Studies on empirical analysis of macroeconomic models with heterogeneous agents

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Abstract

The dissertation consists of four chapters studying the nonlinear stochastic dynamic optimization model with heterogeneous agents.

Chapter 2 is based on the joint work with Makoto Nirei and Sanjib Sarker. In this chapter, we examine the response of aggregate consumption to active labor market policies that reduce unemployment. We develop a dynamic general equilibrium model with heterogeneous agents and uninsurable unemployment risk as well as policy regime shocks to quantify the consumption effects of policy. By implementing numerical experiments using the model, we demonstrate a positive effect on aggregate consumption even when the policy serves as a pure transfer from the employed to the unemployed. The positive effect on consumption results from the reduced precautionary savings of households who indirectly benefit from the policy by decreased unemployment hazard in future.

Chapter 3 presents a structural estimation method for nonlinear stochastic dynamic models of heterogeneous firms. As a result of technical constraints, there is still no consensus on the parameters of a productivity process. In order to estimate the parameters, I propose a Bayesian likelihood-free inference method to minimize the density difference between the cross-sectional distribution of the observations and the stationary distribution of the structural model. Because the stationary distribution is a nonlinear function of a set of the structural parameters, we can estimate the parameters by minimizing the density difference. Finally, I check the finite sample property of this estimator using Monte Carlo experiments, and find that the estimator exhibits a comparatively lower root mean squared error in almost all the experiments.

Chapter 4 studies a structural estimation method for the nonlinear stochastic dynamic optimization model with heterogeneous households, and then conducts the empirical research about the household asset allocation behavior. The analysis of household finance has non-negligible implications in asset pricing literature, but empirical research on this topic is challenging. To solve the equity premium puzzle, I consider two kinds of heterogeneity across households: wealth heterogeneity and limited stock market participation. Then, I summarize the empirical facts about household investment portfolio with the National Survey of Family Income and Expenditure, a cross-sectional Japanese household survey. Because we cannot observe the dynamics of the individual portfolio with the cross-sectional data, we cannot estimate the structural parameters of the dynamic model. I propose the Bayesian likelihood-free inference method to minimize both the density difference and the distance between policy functions between the observed and the simulated values. Because the stationary distribution and the policy function are nonlinear functions of a set of structural parameters, we can estimate the parameters by minimizing the density difference and the distance between policy functions. We can find that the estimated relative risk aversion is around four. The estimation outcome implies that the model can mimic the observed household finance behavior well and the equity premium puzzle comes of a biased estimate caused by the representative agent assumption.

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[1]This chapter is based on joint work with Makoto Nirei and Sanjib Sarker (Yamana, Nirei and Sarker (2016))

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Chapter 1

Introduction

This dissertation consists of four chapters studying the nonlinear stochastic dynamic optimization model with heterogeneous agents. In general, heterogeneity in economics is generally categorized into three groups following Browning, Hansen and Heckman (1999) and Blundell and Stoker (2005): (i) heterogeneity in individual tastes and incomes, (iii) heterogeneity in wealth and income risks faced by individuals, and (iii) heterogeneity in market participation. Though this classification is empirically useful, when modeling a micro-founded heterogeneous agents behavior, it is not necessarily clear where the line between exogenous factors and endogenous outcomes lies (Heathcote, Storesletten and Violante (2009)). This is explained by the fact that the observed heterogeneity generates as a compound of exogenous innate characteristics (ex-ante heterogeneity), exogenous or endogenous subsequent stochastic shocks, and endogenous rational choices based on individual states. I mainly focus on the second heterogeneity which builds on an incomplete market structure where agents are ex-ante homogeneous and ex-post heterogeneous through exogenous idiosyncratic shock history across the agents. Idiosyncratic shock is not directly insurable but is insured partially by trading an asset subject to a limit or accumulating the asset as a buffer stock (self-insurance). That is, we take heterogeneity in earnings history as given and generate the endogenous heterogeneity in consumption and wealth.[1] This specification is appealing because it enables us to disentangle quantitatively how much we can account for the ex-post heterogeneity by incomplete markets without assuming ex-ante unobservable heterogeneity (e.g. preference).

Krusell and Smith (2006) discussed that it is important to consider a heterogeneous population structure for at least two reasons, a robustness check on the representative-agent model and a growing interest in distributional issues. The robustness check is done on the representative-agent assumption which treats all agents as identical and idiosyncratic risks as perfectly diversifiable. Since there is a possibility that ignoring heterogeneity may affect the aggregate implication or cause aggregation bias, robustness must be checked both theoretically and empirically. Guvenen (2011) discussed that this use of heterogeneity is less obvious because theoretical and numerical studies have already confirmed that certain types of heterogeneity do not change the aggregate implication. Levine and Zame (2002) theoretically showed that if we assume an exchange economy with a single consumption good and incomplete markets where infinitely-lived agents have an access to a single risk-free asset and share the common subjective discount factor, and there exists transitory idiosyncratic risk but there is neither extremely persistent idiosyncratic risk (Constantinides and Duffie (1996)) nor aggregate risk, the effect of the incomplete markets will vanish in the long run. A similar result was confirmed numerically by Krusell and Smith (1998) for the imperfect insurance economy and by Rios-Rull (1996) for the finitely-lived overlapping generations economy. These results depend on the fact that an individual's consumption policy function is approximately linear with respect to wealth even with

[1] Rubinstein (1974) studied a dynamic economy with no idiosyncratic shocks and assumed preference homogeneity for aggregation. Constantinides (1982) studied a dynamic economy with idiosyncratic shocks, not assuming preference homogeneity but assuming complete markets for aggregation. Both Huggett (1993) and Aiyagari (1994) studied dynamic general equilibrium economy with idiosyncratic shocks and incomplete markets. Huggett (1993) studied the exchange economy of a household bond; meanwhile, Aiyagari (1994) studied the production economy of a capital of the firm.

the existence of uninsured idiosyncratic risk, except for the wealth levels near the borrowing constraint.[2]

The second reason for considering a heterogeneous population structure is a growing interest in distributional issues or inequality (disparity), especially in conjunction with macroeconomic forces or policies, that leads to different policy implications. For example, business cycles and inflation are likely to have asymmetric welfare effects across agents depending on their respective wealth levels and compositions. So, when evaluating policy implications, we should take into consideration not only traditional general equilibrium effects, but also asymmetric responses caused by inequality. Following these studies, we can finally evaluate (i) how a stabilization policy designed to lessen the aggregate time-series volatility can affect cross-sectional distribution and (ii) how reallocation policy designed to lessen cross-sectional inequality (disturbance) can affect the aggregate time-series volatility, as discussed by Heckman (2001), Lucas (2003), and Heathcote et al. (2009).

In addition to the two traditional reasons discussed above, I point out a third reason to consider a heterogeneous population structure, which enables us to use rich micro data for structural estimation. By employing micro data (especially, panel data) instead of aggregate time series data, we can exclude potential aggregation biases from fundamental microeconomic dynamics and can consider heterogeneity (Bond (2002)). I try to explain the advantage by comparing the representative-agent formulation with a heterogeneous population structure. We first consider the calibration or estimation of parameters in the representative-agent formulation. Conditional on exogenous shocks, the representative-agent formulation can compute the unique one-dimensional steady-state aggregate capital stock level, and then generate a joint probability distribution for endogenous variables such as output and consumption.

[2] We can also find a case where aggregate approximation cannot function. For example, Chang and Kim (2007), Takahashi (2014), Chang and Kim (2014), and An, Chang and Kim (2009) studied indivisible labor supply.

Therefore, we can use the aggregate statistics and their time series as empirical counterparts of the endogenous values for estimation. In contrast, the incomplete market structure with no aggregate risk can generate the unique stationary cross-sectional wealth distribution as an infinite-dimensional equilibrium object. Therefore, we can employ not only one-dimensional aggregate statistics, but also N-dimensional individual statistics as empirical counterparts of the endogenous values for estimation. It implies that if we adopt the incomplete market structure, we can make the most of rich micro data sources, —ranging from cross-sectional surveys to panel data,— to calibrate or estimate the structural parameters.

A common strategy for parametrization in the incomplete market literature is a combination of external calibration with moment matching: to minimize the distance between simulated moments and empirical moments based on N-dimensional individual statistics. One of the drawbacks in the strategy is that it cannot exploit all the available information from the data, especially the distribution. This disadvantage is clearly revealed when the stationary equilibrium distribution is a mixture distribution where the moments are not the right statistics to summarize the distribution. In contrast, density estimators can provide more information than estimators using the mode or a finite set of moments (Liao and Stachurski (2015)). So, there is a need to develop the structural density estimation method for the incomplete market model to take advantage of the distributional information in rich micro data sources.

With respect to the estimation method, a further challenging task is to estimate the parameters of the incomplete markets model with aggregate risk. When there is aggregate risk, equilibrium prices are not constants but are functions of the infinite-dimensional wealth distribution. Therefore, we are required to know the law of motion of the cross-sectional distribution to solve the model. Krusell and Smith (1998) proposed the "approximate aggregation" method to solve the computational problem which approximates the law of motion of the infinite-dimensional wealth distribution to a law of motion of a finite number of moments of the distribution rather than the entire distribution itself. While we can appreciate the computational efficiency, there are some theoretical and methodological disadvantages. The theoretical disadvantage is that approximate aggregation gives us a local solution around the stationary equilibrium depending on the fact that the consumption policy function is linear with respect to wealth. Thus, the approximation can give us an inaccurate equilibrium function when the nonlinearities of the model are quantitatively large, i.e. wealth is very unequally distributed (Carroll (2000)), or if the initial value is set far away from the stationary equilibrium values. This kind of inaccuracy is essentially the same as the one which occurs during the linearization process of the dynamic equilibrium model with a representative agent as Rubio-Ramírez and Fernández-Villaverde (2005) discussed. The methodological disadvantage is that considering the aggregate risk is equivalent to enriching the standard incomplete markets model with latent aggregate states which often follows discrete Markov process. Since the computation of the marginal likelihood for this kind of model is subject to the path dependence problem, it is difficult to estimate the parameters, which is discussed by Bauwens, Dufays and Rombouts (2014) who studied the estimation method of Markov-switching GARCH. The path dependence problem occurs because the observed distribution depends on the entire sequence of regimes. Because the regimes and their path are latent, we should integrate over all possible paths when computing the likelihood. However, we cannot just do that as the number of paths increases exponentially with t. Since we have not been able to estimate structural parameters in the incomplete markets model with aggregate risk, I do not estimate but instead calibrate parameters of this kind of model in the first chapter, though the estimation of parameters is a promising field to research on.

The idea of a structural estimation strategy to minimize the difference between the empirical distribution and simulated distributions looks simple, but its implementation is rather difficult. The biggest impedance is the fact that the stationary distribution of the incomplete market model has no analytical expression and accordingly, we cannot employ the standard maximum likelihood (ML) procedure.[3] Instead of using the ML procedure, previous works in macroeconomics literature calibrated parameters with relevant microeconometric reduced-form estimates or moment matching indirect inference (II) type estimates[4]. However, there are several problems in using these methods. When calibrating the parameters with reduced-form estimates, (i) we cannot incorporate the theoretical restriction into the estimation procedure and (ii) there is little guidance from econometric theory to choose an estimation technique, each of which makes different assumptions on the error term. When calibrating the parameters with II-type estimates, we can perform a calibration and its statistical test simultaneously, however, (iii) the finite sample properties of estimates are poor and (iv) the ignorance of distribution may lead to biased estimates, except for the first moment.

To implement the algorithm to minimize the density difference, I use the approximate Bayesian computation (ABC) algorithm. ABC is a Bayesian statistical method for likelihood-free inference. When estimating the structural parameters of the incomplete market model, we need to specify the data generating process. In other words, equilibrium values can be generated conditional on parameters, but that is all we know about the likelihood; we have no information of the likelihood itself. ABC is the optimal estimation method in such a case, and by using ABC to minimize the difference between distributions, (i) we can incorporate theoretical restrictions into the estimation procedure, (ii) we can exclude the arbitrary process of selecting estimation methods, (iii) we can appreciate nice finite sample property, and (iv) we can

[3]This is because we cannot calculate the likelihood.

[4]Specifically, indirect inference estimator, simulated method of moments estimator (SMM or MSM), and efficient method of moments estimator (EMM) are classified into this class of estimators. These take the form of continuous-updating generalized method of moments estimator (GMM) and asymptotically equivalent. (See chapter 3)

employ the distributional information of the sample, not only the mode or a set of finite moments.

The key insight of ABC is that the calculation of likelihood can be replaced by a comparison process between observations and simulated values. For high dimensional data spaces, we rarely match these values and thus we usually compress them into a finite set of summary statistics. Since the estimation accuracy depends on how well summary statistics epitomize the data, the abstraction of the infinite-dimensional distribution is significant. To summarize the density difference, we first consider a naive two-step approach where we first separately estimate each density and then compute their distance using measures such as the Kullback-Leibler divergence (KLd). However, there are some problems in this two-step approach. First, because the estimation in the first step does not consider that in the second step's computing process, an estimation error which comes from the neglect of the second step can generate a big estimation error. Second, although minimizing the KLd is statistically equivalent to maximizing likelihood, it cannot satisfy the properties of a mathematical metric such as the symmetric property and triangle inequality, it is not robust to outliers, and is numerically unstable. So, instead of using the naive two-step approach, we should directly estimate the L^2-distance by least-squares density-difference which Sugiyama, Suzuki, Kanamori, du Plessis, Liu and Takeuchi (2013) proposed to minimize the density difference.

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Chapter 2

Time-Varying Employment Risks, Consumption Composition, and Fiscal Policy[1]

2.1 Introduction

The impact of the recent recession on the labor market was so severe that the unemployment rate in the U.S. is still above normal and the duration of unemployment remains unprecedentedly large. There is a growing interest in labor market policies as effective macroeconomic policy instruments to combat such high unemployment (Nie and Struby (2011)) that has been used conservatively to help the unemployed. Two major questions presented in this literature are as follows: (i) What is the effect of the policy on the labor market performance of program participants? and (ii) What is the general equilibrium consequence of such policy? While there have been extensive microeconometric evaluations and discussions that have led to a consensus on the

[1] This chapter is based on joint work with Makoto Nirei and Sanjib Sarker (Yamana et al. (2016))

first question,[2] the second question is unanswered because the indirect effects of the programs on nonparticipants via general equilibrium adjustments are inconclusive. Heckman, Lalonde and Smith (1999) pointed out that the commonly used partial equilibrium approach implicitly assumes that the indirect effects are negligible and can therefore produce misleading estimates when the indirect effects are substantial. Moreover, Calmfors (1994) investigated several indirect effects, and concluded that microeconometric estimates merely provide partial knowledge about the entire policy impact of such programs.

This study investigates the indirect effects of labor market policy by focusing on the aggregate consumption response. Previous research has identified several kinds of indirect effects, such as the deadweight effect, displacement effect, substitution effect, tax effect, and composition effect.[3] In this study, we concentrate on the effect of reduced unemployment risk on aggregate consumption. When the unemployment rate is lowered because of the labor market program, the expected future wealth of workers increases and therefore the need for present precautionary savings decreases not only for the program participants, but also for the nonparticipants. We numerically analyze the precautionary savings channel for the impact of this reduced unemployment risk and quantify the indirect effect on the consumption of nonparticipants.

Our analysis is based on a general equilibrium model with uninsurable idiosyncratic shocks and aggregate shocks as proposed by Krusell and Smith (1998) (henceforth referred to as KS). The KS economy features both aggregate and idiosyncratic

[2] According to Card, Kluve and Weber (2010), there is a great amount of micro econometric research discussing the individual treatment effect. Heckman, Lalonde and Smith (1999) summarize approximately 75 empirical studies; Kluve (2010) includes about 100 studies in his study, Greenberg, Michalopoulos and Robins (2003) survey includes 31 evaluations; and Card et al. (2010) compare 97 studies conducted between 1995 and 2007.

[3] According to Calmfors (1994), the deadweight effect arises from subsidizing the hiring that would have occurred in the absence of the program; the displacement effect arises from job creation by the program at the expense of other jobs; and the substitution effect arises from job creation in a certain category that replaces jobs in other categories because of a change in relative wage costs. The tax effect refers to the situation where higher employment tends to increase the tax base and reduce the sum of the costs of unemployment benefits. The composition effect occurs because the consumption levels of the employed and that of the unemployed are different.

shocks. An aggregate shock cannot be insured, and the markets for idiosyncratic risks are missing in this economy. Households can insure their consumption by accumulating their own wealth; that is, precautionary savings, but they can only partially hedge their consumption fluctuations with a binding borrowing constraint. The demand for precautionary savings is affected by the magnitude of the idiosyncratic unemployment risk that individual households must bear. The magnitude of the unemployment risk changes in tandem with the level of unemployment because a high unemployment rate is associated with a longer average spell of unemployment. Thus, when the rate of unemployment is reduced by the labor market policy, the workers who are currently employed perceive a lower chance of becoming unemployed and the unemployed have a higher chance of finding jobs. This perceived lower risk of future unemployment leads to less demand for precautionary savings and more demand for current consumption even for the households who do not participate in the government program.

The link between the labor market policy and precautionary savings was examined by Engen and Gruber (2001), who found evidence that unemployment insurance reduces household savings. This study investigates the aggregate consequences of the precautionary savings motive when the employment risk fluctuates. In our model, aggregate fluctuations in the economy are driven by a stochastic regime switch between passive and active regimes. In our first set of experiments, we consider direct job creation by government employment as an active policy. In essence, it is a pure transfer policy from the employed to a randomly selected fraction of the unemployed. If there were a complete market for each idiosyncratic employment risk, such a transfer policy would not affect household consumption at all. We are interested in the extent to which the lack of complete markets alters this prediction. In the second set of experiments, we consider employment incentives from a regime switch in the corporate tax rate in an economy with real wage rigidity. In this case, the labor input and thus the goods output varies along with the policy shock. The difference between the first and second set of exercises lies in who hires the additional labor—the public sector or the private sector. To isolate the latent impact of precautionary savings, we vary each of the two policy experiments so that an employed worker's real income is fixed across regimes. With these policy experiments, we analyze the behavior of the employed and unemployed workers with various asset positions, and thereby elicit the nature of the aggregate impact of the employment risks on consumption demand.

The results of our experiments are summarized as follows. We find a limited increase in the aggregate consumption level by the labor market policy. Although the consumption level of the program participants increases, the increase is almost offset by the reduced consumption of the employed nonparticipants who finance such hires (the tax effect) in the case of government employment policy. Therefore, the net increase in the aggregate consumption level largely results from the increased consumption of the unemployed nonparticipants who do not directly benefit from the program, but now have better prospects of future employment according to the program (the unemployment risk effect). To isolate the impact of the reduced unemployment risks from the tax effect, we conduct a modified experiment with a hypothetical international insurance program under which the employed workers face a constant tax over time across regimes. In this experiment, we find that the employed workers also respond strongly to the reduced risks even though they prefer a smoothed consumption path. The two experiments imply that the impact of reduced risks on consumption demand schedule is quantitatively large, even though the realized change in consumption amount is limited. Contrary to the experiment with government employment, the experiment with a corporate tax reduction affects both employment and output through private firms' production decision. In this case, we find that a decrease of employment risk by a tax cut generates considerable growth in both consumption and output. The participants as well as nonparticipants increase their consumption during periods of reduced unemployment risks, and firms increase their supply of goods to meet the higher consumption demand. Finally, sensitivity analyses conducted on the households' risk attitudes, borrowing constraints, and preference specifications confirm our interpretations of the results.

This chapter combines two threads of the literature—the general equilibrium effect of active labor market policies (ALMPs) and a precautionary savings behavior. ALMPs mainly consist of job-search assistance, job-training programs, employment support, direct job creation, and employment incentives, among others. While the first three policies affect the labor supply, the latter two policies (direct job creation[4] and employment incentives[5]) affect the labor demand. Our study investigates the latter set as the policy instruments. Only a few papers have investigated the general equilibrium effect of ALMPs. Calmfors (1994) discussed the several indirect effects of ALMPs which are neglected in the partial equilibrium approach. Meyer (1995) argued that in a bonus program of permanent unemployment insurance, the bonus induces the excess reemployment of claimants at the expense of other job claimants leading to a deadweight effect. Davidson and Woodbury (1993) used a Mortensen-Pissarides search model to evaluate the reemployment bonus program, which encourages the unemployed to accelerate their job-search, leading to a displacement effect. Heckman, Lochner and Taber (1998) used an overlapping generations model to consider the evaluation of tuition subsidy programs, which led to a substitution effect. Our study augments the literature by investigating the unemployment risk effect on consumption.

