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Abstract

This study's primary objective is to examine a capital–labor conflict in the Harrodian model by explicitly accounting for not only reserved army effects but also capital–labor substitution. By modifying the benchmark framework, we attempt to view the debate on the long-run stability condition in the Harrodian model and the effects of various economic policies on fostering economic growth from a different viewpoint. Using this model, we find following results. First, we find that an indirect effect of capacity utilization on investment through the imperfect labor market makes the model stable. Second, the capital–labor substitution does not always have a stabilizing effect. Finally, the reserved army effect becomes stronger and the model unstable (stable) if capital–labor substitution is small (large).

1 Introduction

This study's primary objective is to examine a capital–labor conflict in the Harrodian model by explicitly accounting for not only reserved army effects but also capital–labor substitution. By modifying the benchmark framework, we attempt to view the debate on the long-run stability condition in the Harrodian model and the effects of various economic policies on fostering economic growth from a different viewpoint.

As is well known, the neo-Kaleckian model tends to dominate post-Keynesian macroeconomics, although the latter includes the neo-Kaleckian as well as Harrodian and Sraffian models (Lavoie 2014). The neo-Kaleckian model assumes imperfect competition and discusses the effects of wage demand, although the neoclassical economy considers the cost of wage. Examining these models can provide us with insightful results for the stagnation and wage-led growth regime.[1] Since these results are contrary to a neoclassical economy, the neo-Kaleckian model serves as a theoretical foundation to increase the real wage rate for higher growth and is seen as an alternative to mainstream macroeconomics. By contrast, the Harrodian model, recognized as a post-Keynesian macroeconomics model, has recently received much attention in the literature. Since the Harrodian model assumes the capacity utilization effect, not investment level, and an investment variance, the benchmark Harrodian model is unstable; that is, it is difficult to discuss the effects of economic policy including distribution, fiscal, and monetary policies. As the result, several studies have focused on the neo-Kaleckian model, instead of the Harrodian model.

However, there are two feasibility issues in the neo-Kaleckian model. First, the neo-Kaleckian model regards capitalism as a class and does not consider the conflict between capitalists and labor, but the difference in the saving rate between them. In other words, income is transferred from the stinter to a good spender. As the result, the neo-Kaleckian model does not explicitly consider the effects of a capital–labor conflict; rather, it discusses an income distribution effect on capacity utilization and growth rate. Second is the recent debate on the neo-Kaleckian and Harrodian models.[2] The Harrodian model criticizes the neo-Kaleckian model in terms of its goods market stability condition and investment function. The ongoing debate on the topic is that in the short run, the savings effect of a change in capacity utilization is expected to be stronger than the investment effect, whereas in the long run, the effect of utilization on investment is larger than that on saving. From the viewpoint of understanding capitalism, instability is an important topic.

Although the characteristics of the benchmark Harrodian model are unstable in the long run, owing to assumptions of the investment function[3], several works have proposed a stable Harrodian model. Some typical works include Skott (2010) and Flaschel and Skott (2006).[4] Thus, the Harrodian model is presently in the spotlight and is expected to be a subject for further research. Drawing on the benchmark Harrodian model, we consider a capital–

[1] The exhilaration regime is characterized by the positive relationship between the real wage rate and capacity utilization rate and the wage-led growth regime is the positive relationship between the real wage rate and growth rate.

[2] Researchers are currently debating the fragility of the Kaleckian model (Hein et al. (2011)) in comparison to the Harrodian model (Skott (2012)).

[3] See Yoshida (1999) and Sportelli (2000).

[4] Skott (2010) considers labor market effects (the reserve army of labor) in the Harrodian model and shows that the labor market has a stabilizing effect on the Harrodian model.

labor conflict, including the reserved army effect, because one of the feature of capitalism is capital–labor conflicts, although the benchmark neo-Kaleckian model tends to dismiss it. Second, we analyze the reserved army effects on the investment function, including capita–labor substitution. Using these models, we derive the following results. First, we find that the indirect effect of capacity utilization on investment through the imperfect labor market makes the model stable. Second, capital–labor substitution does not always have a stabilizing effect. Third, the reserved army effect tends to become stronger and the model becomes unstable (stable) if the capital–labor substitution is small (large).

The remainder of this paper is organized as follows. Section 2 presents a benchmark Harrodian model and discusses reserved army effects. In section 3, we modify the investment function and show that a reserved army effect using an investment function. Section 4 discusses the three dynamic equations, including the capacity utilization and accumulation rates as well as the capital–labor ratio. Section 5 concludes.

2 Harrodian model

In this section, we consider a benchmark Harrodian model and discuss reserved army effects. In our closed economy, there are two social classes: capitalists who own the firms and workers. Workers consume their entire wages, while capitalists save a constant fraction of their profits and consume the rest. We divide time into the short and long run. The "short run" is defined as the time over which capacity utilization is decided by a demand constraint and the "long run" is described such that the variation in the accumulation rate is determined by capacity utilization.

2.1 Benchmark Harrodian model

The firm has the following coefficient production function:

$$Y = min\{aN, K/c\}, \qquad K/c > aN.$$

We use Y, N, and K to represent output, employment, and capital stock, respectively. The output of the firm per capital, y, is given by

$$y = an, \tag{1}$$

where n is the labor–capital ratio of each firm and a is labor productivity. We further define the following variables:

$$u = \frac{Y}{Y^*}, \qquad c = \frac{K}{Y^*}, \qquad n = \frac{N}{K}.$$

The first defines the capacity utilization rate, with Y^* representing a firm's full capacity output. The second defines the capital to full output ratio, which we assume depends on technology. We assume that firms hold excess capital. If there is a maximum amount of output that a firm's capital can produce, determined by the maximum capital to output ratio c, then the economy must obey the restriction $Y < K/c$. The third defines the labor–capital ratio.

