

A NOTE ON ENVY AND EARNINGS INEQUALITY UNDER LIMITED LIABILITY CONTRACTS*

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Abstract

The paper analyzes an ex-ante contracting with limited liability constraints when agents feel envious of others' higher wages. We show that depending on the degree of limited liability constraints, the principal requires various distortions in output at both the top and bottom productivity levels for agent's type. Compared to the result without envy, the output gap between efficient and inefficient agents is less spread out. Moreover, when the degree of envy is sufficiently large, bunching can always occur. Hence, the first-best solutions for both types of agent are never obtained with envy regardless of the burden of limited liability.

Keywords: envy, limited liability, ex-ante contracting, distortion

JEL Classification Codes: D82, J31, M21

I. *Introduction*

Traditionally, individuals have generally been assumed to selfishly maximize their own payoffs. However, recent research on social preferences suggests that individual behavior is guided not only by payoff-equalizing motivations but also by concern for overall welfare (Bolton and Ockenfels, 2000; Camerer, 2003; Camerer *et al.*, 2004; Gintis *et al.*, 2005). Specifically, Fehr and Schmidt (1999, 2006) found that the influence of fairness and inequity aversion (i.e., social preferences) depends on how people compare themselves based on the level of payoff of their individual dislikes¹.

The suggestions that social comparisons in the firm affect workers' behaviors are as follows. Festinger (1954) proposed a theory of social comparisons that people compare

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¹ Moreover, Easterlin (1995, 2001) demonstrated that at the country level, an increase in individual wages is closely related to the degree of residents' happiness. However, this correlation does not increase when the country grows richer over time. This indicates that an individual's degree of happiness is related to his/her relative income level rather than to his/her absolute income level.

themselves to others that are similar. In a survey study with interviewing firm managers, Bewley (1999, p. 82) found that “the main function of internal pay structure is to ensure internal pay equity, which is crucial for good morale.” Recently, Bandiera *et al.* (2005) reported on personnel data from the UK fruit farm that switched its payment scheme from relative incentives to piece rates. The productivity of the average worker was at least 50 percent higher under piece rates than under relative incentives, and Bandiera *et al.* (2005) attribute this productivity gap to social preference (i.e., workers internalize the externality that their effort imposes on others under relative incentives). For more experimental evidence, among others, Clark and Oswald (1996) and Brown *et al.* (2008) have emphasized that the role of the degree of happiness on income comparison depends on the position in an organization.

Based on recent experimental evidence of agents’ comparison of their payoffs with those of other agents, this study expands on the theory of the *ex-ante contracting*² model by adopting agent’s preferences as “(self-interested) inequity aversion” as proposed by Fehr and Schmidt (1999), abstracting from empathy. Hereafter, we refer to agents as being envious in the sense that Fehr and Schmidt (1999) suggested that altruistic motivations are dominated by envy, assuming that fair-minded individuals dislike the inequitable distribution of economic resources³. Thus, agents receiving a lower wage are envious of agents receiving a higher wage. The cost of receiving a lower wage is increasing in the wage differential. We analyze whether a competitive agent is motivated by envy and limited liability constraints. Unlike most studies, which have focused on principal-agent model and imply that the principal imposes limited liability constraints on *wages*, we assume that the limited liability constraints cover the envy cost (i.e., the principal imposes limited liability constraints on *rents* as relevant measure).

In light of this envy effect, this paper builds upon the principal-agent model with the following features⁴: (1) the risk-neutral agent can be either efficient or inefficient and the risk-neutral principal never observes the agent’s type; and (2) the two parties sign a contract before the agent knows his type. This uncertainty also includes the existence of a risk-neutral agent who is concerned about other agents’ payoffs. The first feature of our setup is simply one of the problem of adverse selection, in which only the agent is aware of hidden information after the contract is signed. Under the second feature of our setup, the two parties sign a contract when they share the same belief about the agent’s type. This implies that even the agent does not know whether his type is efficient or inefficient at the timing of contracting, and that only the agent discovers his type to be costless after signing a contract. We refer to this situation as “*ex-ante contracting*,” which is a classification concept used in contract theory⁵. As an intuitive example of this timing, we can imagine a scenario in which the employer in a firm (i.e., the

² We borrow this terminology that comes from Laffont and Martimort (2002, pp. 57-58), which will be clear below.

³ According to Itoh (2004, p. 25), “I adopt the distributional approach, and in particular the theory of inequity aversion *à la* Fehr and Schmidt (1999)...it is simple and tractable. Second, it is not an *ad hoc* specification tailored for a particular finding. It is a parsimonious specification of other-regarding preferences, supported by psychological intuition, that can explain various types of experiment results with a single function.” Moreover, Rey-Biel (2008, p. 298) argued that, “Distributional preferences are one of the most frequent explanations for subject’s behavior in a wide variety of experiments....We have chosen Fehr and Schmidt’s (1999) inequity-aversion model as a reduced form of social preference due to its prominence and simplicity.” See also Section II for theoretical models with social preference.

⁴ For the purposes of identification, it is assumed that the principal is a female and the agent is a male.

⁵ Other terminology includes either “moral hazard with hidden information” or the “hybrid model” (i.e., hidden actions and hidden information).

principal) offers two possible units (large or small) of the good to a job candidate (i.e., the agent). In particular, at the timing of contracting, even an inefficient agent believes with some probability that he may be an efficient supplier; then, by taking this possibility into account, the agent signs the contract⁶. This situation may be fitted that if the costs for training technological skills develop rapidly in a new industry, the contract remains unaltered.

We now present some rationale for discussing why it is necessary to consider limited liability constraints under an ex-ante contracting model, along with a version of inequity aversion of social preferences. Harris and Raviv (1979) showed that the principal can implement the first-best outcome by making the agent bear all the risks of the transaction at some fixed price when an agent's state of nature can be observed before he exerts his level of effort but after he signs a contract. Although this contract promises the agent's reservation level of the expected utility, when the agent turns out to be inefficient, he can do no better under this contract than suffer a loss in utility below the level achieved in autarky. In such a case, the agent would like to breach the contract, but Harris and Raviv (1979) simply assumed the existence of a strong law-enforcement institution that prevents him from doing so. In the absence of such an institution, it is important to examine the properties of the contract that emerge between the two parties when limits are imposed on the maximum loss that the agent can be forced to bear. Such contracts are said to be subject to limited liability constraints. Sappington (1983) characterized the optimal contract under limited liability and showed that it never achieves the first-best outcome⁷. We investigate whether the efficiency of a contract increases or decreases if social preferences are introduced within a framework of limited liability constraints. By adding the bearable maximum amount of loss as limited liability constraints, we can thus examine the issue of incentive contracts between a principal and risk-neutral agents with inequity aversion.

