Hitotsubashi Journal of Economics 55 (2014), pp.89-108.

MARKET STRUCTURE, PRODUCTION EFFICIENCY, AND PRIVATIZATION*

Ya-Po Yang

Institute of Business and Management, National University of Kaohsiung
Kaohsiung 81148, Taiwan
yapo@nuk.edu.tw

Shih-Jye Wu

Department of Political Economy, National Sun Yat-Sen University
Kaohsiung 80424, Taiwan
shihjye@mail.nsysu.edu.tw

Jin-Li Hu**

Institute of Business and Management, National Chiao Tung University
Taipei 10044, Taiwan
jinlihu@mail.nctu.edu.tw

Received May 2013; Accepted December 2013

Abstract

In order to analyze the optimal degree of privatizing an upstream public firm, this paper sets up a vertically related market that consists of an upstream mixed oligopoly with one public firm and m private firms and a downstream oligopoly with n private firms. The major findings of this paper are as follows: If the marginal production cost of input increases slowly (rapidly), then the optimal degree for privatizing a public upstream firm increases (decreases) with the number of downstream firms. If the marginal production cost of input increases moderately, then the optimal degree for privatizing the public upstream firm first increases and then decreases with the number of downstream firms. If the marginal production cost of input is constant, then the optimal degree for privatizing a public upstream firm always increases with the number of downstream firms.

Keywords: vertically related market upstream market, intermediate goods, mixed Oligopoly, privatization

JEL Classification Codes: L22, L33

---

* The authors thank two anonymous referees of this journal, Leonard F.S. Wang, and Hong Hwang, for their valuable comments. The usual disclaimer applies. Financial support from Taiwan's National Science Council (NSC-100-2410-H-390-017 and NSC99-2410-H-009-063) is gratefully acknowledged.

** Corresponding author

I. *Introduction*

Privatization has been a worldwide trend since the late 1970s, with one famous example being that of British Rail under the leadership of Prime Minister Margaret Thatcher in 1993 (Railway Britain 2008). Many privatized firms are in downstream markets where they directly face the consumers, but in many developing economies such as mainland China and Taiwan, those that have been privatized are upstream firms in the industries of petroleum, electricity, minerals, steel, glass, ship construction, etc. (Lee 2009; Pao et al. 2008). Most of the literature on privatization looks at a mixed oligopoly that is not embedded in a vertically related market environment, hence providing no sufficient analysis of privatization in an upstream mixed oligopoly market structure. The main purpose of this paper is to analyze the optimal privatization of an upstream public firm in an upstream mixed oligopoly market and to compare it with the previous literature on privatization. This paper sets up a vertically related market consisting of an upstream mixed oligopoly and a downstream oligopoly to analyze the optimal degree for privatizing a public firm.

A market where public firms and private firms co-exist is regarded as a mixed market. The literature on a mixed oligopoly can be traced back to Merrill and Schneider (1996). Recently, the literature on a mixed oligopoly structure has developed fast and has extended to an open economy and spatial competition market.[1]

The literature on partial privatization of a downstream public firm includes Fershtman (1990), Matsumura (1998), Lee and Hwang (2003), Matsumura and Kanda (2005), Fujiwara (2007), Lu and Poddar (2007), Matsumura and Shimizu (2010), Han and Ogawa (2012), etc. Matsumura (1998) finds that neither full privatization nor full nationalization is optimal under moderate conditions. By extending the model of Matsumura (1998) and taking the inefficiency caused by public management into account, Lee and Hwang (2003) prove that partial privatization is a reasonable decision-making outcome, no matter under a monopoly or a mixed oligopoly. Matsumura and Kanda (2005) allow free entry and find, in contrast to the case of a fixed number of private firms, that welfare-maximizing behavior by the public firm is always optimal. Lu and Poddar (2007) study the impact of firm ownership in a differentiated industry. Fujiwara (2007) applies the horizontal differentiated mixed oligopoly model to study free-entry and non-free-entry effects of product differentiation upon the optimal degree of privatization. Matsumura and Shimizu (2010) set up a mixed oligopoly with m public firms and N-m private firms to examine the welfare of sequential privatizing public enterprises. Under plausible assumptions, the social welfare function is convex on the number of public firms. Therefore, if the number of privatized firms reaches some point, then this can improve social welfare.

Papers looking at the privatization of an upstream public firm in a vertically related market structure include Vickers (1995), Lee (2006), Gangopadhyay (2005), Willner (2008), De Fraja

[1] The literature on the welfare effects of privatization encompasses De Fraja and Delbono (1989), De Fraja and Delbono (1990), Fershtman (1990), Cremer et al. (1989), Husain (1994), White (1996), George and Manna (1996), Mujumdar and Pal (1998), etc. The literature on the welfare effects of privatization in an open economy includes Fjell and Pal (1996), Pal and White (1998), Chang (2005), Chao and Yu (2006), Dadpay and Heywood (2006), Han and Ogawa (2007), Mukherjee and Suetrong (2009), Matsumura et al. (2009), Wang and Cheng (2010), Wang and Cheng (2011), etc. Furthermore, Cremer et al. (1991), Anderson et al. (1997), Matsushima and Matsumura (2003), and Matsushima and Matsumura (2006) study spatial competition under a mixed oligopoly.

and Roberts (2009), Stähler and Traub (2009), Wen and Yuan (2010), Ceriani and Florio (2011), Ohori (2012), and Bose and Gupta (2013). Vickers (1995), Gangopadhyay (2005), and Stähler and Traub (2009) analyze whether or not a natural monopolist in a vertically integrated market should also be allowed to participate in a competitive downstream market by considering the tradeoffs between privatization and keeping a public firm at different vertical stages. In a vertical structure of the telecommunications industry, Lee (2006) examines the welfare effects of privatization on the upstream public enterprise, showing that the cost advantage of the independent rivals improves welfare post privatization. Willner (2008) investigates a market with an upstream bottleneck monopoly and downstream activity that may either be vertically integrated or separated. He finds that separation always reduces consumer surplus as well as total surplus unless there are large cost reductions.

De Fraja and Roberts (2009) use a vertically related model to discuss the sequence of privatization on vertical integrated public firms in Poland. Wen and Yuan (2010) examine restructuring, divestiture, and deregulation of a vertically integrated public firm from a public finance perspective, finding that the optimal restructuring plan for the utility depends on the cost of public funds and on the X-efficiency gains from privatization. Ohori (2012) investigates the optimal rate of an environmental tax and the level of privatization in a vertical relationship between one partially privatized producer and two private sellers. Considering a public monopolist, Ceriani and Florio (2011) study the effects of a sequence of reforms on consumer surplus within the network industry. Their results depend on the X-inefficiency of the public monopolist, allocative inefficiency of the private monopolist, cost of unbundling, and cost of establishing a competitive market. Bose and Gupta (2013) look at the optimal sequence of privatization of a public bilateral monopoly.

All of the above studies on the privatization of an upstream public firm confine themselves to regimes where upstream public firms face no private competitors, or to put it differently, the upstream market is not a typical mixed oligopoly. There are many industries in the real world with an upstream mixed oligopoly, such as the petroleum and steel industries in the developing economy of Taiwan, and some of the upstream public firms of these industries are already privatized or are going to be privatized. To the best of our knowledge, no study exists in the literature that looks at this topic, except Matsumura and Matsushima (2012) who examine the optimal privatization of upstream public firms in an upstream mixed oligopoly set-up. Matsumura and Matsushima (2012) provide a model of an upstream (airport) duopoly with two downstream (airline) companies that compete internationally. They show that the privatization of both airports is always an equilibrium, but they do not consider the plausibility of partial privatization and the relevant impacts of the market structure and technology on the optimal privatization degree of an upstream public firm. Thus, our model is completely different from theirs.

In order to fill this gap in the literature, the purpose of this paper is to look closely at the optimal privatization degree of an upstream public firm. This paper sets up a model with a vertically related market structure, whereby the upstream (intermediate good) market contains one public firm and m private firms, and the downstream (final good) market has n homogeneous private firms. This model allows us to analyze the optimal degree of privatization of a public upstream firm and the influence of the downstream market structure on the resultant privatization policy.

