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ROMER MEETS HETEROGENEOUS WORKERS IN AN ENDOGENOUS GROWTH MODEL*

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Abstract

This paper extends Romer's (1990) endogenous growth model by incorporating the heterogeneity of skills among workers. Based on this heterogeneous characteristic, our model has an endogenous labor allocation mechanism determined by the shape of the skill distribution of the workers. Workers are divided between the research and production sectors according to the demand and supply conditions of the economy for their specific skills. We also show that the long-run growth rate of the economy can be characterized by the allocation mechanism.

Keywords: endogenous growth, heterogeneity, labor allocation
JEL Classification Codes: E24, F15, F41

I. *Introduction*

This paper analyzes the effects of endogenous labor allocation on the long-run growth of an economy by incorporating the heterogeneity of worker skills into the Romer (1990) model. The relationship between endogenous accumulation of productive inputs and productivity growth of the economy has been the subject of an important branch in endogenous growth models. However, less attention is paid to the allocation of productive inputs – in particular labor. As discussed in Romer (1990), homogenous workers in a multi-sector model are

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randomly assigned to each sector without incurring any loss of the model's general implications since wages are the same regardless of the sector. However, in labor economic literature, there is incontrovertible evidence of workers' endogenous decisions on their jobs, necessitating growth models that incorporate these features.

Apart from Romer (1990), we analyze the heterogeneity of workers' abilities and characterize the steady-state growth rates. By incorporating workers' heterogeneity into the Romer model, we investigate a labor allocation mechanism between research (skilled) and production (less-skilled) sectors. Workers are self-sorted by individual optimizing behaviors conditional on their skill levels. This labor allocation is determined by the equilibrium threshold skill level, reflecting the skill demand and supply conditions of the economy.

Our model follows up on two seminal works in endogenous growth literature: Romer (1990) and Acemoglu (1998). In Romer (1990), the equilibrium ratio of workers between research and production sectors is determined only by the demand side of the economy. Thus, any change in the distribution of skills among the workers has no effect on this ratio. On the contrary, in Acemoglu (1998), the equilibrium ratio is directly determined by the given distribution of skills among the workers, and the demand side conditions play no role in determining the equilibrium ratio because the model assumes that all of the skilled workers are always employed in the skilled sector.

In contrast to the two models, our model explains that the equilibrium ratio is determined by both the skill demand and supply conditions in the labor market. We also show that the growth rate of productivity along with the balanced growth path (BGP) is expressed as a function of the equilibrium threshold skill level. Since the equilibrium threshold skill level is affected not only by the total quantity of human capital in the economy but also by the distribution of the worker's skill level, our model is effective in explaining the role of human capital distribution in TFP growth, whereby productivity growth is closely related to the increase in skilled workers rather than the increase in the gross amount of human capital of the economy.[1]

The strategy of modeling endogenous skill acquisition in our model is quite similar to that of Dinopoulos and Segarstrom (1999) in that individuals differ in their ability and decide to become a skilled or unskilled worker based on that ability. It is also similar in that the model has an equilibrium threshold skill level that divides workers into the skilled and unskilled, and the productivity of the economy depends on the equilibrium threshold skill level. However there are several distinctions between the models. For instance, our model focuses on the effect of skill distribution on the long-run productivity growth rate of the economy while Dinopoulos and Segarstrom (1999) study the effect on trade liberalization. The education cost in Dinopoulos and Segarstrom (1999) is an opportunity cost rooted in the training period. Our model explicitly considers the education sector and analyzes the social optimum by government policy.

The heterogeneous model provides for the possibility that the effect of human capital on the long-run growth rate of productivity could be smaller than that predicted in Romer (1990). Our model shows that the effect of increasing human capital on the long-run growth rate of productivity should be affected by both skilled labor supply and demand. This feature of the model is advantageous in explaining the education puzzle on human capital and long-run

[1] There is vast literature on the composition effect of human capital on TFP growth. See Iranzo and Peri (2006), Vandenbussche, Aghion and Meghir (2006), and Papagiouge (2003).

growth rate of the economy. Pritchett (2001) reports that the rate of return on education is estimated to be quite high from the standpoint of individuals, but weaker correlations between the increase in human capital and output per worker are found in cross-national data. In an attempt to explain this puzzle, Pritchett (2001) argues that the effect of increasing the supply of human capital from education on the long-run growth rate of output per worker could be limited when the demand for human capital is stagnant. This argument is supported by the fact that countries with high growth rates in both human capital and employment of the skilled sector exhibit relatively high growth rates of output per worker. Our model is consistent with this line of argument. The prediction of the model shows that the share of skilled labor plays a crucial role in explaining the long-run growth of an economy.

The basic structure of the model is as follows. The technological level of an economy is represented by the number of varieties of intermediate goods. The economy employs three types of workers; final good producers, researchers and teachers. The researchers invent new designs of intermediate goods. The final good producers do so by inputting labor and intermediate goods under the given level of technology. Teachers influence newly born agents to obtain education to acquire skills. Workers are allowed to match their heterogeneous skill characteristics to a working sector. This labor allocation based on the heterogeneity of workers is affected by the supply and demand conditions of the economy for those skills. The long-run growth rate of the economy is characterized by the labor allocation.

In the presence of externalities in R&D according to Romer (1990), optimal growth paths will not coincide with competitive equilibrium paths. To see this, we solve for the first-best equilibrium from the centralized planned economy and compare the outcomes with the results from a parallel problem in a decentralized economy. We find that the decentralized economy under-provides skilled workers and, as a result, experiences lower growth, which is consistent with Romer (1990). This finding may justify the widespread prevalence of education subsidies in many countries.

The remainder of the paper is organized as follows. Section II describes the model and characterizes the balanced growth path equilibrium of the model. Section III presents comparative static results. Section IV shows the efficient equilibrium under a centrally planned economy and characterizes government policies to attain the social optimum. Section V concludes our research.

II. *The Model*

1. Description of the Model

The basic structure of the model is the same as the standard R&D based endogenous growth models. The economy consists of three sectors: final good, intermediate good and research. All workers belong to either the less-skilled (final good) or skilled (research) sector. In contrast to Romer (1990), workers are heterogeneous in their innate abilities. Each agent draws from her innate ability when she is born. Based on this innate ability, each worker pursues an optimal level of investment in education and acquires skills. Technological progress takes the form of an increasing number of varieties of intermediate goods. Final goods serve as the numeraire and the only consumable good, which is produced in the final good sector by inputting the labor of production workers and the variety of intermediate goods.

The final good sector hires production workers and produces the final good. The research sector hires researchers and teachers. The researchers invent new intermediate good designs, and the teachers instruct newly born agents who are willing to acquire skills. With licenses, intermediate good producing firms produce intermediate goods with one-to-one transfer technology from final good to intermediate goods. Labor supply is inelastic since it is assumed that there is no utility from leisure, and all the workers are employed. The labor allocation between the two working sectors is guaranteed because the worker with a relatively higher skill level has a comparative advantage working in the research sector, and the worker with the lower skill level has a comparative advantage working in the final good sector. This comparative advantage condition matches workers who are optimally allocated between the research and production sectors.[2] Individual optimality will then enable workers to self-sort into the research and production sectors based on their skill level.

To show variations in how important an individual's skill level is to the worker's productivity in different kinds of jobs, it is assumed that each worker's productivity in the research sector is an increasing function of the individual's skill level. On the other hand, the production worker's productivity is independent of the individual's skill level.[3] Given this setting, the worker's skill level is closely related to his labor productivity when he works in the research sector rather than the production sector. For the sake of simplicity, we assume that the population is constant and normalized to one, enabling us to drop the distinction between aggregate and individual quantities in equilibrium. We assume that every agent has perfect foresight. As we are only interested in the long-run growth path of the economy, this assumption will be innocuous for our purpose. Detailed descriptions of the formal model of the economy are presented below.

2. Structure of the Economy

According to literature on the R&D-based endogenous growth model, the structure of the production side of the economy is standard where the production side of the economy consists of three sectors: final good, intermediate good and research.

First, the final good sector produces the final good, and the instantaneous production function for the final good is given as equation (1). The final good market is assumed to be perfectly competitive, and the price of the final good is normalized to one.

$$Y_t \equiv A_t \cdot L_{Y,t}{}^{1-\alpha} \cdot \int_0^{N_t} x_{j,t}{}^{\alpha} dj \quad (0 < \alpha < 1) \tag{1}$$

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[2] Assume that for $\forall z \in [0,\infty)$,

(i) $\dfrac{\partial \varphi_R(z)}{\partial z} \geq 0$ *and* $\dfrac{\partial \varphi_Y(z)}{\partial z} \geq 0$ (ii) $\dfrac{\partial \varphi_R(z)}{\partial z}\dfrac{1}{\varphi_R(z)} > \dfrac{\partial \varphi_Y(z)}{\partial z}\dfrac{1}{\varphi_Y(z)}$

where $\varphi_R(z)$ and $\varphi_Y(z)$ represent the z-skilled worker's productivity when she works in the research and production sectors, respectively. Under this assumption, we may have a super-modular surplus function which matches each worker to her working place. The only stable matching under this super-modularity condition is assortative matching which maximizes the total amount of surplus (i.e., maximizes the aggregate labor productivity of the economy)

[3] This strategy of modeling follows Dinopoulos and Segarstrom (1999). They assume that all unskilled workers earn the same wage independent of their ability, but skilled workers with higher abilities earn higher wages.

where A_t is exogenously given an economy-wide productivity parameter, $L_{Y,\,t}$ is the number of workers in the production sector at time t, N_t is the variety of intermediate goods, and $x_{j,\,t}$ is the total amount of input of j-th intermediate good at time t. Note that each worker's heterogeneous skill level does not appear in the final good production function since the final good output depends only on the number of workers in the final good sector.

