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Treading a Fine Line: (Im)possibilities for Nash Implementation with Partially-honest Individuals

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Treading a fine line: (Im)possibilities for Nash implementation with partially-honest individuals*

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Abstract

This paper investigates the robustness of Dutta and Sen's (2012) Theorem 1 to reductions in the strategy space of individuals in relation to preference announcements. Specifically, it considers the Saijo-type's (1988) simplification of Maskin's canonical mechanism, according to which each individual's strategy choice includes her own preference and those of her k 'neighbor' individuals. This paper refers to this type of mechanisms as q-mechanisms where $q = k + 1$. A partially-honest individual is an individual who strictly prefers to tell the truth whenever lying has no effect on her material well-being. When there is at least one partially-honest participant, it offers a necessary condition for Nash implementation by q-mechanisms, called partial-honesty monotonicity, and shows that in an independent domain of preferences that condition is equivalent to Maskin monotonicity. It also shows that the limitations imposed by Maskin monotonicity can be circumvented by a q-mechanism provided that there are at least $n - q + 1$ partially-honest participants.

JEL classification: C72; D71; D82.
Keywords: Nash implementation; partial-honesty; non-connected honesty standards, independent domain, q-mechanisms.

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Introduction

The implementation problem is the problem of designing a mechanism or game form with the property that for each profile of participants' preferences, the equilibrium outcomes of the mechanism played with those preferences coincide with the recommendations that a given social choice rule (SCR) would prescribe for that profile. If that mechanism design exercise can be accomplished, the SCR is said to be implementable. The fundamental paper on implementation in Nash equilibrium is thanks to Maskin (1999; circulated since 1977), who proves that any SCR that can be Nash implemented satisfies a remarkably strong invariance condition, now widely referred to as Maskin monotonicity. Moreover, he shows that when the mechanism designer faces at least three individuals, a SCR is Nash implementable if it is Maskin monotonic and satisfies the condition of no veto-power, subsequently, Maskin's theorem.

Maskin (1999) obtains his original result by means of a mechanism that requires each individual to report, besides two auxiliary data, the whole description of the state. In a preference model, this means that each participant is asked to report preferences that members of the society have (preference profile). It is natural to explore the possibility of simplifying the strategy space of the individuals. Saijo (1988) addressed this question. While still retaining the generality of the implementation model, Saijo (1988) proves that when monotonicity and no veto-power are satisfied, it is enough to arrange agents in a directed circle and ask each of them to report, besides two auxiliary data, her own preference and that of her successor in the circle. Nash implementation is actually equivalent to implementation with these restricted mechanisms (Lombardi and Yoshihara, 2013).

Since Maskin's theorem, economists have also been interested in understanding how to circumvent the limitations imposed by Maskin monotonicity by exploring the possibilities offered by approximate (as opposed to exact) implementation (Matsushima, 1988; Abreu and Sen, 1991), as well as by implementation in refinements of Nash equilibrium (Moore and Repullo, 1988; Abreu and Sen, 1990; Palfrey and Srivastava, 1991; Jackson, 1992) and by repeated implementation (Kalai and Ledyard, 1998; Lee and Sabourian, 2011; Mezzetti and Renou, 2012). One additional way around those limitations is offered by implementation with partially-honest individuals.

A partially-honest individual is an individual who deceives the mechanism designer when the truth poses some obstacle to her material well-being. Thus, she does not deceive when the truth is equally efficacious. Simply put, a partially-honest individual follows the maxim, "Do not lie if you do not have to" to serve your material interest.

In a general environment, a seminal paper on Nash implementation problems involving partially-honest individuals is Dutta and Sen (2012), whose Theorem 1 (p. 157) shows that for implementation problems involving at least three individuals and in which there is at least one partially-honest individual, the Nash implementability is assured by no veto-power. Similar positive results are uncovered in other environments by Matsushima (2008a,b), Kartik and Tercieux (2012), Kartik et al. (2014), Lombardi and Yoshihara (2014), Saporiti (2014) and Ortner (2015). Thus, there are far fewer limitations for Nash implementation when there are partially-honest individuals.[1]

As in Maskin's (1999) original result, Dutta and Sen's (2012) Theorem 1 uses a mechanism that asks participants to report, among two auxiliary data, the whole preference profile. Moreover, according to Dutta and Sen's (2012) definition of honesty, a participant's play is honest if she plays a strategy choice which is veracious in its preference profile announcement component. The current paper bridges the gap between Theorem 1 of Dutta and Sen and the literature on strategy space reduction by asking the following question: If we use a mechanism that asks participants to report, among other auxiliary data, only part of the whole preference profile, can this 'simpler' communication scheme have a significant impact on Nash implementation with partially-honest individuals?

To answer this question, the paper presents an implementation model which reduces straightforwardly to a model with strategy space reduction. Specifically, the paper presents a model in which each partially-honest individual cares about telling the truth about the preferences of an idiosyncratic subset of individuals, which includes herself. We call this set as *individual honesty standard*. One interpretation is that participant i concerns herself with the truth-telling of preferences of individuals who are in her honesty standard when she plays a strategy choice, and such a subset represents the individuals whose truthful information is relevant to retain her self-image as an honest individual. Then, on the basis of this definition, the paper looks at what SCR can be Nash implemented in a society involving partially-honest individuals.

First, without any restriction on the available class of mechanisms, we show that any SCR that can be Nash implemented with partially-honest individuals satisfies a variant of Maskin monotonicity, called *partial-honesty monotonicity*. The idea of this axiom is quite intuitive. If x is one of the outcomes selected by a given SCR at one preference profile but is not selected when there is a monotonic change of preferences around x, then that monotonic change has altered preferences of individuals in the honesty standard of a partially-honest individual.

Second, we consider what we call *non-connected honesty standards*. Simply put, individual honesty standards are connected if some participant is in the honest standard of

[1] A pioneering work on the impact of decency constraints on Nash implementation problems is Corchón and Herrero (2004). These authors propose restrictions on sets of strategies available to agents that depend on the state of the world. They refer to these strategies as decent strategies and study Nash implementation problems in decent strategies. For a particular formulation of decent strategies, they are also able to circumvent the limitations imposed by Maskin monotonicity.

every other participant. When that is not the case, we call them non-connected honesty standards. In other words, they are non-connected if every participant is excluded from the honesty standard of another participant.

In an independent domain of preferences, where the set of the profiles of participants' preferences takes the structure of the Cartesian product of individual preferences, we show that partial-honesty monotonicity is equivalent to Maskin monotonicity whenever there exists at least one partially honest individual and all of such individuals share non-connected honesty standards in the society. Thus, under those hypotheses, Maskin's theorem provides an almost complete characterization of SCRs that are Nash implementable in the society with partially-honest individuals.

The above results is derived without imposing any restriction on the implementing mechanism as well as on the basis that in every state a strategy choice of an individual is truthful if it encodes information of individuals' preferences *consistent* with that state for members of society in her honesty standard (Definition 1 below). This implies that if we arrange agents in a directed circle and ask them to report their own preferences and those of their successor(s) in the circle, and the honesty standard of every individual includes herself and her successor(s), then this 'simpler' mechanism would impair the ability of the mechanism designer to escape the limitations imposed by Maskin monotonicity. Then, a natural question that arises immediately is: Under what conditions would the positive sufficiency result of Dutta and Sen (2012) be restored? Our answer is that the mechanism designer who knows that $\alpha(\geq 1)$ members of society have a taste for honesty can expect to do well if no participant has a veto-power by structuring communication with participants in a way that each of them reports her own preference and those of other $n - \alpha$ successors who are in the honesty standard of her.

The remainder of the paper is divided into five sections. Section 2 presents the theoretical framework and outlines the implementation model, with the necessary condition presented in section 3. Section 4 presents the equivalence result. Section 5 presents sufficient conditions for the restoration of Dutta-Sen's positive result. Section 6 concludes.

2. Preliminaries

2.1 Basic framework

We consider a finite set of individuals indexed by $i \in N = \{1, \cdots, n\}$, which we will refer to as a society. The set of outcomes available to individuals is X. The information held by the individuals is summarized in the concept of a state. Write Θ for the domain of possible states, with θ as a typical state. In the usual fashion, individual i's preferences in state θ are given by a complete and transitive binary relation, subsequently an ordering, $R_i(\theta)$ over the set X. The corresponding strict and indifference relations are denoted by $P_i(\theta)$ and $I_i(\theta)$, respectively. The preference profile in state θ is a list of orderings for individuals in N that are consistent with that state and is denoted by $R_N(\theta)$.

We assume that the mechanism designer does not know the true state. We assume, however, that there is complete information among the individuals in N. This implies that the mechanism designer knows the preference domain consistent with the domain Θ. In this paper, we identify states with preference profiles.

The goal of the mechanism designer is to implement a SCR $F : \Theta \twoheadrightarrow X$ where $F(\theta)$ is non-empty for any $\theta \in \Theta$. We shall refer to $x \in F(\theta)$ as an F-optimal outcome at θ. Given that individuals will have to be given the necessary incentives to reveal the state truthfully, the mechanism designer delegates the choice to individuals according to a mechanism $\Gamma \equiv \left(\prod_{i \in N} M_i, g\right)$, where M_i is the strategy space of individual i and $g : M \to X$, the outcome function, assigns to every strategy profile $m \in M \equiv \prod_{i \in N} M_i$ a unique outcome in X. We shall sometimes write (m_i, m_{-i}) for the strategy profile m, where $m_{-i} = (m_1, \cdots, m_{i-1}, m_{i+1}, \cdots, m_n)$.

