Summary

This dissertation consists of four chapters studying the nonlinear stochastic dynamic optimization model with heterogeneous agents. In general, heterogeneity in economics is generally categorized into three groups following Browning, Hansen and Heckman (1999) and Blundell and Stoker (2005): (i) heterogeneity in individual tastes and incomes, (iii) heterogeneity in wealth and income risks faced by individuals, and (iii) heterogeneity in market participation. Though this classification is empirically useful, when modeling a micro-founded heterogeneous agents behavior, it is not necessarily clear where the line between exogenous factors and endogenous outcomes lies (Heathcote, Storesletten and Violante (2009)). This is explained by the fact that the observed heterogeneity is generated as a compound of exogenous innate characteristics (ex-ante heterogeneity), exogenous or endogenous subsequent stochastic shocks, and endogenous rational choices based on individual states. I mainly focus on the second heterogeneity which builds on an incomplete market structure where agents are ex-ante homogeneous and ex-post heterogeneous through exogenous idiosyncratic shock history across the agents. Idiosyncratic shock is not directly insurable but is insured partially by trading an asset subject to a limit or accumulating the asset as a buffer stock (self-insurance). That is, we take heterogeneity in earnings history as given and generate the endogenous heterogeneity in consumption and wealth. This specification is appealing because it enables us to disentangle quantitatively how much we can account for the ex-post heterogeneity by incomplete markets without assuming ex-ante unobservable heterogeneity (e.g. preference).

Krusell and Smith (2006) discussed that it is important to consider a heterogeneous population structure for at least two reasons, a robustness check on the representative-agent model and a growing interest in distributional issues. The robustness check is done on the representative-agent assumption which treats all agents

as identical and idiosyncratic risks as perfectly diversifiable. Since there is a possibility that ignoring heterogeneity may affect the aggregate implication or cause aggregation bias, robustness must be checked both theoretically and empirically. Guvenen (2011) discussed that this use of heterogeneity is less obvious because theoretical and numerical studies have already confirmed that certain types of heterogeneity do not change the aggregate implication. Levine and Zame (2002) theoretically showed that if we assume an exchange economy with a single consumption good and incomplete markets where infinitely-lived agents have an access to a single risk-free asset and share the common subjective discount factor, and there exists transitory idiosyncratic risk but there is neither extremely persistent idiosyncratic risk (Constantinides and Duffie (1996)) nor aggregate risk, the effect of the incomplete markets will vanish in the long run. A similar result was confirmed numerically by Krusell and Smith (1998) for the imperfect insurance economy and by Rios-Rull (1996) for the finitely-lived overlapping generations economy. These results depend on the fact that an individual's consumption policy function is approximately linear with respect to wealth even with the existence of idiosyncratic risk, except for the wealth levels near the borrowing constraint.[1]

The second reason for considering a heterogeneous population structure is a growing interest in distributional issues or inequality (disparity), especially in conjunction with macroeconomic forces or policies, that leads to different policy implications. For example, business cycles and inflation are likely to have asymmetric welfare effects across agents depending on their respective wealth levels and compositions. So, when evaluating policy implications, we should take into consideration not only traditional general equilibrium effects, but also asymmetric reactions caused by inequality. Following these studies, we can finally evaluate (i) how a stabilization policy designed to

[1] We can also find a case where aggregate approximation cannot function. For example, Chang and Kim (2007), Takahashi (2014), Chang and Kim (2014), and An, Chang and Kim (2009) studied indivisible labor supply.

lessen the aggregate time-series volatility can affect cross-sectional distribution and (ii) how reallocation policy designed to lessen cross-sectional inequality (disturbance) can affect the aggregate time-series volatility, as discussed by Heckman (2001), Lucas (2003), and Heathcote et al. (2009).

In addition to the two traditional reasons discussed above, I point out a third reason to consider a heterogeneous population structure, which enables us to use rich micro data for structural estimation. By employing micro data (especially, panel data) instead of aggregate time series data, we can exclude potential aggregation biases from fundamental microeconomic dynamics and can consider heterogeneity. (Bond (2002)) I try to explain the advantage by comparing the representative-agent formulation with a heterogeneous population structure. We first consider the calibration or estimation of parameters in the representative-agent formulation. Conditional on exogenous shocks, the representative-agent formulation can compute the unique one-dimensional steady-state aggregate capital stock level, and then generate a joint probability distribution for endogenous variables such as output and consumption. Therefore, we can use the aggregate statistics and their time series as empirical counterparts of the endogenous values for estimation. In contrast, the incomplete market structure with no aggregate risk can generate the unique infinite-dimensional stationary cross-sectional wealth distribution as an equilibrium object. Therefore, we can employ not only one-dimensional aggregate statistics, but also N-dimensional individual statistics as empirical counterparts of the endogenous values for estimation. It implies that if we adopt the incomplete market structure, we can make the most of rich micro data sources, —ranging from cross-sectional surveys to panel data,— to calibrate or estimate the structural parameters.