The instantaneous wage of production workers is determined by their productivity. Since the productivity of workers in the production sector is the same independent of their skill levels, their wages should be the same as equation (2). For all types of intermediate good j, the demands are symmetric as equation (3).

$$MP_{L_Y}=\frac{\partial Y}{\partial L_Y}=(1-\alpha)\frac{Y}{L_Y}=w^Y \tag{2}$$

$$MP_{x_j}=\frac{\partial Y}{\partial x_j}=AL_Y^{1-\alpha}\alpha\cdot x_j^{\alpha-1}=p_j \quad\Rightarrow\quad x_j=L_Y\left(\frac{A\alpha}{p_j}\right)^{\frac{1}{1-\alpha}} \tag{3}$$

where p_j is the price of intermediate good j.

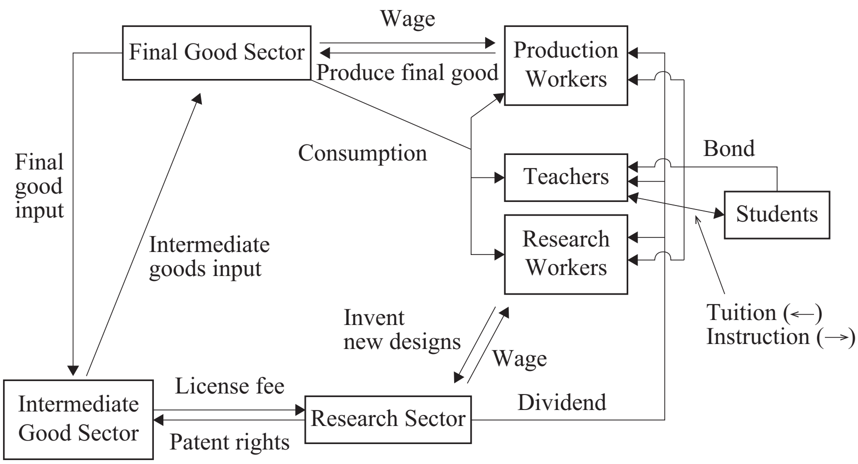
Second, the intermediate goods sector consists of a number of monopolistic intermediate good producing firms. It is assumed that one unit of intermediate good is produced by transferring one unit of final good. No labor is needed in this process. To produce each intermediate good, the intermediate good producing firm would have a license for using each design of the intermediate good. Hence, they would pay license fees to the research sector. Since the price of the final good is normalized to one, the marginal cost for producing one unit of an intermediate good is one. Under the assumption of free entry and perfect patent protection, for each intermediate good, only one firm produces the good. The profit maximizing prices of intermediate goods are shown as equation (4), and the instantaneous profit from each intermediate good j can be derived as equation (5).

$$\underset{p_j}{Max}\,[p_j-1]\cdot x_{j,t} \quad\Rightarrow\quad p_j=\frac{1}{\alpha} \qquad \text{for all } j\in N_t \tag{4}$$

$$\pi_t^j=\left(\frac{1-\alpha}{\alpha}\right)\cdot A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\cdot L_{Y,\,t} \qquad \text{for all } j\in N_t \tag{5}$$

Third, the research sector employs researchers and teachers. The researchers invent new designs of the intermediate goods. Each researcher with a skill level of z invents $\delta(z)\cdot N_t$ number of new designs at each time t. Let $\delta(z)=e^{\gamma z}$ denote the labor productivity of the researcher with skill level z, and N_t be the total number of designs in the economy at time t. The "γ" is the productivity parameter which represents an economy-wide degree of skill biased technology because the higher value of the parameter causes higher productivity of researchers. But the productivity of the final good producing worker is independent of the parameter value. In sum, a researcher's productivity is determined by three factors; individual skill level ($z\in[0,\infty)$), skill biased technology parameter (γ), and the number of varieties in the economy (N). Each researcher's instantaneous wage (w^R) is equal to the total market value of designs

Fig. 1. Structure of the Model



invented by the researcher.

$$w^R(z)=p_A^j\cdot e^{\gamma z}\cdot N_t \qquad \text{for all } j\in N_t,\ z\in[z_1,\ \infty) \tag{6}$$

where p_A^j is the present value of patent right for the j-th intermediate good design.

The teachers instruct newly born agents who are willing to acquire skills. The productivity of the teachers also depends on their skill level. It is assumed that the teachers are randomly selected among the workers in the research sector. This assumption does not lead to any loss of generality since their wages are the same, regardless of their positions of researcher or teacher.[4]

The researcher's wage is the only cost for inventing new designs. The research sector owns all the existing patent rights as equity assets owned by individual workers. At each time, the research sector rents the patent rights of designs to intermediate good producing firms and earns license fees. This instantaneous license fee may exceed the total payments of wages for researchers. The research sector then distributes all of the net profits to its shareholders as dividends. Figure 1 summarizes how these three sectors are linked each other.

3. Individual Behavior

There is a continuum of heterogeneous agents in terms of innate ability. We assume that every agent draws her innate ability from a given decreasing exponential distribution ($f(q)=\lambda e^{-\lambda q}$, $\lambda>0$, $q\in[0,\ \infty)$) when born. The individual agent i's innate ability is indexed by q_i. This innate ability is interpreted as a maximum attainable skill level that each worker can reach through education. The setup for individual optimization is based on the perpetual youth model introduced by Yaari (1965) and Blanchard (1985). Every agent faces a constant instantaneous probability of death p (≥ 0). It is assumed that the size of the cohort of newborns is exactly equal to the size of the dead at any point in time. Hence, the total population size is

[4] Further explanation on this provided in section II.3.

FIG. 2. LIFE-TIME STRUCTURE OF t_0-GENERATION

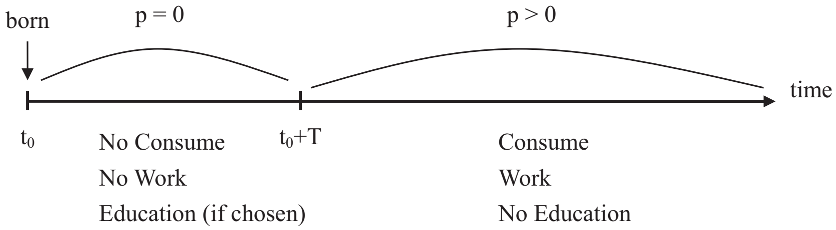
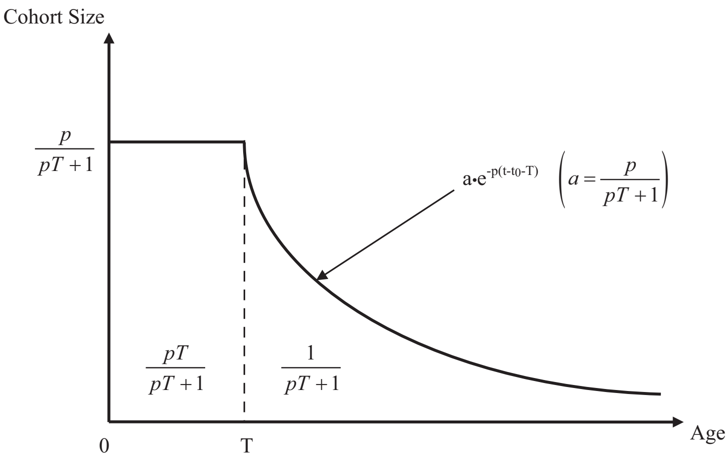


FIG. 3. DEMOGRAPHIC PROFILE OF THE ECONOMY[6]



constant over time. Further, to simplify, it is assumed that every agent has a special period of time at the beginning of her life from birth to age T.[5] During this special period, the agent does not die (i.e., p=0), consume, or work. The only possible activity during this period is to learn to acquire job skills. If an agent pursues education, she uses this period for learning. Otherwise, this time is idle. Figure 2 illustrates the life-time structure for a representative agent who is born at time t_0 based on the assumptions as described above. Figure 3 shows the demographic profile of the economy at each time t in terms of the agent's ages. The demographic profile can be drawn from the set-up of the model. To normalize the total population size of the economy and maintain this population size over time, at each instant of time, $p/(pT+1)$ number of agents die, and the same number of agents are born. Since all the agents do not die before their age T, the population of the agents under age T is $pT/(pT+1)$.

The agents who choose education need to pay tuition. All the tuition goes to the teachers

[5] That is, for an agent who is born at time t_0, $t \in [t_0, t_0+T]$.

