Hitotsubashi Journal of Economics 55 (2014), pp.1-20.

SPATIAL AGGLOMERATION AND DISPERSION: REVISITING THE HELPMAN MODEL

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Received October 2012; Accepted July 2013

Abstract

This paper presents a modified Helpman model (1998) with an added tradable agriculture good, and modifies the manufacturing production function according to Forslid and Ottaviano (2003) to identify all possible spatial configurations of a two-region economy. Moreover, the current work neatly separates the four spatial shaping effects: market size effect, market crowding effect, cost of living effect, and urban congestion effect, and diagrammatically exposes how these forces shape spatial configurations as the degree of trade freeness increases.

Keywords: core-periphery model, agglomeration and dispersion, bell-shaped core-periphery, dispersion black hole
JEL Classification Codes: F12, F22, R12

I. *Introduction*

The pioneering work of Krugman (1991) created a famous core-periphery (CP) model showing an endogenously monotone decreasing relationship between industry sector agglomeration rate and transportation costs. Various modifications of the CP model challenge the monotonic relationship of industrial agglomeration and transportation costs, particularly, as Murata and Thisse (2005) argues that the main real world dispersion force should not be "market crowding effect" only, but that "urban costs" borne by workers concentrating in core region also matter. Given this perspective, Helpman (1998) and Tabuchi (1998) are the most

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important literatures of origin. Fujita and Thisse (2002) mentioned that "positive urban costs", in the form of housing and commuting costs considered by Helpman (1998), Tabuchi (1998), Ottaviano, Tabuchi and Thisse (2002), may generate a bell-shaped relationship between industry sector agglomeration rate and transportation costs, instead of decreasing monotone.[1] Recently, Murata and Thisse (2005) clarify that what really matters for space-economy structure is not just transportation costs of commodities, but also commuting costs borne by workers à la Tabuchi (1998). Süedekum (2006) shows that a CP structure can endogenously emerge in which the core has a higher aggregate cost of living index combining with housing sector and manufacturing sector. Pflüger and Süedekum (2008) use a quasi-linear utility function to analyze the 'bubble-shaped' market equilibrium and social welfare for CP structure which the model is also concerned with housing sector. All these important studies do not further clarify how/why the various core-periphery patterns arise when adding urban costs in the model.

In particular, from an empirical perspective, Helpman (1998) adding a housing sector is generally more suited for empirical validation than the seminal CP model. Several empirical works (directly and indirectly) use the Helpman model to avoid the non-realistic equilibrium of the seminal CP model, where industry completely disappears from the periphery region and to satisfy the real wage equalization assumption (Brakman et al., 2004; Mion, 2004; Hanson, 2005; Ottaviano and Pinelli, 2006; Fingleton, 2007; Redding and Sturm, 2008; Partridge et al., 2009). Moreover, empirical studies show that Helpman (1998) model may be supported with more evidences than the basic Krugman model. As Hanson (2005, p.20) notes: "In all regressions, the data reject the strict of the Krugman model in favor of Helpman's (1998) extension of this model." Although a rich body of empirical research based on the Helpman model has now emerged, Helpman (1998) has not been fully explored from a theoretical perspective.

Conceptually, agglomeration forces (centripetal forces) and dispersion forces (centrifugal forces) clearly determine the spatial economy structure. According to Fujita, Krugman and Venables (1999, chapter 19), agglomeration forces contain linkages, thick labor markets, knowledge spillovers and other pure external economies; dispersion forces contain immobile factors, land, commuting, congestion and other pure diseconomies; namely, both agglomeration and dispersion forces are multiple faceted. This assertion may reasonably claim that builders consider as many forces as possible in CP model construction. However, existing CP models in the literature typically include limited agglomeration and dispersion forces, confined to two or three sectors, and two types of labors, to keep the model solvable or mathematically tractable. Helpman (1998), for example, considered two regions, two goods, two agglomeration forces (the market size effect and the cost-of-living effect), and two dispersion forces (market crowding effect and housing as an urban-cost effect); Tabuchi (1998) introduced commuting cost effect as an urban cost force into the Helpman model. Given this limitation, this work argues that existing CP models still have much room for improvement.

Existing CP models though simple, are still complicated for obtaining complete analytic solutions. Most results have been established under restricted assumptions or analysed by means of numerical simulations. Based on these literature results, the current study argues the difficulty for general readers to arrive at answers for the following questions: (1) What are the

[1] The Helpman model (1998) in its original form is seemingly unable to derive the bell-shaped relationship between agglomeration rate of the industry sector and transportation costs claimed by Fujita and Thisse (2002).

relative strengths of agglomeration and dispersion forces in shaping economic geography? (2) Under what conditions do the following spatial configurations exist: full agglomeration, a black-hole, one breaking point, two breaking points or a bell-shaped core-periphery structure? (3) What are the relationships between various agglomeration-dispersion forces and transportation costs or the degree of trade freeness? This study slightly modifies the Helpman model, rather than greatly extending the CP model, to illustrate possible answers for these complicated spatial equilibrium problems. The simplified demonstration this research offers is hopefully informative for general readers to appreciate CP models.

This paper introduces a shipping-free homogenous tradable agriculture good into the Helpman model and modifies the differentiated manufacture production function a la Forslid and Ottaviano (2003). This modified Helpman model allows clear examination of all possible equilibrium spatial configurations, and neatly separates agglomeration forces (including market size effect and cost-of-living effect) and dispersion forces (including market crowding effect and urban congestion effect), and clarifies the relationships of these forces and the degree of trade freeness. Consequently, the aforementioned problems can be clarified.

The rest of this paper is arranged as follows. Section 2 presents a modified Helpman model (1998). Section 3 examines the long run spatial equilibrium to identify all possible equilibrium spatial configurations in the system. Section 4 diagrammatically illustrates the various equilibrium spatial configurations by means of numerical analysis. Section 5 makes concluding remarks.

II. *The Model*

The model presented below is a modified version of the Helpman model (1998). The modifications include: (1) Introduce a shipping-free tradable homogenous agriculture good into the model, besides the tradable differentiated manufacturing sector and immobile housing spaces (2) Modify the manufacturing production function a la Forslid and Ottaviano (2003). Specifically, this work assumes that (1) There are two regions, denoted by i and j, in the system; (2) Each region produces three kinds of goods, a homogenous agricultural good (A), a horizontally differentiated manufacture good (M) and housing in terms of floor space (F); (3) Two production factors include skilled labor (H) and unskilled labor (L), skilled labors can only work for the manufacturing sector, and can cost-free migrate between the two regions, unskilled labors can work for either the manufacturing sector or the agricultural sector, but are immobile between regions; (4) The agricultural goods market is competitive and freely trades between two regions and employs only unskilled labors; (5) Agriculture production is subject to constant returns to scale with one unit output requiring one unit of unskilled labor only, while manufacturing goods are subject to monopolistic competition with increasing returns to scale and costly trade between two regions; (6) Manufacturing firms employ both skilled and unskilled labor, each uses one unit of skilled labor as fixed cost and β unit of unskilled labor as variable cost; (7) Housing floor stock is fixed and equal in two regions, each region provides α proportion of housing floors for skilled labor and $(1-\alpha)$ proportion of housing floors for unskilled labor; (8) The total number of skilled labors (H) and unskilled labors (L) are fixed, and $H_i+H_j=H$, $L_i=L_j=L/2$.

In this economic system, only skilled labors are mobile between regions, hence only discussing skilled labors' consumption behavior and their utility level is necessary. Skilled labors are identical to the Cobb-Douglas utility function specified as

$$U_i = M_i^{\mu} F_i^{\gamma} A_i^{1-\mu-\gamma}, \tag{1}$$

where A_i is the consumption of agricultural good in region i, M_i is the consumption of manufactured goods in region i, and F_i is consumption of housing floor in region i. M is a differentiated good characterized by

$$M_i \equiv \left[\int_0^{n_i} d_{ii}(c_i)^{\frac{\sigma-1}{\sigma}} \mathbf{d}c_i + \int_0^{n_j} d_{ji}(c_j)^{\frac{\sigma-1}{\sigma}} \mathbf{d}c_j \right]^{\frac{\sigma}{\sigma-1}}, \quad 0<\mu,\ \gamma<1<\sigma, \tag{2}$$

where n_i and n_j are the manufactured varieties produced in region i and j , respectively; $d(c_i)$ and $d(c_j)$ are consumptions of varied manufactured goods c_i and c_j, respectively; σ is substitution elasticity between different varieties of manufactured goods, and also is demand elasticity of any variety.