Most analyses that apply social preferences to the principal-agent model do not fully compare the optimal solutions with the first-best output (or the first-best level of effort) in the presence of limited liability constraints⁸. However, this paper finds that if an agent is envious of others' payoffs (i.e., wages) with limited liability constraints, the first-best solution is never obtained before the agent discovers his type regardless of the burden of limited liability. Our main contribution of this paper is the failure to achieve the first-best outcome that ex-post Pareto efficiency does not hold for the efficient type of the agent. This exhibits a contrast with the result of Sappington (1983), who showed that ex-post Pareto efficiency holds for the efficient type but does not hold for the inefficient type. In this study, depending on the degree of limited liability, the principal who takes the existence of envy into account requires various distortions in both the top and bottom output levels according to each agent's type. However, if

⁶ Another example of ex-ante contracting is given in Macho-Stadler and Perez-Castrillo (2001), who consider an import-export agent who works for a firm and does not know how the product will perform in the foreign market until he begins working there. However, he must choose the best strategy for the firm in the foreign market based on the available information.

⁷ See Macho-Stadler and Perez-Castrillo (2001, pp. 64-66) for an intuitive explanation. Moreover, Innes (1990), Park (1995), Kim (1997), and Lewis and Sappington (2001) derived optimal contracts under moral hazard and limited liability constraints.

⁸ Grund and Sliwka (2005) studied that the first-best effort is not implementable under the rank-order tournament among inequity-averse agents. They do not impose the limited liability constraints on contracts; hence, the participation constraint binds and plays a crucial role.

the degree of envy is sufficiently large, we show that bunching may always occur regardless of the degree of limited liability.

There are several reasons why the contract efficiency decreases. Suppose that bunching does not occur. First, if the limited liability constraint on the inefficient agent is sufficiently stringent, the principal will try to reduce her loss resulting by increasing the inefficient agent's output above the usual level with limited liability constraints. This is because the principal bears the additional cost owing to the existence of envy in order to ensure that all envious agents participate in contracts. Then, for the principal, in order to overcome the increase in the inefficient type's output, the efficient type's output is decreased below the first-best level, implying that envy makes the efficient type weaken the incentive provision. Thus, in comparison to the solutions of the canonical ex-ante contracting model, the existence of envy leads to a decrease (an increase) of production by efficient (inefficient) agents.

Second, if the inefficient agent's limited liability constraints are relaxed, the ex-ante participation constraint can be binding. This effect leads to an increase in the inefficient agent's output above the first-best level because the information revelation becomes costly with the envy cost in the viewpoint of the principal. However, the efficient agent's output is then reduced in order to save on additional costs of the principal with higher information rent because of the distorting of the inefficient type's output upward as compared to the first-best level. Consequently, the principal offers contracts involving an output level that is different from the first-best level, resulting in further downward distortions for efficient agents and upward ones for inefficient agents.

Lastly, if the burden of limited liability is sufficiently small for both types of agents, there may be some asymmetric risk in the distribution of information rents because agents' envy generates an additional cost to the principal. Thus, distorting the inefficient type's output upward relaxes the limited liability constraint. In this case, envy will reduce the efficiency of efficient agents, leading to the possibility of distortions in both the top and bottom output levels with an additional cost to the principal.

II. *Related Literature*

Several relevant studies have investigated social preference with or without limited liability contracts. For instance, Itoh (2004) and Rey-Biel (2008) analyzed optimal contracts by applying social preferences and limited liability constraints to the moral hazard model⁹. Itoh (2004) demonstrated that the principal's profit decreases if the agent's concern for equity increases. When a number of identical agents are concerned about the others' material payoffs, the principal can optimally exploit their inequity aversion by designing an appropriate interdependent contract. Rey-Biel (2008) showed how equal and unequal rewards must be offered and unequal rewards offered to provide extra incentives to work hard, even if employers are restricted by limited liability (i.e., when negative bonuses are not possible). However, this leads to a different result from that of the asymmetric information of ex-ante contracting described in

⁹ Several papers have introduced inequity aversion into moral hazard problems, including Demougin *et al.* (2006), Dur and Glazer (2008), Grund and Sliwka (2005), Bartling and von Siemens (2010), and Englmaier and Wambach (2010). See Englmaier (2005), and the references therein.

the present study. In contrast to Itoh (2004) and Rey-Biel (2008), taking into account the existence of envy under an ex-ante contracting model, we find that depending on the restrictions on limited liability constraints for agents, the principal requires varying distortions of output levels at both the top and bottom for each agent's type.

In addition, Neilson and Stowe (2010) analyzed the linear wage of multiple agents with social preferences where the contract proposed by the principal depends on each agent's performance. von Siemens (2004, 2011, 2012) analyzed optimal employment contracts in adverse selection, in which agents with social preferences know their types before signing the contract. Desiraju and Sappington (2007) focused on the condition under which inequity can be avoided, without cost by changing the provision of incentives. As in von Siemens (2004, 2011, 2012) and Desiraju and Sappington (2007), the agents in their study know only their types at the time of the contract and these models examined contracts offered at the interim stage without limited liability constraints. The present study of envy differs because it proposes that varying distortions in both the top and bottom output levels for each agent's type are possible even after the imposition of limited liability constraints.

III. *The Model*

1. **Productions, Costs, and Utility**

We consider an extension of the Laffont and Martimort (2002) model of ex-ante contracting. Consider a risk-neutral principal facing a continuum of agents with measure one. The principal wants to delegate the production of x units of a good to an agent. For the principal, the value of these x units is $S(x)$, where $S'(x) > 0$, $S''(x) < 0$ and $S(x) = 0$. To ensure that some production occurs for every type, we assume that $S'(x) = \infty$, and to ensure that production is always finite, we assume that $\lim_{x \rightarrow \infty} S'(x) = 0$.

Assume that the production cost of the agent is unobservable by the principal, but that it is common knowledge that there are two types of agents, $T = \{t_0, t_1\}$ with $0 < t_0 < t_1$. We assume the two-type case, where agents can be either efficient (t_0) or inefficient (t_1) with respective probabilities of p_0 and p_1 . For simplicity, let us denote $p_0 = p$ and $p_1 = 1 - p$. In other words, the agent has the cost function $C(x_i, t_i) = t_i x_i$, $i = 0, 1$. To focus on the envy effect, we assume that a risk-neutral (instead of risk averse) agent with a utility function U_i defines his monetary gains as $w_i - t_i x_i$ where the wage that the principal has to pay is denoted by w_i : $U_i = w_i - t_i x_i$, $i = 0, 1$ without envy.