Another related topic in the literature is the precautionary savings effect on the aggregate consumption. The macroeconomic effects of precautionary savings have been analyzed by Aiyagari (1994), Carroll (2001), Huggett (1997), and Lusardi (1997), among others. Krusell and Smith (1998) formalized a dynamic general equilibrium model with incomplete markets and aggregate and idiosyncratic shocks. They found

[4]Direct job creation is a policy that creates nonmarket jobs in the public sector.

[5]An employment incentive is a policy that subsidizes the private sector to hire new employees.

that the consumption function in such an economy is almost linear in terms of wealth, which implies that the aggregate consequence of incomplete markets in the business cycle frequency is limited. Carroll (2001) argued that the KS model underestimates the precautionary savings effect because it generates a fairly centered wealth distribution, while the nonlinearity of the consumption function concentrates on low levels of wealth. Heathcote (2005) found a quantitatively significant impact of tax changes on consumption in the KS economy. This study investigates a new consumption effects mechanism in the KS framework by focusing on the time-varying unemployment hazard perceived by workers when the unemployment level fluctuates over time.

As a benchmark case of the consumption response to ALMPs, our first policy experiment features a pure transfer to the unemployed workers. Such a transfer constitutes an important fraction of the various fiscal expenditures that relate to purchases. Empirically, Oh and Reis (2012) and Cogan and Taylor (2012) reported that approximately three-quarters of the U.S. stimulus package from 2007Q4 to 2009Q4 was allocated to transfers. The transfer in our model is represented by the government employment of workers. Our study shows that there is a positive aggregate consumption response to ALMPs.

Finally, this study is also related to the literature about the co-movements of consumption and government expenditures. Empirical analyses using war-time events typically find a negative co-movement between consumption and government expenditures (Ramey and Shapiro (1998); Edelberg, Eichenbaum and Fisher (1999); Burnside, Eichenbaum and Fisher (2004)). Other analyses have found a positive correlation between consumption and government spending in identified VAR estimates (Blanchard and Perotti (2002); Mountford and Uhlig (2009); Galí, López-Salido and Vallés (2007)). Galí et al. also proposed a rule-of-thumb consumer to account for the positive comovement between consumption and government expenditures. Ramey (2011) has recently provided an account of these empirical differences. Moreover, incomplete markets and idiosyncratic employment risks are important factors in accounting for these co-movements. For example, Challe and Ragot (2011) analyzed the quantitative effects of transitory fiscal expansion in an economy where public debt serves as the liquidity supply, as in Aiyagari and McGrattan (1998) and Floden (2001). In this study, to examine the fiscal stimulus impact on consumption, we focus our attention on unemployment risks rather than liquidity effects.

The remainder of the chapter is organized as follows. The next section presents the model where we modify the Krusell-Smith model to incorporate government labor expenditures as a fundamental aggregate shock. Section 3 shows our numerical results. Sections 3.1 and 3.2 deal with the benchmark transfer policy, while Section 3.3 is concerned with corporate tax policy. Section 3.4 discusses the robustness of the results. Section 4 concludes the chapter. The details of our computational methods and numerical results are mentioned below in the Appendix.

2.2 Model

2.2.1 Model specification

We consider a dynamic stochastic general equilibrium model with incomplete markets, uninsurable employment shocks, and aggregate shocks as in KS. The economy is populated by a continuum of households with the population normalized to one. The households maximize their utility subject to budget constraints as follows:

$$\max_{c_{i,t},k_{i,t+1}\geq -\phi} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t c_{i,t}^{1-\sigma}/(1-\sigma) \tag{2.1}$$

$$\text{s.t.} \qquad c_{i,t} + k_{i,t+1} = (r_t + 1 - \delta)k_{i,t} + \iota(h_{i,t})w_t - \tau(h_{i,t}, z_t), \quad \forall t \tag{2.2}$$

$$k_{i,t+1} \geq -\phi, \quad \forall t \tag{2.3}$$

where $c_{i,t}$ is consumption, $k_{i,t}$ is capital assets, $h_{i,t}$ is the employment status, $\tau(h_{i,t}, z_t)$ is the lump-sum tax, r_t is the net return to capital, and w_t is the real wage in which the consumption good is the numeraire. Capital depreciates at the rate of δ, and the future utility is discounted by β. The households are subject to a borrowing constraint with a debt limit ϕ. The households are either unemployed ($h_{i,t} = 0$) or employed ($h_{i,t} = 1$), and $h_{i,t}$ follows an exogenous process, as discussed below. The households receive wage income when employed, whereas they depend on unemployment insurance when unemployed:[6]

$$\iota(h_{i,t}) = \begin{cases} 1 & h_{i,t} = 1 \\ 0.2 & h_{i,t} = 0. \end{cases}$$

This unemployment insurance is financed by taxation of the employed.

The representative firm produces goods with the technology specified by a Cobb-Douglas production function with constant returns to scale $Y_t = K_t^{\alpha} H_t^{1-\alpha}$, where Y_t represents the aggregate goods produced and K_t and H_t represent the aggregate capital and labor, respectively. The firm maximizes its profit in a competitive market, where the following conditions hold:

$$r_t = \alpha(K_t/H_t)^{\alpha-1} \quad (2.4)$$

$$w_t = (1-\alpha)(K_t/H_t)^{\alpha}. \quad (2.5)$$

[6] This represents an exogenous income support for the unemployed and it is common to technically include this lower limit in the literature of KS models. While there are various interpretations in the literature, a standard value is 10%. KS sets the value at about 9% of the average wage of the employed and Mukoyama and Şahin (2006) adopt the household production parameter, which is equal to 0.1. In our experiment, the ratio is interpreted as the unemployment insurance replacement rate and we set it at 20% because the average net unemployment benefit replacement rate in the 2000s (before 2008) is approximately 20%, according to the DICE Database (2013), "Unemployment Benefit Replacement Rates, 1961 - 2011," Ifo Institute, Munich. We notice that this OECD summary measure of benefit entitlements is not close to the initial replacement rate, which was legally guaranteed for the unemployed. For further discussion, see Martin (1996).

Our model features a fiscal expansion that affects the labor market as an aggregate shock. We first consider a government employment program. The fiscal policy z_t follows a Markov process with two states $\{0, 1\}$ and a transition matrix $[\pi_{zz'}]$. The labor market policy is passive in state $z_t = 0$ and the government supplies only the unemployment insurance. The lump-sum tax is determined as

$$\tau(1, 0) = 0.2 w_t u_0/(1 - u_0) \tag{2.6}$$

and aggregate unemployment stays at a high rate, u_0. In state $z_t = 1$, the government employs a fraction of the unemployed at the wage rate w_t as well as supplies the unemployment insurance. The fraction of the unemployed nonparticipants amounts to u_1, which is strictly less than u_0. The government employment program is financed by a lump-sum tax on the employed workers so that the government budget is balanced in each period. Thus, the tax is determined as

$$\tau(1, 1) = 0.2 w_t u_1/(1 - u_1) + w_t(u_0 - u_1)/(1 - u_1). \tag{2.7}$$

The unemployed do not pay tax for any z_t: $\tau(0, z_t) = 0$. Note that the aggregate labor supply for firms is exogenously constant at $H_t = 1 - u_0$ for any t regardless of z_t, whereas the total number of workers employed by firms or government is either $1 - u_0$ or $1 - u_1$, depending on z_t. We assume that the government is non-productive and its employment does not produce goods.

We allow the aggregate state z_t to affect the transition probability of the individual employment state, $h_{i,t}$. Let Π denote the transition matrix for the pair comprising the employment status and fiscal policy states, $(h_{i,t}, z_t)$. The transition probability from (h, z) to (h', z') is denoted by $\pi_{hh'zz'}$. In our model, the aggregate shock z determines both the labor market policy regime and employment level, whereas in the original KS model, the aggregate state only determines the employment level.

A recursive competitive equilibrium is defined as follows. The household's maximization problem is written as a dynamic programming problem with state variables (k, h, z, Γ), where Γ is the cross-sectional distribution of (k_i, h_i) across households $i \in [0, 1]$. The law of motion for (h, z) is determined by the exogenous transition matrix Π. We define the transition function T that maps Γ to the next period distribution as Γ'. The recursive competitive equilibrium is defined by the value function, $V(k, h, z, \Gamma)$; the households' policy function, $F(k, h, z, \Gamma)$; and the transition function, T; such that V and F solve the households' problem under T. The competitive factor prices that satisfy equations (2.4) and (2.5) are consistent with the market clearing conditions $K = \int k_i d\Gamma$, and H is equal to the measure of workers employed by the firms,[7] and T is consistent with F and Π. By Walras' law, the goods market clears; that is, $C + K' - (1-\delta)K = Y$, where $C = \int c_i di$ is the aggregate consumption.

KS approximates the state variable Γ, which includes a capital distribution function by a finite vector of capital moments. They then show that the mean capital alone is sufficient for the approximation. We follow their approach and denote the approximate policy function for consumption by $c(k, h, z, K)$. We also approximate the transition function T by a linear mapping of $\log K$. Following Maliar, Maliar and Valli (2010), we show that both the slope of the function and the constants can vary across z:[8]

$$\log K' = a_z + b_z \log K_z + \epsilon, \quad z \in \{0, 1\}. \tag{2.8}$$

Simulations show that as in KS the linear transition function on the first moment provides a sufficiently accurate forecast for the future aggregate capital.

[7] H depends on the kind of policy. $H = \int h_i d\Gamma - (u_0 - u_z)$ in the government employment policy and $H = \int h_i d\Gamma$ in the employment incentives policy.

[8] This method is different from Mukoyama and Şahin (2006). They specify that the slope of the function is common, but the constants can vary across z.

2.2.2 Calibration

We assume that the unemployment rate follows an exogenous regime-switching process of labor policy. The policy regime determines the unemployment rate on a one-to-one basis. Thus, the unemployment rate can take only two values. The difference in the two unemployment rates corresponds to the effect of the labor policy. In this study, we set the Jobs and Growth Tax Relief Reconciliation Act (JGTRRA) in 2003 as our calibration target policy. The Economic Growth and Tax Relief Reconciliation Act (EGTRRA) in 2001 and JGTRRA are collectively called the Bush tax cuts. The JGTRRA is a policy that consists of tax reductions in both labor and capital incomes, and it has been successful in reducing unemployment and increasing the level of consumption (House and Shapiro (2006)).[9]

We set the mean interval of policy changes as two years, considering that the U.S. general elections are held at that interval, and that it took two years after EGTRRA to implement JGTRRA, which was intended to accelerate the EGTRRA tax cuts. The average two-year interval (or equivalently, eight quarters) pins down the symmetric transition matrix for policy regime z.[10] The unemployment rates in the different policy regimes, u_0 and u_1, are set so that the impact of the exogenous policy shock is comparable with that of JGTRRA. House and Shapiro (2006) argue that both the production and employment levels recovered sharply in response to JGTRRA, and they estimate that the tax cuts raised the employment rate above the trend by about 1.25%. We calibrate the unemployment rate in the passive policy regime u_0 at 6%,

[9] The American Recovery and Reinvestment Act of 2009 (ARRA) by the Obama administration could also be a calibration target for our research objective. However, implementing this calibration is difficult at this time, because its estimated employment effects are still under review.

[10] Denoting the transition probability from z to z' by $\pi_{zz'}$, the average duration is written as $\sum_{k=1}^{\infty} k\pi_{zz'}^{k-1}(1-\pi_{zz'})$. The average duration of each regime in the benchmark calibration is eight quarters. Therefore, the regime-switching probability is $\pi_{zz'} = 7/8 (= 0.875)$. Hence we obtain:

$$\pi = \begin{bmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{bmatrix} = \begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix}.$$

which matches the unemployment rate before mid-2003, according to the Labor Force Statistics from the Current Population Survey.[11] Thus, the unemployment rate in the active policy regime is set as $u_1 = 1 - (1 - 0.06) \times 1.0125 \simeq 0.0483$.

The transition matrix Π must satisfy

$$u_z(\pi_{00zz'}/\pi_{zz'}) + (1 - u_z)(\pi_{10zz'}/\pi_{zz'}) = u_{z'}, \quad z, z' \in \{0, 1\} \tag{2.9}$$

to be compatible with the exogenous aggregate labor employed by the government or firms, $1 - u_z$. Π is also restricted by the mean duration of unemployment for each state, which we calibrate as 2.5 quarters for state 0 and 1.5 quarters for state 1 following KS. This calibration is compatible with the average duration of unemployment reported by the Current Population Survey from 1995 to 2010.[12] We divide the sample years according to whether the duration exceeded or fell short of the total average. The averages of the sub-sample are 22.7 and 15.4 weeks, respectively, whereas the total average is 17.8 weeks. These values are comparable to the KS calibration. Other authors provide different calibrations for the duration of unemployment; for example, İmrohoroğlu (1989) assumes 14 and 10 weeks for states 0 and 1, respectively. However, Del Negro (2005) argues that the implication for aggregate unemployment is almost independent of the calibrated values as long as the assumed unemployment duration is not too different from that previously assumed in the literature. In this chapter, we therefore choose to follow the KS calibration. We also follow the KS calibrations, $\pi_{0001} = 0.75\pi_{0011}$ and $\pi_{0010} = 1.25\pi_{0011}$. This implies that the job-finding rate when the policy switches from 0 to 1 overshoots the rate when the policy stays active in state 1, while it drops when the policy switches back to a passive state. These

[11] http://data.bls.gov/timeseries/LNS14000000
[12] http://research.stlouisfed.org/fred2/series/UEMPMEAN/

Description	Symbol	Value
Capital share	α	0.36
Discount factor	β	0.99
Depreciation rate	δ	0.025
Risk aversion	σ	1
Debt limit	ϕ	3
Unemployment rate in the passive regime	u_0	6%
Unemployment rate in the active regime	u_1	4.83%

Table 2.1: Parameter values

restrictions fully determine Π:

$$\Pi = \begin{bmatrix} 0.5250 & 0.3500 & 0.0313 & 0.0938 \\ 0.0223 & 0.8527 & 0.0044 & 0.1206 \\ 0.0938 & 0.0313 & 0.2917 & 0.5833 \\ 0.0031 & 0.1219 & 0.0296 & 0.8454 \end{bmatrix}. \tag{2.10}$$

The debt limit ϕ is set at 3, which is roughly equal to three months' average income. This value is chosen so that the gap between the consumption growth rates of the low and high asset holders roughly matches Zeldes' estimate (Zeldes (1989); Nirei (2006)). The other parameters are set at $\alpha = 0.36$, $\beta = 0.99$, and $\delta = 0.025$ to match the quarterly U.S. statistics on the share of capital in production, the rate of depreciation, and the steady-state annual real interest rate (KS and Hansen (1985)). The risk-aversion parameter is set at $\sigma = 1$ and put to a robustness check in Appendix C.1. Table 2.1 summarizes the parameter values.

2.3 Results

2.3.1 Government employment with balanced budget

Government employment as a pure transfer policy

In this section, we numerically compute the equilibrium defined in the previous section. The model represents an economy with government employment financed by a contemporaneous lump-sum tax, Equation (2.7), leaving the government budget balanced in every period. The government provides both the unemployment insurance and the additional employment in state 1, whereas it only provides the unemployment insurance in state 0. The government employment program functions as a pure transfer, levying a lump-sum tax on the employed workers and distributing the proceeds to a fraction $u_0 - u_1$ of the randomly selected unemployed workers. Following the microeconometric literature on active labor market policies, we call the selected unemployed as the treatment group and the other unemployed who are not selected by the government as the control group. Since the government employment is non-productive, the aggregate production is not affected by this policy, unless the capital level changes.

The household policy functions and the exogenous state transition Π constitute our generating process for household data. We generate a simulated path of an economy with $N = 10,000$ households for 3,000 periods. The first 1,000 periods are discarded when computing the time-average of the aggregate variables. The standard errors of the time-average aggregates are computed from 50 simulated paths.

Simulated aggregate consumption paths

Table 2.2 shows the simulation results of the time-averaged aggregate consumption C_z^h for different employment statuses, $h \in \{e, u\}$, and policy regimes, $z \in \{0, 1\}$. C_z is the time-averaged aggregate consumption during policy regime z. The column GE

z	GE I			GE II		
	C_z^e	C_z^u	C_z	C_z^e	C_z^u	C_z
0	2.5974	2.4682	2.5896	2.5699	2.3533	2.5569
	(0.0001)	(0.0012)	(0.0001)	(0.0005)	(0.0065)	(0.0008)
1	2.5942	2.5188	2.5905	2.5722	2.4494	2.5662
	(0.0001)	(0.0008)	(0.0001)	(0.0006)	(0.0042)	(0.0007)
log diff.	-0.0012	0.0199	0.0004	0.0009	0.0400	0.0037
	(0.0000)	(0.0005)	(0.0000)	(0.0002)	(0.0017)	(0.0002)

Table 2.2: Simulated average consumption for workers in different employment statuses, ($h \in \{e, u\}$) and policy regimes, ($z \in \{0, 1\}$). GE I is the case of transfers with a balanced budget, while GE II is the case of transfers with a constant tax.

I in the table corresponds to the current benchmark model specification, where "GE" stands for government employment. We observe that when the policy regime is active ($z = 1$), the aggregate consumption level is higher ($C_1 > C_0$), the consumption level of the employed is lower ($C_1^e < C_0^e$), and the consumption level of the unemployed is higher ($C_1^u > C_0^u$) than when the policy regime is passive ($z = 0$).

The results show that the aggregate consumption increases slightly under an active labor market policy. This conforms to the standard intuition associated with a general equilibrium model with incomplete markets. If there are complete markets for individual unemployment risks, a pure transfer from the employed to the unemployed has no impact on aggregate consumption, because the consumption responses of the employed and unemployed get negated.When the unemployment risk is uninsurable, as in our model, the increased consumption by the unemployed may overwhelm the decreased consumption by the employed. This is because the precautionary motives of savings affect the low-wealth group more than the high-wealth group, whereas the low-wealth group has a greater fraction of unemployed workers than the high-wealth group. The results of our baseline simulation above show the effect of this pure transfer.

Using the simulated average consumption for each group, we can determine the positive treatment effect, which is calculated by the difference between the consumption change of the treatment group and that of the control group: $\log(2.5942/2.4682) - \log(2.5188/2.4682) = 0.0295$. Since the treatment group constitutes 1.25% of the labor force, the aggregated treatment effects amount to a 0.037% increase in aggregate consumption. Although the magnitude roughly matches with that of the slight increase in aggregate consumption in our simulation (0.04%), this can be a mere coincidence. To accurately understand where the impact on the aggregate consumption comes from, we need to analyze the consumption responses of the other households, which we further explain.

Precautionary savings

Figure 2.1 shows the policy function, $c(k, h, z, K)$, for the idiosyncratic states, $h \in \{u, e\}$, and the aggregate states, $z \in \{0, 1\}$, while the aggregate capital is fixed at a simulated time-average level, $\bar{K}$. As can be seen from Figure 2.1, household consumption nonlinearly depends on the household wealth level, k, especially in the domain of low-wealth. The concave consumption function is analytically shown under a borrowing constraint by Carroll and Kimball (1996). The observed concavity is interpreted as the precautionary saving motive of households. The households consume less and save more when their wealth levels are insufficient to insure against future unemployment risks. In Appendix C.1, we confirm this interpretation of the concave consumption function by a sensitivity analysis on risk aversion. In addition, we also find that the upward shift of the consumption function caused by active policy is most prominent for the low-wealth unemployed group.[13] This indicates that an active policy decreases the precautionary savings of the unemployed: the government employment program shortens the expected unemployment duration, leading the unemployed to save less and consume more in the current period.

[13] The consumption of the extremely low-wealth group is rather insensitive because at this level households are constrained by a debt limit and cannot increase their consumption above the level that is financially supported by unemployment insurance.