The budget constraint for a skilled labor in region i is given by

$$\int_0^{n_i} p_{ii}(c_i)\ d_{ii}(c_i)\ \mathbf{d}c_i + \int_0^{n_j} p_{ji}(c_j)\ d_{ji}(c_j)\ \mathbf{d}c_j + p_{iF}F_i + p_{iA}A_i = w_{iH}, \tag{3}$$

where w_{iH} is the endogenous wage rate of skilled labor in region i; p_{iF} is the rental rate of housing floor space, p_{iA} is the price of the agricultural good, $d_{ji}(c_j)$ is the demand of variety c_j produced in location j by a skilled labor in location i, $p_{ji}(c_j)$ is the price of variety c_j produced in region j sold in i, $d_{ii}(c_i)$ and $p_{ii}(c_i)$ are defined in the same way.

Following the standard procedures in CP literature (for example, Fujita, Krugman and Venables, 1999; Tabuchi, 1998), this work obtains skilled labor's demand for c_i and c_j in region i as follows:

$$d_{ji}(c_j) = \frac{p_{ji}(c_j)^{-\sigma}}{\mathbb{P}_i^{1-\sigma}} \mu\, w_{iH}\ ;\ d_{ii}(c_i) = \frac{p_{ii}(c_i)^{-\sigma}}{\mathbb{P}_i^{1-\sigma}} \mu\, w_{iH}, \tag{4}$$

where $\mathbb{P}_i$ and $\mathbb{P}_j$ are the locally manufactured price index of region i and j, respectively. They are the appropriately chosen price index of differentiated goods in terms of a numeraire and are associated with (2)

$$\mathbb{P}_i = \left[n_i p_{ii}(c_i)^{1-\sigma} + n_j p_{ji}(c_j)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \quad ; \quad \mathbb{P}_j = \left[n_i p_{ij}(c_i)^{1-\sigma} + n_j p_{jj}(c_j)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{5}$$

$$M_i = \mu \frac{w_{iH}}{\mathbb{P}_i}\ ; \quad F_i = \gamma \frac{w_{iH}}{p_{iF}} = \frac{\alpha F}{H_i}\ ; \quad A_i = (1-\mu-\gamma)\frac{w_{iH}}{p_{iA}}. \tag{6}$$

Substituting (6) into (1), we obtain the indirect utility function of skilled labor in both regions as

$$u_{iH} = \mu^{\mu}(1-\mu-\gamma)^{1-\mu-\gamma}(\alpha F)^{\gamma} \frac{(w_{iH})^{1-\gamma}}{(\mathbb{P}_i)^{\mu}(H_i)^{\gamma}(p_{iA})^{1-\mu-\gamma}}, \tag{7-1}$$

$$u_{jH}=\mu^{\mu}(1-\mu-\gamma)^{1-\mu-\gamma}(\alpha F)^{\gamma}\frac{(w_{jH})^{1-\gamma}}{(\mathbb{P}_j)^{\mu}(H_j)^{\gamma}(p_{jA})^{1-\mu-\gamma}}. \tag{7-2}$$

The indirect utility function in (7-1) or (7-2) adds a new item to the list of location effects. As the number of skilled labor (H_i) rises, its utility level decreases, which exerts a dispersion force in the system.

The manufacturing sector is a standard Dixit-Stiglitz monopolistic competition sector, where manufacturing firms employ both skilled and unskilled labor to produce differentiated goods subject to increasing returns to scale. Together with assumed costless differentiation this ensures that each firm will produce only its own variety, that is, a one-to-one relation between firms and varieties. Using Forslid and Ottaviano (2003), total production of each variety $X_i(c_i)$ units requires one unit of skilled labor as a fixed input and $\beta X_i(c_i)$ units of unskilled labor as a marginal input. Each firm uses the same production technology. The production for maximum profit of a firm in location i, is thus given as

$$\underset{p_{ii},\, p_{ij}}{\text{Max}} \quad \pi_i(c_i)=p_{ii}(c_i)d_{ii}^{T}(c_i)+p_{ij}(c_i)d_{ij}^{T}(c_i)-\left\{w_{iH}+w_{iL}\beta\left[d_{ii}^{T}(c_i)+\tau d_{ij}^{T}(c_i)\right]\right\}, \tag{8}$$

where $d_{ii}^{T}(c_i)$ and $d_{ij}^{T}(c_i)$ are the total demands of c_i in region i and j, respectively; $\tau\, d_{ij}^{T}(c_i)$ represents c_i supplied by region j including the "melting iceberg cost" (Samuelson, 1954). Melting iceberg cost means that for one unit of the manufactured good available for consumption in other region, $\tau\in[1, +\infty)$ units of that good must be shipped. Hence, under the market clearing condition, $X_i(c_i)=d_{ii}^{T}(c_i)+\tau d_{ij}^{T}(c_i)$, i.e. the total production of each variety c_i is equal to total demand of the corresponding variety. Moreover, under (4), total demand for each variety c_i in region i is

$$d_{ii}^{T}(c_i)=\frac{p_{ii}(c_i)^{-\sigma}}{\mathbb{P}_i^{1-\sigma}}\mu\, Y_i^{N} \quad ; \quad d_{ij}^{T}(c_i)=\frac{p_{ij}(c_i)^{-\sigma}}{\mathbb{P}_j^{1-\sigma}}\mu\, Y_j^{N}, \tag{9}$$

where Y_i^N and Y_j^N are the aggregate incomes of region i and j, respectively, each consists of skilled labor wage (w_{iH}, w_{jH}) and unskilled labor wages ($w_{iL}=w_{jL}=1$):

$$Y_i^N=\frac{1}{2}L+w_{iH}H_i \quad ; \quad Y_j^N=\frac{1}{2}L+w_{jH}H_j. \tag{10}$$

According to (8), we obtain:

$$p_{ii}(c_i)=p_{jj}(c_j)=w_{iL}\frac{\beta\sigma}{\sigma-1}=\frac{\beta\sigma}{\sigma-1} \quad ; \quad p_{ij}(c_i)=p_{ji}(c_j)=w_{iL}\frac{\tau\,\beta\sigma}{\sigma-1}=\frac{\tau\,\beta\sigma}{\sigma-1}. \tag{11}$$

Plugging (11) and skilled labor market clearing condition[2] into (5), the locally manufactured price indexes become

$$\mathbb{P}_i=\frac{\beta\sigma}{\sigma-1}H^{\frac{1}{1-\sigma}}\left[h+\phi(1-h)\right]^{\frac{1}{1-\sigma}} \quad ; \quad \mathbb{P}_j=\frac{\beta\sigma}{\sigma-1}H^{\frac{1}{1-\sigma}}\left[1-h+\phi h\right]^{\frac{1}{1-\sigma}}, \tag{12}$$

[2] According to Forslid and Ottaviano (2003), the fixed input requirement is given as one unit of skilled labor, the number of manufacturing firms in equilibrium is determined under skilled labor market clearing condition: $n_i=H_i$; $n_j=H_j$, so that the number of firms in a region is the same as its skilled residents in that region.

where $\phi \equiv \tau^{1-\sigma} \in (0, 1]$ denotes the freeness of trade, which is equal to one when transportation cost is non-existent (trade is free), and equals zero when trade of manufactured goods is impossible. $h \equiv (H_i/H) \in [0, 1]$ is the share of skilled worker that resides in region i, H is total endowments of skilled labor in the whole system. Plugging (9), (11) and (12) into the market clearing condition, the total production of each variety produced in region i and j is

$$X_i = \frac{\mu(\sigma-1)}{\beta\sigma H}\left[\frac{Y_i^N}{h+\phi(1-h)} + \frac{\phi Y_j^N}{1-h+\phi h}\right];\ X_j = \frac{\mu(\sigma-1)}{\beta\sigma H}\left[\frac{Y_j^N}{1-h+\phi h} + \frac{\phi Y_i^N}{h+\phi(1-h)}\right]. \quad (13)$$

Due to free entry and exit in a monopolistically competitive market, each firm has no profits in the equilibrium. Hence, the nominal wages of skilled labor in two regions are determined as follows:

$$w_{iH} = \frac{\beta}{\sigma-1} X_i \quad ; \quad w_{jH} = \frac{\beta}{\sigma-1} X_j. \quad (14)$$