An *honesty standard of individual* i, denoted by $S(i)$, is a subgroup of society with the property that $i \in S(i)$. Thus, given a state θ, $R_{S(i)}(\theta)$ is a list of orderings consistent with θ for individuals in the honesty standard $S(i)$ of individual i. An *honesty standard of society* is a list of honesty standards for all members of society. Write $S(N)$ for a typical honesty standard of society.

2.2 *Intrinsic preferences for honesty*

We assume that in a state θ, every truthful strategy choice of individual i is to encode information of individuals' orderings consistent with that state for members of society in her honesty standard $S(i)$. Moreover, if in two different states, say θ and θ', the orderings consistent with those two states for individuals in $S(i)$ are the same, then the sets of individual i's truthful strategy choices for those two states need to be identical according to her honesty standard $S(i)$. This is captured by the following notion of truth-telling correspodence:

Definition 1 For each Γ and each individual $i \in N$ with an honesty standard $S(i)$, individual i's *truth-telling correspondence* is a (non-empty) correspondence $T_i^{\Gamma}(\cdot; S(i)) : \Theta \twoheadrightarrow M_i$ with the property that for any two states θ and θ', it holds that

$$T_i^{\Gamma}(\theta; S(i)) = T_i^{\Gamma}(\theta'; S(i)) \iff R_{S(i)}(\theta) = R_{S(i)}(\theta').$$

Strategy choices in $T_i^{\Gamma}(\theta; S(i))$ will be referred to as truthful strategy choices for θ according to $S(i)$.

In modeling intrinsic preferences for honesty, we adapt the notion of partially-honest individuals of Dutta and Sen (2012) to our research question. First, a partially-honest individual is an individual who responds primarily to material incentives. Second, she strictly prefers to tell the truth whenever lying has no effect on her material well-being. That behavioral choice of a partially-honest individual can be modeled by extending an individual's ordering over X to an ordering over the strategy space M, because that individual's preference between being truthful and being untruthful is contingent upon announcements made by other individuals as well as the outcome(s) obtained from them. By following standard conventions of orderings, write $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ for individual i's ordering over M in state θ whenever she is confronted with the mechanism Γ and has set her honesty standard at $S(i)$. Formally, our notion of a partially-honest individual is as follows:

Definition 2 For each Γ, individual $i \in N$ with an honesty standard $S(i)$ is *partially-honest* if, for all $\theta \in \Theta$, her intrinsic preference for honesty $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ on M satisfies the following properties: for all m_{-i} and all $m_i, m_i' \in M_i$, it holds that

(i) If $m_i \in T_i^{\Gamma}(\theta; S(i))$, $m_i' \notin T_i^{\Gamma}(\theta; S(i))$ and $g(m) R_i(\theta) g(m_i', m_{-i})$, then $m \succ_i^{\Gamma,\theta,S(i)} (m_i', m_{-i})$.

(ii) In all other cases, $m \succcurlyeq_i^{\Gamma,\theta,S(i)} (m_i', m_{-i})$ if and only if $g(m) R_i(\theta) g(m_i', m_{-i})$.

Intrinsic preference for honesty of individual i is captured by the first part of the above definition, in that, for a given mechanism Γ, honesty standard $S(i)$ and state θ, individual i strictly prefers the message profile (m_i, m_{-i}) to (m_i', m_{-i}) provided that the outcome $g(m_i, m_{-i})$ is at least as good as $g(m_i, m_{-i})$ according to her ordering $R_i(\theta)$ and that m_i is truthful for θ and m_i' is not truthful for θ, according to $S(i)$.

If individual i is *not* partially-honest, this individual cares for her material well-being associated with outcomes of the mechanism and nothing else. Then, individual i's ordering over M is just the transposition into space M of individual i's relative ranking of outcomes. More formally:

Definition 3 For each Γ, individual $i \in N$ with an honesty standard $S(i)$ is *not partially-honest* if, for all $\theta \in \Theta$, her intrinsic preference for honesty $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ on M satisfies the following property: for all $m, m' \in M$, it holds that

$$m \succcurlyeq_i^{\Gamma,\theta,S(i)} m' \iff g(m) R_i(\theta) g(m').$$

2.3 Implementation problems

In formalizing the mechanism designer's problems, we first introduce our informational assumptions and discuss their implications for our analysis. They are:

Assumption 1 There exists at least one partially-honest individual in the society N.

Assumption 2 The mechanism designer knows the honesty standard of the society N.

The above two assumptions combined with the assumption that there is complete information among the individuals imply that the mechanism designer only knows the set Θ, the fact that there is at least one partially-honest individual among the individuals and the honesty standard of society, but he does not know either the true state or the identity of the partially-honest individual(s). Indeed, the mechanism designer cannot exclude any member(s) of society from being partially-honest purely on the basis of Assumption 1. Therefore, the following considerations are in order from the viewpoint of the mechanism designer.

An environment is described by three parameters, $(\theta, S(N), H)$: a state θ, an honesty standard of society $S(N)$ and a conceivable set of partially-honest individuals H. We denote by H a typical conceivable set of partially-honest individuals in N, with h as a typical element, and by $\mathcal{H}$ the class of conceivable sets of partially-honest individuals.

A mechanism Γ and an environment $(\theta, S(N), H)$ induce a strategic game $\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$, where

$$\succcurlyeq^{\Gamma,\theta,S(N),H} \equiv \left(\succcurlyeq_i^{\Gamma,\theta,S(i)}\right)_{i\in N}$$

is a profile of orderings over the strategy space M as formulated in Definition 2 and in Definition 3. Specifically, $\succcurlyeq_i^{\Gamma,\theta,S(i)}$ is individual i's ordering over M as formulated in Definition 2 if individual i is in H, whereas it is the individual i's ordering over M as formulated in Definition 3 if individual i is not in H.

A (pure strategy) Nash equilibrium of the strategic game $\left(\Gamma, \succcurlyeq^{\Gamma,\theta,H,S(N)}\right)$ is a strategy profile m such that for all $i \in N$, it holds that

$$\text{for all } m_i' \in M_i : m \succcurlyeq_i^{\Gamma,\theta,S(i)} (m_i', m_{-i}).$$

Write $NE\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ for the set of Nash equilibrium strategies of the strategic game $\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ and $NA\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$ for its corresponding set of Nash equilibrium outcomes.

The following definition is to formulate the mechanism designer's Nash implementation problem involving partially-honest individuals in which the society maintains the standard of honesty summarized in $S(N)$.

Definition 4 Let Assumption 1 be given. Let the honesty standard of society be summarized in $S(N)$. A mechanism Γ *partially-honestly Nash implements* the SCR $F : \Theta \twoheadrightarrow X$ provided that for all $\theta \in \Theta$ and $H \in \mathcal{H}$ there exists for any $h \in H$ a truth-telling correspondence $T_h^{\Gamma}(\theta; S(h))$ as formulated in Definition 1 and, moreover, it holds that $F(\theta) = NA\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$. If such a mechanism exists, F is said to be *partially-honestly Nash implementable.*

The objective of the mechanism designer is thus to design a mechanism whose Nash equilibrium outcomes, for each state θ as well as for each conceivable set of partially-honest individuals H, coincide with $F(\theta)$. Note that there is no distinction between the above formulation and the standard Nash implementation problem as long as Assumption 1 is discarded.

3. A necessary condition

In this section, we discuss a condition that is necessary for the partially-honest Nash implementation where the honesty standard of society is prescribed by $S(N)$.

A condition that is central to the implementation of SCRs in Nash equilibrium is Maskin monotonicity. This condition says that if an outcome x is F-optimal at the state θ, and this x does not strictly fall in preference for anyone when the state is changed to θ', then x must remain an F-optimal outcome at θ'. Let us formalize that condition as follows. For any state θ, individual i and outcome x, the weak lower contour set of $R_i(\theta)$ at x is defined by $L_i(\theta, x) = \{x' \in X | x R_i(\theta) x'\}$. Therefore:

Definition 5 The SCR $F : \Theta \twoheadrightarrow X$ is *Maskin monotonic* provided that for all $x \in X$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta)$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for all $i \in N$, then $x \in F(\theta')$.

An equivalent statement of Maskin monotonicity stated above follows the reasoning that if x is F-optimal at θ but not F-optimal at θ', then the outcome x must have fallen strictly in someone's ordering at the state θ' in order to break the Nash equilibrium via some deviation. Therefore, there must exist some (outcome-)preference reversal if an equilibrium strategy profile at θ is to be broken at θ'.

Our variant of Maskin monotonicity for Nash implementation problems involving partially-honest individuals where the standard of honesty in a society is represented by $S(N)$ can be formulated as follows:

Definition 6 The SCR $F : \Theta \twoheadrightarrow X$ is *partial-honesty monotonic* given the standard $S(N)$ provided that for all $x \in X$, all $H \in \mathcal{H}$ and all $\theta, \theta' \in \Theta$, if $x \in F(\theta) \backslash F(\theta')$ and $L_i(\theta, x) \subseteq L_i(\theta', x)$ for all $i \in N$, then for one $h \in H : R_{S(h)}(\theta) \neq R_{S(h)}(\theta')$.

This says that if x is F-optimal at θ but not F-optimal at θ' and, moreover, there is a monotonic change of preferences around x from θ to θ' (that is, whenever $xR_i(\theta)x'$, one has that $xR_i(\theta')x'$), then that monotonic change has altered preferences of individuals in the honesty standard of a partially-honest individual $h \in H$ (that is, $R_{S(h)}(\theta) \neq R_{S(h)}(\theta')$). Stated in the contrapositive, this says that if x is F-optimal at θ and there is a monotonic change of preferences around x from θ to θ' and, moreover, for any conceivable partially-honest individual h in H that change has not altered preferences of individuals in her honesty standard $S(h)$, then x must continue to be one of the outcomes selected by F at the state θ'. Note that if x is F-optimal at θ but not F-optimal at θ', one has that $R_N(\theta) \neq R_N(\theta')$, and thus any SCR is partial-honesty monotonic whenever the honesty standard of society is such that $S(i) = N$ for all $i \in N$.