The labor input of firms in the short run is determined to fulfill the goods market-clearing condition. Therefore, the dynamic equation of capacity utilization is as follows:

$$\frac{\dot{u}}{u} = \lambda_1 \left(\frac{I}{K} - \frac{S}{K} \right), \qquad \lambda_1 > 0. \tag{2}$$

I and S denote investment and saving and λ_1 is the speed of adjustment of the goods market. Equation (2) shows that excess demand increases the rate of capacity utilization, while excess supply decreases it. If $\frac{S}{K}$ is equal to $\frac{I}{K}$, capacity utilization is constant. Rearranging the above equation, we obtain the goods market equilibrium condition as follows:

$$g = s\pi u \qquad \rightarrow \qquad u = \frac{g}{s\pi}. \tag{3}$$

g, s, and π denote the growth rate of capital stock, saving rate of a capitalist, and profit share, respectively. For simplicity, we ignore capital depreciation. Since we assume $Y^* = K$, the profit rate is πu. We assume that the dynamic equation of growth rate depends on the difference between the actual capacity utilization rate and the desired capacity utilization rate u^*.

$$\dot{g} = \beta(u - u^*), \qquad \beta > 0, \quad u^* < 1. \tag{4}$$

The benchmark Harrodian specification in the above equation implies that the accumulation rate becomes a state variable and there is no immediate impact of changes in utilization on investment. The stability condition in the long run is as follows.

$$\frac{\partial \dot{g}}{\partial g} = \frac{\beta}{s\pi} > 0. \tag{5}$$

Therefore, the benchmark Harrodian model is unstable. The mechanism is as follows. When $u > u^*$, the growth rate increases; capacity utilization also increases because of the goods market equilibrium condition. Further, there is a widening gap between u and u^*. Therefore, the model is unstable.

2.2 Benchmark Harrodian model with reserved army effects

Next, we induce the reserved army effect in the benchmark Harrodian model as a capital–labor conflict. We assume the following function from Bowles (2012) using a non-sharking condition (See the Appendix):

$$\begin{aligned} &\pi = \pi(z = xu, b), \qquad z = \frac{N}{N_s} = xu = \frac{K}{N_s}\frac{Y^*}{K}\frac{N}{Y}\frac{Y}{Y^*} \\ &\frac{\partial \pi(xu, b)}{\partial z} = \pi_z(xu, b) < 0, \qquad \frac{\partial \pi(xu, b)}{\partial b} = \pi_b(xu, b) < 0, \end{aligned} \tag{6}$$

where b is the parameter for the power of a worker including unemployment compensation. We define $u = \frac{Y}{Y^*}$, $x = \frac{K}{N_s}\frac{Y^*}{K}\frac{N}{Y}$, $Y^*/K = c$, and z as the employment rate such that $z = xu$. When the employment rate increases, reserved wage increases. We assume when $|\pi_z(xu, b)|$

increases, the reserved army effect increases.[5] A capitalist increases the real wage rate for workers who are not idling on the job, and consequently, capital share decreases. When $|\pi_z(xu,b)|$ is large, an increase in the employment rate drastically decreases capital share. This situation occurs in a tight labor market.

Since the dynamic equation of capacity utilization in the short run is (2), the goods market equilibrium condition is

$$g = s\pi(xu,b)u \tag{7}$$

and the stability condition in the short run is

$$\pi_z(xu,b)xu + \pi(xu,b) > 0. \tag{8}$$

As $\pi(xu,b) + \pi_z(xu,b)xu > 0$ is $\frac{\partial \pi(xu,b)u}{\partial u} > 0$, excess demand automatically decreases with an increase in the capacity utilization rate.[6] The effect of an increase in capacity utilization on the profit rate through a capital share is smaller than that of an increase in capacity utilization. In this case, the reserved army effect is small such that an increase in capacity utilization increases the profit rate. On the other hand, the effect of an increase in capacity utilization on the profit rate through a capital share is smaller than that of an increase in capacity utilization. In this case, the reserved army effect is large such that an increase in capacity utilization decreases the profit rate. Such an unstable case can occur in a tight labor market, particularly in the case of an economic boom.

Here we find the following proposition.

Proposition 1 The Harrodian model with reserved army effects is unstable.

Proof: Using (4), we determine the stability condition in the long run as follows.

$$\frac{\partial \dot{g}}{\partial g} = \frac{\beta}{s\pi(xu,b) + s\pi_z(xu,b)xu}. \tag{9}$$

Q.E.D.

Since $\pi(xu,b) + \pi_z(xu,b)xu > 0$ because of the goods market equilibrium condition, the benchmark Harrodian model with reserved army effect is unstable. This result is contrary to that of Skott (2010), who stresses that the imperfect labor market stabilizes the model. Therefore, the stability condition of the benchmark Harrodian model with an imperfect labor market is the same as that of the benchmark Harrodian model.

3 Stable Harrodian model with modified investment function

In the previous section, we induced an imperfect labor market in the benchmark Harrodian model. However, even though we assumed capital share to be an endogenous variable, it

[5]Bowles and Boyer (1988) demonstrate the same relationship between π and xu using the efficiency wage framework.

[6]$|\pi_z|$ is needed to be small in the case of a stable model.

has no effect on a firm's investment behavior. As a result, the stability condition remains unchanged in the model, irrespective of the labor marker. However, in capitalism, a change in capital share has multiple effects on the economy, including capital accumulation and capital–labor conflict. When capital share decreases, a firm decreases capital stock by decreasing investment. On the other hand, a firm substitutes labor with capital by introducing a new machine (labor saving technology). Therefore, the movement of capital share caused by an imperfect labor market is an important topic in terms of the Harrodian model. In this section, we consider the effects of capital share on investment for a stability condition.