A common strategy for parametrization in the incomplete market literature is a combination of external calibration with moment matching: to minimize the distance between simulated moments and empirical moments based on N-dimensional individual statistics. One of the drawbacks in the strategy is that it cannot exploit all the available information from the data, especially the distribution. This disadvantage is clearly revealed when the stationary equilibrium distribution is a mixture distribution where the moments are not the right statistics to summarize the distribution. In contrast, density estimators can provide more information than estimators using the mode or a finite set of moments (Liao and Stachurski (2015)). So, there is a need to develop the structural density estimation method for the incomplete market model to take advantage of the distributional information in rich micro data sources.

The idea of a structural estimation strategy to minimize the distance between the empirical distribution and simulated distributions looks simple, but its implementation is rather difficult. The biggest impedance is the fact that the stationary equilibrium distribution of the incomplete market model has no analytical expression and accordingly, we cannot employ the standard maximum likelihood (ML) procedure because we cannot calculate the likelihood. Instead of using the ML procedure, previous works in macroeconomics literature calibrated parameters with relevant microeconometric reduced-form estimates or moment matching indirect inference (II) type estimates[2]. However, there are several problems in using these methods. When calibrating the parameters with reduced-form estimates, (i) we cannot incorporate the theoretical restriction into the estimation procedure and (ii) there is little guidance from econometric theory to choose an estimation technique, each of which makes different assumptions on the error term. When calibrating the parameters with II-type estimates, we can perform a calibration and its statistical test simultaneously, however, (iii) the finite sample properties of estimates are poor and (iv) the ignorance of distribution may lead to biased estimates, except for the first moment.

[2]Specifically, indirect inference estimator, simulated method of moments estimator (SMM or MSM), and efficient method of moments estimator (EMM) are classified into this class of estimators. These take the form of continuous-updating generalized method of moments estimator (GMM) and asymptotically equivalent. (See chapter 3)

To implement the density matching algorithm, I propose applying the Approximate Bayesian Computation (ABC) algorithm to the empirical cross-sectional distribution. ABC is a Bayesian statistical method for likelihood-free inference. When estimating the structural parameters of the incomplete market model, we only specify the data generating process. In other words, equilibrium values can be generated conditional on parameters, but that is all we know about the likelihood; we have no information of the likelihood itself. ABC is the optimal estimation method in such a case, and by using ABC to minimize the distance of distributions, (i) we can incorporate theoretical restrictions into the estimation procedure, (ii) we can exclude the arbitrary process of selecting estimation methods, (iii) we can appreciate nice finite sample property, and (iv) we can employ the distributional information, not only the mode or a set of finite moments.

The key insight of ABC is that the calculation of likelihood can be replaced by a comparison process between observations and simulated values. For high dimensional data spaces, we rarely match these values and thus we usually compress these values into a finite set of summary statistics. Since the estimation accuracy depends on how well summary statistics epitomize the data, the summarization of the infinite-dimensional distribution is significant. To summarize the distribution, we first consider a naive two-step approach where we first separately estimate each density and then compute their distance using measures such as the Kullback-Leibler divergence. However, there are some problems in this two-step approach. First, it is not effective in that the first step estimation does not consider the second step's computing process. An estimation error which comes from the neglect of the second step can generate a big estimation error. Second, although minimizing the Kullback-Leibler divergence is statistically equivalent to maximizing likelihood, it cannot satisfy the properties of a mathematical metric such as the symmetric property and triangle inequality, it is not robust to outliers, and is numerically unstable. So, instead of using the naive two-step approach, I employ the L^2-distance approximation method (Sugiyama, Suzuki, Kanamori, du Plessis, Liu and Takeuchi (2013)).

Chapter 2 is based on the joint work with Makoto Nirei and Sanjib Sarker (Yamana, Nirei and Sarker (2016)). In this chapter, we examine the response of aggregate consumption to active labor market policies that reduce unemployment. We develop a dynamic general equilibrium model with heterogeneous agents and uninsurable unemployment risk as well as policy regime shocks to quantify the consumption effects of policy. By implementing numerical experiments using the model, we demonstrate a positive effect on aggregate consumption even when the policy serves as a pure transfer from the employed to the unemployed. The positive effect on consumption results from the reduced precautionary savings of households who indirectly benefit from the policy by decreased unemployment hazard in future.