When privatizing an upstream public firm, the government's motive is to correct upstream production distortions (the previous literature calls this the 'cost saving effect') and both upstream and downstream oligopolistic distortions (known as the 'quantity reduction effect').[2] A greater (smaller) cost saving effect increases (decreases) the incentive for the government to privatize the public firm to a greater degree, while a greater (smaller) quantity in the reduction effect decreases (increases) the incentive to privatize the public firm. This paper finds that the number of upstream and downstream firms and the efficiency of production technology both play key roles in determining the relative size of these two effects and the optimal degree of privatization of an upstream public firm. If the marginal production cost of input increases slowly (rapidly), then the optimal degree for privatizing a public upstream firm increases (decreases) with the number of downstream firms. If the marginal production cost increases moderately, then the optimal degree for privatizing the public upstream firm first increases and then decreases with the number of downstream firms. When marginal production cost is constant, the optimal degree of privatization always decreases with the number of downstream firms. This result is quite different from the case of an increasing marginal cost.

This paper is organized as follows. Section II provides the basic model. Section III discusses the optimal degree of privatization of an upstream public firm with an increasing marginal cost. Section IV analyzes the case of a constant marginal cost. Section V concludes.

II. *The Basic Model*

In a vertically related market, the upstream intermediate goods market is a mixed oligopoly where one public firm (denoted as firm 0) and m private firms (denoted as firm j, for $j=1, 2, ..., m$) co-exist and supply homogeneous intermediate goods to n downstream private firms (denoted as firm i, for $i=m+1, m+2, ..., m+n$). These n firms use the intermediate goods to produce homogeneous final goods to supply the final goods market. All upstream and downstream firms engage in Cournot competition.

Before privatization, firm 0 is a welfare-maximizing pure public firm with 100% public shareholdings. However, after a proportion of λ shares are released to the private sector, the public shareholder wants to maximize the social welfare, while the private shareholders want to maximize profit. As a result, privatized firm 0's objective function Ω is a weighted average of its own profit and social welfare:

$$\Omega=\lambda\pi_0+(1-\lambda)SW, \tag{1}$$

[2] Production distortion comes from the marginal production cost of an upstream public firm being different from that of the upstream private firms. If the public firm is not fully privatized, then its output will be greater than each upstream private firm, and hence its marginal production cost will be greater than the private firms. Privatizing the public firm can shift some output from the public firm to the private firms and reduce the total production cost of the industry, which is the cost saving effect of privatization. Thus, privatizing the public firm can save the production cost of the industry and improve social welfare. In contrast, oligopolistic distortionscome from firms not producing at a price equal to the marginal production cost. In other words, the total output of the industry is not equal to the first best output level of the sociality. When the public firm is totally nationalized, the total output level is still less than the first best result, because only the public firm produces at a price equal to marginal cost. Once the government initiates the process of privatization, the total output of the industry will be farther away from the first best level, which is the output reduction effect in the literature. Therefore, from the viewpoint of correcting the oligopolistic distortion, reducing the privatization degree of the public firm increases the industry's total output and improves social welfare.

where π_0 is the profit of privatized firm 0; SW is social welfare as a sum of consumer surplus and the profits of $m+1$ upstream firms and n downstream firms; $\lambda \in [0, 1]$ is the proportion of private shareholdings. The value of λ represents the degree of privatization. When $\lambda = 1(0)$, the privatized firm 0 is a pure private (public) firm that pursues profit maximization (social welfare). When $0 < \lambda < 1$, firm 0 is a partially privatized firm.

For simplicity of exposition, we assume that the final goods demand function is $p = a - Q$, where p and Q are the price and the demanded quantity of the final good, respectively. The production function of each downstream firm is $q_i = y_i$, where q_i (y_i) denotes the output (intermediate goods) of firm i,[3] which represents one unit of intermediate goods producing one unit of final goods for all n firms. We also assume that downstream firms need no other complementary inputs to produce final goods. Thus, the cost function of downstream firm i can be expressed as $TC_i = wq_i$, where w denotes the price of the intermediate goods.

Following the set-up of most studies in the literature on a mixed oligopoly, we assume that upstream firm 0 and firm j (for $j = 1, 2, ..., m$) have a cost function that is $TC_0 = c_0 x_0 + k_0 x_0^2/2$ and $TC_j = c_j x_j + k_j x_j^2/2$,[4] respectively, where x_0 (x_j) is the intermediate goods produced by the upstream firm 0 (j). We discuss first the case when the marginal production costs of all upstream firms are increasing (i.e., $k_0 = k_j = k > 0$ and $c_0 = c_j = 0$) and then the case when their marginal costs are constant (i.e., $c_0 \neq 0$, $c_j \neq 0$, and $k_0 = k_j = k = 0$).[5]

Based on those assumptions, the model is a three-stage game. In stage 1, the government chooses the optimal degree of privatization, λ, of the upstream public firm 0, in order to maximize social welfare. In stage 2, the $m+1$ upstream firms simultaneously choose their output quantities, x_0 and x_j (for all $j = 1, 2, ..., m$), to maximize their own objective function. In stage 3, the n downstream firms simultaneously choose their output quantities, q_i (for all $i = m+1, m+2, ...m+n$), to maximize their own profits.[6]

We apply the solution concept of the subgame-perfect Nash equilibrium (SPNE) to solve this game and therefore follow the backward induction approach to find the SPNE. We first solve for the Cournot equilibirum output level of the n downstream firms, then work out the equilibrium outputs of the two upstream firms in the second stage, and finally, characterize the social optimal privatization degree in the first stage of the model.

---

[3] One unit of output requiring one unit of input is a typical assumption in the literature on a vertically related market.

[4] Those papers assuming that the public firm and private firms have the same quadratic cost function include De Fraja and Delbono (1989), Barcena-Ruiz and Garzon (2003), Fjell and Pal (1996), White (1996), Pal and White (1998), Han and Ogawa (2012), etc. Matsumura and Shimizu (2010) assume that the public firm and private firms have asymmetric quadratic cost functions, whereas Nishimori and Ogawa (2002) assume that the public firm and private firms have asymmetric constant marginal costs. Chang (2005) discusses both the case of asymmetric quadratic cost functions and the case of asymmetric constant marginal costs.

[5] We also present the more general case where $k_0 \neq k_j$ in Section III.3.

[6] Our model follows the conventional set-up of the literature on a vertically related industry, where the intermediate firms co-determine the input price and the final-good firms act as price takers, such as in Greenhut and Ohta (1979), Spencer and Jones (1991, 1992), etc.

III. *The Optimal Degree of Privatization of an Upstream Public Firm with an Increasing Marginal Cost*

This section discusses the optimal degree of privatization of an upstream public firm when the marginal production cost of all the upstream firms is increasing. We solve the subgame perfect equilibrium of the game through backward induction.

1. Equilibria of the Downstream Market

In stage 3, all n downstream firms take λ and w as given to maximize profit. According to the setting in Section II, we express the profit function of downstream firm i as:

$$\pi_i=[(a-\sum_{l=m+1}^{m+n} q_l)-w]q_i,\ i=m+1,\ m+2...,\ m+n. \tag{2}$$

Differentiating Equation (2) with respect to q_i, we have first-order conditions and solve the symmetric equilibrium output for every downstream firm as $q_i=q=\dfrac{a-w}{n+1}$. Because the production function is $q_i=y_i$, $y_i=q_i=q=\dfrac{a-w}{n+1}$ is also the derived demand function for every firm i. By aggregating the derived demand function of the downstream n firms, we get the total derived demand function as $X=\sum_{i=m+1}^{m+n} y_i=\sum_{i=m+1}^{m+n} q_i=nq=\dfrac{n}{n+1}(a-w)$, where X is the total intermediate good demand quantities.

2. Upstream Market Equilibria

In stage 2, these $m+1$ upstream firms take the privatization degree, λ, as given in order to maximize their objective function. Let us further denote the total quantity supplied of the intermediate good by $x_0+\sum_{h=1}^{m} x_h$. After rearranging the total demand function of the intermediate good, we have the inverse derived demand function as $w=a-\dfrac{n+1}{n}X$. Thus, upstream private firm j's profit functions can be expressed as:

$$\pi_j=[a-\frac{n+1}{n}(\sum_{h=1}^{m} x_h+x_0)]x_j-\frac{k}{2}x_j^2,\ \text{for all } j=1,\ 2,\ ...,\ m. \tag{3}$$

Using Equation (3), privatized public upstream firm 0's objective function, Ω, can be written as:

$$\Omega=\lambda\pi_0+(1-\lambda)][CS+\pi_0+\sum_{j=1}^{m}\pi_j+\sum_{i=m+1}^{m+n}\pi_i]. \tag{4}$$

The first item and the items in the parentheses on the right-hand side of Equation (4) are respectively firm 0's profit and social welfare (including consumer surplus and all upstream and downstream profits), where $CS=\frac{Q^2}{2}=\frac{n^2(a-w)^2}{2(n+1)^2}=\frac{n^2(\frac{n+1}{n}X)^2}{2(n+1)^2}=\frac{X^2}{2}$ and $\sum_{i=m+1}^{m+n}\pi_i=n\frac{(a-w)^2}{(n+1)^2}$ $=n\frac{(\frac{n+1}{n}X)^2}{(n+1)^2}=\frac{X^2}{n}$. In other words, except for its own profit, firm 0 also cares about the profit of the other $m+n$ upstream and downstream firms.