[6] Total population size is normalized as unity. At each instant of time, $p/(pT+1)$ number of agents dies, and the same number of agents is newly born. The share of agents who are in the juvenile period is $pT/(pT+1)$. This demographic profile does not change over time.

for salary. The education period is fixed and the same for everybody in a cohort of those born at t_0 as $t \in [t_0, t_0+T]$ regardless of individual degree of learning. But the agent who seeks more learning to reach a higher skill level needs to pay more tuition. That is, the instantaneous tuition at each time for an agent who seeks to acquire z_i skill level is "$\tau \cdot w_t^R(z_i)$" where t (0<t<1) denotes the amount of tuition in terms of expected wage in the research sector with that skill level.[7]

Teachers are randomly selected among the educated workers in the research sector, and one teacher takes a class of "$1/\tau$" number of students in terms of efficient unit of labor. The efficient unit of labor takes into account the different productivities of the teachers who have different skill levels. That is, a teacher who has skill level z_i can instruct "$1/\tau$" number of students who seek z_i skill level. In other words, a teacher who has skill level $2z_i$ can instruct either "$2/\tau$" number of students who seek z_i skill level or "$1/\tau$" number of students who seek $2z_i$ skill level. Since the instantaneous tuition for the agent who seeks z_i skill level at time t is "$\tau \cdot w_t^R(z_i)$" and this tuition is continuously paid during the whole education period ($t \in [t_0, t_0+T]$), the teacher who has z_i skill level will collect a total of "$w_t^R(z_i)$" from her educational services. Therefore, there is no loss of generality in the random selection of teachers from the pool of workers in the research sector because their wages are the same, regardless of their position as researcher or teacher.

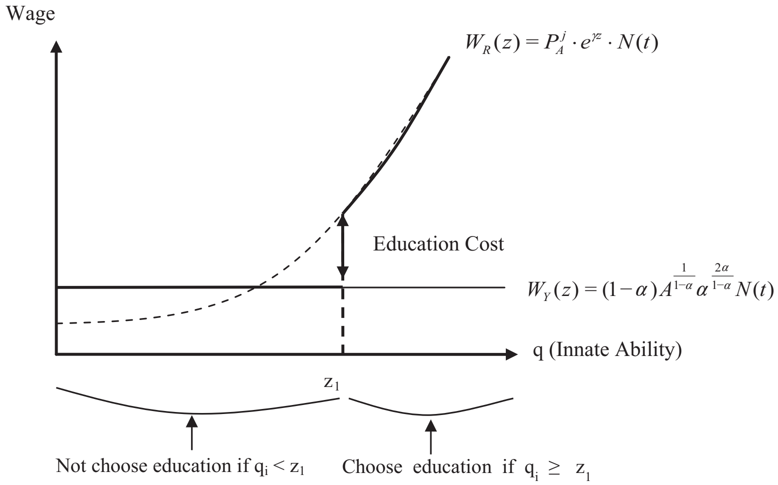
Since there is no income during the education period, education cost should be funded by issuing bonds. Consequently, the agent who chooses z_i level of education will make "$b_{t_0+T}(z_i) = \int_{t_0}^{t_0+T} \tau \cdot w_t^R(z_i) \cdot e^{-\int_{t_0+T}^{t} r(s)ds} dt$" amounts of debt at time t_0+T. Each agent can obtain a higher level of skills by investing more up to her maximum level of innate ability. For all the agents, the instantaneous tuition "$\tau \cdot w_t^R(z_i)$" is a constant share of the agent's expected wage "$w_t^R(z_i)$". Thus, we can see once that when an agent decides to invest in her education, the optimal choice of investment is to reach her maximum innate level. Therefore, the individual *ex-post* skill level after education will be equal to her maximum level of innate ability, once the agent chooses education. Conversely, for the agent who does not decide to get an education, her skill level is equal to zero regardless of her innate ability. Note that each worker's wage is determined by her labor productivity, and this labor productivity is determined by not only her *ex-post* skill level, but also her working sector. The labor productivity of a worker who works in the research sector is proportional to her skill level, but that of a worker in the final good sector is the same as any other in the sector. Since every agent is assumed to have perfect foresight, only the agent who decides to work in the research sector (either as a researcher or teacher) will choose education in the equilibrium. This kind of self-sorting of the labor is determined by individual optimizing behavior according to their heterogeneous innate abilities based on the wages in each working sector as drawn in equation (2) and (6). There exists a unique equilibrium threshold ability (z_1) which divides workers into the production and research sectors. This will be confirmed in section II.4. Figure 4 illustrates the wage profile of the economy and shows conceptually how the skill acquisition decisions are chosen by agents according to their innate abilities.[8]

Therefore, if there is an equilibrium threshold level that divides the workers into the

[7] τ is a constant number.

[8] More detailed illustrations provided in Figure 5.

Fig. 4. Innate Ability and Human Capital Acquisition Decision



research and production sectors, then the relationship between individual innate ability (q_i) and *ex-post* skill level (z_i) can be expressed as equation (7).[9]

$$\begin{cases} z_i = 0 & \text{if } q_i \in [0, z_1) \\ z_i = q_i & \text{if } q_i \in [z_1, \infty) \end{cases} \tag{7}$$

Although the workers are heterogeneous in their skill and income level, it is assumed that their utility functions are all the same. Every agent chooses her optimal flow of consumption to maximize lifetime utility. No bequest motive is assumed here, but there is an unintended bequest because agents do not know when they die. It is assumed that all of the remaining assets of the agents who die at time $t_0 + T$ are gathered and equally distributed to the new generation who are born at time t_0 at the point of time $t_0 + T$.[10] The dynamic optimization problem for each agent born at time t_0 is expressed as follows.

[9] Thus, given the exponential distribution ($f(q) = \lambda e^{-\lambda q}$, $\lambda > 0$, $q \in [0, \infty)$), the distribution function for all individual *ex-post* skill-levels will be

$$h(z) = \begin{cases} 1 - e^{-\lambda z_1} & \text{if } z = 0 \\ 0 & \text{if } z \in (0, z_1) \\ \lambda e^{-\lambda z} & \text{if } z \in [z_1, \infty) \end{cases}$$

[10] Alternatively, we may consider a competitive insurance market that provides a reverse life insurance. That is, each agent receives $p \cdot a_t$ at each instant of time if she survives, but pays her entire asset (a_t) when she dies. The insurance companies gather all of the remaining assets from the dead and distribute the assets to the survivors according to their asset amount (i.e. each survivor i gets $p \cdot a^i(t)$). The main implication of the model is still preserved under this alternative setting. The only difference in this case is that the BGP growth rate of the economy is "r-ρ" rather than "r-ρ-p." See Appendix A for proof.

$$Max\int_{t_0+T}^{\infty}\log c_t\cdot e^{-(\rho+p)(t-t_0-T)}dt$$
$$s.t. \qquad \frac{da_t}{dt}=r\cdot a_t+w_t-c_t \tag{8}$$

where "ρ" is the pure rate of individual time preference, "p" denotes the instantaneous probability of death, "r" is the rate of return, "c_t" is consumption, "a_t" is asset holding,[11] and "w_t" is wage.

For every instant of time, the size of newly born agents is the same as the size of agents who die, so the population size is fixed over time. Therefore, regardless of the individual heterogeneous skill level, the amount of assets owned by each production worker at time t_0+T is

$$a^Y_{t_0+T}=V_{t_0+T}+B_{t_0+T} \tag{9}$$

where V_{t_0+T} and B_{t_0+T} denote the total value of equity in the research sector and the total value of bonds in the economy, respectively. Unintended bequests are included in bonds. As the total size of population is normalized to one, the total amount of assets of the economy is the same as the amount of assets per capita.

For the agent who chooses education, the amount of assets at time t_0+T is different for the production workers who do not choose education because they have to pay the education cost which will be funded by issuing bonds. The education cost will vary with workers according to their desired skill levels (z_i). Thus, the amount of assets held by the researcher (or teacher) who acquires z_i skill level at time t_0+T can be expressed as follows:[12]

$$a^R_{t_0+T}(z_i)=V_{t_0+T}+B_{t_0+T}-b_{t_0+T}(z_i) \tag{10}$$
$$\text{where } b_{t_0+T}(z_i)=\int_{t_0}^{t_0+T}\tau\cdot w^R_t(z_i)\cdot e^{-\int_{t_0+T}^{t}r(s)ds}dt$$

4. Balanced Growth Path Equilibrium of the Model

The competitive equilibrium of the economy is defined by the following objects; the time paths of aggregate quantities of consumption, final good production and intermediate good production $\{C_t, Y_t, X_t\}_{t=0}^{\infty}$, consumption for each agent $\{c^i_t\}_{i\in Z,\, t=0}^{\infty}$, number of designs for intermediate goods in the economy $\{N_t\}_{t=0}^{\infty}$, prices and quantities for each intermediate good and instantaneous profits from each design $\{p^j_t, x^j_t, \pi^j_t\}_{j\in N(t),\, t=0}^{\infty}$, interest rate $\{r_t\}_{t=0}^{\infty}$, each individual's wage and quantity of asset holdings of equity and bonds $\{w^i_t, v^i_t, b^i_t\}_{i\in Z,\, t=0}^{\infty}$, aggregate quantity of

[11] For all $i\in Z$, $a^i(t)=v^i(t)+b^i(t)$. Note that the rate of return for equity is the same as the rate of return for bonds, and there is no uncertainty in this economy. Thus, these two kinds of assets are indifferent for asset holders.

[12] Individual assets are composed of equity and bonds, but we need not be concerned with the individual portfolio of asset holdings between equity and bonds because these assets are indifferent in terms of return and risk under perfect foresight.