3.1 Improved investment function

A benchmark Harrodian model assumes a simple investment function in that a variance in the accumulation rate is affected by capacity utilization. This assumption ignores the effect of the profit rate and labor market on the accumulation rate. If capacity utilization is the same, the accumulation rate of a lower profit rate is the same as that of a higher profit rate. The profit rate is broken down into capacity utilization and profit share such that the accumulation rate may differ under the hetero-profit rate given a different capital share caused by a different employment rate, although capacity utilization is the same. From this viewpoint, we modify investment as follows: variance in the growth rate is affected by not only capacity utilization but also the profit share as a parameter of the capital–labor conflict.[7] Here, we obtain following function:

$$\dot{g} = \beta_1(u - u^*) + \beta_2(\pi(xu, b) - \pi^*), \qquad \beta_1 > 0, \quad \beta_2 > 0. \tag{10}$$

Using the short-run equilibrium condition, we derive the following proposition.

Proposition 2 The modified Harrodian model is stable if $(\beta_1 + \beta_2\pi_z(xu, b)x) < 0$

Proof:

$$\frac{\partial \dot{g}}{\partial g} = \frac{\beta_1 + \beta_2\pi_z(xu, b)x}{s\pi(xu, b) + s\pi_z(xu, b)xu} < 0. \tag{11}$$

This is needed to satisfy the stability condition in the long run. Therefore, we need the following combination for the model to be stable.

1. $(\beta_1 + \beta_2\pi_z(xu, b)x) > 0, \quad s\pi(xu, b) + s\pi_z(xu, b)xu < 0$
2. $(\beta_1 + \beta_2\pi_z(xu, b)x) < 0, \quad s\pi(xu, b) + s\pi_z(xu, b)xu > 0$

The goods market equilibrium condition is $s\pi + s\pi_z(xu, b)xu > 0$; thus, $(\beta_1 + \beta_2\pi_z(xu, b)x) < 0$ is needed to satisfy the stability condition. Q.E.D.

$(\beta_1 + \beta_2\pi_z(xu, b)x) < 0$ indicates $\frac{\partial \dot{g}}{\partial u} < 0$; the indirect effect of the capacity utilization rate through the labor market on investment $(\beta_2\pi_z(xu, b)x)$ is larger than the direct effect of capacity utilization on investment (β_1). Comparing this with the previous section, we find a difference stability condition because $\beta_2 > 0$. As $|\pi_z(xu, b)|$ increases, the model becomes stable.

[7] If the capital share is constant, the investment function is independent of the variance in the capital share.

3.2 Long-run Harrodian model with reserved army effects

Next, we consider the long-run equilibrium as the growth rate converges to the natural growth rate. We assume $x = \frac{K}{N_s}\frac{Y^*}{K}\frac{N}{Y}$ and $\frac{\dot{N}_s}{N_s} = g_n$. Hence, the rate of change in x yields

$$\dot{x}/x = g - g_n.$$

Therefore, we take g_n as exogenous. We have the following two-dimensional system:

$$\dot{x}/x = g - g_n \tag{12}$$
$$\dot{g} = \beta_1(u - u^*) + \beta_2(\pi(xu, b) - \pi^*). \tag{13}$$

Assuming the existence of a steady-growth solution, local stability is determined by the Jacobian[8]

$$\begin{pmatrix} \frac{\partial \dot{x}/x}{\partial x}, & \frac{\partial \dot{x}/x}{\partial g} \\ \frac{\partial \dot{g}}{\partial x}, & \frac{\partial \dot{g}}{\partial g} \end{pmatrix} = \begin{pmatrix} 0, & 1 \\ \beta_1 u_x + \beta_2 \pi_z(xu, b)(u + xu_x), & \beta_1 u_g + \beta_2 \pi_z(xu, b) u_g x \end{pmatrix}, \tag{14}$$

$$\frac{\partial u}{\partial g} = u_g = \frac{1}{s\pi_z(xu, b)xu + s\pi(xu, b)} > 0, \tag{15}$$
$$\frac{\partial u}{\partial x} = u_x = \frac{-s\pi_z u^2}{s\pi_z(xu, b)xu + s\pi(xu, b)} > 0, \tag{16}$$

with

$$(trace) = \frac{\beta_1 + \beta_2 \pi_z(xu, b)x}{s\pi_z(xu, b)xu + s\pi(xu, b)} < 0,$$
$$(det) = \frac{s\pi_z(xu, b)u}{s\pi(xu, b) + s\pi_z(xu, b)ux}(u\beta_1 - \pi(xu, b)\beta_2) > 0.$$

Therefore, we need three constrains for the model to be stable in the long run: $s\pi(xu, b) + s\pi_z(xu, b)ux > 0$, $u\beta_1 - \pi(xu, b)\beta_2 < 0$, and $\beta_1 + \beta_2\pi_z(xu, b)x < 0$. In comparison with the previous model, the stability condition needs one more constrain, that is, $u\beta_1 < \pi(xu, b)\beta_2$.

According to $u\beta_1 - \pi(xu, b)\beta_2 < 0$, we get

$$\frac{u}{\pi(xu, b)} < \frac{\beta_2}{\beta_1}. \tag{17}$$

According to $\beta_1 + \beta_2\pi_z(xu, b)x < 0$, we get

$$-\frac{1}{\pi_z(xu, b)x} < \frac{\beta_2}{\beta_1}. \tag{18}$$

[8] The long-run equilibrium is described by following three equations:

$$g = g_n,$$
$$\beta_1(u - u^*) = -\beta_2(\pi(xu, b) - \pi^*),$$
$$s\pi(xu, b)u = g,$$

obtained by solving for the three unknowns g, u, and x.

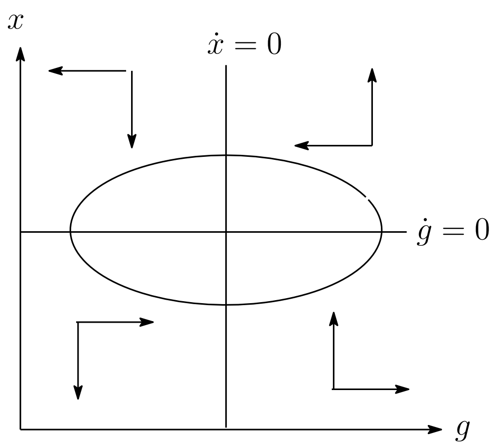

Figure 1: Model as a limit circle

In addition, from $s\pi(xu,b) + s\pi_z(xu,b)ux > 0$, we get

$$\frac{u}{\pi(xu,b)} < -\frac{1}{\pi_z(xu,b)x}. \tag{19}$$

Therefore, only $s\pi(xu,b)+s\pi_z(xu,b)ux > 0$ and $\beta_1+\beta_2\pi_z(xu,b)x < 0$ are needed for a stable model, and this condition is the same as the previous one. As shown above, the stability condition when we assume a reserved army effect and a natural growth rate is the same as the case in which we assume the reserved army effect.