Differentiating Equations (3) and (4) with respect to x_j and x_0, respectively, we have the first-order conditions of maximization for upstream firm j and firm 0 as:

$$\frac{d\pi_j}{dx_j}=a-\frac{n+1}{n}(\sum_{h=1}^{m}x_h+x_0)-\frac{n+1}{n}x_j-kx_j=0,\text{ for } j=1,2,\ldots\ldots,m. \tag{5}$$

$$\begin{aligned}\frac{d\Omega}{dx_0}&=a-\frac{n+1}{n}(\sum_{h=1}^{m}x_h+x_0)-\frac{n+1}{n}x_0-k_0x_0\\&+(1-\lambda)[\sum_{h=1`}^{m}(-\frac{n+1}{n}x_h)+(\sum_{h=1`}^{m}x_h+x_0)+\frac{2}{n}(\sum_{h=1`}^{m}x_h+x_0)]=0.\end{aligned} \tag{6}$$

Solving for x_0 and x_j by Equations (5) and (6),[7] we obtain the equilibrium outputs of the upstream firms as $x_0^*=x_0^*(\lambda,n,m)$ and $x_j^*=x^*=x^*(\lambda,n,m)$, for all $j=1,2,\ldots,m$,[8] respectively. From the equilibrium output level of the intermediate good, we present the comparative statics in Lemma 1.

Lemma 1.

(i) Given n and m, $\frac{\partial x^*}{\partial\lambda}>0$, $\frac{\partial x_0^*}{\partial\lambda}<0$, *and* $\frac{\partial X}{\partial\lambda}<0$.

(ii) Given λ *and m,* $\frac{\partial x^*}{\partial n}>0$; $\frac{\partial x_0^*}{\partial n}>(<)0$, *if* λ *is high (low);* $\frac{\partial X}{\partial n}>0$.

(iii) Given λ *and n,* $\frac{\partial x^*}{\partial m}<0$, $\frac{\partial x_0^*}{\partial m}<0$, $\frac{\partial X}{\partial m}>0$. *(Proof See Appendix A.)*

The economic intuition of *Lemma 1 (i)* is as follows. Given the original privatization degree λ, firm 0 puts some weight on social welfare and some weight on its own profit, and hence it will produce more than the profit maximization output level of private firm j. As the privatization degree increases, firm 0 raises the weight on its own profit and reduces the weight on social welfare in the objection function. Thus, it reduces output to improve its own profit, causing all other private firms facing an increased perceived derived demand to raise their output. However, the decreased amount of firm 0's output dominates the total increased amount of all the private firms' output, leading to a decline in the total output of the upstream industry.

The economic intuition of *Lemma 1 (ii)* is as follows. Other things being equal, an increase in n reduces the downstream oligopoly distortion and raises the derived demand faced by upstream private firm j. Therefore, firm j has an incentive to increases its output level;

---

[7] We assume that the second-order and stability conditions in the upstream market are all satisfied.

[8] Since the equilibrium quantities for each upstream firms are symmetric, we have $x_j^*=x^*=x^*(\lambda,n,m)$, for all $j=1,2,\ldots,m$.

however, firm 0 has some incentive to reduce its output level, because of the reduction in downstream oligopoly distortion. If λ is at a low level, then firm 0 puts a great weight on social welfare, and the incentive to cut down its output is relatively strong and outweighs the incentive to raise its output, causing firm 0 to reduce output. On the contrary, if λ is at a high level, then the latter incentive outweighs the former incentive, leading firm 0 to raise output.[9] The total output of all upstream firms will increase, because the total increased amount of the upstream private firm's output is always greater than the decreased amount of firm 0's output.[10]

The economic intuition of *Lemma 1 (iii)* is more straightforward. When the number of upstream private firms increases, the upstream mixed oligopoly market becomes more competitive and every incumbent private firm will decrease its output. However, total upstream output will increase, because the increased output from the new entrants is greater than the decreased output of all the incumbent firms.

3. The Optimal Degree of Privatization of an Upstream Public Firm

Based on the equilibria of the final two stages, this section discusses the optimal degree of privatization of an upstream public firm. The government, in stage 1, chooses the privatization degree to maximize social welfare, expecting the best responses of all upstream and downstream firms in the following stages. The government's objective function is the social welfare function, expressed as:

$$\max_{\lambda} \quad SW(\lambda)=CS+m\pi^{U}+\pi_0+n\pi^{D}, \tag{7}$$

where $CS=\dfrac{(mx^*+x_0^*)^2}{2}$ is consumer surplus, $\pi^U=\pi_j=[a-\dfrac{n+1}{n}(mx^*+x_0^*)]x^*-\dfrac{kx^{*2}}{2}$ is the equilibrium profit of all the m upstream private firms, $\pi_0=[a-\dfrac{n+1}{n}(mx^*+x_0^*)]x_0^*-\dfrac{kx_0^{*2}}{2}$ is the equilibrium profit of upstream firm 0, and $\pi^D=\dfrac{1}{n^2}(mx^*+x_0^*)^2$ is the equilibrium profit of all the n downstream firms. Totally differentiating Equation (7) with respect to λ, we have the following first-order condition:

$$\frac{dSW}{d\lambda}=m(\frac{\partial\pi^U}{\partial x}\frac{\partial x^*}{\partial\lambda}+\frac{\partial\pi^U}{\partial x_0}\frac{\partial x_0^*}{\partial\lambda})+\frac{\partial\pi_0}{\partial x}\frac{\partial x^*}{\partial\lambda}+\frac{\partial\pi_0}{\partial x_0}\frac{\partial x_0^*}{\partial\lambda}+(mx^*+x_0^*)(1+\frac{2}{n})(m\frac{\partial x^*}{\partial\lambda}+\frac{\partial x_0^*}{\partial\lambda})=0 \tag{8}$$

From Equation (8), we can solve the optimal privatization degree as (*Proof See Appendix*

[9] Taking the extreme cases for example, when $\lambda=0$, firm 0 puts its whole weight on social welfare in the objective function, it has no incentive to raise output, and the former effect will dominate the latter, causing it to cut output when the derived demand increases, whereas when $\lambda=1$, firm 0 puts all weight on its own profit, and the latter effect dominates, causing it to raise output.

[10] In other words, if firm 0 is a pure public firm with maximizing social welfare as its objective, then it faces a vertically integrated market demand that is not affected by the number of n, and so it has no incentive to raise output. When firm 0 is partially privatized, it faces a weighted average of the perceived derived demand and perceived integrated market demand to maximize a weighted average of its own profit and social welfare. When the derived demand increases, the firm has some incentive to raise output. The higher the degree of privatization, the stronger the incentive to raise output will be.

B):

$$\lambda^U = \frac{m(n+1)nk}{n^2k^2 + kn(n+1)(m+2) + (n+1)^2}. \tag{9}$$

Equation (9) shows that the value of λ^U depends on m, n, and k. Based on Equation (9), we get Proposition 1.

Proposition 1. *When the marginal production cost of input is increasing, the optimal degree of privatization of the upstream public firm* (λ^U) *has the following properties:*

(i) $0 < \lambda^U < 1$; *(ii) Given k and n,* $\frac{d\lambda^U}{dm} > 0$; *(iii) Given k and m, if* $k \leq 1$, *then* $\frac{d\lambda^U}{dn} > 0$; *if* $1 < k < 2$, *then* $\frac{d\lambda^U}{dn} \gtrless 0$ *as* $n \lessgtr \frac{1}{k-1}$; *if* $k \geq 2$, *then* $\frac{d\lambda^U}{dn} < 0$; *(iv) Given m and n, if* $k \lessgtr 1 + \frac{1}{n}$, *then* $\frac{d\lambda^U}{dk} \gtrless 0$. *(See Appendix B for proof.)*

The economic intuitions of Proposition 1 are as follows.

