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A NOTE ON UNIFORM PRICING IN THE
MOTION-PICTURE INDUSTRY*

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Abstract

Uniform pricing in the motion-picture industry is puzzling in light of the potential profitability of prices that vary with demand characteristics. Considering a model à la Hotelling in which moviegoers form their beliefs about movie quality through pricing schemes to which an exhibitor commits, we characterize the conditions under which committing to uniform pricing is more profitable than committing to variable pricing. Some extensions of the model are discussed as well.

Keywords: uniform pricing, variable pricing, movie quality, beliefs

JEL Classification Codes: D42, L82

I. Introduction

Movie theaters in the United States, Japan, and Korea implement several price discrimination schemes such as discounts for seniors and students, while they charge the same admission fee for all movies.\(^1\) Such price uniformity across movies is a puzzling phenomenon because price variation over differentiated movies can be a profit-maximizing solution corresponding to different demand characteristics. One would expect that exhibitors could increase their profits by charging more for blockbusters. In the case of the digital music industry, using survey data on individuals’ valuations of popular songs at iTunes where until recently most songs sold for $0.99, Shiller and Waldfogel (2011) find that alternatives to uniform pricing such as song-specific pricing, bundling, two-part tariffs, and nonlinear pricing can raise both producer and consumer surplus.

Despite the extensive economic literature on pricing for differentiated products, there are surprisingly few studies focusing on why movie theaters employ uniform pricing. Orbach and

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* I am grateful to Eric Bahel and an anonymous referee for their helpful comments and suggestions. All remaining errors are my responsibility.

\(^1\) This phenomenon is referred to as the movie puzzle. Another puzzle in the motion-picture industry is the showtime puzzle, which refers to the lack of price variation between weekdays and weekends or across seasons (Orbach and Einav, 2007).
Einav (2007) conclude that exhibitors could increase profits by engaging in variable pricing and that the legal constraints on vertical arrangements between distributors and exhibitors make it difficult to engage in profitable price differentiation.² Chen (2009) considers the agency problem associated with concession sales between the exhibitors’ profit maximization and the distributors’ revenue maximization. He finds that the high profit mark-up from movie theaters’ concession sales makes uniform pricing the profit-maximizing solution for exhibitors and that unless many successful event movies are expected, tiered pricing over regular and event movies will not benefit either exhibitors or distributors. Finally, Courty (2011) shows that a monopolist charges the same price for differentiated products when high quality products are likely to be assigned to low valuation consumers.

In the area of monopoly pricing, demand uncertainty plays an important role in explaining some pricing phenomena that are otherwise difficult to understand. Lewis and Sappington (1994) show that a monopolist strategically chooses to either perfectly inform buyers about their valuation for its product or provide no information at all. Considering consumers who initially do not know their valuation for a product but over time become informed, DeGraba (1995) shows that a monopolist can make more profit by committing to sell fewer units than the number of consumers, which induces customers to purchase while uninformed. In Courty (2003), a monopolist can sell early to uninformed consumers and/or late to informed consumers, or it can ration tickets and allow ticket holders to resell. His main finding is that selling both far from and close to the event date is never optimal.

Incorporating uncertainty into movie ticket pricing, the present study tries to answer the question of why movies are priced uniformly, regardless of their popularity. Specifically, we consider two kinds of uncertainty associated with the motion-picture industry. First, movie theaters do not know which movie will be a hit or flop at the time of setting ticket prices. By showing that the probability distributions of movie box-office revenues and profits are characterized by heavy tails and infinite variance, De Vany and Walls (1999) conclude that there are no formulas for success in the motion-picture industry and that no amount of star power or marketing hype can make a movie a hit. Hence, movie theaters have incentives to use either uniform or variable pricing for reasons that will be discussed in this paper. If exhibitors can observe the exact quality of movies, it will be more profitable to adopt variable pricing where a high-quality movie is priced higher than a low-quality one.