	Log diff	K effect	Risk effect		
			$(1-u_0)\log c_1^e/c_0^e$	$u_1 \log c_1^u/c_0^u$	$(u_0-u_1)\log c_1^e/c_0^u$
GE I	0.0004	0.0000	-0.0005	0.0002	0.0005
	(0.0000)	(0.0000)	(0.0000)	(0.0000)	(0.0000)
GE II	0.0037	0.0012	0.0009	0.0002	0.0006
	(0.0002)	(0.0002)	(0.0000)	(0.0000)	(0.0000)

Table 2.3: Decomposition of aggregate consumption growth

The decrease in the precautionary savings of the low-wealth unemployed leads to a decline in the aggregate capital level, K. The decline of K increases the factor prices, and thus affects the household incomes. Hence, the simulated consumption responses consist of the effects of transfers across households and varied K level. Because we are interested in the consumption response in a reduced-risk environment, we isolate the effect of the shift in K. To do so, we regress a simulated time series, C_z on K_z for each regime z and interpolate $\hat{C}_z$ at the time-averaged aggregate capital level, $\bar{K}$. The column labeled "K effect" in Table 2.3 shows the difference between $\log C_1/C_0$ and $\log \hat{C}_1/\hat{C}_0$. We find that the K effect is almost zero. This is due to the fact that the movement of aggregate capital is quantitatively small in our GE I experiment. The log difference subtracted by the K effect; that is, $\log(\hat{C}_1/\hat{C}_0)$, gauges the shift in aggregate consumption caused by a transfer policy where K is kept constant at $\bar{K}$.

Decomposition of the risk effect

To understand the remaining increase in aggregate consumption in the active regime of government employment, we analyze the consumption of three worker groups: the program participants, the employed nonparticipants, and the unemployed nonparticipants. When the policy switches from a passive to active regime, there are five movements in the employment status (i) employed to employed, (ii) unemployed to unemployed, (iii) unemployed to employed by the government, (iv) unemployed to employed by firms, and (v) employed to unemployed. The combined effect of one worker in (iv) and another in (v) is similar to that of (i) and (ii). Given that the inflow and outflow of the unemployment pool is always balanced in this model, the effect of all workers in (iv) and (v) is proportional to that of (i) and (ii), while the workers in (iv) and (v) comprise only about 4% of those in (i) and (ii). Thus, we present the cases (i) to (iii) in Table 2.3.

We compute a consumption change by the transfer policy for each group based on the shift of policy functions in Figure 2.1. We do not use the simulated statistics reported in Table 2.2 because the simulated consumption is affected by shifts in K. We first evaluate the policy function at $(h, z, \bar{K})$ and $(h', z', \bar{K})$ at the time-averaged aggregate capital $\bar{K}$, and then take a log-difference $\log c_{z'}^{h'}/c_z^h$, where c_z^h denotes $c(\bar{k}_z^h, h, z, \bar{K})$ and $\bar{k}_z^h$ is the simulated average capital value in state $(h, z, \bar{K})$. The computed log-difference measure reflects the consumption response independent of the shift in K. The columns labeled under "Risk effect" in Table 2.3 show the consumption increase of each group in aggregate measured by the log-difference, $\log c_1^{h'}/c_0^h$, weighted by the fraction[14] of each group associated with movements (i), (ii), and (iii).

First, we consider a change in the behavior of the program participants in (iii). The program participants are the workers whose employment status changes from unemployed to employed by the introduced government program; that is, the treatment group. We observe in the log-difference measure that their consumption level increases by 0.05% because their present and expected future incomes increase.

Second, we consider the employed nonparticipants whose employment status (i) is unchanged under both regimes. The log-difference measure shows that their consumption level decreases by 0.05% with the regime switch. The behavior of this group of households is affected by the active policy in two ways. First, their tax burden increases. The cost of the passive policy (unemployment insurance) is reduced, but this reduction is outweighed by the increase in the cost of the active policy (govern-

[14] The fractions of the groups are $1 - u_0 = 94\%$ for the employed nonparticipants, $u_1 = 4.83\%$ for the unemployed nonparticipants, and $u_0 - u_1 = 1.17\%$ for the program participants, respectively.

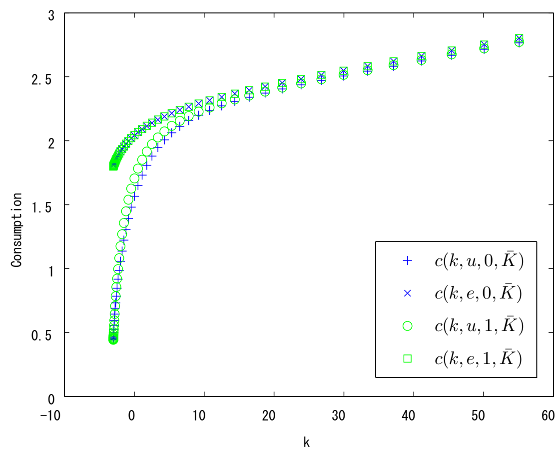

Figure 2.1: The approximated policy function for consumption. Given the average aggregate capital, K, the policy function of the unemployed in state $z_t = 0$ is shown by the + line, that of the employed in state $z_t = 0$ by the $\times$ line, that of the unemployed in state $z_t = 1$ by the circle line, and that of the employed in state $z_t = 1$ by the square line.

ment employment). Second, their future expected labor income increases because the unemployment duration is reduced by the active policy. The negative response of the simulated consumption implies that the negative tax effect outweighs the expected positive income effect.

Third, we consider the unemployed nonparticipants whose employment status (ii) is unchanged under both regimes, which we called the control group. Similar to the employed nonparticipants, there are no direct concurrent benefits to them from the additional employment program. Nevertheless, the regime switch increases the expected job finding rate and hence increases the expected labor income. So even though there is no income increase in the current period, the active policy increases the consumption of this group of households. This positive effect is confirmed by our simulation, which shows that their consumption level increases by 0.02%.

Our analysis of Table 2.3 confirms our previous analysis of the simulated data. Table 2.3 shows that the fall in consumption of the first group is roughly canceled out by the increase in consumption of the third group. This is natural because the active policy functions as a transfer of wealth from the first group to the third group. This corresponds to the direct effect of a pure wealth transfer. The net increase in total consumption is explained by the consumption increase of the second group. The second group is not involved in the transfer because it does not receive the transfer and is not taxed under the new policy. The second group consumes more because it now faces a reduced unemployment risk and begins to dissave its precautionary wealth.

In total, "Log diff" in Table 2.3 summarizes the general equilibrium effect of the transfer policy. We observe a positive but limited impact on aggregate consumption. Log diff can be decomposed into a K and Risk effect, and the latter effect can be decomposed into the consumption responses of three groups. By this decomposition, we find that the control group that does not directly benefit by the policy plays an important role in the increase in aggregate consumption; the positive treatment effect is offset by the decrease in consumption of the employed nonparticipants. The unemployed nonparticipants increase their consumption despite the fact that their present income does not increase, because they perceive a reduction of future unemployment risks and dissave their precautionary wealth.

2.3.2 Government employment financed by a constant tax over time

In the previous section, an active transfer policy should encourage the consumption of not only the program participants, but also the nonparticipants by reducing the risk of unemployment and thereby increasing the expected discounted income. However, we could not directly observe how the employed nonparticipants benefit from a reduced unemployment risk in the previous model, because the tax burden on the employed group increases during the period of active policy. This implies that we should observe the positive consumption response of the employed nonparticipants if the policy is financed by a tax that is constant over time across regimes.

This notion motivates our second model specification in which the transfer is financed by a constant tax and the government budget is allowed to have temporal imbalances. To finance a temporary transfer policy through constant taxation, we assume that the government has access to an international insurance market, which only requires the government budget to be balanced on average. In the international insurance market, our propopsed government agrees to pay out the tax revenue it collects in every period, while it receives the necessary funds for the transfer policy when the policy randomly switches to an active regime. Specifically, the government swaps a stochastic transfer payment sequence, $\{\varepsilon_t\}$, for a fixed insurance cost sequence, $\{T\}$, such that $E(\varepsilon_t) = T$. The international insurance market is completely hedged by the law of large numbers that applies to the many participating governments. Admittedly, this specification has undesirable features; for example, the moral hazard problem of the government is assumed away through the exogenous regime-switching process. However, at the cost of incorporating the insurance contract, we can isolate the response of the employed to a reduced unemployment risk, which is not feasible in the benchmark model.

The simulation results are reported under "GE II." Table 2.2 shows that both the employed and unemployed workers increase their consumption level when the policy switches to an active regime. "Log diff" in Table 2.3 shows that the policy switch results in a 0.37% increase in aggregate consumption. A decomposition of Table 2.3 shows that the employed workers significantly increase their consumption by 0.09%, accounting for 52.9% of the consumption increase in response to a lower unemployment risk. Since a policy switch does not affect the tax paid by workers in each period and K is set to be a constant, an increase in the expected lifetime income largely stems from the prospect of less unemployment risk. Therefore, a significant rise in the consumption level of the employed workers validates our argument that a reduced unemployment risk enhances the consumption demand of not only the unemployed but also the employed workers.

2.3.3 An alternative policy experiment: corporate tax reduction

In the previous section, we showed that an aggregate consumption level responds to a considerable change in employment risk for both the unemployed and employed nonparticipants. In this section, we consider employment incentives as an alternative active labor market policy. In particular, we consider a regime-switching corporate tax rate, as in Davig (2004). By this policy, the government imposes a lower corporate tax on firms to induce a larger labor demand. Therefore, the program participants of this employment incentive policy are employed by private firms rather than by the government, as was the case in the previous model. Since the newly generated employment is productive, output varies endogenously as the policy regime switches.

We consider a case in which the government levies a flat-rate tax on the revenue of firms. The corporate tax rate, ξ_z, fluctuates between two states according to the Markov process specified by Π. In addition, we also assume an exogenous aggregate employment process that fluctuates between two states, u_0 and u_1, along with the policy status, $z \in \{0, 1\}$. The mechanism underlying the employment incentives policy is that labor demand shifts out and employment increases when the tax rate is low. To implement such a mechanism in a simple model, we assume a particular kind of real wage rigidity: the after-tax real wage is held constant by an exogenously imposed norm in the labor market. As the tax rate changes, the employment level also changes so that the marginal product of labor is equal to the fixed after-tax real wage. We calibrate the tax rates such that the implied unemployment rates are equal to u_0 and u_1, as follows.

We set the constant after-tax real wage equal to the full-employment marginal product level $w = (1 - \alpha)K^{\alpha}$. In each period, the production factors are paid for by their after-tax marginal products: $r = (1-\xi_z)\alpha(K/(1-u_z))^{\alpha-1}$ and $w = (1-\xi_z)(1-\alpha)(K/(1-u_z))^{\alpha}$. Then, we obtain the corporate tax rates that are consistent with our calibrated unemployment rates:

$$\xi_z = 1 - (1-u_z)^{\alpha}, \quad z = 0, 1. \tag{2.11}$$

When $z_t = 0$, the tax is high at ξ_0 and the unemployment level is high at u_0. When $z_t = 1$, the tax is low at ξ_1 and the unemployment level is low at u_1. This specification can be used to interpret the numerical results, because we can eliminate the impacts of any after-tax wage fluctuations on the expected lifetime income, which directly reflects the changes in the magnitude of the unemployment risk.

Let us now consider two cases of employment incentives. In the first case, which we call "Tax I," the tax proceeds are rebated to the households in a lump-sum manner. By abuse of notation, we redefine $-\tau_t$ as the lump-sum transfer. Then, $-\tau_t = \xi_z Y_t$. From this notation, the household's budget constraint can continue to be written as Equation (2.2). In the second case ("Tax II"), the tax proceeds are used by the government for non-productive activities (that is, "thrown into the ocean"). Here, the transfer, τ_t, is zero for every t and government expenditure, G_t, is equal to the tax proceeds, $\xi_t Y_t$. Government expenditure appears on the demand side of the goods-market clearing condition; that is, $C + K' - (1-\delta)K + G = Y$. The Tax II specification serves a similar purpose as GE II. By holding the household income constant across regimes, this specification is useful for isolating the effects of a reduced unemployment risk.

Table 2.4 shows the consumption for various states. Note that consumption increases in the periods of low tax for both the employed and unemployed workers in Tax I as well as Tax II. Table 2.5 shows the decomposition of the total consumption growth in terms of the contribution of the worker groups according to their employ-

z	Tax I C_z^e	C_z^u	C_z	Tax II C_z^e	C_z^u	C_z
0	2.6010	2.4552	2.5923	2.5305	2.3876	2.5220
	(0.0008)	(0.0023)	(0.0008)	(0.0048)	(0.0013)	(0.0015)
1	2.6021	2.5161	2.5980	2.5353	2.4512	2.5312
	(0.0009)	(0.0017)	(0.0009)	(0.0014)	(0.0034)	(0.0015)
log diff.	0.0004	0.0245	0.0022	0.0019	0.0263	0.0037
	(0.0002)	(0.0008)	(0.0002)	(0.0003)	(0.0010)	(0.0037)

Table 2.4: Consumption changes in policy transitions for the average workers in different groups. Tax I is the case of a corporate tax with lump-sum rebates and Tax II is the case of a corporate tax and wasteful government spending.

	Log diff	K effect	Risk effect $(1-u_0)\log c_1^e/c_0^e$	$u_1 \log c_1^u/c_0^u$	$(u_0-u_1)\log c_1^e/c_0^u$
Tax I	0.0022	0.0012	0.0001	0.0002	0.0005
	(0.0002)	(0.0002)	(0.0000)	(0.0000)	(0.0000)
Tax II	0.0037	0.0018	0.0009	0.0002	0.0005
	(0.0002)	(0.0003)	(0.0000)	(0.0000)	(0.0000)

Table 2.5: Decomposition of aggregate consumption growth

ment status. The first group (employed to employed) accounts for 13% and the third group (unemployed to employed) accounts for 63% of the consumption variation in response to less unemployment risk.

In Tax I, the tax proceeds are rebated back to the households and the tax is therefore a distortionary transfer from firms to households. The lower tax rate induces a higher labor demand and larger output. Given the real wage rigidity, the lump sum transfer to the households is reduced during the low-tax active policy periods. The reduced transfer income negatively affects the consumption demand of the unemployed. Nonetheless, the unemployed group positively contributes to the increase in consumption by 0.02% through the tax reduction, as shown in Table 2.5. This implies that the wealth effect of a lower unemployment risk overwhelms the effect of a reduced transfer income.

The wealth effect can be more directly observed in Tax II. Here, both the real wage and government transfers (zero) are fixed during the policy transitions. Hence, the contemporaneous income of the employed workers is not affected by the policy at all. Therefore, the consumption increase is due to a policy switch for the employed (0.09%) indicates a pure effect of the reduced unemployment risk. This effect is larger than that in Tax I (0.01%). While a tax cut is always accompanied by a reduced rebate in Tax I, there is no rebate in Tax II. Therefore, we expect a larger impact of a policy switch in Tax II, and the numerical result confirms our belief.

2.3.4 Robustness check

In this section, we check the robustness of our outcomes by conducting three types of sensitivity analysis in terms of the risk aversion, debt limits, and endogenous labor supply. In all of these dimensions, we find our computation results to be robust.

Risk aversion First, we change the risk-aversion parameter σ from 1 to 2 and 5 for GE I. We find an increase in the mean capital level as the risk aversion rises, which is consistent with the theoretical prediction that risk aversion implies more precautionary savings and a lower consumption demand. Since a higher level of capital contributes to a positive income effect, the aggregate consumption response toward various risk aversions depends on the relative strength of these two opposing forces: a lower consumption demand and a positive income effect. In addition, we confirm a stronger nonlinearity in the consumption function as the households become more risk-averse. The results are shown in Appendix C.1.

Debt limits In the second sensitivity analysis, we change the level of a debt limit. In the benchmark case, ϕ is set at three months' worth of wage income; that is, $\phi = 3$. We change this to $\phi = 0$; that is, no debt limit at all. The results are shown in Appendix C.2. We note that the aggregate consumption level decreases as the debt limit is relaxed. When the borrowing constraint is relaxed, the households save less owing to diminished precautionary motives, and therefore the aggregate capital level decreases. This leads to a decrease in the production level and hence to further decreases in the aggregate consumption level.

In every simulation, we find no agents who are bound by debt limits. This does not imply that the borrowing constraint has no effect on household behavior. Since the households are highly concerned with the possibility of a binding debt limit and zero consumption, they begin to severely reduce their consumption level when their wealth is well above the debt limit. Thus, the effect of a debt limit manifests itself in the form of nonlinear consumption functions rather than constrained agents.

Endogenous labor supply In the third sensitivity analysis, we generalize the preference specification to incorporate the utility from leisure. The utility function is generalized, as shown in Appendix C.3, where the Frisch elasticity varies with the new parameter ψ. The benchmark specification correspond to the case where $\psi = 0$. If the labor supply is exogenous, the inclusion of the disutility of labor does not change the equilibrium outcome under the log utility setup where $\sigma = 1$ as in the benchmark models. Thus, we focus on the case of an endogenous labor supply, where households choose the hours that they work when they are employed. The simulation results when $\psi = 0.1$ show that the contribution of leisure lowers the consumption level, because the precautionary motive is weakened by increased leisure when people are unemployed. However, the qualitative pattern of the consumption response to the regime switch is unchanged from the benchmark model.

2.4 Conclusion

This study quantitatively examines a dynamic stochastic general equilibrium model with idiosyncratic employment and aggregate risk. We consider two kinds of labor demand policies and find the general equilibrium effects of these policies on aggregate consumption demand as labor market policy switches stochastically between the two regimes. The direct job creation by the government employment model provides a simple case that facilitates the interpretation of the basic mechanisms and numerical results, whereas the model with employment incentives because of a corporate tax reduction examines how an active labor market policy directly affects production activities in the private sector.

We decompose the consumption response into three effects; the increased number of employed who are program participants, the tax effect on the employed, and the unemployment risk effect on all households. This decomposition shows that the effect of the reduced unemployment risks of the employed nonparticipants is quite large, provided the tax burden of the employed is kept constant across regimes. As a result, the effect of the reduced unemployment risks on the overall consumption demand can be large because it affects not only the unemployed but also a wide range of employed households. This unemployment risk effect, which we identify in this study, is a new general equilibrium effect of active labor market policies. Our result contrasts with the effect of a windfall income, which has been extensively studied in the literature on precautionary savings. The impact of a windfall income on aggregate consumption may be limited, because it affects only a small fraction of workers whose asset holdings are close to the debt limit.

Our numerical simulations show that the general equilibrium effect of a pure transfer in an active labor market policy on realized aggregate consumption is positive, but small. In an experiment in which the government finances the transfer policy with a constant level of taxation, we observe a positive consumption response by the employed nonparticipants to the reduced risks and a large effect on aggregate consumption. A quantitatively similar impact of such policy is observed in our experiment using a reduced corporate tax rate. The tax cut results in higher employment in the production sector and a lower unemployment risk for the workers. The workers respond to this lower risk by reducing their precautionary savings and shifting their consumption demand upwards. As the increased consumption demand is met by an increased output by firms, the equilibrium aggregate consumption increases. By these four experiments, we find that active labor market policies can lead to a quantitatively large increase in the aggregate consumption demand, which can further lead to an increase in the aggregate consumption level in an environment where the supply of goods elastically conforms to the increase in consumption demand.

Chapter 3

Estimation method for dynamic equilibrium models of heterogeneous firms

3.1 Introduction

The dynamics of entries and exits by firms is widely used in theoretical literature (e.g., the general equilibrium model of Hopenhayn and Rogerson (1993); the financial markets model of Cooley and Quadrini (2001); the aggregate dynamics of Palazzo and Clementi (2010)) . Hopenhayn (1992) first studied a firm's nonlinear dynamic optimization problem. Being consistent with the empirical heterogeneity in productivity across firms, existing models usually assume idiosyncratic productivity shocks, which typically follow an AR(1) process. It is important to estimate the parameters specifying this stochastic process of productivity for two main reasons. First, they determine the risk that each firm faces and the resource reallocation outcome, which may lead to a general equilibrium outcome (Gourio (2008)). Second, counter-factual simulations using inappropriate calibrations result in questionable quantitative implications. For example, Hopenhayn and Rogerson (1993) and Veracierto (2001) studied the effects of firing taxes, Restuccia and Rogerson (2008) studied the effects of misallocations across firms with heterogeneous productivity, and Rossi-Hansberg and Wright (2007) studied the relation between establishment size dynamics and human capital accumulation.