The transition dynamics is illustrated in Figure 2. $\dot{g} = 0$ is downward sloping and $\dot{x} = 0$ is vertical sloping. $(det) > 0$ means $\dot{g} = 0$ is flatter than $\dot{x} = 0$.

In addition, $\beta_1 + \beta_2\pi_z(xu,b) = 0$, that is, the model is a limit circle.[9]

3.3 Labor productivity

Next, we consider the effect of a change in labor productivity as a capital–labor substitution. We define $x = \frac{K}{N_s}\frac{Y^*}{K}\frac{N}{Y} = \frac{K}{N_s}\frac{c}{a}$ and the employment rate is $z = xu$. Since $Y^*/K = c$ is constant, the dynamic equation of x is

$$\frac{\dot{x}}{x} = g - g_n - g_a, \qquad g_a = \frac{\dot{a}}{a}, \qquad g = \frac{\dot{K}}{K}, \qquad g_n = \frac{\dot{N}_s}{N_s}. \tag{20}$$

An increase in g_a decreases the employment rate. Although Bhaduri (2006), Dutt (2006), and Sasaki (2013) assume that the growth rate of labor productivity positively depends on

[9]Here, we consider the case of $\beta_1+\beta_2\pi_z(xu,b)x = 0$. Under this condition, the effect of capacity utilization on the variance in the capital accumulation rate is completely unaffected.

$$\begin{pmatrix} \frac{\partial \dot{x}/x}{\partial x}, & \frac{\partial \dot{x}/x}{\partial g} \\ \frac{\partial \dot{g}}{\partial x}, & \frac{\partial \dot{g}}{\partial g} \end{pmatrix} = \begin{pmatrix} 0, & 1 \\ \beta_1 u_x + \beta_2\pi_z(xu,b)(u + xu_x), & 0 \end{pmatrix}.$$

Since $(trace) = 0$, the model is a limit circle. When the indirect effect of the capacity utilization rate through the labor market on investment ($\beta_2\pi_z(xu,b)x$) is the same as the direct effect of the capacity utilization rate on investment (β_1), the model is a limit circle (See Figure 1).

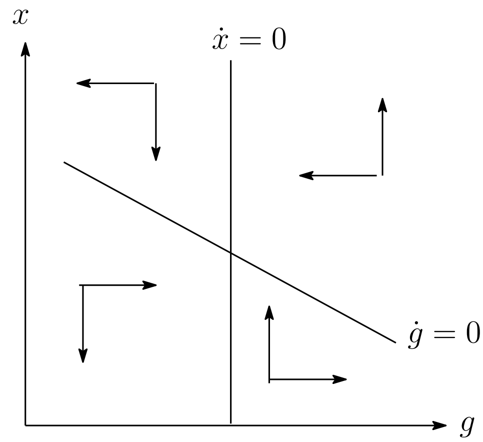

Figure 2: Stable Harrodian model

the employment rate, in this study, we assume g_a is a function of capital share because of capital–labor substitution. Thus, we get

$$g_a = \beta_3(1 - \pi(xu, b)), \qquad \beta_3 > 0. \tag{21}$$

We assume $-\beta_3\pi_z(xu, b) > 0$. The mechanism is as follows. When the employment rate increases, the wage rate increases because of the labor market. Under such a situation, the firm substitutes labor with capital for higher profit, which in turn increases labor productivity. This also leads to decreased employment.[10] The dynamic equation is as follows:

$$\frac{\dot{x}}{x} = g - g_n - \beta_3(1 - \pi(xu, b)), \tag{22}$$

$$\dot{g} = \beta_1(u - u^*) + \beta_2(\pi(xu, b) - \pi^*), \tag{23}$$

These equations define a two-dimensional system of differential equations. Here, we obtain the following proposition about the stability condition.

Proposition 3 β_3 must lie within a specific range for the model to be stable.

Proof: Evaluated at a stationary point, the Jacobian of the system is given by

$$\begin{pmatrix} \frac{\partial \dot{x}}{\partial x}, & \frac{\partial \dot{x}}{\partial g} \\ \frac{\partial \dot{g}}{\partial x}, & \frac{\partial \dot{g}}{\partial g} \end{pmatrix} = \begin{pmatrix} \beta_3\pi_z(xu, b)(u + \frac{\partial u}{\partial x}x), & 1 + \beta_3\pi_z(xu, b)x\frac{\partial u}{\partial g} \\ (\beta_1 + \beta_2\pi_z(xu, b)x)\frac{\partial u}{\partial x} + \beta_2\pi_z(xu, b)u, & (\beta_1 + \beta_2\pi_z(xu, b)x)\frac{\partial u}{\partial g} \end{pmatrix}, \tag{24}$$

and we need

$$(trace) = \frac{s\beta_3 u\pi(xu, b)\pi_z(xu, b) + \beta_1 + \beta_2\pi_z(xu, b)x}{s(\pi(xu, b) + \pi_z(xu, b)xu)} < 0$$

$$(det) = \frac{s\pi_z(xu, b)u(\beta_1 u - \beta_2\pi(xu, b)) + \beta_1\beta_3\pi_z(xu, b)u}{s(\pi(xu, b) + \pi_z(xu, b)xu)} > 0$$

for a stable model.[11] Therefore, β_3 must lie within a specific range, that is,

[10] Sasaki (2013) and Ohno (2009) discuss capital–labor substitution in the neo-Kaleckian model.