(*i*) A production distortion in the upstream market and oligopolistic distortions in both the upstream and downstream markets occur. The greater the former (latter) distortion is, the more (less) the incentive is for the government to privatize firm 0. When firm 0 is a fully public firm (that is, $\lambda = 0$), the former incentive will dominate the latter, and thus partially privatizing firm 0 can improve social welfare. On the contrary, when firm 0 is fully privatized (that is, $\lambda = 1$), there is only oligopolistic distortion, which provides the incentive for the government to nationalize firm 0, and thereby partial privatization is better than full privatization.[11] This is why neither full nationalization nor full privatization is the best policy. Moreover, the optimal privatization degree λ^U emerges when the marginal effects of λ^U on production distortion and oligopolistic distortion are equal.

(*ii*) Given n, k, and the initial optimal λ^U, when the number of upstream private firms m increases, the total output of the industry will rise by *Lemma 1 (ii)* and hence oligopolistic distortion drops, whereas the outputs of firm 0 and every incumbent upstream private firm all fall, with the latter decreasing more than the former, leading to a greater marginal cost difference between firm 0 and firm j and hence an increased production distortion. Both a reduced oligopolistic distortion and an increased production distortion call for privatizing firm 0 further. Therefore, the entry of upstream private firms definitely results in a greater λ^U.

(*iii*) The relationship between n and λ^U is not so intuitive and depends on the speed of the increase in the marginal production cost. If the marginal production cost increases slowly ($k \leq 1$), n is at a low level (for example, $n = 1$), and firm 0 is a fully public firm initially, then all upstream private firms face a more inelastic perceived derived demand to maximize profit, whereas firm 0 puts the whole vertically integrated industry's final demand into its objective function. Thus, firm 0 faces a more elastic perceived final good demand than each upstream

[11] If the marginal cost of the output of firm 0 is greater than that of the upstream private firms, then it results in a production distortion that provides the incentive for privatization. This is because initiating privatization can reduce the difference in the marginal production cost between firm 0 and firm j and hence the production distortion.

private firm and thus produces much more output than each private firm does.[12] However, as firm 0 uses the same efficient production technology as each upstream private firm, the marginal production cost difference between firm 0 and any upstream private firm is not very great, and hence the production distortion is small (that is, the cost saving effect of privatizing firm 0 is small) and results in a small optimal λ^U. Given the initial optimal λ^U, as n increases from a low level, the upstream private firmswill face an increased perceived derived demand, and they will produce more than before (from *Lemma 1(ii)*), leading to a smaller oligopolistic distortion than before (that is, a smaller output reduction effect).

The government has an incentive to lift up the degree of privatizing firm 0, but because the government initially owns a large amount of shares of firm 0, the magnitude of the increased output of firm 0 (which may even decrease by *Lemma 1(ii)*) will be less than that of the upstream private firm j. Thus, the marginal production cost difference between firm 0 and firm j narrows down (that is, a smaller production distortion or a smaller cost saving effect), which raises the incentive for the government to privatize less of firm 0. Because the marginal production cost increases slowly, the former incentive always dominates the latter incentive, leading the government to privatize more of firm 0 in order to improve social welfare.

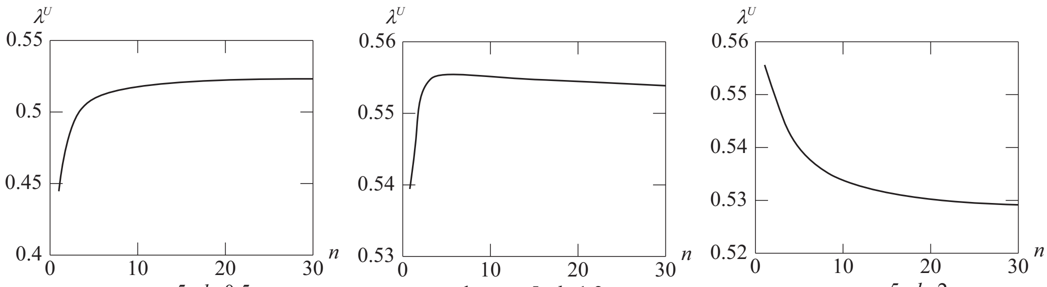
If the marginal production cost increases moderately ($1<k<2$), when n is at a low level and $\lambda=0$ initially, then the aforementioned positive incentive (that is, the decreased output reduction effect) initially dominates the negative incentive (that is, the decreased cost saving effect). As n passes over some critical level, the former is dominated by the latter, causing the government to privatize less of firm 0. This is because the increase in n from a low level causes the government to privatize more of firm 0. The higher the degree of privatization is, the greater the shrinkage is in the output difference between firm 0 and firm j and the smaller the production distortion is (that is, the cost saving effect). As n reaches some critical level, the negative incentive to privatize (i.e., the smaller cost saving effect) turns out to be stronger than the positive incentive (that is, the smaller oligopolistic distortion), causing the government to reduce the degree of privatization λ^U to improve social welfare. Therefore, the optimal degree of privatization λ^U first increases and then decreases with n when the marginal production cost increases moderately.[13]

When the marginal production cost increases rapidly ($k>2$), if n is small and $\lambda=0$, then the output difference between firm 0 and firm j is relatively small (that is, the production distortion is small). At the same time, the oligopolistic distortion is also small, because all the firms have the same inefficient technology, and the total output level is not very far away from the first best level. Therefore, the government will privatize more shares of firm 0. As n increases, it leads to a smaller production distortion and a smaller oligopolistic distortion. The former always dominates the latter as n increases continuously, and this is why the optimal

[12] It should be clarified that the objective of a full public firm 0 is social welfare, and thus it faces vertically integrated perceived demand - that is, the demand of the final good - and not derived demand. This can be seen from Equation (6). Substituting $\lambda=0$ into Equation (6), we have $[a-x_0-\sum_{h=1}^{m} x_h]-kx_0=0$, in which the expression in the bracket represents market demand, and thus firm 0 faces the final demand rather than the derived demand faced by other private upstream firms.

[13] Because firm 0's output is always greater than firm j's output, redistributing some of firm 0's output to firm j via privatization can save on production cost, but as the difference in output levels goes down, the cost saving effect becomes smaller.

FIG. 1.



a. m=5, k=0.5 b. m=5, k=1.2 c. m=5, k=2

degree of privatization λ^U decreases with n when the production technology is less efficient.

The policy implication of Proposition 1 *(iii)* is that a more competitive downstream market (that is, a larger n) may not call for a higher degree of privatization.

(*iv*) For any given n, if the production technology is more efficient (k is low), then the government will not privatize too much of firm 0, because of a relatively small production distortion. Given the initial λ^U, as k increases from a low level, the marginal production costs of all firms will rise. Both firm 0 and firm j will reduce their output, but the former reduces less than the latter does and firm 0 puts some weight on social welfare in its objective function. When k is low, the output difference in firms 0 and j will magnify. Therefore, the production distortion will go up, which increases the incentive to privatize firm 0. At the same time, the output reduction of all firms also magnifies the output reduction effect, which decreases the incentive to privatize. The former dominates the latter, causing the government to privatize more of firm 0, and therefore λ^U will increase with k initially. As k continuously increases to reach a critical value, the magnification of the output difference between firm 0 and firm j begins to mitigate. The incentive to privatize more turns out to be dominated by the incentive to privatize less, causing the government to begin to decrease λ^U. Hence, for a given n, λ^U first increases with k, but as k passes over a critical value, λ^U will decrease with k. In other words, λ^U is concave in k.

The implication of *(iv)* is that, if the efficiency of production technology improves, then a higher degree of privatization of the public firm may be called for. This is in fact counter-intuitive.

We use some numerical examples and Figure 1 to check Proposition 1. In Figure 1, we take $m = 5$ as the same parameter and respectively take k=0.5, 1.2, and 2 (as different efficiencies of the production technology) in a, b, and c to see the relationship between λ^U and n. When the production technology is more efficient (k=0.5), the optimal degree of privatization of an upstream public firm increases with the number of downstream firms and is always less than that of a downstream public firm. When the production technology is intermediately efficient (k=1.2), the optimal degree of privatization of an upstream public firm first increases and then decreases with the number of downstream firms. When the production technology is less efficient (k=2), the optimal degree of privatization of an upstream public firm decreases with the number of downstream firms.

