that an extension of Che (1993)'s scoring rule cannot implement the buyer's optimal direct mechanism. The scoring rule is additively separable in quality. This implies that the buyer should carefully design scoring rules which are additively separable in quality attributes.

In Chapter 3, we have compared the performance of a bundling method with that of an unbundling method in an auction model with risks. An important feature of our model is that the risk factors are categorized into two groups: aggregate risk and idiosyncratic risk. Our results show that each risk factor has a different effect on a public authority's optimal choice between the two methods. In the bundling method, a public authority must pay a high risk premium in exchange for the burden of risks on a private party. As a result, a low aggregate risk is a strong reason to choose the bundling method. A decrease in the idiosyncratic risk may encourage the public authority to choose the unbundling method, by reducing information rents of private parties.

In Chapter 4, we have proved that, in a bilateral trade environment, the mechanism-selection game has a separating equilibrium with a reasonable posterior belief. A fundamental notion in our analysis is the LCS (lease-cost-separating) mechanism. In our model, the LCS mechanism is equivalent to the RSW (Rothschild-Stiglitz-Wilson) mechanism defined by Maskin and Tirole (1992). We show that the seller's interim payoff vector in any separating equilibrium is uniquely determined by that in the LCS mechanism. We also provide sufficient conditions on the seller's cost function under which the allocation rule is distorted upward or downward compared to the optimal direct mechanism when the seller's type is commonly known. Accordingly, the buyer is weakly better off or worse off than in the optimal direct mechanism. This is in contrast to the case in which only the principal has private information.

An important question for future research is whether it is optimal for the buyer to choose negotiations rather than auctions. Indeed, as shown in Section 2.5, a scoring auction may not implement the optimal outcome for the buyer. Then, some form of negotiations can outperform auctions. Bulow and Klemperer (1996) address the issue and show that the principal prefers an auction with no reserve price to an optimally-structured negotiation with one less bidder. Bajari et al. (2009) consider several possible determinants which may influence the choice of auctions versus negotiations, and test their hypotheses using a data set of private sector building contracts. Addressing the issue based on the previous studies is left for future research.

Finally, I hope that our analyses in this thesis make some progress in game theory.

References

G.A. Akerlof. The market for "lemons": Quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84(3):488–500, 1970.

J. Asker and E. Cantillon. Properties of scoring auctions. *RAND Journal of Economics*, 39(1):69–85, 2008.

J. Asker and E. Cantillon. Procurement when price and quality matter. *RAND Journal of Economics*, 41(1):1–34, 2010.

E. Auriol and J.J Laffont. Regulation by duopoly. *Journal of Economics & Management Strategy*, 1(3):507–533, 1992.

P. Bajari, R. McMillan, and S. Tadelis. Auctions versus negotiations in procurement: An empirical analysis. *Journal of Law, Economics, and Organization*, 25(2):372–399, 2009.

F. Balestrieri and S. Izmalkov. Informed seller in a hotelling market. *mimeo*, 2012.

D. Balkenborg and M. Makrisz. An undominated mechanism for a class of informed principal problems with common values. *mimeo*, 2013.

M. Bichler. An experimental analysis of multi-attribute auctions. *Decision Support Systems*, 29(3):249–268, 2000.

F. Branco. The design of multidimensional auctions. *RAND Journal of Economics*, 28 (1):63–81, 1997.

J. Bulow and P. Klemperer. Auction versus negotiation. *American Economic Review*, 86 (1):180–194, 1996.

H. Cai, J. Riley, and L. Ye. Reserve price signaling. *Journal of Economic Theory*, 135 (1):253–268, 2007.

Y.K. Che. Design competition through multidimensional auctions. *RAND Journal of Economics*, 24(4):668–680, 1993.

C.H. Chen-Ritzo, T.P. Harrison, A.M. Kwasnica, and D.J. Thomas. Better, faster, cheaper: An experimental analysis of a multiattribute reverse auction. *Management Science*, 51(12):1753–1762, 2005.

I.K. Cho and D.M. Kreps. Signaling games and stable equilibria. *Quarterly Journal of Economics*, 102(2):179–221, 1987.

P. Cramton, R. Gibbons, and P. Klemperer. Dissolving a partnership efficiently. *Econometrica*, 55(3):615–632, 1987.

ENR. The top 100 design-build firms, 2012. `http://enr.construction.com/toplists/DesignBuild/001-100.asp`.

G.E.J. Gibson, J.T. O'Connor, G. Migliaccio, and J. Walewski. *Key implementation issues and lessens learned with design-build projects*. American Society of Civil Engineers, 2007.

V. Grimm. Sequential versus bundle auctions for recurring procurement. *Journal of Economics*, 90(1):1–27, 2007.

K.M. Hagerty and W.P. Rogerson. Robust trading mechanisms. *Journal of Economic Theory*, 42(1):94–107, 1987.

M. Hanazono, J. Nakabayashi, and M. Tsuruoka. A theoretical analysis of scoring auctions with a general form of scoring rules. *mimeo*, 2011.

G. Hanoch and H. Levy. The efficiency analysis of choices involving risk. *Review of Economic Studies*, 36(3):335–346, 1969.