TABLE 1. SUMMARY OF INDIVIDUAL WORKER'S OPTIMAL SOLUTION

Workers in the production sector ($\forall i,\ q_i < z_1$)	**Workers in the research sector** ($\forall i,\ q_i \geq z_1$)
For all $t \geq t_0 + T$, from equation (15) and (16), ($g = r\text{-}\rho\text{-}p$, BGP growth rate of the economy)	
$w_t^Y(z_i) = w_t^Y = w_{t_0+T}^Y \cdot e^{g(t-t_0-T)}$ $c_t^Y = (\rho+p)[a_t^Y + h_t^Y]$ where $h_t^Y = \int_t^\infty w_s^Y \cdot e^{-r(s-t)} ds = \frac{1}{\rho+p} \cdot w_t^Y$	$w_t^R(z_i) = w_{t_0+T}^R(z_i) \cdot e^{g(t-t_0-T)}$ $c_t^R(z_i) = (\rho+p)[a_t^R(z_i) + h_t^R(z_i)]$ where $h_t^R(z_i) = \int_t^\infty w_s^R(z_i) \cdot e^{-r(s-t)} dt = \frac{1}{\rho+p} \cdot w_t^R(z_i)$
Thus, we can get the optimal consumption path as	
$c_t^Y = (\rho+p) \cdot a_t^Y + w_t^Y$	$c_t^R(z_i) = (\rho+p) \cdot a_t^R(z_i) + w_t^R(z_i)$
From the budget constraint of the dynamic optimization problem, the individual optimal growth rate of asset holding is	
$\frac{da_t^Y}{dt} = r \cdot a_t^Y + w_t^Y - (\rho+p) \cdot a_t^Y - w_t^Y$ $= (r-\rho-p) \cdot a_t^Y = g \cdot a_t^Y$	$\frac{da_t^R(z_i)}{dt} = (r-\rho-p) \cdot a_t^R(z_i) = g \cdot a_t^R(z_i)$
Therefore, the BGP consumption, wage income and amount of asset holdings for each individual worker in the production sector can be expressed as follows	
$c_t^Y = c_{t_0+T}^Y \cdot e^{g(t-t_0-T)}$ $w_t^Y(z_i) = w_{t_0+T}^Y \cdot e^{g(t-t_0-T)}$ $a_t^Y = a_{t_0+T}^Y \cdot e^{g(t-t_0-T)}$	$c_t^R(z_i) = c_{t_0+T}^R(z_i) \cdot e^{g(t-t_0-T)}$ $w_t^R(z_i) = w_{t_0+T}^R(z_i) \cdot e^{g(t-t_0-T)}$ $a_t^R(z_i) = a_{t_0+T}^R(z_i) \cdot e^{g(t-t_0-T)}$

equity and bonds $\{V_t, B_t\}_{t=0}^{\infty}$ and threshold level which determines the labor allocation between the research and production sectors $\{z_{1,t}\}_{t=0}^{\infty}$. The solution of the optimization problem is summarized in Table 1.[13]

As with the Romer (1990) model, the simplest way to characterize both optimal and equilibrium paths is to solve for a balanced growth path (BGP) equilibrium in which Y, C, and N grow at the same constant exponential rates. As an endogenous growth model with one kind of input for production, like the AK model and Romer (1990), our model has no transitional dynamics. We focus on characterizing the BGP equilibrium of the economy in this section. The BGP equilibrium is determined by both supply and demand conditions of skilled labor. The supply condition depends on the shape of workers' distribution in terms of their skill levels. The demand condition comes from the preference of agents in the economy which determines how much of the economy's resources are allocated into research and final good production. Note that the workers in the final good sector works for present consumption, while the workers in the research sector works for future consumption. In this sense, we can say that the demand condition is based on the economy-wide decision of optimal inter-temporal resource allocation for consumption smoothing. With the supply and demand conditions, the economy has the unique and time invariant BGP equilibrium threshold level (z_1), which divides workers into research and production sectors. This will be confirmed later in this section. Given the fixed size of the population, the BGP equilibrium number of workers in each working sector is also

[13] The formal derivation is available from the authors upon request.

time invariant to z_1. In other words, given the equilibrium threshold z_1, the number of workers in the final good production sector (L_Y) and the research sector (L_R) are determined as follows:

$$L_Y=\frac{1}{pT+1}\cdot\left[\int_0^{z_1}h(z)\cdot dz\right]=\frac{1}{pT+1}\cdot[1-e^{-\lambda z_1}] \tag{11}$$

$$L_R=\left(\frac{1}{pT+1}\right)\cdot\left[\int_{z_1}^{\infty}h(z)\cdot dz\right]=\left(\frac{1}{pT+1}\right)\cdot[e^{-\lambda z_1}] \tag{12}$$

where *h(z)* is the probability density function of workers' skill distribution.[14]

From equation (5), the instantaneous profit from each design of an intermediate good is also a time invariant function of z_1.

$$\pi_t^j=\left(\frac{1-\alpha}{\alpha}\right)\cdot A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\cdot\frac{1}{pT+1}\cdot[1-e^{-\lambda z_1}] \qquad \text{for all } j\in N_t \tag{13}$$

From equation (1) and (3), the total amount of final good production of the economy at time t can be expressed as equation (14). By combining equation (2) and (6), we can obtain the wages for workers in each production and research sector as equation (15) and (16) respectively.

$$Y_t=A\cdot L_{Y,t}{}^{1-\alpha}\cdot N_t\cdot x_j{}^{\alpha}=A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}\cdot L_{Y,t}\cdot N_t \tag{14}$$

$$w_t^Y(z)=w_t^Y=(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}\cdot N_t \qquad \text{for all } z\in[0,\ z_1) \tag{15}$$

$$w_t^R(z)=P_A^j\cdot e^{\gamma z}\cdot N_t \qquad \text{for all } z\in[z_1,\ \infty),\ j\in N_t \tag{16}$$

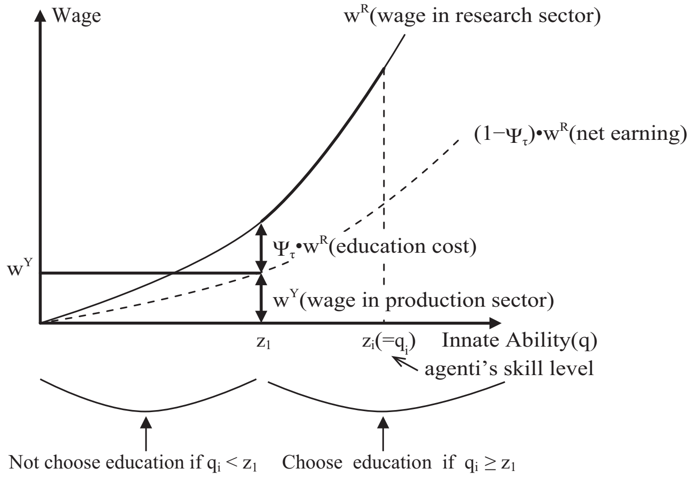
Therefore, we can see that the wages are growing at the same rates on the BGP for all the workers. This growth rate is the same as those for the number of varieties, final good production and aggregate consumption (i.e. $g=\dot{N}/N=\dot{Y}/Y=\dot{C}/C$). Also, the instantaneous tuition "$\tau\cdot w_t^R$" is also growing at the same rate of economic growth.

The equilibrium threshold level z_1 is determined from individual optimizing behavior. Under the assumption of perfect foresight, all agents succeed in complete consumption smoothing over time. Thus, as shown in equation (17), the agents only need to compare instantaneous amounts of consumption between the sectors to decide whether or not to choose education. If the instantaneous amount of consumption for an agent is greater when she becomes a skilled worker in the research sector, she will be a skilled worker. Otherwise she will be a less-skilled worker. The agent who has innate ability z_i will choose education and work in the research sector if and only if [15]

[14] Note that the ratio of the number of teachers and researchers is not determined. Only the ratio of efficient unit of labor between teacher and researcher is determined.
On the BGP with the equilibrium threshold level z_1, the total amount of efficient unit of labor by considering each worker's different skill level is $\tilde{L}_E=\left(\frac{1}{pT+1}\right)\int_{z_1}^{\infty}\bar{\delta}(z)dH(z)=\left(\frac{1}{pT+1}\right)\left[\frac{\lambda}{\lambda-\gamma}e^{(\gamma-\lambda)z_1}\right]$ ($\tilde{L}_E=\tilde{L}_T+\tilde{L}_R$). This total amount of educated worker's efficient unit of labor is divided into the total efficient unit of teacher's labor ($\tilde{L}_T$) and the total efficient unit of researcher's labor ($\tilde{L}_R$) with the ratio of "τpT/(1-τpT)". That is, $\tilde{L}_T=\tilde{L}_E\cdot\tau\cdot pT$.

[15] The formal proof is provided in Appendix B.

FIG. 5. WAGE PROFILE OF THE ECONOMY



$$c^{Y}_{t_0+T}(z_i) \leq c^{R}_{t_0+T}(z_i)$$
$$\Leftrightarrow\ w^{Y}_{t_0+T} \leq w^{R}_{t_0+T}(z_1)\cdot[1-\tau\cdot(e^{(\rho+p)T}-1)] \tag{17}$$

Otherwise, the worker will not choose education and participate in the production sector. Thus, the unique equilibrium threshold level z_1 can be solved from equation (17) as follows:

$$w^{Y}_{t_0+T}(z_1) = w^{Y}_{t_0+T} = w^{R}_{t_0+T}(z_1)\cdot[1-\tau\cdot(e^{(\rho+p)T}-1)] \tag{18}$$

Equation (18) says that all agents who have innate ability $q_i \geq z_1$ will work in the research sector. Moreover, the lower the death rate (p), tuition fee (t), education time (T) or time discount rate (ρ), the more agents will choose education, inferred from equation (18).