In addition to the above, we can also get the more generalized case where firm 0 uses a different production technology from firm j. Under the case of $k_0 \neq k_j = k$, we resolve the three-stage game in the same way and reach the optimal degree of privatization of firm 0 as follows:

$$\lambda^U(k_0, k) = \frac{m(n+1)n[k_0(m+n+1)+k]}{[(m+1)(n+1)+kn][(k_0mn+(n+2)nk]+kn(n+1)(n+m+2)+(n+1)^2(n+2+m)} \quad (10)$$

Equation (10) is a more general optimal degree of privatization than Equation (9). Note that, when $k_0 = k$, $\lambda^U(k_0, k)$ in Equation (10) reduces to λ^U in Equation (9). We thus easily obtain that $\frac{d\lambda^U}{dk_0} > 0$ - that is, the higher the value is of k_0, the greater the value of λ^U will be.

IV. *The Optimal Degree of Privatization of an Upstream Public Firm with a Constant Marginal Cost*

Except for increasing marginal production costs, a constant marginal production cost is also a popular assumption in the literature of a mixed oligopoly.[14] Following the above section, we continue to use backward induction to solve the optimal degree of privatization of an upstream public firm when the marginal production costs of all the upstream firms are constant.

1. Equilibria of the downstream market and upstream market

The downstream market equilibria in stage 3 are the same as those in section III.1. We can directly make use of the previous results to solve the upstream market equilibria in stage 2.

Because the marginal costs of all upstream firms are constant, by substituting $k_0 = k = 0$ into the cost functions of all upstream firms, these cost functions become $TC_0 = c_0 x_0$ and $TC_j = cx_j$. Thus, the first condition of the upstream firms in (5) and (6) changes to be (11) and (12):

$$\frac{d\pi_j}{dx_j} = a - \frac{n+1}{n}\left(\sum_{h=1}^{m} x_h + x_0\right) - \frac{n+1}{n}x_j - c = 0, \text{ for } j = 1, 2, \ldots, m. \quad (11)$$

$$\begin{aligned}\frac{d\Omega}{dx_0} &= a - \frac{n+1}{n}\left(\sum_{h=1}^{m} x_h + x_0\right) - \frac{n+1}{n}x_0 - c_0 \\ &\quad + (1-\lambda)\left[\sum_{h=1}^{m}\left(-\frac{n+1}{n}x_h\right) + \left(\sum_{h=1}^{m} x_n + x_0\right) + \frac{2}{n}\left(\sum_{h=1}^{m} x_n + x_0\right)\right] = 0\end{aligned} \quad (12)$$

From the above two equations, we obtain the equilibrium outputs of the upstream firms as $x_0^* = x^*_0(\lambda, n, m)$ and $x_j^* = x^* = x^*(\lambda, n, m)$, for all $j = 1, 2, \ldots, m$. The comparative statics and the intuition are the same as Lemma 1. *(Please refer to Mathematical Appendix C.)*

As for the first stage, we proceed as before to solve the optimal privatization level:

[14] The constant marginal production cost is also wildly adopted in the literature on privatization issue, for examples, George and Manna (1996), Pal (1998), Matsumura (1998), Nishimori and Ogawa (2002), Chang (2005), and recent works by Matsumura and Ogawa (2012) and Matsumura and Sunada (2013).

$$\lambda^* = \frac{mn(c_0 - c)}{[(n+m+2)(a-c)+(m+1)(c-c_0)(mn+m+n+2)]}.$$

From the above, we now have Proposition 2

Proposition 2. *When the marginal production cost of input is constant, the optimal degree of privatization of the upstream public firm (λ^U) has the following properties: (i) When $c_0 \leq c$, then $\lambda^* = 0$. (ii) When $c_0 > c$, if $c < c_0 < \bar{c}_0 \equiv \frac{a(m+m+2)+m(mm+m+3n+2)c}{(m+1)(mn+m+n+2)+mn}$, then $0 < \lambda^* < 1$, and thus $\frac{d\lambda^*}{dn} > 0$ and $\frac{d\lambda^*}{dm} > 0$; if $c_0 \geq \bar{c}_0$, then $\lambda^* = 1$. (Proof See Appendix C.)*

The result in Proposition 2 differs from Proposition 1 in two respects. One is that, when all the upstream firms use the same production technology with a constant marginal cost, the optimal degree of privatization is zero. This is because no matter how many inputs firm 0 has produced, its marginal cost is always the same as that of private firms, and there is no cost saving effect with privatization under any number of m and n. Thus, the best policy is not to privatize.

The second difference from Proposition 1 is $\frac{d\lambda^*}{dn} > 0$. The economic intuition of $\frac{d\lambda^*}{dn} > 0$ is that, given the initial value of λ^*, an increase in n amplifies the aforementioned positive incentive (the decreased output reduction effect), whereas it also reduces the negative incentive (that is, the decreased cost saving effect), because of a reduction in the output difference among firm 0 and the other upstream firms. The former always dominates the latter owing to a constant marginal cost difference among firm 0 and the other upstream firms.

The results of this paper tell us that the characteristic of marginal production cost and the number of downstream firms both play key roles in determining the optimal degree of privatization of an upstream public firm. In spite of the fact that a greater number of downstream firms increases the derived demand and consequently reduces the downstream oligopolistic distortion, which then increases the incentive to privatize the public firm to a greater degree, upstream private firms may increase more output than upstream public firms do, which also reduces the production distortion and decreases the incentive to privatize more of the public firm. Therefore, a more competitive downstream market is not a sufficient condition for the government to privatize the upstream public firm to a greater degree especially when the marginal production cost is increasing. All the above results tell us that the derived demand of the downstream firms affects the relative strength between oligopolistic distortion (output reduction effect) and production distortion (cost saving effect), which determine the optimal degree of privatization of an upstream public firm.

V. *Conclusion*

This paper establishes a vertically related market model that consists of an upstream mixed oligopoly and a downstream oligopoly to analyze the optimal degree of privatization when privatizing an upstream public firm. The upstream market is a mixed oligopoly containing one public firm and m private firms. The downstream (final goods) market is an oligopoly that contains n homogeneous private firms. This model allows us to analyze the optimal degree of privatization of an upstream public firm. Moreover, we also discuss the influence of the downstream market structure on the optimal degree of privatization of the upstream public firm.

The major findings of this paper are as follows. When the marginal production costs of the upstream public firms are increasing, the relative strength between oligopolistic distortion and production distortion depends on the speed of the increase of the marginal production cost and the number of downstream firms. If the marginal production cost increases slowly (rapidly), then the optimal degree for privatizing a public upstream firm increases (decreases) with the number of downstream firms. If the marginal production cost increases moderately, then the optimal degree of privatization of the public upstream firm first increases and then decreases with the number of downstream firms. Finally, when the marginal production cost is constant, the optimal degree of privatization of an upstream public firm always increases with the number of downstream firms, which is different from the case of an increasing marginal production cost.

This paper has explored the relationship between the degree of privatization and production efficiency in a vertically related market structure. There have been an increasing number of studies in the literature, such as Lin and Matsumura (2012), that take into account the role of foreign investors in a mixed oligopoly market structure. Incorporating downstream foreign firms into the model is an interesting research topic and also provides direction for our future research.

Mathematical Appendices

A. Proof of Lemma 1

2^{nd}-stage equilibrium (upstream market equilibrium)

Based on Equations (5) and (6), we have a symmetric solution for private firms, $x_j = x$, $\forall j = 1, 2, ..., m$. Substituting them into Equations (5) and (6), we obtain the following two rearranged equations.

$$[(n+1)(m+1)+kn]x+(n+1)x_0=an$$
$$(n+\lambda)mx+[(1+k_0)n+\lambda(n+2)]x_0=an$$

By solving x and x_0, we have the equilibrium outputs of the upstream market.

$x_0=\dfrac{[(n+1)(m+1)+kn-(n+\lambda)m]an}{H}$, $x=\dfrac{[(1+k)n+\lambda(n+2)-(n+1)]an}{H}$ and

$X=mx+x_0=\dfrac{[(1+k)mn+\lambda(n+1)m+kn+(n+1)-mn]an}{H}$, where

$H\equiv[(n+1)(m+1)+kn][(1+k)n+\lambda(n+2)]-(n+1)(n+\lambda)m$.

By equilibrium outputs, we have the following comparative statics.

