博士論文要旨

Essays on Structural Breaks in Time Series and Panel Data Models

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Introduction

Time series models with structural breaks have been intensively investigated over the last fifty years, and various kinds of estimation methods and testing procedures have been proposed in the econometric and statistical literature. For the structural change tests, the CUSUM test by Brown, Durbin and Evans (1975) and Ploberger and Kramer (1992), the sup-type test by Andrews (1993), and the mean- and exponential-type tests by Andrews and Ploberger (1994) and Andrews, Lee and Ploberger (1996) are widely used in empirical analyses.

In practice, when we apply structural break tests, we need to take serial correlation into account, and thus we have to estimate the long-run variance of the error term. However, it is known in the literature that the finite sample performance of the tests is poor when we assume serial correlation in the error term. For example, if we estimate the long-run variance under the null hypothesis of no structural breaks, the tests suffer from the so-called "non-monotonic power" problem, as explained in Vogelsang (1999), Crainiceanu and Vogelsang (2007), Deng and Perron (2008) and Perron and Yamamoto (2014). The "non-monotonic power" problem is that the power decreases as the break magnitude increases, so that we cannot detect big structural breaks. The reason for this problem is that the long-run variance estimator using the residuals under the null hypothesis takes extremely large values when the break magnitude is large, and thus the test statistics take small values under the alternative hypothesis. On the other hand, if we estimate the long-run variance under the alternative hypothesis, the tests suffer from the size distortion because the long-run variance estimator has downward bias under the null hypothesis.

In order to cope with the problems, several methods have been proposed in the literature. Sayginsoy and Vogelsang (2011) and Yang and Vogelsang (2011) proposed tests with good size by employing the fixed-b method. Shao and Zhang (2010) applied the self-normalization method to the CUSUM test to improve the size of the tests. However, the fixed-b and the self-normalizing methods use an inconsistent long-run variance estimator, so that the tests suffer from asymptotic power loss. On the other hand, Juhl and Xiao (2009) proposed to estimate the long-run variance using nonparametrically demeaned residuals to mitigate the non-monotonic power problem, but the finite sample performance of their test is very sensitive to the choice of the bandwidth used in the nonparametric estimation. Kejriwal (2009) proposed to estimate the long-run variance using the residuals both under the null and alternative hypotheses, but the test has extremely low power when the error term has strong serial correlation. Overall, the existing methods are not satisfactory, in view of both size and power.

While most of the existing literature consider the time series models, as the macro panel data become available, it is necessary to test for the constancy of parameters in panel data models. For panel data models, we need to consider the cases where the parameters are time-varying and heterogeneous. The tests for slope heterogeneity in panel data models are studied by Swamy (1970), Pesaran and Yamagata (2008) and Juhl and Lugovskyy (2014), but the tests for parameter constancy in the time series direction has not been widely studied in the literature.

In this thesis, we investigate the theoretical properties of structural break models, and propose solutions to the problems associated with structural breaks in time series and panel data models. In Chapter 2, we develop tests for parameter constancy in panel data models, taking heterogeneity into account. In Chapter 3, we derive the bias of the long-run variance estimator in the presence of structural breaks in mean, and propose a bias-corrected long-run variance estimator. In Chapter 4, we propose a bias-corrected test for a shift in mean.

Summary of Chapter 2

In Chapter 2, we propose tests for parameter constancy in the time series direction in the following heterogeneous-slope panel data model:

$$\begin{aligned} y_{it} &= \alpha_i + x_{it}'\beta_{it} + u_{it}, \qquad i = 1, \cdots, N, \quad t = 1, \cdots, T, && (1) \\ \beta_{it} &= \beta_{i,t-1} + e_{it}, && (2) \end{aligned}$$

where α_i is the individual effect, N is the number of cross sections, and T is the number of time series observations. We assume that u_{it} is heteroskedastic across cross-sections, and cross-sectionally dependent. The testing problem which we consider in this chapter is given by

$$H_0: Var(e_{it}) = 0 \qquad \text{vs.} \qquad H_1: Var(e_{it}) > 0.$$

Under the null hypothesis, β_{it} is constant across time, whereas under the alternative, β_{it} is time-varying.

We construct a locally optimal test based on Tanaka (1996) and an asymptotically point optimal test based on Elliott and Müller (2006), and derive the asymptotic distribution of the test statistics as $T \to \infty$ while N is fixed. We also consider the case where the parameter is homogeneous across cross-sections (which we call the "homogeneous-slope" model).

Since the asymptotic distribution depends on N in the heterogeneous-slope case, we need to calculate critical values and the optimal localizing parameter for each values of N. Therefore, we obtain the characteristic function of the limiting distributions, and derive the response surface of the critical values, and the optimal localizing parameter for the point-optimal test.