To study the effects of envy preference on adverse selection in an ex-ante contracting model, we extend our model by allowing agents to compare wages, as proposed by Fehr and Schmidt (1999)¹⁰. Hence, we assume that (A1) all agents experience envy over wages; (A2) agents may have different preferences even with the same level of envy; and (A3) the feeling of envy dominates that of altruism. These assumptions need to be regarded with some cautions. For example, (A2) suggests that agents are more concerned about receiving lower material

¹⁰ For more discussion of comparing wages between agents, we will mention later and do not deal with more complicated forms of social preferences related to reciprocal behavior and intentions. For good surveys on social preferences, see Fehr and Schmidt (2006).

payoffs than they are about receiving higher material payoffs¹¹. Psychologists agree that individuals usually compare themselves with other individuals whom they perceive to have an equal standard of living or a similar job status. Hence, we assume that the reference group of agents consists only of the other agents who work within the same society. In other words, we exclude the comparisons of wages when the agents are not employed, which implies that the agents do not experience envy because not entering a firm means that they receive the lowest possible wage zero and thus experience no envy¹². That is, following Frank (1985), we investigate the implications of the demand for local status, where agents prefer to work in firms which they obtain a higher wage than their co-workers. Comparisons over wages are for wages of “people like me.” Finally, (A3) considers that agents do not like receiving less than other agents do. This seems empirically relevant because the focus here is on envy in inequity aversion.

In our model, like that of von Siemens (2004, 2011, 2012), the following additional notation and definition complete the statement about envy under ex-ante contracting. Assume that the agent is employed and that he has to fulfill the requirements of the contract $\{(w_0, x_0), (w_1, x_1)\}$. Consider an agent who is type t_i but claims to be of type t_k . Thus, let

$$R[t_i, (w_i, x_k)] = w_k - t_i x_k - \alpha \sum_{j=0,1} p_j \max[w_j - w_k, 0] \quad (1)$$

denote the overall utility of the agent, namely his wages minus both the cost function and his concern about the inequality of the comparison wage. An agent compares himself with a type t_i agent after indicating that he is type t_k . The term $\alpha \sum_{j=0,1} p_j \max[w_j - w_k]$ specifies the concern about inequality. In Eq. (1), $\alpha > 0$ is the weight of inequality and p_j is the percentage of agents reporting that they are of type t_j . This Eq. (1) implies that an agent loses his utility when he earns less than the others, implying he feels envious of other agents' wages¹³.

2. Timing

¹¹ If the findings of this study were to be extensively applied, consideration should also be given to a model that takes into account not only agents with social preferences but also selfish ones who are concerned only with their own utility. However, this is beyond the scope of this research. For the case of whether each worker has either a high or low ability, or either selfish or social preferences, see von Siemens (2011, 2012).

¹² As stated in von Siemens (2011, pp. 786-788) and the references therein, “The remaining specification of social preferences is inspired by equity theory....Equity theory further builds on the assumption that individuals mostly compare themselves with similar others....The point is not that employees are not aware of unemployed workers or employees of other firms, but that these workers provide the former with less relevant information concerning their performance in their firm. Further, workers are in more frequent personal contact with direct colleagues than with employees in other firms or the unemployed.” Thus, it has often noted that social comparisons are made locally rather than globally.

¹³ The model can be extended using agents comparing their rent $[w_k - t_i x_k]$ with those of other agents rent $[w_m - t_j x_m]$. The alternative is to assume that types are inequity averse with respect to utility net of inequity costs. Because whether rents/types are compared is an empirical question, to simplify the exposition as much as possible, we assume that envy arises from the difference between high and low wages. This implies that an agent receiving the lower wage feels envious of an agent receiving the higher wage. Cabrales and Calvo-Armengol (2008) assumed that agents do not compare rents, but rather wages. Our qualitative results hold with this alternative specification, although more interesting issues arise when production costs enter the comparison. Ultimately, this is an empirical question that may be context dependent.

The distinguishing characteristics of timing, which have been discussed throughout the paper, are now presented before noting some areas of concern.

- In the first stage, namely at the time of the contract offering the principal and the agent have identical beliefs about the probability distribution of type, t_i . Assume that neither the principal nor the agent knows the agent's type and that the agent is given a contract menu specifying $\mu = \{(w_0, x_0), (w_1, x_1)\}$ including limited liability constraints suggested exogenously by the principal. If the agents are of type t_0 (resp. t_1), the principal pays a wage of w_0 (resp. w_1) for output, x_0 (resp. x_1).
- In the second stage, after observing the terms of the contract, the agent either accepts or rejects the contract. In the latter case the game ends. If the agent accepts the contract, he discovers his type to be costless; that is, there is still asymmetric information between the principal and agent. In this stage, because the principal has already imposed limited liability constraints exogenously, renegotiation is not possible¹⁴.
- In the third stage, the agent begins work and the principal receives a profit even if only a fraction of agents fulfill their contracts. Although the agent's type is unknown, the principal's profit is determined by the ratio of agents, who participate in manufacturing activities, to the group of agents normalized to one. Each agent enjoys a utility level according to the contract, comparing his material benefit with those of others'.

IV. Results as Optimal Contracts

In this section, we examine the asymmetric information situation in which an agent envies another's wage and the agent's type is not observed before contracting.

Suppose that the principal wants to employ both types of agents¹⁵. To analyze the influence of limited liability constraints under the envy effect, we assume that the set of feasible-incentive contracts is constrained by exogenous limits on feasible transfers between the principal and envious agents. These exogenous financial constraints may affect the usual rent-efficiency trade-off. One possible limit is that the rent received by the agent should not be lower than the agent's liabilities, which are fixed at some exogenous value $-l$. We assume that the contract must satisfy the following limited liability constraints on rents:

$$U_0 - \alpha(1-p)\max[w_1 - w_0, 0] \geq -l, \quad (2)$$

$$U_1 - \alpha p \max[w_0 - w_1, 0] \geq -l. \quad (3)$$

These constraints (2) and (3) are a contract with the principal that may lead to negative rents U_i unless those losses are covered by the agent's own liability l .

¹⁴ If the renegotiation is allowed, agents have an incentive of a breach of contract (or may break off the relationship) in the interim stage and the contract will be back to the timing of standard adverse selection. Using the timing of standard adverse selection, von Siemens (2004) has analyzed the case of inequity-averse agents.

¹⁵ Focusing on canonical adverse selection without the limited liability constraints, von Siemens (2011, 2012) showed that envy can create unemployment in a competitive market if agents have private information on both their propensity for social comparisons and their ability. However, the analysis of screening between unemployment and employment is beyond the scope of this research, which is left to future research to develop the analysis more generally.

Typically, wages are expected to be larger than the agent's liabilities, that is, $w_i \geq -l$ (we can rewrite these constraints as $U_i \geq -l - t_i x_i$) as in Itoh (2004) and Rey-Biel (2008). With limited liability constraints on rents as the relevant measure, if the principal imposes limited liability constraints on wages, the agent can use the wage received from the principal to cover the debt level $-l$, because the production cost is sunk. The possibility of using the wage to cover a debt level implies that the agent's wage already becomes low before a comparison of wages among agents. This indirectly implies that costs become observable (i.e., someone has to ensure that the limited liability constraint is satisfied). Thus, all agents can experience envy over the different wages among (in)efficient agents. If we impose limited liability constraints on wages, the results do not seem to be interesting because of the many bunching cases that occur¹⁶. In the first stage, the principal has already imposed limited liability constraints, because neither the principal nor the agent knows the agent's type and the production cost has not been sunk at the time the contract is offered. In this situation, the production cost tx is incurred when the transfer w takes place because the production cost has not been sunk.