$\dfrac{\partial x_0}{\partial \lambda}=\dfrac{-[(n+1)(m+1)+kn][m(kn-1)+(n+2)(kn+m+n+1)]an}{H^2}<0$,

$\dfrac{\partial x}{\partial \lambda}=\dfrac{(n+1)[m(kn-1)+(n+2)(kn+m+n+1)]an}{H^2}>0$,

$\dfrac{\partial X}{\partial \lambda}=\dfrac{\partial x_0}{\partial \lambda}+m\dfrac{\partial x}{\partial \lambda}=\dfrac{-[kn+n+1][m(kn-1)+(n+2)(kn+m+n+1)]an}{H^2}<0$,

$\dfrac{dx_0}{dm}=\dfrac{-an(n+\lambda)[kn+n+1][kn-1+\lambda(n+2)]}{H^2}<0$,

$$\frac{dx}{dm}=\frac{-an(n+1)[kn+\lambda(n+1)][kn-1+\lambda(n+2)]}{H^2}<0,$$

$$\frac{dX}{dm}=\frac{an[(kn+n+1)(kn-1)+\lambda(n+2)(kn+n+1)+(n+1)(1-\lambda)m]}{H^2}>0,$$

$$\frac{\partial x}{\partial n}=\frac{n^2\{k^2(m+2)+k[\lambda(m+1)+1]+\lambda^2+\lambda m+1\}+\lambda(m+2)(2kn+2\lambda n-2\lambda-1)]}{H^2}>0,$$

$$\frac{dX}{dn}=\frac{n^2[k^2(m^2+2m+\lambda m+2)+k[\lambda(\lambda m+1)+(m+1)(2m+3)]+\lambda(\lambda m+1)(2m+3)+(m+2)]}{H^2}$$
$$+\frac{2\lambda n(m+2)[k(m+1)+\lambda m+1]+(\lambda m+1)(m+2)}{H^2}>0,$$

$$\frac{dx_0}{dn}=\frac{n^2\{-(m-\lambda m+1)[k^2+(m+\lambda+2)k+(\lambda+1)(m+1)]+(k+1)[k(1+m+2\lambda)+\lambda(2m+3)+1]\}}{H^2}$$
$$+\frac{\lambda(m+2)[2n(k+1)+m-\lambda m+1]}{H^2}$$
$$\equiv\frac{\Psi(\lambda)}{H^2}.$$

The sign of $\frac{dx_0}{dn}$ is the same as $\Psi(\lambda)$, which depends on the value of λ. By differentiation $\Psi(\lambda)$ with respect toλ, we have:

$\frac{d\Psi}{d\lambda}=n^2[mk(k+m+2+\lambda)+\lambda m(m+1)+(k+1)(2k+m+2)]+(m+2)[2n(k+1)+(m+1)(1-\lambda)]>0$, and thus Ψ increases with λ. Furthermore, we have $\Psi(0)=-n^2m(m+2)(k+1)<0$, and $\Psi(1)=n^2[k^2(m+2)+2k(m+2)+2]+(m+2)[2n(k+1)+1]>0$. By the medium value theorem, these three properties assure that $\Psi(\lambda)<(>)0\Leftrightarrow\frac{dx_0}{dn}<(>)0$, if λ is low (high).

B. Proof of Proposition 1

1st-stage equilibrium (optimal degree of privatization of the upstream public firm).

By substituting $\pi^U=[a-\frac{n+1}{n}(mx+x_0)]x-\frac{kx^2}{2}$, $\pi_0=[a-\frac{n+1}{n}(mx+x_0)]x_0-\frac{kx_0^2}{2}$, $CS=\frac{(mx+x_0)^2}{2}$, and $n\pi^D=\frac{(mx+x_0)^2}{n}$ into Equation (7), then differentiating it with respect to λ, and substituting Equation (6), we have the following two equations.

$$\frac{dSW}{d\lambda}=m(\frac{d\pi^U}{dx}\frac{dx}{d\lambda}+\frac{d\pi^U}{dx_0}\frac{dx_0}{d\lambda})+\frac{d\pi_0}{dx}\frac{dx}{d\lambda}+\frac{d\pi_0}{dx_0}\frac{dx_0}{d\lambda}+(mx+x_0)(1+\frac{2}{n})(m\frac{dx}{d\lambda}+\frac{dx_0}{d\lambda})=0,$$

$$\frac{dSW}{d\lambda}=m\{[a-\frac{n+1}{n}(mx+x_0)-\frac{n+1}{n}mx-kx]\frac{dx}{d\lambda}+(-\frac{n+1}{n}x)\frac{dx_0}{d\lambda}\}+(-\frac{n+1}{n}mx_0)\frac{dx}{d\lambda}$$
$$-(1-\lambda)[-\frac{n+1}{n}mx+\frac{n+2}{n}(mx+x_0)]\frac{dx_0}{d\lambda}+(mx+x_0)(\frac{n+2}{n})(m\frac{dx}{d\lambda}+\frac{dx_0}{d\lambda})=0.$$

Solving the above equation for λ, we have $\lambda^U=\dfrac{m[\frac{n+1}{n}x+\frac{1}{n}(mx+x_0)]}{[\frac{n+1}{n}mx-\frac{n+2}{n}(mx+x_0)\}}\dfrac{\frac{dx}{d\lambda}}{\frac{dx_0}{d\lambda}}$. By substituting $\frac{dx}{d\lambda}$ and $\frac{dx_0}{d\lambda}$ into it, we finally get the reduced form for λ^U as:

$$\lambda^U = \frac{m(n+1)nk}{n^2k^2 + kn(n+1)(m+2) + (n+1)^2}.$$

Proof of Proposition 1.

(i) Because $\lambda^U - 1 = \frac{-[n^2k^2 + 2kn(n+1) + (n+1)^2]}{n^2k^2 + kn(n+1)(m+2) + (n+1)^2} < 0$, $\lambda^U < 1$ is proven.

(ii) From Equation (9), we have $\frac{\partial \lambda^U}{\partial m} = \frac{n(n+1)k(nk+n+1)^2}{\Delta^2} > 0$ where

$\Delta = n^2k^2 + kn(n+1)(m+2) + (n+1)^2$.

(iii) From Equation (9), we have:

$$\frac{\partial \lambda^U}{\partial n} = \frac{km[-k^2n^2 + (n+1)^2]}{\Delta^2} \gtrless 0 \qquad \text{if } kn \lessgtr n+1. \tag{A1}$$

Equation (A1) shows that if $k \leq 1$, then $kn < n+1$ for any $n \geq 1$, and thus $\frac{\partial \lambda^U}{\partial n} > 0$; if $k \geq 2$, then $kn \geq n+1$ for any $n \geq 1$, and thus $\frac{\partial \lambda^U}{\partial n} < 0$; if $1 < k < 2$, then $k-1 > 0$, and thus $kn < (=, >) n+1 \Leftrightarrow n(k-1) < (=, >) 1 \Leftrightarrow n < (=, >) \frac{1}{k-1}$. In other words, given k, if $n < (=, >) \frac{1}{k-1}$, then $\frac{\partial \lambda^U}{\partial n} > (=, <) 0$.

(iv) From Equation (9), we have:

$$\frac{\partial \lambda^U}{\partial k} = \frac{mn(n+1)[-k^2n^2 + (n+1)^2]}{\Delta^2} \gtrless 0 \text{ if } kn \lessgtr n+1. \tag{A2}$$

Because $kn \lessgtr n+1 \Leftrightarrow k \lessgtr 1 + \frac{1}{n}$, and thus given n, if $k \lessgtr 1 + \frac{1}{n}$, then $\frac{\partial \lambda^U}{\partial k} \gtrless 0$.

C. Proof of Proposition 2

By the symmetric property and denoting $x_j = x \ \forall j$, Equations (11) and (12) can be simplified as follows:

$$(n+1)(m+1)x + (n+1)x_0 = an,$$
$$(n+\lambda)mx + [n + \lambda(n+2)]x_0 = an.$$

Solving x and x_0, we have the upstream equilibrium outputs: $x_0 = \frac{n[(n+1)(m+1)l_0 - (n+\lambda)ml]}{(n+1)\Phi}$, $x = \frac{n\{[n+\lambda(n+2)]l - (n+1)l_0\}}{(n+1)\Phi}$, and $mx + x_0 = \frac{n[\lambda m(a-c) + (a-c_0)]}{\Phi}$, where $\Phi \equiv n + \lambda(mn+m+n+2)$, $l \equiv a-c$, $l_0 \equiv a-c_0$.