Second, people do not know how much they will like a movie until they have seen it. Thus, moviegoers decide which movie to see based on factors other than a movie’s quality, which may be in the form of signals. Here we consider the situation where moviegoers form their beliefs about movie quality through pricing schemes to which an exhibitor commits. The recent work of Moretti (2011) considers social learning in consumption of movies where movies’ quality is ex ante uncertain and consumers hold a prior on quality, which they may update based on information from their peers. Using box-office data, he finds that social learning appears to have an important effect on profits in the movie industry.

Employing Hotelling’s model of product differentiation where movies are characterized by variety (genre) and quality, we show that committing to uniform pricing is more profitable for an exhibitor than committing to variable pricing under certain conditions on moviegoer’s

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² Distributors are not allowed to vertically integrate theaters and any involvement of distributors in box-office pricing is prohibited by the Paramount decrees in the US.
beliefs.

The remainder of the paper is organized as follows. Section II sets up the model. Section III presents the results of our work and extends the model by allowing an exhibitor to choose the genre of movies and considering an arbitrary distribution of moviegoers. Section IV concludes.

II. The Model

Consider a multiplex in which an exhibitor is playing two differentiated movies located at the two end points of the Hotelling unit interval, with movie 1 at point 0 and movie 2 at point 1 (we will relax this assumption in Section III). We assume that the marginal cost of screening each movie for an additional audience is zero and that the fixed cost of playing the movies is $K$. There is a continuum of moviegoers uniformly distributed on the interval $[0, 1]$ with a unit mass, which means that moviegoer preferences are heterogeneous in movie genre (an arbitrary distribution will be considered in Section III). Each moviegoer sees at most one movie and a moviegoer’s location on the interval represents her most favorite movie genre. Whenever she sees a movie located some distance away from this point, her utility decreases at a fixed rate. Specifically, a moviegoer located at $x \in [0, 1]$ incurs a disutility of $tx$ when seeing movie 1, and of $t(1-x)$ when seeing movie 2, where $t > 0$ measures the distaste cost of seeing away from her ideal movie genre per unit of distance.

Motion pictures are uncertain products in the sense that it is difficult for movie theaters to estimate which movie will be a hit or flop before screening it (De Vany and Walls, 1999). Suppose thus that each movie is of either high (H) or low (L) quality and that the exhibitor cannot observe the exact quality of the movies prior to their release. However, since movie theaters can predict, to some extent, whether a movie will be popular based on movie stars or marketing hype, we also suppose that the exhibitor expects movie 2 to be of high quality with higher probability than movie 1. This gives the exhibitor incentive to set a higher ticket price for movie 2. Nevertheless, the exhibitor can consider charging the same price for both movies as it still remains uncertain how moviegoers will evaluate them. Indeed, one common explanation for price uniformity across movies is that different ticket prices are likely to be perceived as quality signals and can deter moviegoers from seeing low-priced movies (Orbach and Einav, 2007). Letting $p_i$ denote the ticket price of movie $i \in \{1, 2\}$, we then make the following definition:

**Definition 1.** The exhibitor is said to use uniform pricing (resp. variable pricing) when $p_1 = p_2 = p$ (resp. $p_1 < p_2$).

Due to the symmetry of the model, we will only consider that $p_1 < p_2$ in case of variable pricing. Of course, this comes from the assumption that the exhibitor expects movie 2 is more likely to be of high quality than movie 1. To incorporate the exhibitor’s choice of the pricing schemes into the model, it is assumed that the exhibitor commits to whether it will employ uniform or variable pricing before setting movie ticket prices and that this commitment is...
binding. In what follows, we denote by \( u \) (resp. \( d \)) uniform (resp. variable) pricing.

Likewise, moviegoers are uninformed of the quality of movies before viewing them and thus their decisions about which movie to see rely on factors other than movie quality. Here, we consider the situation where moviegoers form their expectations about movie quality based on the pricing schemes to which the exhibitor commits.

Let \( \mu_j \in (0, 1) \) denote the belief (probability) moviegoers assign to the event that movie \( i \) is of high quality when the exhibitor commits to the pricing scheme \( j \in \{ u, d \} \). Also, let \( s_L \) (resp. \( s_H \)) be the basic value each moviegoer attaches to a low-quality (resp. high-quality) movie, where \( s_L < s_H \). Then a moviegoer indexed by \( x \in [0, 1] \) enjoys (expected) utility

\[
\mu_j s_H + (1 - \mu_j) s_L - t(x - i + 1) - p,
\]

from seeing movie \( i \) under the pricing scheme \( j \). If moviegoers do not see any movie, their utility is zero. We assume that \( s_H \) is sufficiently large, ensuring that the market is covered.