Plugging (13) into (14) gives

$$w_{iH} = \frac{\mu}{\sigma H}\left[\frac{Y_i^N}{h+\phi(1-h)} + \frac{\phi Y_j^N}{1-h+\phi h}\right] \quad ; \quad w_{jH} = \frac{\mu}{\sigma H}\left[\frac{Y_j^N}{1-h+\phi h} + \frac{\phi Y_i^N}{h+\phi(1-h)}\right]. \quad (15)$$

Finally, plugging (10) into (15) gives a system of two equations in w_{iH} and w_{jH}, which can be solved simultaneously to obtain the equilibrium skilled wages as follows:

$$w_{iH} = \frac{L}{2H}\frac{\mu}{\sigma-\mu}\frac{2\sigma\phi h + [\sigma-\mu+(\sigma+\mu)\phi^2](1-h)}{\sigma\phi(h^2+(1-h)^2) + [\sigma-\mu+(\sigma+\mu)\phi^2]h(1-h)}, \quad (16\text{-}1)$$

$$w_{jH} = \frac{L}{2H}\frac{\mu}{\sigma-\mu}\frac{2\sigma\phi(1-h) + [\sigma-\mu+(\sigma+\mu)\phi^2]h}{\sigma\phi(h^2+(1-h)^2) + [\sigma-\mu+(\sigma+\mu)\phi^2]h(1-h)}. \quad (16\text{-}2)$$

III. *Long-run Spatial Equilibrium*

The long-run equilibrium of this two-region system is determined by the condition that the skilled-labor in each region achieves the same maximal attainable level of utility. That is,

$$\Delta u \equiv u_{iH} - u_{jH} = 0. \quad (17)$$

Using (7-1) and (7-2), the difference in utility levels between two regions can be written as

$$\Delta u(h, \phi) = \mu^{\mu}(1-\mu-\gamma)^{1-\mu-\gamma}(\alpha F)^{\gamma} H^{-\gamma}\left(\frac{(w_{iH})^{1-\gamma}}{(\mathbb{P}_i)^{\mu}(h)^{\gamma}} - \frac{(w_{jH})^{1-\gamma}}{(\mathbb{P}_j)^{\mu}(1-h)^{\gamma}}\right). \quad (18)$$

Substituting (12), (16-1) and (16-2) into (18), we obtain

$$\Delta u(h, \phi, \sigma, \mu, \gamma) = \eta \cdot \frac{\left(\frac{L}{2H}\frac{\mu}{\sigma-\mu}\right)^{1-\gamma}\left(\frac{\beta\sigma}{\sigma-1}\right)^{-\mu} H^{\frac{-\mu}{1-\sigma}}}{\left\{\sigma\phi\left(h^2+(1-h)^2\right) + \left[\sigma-\mu+(\sigma+\mu)\phi^2\right]h(1-h)\right\}^{1-\gamma}} \cdot Z(h, \phi, \sigma, \mu, \gamma), \quad (19)$$

TABLE 1. FULL-AGGLOMERATION WITH RESPECT TO ϕ

States	Existence of Full Agglomeration	Reason
$0<\phi<1$	No	$Z(h\to 0,\phi)=+\infty$; $Z(h\to 1,\phi)=-\infty$.
$\phi=0$	Yes, if $\sigma<1+\mu/\gamma$;	$\because$ If $\sigma<1+\mu/\gamma$, then $\lim_{h\to 0} Z(\phi=0)=0$; $\lim_{h\to 1} Z(\phi=0)=0$.
	No, if $\sigma>1+\mu/\gamma$	$\because$ If $\sigma>1+\mu/\gamma$, then $\lim_{h\to 0} Z(\phi=0)=+\infty$; $\lim_{h\to 1} Z(h\to 1,\phi=0)=-\infty$.
$\phi=1$	No	$\lim_{h\to 0} Z(\phi=1)=+\infty$; $\lim_{h\to 1} Z(\phi=1)=-\infty$.

Note: For details, please see Appendix A.

where $\eta\equiv\mu^{\mu}(1-\mu-\gamma)^{1-\mu-\gamma}(\alpha F)^{\gamma}H^{-\gamma}$ is a positive bundling parameter, and

$$Z(h,\phi,\sigma,\mu,\gamma)\equiv\frac{\{2\sigma\phi h+[\sigma-\mu+(\sigma+\mu)\phi^2](1-h)\}^{1-\gamma}}{[h+\phi(1-h)]^{\frac{\mu}{1-\sigma}}\cdot h^{\gamma}}-\frac{\{2\sigma\phi(1-h)+[\sigma-\mu+(\sigma+\mu)\phi^2]h\}^{1-\gamma}}{(1-h+\phi h)^{\frac{\mu}{1-\sigma}}\cdot(1-h)^{\gamma}}. \quad (20)$$

On the right side of (19), η is common to the two regions, which does not affect the utility difference between two regions, the first term in parenthesis is positive, therefore utility difference depends on the second term in parenthesis. That is, utility difference of skilled labor between two regions is determined by $Z(h,\phi,\sigma,\mu,\gamma)$, which in turn is determined by h,ϕ and the three exogenously given parameters σ,μ and γ. The rest of this section first examines possible full-agglomeration equilibrium, and then examines the complete set of interior solutions for core-periphery configurations.

1. Full Agglomeration Equilibrium

The CP-Model literature documents that the fully agglomerated configuration ($h=0$ or $h=1$) is the long-run equilibrium iff $Z(h=0,\phi)<0$ or $Z(h=1,\phi)>0$, and once full agglomeration arises in either region, it is always a stable spatial equilibrium. This study examines the existence of fully agglomerated configurations with the whole range of ϕ values in Table 1 (For details, please see Appendix A) which shows no full-agglomeration equilibrium for the following three cases: $0<\phi<1$, $\phi=1$, and $\phi=0$ with $\sigma>1+\mu/\gamma$. Full-agglomeration equilibrium exists only under the conditions of $\phi=0$ and $\sigma<1+\mu/\gamma$. In this modified model, ϕ is defined as $\phi\equiv\tau^{1-\sigma}$, and $\tau\in[1,+\infty)$, so $\tau=1\Rightarrow\phi=1$; $\tau=\infty\Rightarrow\phi$ approaches zero. That is, ϕ cannot be zero, and hence the full-agglomeration equilibrium does not exist in our model setting. Substitution elasticity between different varieties of manufactured goods (σ) is not sufficiently large (specifically, $\sigma<1+\mu/\gamma$), however, the model approaches full-agglomeration equilibrium when manufactured goods from one region to another is extremely costly.

2. Core-Periphery Configurations—Symmetric and Asymmetric Configurations

This section examines symmetric and asymmetric spatial configurations of this two-region system. Findings show that $Z(h, \phi, \sigma, \mu, \gamma)=0$ has at most three interior solutions for $0<h<1$. One of the three interior solutions can be straightforwardly verified from (19) and (20). Letting $h=1/2$ in (20) gives $Z(h, \phi, \sigma, \mu, \gamma)=0$; that is, $h=1/2$ is an equilibrium. This even geographical distribution of skilled workers and manufacturing firms is an equilibrium independent of the parameters $(\tau, \sigma, \mu, \gamma)$. This solution is stable whenever $Z_h(h=1/2, \phi, \sigma, \mu, \gamma)<0$, where the subscript denotes the partial derivative with respect to h. Forslid and Ottaviano (2003) show the stability of this symmetric solution.

Apart from $h = 1/2$ at most two other interior solutions exist for $h\in(0,1)$ that are symmetrically placed around it. To illustrate, this study takes the partial derivative of $Z(.)$ with respect to h and evaluates it at $h=1/2$ as follows:

$$\left.\frac{\partial Z}{\partial h}\right|_{h=\frac{1}{2}}=-\left(\frac{2^{\frac{2\gamma\mu}{\sigma-1}+1}(\phi+1)^{\frac{\mu-\gamma}{\sigma-1}}}{[\sigma-\mu+(\sigma+\mu)\phi]^{\gamma}\sigma^{1-\gamma}(\sigma-1)}\right)\cdot\Phi(\phi\ , \sigma, \mu, \gamma), \tag{21}$$

where

$$\begin{aligned}\Phi(\phi\ ; \sigma, \mu, \gamma)&\equiv\mu^2(\phi-1)^2+\mu(2\sigma-1)(\phi^2-1)+\sigma(\sigma-1)[1+\phi(\phi+4\gamma-2)]\\ &\equiv a\phi^2+b\phi+c,\end{aligned} \tag{22}$$

and

$$\begin{aligned}a&\equiv-\mu+\mu^2-\sigma+2\mu\sigma+\sigma^2=(\mu+\sigma)(\sigma-1+\mu)>0,\\ b&\equiv-2\mu^2+2\sigma-4\gamma\sigma-2\sigma^2+4\gamma\sigma^2=-2[\mu^2+\sigma(\sigma-1)(1-2\gamma)],\\ c&\equiv\mu+\mu^2-\sigma-2\mu\sigma+\sigma^2=(\mu-\sigma)(-\sigma+1+\mu).\end{aligned} \tag{23}$$