[11] The slope of $\dot{x} = 0$ is $-\frac{\frac{\partial \dot{x}}{\partial g}}{\frac{\partial \dot{x}}{\partial x}}$ and that of $\dot{g} = 0$ is $-\frac{\frac{\partial \dot{g}}{\partial g}}{\frac{\partial \dot{g}}{\partial x}}$.

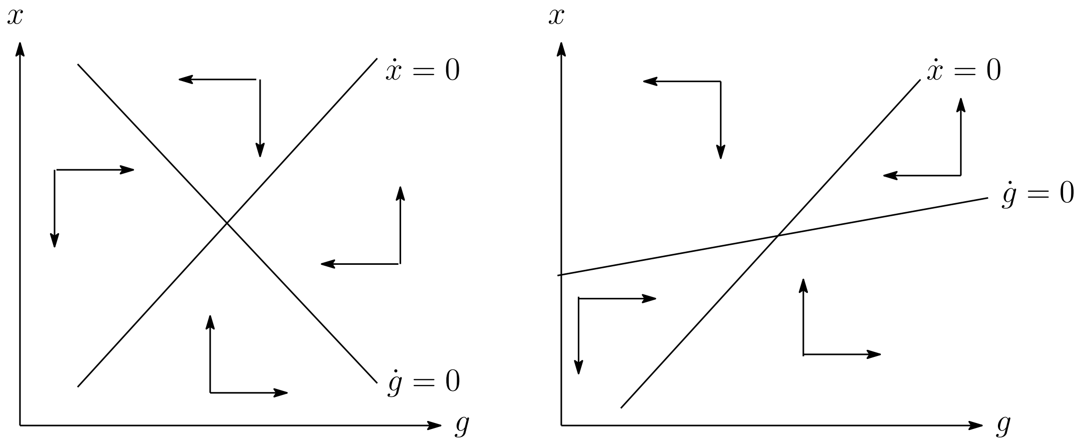

Figure 3: Stable case

$$-\frac{\beta_1+\beta_2\pi_z(xu,b)x}{s\pi(xu,b)\pi_z(xu,b)u}<\beta_3<-\frac{s}{\beta_1}(\beta_1 u-\beta_2\pi(xu,b))$$

to satisfy $(det)>0$ and $(trace)<0$. Q.E.D.

Therefore, we find

$$\beta_1 u-\beta_2\pi(xu,b)<0$$

to satisfy $(det)>0$. To do so, $\beta_1 u-\beta_2\pi(xu,b)$ must be negative and not large.[12] Furthermore, to satisfy $(trace)<0$, $\beta_1+\beta_2\pi_z(xu,b)x$ is negative or positive.

For stability, g_a plays the role of bridging the gap between g and g_n. When g is larger than g_n, the gap between g and g_b continues to widen and it leads to an increase in the capacity utilization and employment rates. A large β_3 is also needed. On the other hand, a oversized β_3 hinders the stability condition.

In addition, we find the following results for the stability condition. β_3 negatively affects $(trace)<0$, but positively affects $(det)>0$. $\pi_z(xu,b)$ also negatively affects $(trace)<0$, but positively affects $(det)>0$. Thus, β_3 and $\pi_z(xu,b)$ must lie within a specific range for stability. On the other hand, β_1 negatively affects both $(trace)<0$ and $(det)>0$. A small β_1 is plausible for stability. β_2 positively affects both $(trace)<0$ and $(det)>0$. A large β_2 is plausible for stability. If $\pi_z(xu,b)=0$ and $(det)<0$, the model is unstable. Therefore, a combination of the reserved army effect, β_3 and β_2, are needed for a Harrodian model.

3.3.1 Graphical illustrations of dynamics.

Figure 3 illustrates the transition dynamics in the two cases. The $\dot{x}=0$ curve is always upward sloping but the $\dot{g}=0$ curve can slope either upward or downward. In case (a), the $\dot{g}=0$ curve is downward and in (b), the $\dot{g}=0$ curve is upward and $\dot{g}=0$ is flatter than $\dot{x}=0$.

In addition, β_3 affects the slope of the $\dot{x}=0$ curve and β_2 affects the slope of $\dot{g}=0$. Therefore, we find that a decrease in β_3 decreases the slope of $\dot{x}=0$. In addition, we find

[12] When $\beta_1=0$, if $-\beta_3\pi_z(xu,b)$ is small so that $(det)>0$ is satisfied, the model is stable.

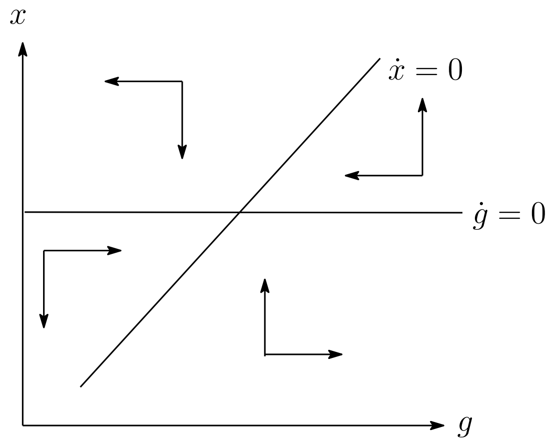

Figure 4: Stable case of $\beta_1 + \beta_2 \pi_z(xu, b)x = 0$

that an increase in β_2 (a decrease in β_1) decreases the slope of $\dot{g} = 0$. Therefore, we find that as β_3 decreases, the slope of $\dot{x} = 0$ becomes small in a clockwise direction, and finally, the model becomes unstable when the slope of $\dot{x} = 0$ becomes smaller than the slope of $\dot{g} = 0$. On the other hand, we find that as β_2 decreases, the slope of $\dot{g} = 0$ becomes large in a counter-clockwise direction, and finally, the model becomes unstable when the slope of $\dot{x} = 0$ becomes larger than the slope of $\dot{g} = 0$.