Our key assumption is that, conditional on the pricing schemes to which the exhibitor commits, moviegoers form their beliefs about the quality of the movies in the following manner:

\[
\left\{ \begin{array}{l}
\mu^*_u = \mu^*_d = b_1 \\
\mu^*_i = b_1 < \mu^*_d = b_2,
\end{array} \right.
\]

(1)

where, without loss of generality, \( b_1 < b < b_2 \).

The belief formation (1) implies that if the exhibitor commits to uniform pricing, moviegoers believe the quality of the two movies is high with equal probability, whereas they believe movie 2 is more likely to be of high quality than movie 1 under a variable pricing commitment. Despite its simplicity, this belief formation captures the important stylized fact that a high price signals high quality.\(^5\)

For simplicity, \( s_L \) is normalized to zero. The utility of a moviegoer indexed by \( x \in [0, 1] \) is then defined by

\[
U_x = \begin{cases} 
bs_H - tx - p & \text{if he sees movie 1; uniform} \\
bs_H - t(1-x) - p & \text{if he sees movie 2; uniform} \\
b_1 s_H - tx - p_1 & \text{if he sees movie 1; variable} \\
b_2 s_H - t(1-x) - p_2 & \text{if he sees movie 2; variable} \\
0 & \text{if he does not see any movie.}
\end{cases}
\]

(2)

In sum, the interaction of the exhibitor and moviegoers is as follows:

\(^4\) An example of such a commitment is to maintain or change customary pricing patterns.

\(^5\) In the context of price signaling, Wolinsky (1983) shows that considering a market in which the exact quality chosen by a firm is known only to the firm itself, prices serve as signals and each price-signal exceeds the marginal cost of producing the quality it signals. Milgrom and Roberts (1986) show that when consumers make repeat purchases, a high price combined with advertising enables a monopoly to signal its quality. High quality could also be signaled by a high price alone but this would reduce current demand, which is the basis of future demand. Bagwell and Riordan (1991) consider a situation where a monopoly signals quality to consumers when some consumers are informed about product quality. They find that in a one-period market, firms first signal high quality with prices higher than full information profit-maximization prices. As information about product quality is diffused, this price distortion decreases. Hence, high and declining prices signal a high quality product due to an increasing number of informed consumers.
Stage 1: The exhibitor commits to whether it will use uniform or variable pricing.

Stage 2: Conditional on the pricing scheme to which the exhibitor commits in stage 1, moviegoers form their beliefs about the movies’ quality. The exhibitor chooses movie ticket prices according to the pricing commitment and then moviegoers decide which movie to see.

The following assumption on the parameters of the model will be maintained throughout the paper:

**Assumption 1.** \( b_2 - b_1 \frac{2t}{s_H} \)

The assumption guarantees that in equilibrium, there always exist moviegoers who prefer seeing movie 1 to 2, even if movie 1 is believed to be of lower quality because of a variable pricing commitment. In addition, the following definition will be useful in discussing our results:

**Definition 2.** A commitment to variable pricing is said to have a negative effect (resp. positive effect) on moviegoer’s beliefs about the quality of the movies if it leads to \( b_1 + b_2 < 2b \) (resp. \( b_1 + b_2 \geq 2b \)).

By Definition 2, the negative effect of committing to variable pricing means that the sum of the movies’ expected values under a variable pricing commitment is lower than under a uniform pricing commitment.

### III. Analysis and Results

To explore how the two pricing commitments affect the exhibitor’s profit, we begin this section with the analysis of the pricing schemes.