Along with the BGP, workers' wages are growing at the same rate as consumption and savings. Hence, the equilibrium threshold level z_1 is time invariant. The equilibrium threshold level satisfies the equation (19) for every time $t \in [0, \infty)$. Thus, the following will also be satisfied for every time t.

$$w^{Y}_{t}(z_1) = (1-\alpha)\frac{Y}{L_Y} = (1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}\cdot N(t) = w^{R}_{t}(z_1)\cdot(1-\Psi_\tau) \tag{19}$$

where $\Psi_\tau \equiv \tau\cdot(e^{(\rho+p)T}-1)\ \ (0<\Psi_\tau<1)$

Equation (19) summarizes the labor market equilibrium condition that determines labor allocation between research and production sectors. Figure 5 illustrates how agents decide on their skill acquisition behavior and working sector based on their innate abilities.

As shown in equation (19) and Figure 5, all agents who have innate ability less than z_1 will choose to work in the final good sector, and all the other agents will choose to work in the research sector as a researcher or teacher. As mentioned above, without loss of generality, teachers are randomly selected among the educated workers in the research sector.

From the demographic profile of the economy shown in Figure 3, the total number of workers in the economy is "1/(pT+1)". Since the total number of agents in the education period is "pT/(pT+1)", the total number of agents who choose education is

$$\left(\frac{pT}{pT+1}\right)\cdot\left[\int_{z_1}^{\infty} h(z)\cdot dz\right]=\left(\frac{pT}{pT+1}\right)\cdot\left[e^{-\lambda z_1}\right] \tag{20}$$

It is assumed that one teacher takes class of "$1/\tau$" number of students in terms of efficient unit of labor. This means that the total amount of efficient unit of labor in the research sector should be divided by teaching and research with the ratio of "τpT/(1-τpT)". Since each z-skilled worker invents $\delta(z)\cdot N_t$ number of new designs at each time, the growth rate of the number of varieties of the economy is expressed as follows. Note that the parameter "γ" stands for the economy-wide skill biased technology level.

$$\frac{\dot{N}_t}{N_t}=\left(\frac{1-\tau pT}{pT+1}\right)\cdot\frac{\lambda}{\lambda-\gamma}e^{(\gamma-\lambda)z_1} \qquad (\lambda>\gamma)^{16} \tag{21}$$

Free entry condition implies that the patent value "P_A^j" should be equal to the cost of inventing one unit of new design. The market value of a new design is the same as the present value of the patent right of the new design. Since the wage of a researcher is "$w_t^R(z)=P_A^j\cdot e^{\gamma z}\cdot N_t$", the present value of the patent right of each intermediate good j should be

$$P_A^j=\frac{(1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}}{e^{\gamma z_1}\cdot(1-\Psi_\tau)} \tag{22}$$

The rate of return of the economy "r" is constant over time on the BGP. The present value of each type of design is equal to the present value of profit flow from the design, i.e.

$$P_{A,t}^j=\int_t^{\infty}\pi_{j,s}\cdot e^{-r(s-t)}ds=\frac{\pi_j}{r} \tag{23}$$

$$\text{where } \pi_{j,s}=\pi_j=\left(\frac{1-\alpha}{\alpha}\right)A^{\frac{1}{1-\alpha}}\alpha^{\frac{2}{1-\alpha}}\cdot\left[1-e^{-\lambda z_1}\right]\cdot\left(\frac{1}{pT+1}\right)$$

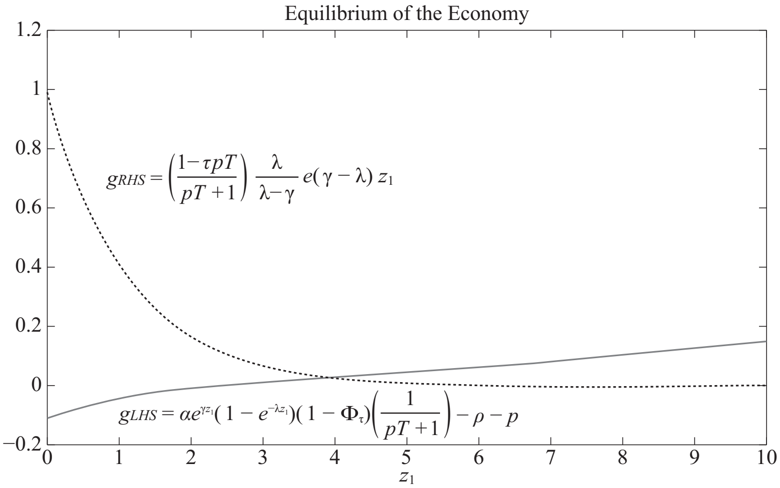
From equation (23) we can observe that the "no arbitrage condition" is satisfied, and the equilibrium value of each design "$P_{A,t}^j$" is time invariant. Thus, the BGP equilibrium rate of return "r" can be determined from equation (22) and (23) as equation (24). The equation (24) summarizes the supply side equilibrium condition which is determined from the labor market equilibrium condition.

$$r=\frac{\pi_j}{P_A^j}=\alpha\delta(z_1)L_Y(1-\Psi_\tau)\left(\frac{1}{pT+1}\right)=\alpha e^{\gamma z_1}(1-e^{-\lambda z_1})(1-\Psi_\tau)\left(\frac{1}{pT+1}\right) \tag{24}$$

The intuition behind equation (24) is straightforward. The higher value of the rate of return "r" means the lower present value of the patent right "P_A^j", so the less demand for researchers. Therefore, the higher value of the rate of return should be related to the higher threshold level z_1.

[16] The condition, $\lambda>\gamma$ is necessary to limit the total amount of effective labor of the economy to be finite.

FIG. 6. EQUILIBRIUM OF THE ECONOMY[17]



The demand side equilibrium condition is derived from the inter-temporal choices of households as in Romer (1990). As expressed in equation (8), workers are heterogeneous in their skill and income levels, but there is no difference in their preference. All the agents have the same logarithm preference. In addition, we can see that every worker's wage is growing at the same rate on the BGP from equation (19). This growth rate of wages is the same as the growth rate of number of varieties (N_t). Therefore, the equilibrium rate of return of the economy is fixed over time.

The equilibrium growth rate of the economy can be expressed as a function of the equilibrium threshold level z_1. The intuition behind this is straightforward. The higher the value of the rate of return means that the economy tends to save more for the future, so there is greater demand for researchers. Thus, the higher value of the rate of return is related to the lower threshold level, z_1, given all the parameter values.

$$g=\frac{\dot{C}}{C}=\frac{\dot{Y}}{Y}=\frac{\dot{N}}{N}=r-\rho-p=\left(\frac{1-\tau pT}{pT+1}\right)\frac{\lambda}{\lambda-\gamma}e^{(\gamma-\lambda)z_1} \tag{25}$$

In sum, the equilibrium threshold level and the rate of return of the economy are determined by equation (24) and (25). Equation (24) is the skill supply condition of the economy, which is from the labor market equilibrium condition. Equation (25) is the skill demand condition, which is from the preference side of the economy. Given all the parameter values, once the equilibrium threshold level and the rate of return are determined, the BGP growth rate of the economy can be found immediately. Figure 6 shows an example of numerical solutions on how the threshold level and the BGP growth rate of the economy are determined by the supply and demand conditions. The upward slope curve in Figure 6 indicates

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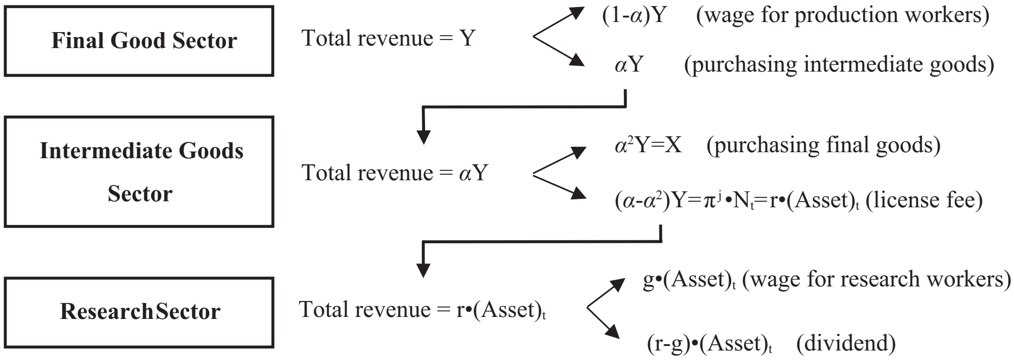
[17] For this simulation, the parameter values are selected as $A=1$, $\alpha=1/3$, $\lambda=1$, $\gamma=0.1$, $\rho=0.01$, $p=0.1$, $\tau=0.1$, $T=1$.