Inspection of the RHS of (21), shows that the term in parenthesis is positive, but $\Phi(\phi, \sigma, \mu, \gamma)$ is indefinite. Therefore, the possible break points (ϕ_{b1} and ϕ_{b2}) can be solved by letting $\Phi(\phi, \sigma, \mu, \gamma)=0$. The solutions of ϕ_{b1} and ϕ_{b2} are given below:

$$\phi_{b1}=\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{\mu^2+\theta(1-2\gamma)-\sqrt{4\theta(1-\gamma)[\mu^2-\gamma\theta]+\mu^2}}{(\sigma+\mu-1)(\mu+\sigma)}, \tag{24}$$

$$\phi_{b2}=\frac{-b+\sqrt{b^2-4ac}}{2a}=\frac{\mu^2+\theta(1-2\gamma)+\sqrt{4\theta(1-\gamma)[\mu^2-\gamma\theta]+\mu^2}}{(\sigma+\mu-1)(\mu+\sigma)}, \tag{25}$$

where $\theta\equiv\sigma(\sigma-1)\in(0, \infty)$.

Under the complicated interaction of the parameters (σ, μ, γ), the solutions of ϕ_{b1} and ϕ_{b2} can be positive, negative, or imaginary. According to (23), a is always positive, $\Phi(\phi=1)=a+b+c=4\gamma\sigma(\sigma-1)>0$ and the sign of $\Phi(\phi=0)=c$ is indefinite, then we have the following results:

(1) Both ϕ_{b1} and ϕ_{b2} are smaller than 1 if $b^2-4ac>0$.
(2) If $b^2-4ac>0$ and $c<0$, $\phi_{b1}<0$ and $\phi_{b2}\in(0, 1)$ hold.
(3) If $b^2-4ac>0$ and $c>0$, there are two possible cases. One is ϕ_{b1} and $\phi_{b2}\in(0, 1)$; the other is ϕ_{b1} and $\phi_{b2}<0$.

Moreover, b can be positive or negative, and $\sqrt{b^2-4ac}$ can be imaginary, so there are *six*

TABLE 2. ALL POSSIBLE SET OF SOLUTIONS FOR ϕ_{b1} AND ϕ_{b2}

Cases	a	b	c	b^2-4ac	ϕ_{b1} and ϕ_{b2}	Notes
(1)	$a>0$	$b>0$	$c>0$	$b^2-4ac<0$	Imaginary roots	Dispersion black hole
(2)	$a>0$	$b>0$	$c>0$	$b^2-4ac>0$	$\phi_{b1}<0$, $\phi_{b2}<0$	Dispersion black hole
(3)	$a>0$	$b<0$	$c>0$	$b^2-4ac<0$	Imaginary roots	Dispersion black hole
(4)	$a>0$	$b<0$	$c>0$	$b^2-4ac>0$	$\phi_{b1}\in(0, 1)$, $\phi_{b2}\in(0, 1)$	Two break points
(5)	$a>0$	$b>0$	$c<0$	$c<0\Rightarrow\sigma<1+\mu$ $\Rightarrow\sqrt{b^2-4ac}>b>0$	$\phi_{b1}<0$, $\phi_{b2}\in(0, 1)$	One break point
(6)	$a>0$	$b<0$	$c<0$	$c<0\Rightarrow\sigma<1+\mu$ $\Rightarrow\sqrt{b^2-4ac}>\lvert b\rvert>0$	$\phi_{b1}<0$, $\phi_{b2}\in(0, 1)$	One break point

Notes: (1) From (23), a is always positive, so that there are only six possible cases of b, c and b^2-4ac required to examine the existence of interior solutions. (2) $c>0$ is equivalent to $\sigma>1+\mu$; $c<0$ is equivalent to $\sigma<1+\mu$.

possible cases of b, c, and $\sqrt{b^2-4ac}$ required for examination for the existence of interior solutions (see Table 2):

(1) $a>0$, $b>0$, $c>0$, and $b^2-4ac<0$; that is, both ϕ_{b1} and ϕ_{b2} are imaginary roots. This is the case of no break points.
(2) $a>0$, $b>0$, $c>0$, and $b^2-4ac>0$; that is, both ϕ_{b1} and ϕ_{b2} are negative. This is also the case of no break points.
(3) $a>0$, $b<0$, $c>0$, and $b^2-4ac<0$, that is, both ϕ_{b1} and ϕ_{b2} are imaginary roots. This is again the case of no break points.
(4) $a>0$, $b<0$, $c>0$, and $b^2-4ac>0$, that is, both ϕ_{b1} and ϕ_{b2} are positive. This is the case that ϕ_{b1} and ϕ_{b2} are two break points provided that $\phi_{b1}\in(0,1)$, $\phi_{b2}\in(0,1)$.
(5) $a>0$, $b>0$, $c<0$. Since $c<0\Rightarrow\sigma<1+\mu\Rightarrow\sqrt{b^2-4ac}>b>0$, $\phi_{b1}<0$, $\phi_{b2}>0$, verified from (24) and (25)). That is, ϕ_{b1} is not a break point, ϕ_{b2} is a break point provided that $\phi_{b2}\in(0,1)$.
(6) $a>0$, $b<0$, $c<0$. This case is similar to case (5) that $\phi_{b1}<0$, $\phi_{b2}\in(0,1)$ since $c<0\Rightarrow\sigma<1+\mu\Rightarrow\sqrt{b^2-4ac}>\lvert b\rvert>0$, That is, ϕ_{b1} is not a break point, but ϕ_{b2} is a break point provided that $\phi_{b2}\in(0,1)$.

Among the above six cases, no break points exist in cases (1)-(3). This study defines these cases as the ***dispersion black hole***, in which dispersion forces are always dominant to agglomeration forces, then the symmetric distribution is a persistently stable equilibrium. Case (4) has two break points which generates a bell-shaped core-periphery structure. Cases (5) and (6) each have just one break point, shown by Helpman (1998).

The solutions of ϕ_{b1} and ϕ_{b2} in the above cases are determined by the values of $a>0$, b, c, $a+b+c>0$ and b^2-4ac. The b and c could be positive or negative, but b^2-4ac is a complicated high-power equation consisting of three parameters (γ, μ and σ). Therefore, it is not easy to derive the ranges of γ, μ and σ which generate the interior solutions (Cases (4)-(6)) and dispersion black holes (Cases (1)-(3)). To determine the ranges or relations of parameters, which lead to the six cases, this work employs the 'Inequality Solve' package of Mathematica 5.0 to solve the systems of inequalities list in Table 2. The solutions provide the conditions in terms of the relationships of γ, μ and σ or $\theta\equiv\sigma(\sigma-1)$ for resulting 'core-periphery

TABLE 3. CONDITIONS FOR CORE-PERIPHERY CONFIGURATIONS AND DISPERSION BLACK HOLE

Type	Break-point	Conditions
I	$0>\phi_{b1}$ $0<\phi_{b2}<1$ One break point	$0<\gamma<1,\quad 0<\mu<1-\gamma,\quad 1<\sigma<1+\mu\quad$ (i.e. $0<\theta<\mu+\mu^2$)
II-1	$\phi_{b1}<0$ $\phi_{b2}<0$ Dispersion black hole	(C.1) $\frac{3-\sqrt{3}}{2}<\gamma<1,\quad 0<\mu<1-\gamma,\quad \mu+\mu^2<\theta<\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$ (C.2) $\frac{1}{2}<\gamma\leq\frac{3-\sqrt{3}}{2},\quad 0<\mu<\frac{1-2\gamma}{2\gamma-2},\quad \mu+\mu^2<\theta<\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$
II-2	$0<\phi_{b1}<1$ $0<\phi_{b2}<1$ Two break points	(C.3) $\frac{1}{2}<\gamma<\frac{3-\sqrt{3}}{2},\quad \frac{1-2\gamma}{2\gamma-2}<\mu<1-\gamma,\quad \mu+\mu^2<\theta<\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$ (C.4) $0<\gamma\leq\frac{1}{2},\quad 0<\mu<1-\gamma,\quad \mu+\mu^2<\theta<\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$
III	Two imaginary roots: Dispersion black-hole	$0<\gamma<1,\quad 0<\mu<1-\gamma,\quad \theta>\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$

Notes: (1) $\theta\equiv\sigma(\sigma-1)$. Since the fourth power equation of σ cannot be solved analytically, we use θ to define a lower power equation of σ to make it solvable.