#### 1. Uniform Pricing

Consider first the case where the exhibitor commits to uniform pricing. Let \( \hat{x}^u \) denote a moviegoer who is indifferent between seeing movie 1 and movie 2. Given a price \( p \) and a belief \( b_1 \), \( \hat{x}^u \) is determined by \( b_{s_H} - t \hat{x}^u - p = b_{s_H} - t(1 - \hat{x}^u) - p \) in (2). Solving this condition gives \( \hat{x}^u = \frac{1}{2} \), which means that all moviegoers indexed on \([0, \frac{1}{2}] \) will see movie 1, whereas all moviegoers indexed on \( \left[\frac{1}{2}, 1 \right] \) will see movie 2. The exhibitor can then maximize its profit by extracting all the surplus of this marginal moviegoer. The equilibrium values for price, profit, and \( \hat{x}^u \) when committing to uniform pricing are thus

\[
p^u = b_{s_H} - \frac{t}{2}
\]

\[
\pi^u = b_{s_H} - \frac{t}{2} - K
\]

\[
\hat{x}^u = \frac{1}{2}
\]
2. Variable Pricing

Suppose now that the exhibitor commits to variable pricing. Recall that in the case of a variable pricing commitment, we have \( b_1 < b_2 \). Let \( \hat{x}^d \) denote a moviegoer who is indifferent between seeing movie 1 and movie 2. Given prices \((p_1, p_2)\) and beliefs \((b_1, b_2)\), we have

\[
\hat{x}^d = \frac{1}{2} + \frac{(b_1 - b_2)s_H}{2t} + \frac{p_2 - p_1}{2t}
\]

from \( b_1 s_H - t \hat{x}^d - p_1 = b_2 s_H - t(1 - \hat{x}^d) - p_2 \) in (2). When the exhibitor chooses prices \( p_1 \) and \( p_2 \) to maximize

\[
\pi^d = p_1 \hat{x}^d + p_2(1 - \hat{x}^d) - K
\]

it extracts all the surplus of the marginal moviegoer indexed by \( \hat{x}^d \). The equilibrium values for prices, profit, and \( \hat{x}^d \) under a variable pricing commitment are then

\[
p_1^d = \frac{(3b_1 + b_2)s_H}{4} - \frac{t}{2}
\]

\[
p_2^d = \frac{(b_1 + 3b_2)s_H}{4} - \frac{t}{2}
\]

\[
\pi^d = \frac{(b_1 + b_2)s_H}{2} + \frac{(b_2 - b_1)^2 s_H^2}{8t} - \frac{t}{2} - K
\]

\[
\hat{x}^d = \frac{(b_2 - b_1)s_H}{4t}
\]

3. Exhibitor’s Profit

To characterize the conditions under which the exhibitor has incentive to commit to uniform pricing, we calculate \( \pi^u - \pi^d \). From (3) and (4) this calculation yields the following:

**Proposition 1.** Committing to uniform pricing is more profitable than committing to variable pricing if moviegoer’s beliefs about the movies’ quality satisfy

\[
\frac{2b - (b_1 + b_2)}{(b_2 - b_1)^2} > \frac{s_H}{4t}.
\]

Figure 1, drawn for \( s_H = 2t \) and \( b = \frac{1}{2} \), shows the range of moviegoer’s beliefs for the model predictions. Given that under a uniform pricing commitment, moviegoers expect the two movies to be of high quality with probability \( \frac{1}{2} \), the exhibitor can be better off by committing to uniform pricing if moviegoers form their beliefs \((b_1 \) and \( b_2)\) under a variable pricing commitment in region A (excluding the boundaries). In this region, we can see that when movie 2 for which the exhibitor commits to charge a high price is believed to be of high quality with somewhat low probability, committing to uniform pricing is more likely to be profitable. However, if a moviegoer’s belief that the quality of movie 2 is high is large enough, then a variable pricing commitment would emerge as an optimal strategy (see, e.g., the point c in Figure 1). Therefore, (committing to) uniform pricing observed in the motion-picture industry reflects that under uncertainty about movie quality, ticket price differentials cannot sufficiently induce moviegoers to see high-priced movies by sending good signals about their quality.

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\( ^6 \) For details of the derivations, see Lemma 2 and its proof.
The following result can be directly obtained from Proposition 1:

**Corollary 1.** Committing to variable pricing is more profitable than committing to uniform pricing whenever it has a positive effect.