The above condition is necessary for partially-honest Nash implementation. This is because if x is F-optimal at θ but not F-optimal at θ' and, moreover, the outcome x has not fallen strictly in any individual's ordering at the state θ', then only a partially-honest individual in the given conceivable set H can break the Nash equilibrium via a unilateral deviation. Therefore, there must exist a partially-honest individual $h \in H$ whose equilibrium strategy to attain x at $(\theta, S(N), H)$ is not a truthful strategy choice at $(\theta', S(N), H)$. It implies $R_{S(h)}(\theta) \neq R_{S(h)}(\theta')$ according to Definition 1. Formally:

Theorem 1 Let Assumption 1 be given. Let the honesty standard of society be summarized in $S(N)$. The SCR $F : \Theta \twoheadrightarrow X$ is partial-honesty monotonic given the standard $S(N)$ if it is partially-honestly Nash implementable.

Proof. Let Assumption 1 be given. Let the honesty standard of society be summarized in $S(N)$. Suppose that $\Gamma \equiv (M, g)$ partially-honest Nash implements the SCR $F : \Theta \twoheadrightarrow X$. For any $x \in X$, consider any environment $(\theta, S(N), H)$ such that $x \in F(\theta)$. Then, there is $m \in NE\left(\Gamma, \succsim^{\Gamma,\theta,S(N),H}\right)$ such that $g(m) = x$.

Consider any state $\theta' \in \Theta$ such that

$$\text{for all } i \in N \text{ and all } x' \in X : xR_i(\theta)x' \implies xR_i(\theta')x'. \tag{1}$$

If there exists an individual $i \in N$ such that $g(m_i', m_{-i}) P_i(\theta') g(m)$, then, from (1), $g(m_i', m_{-i}) P_i(\theta) g(m)$, a contradiction of the fact that $m \in NE\left(\Gamma, \succsim^{\Gamma,\theta,S(N),H}\right)$. Therefore, we conclude that

$$\text{for all } i \in N \text{ and all } m_i' \in M_i : g(m) R_i(\theta') g(m_i', m_{-i}). \tag{2}$$

Suppose that $x \notin F(\theta')$. Then, the strategy profile m is not a Nash equilibrium of $\left(\Gamma, \succsim^{\Gamma,\theta',S(N),H}\right)$; that is, there exists an individual $i \in N$ who can find a strategy choice

$m_i' \in M_i$ such that $(m_i', m_{-i}) \succ_i^{\Gamma,\theta',S(i)} m$. Given that (2) holds, it must be the case that $i \in H$. From part (i) of Definition 2 we conclude, therefore, that

$$m_i \notin T_i^{\Gamma}(\theta'; S(i)) \text{ and } m_i' \in T_i^{\Gamma}(\theta'; S(i)) \tag{3}$$

and that

$$g(m_i', m_{-i}) R_i(\theta') g(m). \tag{4}$$

Note that (2) and (4) jointly imply that

$$g(m_i', m_{-i}) I_i(\theta') g(m). \tag{5}$$

We show that $R_{S(i)}(\theta) \neq R_{S(i)}(\theta')$. Assume, to the contrary, that

$$\text{for all } h \in H : R_{S(h)}(\theta) = R_{S(h)}(\theta'). \tag{6}$$

Definition 1 implies that

$$\text{for all } h \in H : T_h^{\Gamma}(\theta; S(h)) = T_h^{\Gamma}(\theta'; S(h)). \tag{7}$$

From (3) and (7), it follows that

$$m_i \notin T_i^{\Gamma}(\theta; S(i)) \text{ and } m_i' \in T_i^{\Gamma}(\theta; S(i)). \tag{8}$$

Furthermore, given that $i \in S(i)$, by definition of an individual honesty standard, (5) and (6) jointly imply that

$$g(m_i', m_{-i}) I_i(\theta) g(m). \tag{9}$$

Given (8) and (9) and the fact that $i \in H$, Definition 2 implies that $(m_i', m_{-i}) \succ_i^{\Gamma,\theta,S(i)} m$, which is a contradiction of the fact that $m \in NE\left(\Gamma, \succcurlyeq^{\Gamma,\theta,S(N),H}\right)$. Thus, F is partial-honesty monotonic given the honesty standard $S(N)$. ■

4. Equivalence result

The classic paper on Nash implementation theory is Maskin (1999), which shows that where the mechanism designer faces a society involving at least three individuals, a SCR is Nash implementable if it is monotonic and satisfies the auxiliary condition of no veto-power.[2]

[2] Moore and Repullo (1990), Dutta and Sen (1991), Sjöström (1991) and Lombardi and Yoshihara (2013) refined Maskin's theorem by providing necessary and sufficient conditions for an SCR to be implementable in (pure strategies) Nash equilibrium. For an introduction to the theory of implementation see Jackson (2001),

The condition of no veto-power says that if an outcome is at the top of the preferences of all individuals but possibly one, then it should be chosen irrespective of the preferences of the remaining individual: that individual cannot veto it. Formally:

Definition 7 The SCR $F : \Theta \twoheadrightarrow X$ satisfies *no veto-power* provided that for all $\theta \in \Theta$ and all $x \in X$, if there exists $i \in N$ such that for all $j \in N \setminus \{i\}$ and all $x' \in X : xR_j(\theta)\, x'$, then $x \in F(\theta)$.

Proposition 1 (Maskin's Theorem, 1999) If $n \geq 3$ and $F : \Theta \twoheadrightarrow X$ is a SCR satisfying Maskin monotonicity and no veto-power, then it is Nash implementable.

In a general environment such as that considered here, a seminal paper on Nash implementation problems involving partially-honest individuals is Dutta and Sen (2012). It shows that for Nash implementation problems involving at least three individuals and in which there is at least one partially-honest individual, the Nash implementability is assured by no veto-power (Dutta and Sen, 2012; p. 157). From the perspective of this paper, Dutta-Sen's theorem can be formally restated as follows:

Proposition 2 (Dutta-Sen's Theorem, 2012) Let Assumption 1 and Assumption 2 be given. Let the honesty standard of society be summarized in $\overline{S}(N)$, where $\overline{S}(i) \equiv N$ for all $i \in N$. If $n \geq 3$ and $F : \Theta \twoheadrightarrow X$ is a SCR satisfying partial-honesty monotonicity for the standard $\overline{S}(N)$ and no veto-power, then it is partially-honestly Nash implementable.

As already noted in the previous section, any SCR is partial-honesty monotonic whenever the honesty standard of society is such that every individual considers truthful only messages that encode the whole truth about preferences of individuals in society; that is, $S(i) = N$ for all $i \in N$. Clearly, $S(i) = N$ for all $i \in N$ is a particular kind of honesty standard of individuals, and there is no reason to restrict attention to such standards.

In this section, we are interested in understanding the kind of honesty standards of individuals which would make it impossible for the mechanism designer to circumvent the limitations imposed by Maskin monotonicity. To this end, let us introduce the following notion of standards of honesty of a society.

Definition 8 Given a society N involving at least two individuals, an honesty standard of this society is said to be *non-connected* if and only if for all $i \in N$, $i \notin S(j)$ for some $j \in N$.

Given that the honesty standard of individual i includes the individual herself, by definition of $S(i)$, the honesty standard of society is non-connected whenever every one of its

Maskin and Sjöström (2002) and Serrano (2004).

members is excluded from the honesty standard of another member of the society. Simply put, members of a society do not concern themselves with the same individual.

It is self-evident that the kind of honesty standards in Dutta-Sen's theorem are not non-connected, because every individual of the society is interested in telling the truth about the whole society. As another example of honesty standards of a society that are not non-connected, consider a three-individual society where individual 1 concerns herself with herself and with individual 2 (that is, $S(1) = \{1, 2\}$), individual 2 concerns herself with everyone (that is, $S(2) = \{1, 2, 3\}$) and, finally, individual 3 concerns herself with herself and with individual 1 (that is, $S(3) = \{1, 3\}$). The honesty standard of this three-individual society is not non-connected because everyone concerns themselves with individual 1.

Moreover, it is not the case that every non-connected honesty standard of society implies that every individual honesty standard be of the form $S(i) \neq N$, as we demonstrate with the next example. Consider a three-individual society where individual 1 is concerned only with herself (that is, $S(1) = \{1\}$), individual 2 with everyone (that is, $S(2) = \{1, 2, 3\}$) and individual 3 with herself and with individual 2 (that is, $S(3) = \{2, 3\}$). The honesty standard of this society is non-connected given that individual 2 and individual 3 are both excluded from the honesty standard of individual 1 and individual 1 is excluded from the honesty standard of individual 3.

As is the case here, the above definition is a requirement for the honesty standard of a society that is sufficient for partial-honesty monotonicity to be equivalent to Maskin monotonicity when two further assumptions are satisfied. The first assumption requires that the family $\mathcal{H}$ includes singletons. This requirement is innocuous given that the mechanism designer cannot exclude any individual from being partially-honest purely on the basis of Assumption 1.

The second requirement is that the set of states Θ takes the structure of the Cartesian product of allowable independent characteristics for individuals. More formally, the domain Θ is said to be independent if it takes the form

$$\Theta = \prod_{i \in N} \Theta_i,$$

where Θ_i is the domain of allowable independent characteristics for individual i, with θ_i as a typical element. A typical example of independent domain is that each Θ_i simply represents the domain of the preference orderings over X of individual i and so the domain of the profiles of all individuals' preference orderings on X has the structure of the Cartesian product. In such a case, in a state $\theta = (\theta_i)_{i \in N}$, individual i's preference ordering over X depends solely on individual i's independent characteristic θ_i rather than on the profile θ. Given that a characteristic of individual i is independent from those of other individuals, the equivalence result does not hold for the correlated values case.