Despite the importance of these parameters, there is still no consensus on their estimates.[1] There are three primary reasons for differing estimates.

First, estimators (and estimation methods) are chosen arbitrarily by researchers. In general, different estimators mean the different assumptions on the error term, resulting in the varying estimates as summarized in Table 3.1. Because statistical or econometric theory provide little guidance on the choice of estimators, we cannot choose an estimator in a statistically rigorous way. As a result, the choice of estimator is left to the discretion of researchers, who choose different estimators and, thus, report different estimates.

Second, although previous studies usually report a balanced panel estimate, we use an unbalanced panel owing to a firm's exit. In general, statistical inferences based on non-randomly truncated samples can lead to an estimation bias. In order to correct this selection bias, several methods have been proposed (e.g., Heckman (1979) for a cross-section, Wooldridge (1995) for a panel, and Kyriazidou (2001), Gayle and Viauroux (2007), and Semykina and Wooldridge (2013) for a dynamic panel sample selection problem). These frameworks first specify a reduced-form selection rule and then corrects for the truncation. This correction method functions well, but we cannot use it here because the threshold value is determined endogenously by the

[1]Parameters in the stochastic process of productivity are usually estimated using dynamic panel data. Owing to the correlation between explanatory variables and the individual fixed effect, the ordinary least squares (OLS) estimator on dynamic panel data is generally inconsistent. The standard approach is to remove the fixed effect by first-differencing (fixed-effect (within) estimator) and to apply the instrumental variables method. Anderson and Hsiao (1982) first proposed the approach (two stage least squares (2SLS) estimator), Arellano and Bond (1991) considered the more efficient generalized method of moment (GMM) estimator, and Blundell and Bond (1998) proposed the system GMM estimator to alleviate the weak instruments problem. Although the system GMM estimator is more reliable, Ziliak (1997) and Hsiao, Hashem Pesaran and Kamil Tahmiscioglu (2002) reported that it has a downward bias with small samples, and this bias becomes severe as the number of moment conditions increases.

structural model. This means we cannot observe the explanatory variables of the structural selection rule, and if we estimate the reduced-form selection rule for the relationship between endogenous variables, the estimated rule depends on a change in exogenous variables (and, thus, the correction method violates the Lucas critique). Therefore, we need to estimate the structural (deep) parameter, which is independent of a change in exogenous variables and affects the endogenous exit threshold level. Unfortunately, we cannot estimate the structural parameters using a standard panel estimation method. Thus, biased balanced panel estimates are usually reported, and we find inconsistencies across estimates.

Third, the initial condition for all cross-sectional units is usually assumed to follow the stationary distribution of the AR(1) process, though, the initial sample in Hopenhayn (1992) are actually obtained from the stationary mixture distribution composed not only of incumbents (whose productivity follows the AR(1) process), but also of entrants (whose productivity is generated from the entrant's distribution). Since the stationary distribution has no analytical solution, in general, we cannot calculate and correct the likelihood. Accordingly, the estimation errors based on the wrong assumptions of the initial condition generate inconsistencies across estimation outcomes.

Because there are many problems in using dynamic panel estimators, some studies use indirect inference (II) estimation methods[2] instead.[3] However, an empirical value of the parameter remains uncertain, as shown in Table 3.2. This is because a set of moments (cross-sectional) may not provide good summary statistics when the stationary distribution is a mixture distribution. Generally, the p-th moment about zero of a mixture distribution is a weighted average of the p-th moment of each component, and we cannot identify a bundle of parameter estimates with only a finite set of moments of the mixture distribution itself. Therefore, parameter estimates that minimize the distance between simulated moments and data moments can be biased, and may lead to inconsistent estimates. In addition, the property of the estimates based on the method of moments is generally not suitable with small samples and is not robust to higher order distributional features.[4]

[2] II-type methods are simulation-based estimation methods that choose parameters by matching the properties of simulated values to observed values. II-type methods are commonly used, especially when the likelihood function is intractable or difficult to compute. One of the principal benefits of the simulation-based procedure is that we can do a calibration and a statistical test simultaneously. Creel and Kristensen (2013) discussed that the standard II estimator takes the form of a continuous-updating (CU) GMM estimator, which minimizes a GMM-type criterion function for some set of moment conditions. While available with various summary statistics, all methods apply the same principles. When we choose a set of sample moments as summary statistics, it is called a simulated method of moments (SMM or MSM) estimator (McFadden (1989), Pakes and Pollard (1989), Duffie and Singleton (1993), Lee and Ingram (1991)). When choosing the binding function that maps the auxiliary parameter vector to the structural one, it is called an II estimator (Smith (1993); Gouriéroux, Monfort and Renault (1993); Gouriéroux and Monfort (1997)). When choosing the score vector of an ancillary model, it is called an efficient method of moments (EMM) estimator (Gallant and Tauchen (1996)). This literature usually estimate the structural parameters using a SMM estimator. These three estimators are closely related and are asymptotically equivalent (Fackler and Tastan (2008)).

[3] In related literature, rather than estimate, Caballero and Engel (1999) and Bloom (2009) calibrated a geometric random walk process such that the computed distribution follows Gibrat's law. With regard to Gibrat's law, Axtell (2001) reported a range of estimated power law exponents between 0.994 and 1.098, which were less than 2. The power law exponent α is also called the tail index, tail exponent, shape parameter, or characteristic exponent. The power law exponent determines where the tails of the distribution taper off and, therefore, the degree of leptokurtosis. In general, the p-th moment exists only up to the tail exponent $p \leq \alpha$. (e.g., see Farmer (1999) and Haas and Pigorsch (2009))

[4] My research focuses on panels where a large number of firms are observed over a small number of periods. The shortness of the periods makes it difficult to estimate the parameters controlling the dynamic property of the stochastic process, especially when observations are highly correlated. Additionally, GMM estimators are generally not appropriate for small samples, although they are most robust when the specification is correct with large samples (in most cases, CU-GMM estimators

In this study, I propose an algorithm for estimating the structural parameters of the nonlinear dynamic optimization model. By employing the stationary distribution as the summary statistics of the likelihood-free approximate Bayesian computation (ABC) inference, I successfully estimate the structural parameters. There are two primary reasons why I use ABC algorithm. First, although the stationary equilibrium distribution is not analytically tractable, it is relatively easy to simulate. Second, I want to reflect the higher-order features of the distribution in the parameter inference, not just a mode, as is the case in the indirect likelihood inference (Creel and Kristensen (2013)) or the nonparametric simulated maximum likelihood (NPSML) estimation (e.g., see Fermanian and Salani (2004), Kristensen and Shin (2012)), or a set of moments, as several CU-GMM estimators use.

The ABC was first introduced by researchers involved in population genetics (Tavaré, Balding, Griffiths and Donnelly (1997); Pritchard, Seielstad, Perez-Lezaun and Feldman (1999); Beaumont, Zhang and Balding (2002)), and has become widespread in many research areas (e.g., Sisson and Fan (2011), Marin, Pudlo, Robert and Ryder (2012)). In the ABC, the calculation of likelihood is replaced by a comparison process between observations (x_{obs}) and simulated values (x_{sim}), as in other simulation-based estimation methods. For high dimensional data spaces, we rarely match x_{obs} and x_{sim}, and usually introduce summary statistics to compress the data. The choice of summary statistics is one of the most important aspects of a statistical analysis because it has a substantial effect on the estimation accuracy. Although many approximation methods have been proposed to generate low-dimensional and highly informative summary statistics for targeted parameters, there is no consensus on the best method.[5] In this study, I propose employing the

are more efficient than the usual GMM, two-step GMM, and iterative GMM estimators for small samples (e.g., Tauchen (1986); Hansen, Heaton and Yaron (1996); Chistiano and Den Haan (1996))).

[5]For example, Nunes and Balding (2010) proposed a minimum entropy method, Blum and François (2010) proposed a nonlinear regression, and Fearnhead and Prangle (2012) proposed a semi-automatic computation. Blum, Nunes, Prangle and Sisson (2013) reviewed this field of research.

equilibrium objects of the structural model as summary statistics, and check the finite sample property of the estimator using Monte Carlo experiments. Specifically, I use the stationary equilibrium distribution as the summary statistics for the ABC inference.

The remainder of this chapter is organized as follows. Section 2 reviews the theoretical model and its solution algorithm. Section 3 presents the simulation-based estimation algorithm and Section 4 presents the Monte Carlo results. Lastly, Section 5 concludes the chapter.

3.2 Model

In this section, I briefly review Hopenhayn's (1992) model of firm dynamics and its solution algorithm. In this model, industry is composed of firms that produce a homogeneous good. Each firm is a price taker with respect to the price of the good and a labor wage. The firms face idiosyncratic productivity shocks which follow a Markov chain with a finite bound $[0, 1]$. The profit function is given by $f(n, \phi)$, where n represents labor and ϕ denotes productivity, following $F(\phi'|\phi)$. Incumbents must pay a fixed management cost c_f to survive in the market for each period. This cost determines a reservation level of productivity; each firm faces a dynamic real optional decision on whether to exit in each period. In this chapter, I specify the profit function as follows: $\pi(\phi, p, w) = p\phi f(n) - wn - c_f$, where p is an exogenous price, $f(n)$ is a production function, and w is an exogenous labor wage. Then, the Bellman equation is expressed as:

$$v(\phi, p, w) = \pi(\phi, p, w) + \beta \max \left\{ 0, \int_0^1 v(\phi', p, w) \mathrm{dF}(\phi'|\phi) \right\},$$

where v is a value function and β is a discount factor. The solution of the dynamic programming problem determines the cutoff productivity level endogenously, as follows:

$$x = \inf \left\{ \phi \in [0,1] : \int_{\phi' \in [0,1]} v(\phi', p, w) \mathrm{dF}(\phi'|\phi) \geq 0 \right\}.$$

Certainly, we can observe a reduced-form selection given by:

$$\begin{aligned} &\phi' > 0 \text{ when } \phi \geq x \\ &\phi' = 0 \text{ o/w}, \end{aligned}$$

although it is not important to estimate the threshold parameter x because it is determined endogenously, and is not invariant to a change in the structural parameter, namely the fixed management cost c_f. That is, it violates the Lucas critique, and we need to estimate c_f instead. Potential entrants draw productivity ϕ from the initial density ν, whose distribution function is defined as G, and enter the market until the expected entry values are zero. The expected entry values are given by:

$$v_e(p, w) = \int_0^1 v(\phi, p, w) \nu(d\phi).$$

The law of motion of the cross-sectional distribution of productivity is given by the mixture distribution composed of the incumbents' distribution with truncation and the entrants' distribution:

$$\mu' = \int_{\phi \geq x} F(\phi'|\phi) \mu(d\phi) + M' G(\phi'),$$

where M is the mass of entrants.

Under some technical assumptions given by Hopenhayn (1992), there exists a stationary competitive industry equilibrium that consists of a vector $(p^*, w^*, Q^*, n^*, M^*, x^*)$ and we can define the stationary equilibrium distribution

μ^* as follows:

$$\mu^* = \int_{\phi \geq x^*} F(\phi'|\phi)\mu^* + M^* G(\phi'),$$

where Q denotes the aggregate demand, and is equal to the aggregate supply Q^s:

$$Q^* = Q^s(\mu^*, p^*, w^*).$$

Although direct computation with discretization is widely used to solve the model, the size of the resulting simulation error is typically unknown and can affect the estimation outcome. In order to check the performance of the estimation algorithm itself, we need to reduce the estimation error stemming from the Monte Carlo simulation error. In order to do so, I apply the coupling-from-the-past (CFTP) algorithm, which enables us to compute an exact (perfect) sample from the stationary distribution (Kamihigashi and Stachurski (2015)).

3.3 Estimation

When we interpret Hopenhayn's (1992) model of firm dynamics as a data generating process, the empirical counterpart is truncated dynamic panel data, often studied as a dynamic panel Tobit model. Although several methods have been proposed to correct the truncation bias, we cannot apply them here. This is because we cannot observe the explanatory variables of the selection rule, and if we can, the estimation method violates the Lucas critique. Because a set of moments may not summarize a mixture distribution well, I propose a non-moment-based inference routine for the structural parameters based on the ABC algorithm.

3.3.1 Algorithm

I first present the motivation for the ABC methods. Standard inferences in Bayesian statistics depend on the following full posterior distribution:

$$\pi(\Theta|x_{obs}) = \frac{p(x_{obs}|\Theta)\pi(\Theta)}{p(x_{obs})},$$

where x_{obs} denotes the observed data, $\pi(\bullet)$ denotes the prior distribution, $p(x_{obs}|\Theta)$ denotes the likelihood function, and $p(x_{obs})$ denotes the marginal probability of the observations: $p(x_{obs}) = \int_{\Theta} p(x_{obs}|\Theta)\pi(\Theta)\mathrm{d}\Theta$. However, we cannot compute the likelihood and hence, its full posterior distribution. So, the inference relies not on the full posterior distribution, but on an approximation with the partial posterior distribution:

$$\begin{aligned}\pi(\Theta|\eta(x_{obs})) &= \frac{p(\eta(x_{obs})|\Theta)\pi(\Theta)}{p(\eta(x_{obs}))} \\ &\propto p\left(d(\eta(x_{obs}), \eta(x_{sim})) < \epsilon|\Theta\right)\pi(\Theta).\end{aligned}$$

The most primitive ABC is a rejection scheme that first draws a parameter guess from a prior distribution, simulates the model based on the guess, accepts or rejects it with respect to a distance criterion, updates the guess and continues until convergence. Since the proposed density is not informative of the posterior distribution, the rejection scheme is inefficient. Algorithms built on Markov Chain Monte Carlo (MCMC) or Sequential Monte Carlo (SMC) samplers help to sample parameter proposals from high density regions of the posterior distribution (e.g., Marjoram, Molitor, Plagnol and Tavaré (2003); Wegmann, Leuenberger and Excoffier (2009)). In general, an ABC based on the SMC algorithm is more efficient than an ABC based on the MCMC algorithm, because the former can sample from the posterior distribution independently. In this study, I use an ABC algorithm based on the SMC algorithm proposed by Del Moral, Doucet and Jasra (2012), where tolerance levels can be adaptively annealed. The estimation algorithm is as follows:

1. Given the initial distance criterion $\epsilon_0 = \infty$, set the corresponding initial weight $W_0^i = 1/N$ for $i = 1, \ldots, N$.

2. For $i = 1, \ldots, N$, sample a proposal from the prior distribution $\Theta_0^i \sim \pi(\bullet)$ and compute simulated values conditional on the proposal $x_{k,0}^i \sim f(\bullet|\Theta_0^i)$ for $k = 1, \ldots, M$.

$$X = \begin{pmatrix} x_{1,0}^1 & \cdots & x_{1,0}^N \\ \vdots & x_{k,0}^i & \vdots \\ x_{M,0}^1 & \cdots & x_{M,0}^N \end{pmatrix}$$

3. Set $l - 1 \to l$ and if $\epsilon_{l-1} < \epsilon_{target}$, stop; otherwise, compute ϵ_n such that $\text{ESS}(\{W_l^i\}, \epsilon_l) = \alpha_{ABC}\text{ESS}(\{W_{l-1}^i\}, \epsilon_{l-1})$, where ESS denotes effective sample size:

$$\begin{aligned} \text{ESS}(\{W_l^i\}, \epsilon_l) &= \left(\sum_{i=1}^{N} (W_l^i)^2 \right)^{-1} \\ &\propto \left(\sum_{i=1}^{N} \left(W_{l-1}^i \frac{\sum_{k=1}^{M} \mathbb{I}_{A_{\epsilon_l, x_{obs}}}(X_{k,l-1}^i)}{\sum_{k=1}^{M} \mathbb{I}_{A_{\epsilon_{l-1}, x_{obs}}}(X_{k,l-1}^i)} \right)^2 \right)^{-1}, \end{aligned}$$

which takes values between 1 and N and indicates the properness of the weight distribution, α_{ABC} is a quality index of the SMC approximation, x_{obs} denotes the true value (observation), and $A_{\epsilon, x_{obs}}$ denotes the epsilon neighborhood of the true value with respect to the distance function $d(\bullet)$ and summary statistics $\eta(\bullet)$:

$$A_{\epsilon, x_{obs}} \equiv \{z \in D : d(\eta(z), \eta(x_{obs})) < \epsilon\}.$$

The importance weight W^i_{l-1} is updated to W^i_l.

4. If $\text{ESS}(\{W^i_l\}, \epsilon_l) < N_T$, this indicates that the values of weights differ considerably, and thus we increase the alive particles by duplication following the systematic scheme proposed by Kitagawa (1996).

5. For i=1, ..., N, perturb each particle by $(\Theta^i_n, X^i_{1:M,l}) \sim \mathbb{K}_n(\Theta^i_{n-1}, X^i_{1:M,l-1})$, where $\mathbb{K}_n$ is an MCMC kernel. Specifically, I use the normal random walk Metropolis-Hastings algorithm to sample the new proposal, where the standard deviation is calculated to be equal to that of the posterior distribution.

Finally, I smooth Θ^i with $d(\eta(x^i) - \eta(x_{obs})$ using locally weighted scatterplot smoothing (LOWESS) to weaken Monte Carlo simulation error, which is not intended to correct the error due to a positive value of ϵ (Beaumont et al. (2002)[6]).

3.3.2 Summary statistics

The performance of ABC algorithm hinges on the choice of summary statistics. Although lots of approximation methods for summary statistics were proposed, there has been no consensus. In this chapter, I propose using the equilibrium objects of the structural model as summary statistics. The equilibrium objects are a set of locally unique nonlinear function of structural parameters. For Hopenhayn (1992)'s model of firm dynamics, the stationary distribution is an equilibrium object which is infinite-dimensional, it does not have a closed-form expression, and it is empirically fat-tailed.

With respect to the summary statistics of the distributions, Drovandi and Pettitt (2011) discussed that if the data set is quite large and exhibits a substantial amount of skewness and/or kurtosis, the set of octiles or the quantile-based robust measures

[6]It is not possible to use a local-linear regression adjustment because the summary statistics are infinitely-dimensional; we can compute $d(\eta(x^i) - \eta(x_{obs}))$, but we cannot compute $\eta(x^i) - \eta(x_{obs})$.

seem appropriate as summary measures. Dominicy and Veredas (2013) showed that when the density does not have a closed-form solution and/or moments do not exist, the quantile-based approach is effective. In this study, I compute the Kullback-Leibler divergence (KLd) and the L^2-distance as distance metrics to summarize the difference between distributions, instead of comparing a finite set of moments or a mode. Following the algorithm, we can compute the parametric density estimator that minimizes the density difference. In order to compute the KLd, I use a two-step naive approach: first estimating two kernel densities separately and, second, computing the KLd. In order to compute the L^2-distance, I approximate the distance directly using the least-squares density-difference (LSDD) estimation (Sugiyama et al. (2013)). Since the L^2-distance satisfies the definition of mathematical metrics (the KLd does not) and is more robust against outliers, the estimation accuracy is expected to increase.

I call this parametric density estimator, which can minimize the density difference using a simulation-based likelihood-free ABC algorithm, the minimum density difference (MDD) estimator. Because we need only a cross-sectional observation for estimation, we can estimate the dynamic structural parameters without panel data.

3.3.3 Settings

Simulation setting

In this section, I check the finite sample property of the MDD estimator using three Monte-Carlo experiments. Specifically, I compare the root mean squared error (RMSE) of the estimator with those of existing dynamic panel estimators. Because all of the experiments assume that the specification is correct, following the standard fashion of structural econometrics, I do not discuss the robustness of the estimator to a specification error. Throughout these experiments, I set the parameter values $p^* = 1$, $h(n) = 2n$, $w^* = 1$, and $n^* = 1$ in the profit function specified as $\pi(\phi, p^* = 1, w^* = 1) = 2\phi - c_f - 1$.