Now we extend the model by relaxing the two assumptions: (i) movie 1 and movie 2 are located at the endpoints (0 and 1) of the unit interval, and (ii) moviegoers are uniformly distributed on the interval [0, 1].

4. **Genre Choice**

We first allow the exhibitor to choose the location (genre) of the movies before setting movie ticket prices at stage 2. This would apply, for example, to a movie industry that is vertically integrated and where producers (who choose the genre of movies to be made) are also exhibitors. The next lemma gives the exhibitor’s optimal choice of movie location:

**Lemma 1.** Let \( l_i = (x_i^1, x_i^2) \) be the optimal location of movies 1 and 2 when committing to the pricing scheme \( j \in \{ u, d \} \), where \( x_i^j \) denotes a point at which movie \( i \) is located and \( 0 \leq x_i^1 < x_i^2 \leq 1 \). Then, we have

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7 The movie industry in Korea is an example.
Proof. See Appendix. \(\Box\)

Lemma 1 implies that, when committing to the pricing scheme \(j\), the original location of the movies (0, 1) is dominated by \(l^i\) in the sense that \(\pi^j_l \pi^j_i\), where \(\pi^j\) (resp. \(\pi^i\)) denotes the exhibitor's profit under a commitment to the pricing scheme \(j\) with the optimal movie location (resp. original movie location).

We can then examine the profitability of each pricing commitment with its optimal movie location. The following proposition, in line with the previous result, summarizes the result on the exhibitor’s pricing strategy when the movie location is endogenously determined:

**Proposition 2.** Suppose that the exhibitor chooses the location (genre) of the movies prior to setting movie ticket prices. Committing to uniform pricing then emerges as an optimal strategy if \[ \frac{2b_1 - (b_2 + b_2)}{(b_2 - b_1)^2} \geq \frac{3s_H}{8t} \]

Proof. See Appendix. \(\Box\)

5. **Arbitrary Distribution**

Next, suppose that moviegoers are distributed on the interval \([0, 1]\) according to an arbitrary distribution function \(F\) with full support and density \(f\). Here we restrict ourselves to distributions whose median is \(\frac{1}{2}\) for simplicity. Assuming that the density function \(f(x)\) is continuously differentiable and log-concave, we obtain the following results:

**Lemma 2.** Suppose that the exhibitor commits to variable pricing. The indifferent moviegoer is characterized by the solution to the equation

\[ 2\hat{x}^d - 1 = \frac{(b_1 - b_2)s_H}{t} + \frac{1 - 2F(\hat{x}^d)}{f(\hat{x}^d)}. \]  

(6)

The unique equilibrium prices are then given by

\[ p_1^d = b_1s_H - \hat{x}^d \quad \text{and} \quad p_2^d = b_2s_H - t(1 - \hat{x}^d). \]  

(7)

The corresponding outcomes when committing to uniform pricing (\(\hat{x}^u\) and \(p^u\)) can be also obtained by replacing \(b_1\) and \(b_2\) in (6) and (7) with \(b\).

Proof. See Appendix. \(\Box\)

Since \(b_1 < b_2\), \(\frac{1 - 2F(x)}{f(x)}\) is monotonically decreasing, and \(\frac{1 - 2F(x)}{f(x)} = 0\) at \(x = \frac{1}{2}\), we know that \(\hat{x}^d < \frac{1}{2}\), and thus \(F(\hat{x}^d) < F\left(\frac{1}{2}\right) = \frac{1}{2}\). Lemma 2 yields (3) and (4) when \(F\) is a uniform
We can then state the following proposition:

**Proposition 3.** Let $F$ be an arbitrary distribution of moviegoer preferences. Then committing to uniform pricing is more profitable if

$$F(\hat{x}^d) > \frac{1 - \sqrt{M}}{2},$$

where $M = \frac{(2b - b_1 - b_2)s_H(\hat{x}^d)}{t} > 0$ and $\hat{x}^d$ is the indifferent moviegoer under variable pricing.