Because we assume that x_0 and x are always positive before ($\lambda=0$) and after ($\lambda=1$) firm 0 is fully privatized, thus the condition $\frac{n+1}{n} < \frac{l}{l_0} < \frac{m+1}{m}$ must hold - that is, the cost difference between firm 0 and the other upstream private firms (i.e., $l - l_0 = c_0 - c$) should not be too high or too low. Moreover, if $m > n$, then $\frac{n+1}{n} > \frac{m+1}{m}$, the above conditions will not hold, and hence we only focus on the case of $m < n$.

The comparative statics of x_0, x, and X on λ, m, and n are as follows:

$\frac{\partial x_0}{\partial \lambda} = \frac{n(m+1)G}{\Phi^2} < 0$, $\frac{\partial x}{\partial \lambda} = \frac{-nG}{\Phi^2} > 0$, $\frac{\partial (mx + x_0)}{\partial \lambda} = \frac{nG}{\Phi^2} < 0$, where $G \equiv mn(a-c) - (mn+m+n+2)$

$(a-c_0)<0$. This is because the condition for $x_0>0$ (i.e., $(n+1)(m+1)l_0-(n+\lambda)ml>0$) implies $\frac{l}{l_0}<\frac{(n+1)(m+1)}{(n+\lambda)m}<\frac{mn+m+n+2}{mn}$, and thus we have $G=-mnl_0[\frac{(mn+m+n+2)}{mn}-\frac{l}{l_0}]<0$.

$\frac{\partial x_0}{\partial m}=\frac{n\{(n+1)[n+\lambda(n+1)]l_0-(n+\lambda)[n+\lambda(n+2)]l\}}{(n+1)\Phi^2}<0$,

$\frac{\partial x}{\partial m}=\frac{-\lambda n(n+1)\{[n+\lambda(n+2)]l-(n+1)l_0]\}}{\Phi^2}<0$, $\frac{\partial(mx+x_0)}{\partial m}=\frac{\lambda n\{[nl-(n+1)l_0+(n+2)\lambda l\}}{\Phi^2}>0$,

$$\frac{\partial x}{\partial n}=\frac{\{n^2+\lambda^2[2(m+1)(n+1)+n(n+2)+2]\}l+\lambda\{[2n(m+1)(n+1)+n]l-(n+1)^2(m+2)l_0\}}{(n+1)^2[n+\lambda(mn+m+n+2)]^2}>0, \tag{A3}$$

$$\frac{\partial x_0}{\partial n}=\frac{\Gamma}{(n+1)^2\Phi^2}, \tag{A4}$$

where $\Gamma\equiv\lambda(m+1)(m+2)(n+1)^2l_0-ml\{n^2+2\lambda n(mn+m+n+2)-\lambda^2[(m+1)(n^2-1)-1]$ and $\frac{\partial X}{\partial n}=\frac{\lambda(m+2)(\lambda ml+l_0)}{\Phi^2}>0$.

The item $[2n(m+1)(n+1)+n]l-(n+1)^2(m+2)l_0$ in the numerator of (A3) can be rearranged as $[2n(m+1)(n+1)+n]l_0[\frac{l}{l_0}-\frac{(n+1)^2(m+2)}{2n(m+1)(n+1)+n}]>0$, which is positive because of $\frac{l}{l_0}>\frac{n+1}{n}>\frac{(n+1)^2(m+2)}{2n(m+1)(n+1)+n}$, and thus we have $\frac{\partial x}{\partial n}>0$. Because $\frac{\partial\Gamma}{\partial\lambda}=(m+1)(m+2)(n+1)l_0$ $-2mn(mn+m+n+2)l\ +2\lambda l[(m+1)(n^2-1)-1]>0$, $\frac{\partial x_0}{\partial n}(\lambda=0)=\frac{-mn^2l}{(n+1)^2\Phi^2}<0$ and $\frac{\partial x_0}{\partial n}(\lambda=1)=\frac{(n+1)^2(m+2)[(m+1)l_0-ml]}{(n+1)^2\Phi^2}>0$, and there exists a critical $\vec{\lambda}\equiv\frac{2mn(mn+m+n+2)l-(m+1)(m+2)(n+1)l_0}{2[(m+1)(n^2-1)-1]}$ such that if $\lambda\underset{>}{<}\bar{\lambda}$, then $\frac{\partial x_0}{\partial n}\underset{>}{<}0$. Thus, we have $\frac{\partial x_0}{\partial n}\underset{<}{>}0$.

By substituting $p=\frac{n+1}{n}x+c$ into π_0 in Equation (7), the social welfare function can be rewritten as:

$$\begin{aligned}SW&=\frac{1}{2}(mx+x_0)^2+m\,\frac{n+1}{n}x^2+\frac{1}{n}(mx+x_0)^2+(p-c_0)x_0\\&=\frac{n+2}{2n}(mx+x_0)^2+m\,\frac{n+1}{n}x^2+\frac{n+1}{n}\,xx_0+(c-c_0)x_0.\end{aligned}$$

The first-order derivative of SW on λ is:

$$\begin{aligned}\frac{dSW}{d\lambda}&=\frac{(n+2)}{n}(mx+x_0)\frac{\partial(mx+x_0)}{\partial\lambda}+2m\,\frac{n+1}{n}\,x\,\frac{\partial x}{\partial\lambda}+\frac{n+1}{n}(x\frac{\partial x_0}{\partial\lambda}+x_0\frac{\partial x}{\partial\lambda})+(c-c_0)\frac{\partial x_0}{\partial\lambda}\\&=\frac{n^2G}{n\Phi^3}\{\lambda[(m+1)(mn+m+n+2)l_0-m(m+2)(n+1)l]-[mn(c_0-c)]\}.\end{aligned} \tag{A5}$$

From (A5), we have $\frac{dSW}{d\lambda}|_{\lambda=0}=\frac{-nG}{\Phi^3}[mn(c_0-c)]$. Thus, if $c_0\leq c$, then $\frac{dSW}{d\lambda}|_{\lambda=0}<0$, and the best policy is not to privatize firm 0; if $c_0>c$, then $\frac{dSW}{d\lambda}|_{\lambda=0}>0$, and privatization will improve welfare.

We can further see the case that $c_0 > c$ (i.e., $l > l_0$). From (A5), we also have $\frac{dSW}{d\lambda}|_{\lambda=1} = \frac{nG}{\Phi^3}[(m+1)(mn+m+n+2)+mn]l_0 - m[(m+2)(n+1)+n]l\}$. Thus, if $\frac{l}{l_0} \lessgtr \frac{(m+1)(mn+m+n+2)}{m(m+2)(n+1)}$, then $\frac{dSW}{d\lambda}|_{\lambda=1} \lessgtr 0$. It tells us that, given m, n, and c, if the difference in the marginal cost $c_0 - c = l - l_0$ is great enough, then the value of $\frac{l}{l_0} = \frac{a-c}{a-c_0}$ will be high, and fully privatization (i.e., $\lambda = 1$) is optimal; otherwise, partially privatization is the best policy. In other words, if c_0 is greater than a critical $\bar{c}_0$, then $\lambda^* = 1$; if c_0 is less than $\bar{c}_0$, then $0 < \lambda^* < 1$. By setting $\frac{dSW}{d\lambda}|_{\lambda=1} = 0$, we get $\bar{c}_0 \equiv \frac{a(m+m+2)+m(mm+m+3n+2)c}{(m+1)(mn+m+n+2)+mn}$.

When $\frac{dSW}{d\lambda}|_{\lambda=0} > 0$ and $\frac{dSW}{d\lambda}|_{\lambda=1} < 0$, by setting Equation (A5) to be zero, we can solve the optimal degree of privatization as:

$$\lambda^* = \frac{mn(c_0 - c)}{[(n+m+2)l + (m+1)(c-c_0)(mn+m+n+2)]}.$$

The effects of m and n on λ^* are respectively:

$$\frac{d\lambda^*}{dm} = \frac{n(c_0-c)[(m^2n+m^2)(c_0-c)+(n+2)l_0]}{[(n+m+2)l+(m+1)(c-c_0)(mn+m+n+2)]^2} > 0 \text{ and}$$

$$\frac{d\lambda^*}{dn} = \frac{m(c_0-c)(m+2)[(m+1)l_0 - ml]}{[(n+m+2)l-(m+1)(c_0-c)(mn+m+n+2)]^2} > 0.$$

References

Anderson, S.P., A. De Palma and J.F. Thisse (1997), "Privatization and Efficiency in a Differentiated Industry," *European Economic Review* 41, pp.1635-1654.