3.3.2 $\beta_1 + \beta_2 \pi_z(xu, b)x = 0$

For stability, when $\beta_1 + \beta_2 \pi_z(xu, b)x = 0$, we need

$$\begin{aligned}(trace) &= \frac{s\pi(xu,b)u\beta_3\pi_z(xu,b)}{s(\pi(xu,b) + \pi_z(xu,b)xu)} < 0 \\ (det) &= -\beta_2\pi_z(xu,b)u\frac{s\pi + s\pi_z(xu,b)xu + \beta_3 x\pi_z(xu,b)}{s(\pi(xu,b) + \pi_z(xu,b)xu)} > 0\end{aligned}$$

for stability. If β_3 is not large[13], the model is stable. As a result, we find that the stability condition may be satisfied if $\beta_1 + \beta_2 \pi_z(xu, b)x$ is positive and small. Although the direct effect of utilization on investment β_1 is larger than the indirect effect of utilization through the labor market on investment $\beta_2 \pi_z(xu, b)x$, the model is stable because $\beta_3 > 0$. The case of $\beta_1 + \beta_2 \pi_z(xu, b)x = 0$ is illustrated in Figure 4. The slope of $\dot{x} = 0$ is upward and that of $\dot{g} = 0$ is horizontal.

[13] $g'_a(xu) < \frac{1}{\frac{\partial u}{\partial g}}$

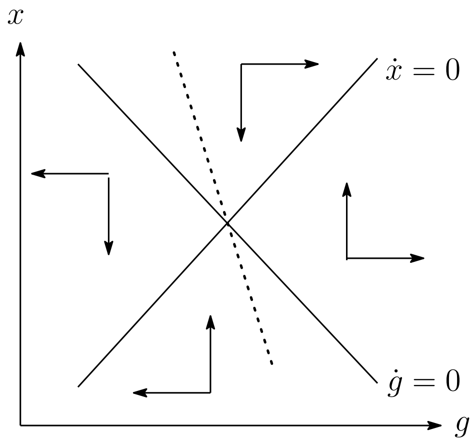

Figure 5: Unstable case (saddle), $\beta_2 = 0$

3.3.3 $\beta_2 = 0$

When we consider $\beta_2 = 0$, the stability condition is not satisfied because $(det) < 0$.

$$(trace) = \frac{s\beta_3 u\pi(xu,b)\pi_z(xu,b) + \beta_1}{s(\pi(xu,b) + \pi_z(xu,b)xu)} < 0$$
$$(det) = \frac{s\pi_z(xu,b)u\beta_1 u + \beta_1\beta_3\pi_z(xu,b)u}{s(\pi(xu,b) + \pi_z(xu,b)xu)} < 0$$

The case under $\beta_2 = 0$ is illustrated in Figure 5. The previous subsection shows that the model may be stable under $\beta_1 + \beta_2\pi_z(xu,b)x > 0$; however, if $\beta_2 = 0$, the indirect effect of capacity utilization on the accumulation rate is ignored and the model becomes unstable. Therefore, we stress that the indirect effect of capacity utilization on the accumulation rate is needed for stability, even though $\beta_1 + \beta_2\pi_z(xu,b)x > 0$.

3.4 Long-run equilibrium

Next, we show the effect of g_n, u^*, and b on the growth and employment rate. We summarize the following three equations for the unknowns of g, u, and x in the long run, when $\dot{x} = 0$ and $\dot{g} = 0$.

$$g = g_n + \beta_3(1 - \pi(xu,b)), \tag{25}$$
$$\beta_1(u - u^*) = -\beta_2(\pi(xu,b) - \pi^*), \tag{26}$$
$$s\pi(xu,b)u = g + \beta_4(1 - \pi(xu,b)), \tag{27}$$

$$\begin{pmatrix} 1, & \beta_3\pi_z(xu,b)x, & \beta_3\pi_z(xu,b)u \\ 0, & \beta_1 + \beta_2\pi_z(xu,b)x, & \beta_2\pi_z(xu,b)u \\ 1, & -s\pi(xu,b) - s\pi_z(xu,b)xu - \beta_4\pi_z(xu,b)x, & -s\pi_z(xu,b)u^2 - \beta_4\pi_z(xu,b)u \end{pmatrix} \begin{pmatrix} dg \\ du \\ dx \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} dg_n + \begin{pmatrix} 0 \\ \beta_1 \\ 0 \end{pmatrix} du^* + \begin{pmatrix} -\beta_3\pi_b(xu,b) \\ -\beta_2\pi_b(xu,b) \\ s\pi_b(xu,b)u + \beta_4\pi_b(xu,b), \end{pmatrix} db$$

$$\Delta = s\pi_z(xu,b)u(\beta_2\pi(xu,b) - \beta_1 u) - \beta_3\pi_z(xu,b)u\beta_1 - \beta_4\beta_1 u\pi_z(xu,b) < 0.$$

$\Delta < 0$ is the same as $(det) > 0$. To satisfy $\Delta < 0$, we need $\beta_2\pi - \beta_1 u > 0$.

Proposition An increase in g_n increases the growth, capacity utilization, and employment rate.

$$\frac{\partial g}{\partial g_n} = s\pi_z(xu,b)u(\beta_2\pi(xu,b) - \beta_1 u - \beta_1\beta_4\pi_z(xu,b)u)/\Delta > 0, \tag{28}$$
$$\frac{\partial u}{\partial g_n} = \beta_2\pi_z(xu,b)u/\Delta > 0, \tag{29}$$
$$\frac{\partial x}{\partial g_n} = -(\beta_1 + \beta_2\pi_z(xu,b)x)/\Delta, \tag{30}$$
$$\frac{\partial xu}{\partial g_n} = -\beta_1 u/\Delta > 0. \tag{31}$$

g_n positively affects the growth and employment rate because the power of labor becomes relatively small.