By Monte Carlo simulations, we find that the tests based on the homogeneous-slope model have serious size distortion when the true model has heterogeneous slopes. On the other hand, the tests based on the heterogeneous-slope model have good size for both the homogeneous- and heterogeneous-slope models, although these tests have lower power when the true model has homogeneous slopes. Therefore, we need to pay careful attention to the existence of heterogeneity in the slopes when we apply these tests.

Summary of Chapter 3

In Chapter 3, we consider the following time series model with multiple shifts in mean:

$$y_t = \begin{cases} \mu_1 + u_t & \text{for } t = 1, \cdots, T_1, \\ \mu_2 + u_t & \text{for } t = T_1 + 1, \cdots, T_2, \\ & \vdots \\ \mu_{m+1} + u_t & \text{for } t = T_m + 1, \cdots, T, \end{cases}$$

and consider estimating the long-run variance of the error term u_t. We estimate the long-run variance by the autoregressive spectral density estimator based on the AR(p) model, which is defined as

$$\hat{\omega}_{AR} = \frac{\hat{\sigma}_\varepsilon^2}{\left(1 - \sum_{j=1}^p \hat{\phi}_j\right)^2},$$

where $\hat{u}_t = \sum_{j=1}^p \hat{\phi}_j \hat{u}_{t-j} + \hat{\varepsilon}_t$ with $\hat{\phi}_j$ $(j = 1, \cdots, p)$ being the OLS estimator, and $\hat{\sigma}_\varepsilon^2 = (T-p)^{-1} \sum_{t=p+1}^T \hat{\varepsilon}_t^2$.

It is known that the autoregressive spectral density estimator has downward bias in finite samples. In this chapter, we derive the bias of the autoregressive spectral density estimator up to $O(T^{-1})$, under the assumption that u_t follows a stationary AR(p) model. In order to derive the first-order bias of the long-run variance estimator, we first obtain the bias of the OLS estimator of the AR(p) regression in the presence of multiple shifts in mean. Then, we obtain the bias of the long-run variance estimator, and propose a bias-correction method. We find that the downward bias of the OLS estimator gets larger as the number of structural breaks increases, which leads to the downward bias of the long-run variance estimator.

When the error term u_t follows a stationary infinite-order autoregressive process, we need to truncate the lag order at p_T to implement the autoregression, and we let p_T go to infinity at an appropriate rate. In this case, we show that the first-order bias of the long-run variance estimator is exactly the same as the case with fixed p, so that our bias correction method can also be applied in such cases.

We perform simulations to investigate the finite sample properties of the long-run variance estimators. We find that the bias-corrected long-run variance estimator has much smaller bias than other estimators, and the mean squared error of the bias-corrected estimator is comparable to that of other estimators. Overall, we can see that our bias correction works well in finite samples.

Summary of Chapter 4

Chapter 4 considers the following mean-shift model:

$$y_t = \mu + \delta DU_t(T_b^0) + u_t, \quad t = 1, \cdots, T,$$

where $DU_t(T_b^0) = 1\{t > T_b^0\}$, and $1\{\cdot\}$ is the indicator function. We are interested in the following testing problem:

$$H_0 : \ \delta = 0 \quad \text{vs.} \quad H_1 : \ \delta \neq 0.$$

Under H_0, there is no shift in mean, while under H_1, there is a one-time break.

When u_t is serially correlated, we need to estimate the long-run variance of u_t for the scale adjustment in order to test for a shift in mean. If we estimate the long-run variance under the null hypothesis of no structural breaks, it is known that the tests suffer from the non-monotonic power problem because the long-run variance is over-estimated when the break magnitude is large (Vogelsang 1999, Crainiceanu and Vogelsang 2007, Perron and Yamamoto 2014). On the other hand, if we estimate the long-run variance under the alternative hypothesis, the tests suffer from the over-size distortion because the long-run variance is under-estimated under the null hypothesis.

In order to improve the finite sample properties of the tests, we propose bias correction to the long-run variance estimator, which is estimated under the alternative hypothesis. First, we derive the bias of the reciprocal of the autoregressive spectral density estimator based on the AR(p) model, under the assumption that the correct specification of u_t is the AR(p) process. Then, we propose bias correction to the test statistics.

We also discuss the cases where u_t follows a stationary AR(∞) process, and we find that the first-order bias is exactly the same as in the AR(p) case.

Simulation results show that the bias-corrected tests have much less size distortion than the tests without bias correction. Moreover, the bias-corrected tests have higher size-adjusted power than the existing tests. Since our proposed tests use a consistent long-run variance estimator, there is no asymptotic power loss due to bias correction. Thus, the bias-corrected tests have good finite sample property, in terms of both size and power.