The latter two requirements and the requirement that the honesty standard of society needs to be non-connected are jointly sufficient for partial-honesty monotonicity to imply Maskin monotonicity. This can be seen as follows:

Consider a two-individual society where Θ is the set of states and X is the set of outcomes available to individuals. Let $S(i)$ be the honesty standard of individual $i = 1, 2$. Consider an outcome x and a state θ such that x is an F-optimal outcome at θ. Consider any other state θ' such that individuals' preferences change in a Maskin monotonic way around x from θ to θ'. Maskin monotonicity says that x must continue to be an F-optimal outcome at θ'. To avoid trivialities, let us focus on the case that $\theta \neq \theta'$, which means that $R_N(\theta) \neq R_N(\theta')$, given that we identify states with preference profiles.

If every individual were concerned with the whole society, we could never invoke (the contrapositive of) partial-honesty monotonicity to conclude that x should remain F-optimal at θ' because $R_N(\theta) \neq R_N(\theta')$. Furthermore, consider the case that individual 1 concerns herself with only herself, that is, $S(1) = \{1\}$, while individual 2 with the whole society, that is, $S(2) = \{1, 2\}$. Reasoning such as the one just used shows that partial-honesty monotonicity cannot be invoked if $R_1(\theta) \neq R_1(\theta')$. The argument for honesty standards of the form $S(1) = \{1, 2\}$ and $S(2) = \{2\}$ is symmetric. Thus, the only case left to be considered is the one in which everyone concerns themselves with only themselves, that is, $S(i) = \{i\}$ for $i = 1, 2$. In this situation, the honesty standard of society is reduced to the non-connected one. Note that standards considered earlier were not.

Suppose that preferences of individual 1 are identical in the two states, that is, $R_1(\theta) = R_1(\theta')$. To conclude that x should be F-optimal at θ' by invoking partial-honesty monotonicity we need to find individual 1 in the family $\mathcal{H}$. The argument for the case $R_2(\theta) = R_2(\theta')$ is symmetric. Thus, if $R_i(\theta) = R_i(\theta')$ for one of the individuals, the requirement that the singleton $\{i\}$ is an element of $\mathcal{H}$ is needed for the completion of the argument.

Suppose that preferences of individuals are not the same in the two states, that is, $R_i(\theta) \neq R_i(\theta')$ for every individual i, though they have changed in a Maskin monotonic way around x from the state θ to θ'. In this case, one cannot directly reach the conclusion of Maskin monotonicity by invoking partial-honesty monotonicity. One way to circumvent the problem is to be able to find a feasible state θ'' with the following properties: i) individuals' preferences change in a Maskin monotonic way around x from θ to θ'' and $R_i(\theta) = R_i(\theta'')$ for an individual i, and ii) individuals' preferences change in that way around x from θ'' to θ' and $R_j(\theta') = R_j(\theta'')$ for individual $j \neq i$. A domain Θ that assures the existence of such a state is the independent domain.

Even if one were able to find such a state θ'' by requiring an independent product structure of Θ, one could not invoke partial-honesty monotonicity and conclude that x must continue to be an F-optimal outcome at θ' whenever the family $\mathcal{H}$ did not have the appropriate structure. This can be seen as in the following argument.

Suppose that Θ is an independent domain. Then, states take the form of profiles of individuals' characteristics, that is, $\theta = (\theta_1, \theta_2)$ and $\theta' = (\theta'_1, \theta'_2)$. Moreover, the characteristic of individual i in one state is independent from the characteristic of the other individual. That is, $R_i(\theta) = R_i(\theta_i)$ and $R_i(\theta') = R_i(\theta'_i)$ for every individual i. The product structure of Θ assures that the states (θ_1, θ'_2) and (θ'_1, θ_2) are both available and each of them has the properties summarized above.

Next, suppose that the family $\mathcal{H}$ has a structure given by $\{\{1\}, \{1,2\}\}$. One can invoke partial-honesty monotonicity for $H = \{1\}$ to obtain that x is one of the outcomes chosen by the SCR F at (θ_1, θ'_2) when the state changes from θ to (θ_1, θ'_2), but he cannot conclude that x remains also F-optimal at θ' when it changes from (θ_1, θ'_2) to θ'. The reason is that partial-honesty monotonicity cannot be invoked again for the case $H = \{2\}$ because the structure of the family $\mathcal{H}$ does not contemplate such a case. The argument for the case that $\mathcal{H}$ takes the form $\{\{2\}, \{1,2\}\}$ is symmetric. Thus, each of our requirements is indispensable, and jointly they lead to the following conclusion:

Theorem 2 Let N be a society involving at least two individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society is non-connected. Partial-honesty monotonicity is equivalent to Maskin monotonicity.

Proof. Let $n \geq 2$, Θ be an independent domain and $\mathcal{H}$ include singletons. Let $S(N)$ be a non-connected honesty standard of N. One can see that Maskin monotonicity implies partial-honesty monotonicity.

For the converse, consider any SCR $F : \Theta \twoheadrightarrow X$ satisfying partial-honesty monotonicity. Consider any $x \in X$ and any state $\theta \in \Theta$ such that x is an F-optimal outcome at θ. Moreover, consider any state θ' such that individuals' preferences change in a Maskin monotonic way around x from θ to θ', that is,

$$\text{for all } i \in N \text{ and all } x' \in X : xR_i(\theta)\, x' \implies xR_i(\theta')\, x'.$$

We show that x remains F-optimal at θ'.

If characteristics of individuals in the honesty standard of individual $i \in N$ are identical in the two states, that is, $R_{S(i)}(\theta) = R_{S(i)}(\theta')$, partial-honesty monotonicity for the case $H = \{i\}$ assures that x is still F-optimal at θ'. Thus, let us consider the case $R_{S(i)}(\theta) \neq R_{S(i)}(\theta')$ for every individual $i \in N$.

To economize notation, for any subset K of N, write K_C for the complement of K in N. Therefore, for any non-empty subset K of N, we can write any non-trivial combination of the states θ and θ' as $\left(\theta_K, \theta'_{K_C}\right)$, where it is understood that θ_K is a list of characteristics of individuals in K at the state θ and θ'_{K_C} is a list of characteristics of individuals in K_C at θ'. Note that any state that results by that combination is available in Θ because of its product structure.

Given that the honesty standard of society is non-connected, there must be an individual $j(1) \in N$ who does not concern herself with the whole society, that is, $S(j(1)) \neq N$. Consider the state

$$\left(\theta_{K(1)}, \theta'_{K(1)_C}\right) \text{ where } K(1) \equiv S(j(1)),$$

and call it θ^1. By construction, individuals' preferences change in a Maskin monotonic way around x from θ to θ^1 and, moreover, $\theta_{K(1)} = \theta^1_{K(1)}$. Partial-honesty monotonicity for the case $H = \{j(1)\}$ assures that the x remains an F-optimal outcome at θ^1.

If there is an individual $i \in N \setminus \{j(1)\}$ who is not concerned with any of the individuals in the honesty standard of individual $j(1)$, that is, the intersection $S(i) \cap S(j(1))$ is empty, then partial-honesty monotonicity for the case $H = \{i\}$ assures that x is still F-optimal at θ'. This is because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^1 to θ' and $\theta^1_{S(i)} = \theta'_{S(i)}$.

Thus, consider any individual $j(2) \in N \setminus \{j(1)\}$, and denote by $K(2)$ the set of individuals who jointly concern individual $j(1)$ and individual $j(2)$ according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(2)}, \theta'_{K(2)_C}\right) \text{ where } K(2) \equiv K(1) \cap S(j(2)),$$

and call it θ^2. By construction, individuals' preferences change in a Maskin monotonic way around x from θ^1 to θ^2 and, moreover, $\theta^1_{S(j(2))} = \theta^2_{S(j(2))}$. Partial-honesty monotonicity for the case $H = \{j(2)\}$ assures that x remains an F-optimal outcome at θ^2.

If there is an individual $i \in N \setminus \{j(1), j(2)\}$ who is not concerned with any of the individuals with whom individuals $j(1)$ and $j(2)$ are jointly concerned, partial-honesty monotonicity for the case $H = \{i\}$ assures that x is also F-optimal at θ'. This is because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^2 to θ' and $\theta^2_{S(i)} = \theta'_{S(i)}$.

Thus, consider any individual $j(3) \in N \setminus \{j(1), j(2)\}$, and denote by $K(3)$ the set of individuals that jointly concern individuals $j(1)$, $j(2)$ and $j(3)$ according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(3)}, \theta'_{K(3)_C}\right) \text{ where } K(3) \equiv K(2) \cap S(j(3)),$$

and call it θ^3. By construction, individuals' preferences change in a Maskin monotonic way around x from θ^2 to θ^3 and, moreover, $\theta^2_{S(j(3))} = \theta^3_{S(j(3))}$. Partial-honesty monotonicity for the case $H = \{j(3)\}$ assures that x remains an F-optimal outcome at θ^3.

As above, if there is an individual $i \in N \backslash \{j(1), j(2), j(3)\}$ who is not concerned with any of the individuals with whom individuals $j(1)$, $j(2)$ and $j(3)$ are jointly concerned, partial-honesty monotonicity for the case $H = \{i\}$ assures that x remains also F-optimal at θ', because, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^3 to θ' and $\theta^3_{S(i)} = \theta'_{S(i)}$. And so on.

Since the society N is a finite set and the above iterative reasoning is based on its cardinality, we are left to show that it must stop at most after $n-1$ iterations.