(2) In Forslid and Ottaviano (2003), when $\sigma<1+\mu$ the system falls into "black-hole condition", meaning that the dispersion force is always dominated by agglomeration force, and the symmetric outcome is never stable.

(3) $\theta=\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}\quad\Rightarrow\quad b^2-4ac=0.$

configurations' and 'dispersion black hole' shown in Table 3.

According to Table 2, the solutions of ϕ_{b1} and ϕ_{b2} are four types. They are: (1) one interior solution of break point ϕ_{b2} (cases (5) and (6)) (corresponding to type I of Table 3), (2) two interior solutions of break point (case (4)) (corresponding to type II-2 of Table 3), (3) dispersion black hole of two negative solutions (case (2)) (corresponding to type II-1 of Table 3), and (4) dispersion black hole of two imaginary solutions (cases (1) and (3)) (corresponding to type III of Table 3). To derive the results of Table 3, we solve the following four systems of inequality condition respectively under $\mu, \gamma\in(0, 1)$ and $\mu+\gamma<1$:

(1) **Type I**: there exists one break point ($0<\phi_{b2}<1$), if and only if $c<0$, which is equivalent to $1<\sigma<1+\mu$ (i.e. $0<\theta<\mu+\mu^2$ where $\theta\equiv\sigma(\sigma-1)$).
(2) **Type II-1**: both ϕ_{b1} and ϕ_{b2} are negative, if $b>0$ (i.e. $-2[\mu^2+\theta(1-2\gamma)]>0$) and $c>0$ (i.e. $\theta>\mu+\mu^2$) and $b^2-4ac>0$ (i.e. $4\mu^2[4\theta(1-\gamma)+1]-16\theta^2\gamma(1-\gamma)>0$).
(3) **Type II-2**: there exists two break points ($0<\phi_{b1}<1$ and $0<\phi_{b2}<1$), if $b<0$ (i.e. $-2[\mu^2+\theta(1-2\gamma)]<0$) and $c>0$ (i.e. $\theta>\mu+\mu^2$) and $b^2-4ac>0$ (i.e. $4\mu^2[4\theta(1-\gamma)+1]-16\theta^2\gamma(1-\gamma)>0$).
(4) **Type III**: both ϕ_{b1} and ϕ_{b2} are imaginary, if $c>0$ (i.e. $\theta>\mu+\mu^2$) and $b^2-4ac<0$ (i.e. $4\mu^2[4\theta(1-\gamma)+1]-16\theta^2\gamma(1-\gamma)<0$).

According to analytical solutions, we obtain the following results under $\mu, \gamma\in(0, 1)$ and $\mu+\gamma<1$ (see Table 3, Figures 1 and 2). Figure 1 shows the total solution sets which are divided into three sub-sets by θ (the horizontal axis) under the restrictions of $\mu, \gamma\in(0, 1)$ and

FIG. 1. THE ROLE OF (σ, μ, γ) IN SHAPING SPATIAL CONFIGURATIONS

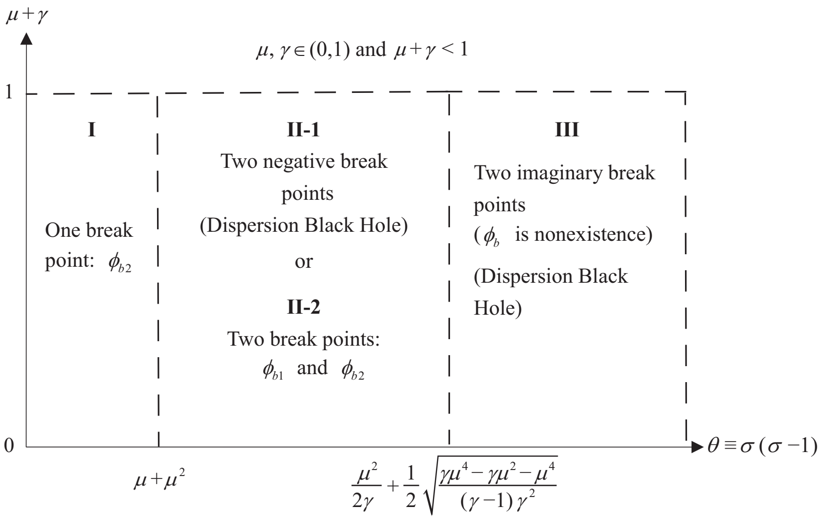
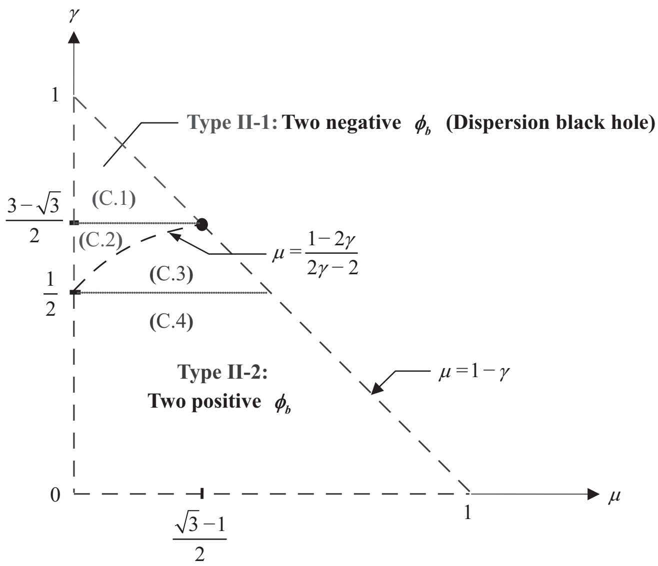
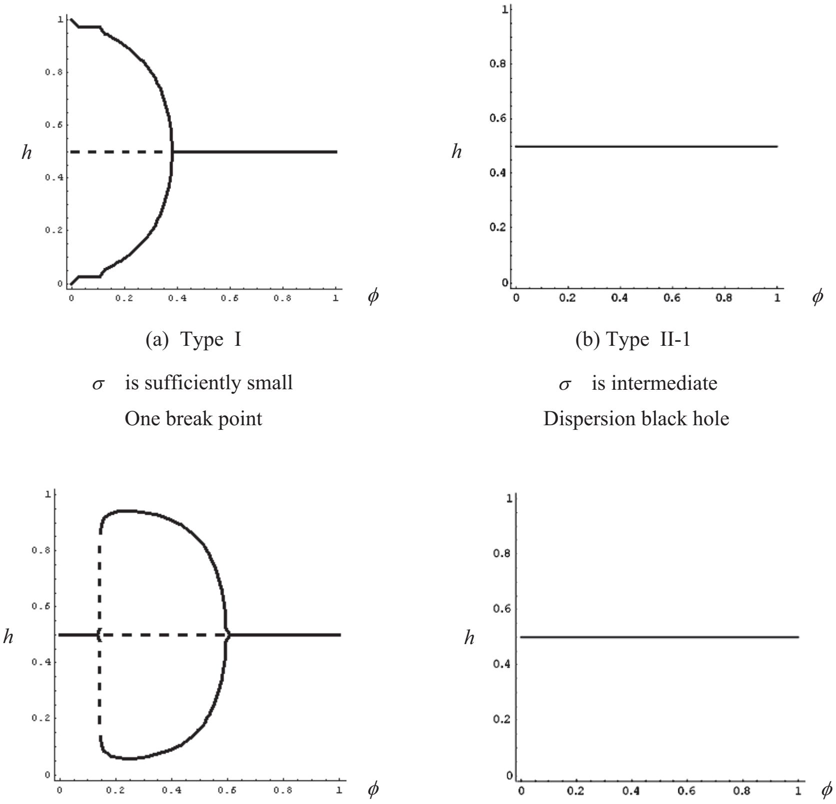


FIG. 2. THE SOLUTION SET OF ϕ_b UNDER $\mu+\mu^2<\theta<\frac{\mu^2}{2\gamma}+\frac{1}{2}\sqrt{\frac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$



$\mu+\gamma<1$ (the vertical axis). Figure 2 shows that how to distinguish the type II-1 and type II-2 under the common range of θ.