Given that the principal-agent contract is signed before the agent discovers his type, the agent's ex-ante participation constraint is written as

$$pU_0 + (1-p)U_1 - \alpha p(1-p)\{\max[w_1 - w_0, 0] + \max[w_0 - w_1, 0]\} \geq 0, \quad (4)$$

where the contract must guarantee the agent's reservation utility, which is taken to be zero. Thus, the agent's expected information rent minus the expected amount of envy must be non-negative to ensure participation. Given such a direct revelation mechanism, the principal's program $[P']$ is given by

$$[P'] : \quad \max_{U_i, x_i} p[S(x_0) - w_0] + (1-p)[S(x_1) - w_1]$$

subject to (2), (3) and (4),

$$\begin{aligned} R[t_0, (w_0, x_0)] &\geq R[t_0, (w_1, x_1)] \\ \Leftrightarrow w_0 - t_0 x_0 - \alpha(1-p)\max[w_1 - w_0, 0] &\geq w_1 - t_0 x_1 - \alpha p \max[w_0 - w_1, 0], \end{aligned} \quad (5)$$

$$\begin{aligned} R[t_1, (w_1, x_1)] &\geq R[t_1, (w_0, x_0)] \\ \Leftrightarrow w_1 - t_1 x_1 - \alpha p \max[w_0 - w_1, 0] &\geq w_0 - t_1 x_0 - \alpha(1-p)\max[w_1 - w_0, 0]. \end{aligned} \quad (6)$$

The incentive compatibility constraint (5) with the envy factor ensures that the efficient agent will not gain by announcing t_1 . The incentive compatibility constraint (6) with envy states that the inefficient agent truthfully reports his private information. After accepting the contract, an agent discovers his type to be costless, which implies that the overall utility of agents is provided by the principal.

1. Benchmark: The Selfish Case

We first introduce the formal definition of the first-best solution when there is no asymmetry of information and no envy between the principal and agents. The efficient levels

¹⁶ The detailed computations of limited liability constraints on wages under envy are available from author upon request.

are obtained by equating the principal's marginal value and the agent's marginal cost¹⁷. Thus, first-best outputs are given by $S'(x_i^{fb}) = t_i$ where by fb denotes the first-best level and $w_i^{fb} = t_i x_i^{fb}$. The optimal output levels are such that $x_0^{fb} > x_1^{fb}$.

Before presenting the results of optimal contracts made under envy, we need to clarify the benchmark of asymmetric information without envy. To distinguish between different notations, we index limited liability (i.e., the second-best optimal contract without envy) with a superscript L and incentive constraints with no envy with (IC_i^L) . As in the textbook case (e.g., Laffont and Martimort, 2002) and using notation with $t_1 - t_0 \equiv \Delta t$, the canonical solutions are as follows.

Proposition 1-A (Laffont and Martimort (2002, Chapter 3.5 Proposition 3.4: pp. 118-121)): *Assume ex-ante contracting and limited liability on rents. Then the optimal contract entails¹⁸:*

(a) *For $l < p\Delta t x_1^{sb}$, only (IC_0^L) and (LL_1^L) are binding. The efficient agent produces efficiently $x_0^L = x_0^{fb}$, and the inefficient agent's production is equal to the second-best output with $x_1^L = x_1^{fb}$, where $S'(x_1^{fb}) = t_1 + \frac{p}{1-p}\Delta t_1 > S'(x_1^{fb}) = t_1 \Leftrightarrow x_1^{sb} < x_1^{fb}$.*

(b) *For $p\Delta t x_1^{sb} < l \leq p\Delta t x_1^{fb}$, (EPC^L) , (IC_0^L) , and (IC_1^L) are only binding. The efficient agent produces efficiently $x_0^L = x_0^{fb}$, and the inefficient agent's production is distorted downward from the first-best $x_1^L \leq x_1^{fb}$ with $x_1^{sb} \geq x_1^L$ and $l = p\Delta t x_1^L$.*

(c) *For $l > p\Delta t x_1^{fb}$, only (IC_0^L) and (EPC^L) are binding and the first-best outcome remains as nonbinding (LL_0^L) and (LL_1^L) . Each ex post information rent is as follows: $U_0 = (1-p)\Delta t x_1^{fb} > 0 > U_1 = -p\Delta t x_1^{fb}$.*

2. Optimal Contracts with Envy and Limited Liability Constraints

Given Subsection IV.1 with the benchmark, we examine the optimal ex-ante contract with the presence of envy. Consider the principal's optimal maximization problem with its program

¹⁷ On the other hand, suppose that there is no asymmetry of information between the principal and agents with envy. Thus, when an agent feels envious of others' wages, the first-best solution is obtained by solving

$$\max_{x_i} S(x_i) - w_i + w_i - t_i x_i - \alpha \sum_{j=0,1} p_j \max[w_j - w_i, 0].$$

Hence, the efficient production levels are obtained by equating the principal's marginal value and the agent's marginal cost even though there is agents' concern about the inequality of the comparison wage. Thus, first-best outputs may be given by $S'(x^{fb}) = t_i$ as in the main context. However, first-best wages may be determined with α : $w_0^{fb} = t_0 x_0^{fb} - \alpha(1-p) \max[w_1 - w_0, 0]$ and $w_1^{fb} = t_1 x_1^{fb} - \alpha p \max[w_0 - w_1, 0]$.

¹⁸ Proposition 1-A is derived directly from the proof of Proposition 1. Such results can be obtained easily by calculating $\alpha = 0$ in the principal's program $[P']$. See Appendix A considering $\alpha = 0$. The intuition without envy effect is as follows. Efficient agents produce more than inefficient agents, and the principal does not leave rent to the agent on average. The contract deters efficient agents from mimicking inefficient agents by giving negative rents to inefficient agents. This is possible because agents do not know whether they are productive or not when accepting the contract. The positive rent given to efficient agents exactly balances the loss of inefficient agents so that an agent accepts the contract ex ante. Agents may, however, be protected by a limited liability clause that sets a lower bound on rents. If this bound is stringent enough, information revelation becomes costly. Depending on its value, the optimal contract can take different forms.

Setting $w_0 = w_1$ means that each agent has absolutely no envy in the optimal contract framework. Hence, an inefficient agent must also prefer an efficient agent's contract to his own. This contract cannot be incentive-compatible. To solve for the maximization of the principal's profit under (2), (3), (4), (5) and (6), we momentarily assume that

$$w_0 > w_1. \quad (7)$$

We later check that this is satisfied using the obtained solution¹⁹.