Suppose that we can observe unbalanced and truncated dynamic panel data. The time-series length of the panel is set to 10 (the average value in the literature) and the panel begins with $10,000$ firms, which follow a stationary distribution. When calculating the dynamic panel estimators to be compared, I estimate the parameters for balanced panel data following previous literature;[7] when computing the MDD estimator, I use the first column of the dynamic panel data as the empirical counterpart of the stationary distribution. The productivity of each firm $\phi_{i,t}$, subscripted by $i = 1, \ldots, I$ and $t = 1, \ldots, T$, is assumed to follow AR(1) with a truncation and a firm-specific time-invariant fixed effect α_i:[8]

$$
\begin{aligned}
\phi_{i,t+1} &= \rho\phi_{i,t} + u_{i,t} \quad if\ \phi_{i,t} > x^* \\
&= 0 \text{ (truncated)} \quad otherwise \\
u_{i,t} &= \alpha_i + \epsilon_{i,t}, \\
\epsilon_{i,t} &\overset{\text{i.i.d}}{\sim} \mathcal{N}(0, \sigma^2),
\end{aligned}
$$

where $\epsilon_{i,t}$ is a purely idiosyncratic disturbance with zero mean and constant finite variance σ^2. In order to conduct the Monte-Carlo experiments, I assume that fixed effect α_i independently follows a Gaussian distribution with zero mean and constant

[7]Note that this comparison is not fair because these estimators assume a balanced panel and thus, are not consistent estimators. Besides, since we cannot identify the parameters with only a finite set of moments of the stationary distribution, I do not compare the estimate with the SMM estimator.

[8]In order to reduce the computational cost, I assume that each firm cannot know its own fixed effect, for any i and t.

finite variance σ_α^2:[9]

$$\alpha_i \overset{\text{i.i.d}}{\sim} \mathcal{N}(0, \sigma_\alpha^2).$$

Finally, new entrants are assumed to draw their productivity from a uniform distribution from 0 to 1: $\nu(\phi) = \mathcal{U}(0, 1)$.

In the first experiment, I assume an environment in which we know the true fixed management cost c_f^0 (and, therefore, the true x_0^*), and that there is no firm-specific time-invariant fixed effect (i.e. $\sigma_\alpha = 0$). The parameters to be estimated are only two-dimensional: $\Theta = (\rho, \sigma)$. In the second experiment, I assume an environment in which we know c_f^0, but there is a firm-specific time-invariant fixed effect, which is unknown. Because we also need to estimate σ_α, the parameters to be estimated are three-dimensional: $\Theta = (\rho, \sigma, \sigma_\alpha)$. In the third experiment, I assume an environment in which we do not know c_f^0, and there is a firm-specific time-invariant fixed effect. In this case, the parameters to be estimated are four-dimensional: $\Theta = (\rho, \sigma, \sigma_\alpha, c_f)$. In the last experiment, I introduce the aggregate exit rate into the summary statistics in order to identify c_f. Here, the exit rate is computed as the integration of an estimated kernel smoothing function from $-\infty$ to x^*, as follows:

$$\text{EXR}(\hat{c_f}) = \int_{-\infty}^{x^*} \hat{\mu}_{sim}(p)dp,$$

where the true exit rate is expressed as:

$$\text{TEXR}(c_f^0) = \int_{-\infty}^{x_0^*} \hat{\mu}_{obs}(p)dp.$$

[9] Although I assume that the fixed effect ia independently and identically distributed across firms, it is not a random effect (orthogonal to the regressor). Instead, it is a fixed effect because the lagged term exists in the regressor.

Thus, the distance criterion to be minimized becomes KLd+$\sqrt{(EXR(\hat{c_f}) - TEXR(c_f^0))^2}$ and L^2+$\sqrt{(EXR(\hat{c_f}) - TEXR(c_f^0))^2}$.

Estimation setting

With respect to prior distributions, I set the flat distributions for each parameter, as follows: $\rho \sim \mathcal{U}(0,1)$, $\sigma \sim \mathcal{U}(0,1)$, $\sigma_\alpha \sim \mathcal{U}(0,1)$, and $c_f \sim \mathcal{U}(0,1)$. In order to avoid a degeneracy problem, the algorithm stops when the acceptance rate is lower than 5%, instead of pegging ϵ_{target} at a specific value. With regard to the ABC algorithm variables, the number of particles is set to $N = 100$, the number of replications to $M = 5$, the quality index is $\alpha_{ABC} = .90$, and the number of firms generated for each iteration is $20,000$. All the estimated outcomes are all calculated using the artificial data, which replicates 50 times. The computation and estimation algorithm is implemented using *Python*, on a system running Windows Server 2008 with 27 GB memory and a quad-core 2.40 GHz CPU (Intel (R) Xeon (R) E5620).

3.4 Monte Carlo results

3.4.1 The case of no fixed-effect with true fixed cost

This is the simplest case, where we know that $\sigma_\alpha^0 = 0$ and c_f^0. Our estimation targets the parameter: $\Theta = (\rho, \sigma)$. I conduct three experiments, where $(\rho_0, \sigma_0, \sigma_\alpha^0, c_f^0)$ is set to $(.30, .10, .00, .00)$, $(.60, .20, .00, .12)$, and $(.90, .30, .00, .36)$, respectively. Table 3.3 summarizes the estimation outcomes, including the posterior mode (map; maximum a posteriori estimate), posterior mean, and 95% credible intervals. Table 3.4 compares the RMSEs of $(\hat{\rho}, \hat{\sigma})$ to check the finite sample property of the MDD estimator. It is not surprising that the MDD estimators achieve the lowest RMSE mainly because of their informational advantage, since we know $\sigma_\alpha^0 = 0$ and c_f^0. Moreover, the RMSE of the MDD with the L^2-distance is lower than that with the KLd in almost all experiments. This is because L^2-distance LSDD estimate is more accurate than two step KLd estimate as a measure of density difference.

Note that the GMM estimator takes a plausible value for some experiments, but takes completely different values for others. This uncertainty across the estimation outcomes stems mainly from the truncation bias. Additionally, we observe that the ABGMM estimator performs poorly for larger values of ρ. This is an example of the well-known weak instruments problem, where if the autoregressive parameters are too persistent or the ratio of the variance of the fixed-effect to that of the idiosyncratic error is too large, the accuracy of the ABGMM estimator decreases.

3.4.2 The case of a fixed-effect with true fixed cost

This is the second case where there exists a fixed-effect with unknown finite variance, given that we know c_f^0. Our estimation targets the parameters: $\Theta = (\rho, \sigma, \sigma_\alpha)$. I conduct two experiments, where $(\rho_0, \sigma_0, \sigma_\alpha^0, c_f^0)$ is set to $(.60, .20, .20, .12)$ and $(.90, .20, .10, .72)$, respectively. Table 3.5 summarizes the estimation outcomes, and Table 3.6 compares the RMSEs. We find that the MDD estimator computes the best estimate in almost all parameter ranges, and the MDD with the L^2-distance estimate looks the most accurate.

3.4.3 The case of a fixed-effect with unknown fixed cost

This is the last case, where we do not know the true values of the fixed cost and the fixed effect. Our estimation targets the parameters: $\Theta = (\rho, \sigma, \sigma_\alpha, c_f)$. I conduct two experiments, where $(\rho_0, \sigma_0, \sigma_\alpha^0, c_f^0)$ is set to $(.60, .20, .20, .12)$ and $(.90, .20, .10, .72)$, respectively. Table 3.7 summarizes the estimation outcomes, and Table 3.8 compares the RMSEs. We find that the MDD estimator again computes the best estimate in almost all parameter ranges, but in this case, the MDD with the KLd looks the most accurate. This is because the KLd and the EXR are computed with the same estimated kernel smoothing function. In contrast, the L^2-distance and the EXR are computed separately. As a result, a small estimation error incurred in the separate estimation can cause a large error in the ABC, because the estimation of the L^2-distance is performed without regard to computing the EXR.

3.5 Conclusion

In this study, I have proposed a structural estimation method for Hopenhayn's (1992) model of firm dynamics. Based on a simulation-based parametric density estimation using the ABC, I successfully estimated the structural parameters characterizing dynamics with a one-shot cross-sectional observation only. I check the finite sample property of the MDD estimator using Monte Carlo experiments and find that the estimator achieves the lowest RMSE for almost all cases. In addition, the L^2-distance LSDD estimate is better than the two step KLd as a distance metric of the density difference.

Because we cannot use a reliable estimate of entrants' initial distribution in Japan, I do not conduct empirical research here. However, future empirical work is required to check the effectiveness of this structural estimation algorithm.

Title	Method	Data	Time	Persistence
Gilchrist and Sim (2007)	FE	Korean manufacturing panel	1993 – 2002	0.223 – 0.325
Abraham and White (2006)	2SLS	LRD	1976 – 1999	0.37 – 0.41
Castro, Clementi and Lee (2011)	N/A	ASM	1972 – 1997	0.439
Lee and Mukoyama (2008)	OLS	ASM	1972 – 1997	0.972
	FE	ASM	1972 – 1997	0.493
	2SLS	ASM	1972 – 1997	0.548
	ABGMM	ASM	1972 – 1997	0.324
	SGMM	ASM	1972 – 1997	0.992

Table 3.1: Various estimation outcomes with several dynamic panel estimators. Gilchrist and Sim (2007) applied the fixed-effect regression to Korean manufacturing panel data sets from 1993 to 2002, Abraham and White (2006) applied the 2SLS to the Longitudinal Research Database (LRD) from 1976 to 1999, Castro et al. (2011) used the the Annual Survey of Manufactures (ASM) data from 1972 to 1997, and Lee and Mukoyama (2008) applied various dynamic panel estimation techniques to the ASM data from 1972 to 1997.

Title	Method	Data	Time	Persistence
Cooper and Ejarque (2003)	SMM	COMPUSTAT	1979 – 1989	0.105 – 0.111
Gomes (2001)	SMM	COMPUSTAT	1999	0.620
Hennessy and Whited (2005)	SMM	COMPUSTAT	1993 – 2001	0.740
Cooper and Haltiwanger (2006)	II	LRD	1972 – 1988	0.885

Table 3.2: Various estimation outcomes with II-type estimators. Cooper and Ejarque (2003) applied SMM to Gilchrist and Himmelberg (1995)'s estimates and empirical moments which uses Standard and Poor's COMPUSTAT data from 1979 to 1989. Gomes (2001) used COMPUSTAT data of 1999 to replicate the second moment of the distribution of investment rate, Hennessy and Whited (2005) applied SMM to COMPUSTAT data from 1993 to 2001, and Cooper and Haltiwanger (2006) applied the indirect inference method to the LRD from 1972 to 1988.

	ρ	σ	σ_α	c_f
True	.3000	.1000	.0000	.0000
Posterior mode(KLd)	.2989	.1014		
	(.0154)	(.0084)		
(L^2)	.3004	.1000		
	(.0064)	(.0048)		
Posterior mean(KLd)	.2988	.1004		
	(.0154)	(.0029)		
(L^2)	.3004	.0999		
	(.0060)	(.0048)		
Credible interval(KLd)	[.2947, .3027]	[.0997, .1011]		
(L^2)	[.2972, .3038]	[.0986, .1010]		

	ρ	σ	σ_α	c_f
True	.6000	.2000	.0000	.1200
Posterior mode (KLd)	.5994	.2007		
	(.0193)	(.0067)		
(L^2)	.6009	.2001		
	(.0086)	(.0036)		
Posterior mean (KLd)	.6008	.2001		
	(.0192)	(.0067)		
(L^2)	.5993	.2008		
	(.0085)	(.0034)		
Credible interval (KLd)	[.5954, .6029]	[.1990, .2026]		
(L^2)	[.5989, .6025]	[.1991, .2011]		

	ρ	σ	σ_α	c_f
True	.9000	.3000	.0000	.3600
Posterior mode (KLd)	.9004	.3059		
	(.0071)	(.0150)		
(L^2)	.8983	.3012		
	(.0058)	(.0061)		
Posterior mean (KLd)	.9004	.3064		
	(.0071)	(.0151)		
(L^2)	.8983	.3012		
	(.0058)	(.0061)		
Credible interval (KLd)	[.8989, .9021]	[.3034, .3105]		
(L^2)	[.8967, .8997]	[.2991, .3035]		

Table 3.3: Posterior summaries on the simulated dataset with parameters Θ_0 : $(\rho_0, \sigma_0, c_f^0) = (.30, .10, .00)$, $(.60, .20, .12)$, and $(.90, .30, .36)$. The credible interval is 95%.

	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.3000	.0000	.1000
OLS	.1958	.4969	.0087	.0914
FE	.4183	-.1178	.0144	.0984
2SLS	.3317	-.0223	.0307	.0694
ML	.2556	.0448	.0307	.0694
ABGMM	.0965	.2048	.0096	.1093
SGMM	.0678	.2466	.0049	.1046
MDD (KLd)	.0117	.2988	.0022	.1004
(L^2)	.0060	.3004	.0048	.0999

	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.6000	.0000	.2000
OLS	.2318	.8318	.0209	.1792
FE	.8924	-.2923	.0120	.1882
2SLS	.6345	-.0259	.0396	.1605
ML	.1620	.7129	.0133	.2021
ABGMM	.1519	.4555	.0440	.2438
SGMM	.1052	.4972	.0405	.2281
MDD (KLd)	.0154	.5993	.0054	.2008
(L^2)	.0085	.6008	.0034	.2001

	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.9000	.0000	.3000
OLS	.0699	.9699	.0876	.2124
FE	1.1194	-.2150	.0861	.2140
2SLS	.7996	.1194	.1078	.1922
ML	.0557	.9557	.0365	.2635
ABGMM	.1076	.5720	.0175	.2825
SGMM	.1127	.7877	.0053	.2950
MDD (KLd)	.0056	.9004	.0122	.3064
(L^2)	.0060	.8983	.0061	.3012

Table 3.4: RMSEs of various estimators calculated on 50 replications. TRUE denotes true parameter sets, OLS denotes Ordinary Least Squared estimator, FE denotes Fixed-effect estimator, 2SLS denotes Anderson-Hsiao Two Stage Least Squares estimator using $\phi_{i,t-2}$ as an instrument for $\Delta\phi_{i,t-1} = \phi_{i,t-1} - \phi_{i,t-2}$, ML denotes Maximum Likelihood estimator, ABGMM denotes Arellano-Bond first-differenced GMM estimator, and SGMM denotes Blundell-Bond System GMM estimator using $\phi_{i,t-3}$ and $\phi_{i,t-4}$ as an instrument. All the reduced-form estimates are computed on balanced panel. The three lowest RMSEs are shaded.

	ρ	σ	σ_α	c_f
True	.6000	.2000	.2000	.1200
Posterior mode (KLd)	.6532	.2023	.1871	
	(.0677)	(.0236)	(.0426)	
(L^2)	.6221	.2095	.1836	
	(.0395)	(.0208)	(.0361)	
Posterior mean (KLd)	.6567	.2016	.1870	
	(.0671)	(.0215)	(.0424)	
(L^2)	.6264	.2103	.1821	
	(.0293)	(.0168)	(.0287)	
Credible interval (KLd)	[.6098, .7040]	[.1843, .2183]	[.1622, .2117]	
(L^2)	[.5896, .6751]	[.1835, .2358]	[.1432, .2202]	
	ρ	σ	σ_α	c_f
True	.9000	.2000	.1000	.7200
Posterior mode (KLd)	.9058	.2075	.0820	
	(.0126)	(.0128)	(.0132)	
(L^2)	.8989	.2022	.0997	
	(.0050)	(.0069)	(.0054)	
Posterior mean (KLd)	.9092	.2067	.0835	
	(.0125)	(.0120)	(.0112)	
(L^2)	.8993	.2017	.0997	
	(.0048)	(.0069)	(.0052)	
Credible interval (KLd)	[.8963, .9290]	[.1998, .2121]	[.0739, .0957]	
(L^2)	[.8943, .9047]	[.1960, .2065]	[.0963, .1034]	

Table 3.5: Posterior summaries on the simulated dataset with parameters Θ_0 : $(\rho_0, \sigma_0, \sigma_\alpha^0, c_f^0)$=(.60, .20, .20, .12) and (.90, .20, .10, .72). The credible interval is 95%.

	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.6000	.0000	.2000
OLS	.3711	.9711	.0021	.2020
FE	.8331	-.2330	.0008	.2001
2SLS	.4620	.1384	.0306	.1694
ML	.3473	.9473	.0502	.2502
ABGMM	.1157	.4851	.0536	.2535
SGMM	.2322	.8319	.0841	.2841
MDD (KLd)	.0690	.6567	.0172	.2016
(L^2)	.0360	.6221	.0195	.2095
	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.9000	.0000	.2000
OLS	.0886	1.0002	.0578	.1422
FE	.9720	-.1995	.0607	.1393
2SLS	.7431	.2963	.0747	.1218
ML	.1040	1.0040	.0182	.1818
ABGMM	.3531	.5471	.0204	.1796
SGMM	.0116	.9112	.0030	.2028
MDD (KLd)	.0111	.9092	.0113	.2067
(L^2)	.0048	.8993	.0070	.2017

Table 3.6: RMSEs of several estimators calculated on 50 replications, same as Table 3.4.

	ρ	σ	σ_α	c_f
True	.6000	.2000	.2000	.1200
Posterior mode (KLd)	.6587	.2019	.1890	.1193
	(.0471)	(.0207)	(.0410)	(.0024)
(L^2)	.6386	.2112	.1872	.1128
	(0365)	(.0156)	(.0221)	(.0107)
Posterior mean (KLd)	.6565	.2011	.1930	.1193
	(.0523)	(.0190)	(.0451)	(.0023)
(L^2)	.6440	.2101	.1884	.1113
	(.0325)	(.0126)	(.0173)	(.0085)
Credible interval (KLd)	[.6043, .7069]	[.1819, .2175]	[.1686, .2259]	[.1177, .1242]
(L^2)	[.6024, .6900]	[.1846, .2356]	[.1531, .2214]	[.0954, .1276]
	ρ	σ	σ_α	c_f
True	.9000	.2000	.1000	.7200
Posterior mode (KLd)	.9080	.2055	.0836	.7175
	(.0144)	(.0101)	(.0110)	(.0014)
(L^2)	.8889	.2287	.0894	.6808
	(.0095)	(.0138)	(.0111)	(.0122)
Posterior mean (KLd)	.9107	.2051	.0844	.7175
	(.0147)	(.0092)	(.0099)	(.0014)
(L^2)	.8896	.2299	.0891	.6806
	(.0069)	(.0101)	(.0091)	(.0099)
Credible interval (KLd)	[.8961, .9336]	[.1969, .2104]	[.0756, .0953]	[.7164, .7225]
(L^2)	[.8762, .9048]	[.2067, .2535]	[.0736, .1042]	[.6504, .7090]

Table 3.7: Posterior summaries on the simulated dataset with parameters Θ_0 : $(\rho_0, \sigma_0, \sigma_\alpha^0, c_f^0) = (.60, .20, .20, .12)$ and $(.90, .20, .10, .72)$. The credible interval is 95%.

	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.6000	.0000	.2000
OLS	.3711	.9711	.0020	.2018
FE	.8336	-.2335	.0010	.1999
2SLS	.4630	.1373	.0308	.1692
ML	.3473	.9472	.0498	.2498
ABGMM	.1145	.4867	.0536	.2535
SGMM	.2325	.8321	.0839	.2839
MDD (KLd)	.0639	.6565	.0142	.2011
(L^2)	.0523	.6440	.0159	.2101
	RMSE	$\bar{\hat{\rho}}$	RMSE	$\bar{\hat{\sigma}}$
TRUE	.0000	.9000	.0000	.2000
OLS	.1001	1.0001	.0575	.1425
FE	1.0993	-.1993	.0605	.1395
2SLS	.6049	.2953	.0779	.1221
ML	.1032	1.0032	.0179	.1821
ABGMM	.3533	.5468	.0201	.1799
SGMM	.0112	.9109	.0034	.2031
MDD (KLd)	.0138	.9107	.0087	.2051
(L^2)	.0125	.8896	.0315	.2299

Table 3.8: RMSEs of several estimators calculated on 50 replications, same as Table 3.4.