Proposition An increase in u^* decreases the growth, capacity utilization, and employment rate. These results are the same as those in the case of π^*.

$$\frac{\partial g}{\partial u^*} = -s\beta_1\beta_3\pi_z(xu,b)\pi(xu,b)u/\Delta < 0, \tag{32}$$
$$\frac{\partial u}{\partial u^*} = \beta_1(-s\pi_z(xu,b)u^2 - \beta_3\pi_z(xu,b)u - \beta_4\pi_z(xu,b)u)/\Delta < 0, \tag{33}$$
$$\frac{\partial x}{\partial u^*} = \beta_1(s\pi(xu,b) + s\pi_z(xu,b)xu + \beta_3\pi_z(xu,b)x + \beta_4\pi_z(xu,b)x)/\Delta, \tag{34}$$
$$\frac{\partial xu}{\partial u^*} = \beta_1 s\pi(xu,b)u/\Delta < 0. \tag{35}$$

An increase in u^* means that the judgment of investment become difficult, and thus, it a decreases investment as well as the growth and employment rate.

Proposition An increase in b decreases the employment rate, but has no effect on the growth rate, profit share, and capacity utilization rate.

$$\frac{\partial g}{\partial b} = 0, \qquad \frac{\partial u}{\partial b} = 0, \qquad \frac{\partial \pi(xu,b)}{\partial b} = 0, \tag{36}$$
$$\frac{\partial x}{\partial b} = \pi_b(xu,b)(\beta_1\beta_4 + s(u\beta_1 - \beta_2\pi(xu,b))/\Delta < 0, \tag{37}$$
$$\frac{\partial xu}{\partial b} = \pi_b(xu,b)u(\beta_1\beta_4 + s(u\beta_1 - \beta_2\pi(xu,b))/\Delta < 0. \tag{38}$$

The power of labor negatively affects the growth and employment rate. The result is the same as the profit-led growth and exhilaration regime.

4 Three dynamic equations

Next, we consider the reserved army effects on the stability condition. Thus far, we assumed $\pi(xu, b) + \pi_z(xu, b)xu > 0$, which means the reserved army effect is weak. To consider the level of the reserved army effect, we must consider the situation $\pi(xu, b) + \pi_z(xu, b)xu < 0$. To this effect, we consider not only the dynamics of the accumulation rate, g and x, but also the dynamics of capacity utilization, u. The dynamic equations are as follows:

$$\frac{\dot{u}}{u} = \lambda_1(g + \beta_4(1 - \pi(xu, b)) - s\pi(xu, b)u), \tag{39}$$
$$\frac{\dot{x}}{x} = g - g_n - \beta_3(1 - \pi(xu, b)), \tag{40}$$
$$\dot{g} = \beta_1(u - u^*) + \beta_2(\pi(xu, b) - \pi^*). \tag{41}$$

This framework does not consider instantaneous output adjustment. The equations define a three-dimensional system of differential equations. Evaluated at a stationary point, the Jacobian of the system is given by

$$\begin{pmatrix} \frac{\partial \dot{u}}{\partial u}, & \frac{\partial \dot{u}}{\partial x}, & \frac{\partial \dot{u}}{\partial g} \\ \frac{\partial \dot{x}}{\partial u}, & \frac{\partial \dot{x}}{\partial x}, & \frac{\partial \dot{x}}{\partial g} \\ \frac{\partial \dot{g}}{\partial u}, & \frac{\partial \dot{g}}{\partial x}, & \frac{\partial \dot{g}}{\partial g} \end{pmatrix}$$

$$= \begin{pmatrix} -\lambda_1 s\pi(xu, b) - s\lambda_1\pi_z(xu, b)xu - \lambda_1\beta_4\pi_z(xu, b)x, & -\lambda_1 s\pi_z(xu, b)u^2 - \lambda_1\beta_4 u\pi_z(xu, b), & \lambda_1 \\ \beta_3 x\pi_z(xu, b), & \beta_3 u\pi_z(xu, b), & 1 \\ \beta_1 + \beta_2\pi_z(xu, b)x, & \beta_2\pi_z(xu, b)u & 0. \end{pmatrix}$$

The necessary and sufficient Routh–Hurwits conditions for local stability are that, evaluated at the equilibrium,

$$b_1 = trace(J) = -s\lambda_1 \left(\pi(xu, b) + \pi_z(xu, b)xu\right) + \beta_3 u\pi_z(xu, b) - \lambda_1\beta_4 x\pi_z(xu, b) < 0, \tag{42}$$
$$\begin{aligned} b_2 &= det(J1) + det(J2) + det(J3) \\ &= -\lambda_1 su\beta_3\pi(xu, b)\pi_z(xu, b) - \beta_2\pi_z(xu, b)u - \lambda_1(\beta_1 + \beta_2\pi_z(xu, b)x) > 0, \end{aligned} \tag{43}$$
$$b_3 = det(J) = \lambda_1\pi_z(xu, b)u(-su\beta_1 + s\pi(xu, b)\beta_2 - \beta_1\beta_4 - \beta_3\beta_1) < 0, \tag{44}$$

$$\begin{aligned} b_1 b_2 - b_3 = &\left(-s\lambda_1 \left(\pi(xu, b) + \pi_z(xu, b)xu\right) + \beta_3 u\pi_z(xu, b) - \lambda_1\beta_4 x\pi_z(xu, b)\right) \\ &* \left(-\lambda_1 su\beta_3\pi(xu, b)\pi_z(xu, b) - \beta_2\pi_z(xu, b)u - \lambda_1(\beta_1 + \beta_2\pi_z(xu, b)x)\right) \\ &- \lambda_1\pi_z(xu, b)u(-su\beta_1 + s\pi(xu, b)\beta_2 - \beta_1\beta_4 - \beta_3\beta_1) < 0. \end{aligned} \tag{45}$$

For $b_1 < 0$, we need

$$s\lambda_1 \left(\pi(xu, b) + \pi_z(xu, b)xu\right) > \underbrace{\beta_3 u\pi_z(xu, b)}_{-} - \underbrace{\lambda_1\beta_4 x\pi_z(xu, b)}_{-} \tag{46}$$