FIG. 3. BIFURCATION DIAGRAMS OF THE SPATIAL CONFIGURATIONS



(a) Type I

σ is sufficiently small

One break point

(b) Type II-1

σ is intermediate

Dispersion black hole

(c) Type II-2

σ is intermediate

Two break Points

(d) Type III

σ is sufficiently large

Dispersion black hole

(1) If $0<\theta<\mu+\mu^2$, then there exists one break point, $\phi_{b2}\in(0, 1)$.

(2) If $\mu+\mu^2<\theta<\dfrac{\mu^2}{2\gamma}+\dfrac{1}{2}\sqrt{\dfrac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$, then there exist two possible types of solution (labeled by (II-1) and (II-2) respectively, Figure 1). One is the existence of a *dispersion black hole*, the other is the existence of *two break points* depending on the relative values of γ and μ. According to analytical solutions, we further clarify the relative values of γ and μ which lead to dispersion black hole or two break points. The results are showed in Figure 2. Under the restrictions of μ, $\gamma\in(0, 1)$, $\mu+\gamma<1$ and $\mu+\mu^2<\theta<\dfrac{\mu^2}{2\gamma}+\dfrac{1}{2}\sqrt{\dfrac{\gamma\mu^4-\gamma\mu^2-\mu^4}{(\gamma-1)\gamma^2}}$, if γ is sufficient large and μ is sufficient small, there exist the dispersion black hole (type II-1); conversely, if γ is sufficient small and μ is sufficient large, there exist two break points (type II-2).

(3) If $\theta > \frac{\mu^2}{2\gamma} + \frac{1}{2}\sqrt{\frac{\gamma\mu^4 - \gamma\mu^2 - \mu^4}{(\gamma-1)\gamma^2}}$, then there exists the dispersion black hole.

Moreover, the following Figure 3 depicts the spatial configurations of the solutions with respect to trade freeness ($\phi \equiv \tau^{1-\sigma} \in (0, 1]$). The current study clarifies bifurcation of the spatial configurations in this two-region system. Along with the frequently observed one break point configuration, this study demonstrates the existence of the two break-point configuration, and the existence of the dispersion black hole. Although the existence of dispersion black hole clearly refers to dispersion forces prevailingly dominant to agglomeration forces, current literature does not formally examine this relationship.

Summing up the above analysis, this investigation concludes that: (1) The even geographical distribution of skilled workers ($h=1/2$) is a stable equilibrium, independent of the parameters (τ, σ, μ, γ). (2) Apart from $h=1/2$ there exist at most two interior solutions of trade freeness ϕ, that is, one break point and two break points solutions. (3) If substitution elasticity between different varieties of manufactured goods (σ) is sufficiently small, one break point exists, and the spatial configuration is toward dispersion (Figure 3(a)). (4) If σ is sufficiently large, no break point exists, and the spatial configurations are dispersion black hole (Figure 3 (d)). (5) If the value of σ is intermediate, then dispersion black hole exists if γ is sufficiently large and μ is sufficiently small (Figure 3(b)); conversely, two break points exist if γ is sufficiently small and μ is sufficiently large (Figure 3(c)). (6) Increasing the degree of trade freeness (i.e. increases the value of ϕ ($\phi \in [0,1]$) eventually resulting in the dispersion configuration.

IV. *Diagrammatic Exposition of the Effects of Agglomeration and Dispersion Forces*

The spatial configurations of the two-region system are determined by relative magnitudes of agglomeration forces and dispersion forces. This paper employs two agglomeration forces and two dispersion forces embodied in the model. They are:

(1) Market Size Effect (MSE): An agglomeration force generated from expanding local expenditure due to increased skilled labor wage (i.e. $\partial w_{iH}/\partial h$).
(2) Market Crowding Effect (MCE): A dispersion effect also resulting from the increased number of skilled labors and hence manufacturing firms ($\partial w_{iH}/\partial h$). Due to high transport cost, products mainly sell in the domestic market, or the own or cross price elasticity of demand for manufactures is large because a firm's demand is quite sensitive to the price index (Forslid and Ottaviano, 2003).
(3) Cost-of-Living Effect (CLE): An agglomeration force resulting from lowering the locally manufactured price index (i.e. $-\partial \mathbb{P}_i/\partial h$).
(4) Urban Congestion Effect (UCE): A dispersion force resulting from average living space with increased skilled workers (h increase).

This section separates the four forces to illustrate the effects of agglomeration forces and dispersion forces on shaping spatial configurations of the economy. Symmetry in this two-region system (raised utility in region i correspondingly reduces the utility in region j, and vice versa (Baldwin et al., 2003)), allows us to limit our investigation to one of the regions.

For exposition, this study omits the common term of indirect utility function (7), i.e. $\mu^{\mu}(1-\mu-\gamma)^{1-\mu-\gamma}(\alpha F)^{\gamma}$, and makes a monotonic transformation of the remaining term as follow:

$$V_{iH}\equiv ln\left(\frac{(w_{iH})^{1-\gamma}}{(\mathbb{P}_i)^{\mu}(h)^{\gamma}}\right)=(1-\gamma)ln(w_{iH})-\mu\, ln(\mathbb{P}_i)-\gamma ln(h). \tag{26}$$

Differentiating (26) with respect to h gives the various agglomeration forces and dispersion forces:

$$\begin{aligned}\frac{\partial V_{iH}}{\partial h}&=\left((1-\gamma)\times\underbrace{\frac{\partial w_{iH}}{\partial h}}_{\text{MSE vs. MCE}}\times\frac{1}{w_{iH}}\right)+\left(\mu\times\underbrace{\frac{-\partial\mathbb{P}_i}{\partial h}}_{\text{CLE}}\times\frac{1}{\mathbb{P}_i}\right)-\left(\gamma\times\underbrace{\frac{1}{h}}_{\text{UCE}}\right)\\&=(1-\gamma)(\text{MSE}-\text{MCE})\frac{1}{w_{iH}}+\mu(\text{CLE})\frac{1}{\mathbb{P}_i}-\gamma(\text{UCE})\\&=\underbrace{\left[(1-\gamma)(\text{MSE})\frac{1}{w_{iH}}+\mu(\text{CLE})\frac{1}{\mathbb{P}_i}\right]}_{\text{Agglomeration forces}}-\underbrace{\left[(1-\gamma)(\text{MCE})\frac{1}{w_{iH}}+\gamma(\text{UCE})\right]}_{\text{Dispersion forces}}.\end{aligned} \tag{27}$$

The $\partial w_{iH}/\partial h$ contains MSE and MCE. We treat the positive terms of $\partial w_{iH}/\partial h$ as MSE, and the negative terms of $\partial w_{iH}/\partial h$ as MCE. The four forces are derived and presented below (for detailed derivation of the four forces, please see Appendix B):

$$\text{MSE}\equiv\lambda\cdot\left\{\frac{2(h\mu+\sigma)(\mu+h\sigma)+\sigma[\sigma+4h(\mu+h\sigma)]\phi+2[(1+h^2)\mu^2+2h\sigma^2]\phi^2+(1+4h^2)\sigma(\mu+\sigma)\phi^3+2h(\mu+\sigma)^2\phi^4}{[(h-1)h(\mu-\sigma)+[1+2(h-1)h]\sigma\phi-(h-1)h(\mu+\sigma)\phi^2]^2}\right\},$$

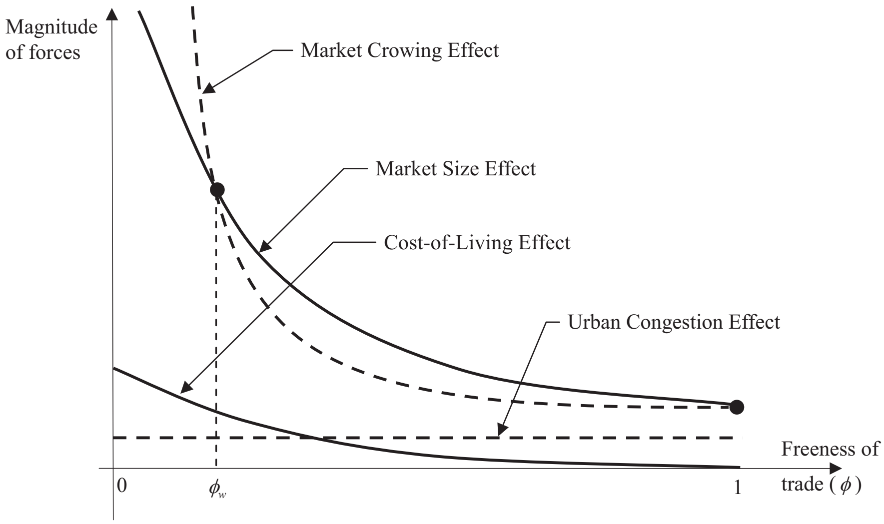
$$\text{MCE}\equiv\lambda\cdot\left\{-\frac{-\mu\sigma[\phi+2\phi^4+4h(1+\phi^3)+2h^2\phi(2+\phi^3)]-\mu^2[1+4h\phi^2+\phi^4+h^2(1+\phi^4)]-\sigma^2[1+\phi^4+4h(\phi+\phi^3)+h^2(1+6\phi^2+\phi^4)]}{[(h-1)h(\mu-\sigma)+[1+2(h-1)h]\sigma\phi-(h-1)h(\mu+\sigma)\phi^2]^2}\right\},$$

$$\text{CLE}\equiv\frac{-\partial\mathbb{P}_i}{\partial h}=\frac{\beta\sigma}{(\sigma-1)^2}H^{\frac{1}{1-\sigma}}(1-\phi)[h+(1-h)\phi]^{\frac{1}{1-\sigma}-1},$$

$\text{UCE}=\dfrac{1}{h}$, which is a positive value.