Incorporating the definition $U_i = w_i - t_i x_i$ into the principal's program $[P']$ yields the principal's program $[P]$ as follows:

$$[P]: \max_{U_i, x_i} p[S(x_0) - w_0] + (1-p)[S(x_1) - w_1] - [pU_0 + (1-p)U_1 - \alpha p(1-p)(w_0 - w_1)]$$

$$\text{subject to } pU_0 + (1-p)U_1 - \alpha p(1-p)(w_0 - w_1) \geq 0, \quad (8)$$

$$U_0 \geq -l, \quad (9)$$

$$U_1 - \alpha p(w_0 - w_1) \geq -l, \quad (10)$$

$$w_1 - t_1 x_1 \geq w_0 - t_1 x_0 + \alpha p(w_0 - w_1), \quad (11)$$

$$w_0 - t_0 x_0 \geq w_1 - t_0 x_1 - \alpha p(w_0 - w_1). \quad (12)$$

The incentive constraint of the efficient agent, (12), necessitates that he receives a higher wage in addition to information rent. When the efficient agent does not report his type honestly, it is less profitable for that agent to pretend to be inefficient because he will receive an inequitable wage. Thus, an efficient agent wants to maintain a balance between receiving an inequitable wage and receiving information rent in order to preserve his utility.

To solve for the maximization of $[P]$ under (8), (9), (10), (11), and (12), we momentarily ignore (11). We check ex post that the omitted constraint (11), and the assumption of (7) are strictly satisfied. Moreover, we introduce the following assumption.

$$\Delta t x_1 > \alpha(w_0 - w_1). \quad (13)$$

Assumption (13) implies that the effect of the information rent is larger than the envy cost (i.e., $\alpha(w_0 - w_1)$) from the wage difference. Otherwise, the incentive constraint for the efficient agent cannot be incentive-compatible²⁰. To distinguish between the notations, we index another second-best optimal contract with envy with superscript "e." Furthermore, superscript "fb" indicates the first-best output, and "sb" indicates the second-best output in the standard case without envy.

Given that the first-best outputs are defined by $S'(x_i^{fb}) = t_i$, the solution of the program $[P]$ leads to the following proposition:

Proposition 1: *Suppose that agents feel envious of others' wages. When the agent is risk neutral and when contracting takes place ex ante with limited liability on rents, the optimal menu of contracts entails:*

¹⁹ Similarly, we can check that $w_0 \geq w_1$ by using von Siemens' (2004) Lemma 3.

²⁰ From (12), we can check that if the assumption (13) does not hold, (12) implies that $U_0 \geq U_1 + \Delta t x_1 - \alpha p(w_0 - w_1) \leq U_1$.

(i) For $l < p\Delta tx_1^e$ when $\alpha < \alpha^\dagger \equiv \frac{\Delta t}{t_0 - pt_1}$, only (10) and (12) are binding. The efficient agent's production is distorted downward from the first-best $x_0^e < x_0^{fb}$, and the inefficient agent's production is distorted downward from the first-best $x_1^{fb} > x_1^e > x_1^{sb}$. The efficient (resp. inefficient) type gains a strictly positive (resp. negative) ex-post information rent, $U_0^e = -l + \Delta tx_1^e - \alpha p(w_0^e - w_1^e) > 0 > U_1^e = -l + \alpha p(w_0^e - w_1^e)$.

However, for $l < p\Delta tx_1^e$ when $\alpha \geq \alpha^\dagger \equiv \frac{\Delta t}{t_0 - pt_1}$, the principal offers both types of agents a single contract, $x_0^e = x_1^e$ and $U_0^e = U_1^e = 0$.

(ii) For $p\Delta tx_1^e \leq l \leq p\Delta tx_1^{fb}$ when $\alpha < \hat{\alpha} \equiv \frac{\Delta t}{t_0}$, only (8), (10) and (12) are binding. The efficient agent's production is distorted downward from the first-best $x_0^e < x_0^{fb}$ and the inefficient agent's production is distorted upward from the first-best $x_1^e > x_1^{fb}$ (i.e., $x_0^{fb} > x_0^e > x_1^e > x_1^{fb} > x_1^{sb}$) and $p\Delta tx_1^e = l$. Thus, when $\alpha < \hat{\alpha} \equiv \frac{\Delta t}{t_0}$, the efficient (resp. inefficient) type gains a strictly positive (resp. negative) ex-post information rent, $U_0^e = (1-p)\Delta tx_1^e > 0 > U_1^e = -p[\Delta tx_1^e - \alpha(w_0^e - w_1^e)]$.

However, for $p\Delta tx_1^e \leq l \leq p\Delta tx_1^{fb}$ when $\alpha \geq \hat{\alpha} \equiv \frac{\Delta t}{t_0}$, only (8) is binding. Then, the principal offers both types of agents a single contract, $x_0^e = x_1^e$ and $U_0^e = U_1^e = 0$.

(iii) For $l > p\Delta tx_1^e$ when $\alpha < \alpha^* \equiv \frac{\Delta t}{t_0 - p\Delta t}$, only (8) and (12) are binding. The efficient agent's production is distorted downward from the first-best $x_0^e < x_0^{fb}$, and the inefficient agent's production is distorted upward from the first-best $x_1^e > x_1^{fb}$ (i.e., $x_0^{fb} > x_0^e > x_1^e > x_1^{fb} > x_1^{sb}$). The efficient (resp. inefficient) type gains a strictly positive (resp. negative) ex-post information rent, $U_0^e = (1-p)\Delta tx_1^e > 0 > U_1^e = -p[\Delta tx_1^e - \alpha(w_0^e - w_1^e)]$.

However, for $l > p\Delta tx_1^e$ when $\alpha \geq \alpha^* \equiv \frac{\Delta t}{t_0 - p\Delta t}$, only (8) is binding. The principal offers both types of agents a single contract, $x_0^e = x_1^e$ and $U_0^e = U_1^e = 0$.

Proof: See Appendix A. Q.E.D.

Proposition 1 provides that a decrease in the efficiency of ex-ante contracting is possible given certain limited liability constraints, which differs from the result in canonical ex-ante contracting with no envy. Moreover, as envy becomes greater than some critical value, bunching may always occur regardless of the degree of the burden of limited liability.