Chapter 4

Structural household finance

4.1 Introduction

Although household asset allocation behavior is disproportionately important in asset pricing and other areas (e.g. tax rate on capital gains and re-distributional effects of inflation (Doepke and Schneider (2006))), research on household finance has not developed sufficiently. According to Campbell (2006), there are two challenges with regard to household finance: how to measure the household portfolio choice precisely and how should the decision-making be modeled adequately. Additionally, I think there is a third challenge, that is, how to estimate the structural parameters of the theoretical model with the data of household portfolio choice.

With respect to the first point, the most reliable survey on financial wealth in the U.S. is the Survey of Consumer Finances (SCF). The SCF is a triennial cross-sectional survey on financial wealth conducted since 1962. It has excellent coverage by both age and wealth, and the sample size of the survey is about 6,000 families. Although we do not know about the asset diversification (e.g. we know the total amount of stock, but do not know the holdings of individual stocks), we know about the asset allocation because it includes the balance of safe assets (deposits and bond holdings) and risky assets (stocks and mutual funds). The biggest challenge for an empirical analysis is that the survey does not track each household but refreshes a household sample each time. Therefore, we cannot employ dynamic panel estimation techniques to calibrate the structural parameters in the dynamic model. The situation is almost the same in different countries, including Japan.[1]

With respect to the second point, the question of how to model the household portfolio choice has mainly been discussed in the asset pricing literature. However, the research interest has not been to model individual household portfolio choice decision-making, but to explain aggregate stock market behavior. These theoretical challenges are collectively dubbed consumption-based asset pricing models (C-CAPM; e.g. Ludvigson (2015)). Formally, the C-CAPMs are built on the representative agent formulation where structural parameters are calibrated by aggregate statistics. As symbolized by the equity premium puzzle first introduced by Mehra and Prescott (1985), the standard representative agent model comes out to be failure when attempting to explain a number of facts about asset pricing (Campbell (2003)). Although various extensions (e.g. habit or recursive utility (Epstein and Zin (1989), Weil (1989)), rare event (Barro (2006), Julliard and Ghosh (2012))) were invented to improve the performance, they cannot fully resolve the equity premium puzzle.

A different strand of literature focuses on heterogeneity across households. This literature is generally classified into two groups. One group focuses on the interactions of heterogeneous agents who can partially insure against idiosyncratic risks. Since there exists an incompleteness in the insurance market and agents are not identical in wealth levels, neglected heterogeneity can alter asset pricing implication induced by the representative agent economy. The other group focuses on the fact that not everyone participates in the stock market, and therefore stock price depends only on stock market participants; on the other hand, bond price depends on all the

[1] There are a very few exceptions such as Italy and the Netherlands.

households. This limited participation also has different asset pricing implications from the representative framework.

The first group considers the following precautionary saving mechanism. In complete insurance markets, households can completely hedge their individual risk and each consumption level is proportional to the aggregate consumption level. But, in incomplete insurance markets, the volatility of each consumption level can be higher than the aggregate, and the asset pricing mechanism can vary. Telmer (1993) and Lucas (1994) considered the general equilibrium economy with transitory idiosyncratic shocks and borrowing or short-sales constraints, and concluded that the incompleteness itself cannot affect pricing, because households who face uninsured idiosyncratic risks can hedge their risk by trading assets through the financial market (self-insurance). Aiyagari and Gertler (1991) and Heaton and Lucas (1996) considered a similar economy but with trading costs. By introducing frictions such as trading costs, households have some limitations in hedging their own risk via trading, and accumulate precautionary savings as a buffer stock (Deaton (1991)). They concluded that the equity premium puzzle can be explained only when the trading costs are set to be unrealistically high. In contrast with transitory idiosyncratic shocks, Constantinides and Duffie (1996) studied permanent idiosyncratic shock. When idiosyncratic shocks are permanent, households have less incentive to trade because such trades cannot hedge their individual risk. Accordingly, the market leads to a no-trade equilibrium, the need for all assets increases, and hence the return on each asset decreases. Although the no-trade equilibrium cannot explain the observed risk premium by itself, the puzzle can be resolved when the aggregate shock and the volatility of idiosyncratic shocks are negatively correlated. Krusell and Smith (1997) studied whether the research outcomes relied on realistic heterogeneity or not. There are two types of model setups: two infinitely lived agents [Telmer (1993), Lucas (1994), and Heaton and Lucas (1996)], or a continuum of agents [Aiyagari and Gertler (1991) and Constantinides and Duffie (1996)]. In lieu of using the two agent setup, which is easy to compute but makes it hard to match their outputs with cross-sectional observations (e.g. no trade equilibrium of Constantinides and Duffie (1996) generates unrealistic degenerate distributions.), Krusell and Smith (1997) constructed the same mechanism on the realistic richer population structure. They concluded that the puzzle is not in conformity with realistic wealth heterogeneity.

The second group focuses on limited participation, which was first stressed by Mankiw and Zeldes (1991). Mankiw and Zeldes (1991) and Attanasio, Banks and Tanner (2002) empirically found that the consumption growth of stockholders is systematically bigger than that of non-stockholders. This might imply that the consumption growth of non-stockholders does not depend on stock returns, which is different from the assumption of the standard representative agent formulation. Therefore, the estimates based on the standard C-CAPM can lead to inconsistent estimates. Vissing-Jørgenson (2002) and Paiella (2004) studied the representative economy only with stockholders or a representative stockholder economy; meanwhile, Guvenen (2009) and Attanasio and Paiella (2011) studied the two infinitely-lived agents economy with stockholders and non-stockholders. The main difference between these papers is whether limited participation was exogenous or endogenous. Despite the differences in setup, these papers showed that accounting for limited participation can serve to reconcile theoretical outcomes with empirical evidence.

With respect to the third point, i.e. how to estimate the structural parameters of an incomplete market model with limited participation is statistically challenging. In general, an empirical test of the theory about the households asset allocation behavior requires disaggregated household-level panel data about the portfolio holdings. However, we cannot use the panel data about the household portfolio for estimation because the SCF refreshes the sample every survey, as described above. Instead of using the household portfolio panel data, some studies used the household income panel data to test only the incomplete market implications. For example, Storesletten, Telmer and Yaron (2004) used the Panel Study of Income Dynamics (PSID); on the other hand, Brav, Constantinides and Geczy (2002), Cogley (2002), Vissing-Jørgenson (2002), Balduzzi and Yao (2007), and Kocherlakota and Pistaferri (2009) used the Consumer Expenditure Survey (CEX). There are a few problems in following their estimation method, aside from their mixed implications. First, consumption data in PSID is available only for food. Thus, there is a general concern about its legitimacy as an empirical counterpart of the dynamic general equilibrium object. Second, CEX is a rotation panel which tracks each individual household only for four consecutive quarters. Because of its limited time-series dimension, most studies focused on cross-sectional moments of consumption growth and estimated the Euler equation. But, Toda and Walsh (2015) pointed out that the existence of higher-order moments is not guaranteed in general. Therefore, estimates based on these moments are not compatible. Thirdly, consumption data from household-level surveys is only available with a variety of measurement errors. When we use the Euler equation estimation, the measurement error is raised to a power and thus leads to larger specification errors. Fourth, they could not use the information on portfolio compositions for estimation. Since the previous literature focused on whether the proposed model could explain the observed equity premium level or not, their Euler equation estimation was sufficient to test the empirical validity of the asset pricing. From the viewpoint of household finance, however, how households compose their financial portfolio is also important because it exhibits the household risk attitude. So, I should care not only whether the simulated distribution mimics the empirical one, but also care whether the asset allocation policy mimics the experiential one. Gomes and Michaelides (2008) also tried to match the stock allocation, but they focused only on the average share and not on the policy.

In this chapter, I consider two kinds of heterogeneity at the same time: wealth heterogeneity from uninsured idiosyncratic risk and limited participation. I summarize the cross-sectional household portfolio survey data and then, implement a structural estimation algorithm which enables us to estimate the parameters characterizing dynamics only with the cross-sectional data. Finally, I estimate the structural parameters of the model by applying the method to the Japanese household portfolio data from the National Survey of Family Income and Expenditure, a cross-sectional survey on the overall family budget structure.

Theoretically, one of the critical drawbacks in the model of limited participation is the outcome relying on unrealistic wealth heterogeneity, which Krusell and Smith (1997) criticized; while, the agents in the Aiyagari-style general equilibrium model are homogeneous with respect to stock market participation. So, structural estimations should be run on the unified framework, and otherwise leads to biased estimates. When considering participation heterogeneity, we need to choose to take the participation given or not, as also discussed in Heathcote et al. (2009). With respect to that point, Guvenen (2009) endogenize participation by exogenously assuming heterogeneity in the elasticity of intertemporal substitution (EIS) in consumption and Attanasio and Paiella (2011) assumed a participation cost, which was first studied by Luttmer (1999). But, Haliassos and Bertaut (1995) discussed that these factors cannot account for the participation puzzle empirically. In addition, Aiyagari and Gertler (1991)'s transaction costs mechanism can endogenize participation, but Vayanos (1998) empirically found that the costs were too small to explain the puzzle. Cao, Wang and Zhang (2005) introduced Knightian uncertainty into the distribution of the asset payoff to endogenize the participation, but the empirical validity of the assumption remains in question. Thus, I treat participation as given following Vissing-Jørgenson (2002) and Paiella (2004), and employ the heterogeneous agents dynamic model to explain the stockholder's portfolio choice behavior. Hence, my model can be termed as the heterogeneous stockholders dynamic model.

By using the heterogeneous agents framework, we can numerically compute the stationary distribution. Since the distribution contains parametric information characterizing the dynamics, we can estimate the true posterior distributions of structural parameters by minimizing the density difference between the stationary distribution and the observed cross-sectional distribution. Because we cannot calculate the analytical expression of the distribution (and thus its likelihood), we cannot employ the maximum likelihood procedures to estimate the structural parameters. Instead of using the maximum likelihood method, I alternatively employ the likelihood-free inference procedure named Approximate Bayesian Computation (ABC). We can estimate the posterior distribution because ABC replaces the process of likelihood evaluation with a process of summary statistics comparison. Owing to the proposed estimation algorithm, we can avoid the powered measurement error problem and can use the portfolio composition for estimation. Brav et al. (2002) performed a similar study to mine, which also considered incomplete markets and limited participation. However, their theory depended on Constantinides and Duffie (1996)'s unrealistic wealth heterogeneity and their estimation could not avoid the powered measurement error problem.

The remainder of this chapter is organized as follows. The next section lays out the empirical facts about the Japanese household portfolio. Section 3 proposes the stochastic dynamic heterogeneous stockholders model, discusses the solution algorithm and calibration. Section 4 summarizes the estimation algorithm and empirical outcomes. Finally, section 5 concludes this chapter.

4.2 Data

This section describes the Japanese household portfolio, following Bertaut and Starr-McCluer (2000) and Campbell (2006). In Japan, one of the most extensive surveys on financial wealth is National Survey of Family Income and Expenditure ("Zensho" in Japanese and hereafter, NSFIE). NSFIE is a quinquennial cross-sectional survey on the overall family budget structure conducted since 1959. The sample size is about 57,000 households including 4,400 one-person households for the 2009 survey. As in the U.S., panel data is not available.

There are a few studies about the Japanese household portfolio choice using cross-sectional survey data. For example, Iwaisako (2009) used the Nikkei Radar to summarize household portfolio allocation in Japan. Although the Nikkei Radar is the only survey that asks households their real estate wealth, their observations are limited to the Tokyo metropolitan district and the age composition is biased toward the young. Fujiki, Hirakata and Shioji (2012) also discussed portfolio choice using the Survey of Household Finances (SHF), which is the equivalent of the SCF in the U.S. Certainly, these surveys ask households about qualitative items such as financial knowledge which is not available in NSFIE, though their sample sizes are much smaller than that of the NSFIE.[2] Because this chapter focuses on the asset allocation between stocks and bonds, and not on the diversification and some qualitative factors, the NSFIE is the best data for my research interest.

Figure 4.1 presents the cross-sectional financial wealth distribution, the financial level for each percentile and histogram. The horizontal axis in the left figure shows the percentiles of the distribution and the vertical axis reports yen on a log scale. Financial wealth is defined as the sum of risky and safe assets. In this data, risky

[2] The sample size of the NSFIE is about 57,000 households (about 53,000 households with two or more people). On the other hand, the Nikkei Radar surveys from 1,500 to 3,000 households; the SHF targets 8,000 households of two or more people and 4,032 households responded for the 2010 survey.

assets are made up of stocks and mutual funds while safe assets consist of deposits and bonds.

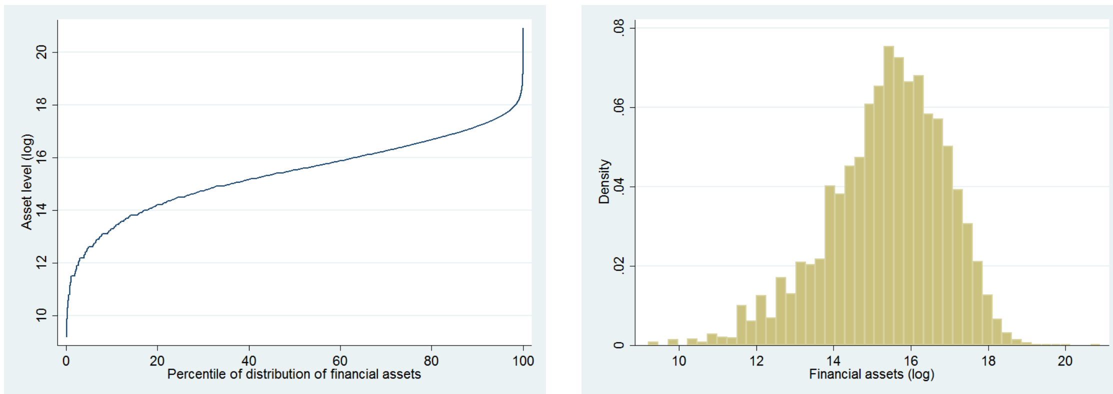

Figure 4.1: **Japanese financial wealth distribution.** The cross-sectional distribution of financial assets in the 2009 National Survey of Family Income and Expenditure.

Table 4.1 presents the summary statistics of Japanese financial wealth distribution. The median household has financial assets of 4.9 million yen and the mean has 10.65 million yen. It is clear that many households possess substantial financial assets and its distribution is highly skewed. Owing to the skewness, aggregate statistics and asset pricing highly depends on wealthy households. Thus, we cannot learn individual household financial decision making from the aggregate statistics.

Obs	Mean	Std. Dev.	Skewness	Kurtosis	Median
48828	1064.68	1753.58	10.66	485.17	490.00

Table 4.1: **Summary statistics.** The summary statistics of the cross-sectional financial wealth distribution(10,000 yen)

Figure 4.2 presents the participation decisions of households with different wealth positions. The horizontal axis is the same as the left side of Figure 4.1 and the vertical axis is the participation rate in different classes of assets. Financial assets are classified into four types: stocks, bonds, ordinary deposits, and fixed deposits. Mutual funds are classified into stocks or bonds, depending on the category of the investment asset. As found for U.S. households by Campbell (2006), most Japanese households did not participate in risky financial markets and have only deposits. A fixed deposit is similar to bonds in that both guarantee depositors or investors with a higher rate of return than that of an ordinary deposit in compensation for less liquidity. The only difference between these two is whether the principal is guaranteed or not. The ordinary deposit participation rate is almost independent of wealth level; meanwhile, fixed deposits increase with wealth level. Low participation in the stock market ($\simeq 20\%$ in aggregate) is the well-known stock holding puzzle and the implicit participation cost may be the key to solve the problem. One of the biggest challenges for the financial theory is the observed limited participation among wealthy households.

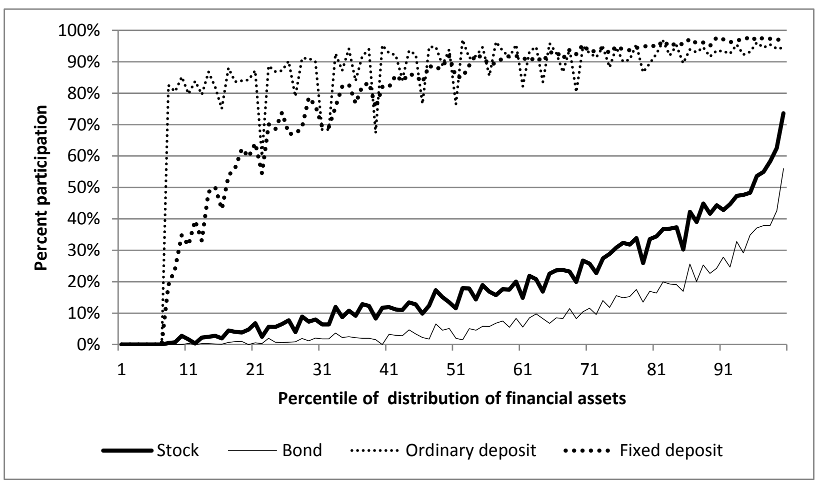

Figure 4.2: **Participation rates by asset class.** The cross-sectional distribution of asset class participation rates for the 2009 survey.

Figure 4.3 presents the allocation decisions of households with different wealth positions. The horizontal axis is the same as Figure 4.1 (left) and Figure 4.2, and the vertical axis shows the asset composition. The figure demonstrates that deposits play a dominant role in household financial wealth. Specifically, the share of ordinary deposits decreases with wealth level; on the other hand, the share of fixed deposits increases with wealth level up to around 60%. It can be seen that as households become wealthier, they tend to hold stock but its share is very limited. This limited share ensures the positive correlation between wealth and participation.

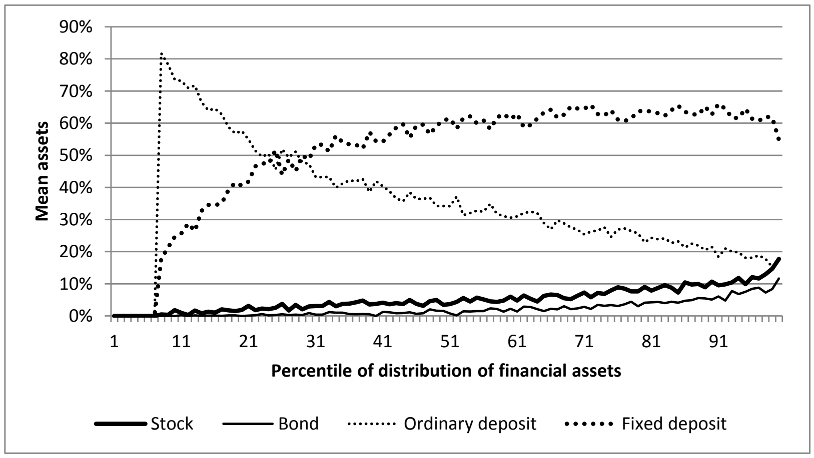

Figure 4.3: **Asset class shares in household portfolios.** The share of each asset class in the financial portfolio of households for each percentile, in the 2009 survey.

The NSFIE also reveals demographic factors which could affect household participation decisions and asset allocations. Age, income, sex of head($\in \{0, 1\}$, where 0 denotes women and 1 denotes men), non-labor force status($\in \{0, 1\}$), and the number of children under 18 are available in the 2009 survey.[3] Table 4.2 summarizes the effects of various factors on stock market participation and asset allocation implications without one-person households, following the specification of Campbell (2006) and Jin (2011).[4] First, I use logistic regressions to estimate the contributions of income,

[3] "Family units" is also available, though it is highly correlated with number of children and can cause the multicollinearity, hence I dropped it from explanatory variables in this analysis.