To this end, suppose that we have reached the start of the $n-1$th iteration. Thus, consider any individual $j(n-1) \in N$, with $j(n-1) \neq j(r)$ for $r = 1, \cdots, n-2$, and denote by $K(n-1)$ the set of individuals that jointly concern individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ according to their individual honesty standards. Furthermore, consider the state

$$\left(\theta_{K(n-1)}, \theta'_{K(n-1)_C}\right) \text{ where } K(n-1) \equiv K(n-2) \cap S(j(n-1)),$$

and call it θ^{n-1}. As above, by construction, individuals' preferences change in a Maskin monotonic way around x from $\theta^{n-2} \equiv \left(\theta_{K(n-2)}, \theta'_{K(n-2)_C}\right)$ to θ^{n-1} and, moreover, $\theta^{n-2}_{S(j(n-1))} = \theta^{n-1}_{S(j(n-1))}$. Partial-honesty monotonicity for the case $H = \{j(n-1)\}$ assures that x is an F-optimal outcome at θ^{n-1}.

At this stage there is only one individual in N who is left to be considered. Call her $j(n)$. Suppose that this individual is concerned with one of the individuals for whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned. In other words, suppose that the intersection $K(n-1) \cap S(j(n))$ is non-empty. Then, the whole society concerns itself with one of its member, and this contradicts the fact that the honesty standard of society is non-connected. Therefore, it must be the case that individual $j(n)$ is not concerned with any of the individuals with whom individuals $j(1)$, $j(2), \cdots, j(n-2)$ and $j(n-1)$ are jointly concerned according to their individual honesty standards. Partial-honesty monotonicity for the case $H = \{j(n)\}$ assures that x remains also F-optimal at θ' given that, by construction, individuals' preferences change in a Maskin monotonic way around x from θ^{n-1} to θ' and $\theta^{n-1}_{S(j(n))} = \theta'_{S(j(n))}$.

The iterative reasoning would stop at the rth $(< n-1)$ iteration if there were an individual $i \in N \backslash \{j(1), \cdots, j(r)\}$ who did not concern itself with any of the individuals in $K(r)$, that is, if the intersection $S(i) \cap K(r)$ were empty. If that were the case, then the desired conclusion could be obtained by invoking partial-honesty monotonicity for $H = \{i\}$ because, by construction, it would hold that individuals' preferences change in a Maskin monotonic way around x from θ^r to θ' and that $\theta^r_{S(i)} = \theta'_{S(i)}$. ■

In light of Theorem 1 and Maskin's theorem, the main implications of the above conclusion can be formally stated as follows:

Corollary 1 Let N be a society involving at least two individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society is non-connected. Let Assumption 1 be given. The SCR $F : \Theta \twoheadrightarrow X$ is Maskin monotonic if it is partially-honestly Nash implementable.

Corollary 2 Let N be a society involving at least three individuals, Θ be an independent domain and $\mathcal{H}$ include singletons. Suppose that the honesty standard of the society is non-connected. Let Assumption 1 be given. Any SCR $F : \Theta \twoheadrightarrow X$ satisfying no veto-power is partially-honestly Nash implementable if and only if it is Maskin monotonic.

Remark 1 In a related but not identical setting, Kartik and Tercieux (2012) study Nash implementation problems where agents can choose to provide evidence as part of their strategies. In this setup, they show that any social choice function satisfying a weaker variant of Maskin monotonicity, called evidence-monotonicity, and no veto-power is Nash implementable. In an environment where there are partially-honest individuals, they show that even small intrinsic costs of lying create a substantial wedge between evidence-monotonicity and Maskin monotonicity, in the sense that every social choice function is evidence-monotonic. Under the assumptions of Theorem 2 and suitable specifications which resemble those of Example 2 in Kartik and Tercieux (2012; p. 333), one can show that this wedge disappears when participants are allowed/forced to produce partial evidence of the true state according to a non-connected (evidence) standard $S(N)$.[3]

5. Restoration of Dutta-Sen's theorem on Nash implementation with strategy space reduction

In an environment in which knowledge is dispersed, how individuals will interact with the mechanism designer is a natural starting point when it comes to Nash implementing a

[3] To see it, let us suppose that individuals have separable preferences in the sense of Kartik and Tercieux (2012; p. 238). That is, suppose that each agent's (extended) preference ordering $R_i(\theta)$ over the outcome-evidence space $X \times E_i$ is represented by a utility function of the form $U_i(x, e_i, \theta) = u_i(a, \theta) - c_i(e_i, \theta)$, where $c_i(e_i, \theta)$ represents agent i's cost of producing evidence e_i. Fix any $S(N)$ and let the domain Θ be independent. For each individual i, let the evidence space be $E_i = \prod_{j \in S(i)} \Theta_j$. Fix any set H. For each $h \in H$, let the cost function be $c_h(\theta, \theta') = 0$ if $R_{S(h)}(\theta) = R_{S(h)}(\theta')$, otherwise, $c_h(\theta, \theta') = \varepsilon > 0$, where ε can be arbitrarily small. For each $i \notin H$, let $c_i(\theta, \theta') = 0$ for every θ and θ'. This structure implies that the set of the least-evidence cost for $h \in H$ given the pair (x, θ) is $E_h^{\ell}(x, \theta) = \{\theta_{S(h)}\}$ while it is $E_i^{\ell}(x, \theta) = E_i$ for every $i \notin H$. Let the evidence function of invididual $h \in H$ be $e_h^*(\theta) = \{R_{S(h)}(\theta)\}$ for every $\theta \in \Theta$. Under these specifications, one can now see from the proof of Theorem 2 that evidence-monotonicity (stated for each $H \in \mathcal{H}$) is equivalent to Maskin monotonicity.

SCR. A particular kind of communication is, as we have done so far, to ask participants to report preferences of the entire society. However, there is no reason to restrict attention to such schemes.

Indeed, there may be sufficiently strong reasons that make it necessary for the mechanism designer to employ communication schemes that are simpler than the type of communication studied so far, and under which individuals behave *as if* their honesty standards were non-connected. In light of Theorem 2, in cases like this, the predicted result is that the mechanism designer may not be able to escape the limitations imposed by Maskin monotonicity and he is thus expected to do poorly. A natural question that arises immediately is: Under which conditions would the positive sufficiency result of Dutta and Sen (2012) be restored? Given our abstract framework, we answer this question by placing it within the literature on strategy space reduction in Nash implementation. A pioneering work in this respect is Saijo (1988).[4]

The basic idea behind the literature on strategy space reduction is to reduce the informational requirements in the preference announcement component of strategy choices. For example, individual i may be required to choose only her own characteristics as part of her strategy choice, or individual i can be required to choose her own characteristics and those of her neighbor individual $i+1$, and so on. A way to proceed is to arrange individuals in a circular fashion numerically clockwise - facing inward, and to require that each individual i announces her own characteristics together with the characteristics of $q-1$ individuals standing immediately to her left, where $1 \leq q \leq n-1$. Following this literature, a q-mechanism can be defined as follows:

Definition 9 For each $q \in N$, a mechanism $\Gamma_q = (M, g)$ is a q-mechanism if, for each $i \in N$, $M_i \equiv \prod_{k=i}^{q+i-1} \Theta_k \times X \times N$, with the convention that $n+p=p$ for $p \in N$.

In this section, we assume that the actual honesty standard of participant $i \in N$ is $S(i) = N$. It is simply motivated by convenience, and the following arguments still hold for some types of non-connected honesty standards of the society as noted in Remark 4 below.

Let us imagine that the mechanism designer knows that participant i feels honest when she is truthful about characteristics of the entire society and that she is forced to govern the communication with individuals by a 2-mechanism; that is, by a communication scheme that requires each individual i to choose her own characteristics and those of her adjacent individual $i+1$. Although the honesty standards of society are connected, this type of communication scheme makes individual i behave *as if* her honesty standard was of the form $S(i;2) \equiv \{i, i+1\}$, and, thereby, it makes the society behave *as if* its honesty standard was

[4] See also McKelvey, 1989; Tatamitani, 2001; Lombardi and Yoshihara, 2013.

non-connected: $S(N;2) \equiv (S(i;2))_{i\in N}$. The reason is that each individual is requested to communicate on a subset of her actually honesty standard. On this basis, let us formalize the mechanism designer's partially-honest Nash implementation problem by a q-mechanism. Write $S(i;q)$ for a *relevant* honesty standard of participant i when a q-mechanism is applied, and write $S(N;q)$ for a typical relevant honesty standard of society N when a q-mechanism is applied. Therefore:

Definition 10 Let Assumption 1 and Assumption 2 be given. Let $S(i) = N$ for each $i \in N$. A q-mechanism Γ_q *partially-honestly Nash implements* the SCR $F : \Theta \twoheadrightarrow X$ provided that for all $\theta \in \Theta$ and $H \in \mathcal{H}$ there exists for any $h \in H$ a truth-telling correspondence $T_h^{\Gamma_q}(\theta; S(h;q))$ as formulated in Definition 1 and, moreover, it holds that $F(\theta) = NA\left(\Gamma_q, \succsim^{\Gamma_q,\theta,S(N;q),H}\right)$, where $S(N;q) \equiv (S(i;q))_{i\in N}$ is the relevant honesty standard of N in the mechanism. If such a mechanism exists, F is said to be *partially-honestly Nash implementable by* a *q-mechanism*.

It can be verified by means of Theorem 1 that for any given (relevant) honesty standard $S(N;q)$, partial-honesty monotonicity with respect to $S(N;q)$ is a necessary condition for partially-honest Nash implementation by a q-mechanism. Furthermore, it can also be verified that the relevant honesty standard $S(N;q)$ of society N is non-connected as long as $q \neq n$. Thus, this reduction would impair the ability of the mechanism designer to escape the limitations imposed by Maskin monotonicity when some further assumptions of Theorem 2 are met.