First, in the case of Proposition 1(i), if the limited liability constraints placed on inefficient agents are stringent enough when $\alpha < \alpha^\dagger$, the principal offers a contract for an inefficient agent with an output level that is higher than the second-best level in a standard ex-ante contracting model. The intuition for Proposition 1(i) is as follows. To ensure that all agents participate in contracts, the principal bears the additional cost owing to the existence of envy. This implies that (8) is strictly satisfied, and thus, (10) is binding. That is, inefficient agents suffer a utility loss if they receive wages lower than those of others; however, they are protected by a limited liability clause that sets a lower bound on rents with envy cost---that is, the cost of receiving a lower wage (i.e., the envy cost) is increasing in the wage differential. Therefore, as long as the

principal insists on a positive output, x_1^e , from the inefficient type, the principal should attempt to generate another distortion, $x_1^{sb} < x_1^e < x_1^{fb}$, to reduce the efficient agent's information rent. Consequently, the downward distortion for the inefficient agent is increased by the presence of envy. On the other hand, the incentive constraint is such that the efficient type is indifferent between accepting the contract designed for the efficient type and that for the inefficient type. As an agent earns a higher wage, he suffers from inequity aversion if he accepts the contract of the lower wage type through the term of the envy cost of the inefficient agent, $-ap(w_0^e - w_1^e)$. This in turn implies that by decreasing the output beyond the first-best level, the efficient type needs to be compensated since an increase in wage will simultaneously increase suffering from inequity aversion if the agent accepts the contract of the inefficient type. Thus, envy weakens the incentive provision, which is related to a new downward distortion for the efficient type. This leads to the possibility of distortions in agents' outputs at both the top and bottom of the productivity scale.

An important insight that follows from Proposition 1(i) is that the incentive problem will cause the principal to assign the same output level to all agents. As envy increases, while $\alpha \geq \alpha^\dagger$, different ex-post information rents are not realized because of the convergence in output levels. The principal cannot indefinitely raise the inefficient agent's output level without causing conflict with the implementation condition. Hence, bunching may occur. That is, the output of efficient agents decreases and that of inefficient agents increases when α approaches $\alpha^\dagger$. This means that if $\alpha \geq \alpha^\dagger$, it is optimal for the principal to propose a single contract (i.e., one involving the same level of output for both types).

In the case of Proposition 1(ii), if the condition $\alpha < \hat{\alpha}$ holds, the efficiency of the contract changes in a different way. This is because the limited liability constraints are relaxed with the existence of envy, which allows inefficient agents to improve their wage level (see Equation (A-2) in Appendix A). This implies that (8) and (10) are binding to facilitate an incentive provision for the inefficient type. Therefore, distorting the inefficient type's output upward relaxes this limited liability constraint, which implies that the inefficient agent is indeed compensated for his envy cost from the limited liability constraint of the inefficient agent (i.e., $U_1^e = -p\Delta tx_1^e + ap(w_0^e - w_1^e)$). However, owing to the relaxation of the limited liability constraints, part of the efficient agent's ex-post information rent with $l = p\Delta tx_1^e$ is extracted. This serves as an incentive for efficient agents to choose a lower output level. Therefore, the principal offers contracts involving an output level that is different from the first-best level, which results in new distortions: downward for efficient agents and *upward* for the inefficient agents. As stated in Proposition 1(i), the principal wants to assign the same output level to all agents when $\alpha \geq \hat{\alpha}$. Thus, it is optimal for the principal to propose a single contract.

Finally, in Proposition 1 (iii), when the limited liability is sufficiently small, analogous output levels to those in Proposition 1 (ii) are obtained for both agents, but through a different process, explained as follows. If the condition $\alpha < \alpha^*$ holds and the limited liability of both types of agents is sufficiently reduced, the principal wants to impose an expected rent that is equal to the expected amount of envy. If (8) is binding, the principal must structure the rents U_0^e and U_1^e to ensure that the gap between the two different output levels is such that (11) and (12) with envy remain satisfied. The limited liability constraint then implies an ex-ante information rent. With such a rent distribution, the optimal contract gives distortions at the top

and bottom productivity levels with an *additional cost* faced by the principal---that is, in the contract defined by $U_0^e = (1-p)\Delta tx_1^e > 0 > U_1^e = -p[\Delta tx_1^e - \alpha(w_0^e - w_1^e)]$ as compared to $U_0 = (1-p)\Delta tx_1^b > 0 > U_1 = -p\Delta tx_1^b$ in Proposition 1-A. From the contracts U_i^e in Proposition 1 (iii), the agent is rewarded when he is efficient and punished when he is inefficient. There may exist some *asymmetric* risk in the distribution of information rents because agents' envy incurs an additional cost to the principal. Thus, distorting the inefficient type's output upward relaxes the limited liability constraint²¹. When the condition $\alpha < \alpha^*$ holds, the presence of envy may raise the information rent of efficient agents (i.e., because of $x_1^e > x_1^{sb}$), but it will inefficiently raise the output level of inefficient agents. Moreover, envy will reduce the efficiency of efficient agents. This leads to the possibility that distortions exist at both the top and bottom productivity levels. However, as stated in Proposition 1(ii), the principal wants to assign the same output level to all agents when $\alpha \geq \alpha^*$.

With limited liability constraints on rents, the present paper is probably most closely connected to Itoh (2004), who showed that when agents have other-regarding preferences in the case of moral hazard, the optimal contract can be a team contract or a relative performance contract in the case of multiple symmetric agents. However, Proposition 1 differs from the results of Itoh (2004), in which the limited liability constraints on wages did not explicitly play a crucial role in requiring an agent's output level, and in which it is exogenously assumed that some limited liability constraints are binding. On the other hand, Desiraju and Sappington (2007) and von Siemens (2011, 2012) examined contracts offered at the interim stage without limited liability constraints. Desiraju and Sappington (2007) demonstrated that the outputs by both efficient and inefficient agents can be distorted; and that these distortions can be upwards and downwards. von Siemens (2011, 2012) showed that the output of efficient agents is never distorted and that social preferences aggravate the downwards distortion of the output of inefficient agents. The present study shows that depending on the burden of limited liability, the output gap between efficient and inefficient types differs from the canonical ex-ante contracting. This is because the present study proposes that two types of distortions in both the top and bottom output levels for each agent's type are possible even after the imposition of limited liability constraints. Furthermore, compared with agents who have private information but are not inequity averse (i.e., selfish and risk-neutral agents), Proposition 1 implies that each type of risk-neutral agent never attains the first-best output but receives different ex-post information rents under certain conditions.

The optimal contracts derived in Proposition 1 can thus be compared with the standard optimal contracts in the situation in which there is no envy on the part of agents (see Subsection IV.1). The results are shown in Corollary 1:

Corollary 1: *Suppose that agents compare wages. When the agent is risk neutral under ex-ante contracting with limited liability on rents, the presence of envy makes the optimal contract distort production more strongly.*

Proof: The criterion for the efficient contract is how the first-best level of output is obtained. From this criterion, Proposition 1 always leads to the possibility that distortions exist at both the top and bottom for the agent, while Proposition 1-A obtains the first-best level of the

²¹ This implies that the agent's expected utility becomes $U = -l + p\Delta tx_1^e + \alpha p(1-p)(w_0^e - w_1^e) > 0$ with envy, while it becomes $U = -l + p\Delta tx_1^{sb} > 0$ without envy.

efficient agent, for which distortion exists at the bottom for the inefficient agent. Q.E.D.

Corollary 1 can be understood as follows. Taking into account the existence of envy, the principal requires different distortions of output levels at both the top and bottom for each agent's type depending on the degree of the restriction on limited liability constraints for agents. However, when the degree of envy is sufficiently large, bunching may occur. With the presence of envy, the efficiency of the contract is always decreased regardless of the burden of limited liability. Therefore, Corollary 1 compares output levels with the first-best level of a model without envy.