Chapter 3 presents a structural estimation method for nonlinear stochastic dynamic models of heterogeneous firms. As a result of technical constraints, there is still no consensus on the parameters of a productivity process. In order to estimate the parameters, I propose a Bayesian likelihood-free inference method to minimize the density difference between the cross-sectional distribution of the observations and the stationary distribution of the structural model. Because the stationary distribution is a nonlinear function of a set of the structural parameters, we can estimate the parameters by minimizing the density difference. Finally, I check the finite sample property of this estimator using Monte Carlo experiments, and find that the estimator exhibits a comparatively lower root mean squared error in almost all the experiments.

Finally, chapter 4 studies a structural estimation method for the nonlinear stochastic dynamic optimization model with heterogeneous households, and then conducts the empirical research about the household asset allocation behavior. The analysis of household finance has non-negligible implications in asset pricing literature, but empirical research on this topic is challenging. To solve the equity premium puzzle, I consider two kinds of heterogeneity across households: wealth heterogeneity and limited stock market participation. Then, I summarize the empirical facts about household investment portfolio with the National Survey of Family Income and Expenditure, a cross-sectional Japanese household survey. Because we cannot observe the dynamics of the individual portfolio with the cross-sectional data, we cannot estimate the structural parameters of the dynamic model. I propose the Bayesian likelihood-free inference method to minimize both the density difference and the distance between policy functions, between the observed and the simulated values. Because the stationary distribution and the policy function are nonlinear functions of a set of structural parameters, we can estimate the parameters by minimizing the density difference and the distance between policy functions. We can find that the estimated relative risk aversion is around four. The estimation outcome implies that the model can mimic the observed household finance behavior well and the equity premium puzzle comes of a biased estimate caused by the representative agent assumption.

Bibliography

An, Sungbae, Yongsung Chang, and Sun-Bin Kim, "Can a representative-agent model represent a heterogeneous-agent economy," *American Economic Journal: Macroeconomics*, 2009, *1* (2), 29–54.

Blundell, Richard and Thomas M. Stoker, "Heterogeneity and aggregation," *Journal of Economic Literature*, 2005, *43* (2), 347–391.

Bond, Steve, "Dynamic panel data models: a guide to microdata methods and practice," CeMMAP working papers CWP09/02, Centre for Microdata Methods and Practice, Institute for Fiscal Studies 2002.

Browning, Martin, Lars Peter Hansen, and James J. Heckman, "Micro data and general equilibrium models," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics* 1999, chapter 8, pp. 543–633.

Chang, Yongsung and Sun-Bin Kim, "Heterogeneity and aggregation: implications for labor-market fluctuations," *American Economic Review*, 2007, *97* (5), 1939–1956.

_ and _ , "Heterogeneity and aggregation: implications for labor-market fluctuations: reply," *American Economic Review*, 2014, *104* (4), 1461–66.

Constantinides, George M and Darrell Duffie, "Asset Pricing with heterogeneous consumers," *Journal of Political Economy*, 1996, *104* (2), 219–40.

Guvenen, Fatih, "Macroeconomics with heterogeneity: a practical guide," *Economic Quarterly*, 2011, (3Q), 225–326.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, "Quantitative macroeconomics with heterogeneous households," *Annual Review of Economics*, 2009, *1* (1), 319–354.

Heckman, James J., "Micro data, heterogeneity, and the evaluation of public policy: Nobel lecture," *Journal of Political Economy*, 2001, *109* (4), 673–748.

Krusell, Per and Anthony A. Smith, "Income and wealth heterogeneity in the macroeconomy," *Journal of Political Economy*, 1998, *106*, 867–896.

— **and** —, "Quantitative macroeconomic models with heterogeneous agents," in "Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress" 2006.

Levine, David K. and William R. Zame, "Does market incompleteness matter?," *Econometrica*, 2002, *70* (5), 1805–1839.

Liao, Yin and John Stachurski, "Simulation-based density estimation for time series using covariate data," *Journal of Business & Economic Statistics*, 2015, *33* (4), 595–606.

Lucas, Robert E. Jr., "Macroeconomic priorities," *American Economic Review*, 2003, *93* (1), 1–14.

Rios-Rull, Jose-Victor, "Life-cycle economies and aggregate fluctuations," *Review of Economic Studies*, 1996, *63* (3), 465–89.

Sugiyama, M., T. Suzuki, T. Kanamori, M. C. du Plessis, S. Liu, and I. Takeuchi, "Density-difference estimation," *Neural Computation*, 2013, *25* (10), 2734–2775.

Takahashi, Shuhei, "Heterogeneity and aggregation: implications for labor-market fluctuations: comment," *American Economic Review*, 2014, *104* (4), 1446–60.

Yamana, Kazfumi, Makoto Nirei, and Sanjib Sarker, "Time-varying employment risks, consumption composition, and fiscal policy," *Economics Bulletin*, 2016, *36* (2), 802–12.