[4] Since it is difficult to identify the age effect due to the cohort effect, I assume the cohort effect to be zero, following previous literature.

wealth, and demographic factors in the stock market participation decision.[5]

$$
\begin{aligned}
\theta_i^* &= \beta_0 + \beta_1 Age_i + \beta_2 Age_i^2 + \beta_3 \ln \text{Income}_i + \beta_4 (\ln \text{Income}_i)^2 \\
&\quad + \beta_5 \ln W_i + \beta_6 (\ln W_i)^2 + \beta_7 \#\text{Children}_i + \beta_8 \text{Sex of head}_i + \beta_9 \text{Non labor force}_i + \varepsilon_i \\
d_i &= 1, \ \ if \ \ \theta_i^* > 0 \\
d_i &= 0, \ \ if \ \ \theta_i^* \leq 0
\end{aligned}
$$

$$
Pr(d_i = 1) = Pr(\theta_i^* > 0) = Pr(\varepsilon_i > -\beta_0 - \beta_1 ...) = F(-\beta_0 - \beta_1 ...)
$$

where θ_i^* denotes a latent optimal stock share and d_i denotes a discrete participation decision. Then, I report the OLS regression outcome of the conditional portfolio stock share on the same variables only for stockholders.

$$
\begin{aligned}
\theta_i &= \beta_0 + \beta_1 Age_i + \beta_2 Age_i^2 + \beta_3 \ln \text{Income}_i + \beta_4 (\ln \text{Income}_i)^2 \\
&\quad + \beta_5 \ln W_i + \beta_6 (\ln W_i)^2 + \beta_7 \#\text{Children}_i + \beta_8 \text{Sex of head}_i + \beta_9 \text{Non labor force}_i + \varepsilon_i
\end{aligned}
$$

The table shows that there was a strong hump-shaped age effect, positive wealth effect, and positive non-labor force effect on participation. The hump-shaped age effect implies younger households tend to buy and older households tend to sell stock. Consistent with Fujiki et al. (2012), participation is positively correlated with the wealth level, but correlation with income level is not robust in this study. The positive correlation with the non-labor force indicates that retirees tend to participate more actively in the stock market. On the whole, we cannot explain the participation behavior with only these proposed explaining variables. This observation is consistent with Haliassos and Bertaut (1995).

[5] In order to check the robustness of the estimation outcome, I use the truncated data limited between the 1_{st} and 99_{th} percentiles of the cross-sectional financial wealth distribution, but the results are similar.

In terms of asset allocation, I find that a strong quadratic wealth effect is quantitatively important in explaining the conditional stock share, which is consistent with Campbell (2006). We also find a strong hump-shaped age effect and a positive correlation with non-labor force, though they are quantitatively less important. The quadratic wealth effect indicates that low-wealth households tend to hold stock if they participate in the stock market. On the other hand, there is a positive correlation between stock share and wealth level in the upper parts of the wealth distribution.

Dependent Variable	Stock market participation Coefficients (Logit)		Portfolio stock share Coefficients (OLS)
Age	0.02**	0.03***	0.004***
	(0.01)	(0.01)	(0.00)
Age squared	-0.0002**	-0.0003***	-0.00003**
	(0.00)	(0.00)	(0.00)
Ln(income)	1.73	1.82	-0.15
	(1.12)	(1.20)	(0.11)
Ln(income) squared	-0.04	-0.04	0.01
	(0.04)	(0.04)	(0.00)
Ln(wealth)	0.24	0.24	-0.45***
	(0.22)	(0.22)	(0.05)
Ln(wealth) squared	0.02**	0.02**	0.01***
	(0.01)	(0.01)	(0.00)
Number of children	-0.01	-0.02	-0.002
	(0.02)	(0.02)	(0.00)
Sex of head	-0.08	-0.08	-0.01
	(0.05)	(0.05)	(0.01)
Non-labor force	-	0.42***	0.02***
	-	(0.04)	(0.01)
Adjusted R^2	0.17	0.17	0.23
Log-likelihood	-19350.14	-19294.03	-
Obs	45184	45184	9467

Table 4.2: **Stock market participation and asset allocation for participants**: The table reports income, wealth, and demographic determinants of stock market participation by Logit analysis, and of portfolio allocation by OLS. Standard errors are reported in parentheses under the estimates. Coefficients significant at 10% are denoted by *, 5% by **, and 1% by ***. The adjusted R^2 reported in the stock market participation is McFadden's pseudo R^2.

4.3 Model

4.3.1 Specification

In this section, I construct the heterogeneous stockholders model in order to generate simulated outcomes that are consistent with empirical findings. Specifically, the key empirical finding is the cross-sectional distribution of household financial wealth presented in Figure 4.1 and the risky asset share presented in Figure 4.3. Participation rates presented in Figure 4.2 are not the research objective because I treat participation as given following Vissing-Jørgenson (2002) and Paiella (2004). Empirically, it is equivalent to using the conditional risky asset share instead of using the unconditional share.

The economy is populated by a continuum of households, who are ex-ante homogeneous and the size of which is normalized to one. Each household maximizes their lifetime expected utility subject to budget constraints, borrowing constraints, and short-selling constraints as follows:

$$\max_{c_{i,t}, b_{i,t} \geq 0, s_{i,t} \geq 0} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{i,t}), \quad \beta \in (0,1) \tag{4.1}$$

$$u(c_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} \tag{4.2}$$

$$\text{s.t.} \quad c_{i,t} + b_{i,t+1} + s_{i,t+1} = w_{i,t} + R_b b_{i,t} + \tilde{R}_{i,s} s_{i,t}, \quad \forall t \tag{4.3}$$

$$a_{i,t} \equiv b_{i,t} + s_{i,t} \tag{4.4}$$

$$b_{i,t} \geq 0 \quad \forall t \tag{4.5}$$

$$s_{i,t} \geq 0 \quad \forall t, \tag{4.6}$$

where i denotes i-th household, $c_{i,t}$ denotes consumption, $b_{i,t}$ denotes risk-free asset (called as "bond") holdings, $s_{i,t}$ denotes risky asset (called as "stock") holdings, $a_{i,t}$ denotes financial wealth composed of bonds and stocks, $w_{i,t}$ denotes exogenous earnings, R_b denotes constant bond return, and $\tilde{R}_{i,s}$ denotes stochastic stock return. Households cannot sell bonds and stocks short. The utility function is assumed to be the CRRA form and σ is the coefficient of relative risk aversion (RRA). We can rewrite equation (4.4) using the stock share $\theta_{i,t} \in [0,1]$:

$$s_{i,t} = \theta_{i,t} a_{i,t} \tag{4.7}$$

$$b_{i,t} = (1-\theta_{i,t}) a_{i,t} \tag{4.8}$$

The earnings follow the exogenous AR(1) process given by:

$$\ln w_{i,t} = \mu(1-\rho) + \rho \ln w_{i,t-1} + \epsilon_{i,t} \tag{4.9}$$

$$\epsilon_{i,t} \sim \text{ i.i.d } N(0, \sigma_w^2) \tag{4.10}$$

The stochastic stock return $\tilde{R}_{i,s}$ independently follows the exogenous three-state Markov process defined by:

$$\tilde{R}_{i,s} = \{R_s^l, R_s^h, R_s^c\}, \quad \forall i \tag{4.11}$$

$$\Pi_{R_s} = \begin{pmatrix} \pi_{ll} & \pi_{lh} & \pi_{lc} \\ \pi_{hl} & \pi_{hh} & \pi_{hc} \\ \pi_{cl} & \pi_{ch} & \pi_{cc} \end{pmatrix}, \tag{4.12}$$

where subscript "l" indicates the state of the low price, "h" indicates the state of the high price, and "c" indicates the crisis state. The crisis state is similar to the rare event discussed by Barro (2005) and Barro (2006). In this chapter, a stock market crash is defined by a stock market price decline in excess of twenty percent within the annual window, following Mishkin and White (2002).

The individual maximization problem can be expressed as the following dynamic programing problem:

$$v_i(a_i, \theta_i; w_i, R_b, R_{i,s}) = \max_{c_i, \theta'_i \in [0,1]} \left\{ u(c_i) + \beta \mathbb{E}[v'_i(a'_i, \theta'_i; w'_i, R_b, R'_{i,s}) | w_i, R_{i,s}] \right\} \tag{4.13}$$

subject to

$$c_i + b'_i + s'_i = w_i + R_b b_i + \tilde{R}_{i,s} s_i \tag{4.14}$$

$$s_i = \theta_i a_i \tag{4.15}$$

$$b_i = (1 - \theta_i) a_i \tag{4.16}$$

$$\theta_i \in [0, 1] \tag{4.17}$$

where the apostrophe $'$ denotes the next state and $v(a, \theta; w, R_b, R_s)$ denotes the value function. The Euler equation for consumption is

$$u'(c_i) = \mathbb{E}[\beta \mathcal{R}'_i u'(c'_i)] \tag{4.18}$$

where $\mathcal{R}'_i = R_b + (R'_s - R_b)\theta_i$, and the first order condition with respect to the stock share is

$$0 = a_i \mathbb{E}[u'(c'_i)(R'_s - R_b)]. \tag{4.19}$$

Optimal decision rules are defined by the value function and the two policy functions:

$$c = f_c(a, \theta; w, R_b, R_s) \tag{4.20}$$

$$\theta' = f_{\theta'}(a, \theta; w, R_b, R_s). \tag{4.21}$$

We can define the cross-sectional distribution of financial wealth Γ, and there exists a stationary distribution Γ^*. In my framework, however, the risk-free rate and the risk premium are set exogenously and independent of Γ^*, because the bond market consists of both stock market participants and non-participants.[6]

The most popular algorithm to solve a stochastic dynamic optimization problem is the value function iteration (VFI) approach. VFI is time-consuming and is subject to the curse of dimensionality so that it does not seem suitable to function it as the inner loop within an estimation loop. Carroll (2006) proposed a faster algorithm named the endogenous grid-points method (EGM). One of the key ideas of EGM is to rewrite the optimization problem by employing all the available resources (which we call cash on hand and define as m_i) as a one-dimensional state variable.

$$v_i(m_i) = \max_{c_i, \theta_i'} \{u(c_i) + \beta \mathbb{E}(v_i'(m_i'))\} \tag{4.22}$$

$$m_i = w_i + \mathcal{R}_i a_i \tag{4.23}$$

Since the original EGM can only handle the problem if it has only one control variable, it is impossible to solve my model where there are two control variables. Barillas and Fernández-Villaverde (2007) combined EGM with a standard VFI which they called the generalized EGM (GEGM) to handle an optimization problem with more than one control variable. In this chapter, I apply their GEGM to solve the model, which is similar to Nirei and Aoki (2009)'s two step algorithm. Algorithm 1 gives a pseudo code to implement GEGM.

[6] To endogenously determine the risk-free rate, we need to employ the general equilibrium framework to incorporate non-participants like Guvenen (2009) or Attanasio and Paiella (2011), though their doubtful theoretical assumption may cause a serious specification error.

Algorithm 1

Number states from n to N
Initialize θ
repeat
  for n to N **do**
    Compute the optimal consumption/saving policy $f_c(a, \theta; w(n), R_s(n))$ by EGM
  end for
  Update a'
  for n to N **do**
    Compute the optimal allocation policy $f_\theta(a, \theta; w(n), R_s(n))$ by FOC
  end for
  Update θ
until θ converges

4.3.2 Calibration

In order to solve the stochastic dynamic optimization problem, we need to specify the exogenous parameter sets. In my model, the parameters to be calibrated are β (discount factor), R_b (bond return), $\{R_s^l, R_s^h, R_s^c\}$, and Π_{R_s} (stochastic stock return and its transition), and the parameters to be estimated are σ (RRA) and $\{\mu, \rho, \sigma_w\}$ (dynamic earnings process). The discount factor is set at the standard value for matching annual aggregate statistics: $\beta = 0.96$. In my model, "bond" summarizes all risk-free assets including ordinary and fixed deposits. The interest rate of a bond is calibrated with the annual yield of a one-year bond, using the data from 1980 to 2009, given by the Ministry of Finance, Japan. In the same way, "stock" summarizes all risky assets including mutual funds, real estate, and private businesses. Ideally, we should specify the covariance structure in the risky assets and compute the aggregate risky asset return and its volatility, though we cannot know how much of individual risky assets each household possesses. So, I instead presume the annual return of the Nikkei 225 to be the return of aggregate risky assets,[7] using the data from FRED. Moreover, I assume that the return of each household portfolio is independent across households, though they are somewhat correlated in reality. To classify the phase from

[7] I do not consider the dividend contribution to the return in the experiment.

1980 to 2009 into three states, I first split the phase whose annual return dropped by over 20% as a crisis state, following Mishkin and White (2002). Specifically, annual returns in 1990, 1992, 1997, 2000, 2001, and 2008, dropped by over 20% and their average is about −30%. Then the residual years whose average return is 12.0% and the standard deviation is 16.9% is split into two states. Calibrated values and transitions are summarized in Table 4.3.

Description	Symbol	Value
Discount factor	β	0.96
Risk-free rate	R_b	0.029
Risky asset return	$\tilde{R}_s$	$(-0.015, 0.255, -0.300)$
Transition	Π_{R_s}	$\begin{pmatrix} 0.4143 & 0.4143 & 0.1714 \\ 0.4143 & 0.4143 & 0.1714 \\ 0.4143 & 0.4143 & 0.1714 \end{pmatrix}$

Table 4.3: **Calibrated parameter values.**

4.4 Estimation

4.4.1 Method

In this chapter, I try to estimate the parameters of RRA: σ and of the dynamic earnings process which each household faces: $\{\mu, \rho, \sigma_w\}$. With respect to the RRA, we find much of the empirical literature is based on the representative agent economy using Japanese data (e.g. Hamori (1992a), Hamori (1992b), Nakano and Saito (1998), and Campbell (2003)), while we cannot find any empirical studies based on the heterogeneous agents economy. Since we can observe the financial wealth distribution and limited participation in the previous sections, disregarding these two kinds of heterogeneity can lead to biased estimates.

In general, we need disaggregated household-level portfolio panel data to estimate these structural parameters in the proposed dynamic heterogeneous households' portfolio choice model. Although we cannot use the portfolio panel data as in the U.S., we can use the CEX-like household income panel data for estimation. One of the important candidates is the Family Income and Expenditure Survey (hereafter, FIES), which is a rotating panel that tracks each individual household for six consecutive months. Because we use the same data structure as CEX, (i) limited time-series dimension, (ii) non-existence of higher order moments, (iii) powered measurement error, and (iv) ignorance of portfolio composition are still problems. In addition, it is difficult to adjust the seasonality of the data, which is a problem specific to the FIES.

In order to overcome these estimation problems, I employ a simulation-based Bayesian structural estimation technique which is based on the adaptive Sequential Monte Carlo Approximate Bayesian Computation algorithm (aSMC-ABC) proposed by Del Moral et al. (2012). In general, observed cross-sectional distribution of endogenous variables can be considered as an empirical counterpart of the theoretical stationary distribution. If we control input parameters to minimize the density difference between the stationary distribution and the observed cross-sectional distribution, the input parameters will be good estimates of true structural parameters. This is because a stationary distribution is a nonlinear function of structural parameters. Since we cannot solve the stationary distribution analytically in general, we employ the likelihood-free simulation-based inference instead of using the maximum likelihood procedure. By following the proposed algorithm, (i) the estimation outcome is independent of the time-series dimension because we use only cross-sectional statistics, (ii) the estimation outcome is independent of the existence of higher-order moments, (iii) measurement error is not powered, and (iv) we can employ the portfolio composition for estimation because we can use cross-sectional portfolio survey data.

The estimation strategy is as follows: First, we construct the theoretical model as the data generating process. In this chapter, I construct the dynamic heterogeneous investors' portfolio choice model in the previous section. Then, we sample the candidates of parameters from prior distributions, and compute the equilibrium outcome based on the parameters using the data generating process. Next, we compare the outcome with the observation for each parameter proposal. I use the summary statistics of the NSFIE as the observation. Finally, we continue perturbing the proposals based on the comparison results following the aSMC-ABC algorithm until the convergence criteria are met.

Since the ABC algorithm replaces the likelihood evaluation with the comparison process, the choice of summary statistics to be compared is vital. In this chapter, I use two kinds of summary statistics: statistics about the distribution and about the stock holding policy. When it comes to measuring the distance between distributions, we first come up with a two step approach that estimates the distribution for each sample at first, and then measures the distance between the estimated densities such as the Kullback-Leibler divergence (KLd). Though minimizing the KLd is statistically equivalent to the maximum likelihood procedure, the KLd cannot satisfy the properties of mathematical metrics such as the symmetric property and triangle inequality. It is not robust to the outliers, and is numerically unstable. In addition, Sugiyama et al. (2013) discussed that the first density estimation process is applied without considering the second process. This separation generates a small estimation error and can result in a big error in the last process. So, I employ the L^2-distance approximation method proposed by Sugiyama et al. (2013) to minimize this kind of error. L^2-distance is a standard metric to measure the distance between distributions, defined as

$$L^2(p, p') \equiv \int (p(x) - p'(x))^2 \mathrm{d}x \tag{4.24}$$

from i.i.d samples $\chi := \{x_i\}_i^n$ and $\chi' := \{x'_{i'}\}_{i'}^{n'}$, where p and p' are probability density functions. I use L^2-distance as the distance metric because there are some advantages in that it satisfies the definitions of mathematical metrics, it is more robust against outliers than the KLd, and it can be easily estimated. Since L^2-distance cannot be directly computed, it is approximated by the least-squares density-difference (LSDD) estimation where the optimal bandwidth of the Gaussian kernel is computed by K-fold cross validation (Härdle, Müller, Sperlich and Werwatz (2004)).

The second summary statistics is the stock holding policy. Because the portfolio stock share of participants is not only affected by their wealth levels, but also affected by demographic factors that I cannot model, such as age, sex of the head of the family, and employment status, I linearly modify the conditional portfolio stock share to eliminate these heterogeneities with OLS coefficients. The properties of the head of the family of the representative household are set to be employed men with average income, average age, and an average number of children. The linearly modified conditional portfolio stock share is computed as:

$$\begin{aligned}\tilde{\theta}_i = \hat{\beta}_0 + \hat{\beta}_1 \overline{Age_i} + \hat{\beta}_2 \overline{Age_i}^2 + \hat{\beta}_3 \ln \overline{\text{Income}_i} + \hat{\beta}_4 (\ln \overline{\text{Income}_i})^2 \\ + \hat{\beta}_5 \ln W_i + \hat{\beta}_6 (\ln W_i)^2 + \hat{\beta}_7 \overline{\#\text{Children}_i} + \hat{\beta}_8 \times 1 + \hat{\beta}_9 \times 0, \qquad (4.25)\end{aligned}$$

where $\hat{\beta}_i$ is the OLS coefficient estimate for each explained variable. The modified portfolio share is summarized as a discretized grid on true financial wealth quintiles, as displayed in Figure 4.4.

Finally, the distance to be minimized is a simple sum of the estimated density difference and the distance of policy functions:

$$\hat{L}^2(\Gamma_{obs}(\ln W), \Gamma^*(a)) + \sqrt{\sum_j^{19} \left\{ \tilde{\theta}_i(Q(p_j)) - \mathfrak{f}_{\theta'}(\mathfrak{Q}(p_j), \theta; w, R_b, R_s) \right\}^2},$$

where $Q(p_j)$ and $\mathfrak{Q}(p_j)$ denote a quantile function $Q(p) = \inf\{\ln W_i : p \leq F(\ln W_i)\}$ and $\mathfrak{Q}(p) = \inf\{a_i : p \leq \mathfrak{F}(a_i)\}$, p_j is a j-th element of a quintile, F and $\mathfrak{F}$ denote an empirical CDF, and $\mathfrak{f}_{\theta'}$ denotes a weighted sum of the policy functions where the weights reflect stationary probability of each state. Algorithm 2 gives a pseudo code to implement the estimation algorithm.

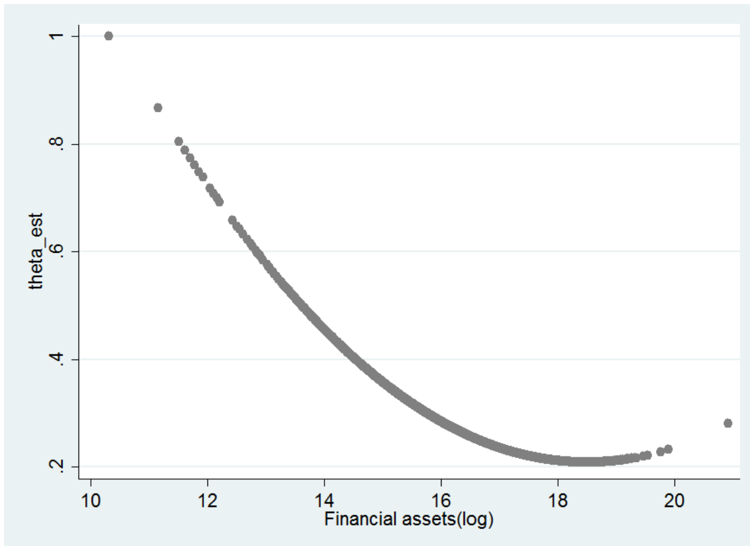

Figure 4.4: **Modified conditional portfolio stock share.** The figure shows the portfolio stock share, linearly modified with OLS coefficients.