FIG. 7. SUMMARY OF RESOURCE FIOW OF THE ECONOMY



that the higher rate of return (hence, higher growth rate of the economy) corresponds to the higher threshold level from equation (24). The downward slope curve indicates that the higher rate of return corresponds to the lower threshold level from the right side of equation (25). Since the supply and demand functions are monotonic increasing and decreasing, there exists unique equilibrium threshold level z_1.

5. Market Clearing

The aggregate resource constraints of the economy can be described as equation (26). The total present value of patent rights of designs is equal to the market value of the research sector. Households own the research sector as equity assets, v(t). $(Asset)_t$ $(=P_A^j \cdot N_t)$denotes the total asset of the economy at time t, which is equal to the market value of the research sector because the total net value of the bond in the economy is zero. Household income consists of wages and return on assets. All the income is spent for consumption and saving. The resource flow of the economy can be summarized as Figure 7.

$$\begin{aligned}\frac{d(Asset)_t}{dt}&=P_A^j \cdot \frac{dN_t}{dt}=w_t^Y L_Y+\left(\frac{1-\tau pT}{pT+1}\right)\int_{z_1}^{\infty} w_t^R(z)\cdot dH(z)+r\cdot(Asset)_t-C_t\\&=(1-\alpha)Y_t+P_A^j\cdot\frac{dN_t}{dt}+\alpha Y_t-X_t-C_t\\&=P_A^j\cdot\frac{dN_t}{dt}+Y_t-X_t-C_t\quad (Y_t-X_t-C_t=0)^{18}\end{aligned}\tag{26}$$

where $w_t^Y L_Y=(1-\alpha)Y_t$

$$\left(\frac{1-\tau pT}{pT+1}\right)\cdot\int_{z_1}^{\infty} w_t^R(z)\cdot dH(z)=P_A^j\cdot\frac{dN_t}{dt}$$

$$r\cdot(Asset)_t=r\cdot P_A^j\cdot N_t=\alpha(1-\alpha)Y_t=\alpha Y_t-X_t$$

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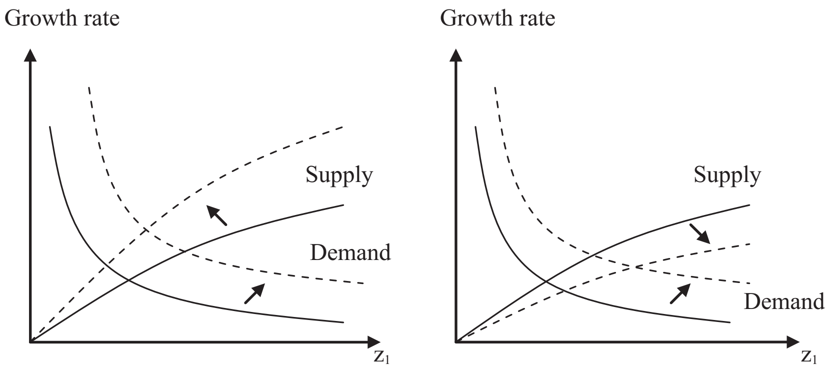
[18] Since all the final goods are used either by consumption or production of intermediate goods, $Y_t=AL_Y^{1-\alpha}x_j^{\alpha}N_t=A^{\frac{1}{1-\alpha}}\alpha^{\frac{2\alpha}{1-\alpha}}L_Y N_t=X_t+C_t$ where $X_t=\int_0^{N(t)} x_j\cdot dj=\alpha^2 Y_t$

Fig. 8. Comparative Analyses

(i) when γ increases (ii) when λ dcereases



III. *Comparative Analysis*

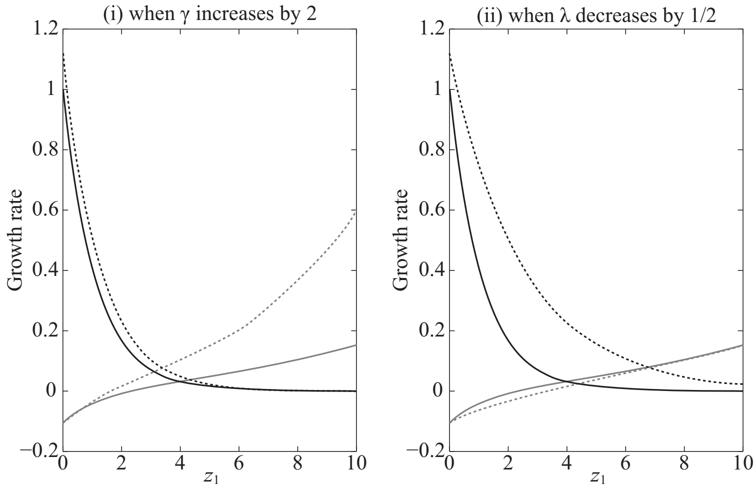
One of the interesting features of the model is that it can predict the effect of (i) increasing the degree of skill biased technology (increasing value of g) and (ii) increasing the supply of skilled workers (decreasing value of λ) on the equilibrium threshold "z_1" and the BGP growth rate of the economy "g". This comparative analysis is based on the supply and demand conditions shown as equation (24) and (25). Note that the growth rate along with the BGP is proportional to the rate of return of the economy "r" as shown in equation (25). Figure 8 illustrates conceptually the comparative analysis.

Intuitively, the results of the comparative analyses can be interpreted as follows. When γ increases, the labor supply curve shifts upward because the higher γ increases the relative productivity of researchers. Hence, more workers are induced to work in the research sector. This makes the equilibrium threshold level, z_1, moves to the left. At the same time, the labor demand curve shifts upward because the higher γ means that more designs can be invented even with the same number of researchers. So depending on the demand for research works in the economy, the economy may have an incentive to reduce the number of researchers. This makes the equilibrium threshold level move to the right.

The intuition of the effect of decreasing λ can be explained in the similar manner. When λ decreases (i.e. exogenous shock of the supply of more skilled workers), the supply curve shifts downward because an increased supply of skilled workers leads to a stricter selection process in the labor market for entering the research sector. Thus, the minimum skill level required for working in the research sector increases when λ decreases in a given demand for research workers in the economy. This makes the equilibrium threshold level move to the right. At the same time, the demand curve shifts upward because the decreasing λ implies more researchers are employed for fixed z_1. Thus, to maintain the same amount of research work, the economy has to take a higher value of the equilibrium threshold level when λ decreases.

Figure 8 illustrates this reasoning. When γ is increasing (i.e., increasing degree of skill biased technology), the rate of return and the growth rate of the economy will increase, but the

FIG. 9. COMPARATIVE ANALYSES[19]



net effect on the threshold level is ambiguous. Conversely, when λ is decreasing (i.e., increasing skilled labor supply), the threshold level will increase, but the net effect on the rate of return and the growth rate of the economy is ambiguous. These contrasting results indicate that the effects of a skill supply shock on economic growth and labor allocation might be affected by the nature of the shock. The skill biased technological change (i.e. increasing γ) and the exogenous supply shock of skilled workers (i.e. decreasing λ) have different impacts on the economy. For instance, recent progress in computing and information technologies could be an example of the skill biased technological change. Comparative analysis has revealed that this type of shock has a relatively greater impact on economic growth but a relatively smaller impact on the proportion of workers in the research sectors. On the other hand, increasing the supply of college graduates can be regarded as an example of decreasing λ. Contrary to the previous case of technological change, this shock has a relatively smaller impact on growth but a greater impact on the proportion of workers in the research sectors.

Figure 9 shows an example of numerical solutions on the comparative analysis when the γ value increases by 2 (i.e., from 0.1 to 0.2) and the λ value decreases by 1/2 (i.e., from 1 to 0.5), respectively. The solid line shows the supply and demand curve based on initial parameter values, and the dotted line shows the curves when the γ and λ values are changed. The closed form solution for the BGP equilibrium cannot be obtained due to the complexity of the model. Thus, we focus on characterizing how the BGP equilibrium of the economy is affected by the exogenous changes of the γ and λ values, that is, an increasing degree of skill biased technology and an increasing supply of skilled workers.

[19] For this simulation, the parameter values are initially set as $A=1$, $\alpha=1/3$, $\lambda=1$, $\gamma=0.1$, $\rho=0.01$, $p=0.1$, $\tau=0.1$, $T=1$.

IV. *Social Optimum and Government Policy*

In this section, we characterize the government policy that will enable the decentralized economy to attain the first-best equilibrium of the centrally planned economy. The outcomes in the decentralized economy are not Pareto optimal as the non-rivalry property of N is not fully utilized. We will assess Pareto optimality by comparing the previous results with the results from the parallel problem for a hypothetical social planner.

The social planner maximizes the household utility as given in equation (8) with the following economy-wide resource constraint and technology level.

$$Y_t = A_t \cdot L_Y^{1-\alpha} \cdot N_t \cdot x_t^{\alpha} = N_t \cdot x_t + C_t \tag{27}$$

$$\text{where} \quad \dot{N}_t = \frac{1-\tau pT}{pT+1}\left[\int_{z_1}^{\infty} e^{\gamma z} \cdot \lambda e^{-\lambda z} dz\right] \cdot N_t = \frac{1-\tau pT}{pT+1}\,\frac{\lambda}{\lambda-\gamma} \cdot e^{(\gamma-\lambda)z_1} \cdot N_t \quad (\lambda > \gamma)$$

We have used the same production functions as in equation (1), but we have already imposed the condition that the quantity of intermediaries is the same for all firms and intermediate products. We can set up the Hamiltonian formula for the social planner's problem by maximizing the household utility subject to equation (27)[20].