The four forces can be grouped into the aggregate agglomeration force vs. the aggregate dispersion force. Numerical calculations illustrate the relationships between each force and trade freeness (ϕ). Figure 4 depicts the results of numerical calculations. Findings show that: (1) The MSE, CLE and MCE decrease with an increasing degree of trade freeness, while the UCE is constant over the whole range of trade freeness (which is due to our assumption on the fixed supply of housing floor space); (2) Whenever ϕ is larger than ϕ_w, the region with more skilled workers provides a higher skilled worker nominal wage, since the MSE becomes larger than the MCE (as shown in Forslid and Ottaviano, 2003); (3) As ϕ=1, MSE is offset by MCE, and CLE vanishes, so UCE becomes the only effective force in the system (as verified in equation (28)); That is, even dispersion is the unique stable equilibrium which is the same as stated in

FIG. 4. VARIOUS EFFECTIVE FORCES ERODE WITH TRADE FREENESS



Proposition 3 of Tabuchi (1998).

$$\begin{aligned}\left.\frac{\partial V_{iH}}{\partial h}\right|_{\phi=1} &= \frac{1-\gamma}{w_{iH}}\left(\left.(\text{MSE}-\text{MCE})\right|_{\phi=1}=0\right)+\frac{\mu}{\mathbb{P}_i}\left(\left.\text{CLE}\right|_{\phi=1}=0\right)-\gamma\left(\left.\text{UCE}\right|_{\phi=1}=\frac{1}{h}\right) \\ &= -\frac{\gamma}{h}.\end{aligned} \tag{28}$$

Figure 5 shows the effects of aggregate agglomeration force and aggregate dispersion force on spatial configurations. The figure shows two break points ϕ_{b1} and ϕ_{b2}, which mean that even distribution of skilled workers (firms) is stable equilibrium whenever ϕ is smaller than ϕ_{b1} or larger than ϕ_{b2}. Dispersion equilibrium rises with both low and high degree of trade freeness, and for different reasons. In the former case, firms disperse to meet the final demand of unskilled labor. However, in the latter case, firms disperse as a response to urban congestion effect (since as ϕ is sufficiently large, UCE is the only force in effect). This illustration helps to clearly understand why a bell-shaped core-periphery structure exists.

This section illustrates the existence of black hole. No break point may be present in the system, that is, there is no intersection between curves of aggregate agglomeration and dispersion forces as Figure 6 shows. The aggregate dispersion force is always larger than the aggregate agglomeration force for $\phi \in (0, 1]$. We call this situation the "dispersion black hole", which is different from the black hole of agglomeration found by Krugman (1991) and Forslid & Ottaviano (2003).

Finally, we illustrate why the Helpman model only has a monotone decreasing relationship of spatial agglomeration-trade freeness (i.e. only one break point), but has no bell-shaped relationship. (i) According to Murata (2003), we can verify that in the Helpman model (1998), $\partial w_{iH}/\partial h$ is always positive, and is equal to zero as $\phi=1$, since the model is lacking for the local peasant (unskilled labor) market (Krugman 1980 home market effect); (ii) The CLE is

Fig. 5. Aggregate Agglomeration and Dispersion Forces Erode with Trade Freeness

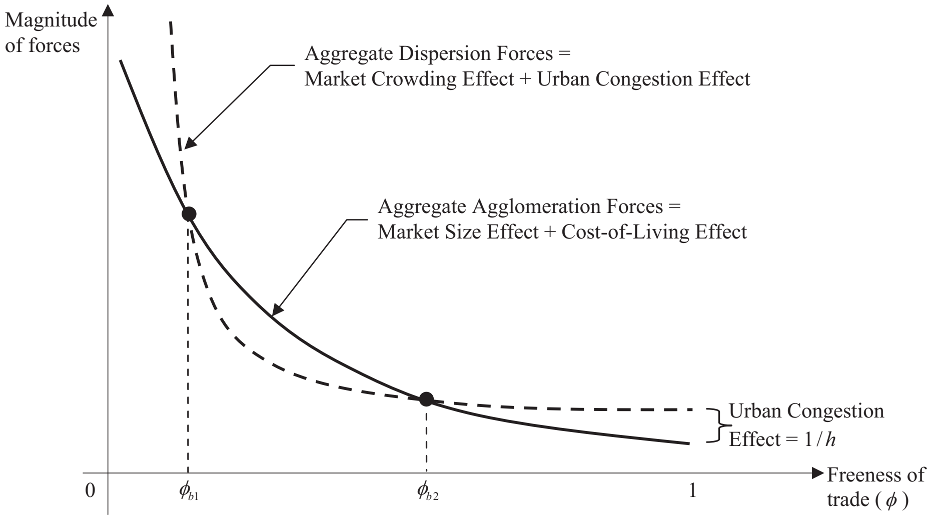
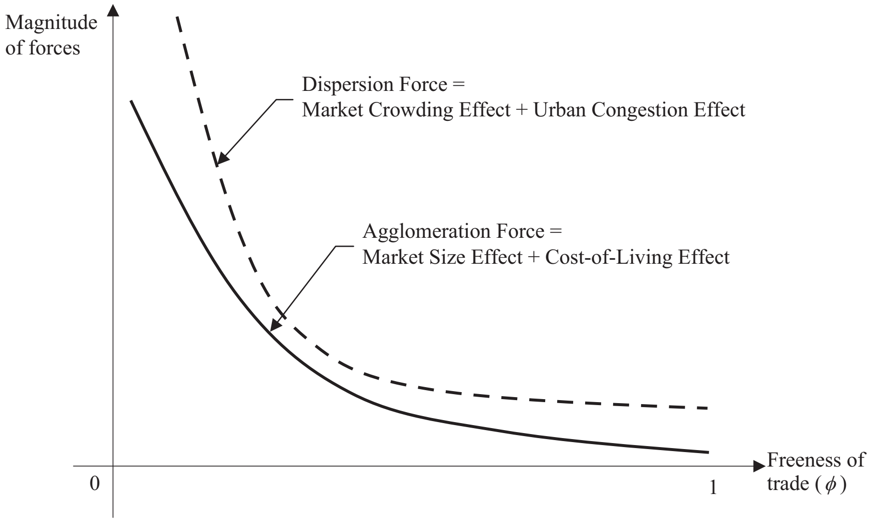
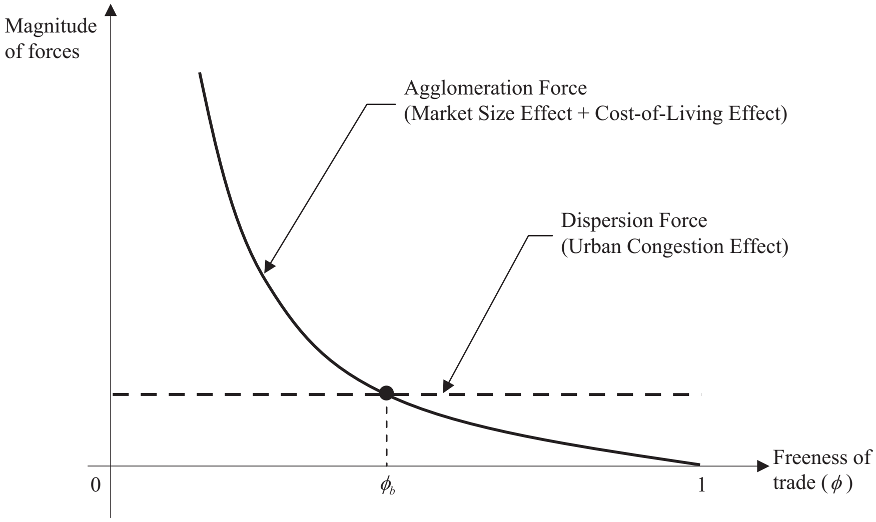


Fig. 6. Dispersion Black Hole



also always positive, and is equal to zero as $\phi=1$. Therefore, the aggregate agglomeration force (MSE + CLE) is a downward-slope curve and ends in zero as Figure 7 shows. On the other hand, the only dispersion force (UCE) is constant in the system. Consequently, the Helpman model has only one break point in response to UCE as Figure 7 shows.