Our next result states that the mechanism designer can circumvent the limitations imposed by Theorem 2 and successfully partially-honest Nash implements SCRs that are not Maskin monotonic by a q-mechanism provided that there are at least $n-q+1$ partially-honest individuals in a society and that no participant has a veto-power.[5] The reason is that $n-q+1$ is the minimal number of partially-honest individuals that assures that for every conceivable set H of partially-honest individuals, the relevant honesty standard $S(N;q)$ forms a covering of society N; that is, $N \subseteq \bigcup_{h\in H} S(h,q)$.

Put differently, it provides a reference for the actual mechanism design: If the mechanism designer knows that $\alpha(\geq 1)$ members of society have a taste for honesty, then he can expect to do well by asking each participant to report her own characteristics and those of $n-\alpha$ individuals and achieve, at most, an overall reduction in the size of the strategy space M equal to $n(\alpha-1)$. The following theorem substantiates our discussion:

[5] Recall that the importance of Dutta-Sen's theorem for Nash implementation is that SCRs that are not Maskin monotonic can be partially-honestly implemented if there is at least one individual who is partially-honest in a society.

Theorem 3 Let $n \geq 3$. Let Assumption 1 and Assumption 2 be given. Let Θ be an independent domain and let $S(i) = N$ for each $i \in N$. Suppose that the SCR $F : \Theta \twoheadrightarrow X$ satisfies no veto-power and that it is not Maskin monotonic. Then, for any $q \in N$ and any environment $(\theta, S(N; q), H)$:
(a) The SCR F is partially-honestly implementable by a q-mechanism if the number of partially-honest individuals in N is at least $n - q + 1$ with $q \neq 1$.
(b) Let the class $\mathcal{H}$ include singletons. The SCR F is not partially-honestly implementable by a q-mechanism if the number of partially-honest individuals in N is lower than $n - q + 1$.

Proof. Let the premises hold. Let us first show the part (a) of the statement. Suppose that the number of partially-honest individuals in society is at least $n - q + 1$. We show that the SCR F is partially-honestly implementable by a q-mechanism if it satisfies no veto-power. A typical strategy played by individual i is denoted by $m_i = \left(\boldsymbol{\theta}^i, x^i, z^i\right)$. For each $(m, \theta, x) \in M \times \Theta \times X$, we say that m is:

(i) *consistent* with (θ, x) if $x^j = x$ and $\boldsymbol{\theta}^j = (\theta_j, \theta_{j+1}, \cdots, \theta_{q+j-1})$ for each $j \in N$.

(ii) for all $i \in N$, m_{-i} *consistent* with (θ, x) if $x^j = x$ and $\boldsymbol{\theta}^j = (\theta_j, \theta_{j+1}, \cdots, \theta_{q+j-1})$ for each $j \in N \backslash \{i\}$, and $x^i \neq x$ or $\boldsymbol{\theta}^i \neq (\theta_i, \theta_{i+1}, \cdots, \theta_{q+i-1})$.

In other words, a message profile m is consistent with (x, θ) if there is no break in the cyclic announcement of characteristics and all individuals announce the outcome x. On the other hand, it is m_{-i} consistent with (x, θ) *either* if there are, at most, q consecutive breaks in the cyclic announcement of characteristics such that these breaks happen in correspondence of the characteristics announced by individual i, and x is unanimously announced *or* if individual i announces an outcome different from the outcome x announced by the others, and there are no more than q consecutive breaks in the cyclic announcement of characteristics such that these breaks (if any) happen in correspondence of the characteristics announced by individual i.

For each individual i, i's truth-telling correspondence for a q-mechanism is defined as follows: For all $\theta \in \Theta$,

$$\left(\boldsymbol{\theta}^i, x^i, z^i\right) \in T_i^{\Gamma_q}(\theta, S(i; q)) \quad \text{if and only if} \quad \boldsymbol{\theta}^i = (\theta_i, \theta_{i+1}, \cdots, \theta_{q+i-1}), \text{ with } n + p = p.$$

As in Lombardi and Yoshihara (2013)'s 2-mechanism, in our q-mechanism individuals make a cyclic announcement of strategies while the profile of characteristics, that is, the state, is determined without relying upon the deviator's announcement. Thus, the outcome function g is defined with the following three rules: For each $m \in M$,

Rule 1: If m is consistent with $\left(\bar{\theta}, x\right)$ and $x \in F\left(\bar{\theta}\right)$, then $g(m) = x$.

Rule 2: If for some $i \in N$, m_{-i} is consistent with $\left(\bar{\theta}, x\right)$ and $x \in F\left(\bar{\theta}\right)$, then $g(m) = x$.

Rule 3: Otherwise, a modulo game is played: identify the individual $i = \sum_{j \in N} z^j \pmod{n}$. This individual is declared the winner of the game, and the alternative implemented is the one she selects.

Let us check that the above q-mechanism partially-honest implements F. Suppose that $\theta \in \Theta$ is the "true" state and that $H \in \mathcal{H}$ is the "true" set of partially-honest individuals. Suppose that $x \in F(\theta)$. Let $m_i = \left(\boldsymbol{\theta}^i, x, z^i\right)$ for each $i \in N$ such that the corresponding strategy profile m is consistent with (θ, x). Then, Rule 1 implies that $g(m) = x$. Note that no unilateral deviation can change the outcome. Also, note that individual i is truthful in the preference announcement component $\boldsymbol{\theta}^i$ of her strategy. Therefore, $x \in NA\left(\Gamma_q, \succcurlyeq^{\Gamma_q, \theta, S(N;q), H}\right)$.

We now show that $NA\left(\Gamma_q, \succcurlyeq^{\Gamma_q, \theta, S(N;q), H}\right) \subseteq F(\theta)$. Let $m \in NE\left(\Gamma_q, \succcurlyeq^{\Gamma_q, \theta, S(N;q), H}\right)$. To avoid triviality, suppose that $|X| \geq 2$.[6] We distinguish three cases.

Suppose that m falls into Rule 1. Take any partially-honest individual $h \in H$. Suppose that $m_h \notin T_h^{\Gamma_q}(\theta, S(h,q))$. Then, it is the case that $\boldsymbol{\theta}^h \neq (\theta_h, \theta_{h+1}, \cdots, \theta_{q+h-1})$. By changing m_h into $m'_h \in T_h^{\Gamma_q}(\theta, S(h,q))$, agent h can induce Rule 2 and obtain $g(m'_h, m_{-h}) = x$. Given that $g(m) = g(m'_h, m_{-h})$, that $m'_h \in T_h^{\Gamma_q}(\theta, S(h,q))$ and that $m_h \notin T_h^{\Gamma_q}(\theta, S(h,q))$, it follows from part (i) of Definition 2 that $(m'_h, m_{-h}) \succ_h^{\Gamma_q, \theta, S(h,q)} m$, which is a contradiction. Therefore, we have established that $m_h \in T_h^{\Gamma_q}(\theta, S(h,q))$, and so $\boldsymbol{\theta}^h = (\theta_h, \theta_{h+1}, \cdots, \theta_{q+h-1})$. Finally, we need to show that $\bar{\theta} = \theta$. Assume, to the contrary, that $\bar{\theta} \neq \theta$. Thus, $\bar{\theta}_j \neq \theta_j$ for some $j \in N$, and so individual j is not truthful in her announcement $\boldsymbol{\theta}^j$. Since every partially-honest individual is truthful, individual j is not a partially-honest individual, that is, $j \notin H$. Furthermore, given that $\bar{\theta}_j \neq \theta_j$, it also follows that at least $q-1$ individuals standing immediately to her right are not partially-honest.[7] Thus, there must be at least q individuals in N who are not partially-honest, which contradicts the fact that there are at least $n-q+1$ partially-honest individuals, and so it must be the case that there are most $q-1$ individuals who are not partially-honest. We conclude that $\bar{\theta} = \theta$.

Suppose that m falls into Rule 2. We proceed according to whether $x^i \neq x$ or not.

Case 1: $x^i \neq x$.

- Suppose that $|X| \neq 2$ or $n \neq 3$. If $|X| > 2$, then every individual $j \neq i$ can induce Rule 3. Thus, we have that $X \subseteq g(M_j, m_{-j})$ for each individual $j \in N \backslash \{i\}$. Otherwise, let us suppose that $|X| = 2$ and that $n \neq 3$. By replacing x with $x^j = x^i$, individual

[6] For a set S, we write $|S|$ to denote the number of elements in S.

[7] Recall that individuals are arranged in a circular fashion clockwise facing inward, and each i is required to announce her own cahracteristics together with the characteristics of $q-1$ individuals standing immediately to her left.

j can make $\left|\left\{\ell \in N | x^\ell = x\right\}\right| \geq 2$ and $\left|\left\{\ell \in N | x^\ell \neq x\right\}\right| \geq 2$. Since the outcome is determined by Rule 3, individual j can attain any outcome in X by appropriately choosing z^j. Again, we have that $X \subseteq g\left(M_j, m_{-j}\right)$ for any $j \in N \backslash \{i\}$. Finally, let us consider the case that $|X| = 2$ and that $n = 3$. Then, let $N = \{i-1, i, i+1\}$, with $n+1 = 1$ and $1-1 = n$. We proceed according to whether or not there exist two distinct individuals $\ell, \ell' \in N$ such that $|\Theta_\ell| \neq 1$ and $|\Theta_{\ell'}| \neq 1$ hold.