Compared to the decentralized economy where intermediates are priced at the monopoly price, the quantity of intermediate goods, x_t, changes as the planner equates to marginal cost. The optimal quantity chosen by the planner leads to the following formula.

$$x_t^{SP} = A_t^{\frac{1}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} L_Y \tag{28}$$

Given L_Y, the quantity of intermediates is supplied less in the decentralized economy as the decentralized solution for intermediates are multiplied by $\alpha^{2/(1-\alpha)}$. Unfortunately, direct comparison of those quantities is not straightforward as L_Y, shown in equation (29), is affected by the threshold skill level, z_1, whose value also differs between the centralized and decentralized solutions. The planner's solution for the optimal threshold skill level is

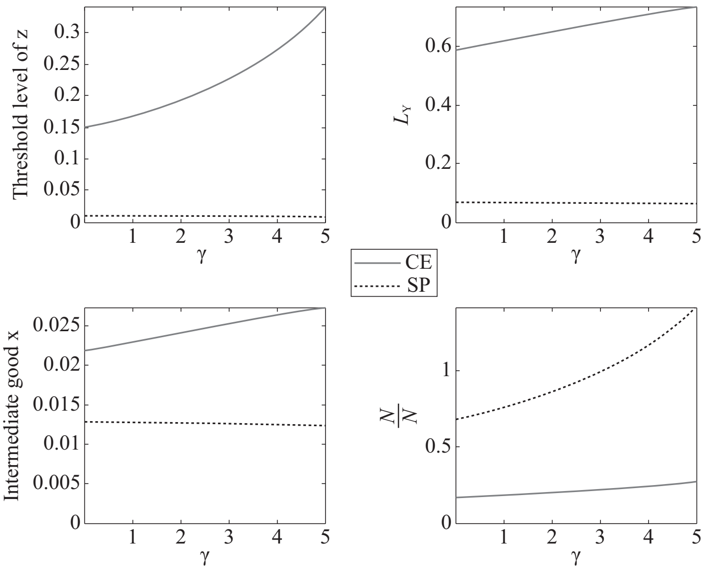
$$2(\rho + p) = \frac{1-\tau pT}{1+pT} e^{\gamma z_1^{SP}} (1 - e^{-\lambda z_1^{SP}}) \tag{29}$$

For the social planner's problem, equation (29) replaces the competitive equilibrium condition derived in equation (25). Again, the threshold skill level of the efficient and the competitive equilibrium cannot be directly compared analytically as the close form solution cannot be found. Hence, we compare them numerically with different values for γ and λ.

Figure 10 shows the relations between γ and other variables, such as threshold skill level, z_1, final good sector employment, L_Y, quantity of intermediates, x and growth rate, $\frac{\dot{N}}{N}$. When we use our benchmark parameter values (A=1, α=0.34, ρ=0.02, p = 1/80, τ=0.1 T= 20), we observe that the equilibrium threshold skill levels in the planner's problem are consistently lower than those in the competitive equilibrium, and higher growth rates are achieved in the

[20] The complete derivation of the planner's problem is provided in Appendix C.

FIG. 10. SOCIAL PLANNER AND COMPETITIVE EQUILIBRIUM: γ



planner's case.

In Figure 11, one can observe similar results when we vary λ with the exception of the quantity of intermediates. This finding contrasts strikingly with that of Romer where the social planner unarguably allocates more resources than the decentralized economy to intermediates. The reason for this discrepancy comes from the heterogeneity of workers.

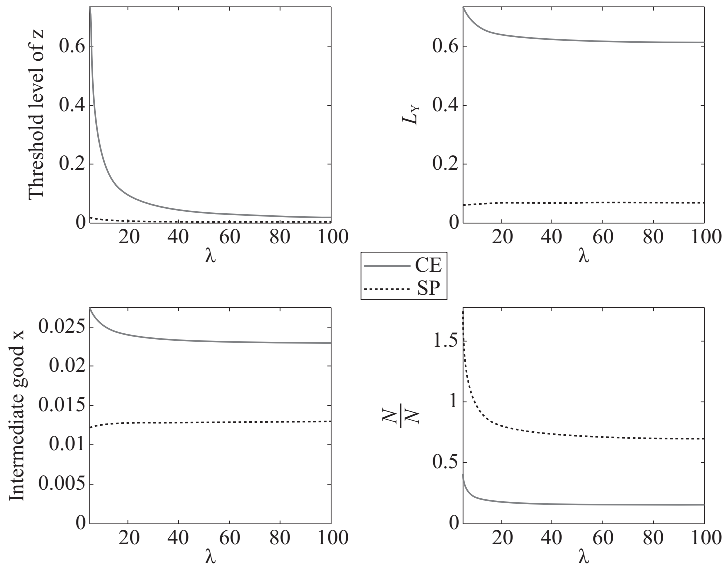
Without the endogenous decision of skill acquisition by workers, there is no variation in the number of workers assigned to L_Y in the planner's and competitive economy. The major role of the social planner in the original Romer model is to fix the efficiency loss of monopoly in the intermediate sector. However, when one begins to consider workers' skill acquisition as another determinant of long-run growth, more workers in the production sector come at the cost of long-run growth potential. There is a clear tradeoff in assigning workers in this case. More (fewer) workers in the final-good sectors will raise (lower) current period production, but less (more) R&D will be achieved. This will lead to lower (more) production in the future. To our knowledge, our findings are new as this perspective has never been addressed in literature.

As the competitive equilibrium is not optimal, the government of a decentralized economy could induce the private sector to pursue the social optimum state if it could design an educational subsidy policy that would encourage more workers to finish their education to join the R&D sector. With the subsidy, the cost of education shown in equation (19) would now become

$$\Psi_{\tau} \equiv \tau(1-s)\cdot(e^{(\rho+p)T}-1) \quad \text{where } s \text{ denotes the subsidy rate} \tag{30}$$

Using this formula, we can identify the optimal educational subsidy by feeding the optimal threshold level of skill found in equation (29) into the competitive equilibrium condition provided in equation (25). Unlike typical solutions, the subsidy rate is not constant and a

FIG. 11. SOCIAL PLANNER AND COMPETITIVE EQUILIBRIUM: λ



function of equilibrium threshold skill level, z_1, and no closed-form solutions can be found. To solve the rate, one has to find the efficient threshold level, z_1^{SP}, from equation (29) for given parameter values and use the value to determine the optimal subsidy rate numerically.

However, an educational subsidy on its own cannot achieve the social optimum because of the ever-present situation of monopoly pricing as a result an undersupply of intermediates. In order to fix this problem, the government also needs to subsidize the purchasing of intermediate goods by the final goods sector at the rate of $1-\alpha$ so that the effective purchase price of intermediates becomes 1, which is the social cost of producing intermediate goods. Having engineered the subsidies, lump-sum taxes should be set to satisfy the government's budget constraints.

V. *Conclusion*

This paper extends Romer's endogenous growth model by incorporating the heterogeneity of worker skills. The main contribution of this research is providing a labor allocation mechanism that divides workers into research and production sectors, and analyzing how labor allocation is determined by the supply and demand conditions of skilled labor in the economy, which is left unanswered in many growth models with representative workers. We also show that the sectoral division of labor is closely related to the long-run growth rate of the economy, i.e., the long-run growth rate of the economy can be expressed as a function of the labor allocation. This paper also provides comparative analyses that explain the effects of skill biased technological changes and increasing supply of skilled workers on the labor allocation and the long-run growth rate of the economy.

A few findings drawn from the model can be summarized as follows. First, the supply of skilled workers may increase or decrease in accordance with the demand conditions of skilled workers in the economy. This change affects the employment structure and long-run growth rate of the economy. Since the equilibrium threshold level is affected not only by the total quantity of human capital in the economy but also by the distribution of the worker's skill levels, our model explains the equilibrium level of human capital determined by both the supply and demand conditions of the economy.

Second, skill biased technological progress, *ceteris paribus*, has a strong positive effect on the long-run growth of the economy, but its effect on labor allocation appears to be relatively negligible. In contrast, an increase in the supply of skilled labor leads to more severe competition in entering the skilled sector, but its effect on the long-run growth rate could be relatively trivial.

Third, an educational subsidy alone will not be sufficient in achieving social optimum as efficiency loss also stems from monopoly pricing in the intermediate sector. Hence, an industry policy that incorporates an educational subsidy is needed to achieve both static and dynamic efficiency in long-run growth.

Appendix

A. Solution for the Dynamic Competitive Optimization Problem

Alternatively, we can assume that there is a competitive insurance market that provides a reverse life insurance. That is, each agent receives $p \cdot a_t$ at that instant in time if she survives, and pays her entire asset (a_t) when she dies. In this case, the dynamic optimization problem for every agent born at time t_0 can be expressed as follows.

$$Max \int_{t_0+T}^{\infty} \log c_t \cdot e^{-(\rho+p)(t-t_0-T)} dt$$

$$s.t. \quad \frac{da_t}{dt} = (r+p) \cdot a_t + w_t - c_t$$

The parameter ρ is the intertemporal discount rate, p is the instantaneous probability of death, c_t is consumption, a_t is asset, and w_t is wage income.