FIG. 7. TWO OPPOSING FORCES IN THE HELPMAN SYSTEM (1998)



V. *Concluding Remarks*

This paper presents a simple modified version of the Helpman Model (1998). This modified model contains two regions, two types of fixed labors: mobile skilled labors, and immobile unskilled labors; three sectors: a differentiated tradable manufacturing sector in which each firm employs both skilled and unskilled labors with increasing returns to scale technology, shipping-free homogenous tradable agriculture goods employing only unskilled labors with constant returns to scale technology, and an immobile housing sector with fixed floor spaces. This modified model allows us to examine the complete set of spatial configurations for a two-region economy: full agglomeration, symmetric distribution, one break point and two break points asymmetric distributions, and dispersion black hole (i.e. persistent dispersion). This investigation shows the conditions for emerging these possible spatial configurations. Moreover, with a logarithm transformation of the CD type utility function, this study clearly decomposes spatial shaping forces into market size effect, market crowding effect, cost of living effect, and urban congestion effect, and diagrammatically exposes how these effects decay at different rates with increased degree of trade freeness, resulting in various spatial configurations.

The results of this paper clarify how the various spatial forces shape spatial configurations and help to understand the conditions for various possible types of spatial equilibrium. This modified model serves as a basic model. The next steps: (1) Specifies the housing production sector with land and labor inputs; (2) Relaxes the fixed housing demand assumption; (3) Removes the shipping-free assumption on agriculture goods; (4) Introduces immobile public goods financed by total differential land rents. The next model of this study will hopefully provide more rigorous results.

Appendix A: Note that

$$\lim_{h\to 0} \; Z(\phi)=\frac{[\sigma-\mu+(\sigma+\mu)\phi^2]^{1-\gamma}}{\phi^{\frac{\mu}{1-\sigma}}\cdot h^{\gamma}}-\frac{(2\sigma\phi)^{1-\gamma}}{1^{\frac{\mu}{1-\sigma}+\gamma}}=+\infty,$$

$$\lim_{h\to 1} \; Z(\phi)=\frac{(2\sigma\phi)^{1-\gamma}}{1^{\frac{\mu}{1-\sigma}+\gamma}}-\frac{[\sigma-\mu+(\sigma+\mu)\phi^2]^{1-\gamma}}{\phi^{\frac{\mu}{1-\sigma}}\cdot(1-h)^{\gamma}}=-\infty \text{ and}$$

$$Z(h,\phi=0)=\frac{[(\sigma-\mu)(1-h)]^{1-\gamma}}{h^{\frac{\mu}{1-\sigma}+\gamma}}-\frac{[(\sigma-\mu)h]^{1-\gamma}}{(1-h)^{\frac{\mu}{1-\sigma}+\gamma}}.$$

Then if $\sigma<1+\dfrac{\mu}{\gamma}$, $\lim\limits_{h\to 0} Z(\phi=0)=0$; $\lim\limits_{h\to 1} Z(\phi=0)=0$,

and if $\sigma>1+\dfrac{\mu}{\gamma}$, $\lim\limits_{h\to 0} Z(\phi=0)=+\infty$; $\lim\limits_{h\to 1} Z(\phi=0)=-\infty$.

Besides, $Z(h,\phi=1)=(2\sigma)^{1-\gamma}\left[\dfrac{1}{h^{\gamma}}-\dfrac{1}{(1-h)^{\gamma}}\right]$,

then $\lim\limits_{h\to 0} Z(\phi=1)=+\infty$; $\lim\limits_{h\to 1} Z(\phi=1)=-\infty$.

Appendix B: Note that

$$w_{iH}=\frac{L}{2H}\frac{\mu}{\sigma-\mu}\frac{2\sigma\phi h+[\sigma-\mu+(\sigma+\mu)\phi^2](1-h)}{\sigma\phi(h^2+(1-h)^2)+[\sigma-\mu+(\sigma+\mu)\phi^2]h(1-h)}.$$

Let $\lambda\equiv\dfrac{L}{2H}\dfrac{\mu}{\sigma-\mu}$; $f(h,\phi\;;\;\sigma,\mu)\equiv\dfrac{2\sigma\phi h+[\sigma-\mu+(\sigma+\mu)\phi^2](1-h)}{\sigma\phi(h^2+(1-h)^2)+[\sigma-\mu+(\sigma+\mu)\phi^2]h(1-h)}$

Then $\dfrac{\partial w_{iH}}{\partial h}=\lambda\cdot\dfrac{\partial f(h,\phi\;;\;\sigma,\mu)}{\partial h}$

$$=\lambda\cdot\left\{-\frac{(\phi-1)[\mu-\sigma+(\mu+\sigma)\phi][-(h-1)^2(\mu-\sigma)+(1-2h^2)\sigma\phi+(h-1)^2(\mu+\sigma)\phi^2]}{[(h-1)h(\mu-\sigma)+[1+2(h-1)h]\sigma\phi-(h-1)h(\mu+\sigma)\phi^2]^2}\right\}$$

Since the denominator is always positive, we only expand the numerator and separate the positive terms and negative terms to define the MSE and MCE respectively:

$$\text{MSE}\equiv\lambda\cdot\left\{\frac{2(h\mu+\sigma)(\mu+h\sigma)+\sigma[\sigma+4h(\mu+h\sigma)]\phi+2[(1+h^2)\mu^2+2h\sigma^2]\phi^2+(1+4h^2)\sigma(\mu+\sigma)\phi^3+2h(\mu+\sigma)^2\phi^4}{[(h-1)h(\mu-\sigma)+[1+2(h-1)h]\sigma\phi-(h-1)h(\mu+\sigma)\phi^2]^2}\right\}$$

$$\text{MCE}\equiv\lambda\cdot\left\{-\frac{-\mu\sigma[\phi+2\phi^4+4h(1+\phi^3)+2h^2\phi(2+\phi^3)]-\mu^2[1+4h\phi^2+\phi^4+h^2(1+\phi^4)]-\sigma^2[1+\phi^4+4h(\phi+\phi^3)+h^2(1+6\phi^2+\phi^4)]}{[(h-1)h(\mu-\sigma)+[1+2(h-1)h]\sigma\phi-(h-1)h(\mu+\sigma)\phi^2]^2}\right\}$$

Besides,

$$\text{CLE}\equiv\frac{-\partial\,\mathbb{P}_i}{\partial h}=\frac{-\partial\left\{\frac{\beta\sigma}{\sigma-1}H^{\frac{1}{1-\sigma}}[h+\phi(1-h)]^{\frac{1}{1-\sigma}}\right\}}{\partial h}=-\frac{\beta\sigma}{\sigma-1}H^{\frac{1}{1-\sigma}}\frac{(1-\phi)[h+(1-h)\phi]^{\frac{1}{1-\sigma}-1}}{1-\sigma}$$

$$=\frac{\beta\sigma}{(\sigma-1)^2}H^{\frac{1}{1-\sigma}}(1-\phi)[h+(1-h)\phi]^{\frac{1}{1-\sigma}-1}.$$

Then, $\dfrac{\partial\,\text{CLE}}{\partial\phi}=\dfrac{\beta\sigma}{(\sigma-1)^2}H^{\frac{1}{1-\sigma}}\dfrac{[\sigma+h(\phi-1)-\phi][h+\phi(1-h)]^{\frac{1}{1-\sigma}-2}}{\sigma-1}$.

$\dfrac{\partial\,\text{CLE}}{\partial\phi}$ is always positive, since $\sigma+h(\phi-1)-\phi$ is always positive.

Besides,

$\text{UCE} = \frac{1}{h}$ is a positive constant (see Equation (27)).

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