Proof. See Appendix. □

Proposition 3 tells us that if, despite movie 1 being believed to be of lower quality than movie 2 due to a variable pricing commitment, the number of moviegoers seeing movie 1 ($F(\hat{x}^d)$) is greater than $\frac{1 - \sqrt{M}}{2}$ (and less than $\frac{1}{2}$), committing to uniform pricing makes the exhibitor better off. However, committing to variable pricing is more desirable for the exhibitor whenever it has a positive effect. Note that this result yields (5) when $F$ is a uniform distribution on $[0, 1]$.

### IV. Conclusion

This work provides a possible explanation of the fact that the anticipated popularity of movies is unpriced. Considering moviegoer's beliefs depending on the pricing schemes to which the exhibitor commits, it shows that there exists a range of moviegoer's beliefs in which committing to uniform pricing is profitable. This range can be characterized by a non-high belief that a movie for which the exhibitor commits to charge a high price is of high quality (equivalently, a relatively high demand for a movie believed to be of lower quality). This reflects that in the film industry, ticket price differentiation across movies cannot sufficiently signal moviegoers to go see high-priced movies. A commitment to variable pricing is, however, more desirable for the exhibitor insofar as it has a positive effect on moviegoer's beliefs. Finally, the profitability of uniform admission fees holds even when introducing the exhibitor's choice of movie location (genre) and an arbitrary distribution of moviegoers' tastes.

### APPENDIX

**Proof of Lemma 1**

Consider first the case of a variable pricing commitment. The optimal locations of the movies are determined such that

$$x^1 = \frac{\hat{x}^d}{2} \quad \text{and} \quad x^2 = \frac{1 + \hat{x}^d}{2},$$

where $\hat{x}^d$ is the indifferent moviegoer under variable pricing.
which results in \((x^*_d, x^*_u) = \left( \frac{1}{4}, \frac{3}{4} \right) \). Then the optimal prices are
\[
\hat{p}^*_d = \frac{7b_1sH + b_2sH}{8} - \frac{t}{4}, \quad \text{and} \quad \hat{p}^*_u = \frac{b_1sH + 7b_2sH}{8} - \frac{t}{4}. \tag{10}
\]

By setting movie 1’s ticket price at \(\hat{p}^*_d\), the exhibitor can extract all the surplus of moviegoers located at 0 and \(x^*_d\). Similarly, the exhibitor can extract all the surplus of moviegoers located at \(x^*_d\) and 1 by charging \(\hat{p}^*_u\) for movie 2.

To show \(x^*_d = \frac{x^*_u}{2}\), suppose first that \(x^*_d < \frac{x^*_u}{2}\). Then it will not gain any new moviegoers on \(\left[ 0, \frac{x^*_u}{2} \right]\) but will lose some of those on \(\left[ \frac{x^*_d}{2}, x^*_u \right]\). In other words, if the exhibitor chooses movie 1 away from \(x^*_d\), the only way it can continue to serve the entire market is by cutting movie 1’s ticket price below \(\hat{p}^*_d\), which leads to lower profit. The same logic can be applied when \(x^*_d > \frac{x^*_u}{2}\). Thus \(x^*_d = \frac{x^*_u}{2}\). The proof of \(x^*_d = \frac{x^*_u + x^*_d}{2}\) is omitted since it is similar to that of \(x^*_d = \frac{x^*_u}{2}\).

In the case of a uniform pricing commitment, we can obtain
\[
(x^*_u, x^*_d) = \left( \frac{1}{4}, \frac{3}{4} \right), \quad \hat{p}^*_u = \frac{b_1sH + b_2sH}{8} - \frac{t}{4},
\]
by replacing \(b_1\) and \(b_2\) in (9) and (10) with \(b\). □

**Proof of Proposition 2**

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<th>Location</th>
<th>Uniform (u)</th>
<th>Variable (d)</th>
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<td>((0, 1))</td>
<td>(\pi^u = bsH - \frac{t}{2} - K)</td>
<td>(\pi^d = \frac{(b + b_1)sH}{2} + \frac{(b_2 - b_1)sH}{8} - \frac{t}{2} - K)</td>
</tr>
<tr>
<td>((x^<em>_u, x^</em>_d))</td>
<td>(\pi^*_u = bsH - \frac{t}{4} - K)</td>
<td>(\pi^*_d = \frac{(b + b_1)sH}{2} + \frac{3(b_2 - b_1)sH}{16} - \frac{t}{4} - K)</td>
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By using the results of Lemma 1, we can easily calculate the exhibitor’s profit when the movie location is endogenously determined (see Table 1). Formally,
\[
\pi^*_u = \hat{p}^*_u x^* + \hat{p}^*_u (1 - x^*) - K
\]
\[
\pi^*_d = \hat{p}^*_d x^*_d + \hat{p}^*_d (1 - x^*_d) - K.
\]