V. Concluding Remarks

The present study analyzed ex-ante contracting that introduces risk-neutral agents while applying envy and limited liability constraints to the problem of asymmetric information. This study differs from other models in that the contract is offered before the agent recognizes his type. The findings demonstrate that the existence of envy makes it impossible to achieve the first-best output regardless of the burden of limited liability: taking into account the existence of envy, the principal requires different output levels at both the top and bottom for each agent's type compared to the case with no distortion at the top under standard ex-ante contracting without envy. Furthermore, when limited liability constraints are added, bunching can occur when the degree of envy is sufficiently large.

Although this study extensively analyzed behavioral contract theory, it focused more on the conditional self-interested utility function (i.e., competitive or status-seeking individuals) than on the conditional altruistic utility function. This regards agents who participate in contracts as responding to material payoffs. In general, the existence of altruism and envy affects the problems of optimal contracts in complicated payments. It is left to future research to develop this analysis more specifically. As a starting point, this study analyzed how incentives are affected by envy. A limitation of this paper is that the proposed model addressed only two types of risk-neutral agents. Therefore, future studies should carry out a comprehensive analysis of continuous types with risk-averse agents when agents feel envious of others' material payoffs (e.g., different sources of envy: wages, types, and rents).

APPENDIX A

Proof of Proposition 1

(12) is binding when agents compare their wages. Otherwise, the principal can increase x_0 and its payoff. This equalizes (12) as $w_0 - t_0 x_0$ decreases, $\alpha \sum_{j=0,1} p_j \max[w_j - w_0, 0]$ is left unchanged (reporting efficient agent truthfully), $w_0 - t_1 x_0$ is left unchanged, and $\alpha \sum_{j=0,1} p_j \max[w_j - w_1, 0]$ is left unchanged (pretending to be an inefficient agent). Equally, increasing x_0 leaves $w_1 - t_1 x_1$ unchanged as well as $\alpha \sum_{j=0,1} p_j \max[w_j - w_1, 0]$. As $w_0 - t_1 x_0$ decreases and $\alpha \sum_{j=0,1} p_j \max[w_j - w_0, 0]$ is left unchanged when the inefficient agent pretends to be an efficient one, (12) is soft. Using the binding (12) and inserting it

into (11) yields $0 > -\Delta t(x_0^e - x_1^e)$.

[Case (i)]: Suppose that $l < p\Delta t x_1^e$. We conjecture that the relevant constraints are (10) and (12). Those constraints are obviously binding to minimize the expected rent $pU_0 + (1-p)U_1 - \alpha p(1-p)(w_0 - w_1)$ left to the agent, which implies that $U_1^e = -l + \alpha p(w_0^e - w_1^e)$ from (10) and $U_0^e = U_1^e + \Delta t x_1^e - \alpha p(w_0^e - w_1^e) = -l + \Delta t x_1^e$ from binding (12). Thus, using (12) with binding and simplifying those yield

$$w_0^e - w_1^e = \frac{t_0(x_0^e - x_1^e)}{1 + \alpha p} > 0, \quad w_1^e = t_1 x_1^e - l + \frac{\alpha p t_0(x_0^e - x_1^e)}{1 + \alpha p}, \quad w_0^e = t_0 x_0^e + \Delta t x_1^e - l. \quad (\text{A-1})$$

Inserting these values into the principal's objective function and optimizing with respect to x_0 and x_1 yields²²

$$\begin{aligned} S'(x_0^e) &= t_0 + \frac{\alpha(1-p)t_0}{1 + \alpha p} > t_0 = S'(x_0^b) \Leftrightarrow x_0^e < x_0^b, \\ S'(x_1^e) &= t_1 + \frac{p\Delta t}{1-p} - \frac{\alpha p t_0}{1 + \alpha p} < S'(x_1^b) = t_1 + \frac{p\Delta t}{1-p} \Leftrightarrow x_1^e > x_1^b. \end{aligned}$$

An analysis of x_1^e supports that the agents' output is strictly positive for all α . $S'(x_1^e)$ is decreasing in α , whereas $S'(x_0^e)$ is increasing in α ; that is, the different wages converge as α increases. Increasing α lowers the output for the efficient agent and raises that for the inefficient agent; and hence, the cutoff level is $\alpha^\dagger \equiv \frac{\Delta t}{t_0 - p t_1}$ such that x_0^e is equal to x_1^e for all $\alpha \geq \alpha^\dagger$. This implies that $w_0^e = w_1^e$ if $\alpha \geq \alpha^\dagger$. As both types can be indifferent between two contracts that set the wage levels equal, envy does not occur when agents compare wages. Furthermore, if $\alpha < \alpha^\dagger$,

$$t_1 = S'(x_1^b) < S'(x_1^e) = t_1 + \frac{p\Delta t}{1-p} - \frac{\alpha p t_0}{1 + \alpha p} \Leftrightarrow x_1^b > x_1^e,$$

and we have the relation $x_0^b > x_0^e > x_1^b > x_1^e > x_1^b$. As with $x_0^e > x_1^e$, which satisfies $w_0^e > w_1^e$ from (A-1) if $\alpha < \alpha^\dagger$. These solutions are valid as long as (8) is strictly satisfied, that is, if $\alpha < \alpha^\dagger$,

$$\begin{aligned} & pU_0 + (1-p)U_1 - \alpha p(1-p)(w_0^e - w_1^e) \\ &= p(\Delta t x_1^e - l) + (1-p) \left[\frac{\alpha p t_0(x_0^e - x_1^e)}{1 + \alpha p} - l \right] - \frac{\alpha p(1-p)t_0(x_0^e - x_1^e)}{1 + \alpha p} \quad \text{from (A-1)} \\ &= p\Delta t x_1^e - l > 0. \end{aligned}$$

Using (12) with binding, (9) is obtained as follows: $U_0^e > -l \Leftrightarrow \Delta t x_1^e > 0$, which implies that (9) is strictly satisfied. Moreover, $U_1^e = -l + \alpha p(w_0^e - w_1^e) \leq 0$. This intuition is as follows. Suppose $U_0^e \leq 0$, then $U_1^e > 0$ can be established as

$$\begin{aligned} 0 &\geq w_0^e - t_0 x_0^e \geq w_1^e - t_0 x_1^e - \alpha p(w_0^e - w_1^e) > w_1^e - t_1 x_1^e - \alpha p(w_0^e - w_1^e) \\ &\Leftrightarrow 0 \geq w_0^e - t_0 x_0^e \geq w_1^e - t_0 x_1^e > w_1^e - t_1 x_1^e. \end{aligned}$$

The second inequality stems from (12), and the third inequality can be established in relation with (11)

²² Straightforward computation is as follows: $\max_{x_0, x_1} p[S(x_0) - \Delta t x_1 - t_0 x_0 + l] + (1-p) \left[S(x_1) - t_1 x_1 + l - \frac{\alpha p t_0(x_0 - x_1)}{1 + \alpha p} \right]$.

and (12), causing a contradiction. On the other hand, if $\alpha \geq \alpha^\dagger$, we obtain $U_0^e = U_1^e = 0$, which implies that (8) is binding, and (9) and (10) holds as long as $x_0^e = x_1^e$.