Therefore, $\pi(xu,b)+\pi_z(xu,b)xu$ may be positive or negative for $b_1<0$. We consider the following combination $(\pi(xu,b)+\pi_z(xu,b)xu, \beta_3 u\pi_z(xu,b)-\lambda_1\beta_4 x\pi_z(xu,b))$=$(+,+)$, $(+,-)$, $(-,-)$ to satisfy $b_1<0$. For $b_2>0$, we need

$$\underbrace{-\lambda_1 su\beta_3\pi(xu,b)\pi_z(xu,b)-\beta_2\pi_z(xu,b)u}_{(+)} > \lambda_1(\beta_1+\beta_2\pi_z(xu,b)x). \tag{47}$$

Therefore, $(\beta_1+\beta_2\pi_z(xu,b)x)$ may be positive or negative for $b_2<0$. For $b_3<0$, we need

$$\underbrace{\beta_1(\beta_4+\beta_3)}_{+} < -s(\beta_1 u-\beta_2\pi(xu,b)).$$

Therefore, $\beta_1 u-\beta_2\pi(xu,b)$ is negative and small to satisfy $b_3<0$.

Hereafter, we obtain $b_1<0$, $b_2>0$, and $b_3<0$. Next, we check the value of $b_1b_2-b_3$.

$$b_1b_2-b_3=$$
$$(s\lambda_1(\pi(xu,b)+\pi_z(xu,b)xu)) * \underbrace{(\lambda_1(\beta_1+\beta_2\pi_z(xu,b)x)+\lambda_1 su\beta_3\pi(xu,b)\pi_z(xu,b)+\beta_2\pi_z(xu,b)u)}_{-}$$
$$-(\beta_3 u\pi_z(xu,b)-\lambda_1\beta_4 x\pi_z(xu,b)) * \underbrace{(\lambda_1(\beta_1+\beta_2\pi_z(xu,b)x)+\lambda_1 su\beta_3\pi(xu,b)\pi_z(xu,b)+\beta_2\pi_z(xu,b)u)}_{-}$$
$$-\underbrace{\lambda_1\pi_z(xu,b)u(-su\beta_1+s\pi(xu,b)\beta_2-\beta_1\beta_4-\beta_3\beta_1)}_{-}.$$

The value of $b_1b_2-b_3$ is ambiguous in sign. When $(\pi(xu,b)+\pi_z(xu,b)xu, \beta_3 u\pi_z(xu,b)-\lambda_1\beta_4 x\pi_z(xu,b))$=$(+,-)$ (the case (1)), $b_1b_2-b_3$ is

$$(-)+(-)-(-).$$

The value may be negative. When $(\pi(xu,b)+\pi_z(xu,b)xu, \beta_3 u\pi_z(xu,b)-\lambda_1\beta_4 x\pi_z(xu,b))$=$(+,+)$ (case (2)), $b_1b_3-b_2$ is

$$(-)+(+)-(-)$$

When $(\pi(xu,b)+\pi_z(xu,b)xu, \beta_3 u\pi_z(xu,b)-\lambda_1\beta_4 x\pi_z(xu,b))$=$(-,-)$ (case (3)), $b_1b_3-b_2$ is

$$(+)+(-)-(-)$$

It is difficult for cases (2) and (3) to be negative in comparison with case (1). Case (1) shows a high possibility of being negative, and cases (2) and (3) have a high possibility of being positive, although it is difficult to exclude the possibility of a negative. Therefore, $b_1b_3-b_2$ is positive and negative. Even though the goods market stability condition is not satisfied, the system may be stable.

Effects of π_z on stability condition Next, we check the effects of π_z on the value of b_1, b_2, b_3, and $b_1b_2-b_3$.

$$\frac{\partial b_1}{\partial\pi_z} = -s\lambda_1 xu+\beta_3 u-\lambda_1\beta_4 x, \tag{48}$$
$$\frac{\partial b_2}{\partial\pi_z} = -\lambda_1 su\beta_3\pi(xu,b)-\beta_2 u-\lambda_1\beta_2 x<0, \tag{49}$$
$$\frac{\partial b_3}{\partial\pi_z} = \lambda_1 u(-su\beta_1+s\pi(xu,b)\beta_2-\beta_1\beta_4-\beta_3\beta_1)>0. \tag{50}$$

Since we need $b_2 > 0$, a decrease in π_z increases b_2. An increase in the reserved army effect (a decrease in π_z) stabilizes the model. Since we need $b_3 < 0$, a decrease in π_z decreases b_3. An increase in the reserved army effect (a decrease in π_z) also stabilizes the model. Although we need $b_1 < 0$, the effect of π_z on b_1 is ambiguous in sign. It depends on the value of $-s\lambda_1 xu + \beta_3 u - \lambda_1\beta_4 x$. In the case of $-s\lambda_1 xu + \beta_3 u - \lambda_1\beta_4 x > 0$, when π_z decreases, b_1 become negative. On the other hand, in the case of $-s\lambda_1 xu + \beta_3 u - \lambda_1\beta_4 x < 0$, as π_z decreases, b_1 becomes positive. Therefore, if β_3 is large (small), $-s\lambda_1 xu + \beta_3 u - \lambda_1\beta_4 x$ is positive (negative); the model is stable (unstable) when π_z decreases. Therefore, the effect of π_z on the stability condition depends on the value of $-s\lambda_1 xu + \beta_3 u - \lambda_1\beta_4 x$.

In addition, $b_1 b_2 - b_3$ is convex downward (upward) when $-s\lambda_1 xu + \beta_3 u - \lambda_1\beta_4 x < (>) 0$. As a result, $b_1 b_2 - b_3$ becomes positive as π_z decreases (increases). Thus, an increase in the reserved army effect makes the model become unstable (stable). [14], [15]

5 Conclusion

This study induces not only the reserved army effect but also a capital–labor substitution to consider the effects of a capital–labor conflict in the Harrodian model. First, we find that an indirect effect of capacity utilization on investment through the imperfect labor market makes the model stable. Second, the capital–labor substitution does not always have a stabilizing effect. Finally, the reserved army effect becomes stronger and the model unstable (stable) if capital–labor substitution is small (large).

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