The solution of the optimization problem is,

$$c_t = (\rho+p)[a_t + h_t] \qquad \text{for all } z_i \in [0, \infty),\ t \geq t_0 + T$$

$$\text{where } h_t = \int_t^{\infty} w_s \cdot e^{-(r+p)(s-t)} ds$$

The optimal time paths for consumption and asset holding for each agent on the BGP can be solved as follows.

(i) Production workers (for every $z_i \leq z_1$)

From equation (15) and (16), for all $t \geq t_0 + T$,

$$w_t^Y(z_i)=w_t^Y=w_{t_0+T}^Y\cdot e^{g(t-t_0-T)}$$ (g = r-ρ, BGP growth rate of the economy),

and from equation (8),

$$c_t^Y=(\rho+p)[a_t^Y+h_t^Y] \text{ where } h_t^Y=\int_t^\infty w_s^Y\cdot e^{-(r+p)(s-t)}ds=\frac{1}{\rho+p}\cdot w_t^Y$$

Therefore, the optimal consumption path is $c_t^Y=(\rho+p)\cdot a_t^Y+w_t^Y$.
Thus, from the budget constraint of the dynamic optimization problem, the individual optimal growth rate of asset holding is

$$\frac{da_t{}^Y}{dt}=(r+p)\cdot a_t^Y+w_t^Y-(\rho+p)\cdot a_t^Y-w_t^Y=(r-\rho)\cdot a_t^Y=g\cdot a_t^Y$$

Therefore, we can get the time paths of consumption, wage income and amount of asset holdings for each individual production worker as follows.

$$c_t^Y=c_{t_0+T}^Y\cdot e^{g(t-t_0-T)},\ w_t^Y(z_i)=w_{t_0+T}^Y\cdot e^{g(t-t_0-T)} \text{ and } a_t^Y=a_{t_0+T}^Y\cdot e^{g(t-t_0-T)}$$

(ii) Research workers (for every $z_i\geq z_1$)

From equation (16), for all $t\geq t_0+T$,

$$w_t^R(z_i)=w_{t_0+T}^Y\cdot e^{g(t-t_0-T)}$$ (g = r-ρ, BGP growth rate of the economy),

and from equation (8),

$$c_t^R(z_i)=(\rho+p)[a_t^R(z_i)+h_t^R(z_i)]$$

$$\text{where } h_t^R(z_i)=\int_t^\infty w_s^R(z_i)\cdot e^{-(r+p)(s-t)}ds=\frac{1}{\rho+p}\cdot w_t^R(z_i)$$

Therefore, the optimal consumption path is $c_t^R(z_i)=(\rho+p)\cdot a_t^R(z_i)+w_t^R(z_i)$.
Thus, from the budget constraint of the dynamic optimization problem, the individual optimal growth rate of asset holding is

$$\begin{aligned}\frac{da_t^R(z_i)}{dt}&=(r+p)\cdot a_t^R(z_i)+w_t^R(z_i)-(\rho+p)\cdot a_t^R(z_i)-w_t^R(z_i)\\&=(r-\rho)\cdot a_t^R(z_i)=g\cdot a_t^R(z_i)\end{aligned}$$

Therefore, we can get the time paths of consumption, wage income and amount of asset holdings for each individual production worker as follows.

$$c_t^Y=c_{t_0+T}^Y\cdot e^{g(t-t_0-T)},\ w_t^Y(z_i)=w_{t_0+T}^Y\cdot e^{g(t-t_0-T)} \text{ and } a_t^Y=a_{t_0+T}^Y\cdot e^{g(t-t_0-T)}.$$

B. Deriving Conditions for the Threshold Level z_1

From equation (17) and <Appendix A>, the threshold level (z_1) satisfies the following equality.

$$\begin{aligned}&(\rho+p)\cdot a_{t_0+T}^Y(z_1)+w_{t_0+T}^Y(z_1)=(\rho+p)\cdot a_{t_0+T}^R(z_1)+w_{t_0+T}^R(z_1)\\&\Rightarrow(\rho+p)\cdot[a_{t_0+T}^Y(z_1)-a_{t_0+T}^R(z_1)]=w_{t_0+T}^R(z_1)-w_{t_0+T}^Y(z_1)\\&\Rightarrow(\rho+p)\cdot b_{t_0}(z_1)\cdot e^{rT}=w_{t_0+T}^R(z_1)-w_{t_0+T}^Y(z_1)\end{aligned}$$

where$b_{t_0}(z_1)=\tau\cdot w^R_{t_0}(z_1)\cdot\frac{1}{\rho+p}\cdot[1-e^{-(\rho+p)T}]$

Therefore,

$$w^Y_{t_0+T}(z_1)=w^R_{t_0+T}(z_1)\cdot[1-\tau\cdot(e^{(\rho+p)T}-1)]$$

C. Solutions for the Social Planner's Problem

The present-value Hamiltonian for the social planner's problem can be written as

$$H=\log C_t\cdot e^{-(\rho+p)t}+\mu[A_t\cdot L_Y{}^{1-\alpha}\cdot N_t\cdot x_t{}^{\alpha}-N_t\cdot x_t-C_t]+\xi\left[\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}\cdot N_t\right]$$

where, $L_Y=\frac{1}{1+pT}\int_0^{z_1}\lambda\cdot e^{-\lambda z}dz=1-e^{-\lambda z_1}$

The first-order necessary conditions are: (transversality condition is omitted for simplicity)

(i) $\frac{1}{C_t}\cdot e^{-(\rho+p)t}=\mu_t$

(ii) $A_t\cdot L_Y{}^{1-\alpha}\cdot N_t\cdot\alpha\cdot x_t{}^{\alpha-1}=N_t$

(iii) $\mu\left[\frac{1}{1+pT}A_t\cdot(1-\alpha)\cdot L_Y^{-\alpha}\cdot\lambda\cdot e^{-\lambda z_1}\cdot N_t\cdot x_t^{\alpha}\right]+\xi\left[\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot(\gamma-\lambda)\cdot e^{(\gamma-\lambda)z_1}\cdot N_t\right]=0$

(iv) $\dot{\xi}-(\rho+p)\cdot\xi=-\mu[A_t\cdot L_Y^{1-\alpha}\cdot x_t{}^{\alpha}-x_t]-\xi\left[\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}\right]$

$$\Rightarrow\dot{\xi}=\xi\cdot\left[(\rho+p)-\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}\right]-\mu[A_t\cdot L_Y{}^{1-\alpha}\cdot x_t{}^{\alpha}-x_t]$$

From (iii), we obtain

(v) $\mu_t\left[\frac{1}{1+pT}A_t\cdot(1-\alpha)\cdot L_Y^{-\alpha}\cdot\lambda\cdot e^{-\lambda z_1}\cdot N_t\cdot x_t^{\alpha}\right]=-\xi_t\left[\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot(\gamma-\lambda)\cdot e^{(\gamma-\lambda)z_1}\cdot N_t\right]$

$\Rightarrow\mu_t[A_t\cdot(1-\alpha)\cdot L_Y^{-\alpha}\cdot x_t^{\alpha}]=\xi_t\cdot(1-\tau pT)e^{\gamma z_1}$

From (ii), we get

$$A_t\cdot L_Y^{1-\alpha}\cdot\alpha\cdot x_t^{\alpha-1}=1$$
$$\Rightarrow x_t=A_t{}^{\frac{1}{1-\alpha}}\cdot L_Y\cdot\alpha^{\frac{1}{1-\alpha}}$$

From (i), we find

$$e^{-(\rho+p)t}=C_t\mu_t$$
$$\Rightarrow\frac{\dot{C}_t}{C_t}=-\frac{\dot{\mu}_t}{\mu_t}-(\rho+p)$$

From the budget constraint

$$A_t\cdot L_Y^{1-\alpha}\cdot x_t^{\alpha}=x_t+\frac{C_t}{N_t}$$

Note that, on the BGP, C, Y and N exhibit the same growth rate

$$\frac{\dot{C}_t}{C_t}=\frac{\dot{Y}_t}{Y_t}=\frac{\dot{N}_t}{N_t}=\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}=r-\rho-p$$

Thus

$$\frac{\dot{C}_t}{C_t}=-\frac{\dot{\mu}_t}{\mu_t}-(\rho+p)=\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}$$

$$\Rightarrow\frac{\dot{\mu}_t}{\mu_t}=-\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}-(\rho+p)$$

From (*iv*)

$$(vi)\quad \frac{\dot{\xi}_t}{\xi_t}=(\rho+p)-\frac{1-\tau pT}{1+pT}\frac{\lambda}{\lambda-\gamma}\cdot e^{(\gamma-\lambda)z_1}-\frac{\mu_t}{\xi_t}[A_t\cdot L_Y^{1-\alpha}\cdot x_t^{\alpha}-x_t]$$

From (v)

$$(vii)\quad \frac{\mu_t}{\xi_t}=\frac{(1-\tau pT)e^{\gamma z_1}}{A_t\cdot(1-\alpha)\cdot L_Y^{-\alpha}\cdot x_t^{\alpha}}$$

Combine (*vi*) and (*vii*), then we yield the following endogenous threshold condition for efficient allocation:

$$2(\rho+p)=(1-\tau pT)e^{\gamma z_1}\cdot L_Y=\frac{1-\tau pT}{1+pT}e^{\gamma z_1}\cdot[1-e^{-\lambda z_1}]$$

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