[Case (ii)]: Suppose that $p\Delta tx_1^e \leq l \leq p\Delta tx_1^b$. We then conjecture that (8) is also binding. In this case, we obtain that

$$pU_0 + (1-p)U_1 - \alpha p(1-p)(w_0^e - w_1^e) = l + p\Delta tx_1^e = 0.$$

We understand that (9) is nonbinding such as the proof of Case (i). Thus,

$$l = p\Delta tx_1^e = p\Delta tx_1^b$$

is held as long as x_1^b is obtained. We can understand that only (8), (10) and (12) are binding. Thus, w_1^e is satisfied as follows

$$w_0^e - w_1^e = \frac{t_0(x_0^e - x_1^e)}{1 + \alpha p} > 0, \quad w_0^e = t_0 x_0^e + (1-p)\Delta tx_1^e, \quad w_1^e = t_1 x_1^e - p\Delta tx_1^e + \frac{\alpha p t_0 (x_0^e - x_1^e)}{1 + \alpha p}. \quad (\text{A-2})$$

Inserting w_i into the principal's objective function and optimizing with respect to x_0 and x_1 yield

$$S'(x_0^e) = \frac{t_0(1+\alpha)}{1+\alpha p} > t_0 = S'(x_0^b) \Leftrightarrow x_0^e < x_0^b,$$

$$S'(x_1^e) = \frac{t_1}{1+\alpha p} < t_1 = S'(x_1^b) < S'(x_1^{sb}) = t_1 + \frac{p\Delta t}{1-p} \Leftrightarrow x_1^e > x_1^b > x_1^{sb}.$$

As stated in proof of the Case (i), an analysis of x_i^e also supports that the agents' output is strictly positive for all α . The different wages converge as α increases ($S'(x_1^e)$ is decreasing, whereas $S'(x_0^e)$ is increasing in α). Increasing α lowers the output for the efficient agent and raises that for the inefficient agent; hence the cutoff level $\hat{\alpha} \equiv \frac{\Delta t}{t_0}$ such that x_0^e is equal to x_1^e for all $\alpha \geq \hat{\alpha}$. This implies that $w_0^e = w_1^e$ if $\alpha \geq \hat{\alpha}$. As both types can be indifferent between two contracts that make the wage levels equal, envy does not occur when agents compare wages. Furthermore, if $\alpha < \hat{\alpha}$, then we have the relation $x_0^b > x_0^e > x_1^e > x_1^b > x_1^{sb}$. As with $x_0^e > x_1^e$, it satisfies $w_0^e > w_1^e$ from (A-2) if $\alpha < \hat{\alpha}$. Therefore, if $\alpha < \hat{\alpha}$, then $U_1^e = -l + \alpha p(w_0^e - w_1^e) = -p\Delta tx_1^e + \alpha p(w_0^e - w_1^e) = -p[\Delta tx_1^e - \alpha(w_0^e - w_1^e)] < 0$ because of $\Delta tx_1^e > \alpha(w_0^e - w_1^e)$ as in assumption of the main text, (13).

Moreover, $U_0^e = -l + \Delta tx_1^e = (1-p)\Delta tx_1^e > 0$ from (12). These rents satisfy that (9) is nonbinding and (10) is binding, respectively. On the other hand, if $\alpha \geq \hat{\alpha}$, then we obtain $U_0^e = U_1^e = 0$, which implies that (8) is binding, and (9) and (10) holds as long as $x_0^e = x_1^e$.

[Case (iii)]: Suppose that $l > p\Delta tx_1^e$. We conjecture that the relevant constraints are (8), (12). Those constraints are obviously binding to minimize the expected rent $pU_0 + (1-p)U_1 - \alpha p(1-p)(w_0^e - w_1^e)$ left to the agent. As stated in proof Case (i), $U_0^e > 0$ can be established on a valid (8) and this can be summarized as follows. Suppose $U_0^e \leq 0$, then $U_1^e > 0$ can be established as

$$0 \geq w_0^e - t_0 x_0^e \geq w_1^e - t_0 x_1^e - \alpha p(w_0^e - w_1^e) > w_1^e - t_1 x_1^e - \alpha p(w_0^e - w_1^e)$$

$$\Leftrightarrow 0 \geq w_0^e - t_0 x_0^e \geq w_1^e - t_0 x_1^e > w_1^e - t_1 x_1^e.$$

The second inequality stems from (12), the third inequality can be established in relation with (11) and

(12), causing a contradiction. Thus, the principal implements as follows:

$$U_0^e = (1-p)\Delta t x_1^e > 0, \quad U_1^e = -p[\Delta t x_1^e - \alpha(w_0^e - w_1^e)] < 0. \quad (\text{A-3})$$

These rents satisfy (8) and (12) with equality. Moreover, (9) and (10) also hold. Furthermore, using (8) and (12) with binding and simplifying those yield $w_0^e - w_1^e = \frac{t_0(x_0^e - x_1^e)}{1 + \alpha p} > 0$. As $x_0^e > x_1^e$, it satisfies $w_0^e > w_1^e$. Inserting w_i into the principal's objective function and optimizing with respect to x_0 and x_1 yield

$$S'(x_0^e) = \frac{t_0(1 + \alpha)}{1 + \alpha p} > t_0 = S'(x_0^b) \Leftrightarrow x_0^e < x_0^b,$$

$$S'(x_1^e) = \frac{t_1 + \alpha p \Delta t}{1 + \alpha p} < t_1 = S'(x_1^b) < S'(x_1^{sb}) = t_1 + \frac{p \Delta t}{1 - p} \Leftrightarrow x_1^e > x_1^b > x_1^{sb}.$$

An analysis of x_i^e supports that the agents' output is strictly positive for all α . As the proof of Case (ii), the cutoff level $\alpha^* \equiv \frac{\Delta t}{t_0 - p \Delta t}$ such that x_0^e is equal to x_1^e for all $\alpha \geq \alpha^*$. This implies that $w_0^e = w_1^e$ if $\alpha \geq \alpha^*$. As both types can be indifferent between two contracts that make the wage levels equal, envy does not occur when agents compare wages. Furthermore, if $\alpha < \alpha^*$, then we have the relation $x_0^b > x_0^e > x_1^e > x_1^b > x_1^{sb}$. On the other hand, if $\alpha \geq \alpha^*$, then we obtain $U_0^e = U_1^e$, which implies that (8) is binding, and (9) and (10) holds as long as $x_0^e = x_1^e$. Q.E.D.

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