O. Hart. Incomplete contracts and public ownership: Remarks, and an application to public-private partnerships. *Economic Journal*, 113:C69–C76, 2003.

B. Holmström and R.B. Myerson. Efficient and durable decision rules with incomplete information. *Econometrica*, 51(6):1799–1819, 1983.

C. Holt. Competitive bidding for contracts under alternative auction procedures. *Journal of Political Economy*, 88(3):433–445, 1980.

E.I. Hoppe, D.J. Kusterer, and P.W. Schmitz. Public-private partnerships versus traditional procurement: An experimental investigation. *Journal of Economic Behavior & Organization*, 89:145–166, 2013.

E. Iossa, G. Spagnolo, and M. Vellez. Contract design in public-private partnerships. *Report for the World Bank*, 2007.

B. Jullien and T. Mariotti. Auction and the informed seller problem. *Games and Economic Behavior*, 56(2):225–258, 2006.

M. Kay. *Transportation megaproject procurement: Benefits and challenges for PPPs and alternative delivery strategies, and the resulting implications for Crossrail*. PhD thesis, Massachusetts Institute of Technology, 2009.

A.N. Kolmogorov and S.V. Fomin. *Introductory real analysis*. Dover Publications, 1975.

J.J. Laffont and D. Martimort. *The theory of incentives: The principal-agent model*. Princeton Univ Press, 2002.

J.J. Laffont and J. Tirole. Using cost observation to regulate firm. *Journal of Political Economy*, 94(3):614–641, 1986.

J.J. Laffont and J. Tirole. Auctioning incentive contracts. *Journal of Political Economy*, 95(5):921–937, 1987.

J.J. Laffont and J. Tirole. *A theory of incentives in procurement and regulation*. MIT press, 1993.

S. Li and J. Yu. Bundling decisions in procurement auctions with sequential tasks. *mimeo*, 2011.

D. Martimort and J. Pouyet. To build or not to build: Normative and positive theories of public-private partnerships. *International Journal of Industrial Organization*, 26(2): 393–411, 2008.

A. Mas-Collel, M.D Whinston, and J. Green. *Microeconomic theory*. Oxford university press, 1995.

E. Maskin and J. Tirole. The principal-agent relationship with an informed principal: The case of private values. *Econometrica*, 58(2):379–409, 1990.

E. Maskin and J. Tirole. The principal-agent relationship with an informed principal, II: Common values. *Econometrica*, 60(1):1–42, 1992.

T. Matsuo. On incentive compatible, individually rational, and ex post efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 49(1):189–194, 1989.

R.P. McAfee and J. McMillan. Competition for agency contracts. *RAND Journal of Economics*, 18(2):296–307, 1987.

P. Milgrom. *Putting auction theory to work*. Cambridge University Press, 2004.

P. Milgrom and I. Segal. Envelope theorems for arbitrary choice sets. *Econometrica*, 70 (2):583–601, 2002.

P. Milgrom and C. Shannon. Monotone comparative statics. *Econometrica*, 62(1):157–180, 1994.

K.R. Molenaar and G. Yakowenko. *Alternative project delivery, procurement, and contracting methods for highways.* American Society of Civil Engineers, 2007.

R.B. Myerson. Mechanism design by an informed principal. *Econometrica*, 51(6):1767–1797, 1983.

R.B. Myerson and M.A. Satterthwaite. Efficient mechanisms for bilateral trading. *Journal of Economic Theory*, 29(2):265–281, 1983.

T. Mylovanov and T. Tröger. Informed-principal problems in environments with generalized private values. *Theoretical Economics*, 7(3):465–488, 2012a.

T. Mylovanov and T. Tröger. Mechanism design by an informed principal: The quasi-linear private-values case. *mimeo*, 2012b.

OECD. *Public-private partnerships – In pursuit of risk sharing and value for money.* OECD, 2008.

OECD. *Size of public procurement market.* OECD, 2012. URL `http://www.oecd-ilibrary.org/governance/government-at-a-glance-2011_gov_glance-2011-en`.

J.G. Riley. Silver signals: Twenty-five years of screening and signaling. *Journal of Economic Literature*, 39(2):432–478, 2001.

M.H. Riordan and D.E.M. Sappington. Awarding monopoly franchises. *American Economic Review*, 77(3):375–387, 1987.

M. Rothschild and J. Stiglitz. Equilibrium in competitive insurance markets: An essay on the economics of imperfect information. *Quarterly Journal of Economics*, 90(4): 629–649, 1976.

A. Shneyerov and P. Xu. Price setting with interdependent values. *mimeo*, 2013.

M. Spence. Job market signaling. *Quarterly Journal of Economics*, 87(3):355–374, 1973.

D.M. Topkis. *Supermodularity and complementarity.* Princeton Univ Press, 1998.

M. Tsuruoka. The impact of scoring auctions in public procurement auctions. *mimeo*, 2013.

E.R. Yescombe. *Public-private partnerships: Principles of policy and finance*. Butterworth-Heinemann, 2007.

O. Yilankaya. A note on the seller's optimal mechanism in bilateral trade with two-sided incomplete information. *Journal of Economic Theory*, 87(1):267–271, 1999.