Barcena-Ruiz, J.C. and M.B. Garzon (2003), "Mixed Duopoly, Merger and Multiproduct Firms," *Journal of Economics* 80, pp.27-42.

Bose, A. and B. Gupta (2013), "Mixed Markets in Bilateral Monopoly," *Journal of Economics* 110, pp.141-164.

Ceriani, L. and M. Florio (2011), "Consumer Surplus and the Reform of Network Industries: A Primer," *Journal of Economics* 102, pp.111-122.

Chang, W.W. (2005), "Optimal Trade and Privatization Policies in an International Duopoly with Cost Asymmetry," *Journal of International Trade & Economic Development* 14, pp.19-42.

Chao, C.C. and E.S.H. Yu (2006), "Partial Privatization, Foreign Competition, and Optimum Tariff," *Review of International Economics* 14, pp.87-92.

Cremer, H. and M. Marchand and J. Thisse (1989), "The Public Firm as an Instrument for Regulating an Oligopolistic Market," *Oxford Economic Papers* 41, pp.283-301.

Cremer, H., M. Marchand and J.F. Thisse (1991), "Mixed Oligopoly with Differentiated Products," *International Journal of Industrial Organization* 9, pp.43-53.

Dadpay, A. and J.S. Heywood (2006), "Mixed Oligopoly in a Single International Market," *Australian Economic Papers* 45, pp.269-280.

De Fraja, G. and F. Delbono (1989), "Alternative Strategies of a Public Enterprise in Oligopoly," *Oxford Economic Papers* 41, pp.302-311.

De Fraja, G. and F. Delbono (1990), "Game Theoretic Models of Mixed Oligopoly," *Journal of Economic Surveys* 4, pp.1-17.

De Fraja, G. and B.M. Roberts (2009), "Privatization in Poland: What Was the Government Trying to Achieve?", *Economics of Transition* 17, pp.531-557.

Fershtman, C. (1990), "The Interdependence between Ownership Status and Market Structure: The Case of Privatization," *Economica* 57, pp.319-328.

Fjell, K. and D. Pal (1996), "A Mixed Oligopoly in the Presence of Foreign Private Firms," *Canadian Journal of Economics* 29, pp.737-743.

Fujiwara, K. (2007), "Partial Privatization in a Differentiated Mixed Oligopoly," *Journal of Economics* 92, pp.51-65.

Gangopadhyay, P. (2005), "The Optimal Access Price in a Vertically Related Industry," *Economic Analysis and Policy* 35, pp.91-102.

George, K. and M.L. Manna (1996), "Mixed Duopoly, Inefficiency, and Public Wwnership," *Review of Industrial Organization* 11, pp.853-860.

Greenhut, M., H. Ohta (1979), "Vertical Integration of Successive Oligopolies," *American Economic Review* 69, pp.137-141.

Han, L. and H. Ogawa (2007), "Partial Privatization and Market-opening Policies: A Mixed Oligopoly Approach," Economic Research Center (Nagoya University), Discussion Paper.

Han, L. and H. Ogawa (2012), "Market-demand Boosting and Privatization in a Mixed Duopoly," *Bulletin of Economic Research* 64, pp.125-134.

Husain, A.M. (1994), "Private Sector Development in Economies in Transition," *Journal of Comparative Economics*19, pp.260-271.

Lee, J. (2009), "State Owned Enterprises in China: Reviewing the Evidence," OECD Working Group on Privatisation and Corporate Governance of State Owned Assets.

Lee, S.H. and H.S. Hwang (2003), "Partial Ownership for the Public Firm and Competition," *Japanese Economic Review* 54, pp.324-335.

Lee, S.H. (2006), "Welfare-improving Privatization Policy in the Telecommunications Ondustry," *Contemporary Economic Policy* 24, pp.237-248.

Lin, M.H. and T. Matsumura (2012), "Presence of Foreign Investors in Privatized Firms and Privatization Policy," *Journal of Economics* 107, pp.71-80.

Lu, Y. and S. Poddar (2007), "Firm Ownership, Production Differentiation and Welfare," *The Manchester School* 75, pp.210-217.

Matsumura, T. (1998), "Partial Privatization in Mixed Duopoly," J*ournal of Public Economics* 70, pp.473-483.

Matsumura, T. and O. Kanda (2005), "Mixed Oligopoly at Free Entry Markets," *Journal of Economics* 84, pp.27-48.

Matsumura, T. and N. Matsushima and I. Ishibashi (2009), "Privatization and Entries of Foreign Enterprises in a Differentiated Industry," *Journal of Economics* 98, pp.203-219.

Matsumura, T. and D. Shimizu (2010), "Privatization Waves," *Manchester School* 78, pp.609-625.

Matsumura, T. and N. Matsushima (2012), "Airport Privatization and International Competition," *Japanese Economic Review* 64, pp.431-450.

Matsumura, T. and A. Ogawa (2012), "Price vs. Quantity in a Mixed Duopoly," *Economics Letters* 116, pp.174-177.

Matsumura, T. and T. Sunada (2013), "Advertising Competition in a Mixed Oligopoly," *Economics Letters* 119, pp.183-185.

Matsushima, N. and T. Matsumura (2003), "Mixed Oligopoly and Spatial Agglomeration," *Canadian Journal of Economics* 36, pp.62-87.

Matsushima, N. and T. Matsumura (2006), "Mixed Oligopoly, Foreign Firms, and Location Choice," *Regional Science and Urban Economics* 36, pp.753-772.

Merrill, W. and N. Schneider (1966), "Government Firms in Oligopoly Industries: a Short Run Analysis," *Quarterly Journal of Economics* 80, pp.400-412.

Mujumdar, S. and D. Pal (1998), "Effects of Indirect Taxation in a Mixed Oligopoly," *Economic Letters* 58, pp.199-204.

Mukherjee, A. and K. Suetrong (2009), "Privatization, Strategic Foreign Direct Investment and Host-country Welfare," *European Economic Review* 53, pp.775-785.

Nishimori, A. and H. Ogawa (2002), "Public Monopoly, Mixed Oligopoly and Productive Efficiency," *Australian Economic Papers* 41, pp.185-190.

Ohori, S. (2012), "Environmental Tax and Public Ownership in Vertically Related Markets," *Journal of Industry, Competition and Trade* 12, pp.169-176.

Pal, D. and M.D. White (1998), "Mixed Oligopoly, Privatization, and Strategic Trade Policy," *Southern Economic Journal* 65, pp.264-281.

Pal, D. (1998), "Endogenous Timing in a Mixed Oligopoly," *Economics Letters* 61, pp.81-185.

Pao, H.W., H.L. Wu and W.H. Pan (2008), "The Road to Liberalization: Policy Design and Implementation of Taiwan's Privatization," *International Economics and Economic Policy* 5, pp.323-344.

Spencer, B.J. and R.W. Jones (1991), "Vertical Foreclosure and International Trade Theory," *Review of International Studies* 58, pp.153-170.

Spencer, B.J. and R.W. Jones (1992), "Trade and Protection in Vertically Related Markets," *Journal of International Economics* 32, pp.31-55.

Stähler, F. and S. Traub (2009), "Privatization and Liberalization in Vertically Linked Markets," Discussion Paper of University Bremen.

Vickers, J. (1995), "Competition and Regulation in Vertically Related Markets," *Review of Economic Studies* 62, pp.1-17.

Wang, L.F.S. and T.L. Chen (2010), "Do Cost Efficiency Gap and Foreign Competitors Matter Concerning Optimal Privatization Policy at the Free Entry Market?", *Journal of Economics* 100, pp.33-49.

Wang, L.F.S. and T.L. Chen (2011), "Privatization, Efficiency Gap and Subsidization with Excess Tax Burden," *Hitotsubashi Journal of Economics* 52, pp.55-68.

Wen, J.F. and L. Yuan (2010), "Optimal Privatization of Vertical Public Utilities," *Canadian Journal of Economics* 43, pp.816-831.

White, M.D. (1996), "Mixed Oligopoly, Privatization and Subsidization," *Economics Letters* 53, pp.189-195.

Willner, J. (2008), "Liberalisation, Competition and Ownership in the Presence of Vertical Relations," *Empirica* 35, pp.449-464.