- Suppose that there are two distinct individuals $\ell, \ell' \in N$ such that $|\Theta_\ell| \neq 1$ and $|\Theta_{\ell'}| \neq 1$. In this case, individual $i-1$ (resp., $i+1$) can always induce Rule 3 by appropriately changing the announcement of her own characteristics or that of her successor, and by carefully choosing the outcome announcement. To attain x^i, individual $i-1$ (resp., $i+1$) has only to adjust the integer index.

- Suppose that for all two distinct individuals $\ell, \ell' \in N$, it holds that $|\Theta_\ell| = 1$ or $|\Theta_{\ell'}| = 1$.

  Suppose that $|\Theta_k| = 1$ for all $k \in N$. Since m falls into Rule 2, it follows that $x \in F(\theta)$, as desired.

  Suppose that there exists $k \in \{i-1, i, i+1\}$ such that $|\Theta_k| \neq 1$.

  If either $|\Theta_{i-1}| > 1$ or $|\Theta_i| > 1$, then individual $i-1$ can induce Rule 3 by changing m_{i-1} to either $m'_{i-1} = \left(\left(\theta^{i-1}_{i-1}, \boldsymbol{\theta}^{i-1}_{-(i-1)}\right), x, z^{i-1}\right)$ with $\theta^{i-1}_{i-1} \neq \bar{\theta}_{i-1}$ (if $|\Theta_{i-1}| > 1$) or $m'_{i-1} = \left(\left(\theta^{i-1}_i, \boldsymbol{\theta}^{i-1}_{-i}\right), x^i, z^{i-1}\right)$ with $\theta^{i-1}_i \neq \bar{\theta}_i$ (if $|\Theta_i| > 1$). Individual $i-1$ can attain x^i by announcing z^{i-1} by which individual i becomes the winner of the modulo game. Therefore, we have that $X \subseteq g\left(M_{i-1}, m_{-(i-1)}\right)$. Suppose that $|\Theta_{i-1}| = |\Theta_i| = 1$. If $q = n$, individual $i-1$ can induce Rule 3 by changing m_{i-1} into $m'_{i-1} = \left(\left(\boldsymbol{\theta}^{i-1}_{-(i+1)}, \theta^{i-1}_{i+1}\right), x, z^{i-1}\right)$, with $\theta^{i-1}_{i+1} \neq \bar{\theta}_{i+1}$. Individual $i-1$ can attain x^i by announcing z^{i-1} by which individual i becomes the winner of the modulo game. Suppose that $q \neq n$. Individual $i-1$ can change m_{i-1} into $m'_{i-1} = \left(\boldsymbol{\theta}^{i-1}, x^i, z^{i-1}\right)$. Note that $\left(m'_{i-1}, m_i\right)$ is consistent with $\left(x^i, \left(\boldsymbol{\theta}^{i-1}, \theta^i_{i+1}\right)\right)$ given that $\theta^{i-1}_i = \theta^i_i$. If $x^i \in F\left(\boldsymbol{\theta}^{i-1}, \theta^i_{i+1}\right)$, then $\left(m'_{i-1}, m_{-(i-1)}\right)$ falls into Rule 2, and so $g\left(m'_{i-1}, m_{-(i-1)}\right) = x^i$. If $x^i \notin F\left(\boldsymbol{\theta}^{i-1}, \theta^i_{i+1}\right)$, then $\left(m'_{i-1}, m_{-(i-1)}\right)$ falls into Rule 3. Individual $i-1$ can attain x^i by announcing z^{i-1} by which she becomes the winner of the modulo game. We have established that $X \subseteq g\left(M_{i-1}, m_{-(i-1)}\right)$ if $|\Theta_k| \neq 1$ for some $k \in \{i-1, i, i+1\}$. Reasoning like that used for individual $i-1$ shows that $X \subseteq g\left(M_{i+1}, m_{-(i+1)}\right)$ if $|\Theta_k| \neq 1$ for some $k \in \{i-1, i, i+1\}$. Thus, $X \subseteq g\left(M_j, m_{-j}\right)$ for each individual $j \in \{i-1, i+1\}$.

From the above arguments, we obtained that $X \subseteq g\left(M_j, m_{-j}\right)$ for each individual $j \in N \backslash \{i\}$. Given that $m \in NE\left(\Gamma_q, \succsim^{\Gamma_q, \theta, S(N), H}\right)$, it follows that $g\left(M_j, m_{-j}\right) = X \subseteq L_j(\theta, x)$ for each $j \in N \backslash \{i\}$. No veto-power implies that $x \in F(\theta)$.

Case 2: $x^i = x$.

Then, it is the case that $\boldsymbol{\theta}^i \neq \left(\bar{\theta}_i, \bar{\theta}_{i+1}, \cdots, \bar{\theta}_{q+i-1}\right)$ given m_{-i} is consistent $\left(x, \bar{\theta}\right)$ and $x^i = x$. We proceed according to whether $q = 2$ or not.

- Suppose that $q \neq 2$. Thus, individual i is a unique deviator. By altering her strategy choice m_j into $m'_j = \left(\boldsymbol{\theta}^j, x^j, z^j\right)$, with $x^j \neq x$, individual j can induce Rule 3. Then, to attain x^j, individual j has only to announce z^j by which she becomes the winner of the modulo game. Since $g(m) = x$ and since, moreover, the choice of $x^j \neq x$ was arbitrary, we have that $X \subseteq g(M_j, m_{-j})$. Thus, we have established that $X \subseteq g(M_j, m_{-j})$ for each individual $j \in N\backslash\{i\}$. Given that $m \in NE\left(\Gamma_q, \succsim^{\Gamma_q,\theta,S(N),H}\right)$, it follows that $g(M_j, m_{-j}) = X \subseteq L_j(\theta, x)$ for each $j \in N\backslash\{i\}$. No veto-power implies that $x \in F(\theta)$.

- Suppose that $q = 2$. We proceed according to the following sub-cases: 1) $\theta_i^i \neq \bar{\theta}_i$ and $\theta_{i+1}^i \neq \bar{\theta}_{i+1}$, and 2) $\theta_i^i \neq \bar{\theta}_i$ and $\theta_{i+1}^i = \bar{\theta}_{i+1}$.[8]

  Suppose that $\theta_i^i \neq \bar{\theta}_i$ and $\theta_{i+1}^i \neq \bar{\theta}_{i+1}$. Thus, individual i is a unique deviator. Reasoning like that used for the case $q \neq 2$ shows that $x \in F(\theta)$.

  Suppose that $\theta_i^i \neq \bar{\theta}_i$ and $\theta_{i+1}^i = \bar{\theta}_{i+1}$. Suppose that $x \notin F\left(\bar{\theta}, \theta_i^i\right)$. Thus, individual i is a unique deviator. Reasoning like that used for the case $q \neq 2$ shows that $x \in F(\theta)$. Let us consider the case that

$$x \in F\left(\bar{\theta}_{-i}, \theta_i^i\right) \bigcap F\left(\bar{\theta}\right). \tag{10}$$

  Then, $i-1$ and i are both deviators.

  Suppose that individual $i \in H$ and that $m_i \notin T_i^{\Gamma_q}(\theta, S(i;q))$. If $(\theta_i, \theta_{i+1}) = \left(\bar{\theta}_i, \bar{\theta}_{i+1}\right)$, then individual i can induce Rule 1 by changing m_i into $m'_i = \left(\left(\bar{\theta}_i, \bar{\theta}_{i+1}\right), x, z^i\right) \in T_i^{\Gamma_q}(\theta, S(i;q))$. Given that $g(m) = g(m'_i, m_{-i})$, that $m'_i \in T_i^{\Gamma_q}(\theta, S(i;q))$ and that $m_i \notin T_i^{\Gamma_q}(\theta, S(i;q))$, it follows from part (i) of Definition 2 that $(m'_i, m_{-i}) \succ_i^{\Gamma_q,\theta,S(i;q)} m$, which is a contradiction. Suppose that $(\theta_i, \theta_{i+1}) \neq \left(\bar{\theta}_i, \bar{\theta}_{i+1}\right)$. By changing m_i into $m'_i = ((\theta_i, \theta_{i+1}), x, z^i) \in T_i^{\Gamma_q}(\theta, S(i;q))$, individual i can induce Rule 2, thus $g(m'_i, m_{-i}) = x$. Given that $g(m) = g(m'_i, m_{-i})$, that $m'_i \in T_i^{\Gamma_q}(\theta, S(i;q))$ and that $m_i \notin T_i^{\Gamma_q}(\theta, S(i;q))$, it follows from part (i) of Definition 2 that $(m'_i, m_{-i}) \succ_i^{\Gamma_q,\theta,S(i;q)} m$, which is a contradiction. We conclude that $m_i \in T_i^{\Gamma_q}(\theta, S(i;q))$ if $i \in H$.

  Suppose that individual $i-1 \in H$ and that $m_{i-1} \notin T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$. If $(\theta_{i-1}, \theta_i) = \left(\bar{\theta}_{i-1}, \theta_i^i\right)$, then individual $i-1$ can induce Rule 1 by changing m_{i-1} into $m'_{i-1} = ((\theta_{i-1}, \theta_i), x, z^{i-1}) \in T_{i-1}^{\Gamma_q}(\theta, S(i-1;q))$. Given that $g(m) = g\left(m'_{i-1}, m_{-(i-1)}\right)$, that

[8] The sub-case $\theta_i^i = \bar{\theta}_i$ and $\theta_{i+1}^i \neq \bar{\theta}_{i+1}$ is not explicitly considered, since it can be proved similarly to the sub-case 2 shown below.