Algorithm 2

Sample N sets of parameters from prior distributions
repeat
  for n to N **do**
    Simulate the model on n-th proposal, compute stationary distribution and stock share on true wealth quintiles by Algorithm 1.
    Compare summary statics of distributions.
    Compare summary statics of stock holding policies.
  end for
  Perturb the parameter proposals following normal random walk Metropolis-Hastings.
  Update the importance weight, anneal convergence criteria via computing effective sample size (ESS).
  Re-sample particles if necessary following the systematic scheme proposed by Kitagawa (1996).
until Convergence criteria are met.

4.4.2 Empirical outcome

First, I summarize the estimation settings and then show the estimation outcomes. With respect to the ABC algorithm, the number of particles is set to $N = 100$, no iteration are performed ($M = 1$), the quality index is set to $\alpha_{ABC} = .90$, convergence criterion is set to $\epsilon = .90$, and prior distributions are set to be such that $\mu \sim \mathcal{N}(15, 2)$, $\rho \sim \mathcal{BETA}(5, 2)$, $\sigma_w \sim \mathcal{IG}(3, 4)$, and $\sigma \sim \mathcal{U}(0, 50)$. Because there is no iteration, ESS is directly proportional to the number of alive particles.

The estimation results are summarized in Table 4.4. They imply that the RRA takes plausible value, compared to the estimates in existing literature. Accordingly, the equity premium puzzle can be interpreted as an upward bias by the specification error in the representative agent economy, which considers neither market incompleteness nor limited participation. In addition, we find that the persistence of the dynamic earnings process is around 0.87 (median) with 95% credible interval: $[0.80, 0.95]$, which is consistent with the estimates of Browning, Ejrnæs and Alvarez (2010) (0.793 (median)) and Gustavsson and Österholm (2014) ([0.81 0.98] (95% (median)).

	μ	ρ	σ_w	σ
Prior mean	14.70	0.70	2.07	23.98
s.d.	1.85	0.16	1.44	15.50
Posterior mode	11.94	**0.84**	0.86	**3.34**
Posterior mean	12.22	**0.87**	1.01	**3.46**
Credible interval	[11.12, 13.56]	**[0.80, 0.95]**	[0.33, 1.75]	**[2.94, 4.01]**

Table 4.4: **Summary outcomes.** This table summarizes prior and posterior mean, mode, standard deviation, and 95% credible intervals.

Figure 4.5 compares the observed wealth distribution and the simulated distribution using posterior means and Figure 4.6 shows the observed stock holding shares and the simulated stock holding policy using posterior means[8], defined on true wealth

[8]The simulated stock holding policy is a weighted sum of the policy functions where the weights reflect stationary probability of each state.

quintiles. We find that the simulated policy matches the observed policy well in Figure 4.6, and that the simulated average conditional stock share matches the observed value in Table 4.5. We also find that the mean of the simulated distribution (16.39) is the same as that of the observed distribution (16.44), the median of the simulated (16.54) is the same as that of the observed (16.56), and the standard deviation is less dispersed (0.77 for the simulated and 1.11 for the observed). This simulated error comes from mainly two reasons. (i) I did not consider preference heterogeneity discussed by Krusell and Smith (1997), and (ii) the model does not care for demographic heterogeneity, such as age. Thus, if the financial wealth level depends on such a heterogeneity, that specification error would affect the simulated outcome.

Finally, I checked the validity of the calibration and estimation with the aggregate consumption-wealth ratio in Table 4.6. There are two reasons to use the aggregate consumption-wealth ratio. First, because the calibration targets only the financial wealth level and stock share, the ratio of the endogenous consumption level of these calibrated values is a good measure of fitness. Secondly, the aggregate consumption-wealth ratio is empirically important. Since the ratio is equivalent to the conditional expectation of difference between returns from the market portfolio and the consumption growth rate, it functions as a strong predictor of excess stock market returns (Lettau and Ludvigson (2001), Lettau and Ludvigson (2010)). As an empirical counterpart of the household consumption level, I utilize household consumption for nondurable goods in the NSFIE. Then, I compute the simulated consumption level with stationary wealth distribution, policy function and randomly generated states reflecting the stationary probability of each state. We find that the log consumption-wealth ratio of the observed and the simulated distributions are almost the same. Thus, the calibration and estimations in this study are valid.

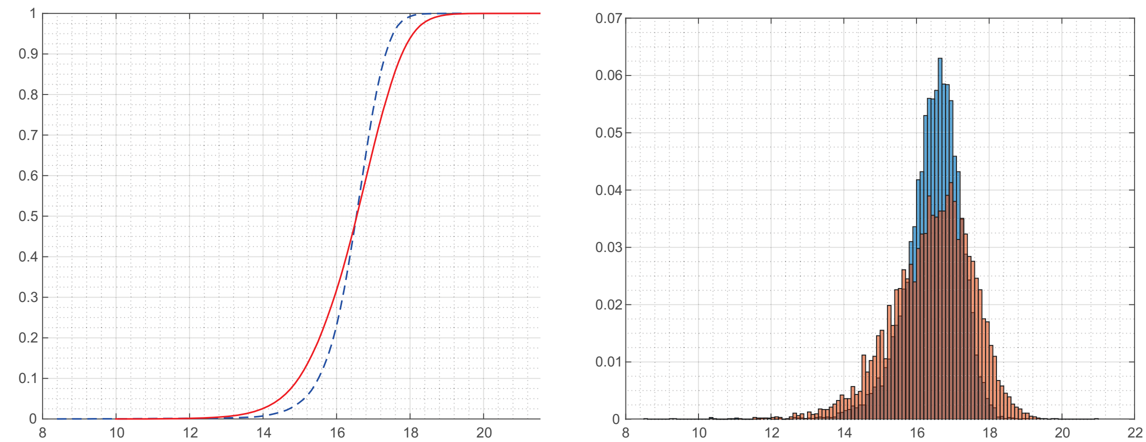

Figure 4.5: **Distribution comparison.** The figure compares the pdfs on the right and the cdfs on the left. The vertical axis measures the density and the horizontal axis measures the financial wealth level(log). The red histogram shows the true asset distribution and the blue one shows the computed stationary distribution. The red line shows the true cdf and the blue dotted line shows the computed cdf.

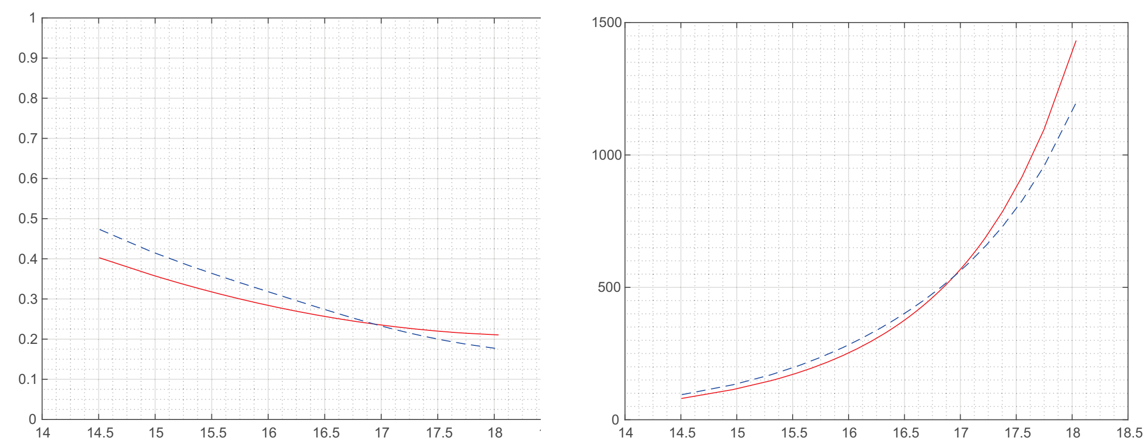

Figure 4.6: **Stock holding policy comparison.** The left figure compares the stock holding policy in each quintile and the right figure compares approximated stock holdings (10,000 yen). In the left figure, the vertical axis measures the portfolio stock share and the horizontal axis measures the quintiles; in the right figure, the vertical axis measures the approximated stock holdings and the horizontal axis measures asset quintiles. The red line denotes the observed values and the blue line shows the simulated values.

4.5 Conclusion

This chapter provides an overview of the empirical facts about portfolio allocation by Japanese households, defines a stochastic dynamic optimization problem with

	$\overline{\theta_i}$
Observed value	0.27
Simulated value	0.28

Table 4.5: **Average conditional stock share.** The table compares the observed and the simulated conditional stock shares. To compute the simulated conditional share, I generate random states following the stationary probability of each state, interpolate the simulated values of the stationary wealth distribution via stock policy functions.

	$\overline{c_i/a_i}$
Observed value	0.86
Simulated value	0.85

Table 4.6: **Log consumption-wealth ratio**. I use the household consumption for nondurable goods in the NSFIE as the observed consumption level; in order to compute the simulated consumption level, I generate the random states following stationary probability of each state, and interpolate the simulated values on the stationary wealth distribution via consumption policy functions.

heterogeneous investors who decide the dynamic portfolio allocation, estimates the structural parameters of the model with the NSFIE, and computes parametric density estimates of structural parameters. Compared to existing studies, I emphasize not only the validity of the distribution (as in Krusell and Smith (1997)), but also that of the allocation. This is because I find that the conditional stock share is negatively correlated with the financial wealth level. To test the theoretical implications about the distribution and the allocation, I construct the model with two kinds of heterogeneity: incomplete market and limited participation, and implement the structural estimation algorithm based on the adaptive sequential Monte Carlo approximate Bayesian computation. The estimation outcome shows that the RRA estimate takes a plausible value relative to the estimates in the existing literature. The estimation outcome implies that considering two kinds of heterogeneity can make the model more reliable, and the equity premium puzzle can be due to upward bias from a specification error associated with the representative agent assumption.

Appendix A

Details of the computation

The solution algorithm follows a modified version of Maliar et al. (2010). The state space for household capital, k_i, is discretized by 100 grids in the range $[-\phi, 1000]$. The upper bound is chosen to be sufficiently high so that the households do not reach the upper bound in the simulated paths. The number of grids is chosen to be sufficiently high so that a further increase of the grid number will not change the simulated mean capital. To capture the curvature of the policy functions, we take the grids densely toward $-\phi$. Specifically, we set $(k_i + \phi)^{0.25}$ to be equally spaced. The state space for the mean capital is discretized by four grids.

Given the approximated law of motion for the joint distribution of the capital holding and employment state, we obtain a policy function by iteration of the Euler equation. To evaluate the policy function at the forecasted mean capital in the next period, we interpolate the policy function in mean capital by the cubic spline method.

Once the policy function is obtained, we simulate the equilibrium path with 10,000 households for 3,000 periods. In each simulation period, the policy function is interpolated at the current mean capital level by the spline method, and the interpolated policy function, which is evaluated at the current mean capital and aggregate state, is further fitted by a quadratic function for each employment state. Fitting by the higher-degree polynomial functions does not alter the results. The fitted function is then used to compute the capital holding for each household in the next period. We use the simulated mean capital path for the last 2,000 periods to estimate the law of motion of the form in Equation (2.8). The convergence criterion for the value function iteration is 1.e-8 in the sup norm. The convergence criterion for the law of motion is 1.e-10 for all coefficients in Equation (2.8).

Appendix B

Other simulated moments of interest

Table B.1 lists the other estimates. C^e and C^u denote the consumption per worker for the employed and unemployed households, respectively, that is time-averaged for all periods through policy transitions. Column C^e/C^u gives the ratio of the average consumption of the employed and unemployed. Although the households partially hedge their unemployment risk by accumulating wealth, this shows that a substantial gap (4.12%) remains uninsured. Table B.2 shows the approximated law of motion for the aggregate capital. The high R^2 shows that the approximation is accurate.

	C^e/C^u	C	I/Y	$\bar{K}$
GE I	1.0412	2.5899	0.2569	35.8107
	(0.0010)	(0.0001)	(0.0000)	(0.0040)
GE II	1.0708	2.5591	0.2647	35.7585
	(0.0034)	(0.0009)	(0.0002)	(0.0072)
Tax I	1.0468	2.5936	0.2521	34.9805
	(0.0011)	(0.0008)	(0.0001)	(0.0150)
Tax II	1.0469	2.5242	0.2719	34.9874
	(0.0021)	(0.0015)	(0.0001)	(0.0207)

Table B.1: Other estimates 1

	$\hat{R}_0^2$	$\hat{a}_0$	$\hat{b}_0$	$\hat{R}_1^2$	$\hat{a}_1$	$\hat{b}_1$
GE I	0.9988	0.0208	0.9942	0.9997	0.0548	0.9847
	(0.0003)	(0.0055)	(0.0015)	(0.0001)	(0.0036)	(0.0010)
GE II	0.9999	0.1653	0.9537	0.9999	0.1540	0.9570
	(0.0000)	(0.0032)	(0.0009)	(0.0000)	(0.0022)	(0.0006)
Tax I	1.0000	0.1402	0.9605	1.0000	0.1378	0.9613
	(0.0000)	(0.0010)	(0.0003)	(0.0000)	(0.0008)	(0.0002)
Tax II	1.0000	0.1358	0.9616	1.0000	0.1353	0.9621
	(0.0000)	(0.0010)	(0.0003)	(0.0000)	(0.0006)	(0.0002)

Table B.2: Other estimates 2

Appendix C

Sensitivity analysis

C.1 Risk aversion

	$\sigma = 1$			$\sigma = 2$			$\sigma = 5$		
z	C^e_z	C^u_z	C_z	C^e_z	C^u_z	C_z	C^e_z	C^u_z	C_z
0	2.5974	2.4682	2.5896	2.5971	2.4856	2.5912	2.6002	2.4994	2.5942
1	2.5942	2.5188	2.5905	2.5943	2.5295	2.5904	2.5979	2.5393	2.5951

Table C.1: Same as Table 2.2

	$\bar{K}$	Log diff	K effect	Risk effect $(1-u_0)\log c^e_1/c^e_0$	$u_1 \log c^u_1/c^u_0$	$(u_0-u_1)\log c^e_1/c^u_0$
$\sigma = 1$	35.8084	0.0004	0.0000	-0.0005	0.0002	0.0005
$\sigma = 2$	35.8841	0.0003	0.0000	-0.0004	0.0002	0.0004
$\sigma = 5$	36.2699	0.0004	0.0000	-0.0004	0.0002	0.0004

Table C.2: Same as Table 2.3

The policy functions (Figure C.1) show that higher risk aversion results in lower consumption levels and stronger nonlinearity (at the consumption levels not influenced by minimum transfer $\iota(0)$). This is because the higher risk aversion induces more precautionary savings and less consumption.

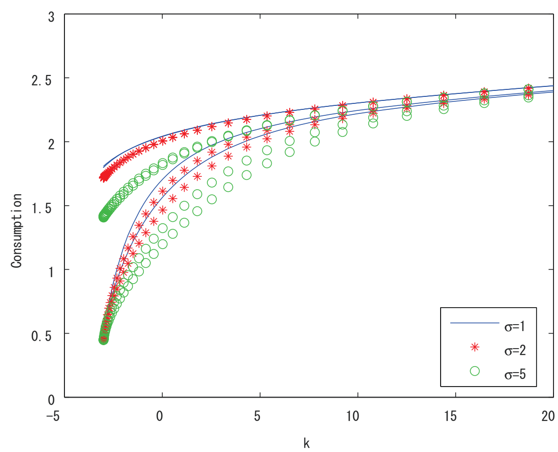

Figure C.1: Policy functions with different risk aversions

	$\bar{K}$	Y	C
$\phi = 0$	35.8112	3.4854	2.5901
$\phi = 3$	35.8093	3.4853	2.5901

Table C.3: Mean capital, aggregate production, and consumption

C.2 Debt limits

	$\phi = 0$			$\phi = 3$		
z	C_z^e	C_z^u	C_z	C_z^e	C_z^u	C_z
0	2.5969	2.4752	2.5896	2.5974	2.4682	2.5896
1	2.5941	2.5218	2.5906	2.5942	2.5188	2.5905

Table C.4: Same as Table 2.2

	Log diff	K effect	Risk effect		
			$(1-u_0)\log c_1^e/c_0^e$	$u_1 \log c_1^u/c_0^u$	$(u_0-u_1)\log c_1^e/c_0^u$
$\phi = 0$	0.0004	0.0000	-0.0004	0.0002	0.0004
$\phi = 3$	0.0004	0.0000	-0.0005	0.0002	0.0005

Table C.5: Same as Table 2.3

The policy function (Figure C.2) shows that the aggregate consumption decreases as the borrowing constraint is relaxed (greater ϕ). This is because a looser credit constraint makes the households less motivated to retain precautionary savings and thus the aggregate capital decreases. The lower aggregate capital results in lower output and the consumption level decreases.

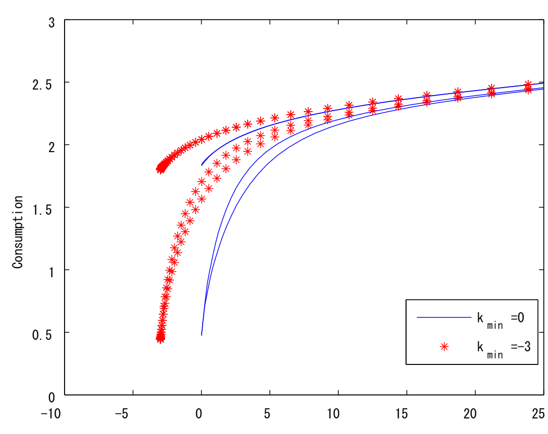

Figure C.2: Policy functions with different borrowing constraints

C.3 Disutility from the labor supply

In order to incorporate the disutility from labor in our analysis, we modify the momentary utility function as $\left((c_t^{1-\psi}(1-h_t)^{\psi})^{(1-\sigma)}-1\right)/(1-\sigma)$. Households decide the hours worked h_t when they are employed. The aggregate hours also become endogenous, and hence, households need to forecast the evolution of the aggregate hours to form expectations on future prices. We approximate the expected aggregate hours as a log-linear function of the contemporaneous mean capital level. In the GE I model, we obtain regression outcomes for $\psi = 0.1$ as:

$$\log L_0 = -0.0765 - 0.0289 \log \bar{K}_0 \quad \bar{R}_0^2 = 0.2447$$
$$\log L_1 = -0.0888 - 0.0253 \log \bar{K}_1 \quad \bar{R}_1^2 = 0.2176.$$

$\bar{R}^2$ is low because the aggregate employment in the productive sector is constant across policies in GE I. Thus, to improve the regression accuracy, we choose to work in TAX I, where the employment in the productive sector changes across policies. The regression results in TAX I are as follows:

$$\log L_0 = -0.0773 - 0.0301 \log \bar{K}_0 \quad \bar{R}_0^2 = 0.9050$$
$$\log L_1 = -0.0763 - 0.0303 \log \bar{K}_1 \quad \bar{R}_1^2 = 0.9149.$$

The inclusion of leisure implies a relatively high utility for the unemployed. This lowers the precautionary savings and aggregate capital leading to a lower consumption level.

	$\psi = 0$			$\psi = 0.1$		
z	C_z^e	C_z^u	C_z	C_z^e	C_z^u	C_z
0	2.6010	2.4552	2.5923	2.3034	2.2293	2.2989
1	2.6021	2.5161	2.5980	2.3069	2.2629	2.3048

Table C.6: Same as Table 2.2

	Log diff	K effect	Risk effect		
			$(1-u_0)\log c_1^e/c_0^e$	$u_1 \log c_1^u/c_0^u$	$(u_0-u_1)\log c_1^e/c_0^u$
$\psi = 0$	0.0004	0.0000	-0.0005	0.0002	0.0005
$\psi = 0.1$	0.0003	0.0000	-0.0005	0.0002	0.0005

Table C.7: Same as Table 2.3

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