Then the result of the proposition is obtained from \(\pi^*_u > \pi^*_d\). □
Proof of Lemma 2

At the optimal choice \((p^d_1, p^d_2)\), the exhibitor extracts the entire surplus from the indifferent moviegoer \(x^{\hat{d}}\):

\[
b_{sH} - tx^{\hat{d}} - p^d_1 = b_{sH} - t(1 - x^{\hat{d}}) - p^d_2 = 0.
\]  

(11)

The problem faced by the exhibitor is given by

\[
\max_{p_1, p_2} \pi = p_1 F\left(\frac{1}{2} + \frac{(b_1 - b_2)s_{H}}{2t} + \frac{b_2 - p_1}{2t}\right) + p_2 \left[1 - F\left(\frac{1}{2} + \frac{(b_1 - b_2)s_{H}}{2t} + \frac{b_2 - p_1}{2t}\right)\right] - K.
\]

Using (11), this problem can be rewritten as

\[
\max_{0 \leq x \leq 1} \pi = (b_1 s_{H} - tx)(F(x) + (b_2 s_{H} - t(1 - x))(1 - F(x)) - K.
\]  

(12)

Differentiating the profit with respect to \(x\) yields

\[
\frac{\partial \pi}{\partial x} = tf(x)\left[1 - 2x + \frac{(b_1 - b_2)s_{H}}{t} + \frac{1 - 2F(x)}{f(x)}\right].
\]

Notice that \(\frac{\partial \pi}{\partial x}|_{x=0} > 0\) and \(\frac{\partial \pi}{\partial x}|_{x=1} < 0\). By Theorem 1 of Bagnoli and Bergstrom (2005), both \(F\) and \(1 - F\) are log-concave so that \(\frac{1 - 2F(x)}{f(x)}\) is monotonically decreasing. Also, \(f(x) > 0 \forall x\) (\(\because\) \(F\) has full support) and \(2x - 1 + \frac{(b_2 - b_1)s_{H}}{t}\) is monotonically increasing. Thus, there exists a unique point \(x^{\hat{d}}\) for which \(\frac{\partial \pi}{\partial x} = 0\), which is characterized by (6). \(\square\)

Proof of Proposition 3

Under the distribution \(F\), Lemma 2 gives \(x^{\hat{u}} = \frac{1}{2}\) and \(F(x^{\hat{u}}) = \frac{1}{2}\). Hence the exhibitor’s profit when committing to uniform pricing is given by

\[
\pi^{u} = bs_{H} - \frac{t}{2} - K.
\]  

(13)

(12) yields the exhibitor’s profit under a variable pricing commitment:

\[
\pi^{d} = (b_1 s_{H} - tx^{\hat{d}})F(x^{\hat{d}}) + (b_2 s_{H} - t(1 - x^{\hat{d}}))(1 - F(x^{\hat{d}})) - K
\]

\[
= b_2 s_{H} + t\left[\left(1 - 2x^{\hat{d}} + \frac{(b_1 - b_2)s_{H}}{t}\right)F(x^{\hat{d}}) + x^{\hat{d}} - 1\right] - K.
\]  

(14)

Subtracting (14) from (13) and using (6), we have

\[
\pi^{u} - \pi^{d} = \frac{2b_2 - b_1 - b_2 s_{H}}{2} - \frac{t\left(1 - 2F(x^{\hat{d}})\right)}{2f(x^{\hat{d}})}.
\]

Here \(1 - 2F(x^{\hat{d}}) > 0\) since \(F(x^{\hat{d}}) < \frac{1}{2}\). Finally, from \(\pi^{u} - \pi^{d} > 0\), we can arrive at (8). \(\square\)
REFERENCES