$m'_{i-1} \in T^{\Gamma_q}_{i-1}(\theta, S(i-1;q))$ and that $m_{i-1} \notin T^{\Gamma_q}_{i-1}(\theta, S(i-1;q))$, it follows from part (i) of Definition 2 that $\left(m'_{i-1}, m_{-(i-1)}\right) \succ^{\Gamma_q,\theta,S(i-1;q)}_{i-1} m$, which is a contradiction. Suppose that $(\theta_{i-1}, \theta_i) \neq \left(\bar{\theta}_{i-1}, \theta^i_i\right)$. By changing m_{i-1} into $m'_{i-1} = ((\theta_{i-1}, \theta_i), x, z^{i-1}) \in T^{\Gamma_q}_{i-1}(\theta, S(i-1;q))$, individual $i-1$ can induce Rule 2, thus $g\left(m'_{i-1}, m_{-(i-1)}\right) = x$. Given that $g(m) = g\left(m'_{i-1}, m_{-(i-1)}\right)$, that $m'_{i-1} \in T^{\Gamma_q}_{i-1}(\theta, S(i-1;q))$ and that $m_{i-1} \notin T^{\Gamma_q}_{i-1}(\theta, S(i-1;q))$, it follows from part (i) of Definition 2 that $\left(m'_{i-1}, m_{-(i-1)}\right) \succ^{\Gamma_q,\theta,S(i-1;q)}_{i-1} m$, which is a contradiction. We conclude that $m_{i-1} \in T^{\Gamma_q}_{i-1}(\theta, S(i-1;q))$ if $i-1 \in H$.

Suppose that individual $j \in H \backslash \{i-1, i\}$ and that $m_j \notin T^{\Gamma_q}_j(\theta, S(j;q))$. Take any $x^j \neq x$. By changing m_j into $m'_j = ((\theta_j, \theta_{j+1}), x^j, z^j) \in T^{\Gamma_q}_j(\theta, S(j;q))$, individual j can induce Rule 3, where z^j satisfies $i = \sum_{k \in N} z^k \pmod{n}$, and thus $g\left(m'_j, m_{-j}\right) = x$. Given that $g(m) = g\left(m'_j, m_{-j}\right)$, that $m'_j \in T^{\Gamma_q}_j(\theta, S(j;q))$ and that $m_j \notin T^{\Gamma_q}_j(\theta, S(j;q))$, it follows from part (i) of Definition 2 that $\left(m'_j, m_{-j}\right) \succ^{\Gamma_q,\theta,S(j;q)}_j m$, which is a contradiction. We conclude that $m_j \in T^{\Gamma_q}_j(\theta, S(j;q))$ if $j \in H \backslash \{i-1, i\}$.

From the above arguments, we obtain that $m_h \in T^{\Gamma_q}_h(\theta, S(h,q))$ for all $h \in H$. Also, note that $i-1 \notin H$ if $i \in H$, given that $\theta^i_i \neq \bar{\theta}_i$. For the same reason, it holds that $i \notin H$ if $i-1 \in H$. Given that there are at least $n-q+1 = n-1$ partially-honest individuals, it is the case that at least one of the deviators is a partially-honest individual. If both deviators are partially-honest, then arguments like those used above for the case $q = 2$ shows that either i or $i-1$ can find a profitable unilateral deviation from the profile $m \in NE\left(\Gamma_q, \succcurlyeq^{\Gamma_q,\theta,S(N),H}\right)$, which is a contradiction. Thus, it is the case that only one of the deviators can be a partially-honest individual. Given that all partially-honest individuals are truthful and given that (10) holds, it follows that $x \in F(\theta)$, as we sought.

Suppose that m falls into Rule 3. By the definition of the outcome function, we have that for each individual i, $g(M_i, m_{-i}) = X$. Given that $m \in NE\left(\Gamma_q, \succcurlyeq^{\Gamma_q,\theta,S(N),H}\right)$, it follows that $g(M_i, m_{-i}) = X \subseteq L_i(\theta, x)$ for each $i \in N$. No veto-power implies that $x \in F(\theta)$.

Let us show the part (b) of the statement. Suppose that the class $\mathcal{H}$ includes singletons and that the number of partially-honest individuals in N is lower than $n-q+1$. Assume, to the contrary, that the SCR F is partially-honestly Nash implementable by a q-mechanism. Given that Assumption 1 assures that there is at least one individual who is partially-honest, an immediate contradiction is obtained if $q = n$. Thus, let us consider the case that $q \neq n$. Given that F is partially-honestly Nash implementable by a q-mechanism, it follows that it is partially-honestly Nash implementable when the actual honesty standard of society is $S(N;q)$. Theorem 1 implies that F is partially-honest monotonic w.r.t. $S(N;q)$. Furthrmore, by assumption, the class $\mathcal{H}$ includes singletons and the domain Θ is indipendent.

Since the honesty standard of the society $S(N;q)$ is non-connected, Corollary 2 implies that the SCR F is Maskin monotonic, which is a contradiction. ■

We make several remarks below regarding Theorem 3.

Remark 2 It is known that Maskin's theorem is robust against Saijo (1988)'s simplification of Maskin's communication scheme. Indeed, the class of Nash implementable SCRs is equivalent to the class of SCRs that are Nash implementable by a q-mechanism provided that $q \geq 2$ (Lombardi and Yoshihara, 2013). In light of Theorem 3, this equivalence relationship no longer holds if there are less than $n - q + 1$ individuals who have a taste for honesty.

Remark 3 Part (b) of the statement continues to hold if $q = 1$, that is, when every individual $i \in N$ is required to choose only her own characteristics as part of her strategy choice, like a self-relevant mechanism (Tatamitani, 2001). It means that if a non-Maskin monotonic SCR F is partially-honestly Nash implementable by this type of mechanism, then all individuals in a society need to be partially-honest. However, if the requirement $q \neq 1$ is dropped, the part (a) of the statement fails to hold. The reason is that the mechanism constructed to prove Theorem 3 detects a participant's lie by relying on the play of other participants. This type of detection is not possible in the case of a self-relevant mechanism. A sufficient condition for the SCR F to be partially-honest Nash implementable by a self-relevant mechanism is that F satisfies no veto-power as well as there is a worst outcome in X for any individual i.

Remark 4 Note that Theorem 3 holds as long as $S(i;q) \subseteq S(i)$ for each $i \in N$. Moreover, when the relevant honesty standard of participant i in a q-mechanism is not a subset of her actual honesty standards, that is, $S(i;q) \nsubseteq S(i)$, then Maskin monotonicity may become again a necessary condition for Nash implementation, though there is at least $n - q + 1$ partially-honest individuals in N. For instance, consider a society with $n = 3$ participants and $q = 2$ for the q-mechanism. By Theorem 3, any SCR satisfying the no veto-power condition is partially-honestly Nash implementable if there are at least two partially-honest individuals, provided that $S(i) = S(i;2)$ for each participant $i \in N$. However, we can show that any SCR satisfying partial-honesty monotonicity should also satisfy Maskin monotonicity in a society with at least two partially-honest individuals if every participant i's honesty standard is $S(i) = \{i\}$.

Remark 5 Part (a) of Theorem 3 can be generalized by allowing the preference announcement component of participant i to depend on her individual honest standard $S(i)$, that is, by allowing participant i to choose $(\theta_k)_{k \in S(i)}$ as her strategy choice (plus two auxiliary data). Specifically, it can be shown that the Nash implementability is assured by no veto-power if (i) the honesty standard of the society $S(N)$ and the family $\mathcal{H}$ are such that for every conceivable set H of partially-honest individuals, the union of individual honesty standards of partially-honest individuals in H forms a covering of N, that is, $N \subseteq \bigcup_{h \in H} S(h)$, and (ii) the honesty standard of the society $S(N)$ is such that every individual's preference is covered at least twice, that is, for every participant i, it holds that $i \in S(j)$ for some participant $j \neq i$. Note that if requirement (ii) is not met, then the idea on which the above definition of the outcome function is based on may not work. Also, note if requirement (i) is met, then the singleton $\{i\}$ is an element of the family $\mathcal{H}$ if $S(i) = N$.[9]

6. Concluding remarks

In an environment in which knowledge is dispersed, how individuals will interact with the mechanism designer is a natural starting point when it comes to Nash implementing a SCR. A particular kind of communication is, as in Dutta and Sen's (2012) Theorem 1, to ask participants to report preferences of the entire society. However, there is no reason to restrict attention to such schemes.

There are multiple other ways for the mechanism designer to structure the exchange of information with individuals, and there is no limit to how imaginative he can be. This paper basically considered the Saijo-type simplifications of Maskin's canonical mechanism and concluded that since those simplifications let individuals behave as if their honesty standards were non-connected, the seminal result of Dutta and Sen's (2012) Theorem 1 does not survive them if there are not enough partially-honest participants.

Postlewaite and Schmeidler (1986), Palfrey and Srivastava (1989) and Jackson (1991) have shown that Maskin's theorem can be generalized to Bayesian environments. A necessary condition for Bayesian Nash implementation is Bayesian monotonicity. In a Bayesian environment involving at least three individuals, Bayesian monotonicity combined with no veto-power is sufficient for Bayesian Nash implementation provided that a necessary condition called closure and the Bayesian incentive compatibility condition are satisfied (Jackson, 1991). Although the implementation model developed in this paper needs to be modified to handle Bayesian environments, we believe a similar equivalence result holds in those environments for suitably defined communication schemes (on this point, see Lombardi and Yoshihara, 2013; section 5). This subject is left for future research.

[9] If $S(N)$ is such that $S(i) = N$ for all $i \in N$, then $\bigcup_{h \in H} S(h) \supseteq N$ holds for any $H \in \mathcal{H}$ and any family $\mathcal{H}$. Therefore, any SCR satisfying the no veto-power condition is partially-honestly Nash implementable without any restriction on the structure of the family $\mathcal{H}$, as per Dutta-Sen (2012)'s Theorem.

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