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A NOTE ON SOCIAL WELFARE ORDERS SATISFYING PIGOU-DALTON TRANSFER PRINCIPLE*

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Abstract

This paper studies the constructive nature of social welfare orders on infinite utility streams defined on $X = Y^{\mathbb{N}}$, satisfying the Pigou-Dalton transfer principle (PD), which are known to be representable (see Alcantud (2010) and Sakamoto (2012)). We describe the restrictions on domain Y for explicit representation or construction of the social welfare orders satisfying (i) PD and monotonicity; or (ii) PD only. We show that the restrictions on Y for either (a) construction; or (b) explicit representation of the social welfare orders are *identical* in both cases.

Keywords: construction, correspondence principle, non-Ramsey set, Pigou-Dalton transfer principle, representation, social welfare orders
JEL Classification Codes: D60, D70, D90

I. *Introduction*

In this paper, we consider the Pigou-Dalton transfer principle (written hereafter as PD), which shows preference for a non-leaky and non-rank switching redistribution of a good from a *rich* person to a *poor* with no-one else being affected. The redistribution is non-leaky in the sense that the gain to the poor person is exactly equal to the loss suffered by the rich person and non-rank switching in the sense that the poor does not end up having more than the other. The redistribution is inequality reducing as was initially hinted at by Pigou (1912, p. 24) as "The Principle of Transfers" and recognized by Dalton (1920, p. 351), as "If there are only two income-receivers and a transfer of income takes place from the richer to the poorer, inequality is diminished.". In the literature on *intergenerational equity*, this scheme of re-distribution is

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known as an illustration of *consequentialist* equity. It is a core principle on measuring inequality in the sense that any index of inequality must register a Pigou-Dalton transfer in income as reducing the measure of inequality. Philosophers have justified Pigou-Dalton transfer as a principle of *distributive justice* (see Adler (2013)).

We focus on the social welfare order (SWO), a complete and transitive binary relation on the set X of infinite utility streams, where X takes the form of $X = Y^{\mathbb{N}}$, with Y being a non-empty sub-set of real numbers $\mathbb{R}$, and $\mathbb{N}$ being the set of natural numbers. Early contributions on the representation of SWO satisfying PD with finitely many agents are Atkinson (1970) and Dasgupta et al. (1973). Dasgupta et al. (1973, Lemma 2) give an elegant statement of the representation theorem, which is based on the theory of mathematical inequalities by Hardy et al. (1952). In recent literature, PD has been applied to more general economies with infinitely many agents (or equivalently to the infinite utility streams). Some of the initial contributions defining the version of PD in the infinite horizon framework are Sakai (2006, p. 44), Hara et al. (2008, p. 183), and Bossert et al. (2007, p. 591), who describe it as "strict transfer principle".

In the general infinite utility setting, PD involves comparing a pair of utility streams (x and y) in which all generations except two have the same utility levels in both utility streams. Regarding the two remaining generations (say, i and j), one of the generations (say i) is better off in utility stream x, and the other generation (j) is better off in utility stream y, thereby setting up a conflict. If for both utility streams, generation j is at least as well off as generation i (so that we have $y_j > x_j \geq x_i > y_i$) and x can be obtained from y by transferring utility from generation j to generation i (so that $y_j - x_j = x_i - y_i$), then x is socially preferred to y.

We take a brief detour to describe results on SWOs satisfying procedural equity (anonymity) and efficiency. In a seminal paper, Diamond (1965) proved that there does not exist any SWO satisfying anonymity (who defined it as "equal treatment" of all generations (present and future)), Strong Pareto and continuity with respect to sup-metric. It could be used to infer that there is no social welfare function (SWF), a real valued map on X, which respects anonymity, Strong Pareto, and sup-metric continuity. Basu and Mitra (2003) refined this result by showing that the impossibility persists even when the continuity axiom is dropped. Svensson (1980) showed that it is possible to escape the negative outcome by discarding both the sup-metric continuity and representability conditions. He proved that there exists a SWO satisfying anonymity and Strong Pareto axioms. However, the existence result was established by using the variant of Szpilrajn's Lemma given in Arrow (1951), which is a non-constructive device.

Fleurbaey and Michel (2003) explored the necessity of use of such non-constructive techniques and conjectured that it may not be possible to explicitly describe the SWO. The conjecture, if found to be true, could have important consequences since the SWO would then be of limited practical use in policy making. The conjecture received wide attention owing to its significance in applications. Lauwers (2010) and Zame (2007) using alternative techniques proved the necessity of reliance on some non-constructive device to establish the existence result, thereby confirming the conjecture. Lauwers (2010) relied upon the existence of non-Ramsey sets to establish the non-constructive nature of the SWOs. Dubey (2011) using technique devised in Lauwers (2010), further refined the results obtained by Lauwers (2010) and characterized the domain Y, for which the conjecture of Fleurbaey and Michel (2003) holds. It led to a correspondence principle that if there is no SWF satisfying the Weak Pareto and anonymity axioms, then there is no SWO, satisfying Weak Pareto and anonymity axioms, which can be constructed.

This set of results relating to procedural equity inspired similar analysis in the case of consequentialist equity conditions. In the literature on consequentialist equity, there are several instances of the existence of equitable and efficient SWOs which are not representable. If the SWO is shown to exist using some form of the *Axiom of Choice* (written hereafter as AC), then it remains an open question if the SWO admits explicit construction. An affirmative answer would enable the social planner to at least be able to do the pair wise ranking of infinite utility streams and would be useful to the policy makers. In this paper, we pursue this line of investigation in the case of PD.

It is well known that SWO satisfying PD exists (Bossert et al. (2007, Theorem 1)). However, this existence result is derived by using the variant of Szpilrajn's Lemma given in Arrow (1951), which is a non-constructive device. Dubey and Mitra (2014b, Proposition 2), show that for $Y=[0,1]$, the existence of SWO satisfying PD implies the existence of a *non-Ramsey* set, which is a non-constructive object. It implies that a SWO satisfying PD cannot be constructed over infinite utility streams when the domain set is $Y=[0,1]$.

In two recent contributions, Alcantud (2010, Proposition 5) and Sakamoto (2012, Proposition 3) have proved the existence of a representable SWO satisfying PD, when the domain set is $Y=[0,1]$. The techniques used by them are similar to that introduced by Basu and Mitra (2007, Proposition 1), who show the existence of a representable SWO satisfying the anonymity and an efficiency principle known as *weak dominance*. However, this existence result uses AC and therefore the representation could potentially be non-constructive.

The possibility of representable SWOs satisfying PD allows us to enrich the research question posed earlier. If the representation admits explicit description, then the SWO itself becomes constructive. A positive answer would demonstrate that the use of AC in the existence proof of equitable SWO is not essential. Negative outcome would on the other hand lead to a version of correspondence principle, i.e., the domains for which a representable SWO satisfying PD admits explicit description is the same as the domains for which the representable SWO satisfying PD is constructive.

Alcantud (2012, Proposition 1) and Sakamoto (2012, Proposition 5) have shown that there does not exist a SWF satisfying PD and Weak Pareto axiom for $Y \supset [0,1]$. Therefore, we combine the weaker efficiency condition of monotonicity with the PD first, and characterize the subsets of $Y=[0,1]$ for which the SWOs are constructive. In the first set of results (Propositions 1-6), we show that the subsets Y for which the monotone SWOs satisfying PD are (i) constructive; or (ii) have a representation, coincide. This leads to a correspondence principle (Theorem 1) in line with similar results in relation to other procedural (Dubey (2011)) as well as consequentialist (Dubey and Mitra (2014a), Dubey and Mitra (2014b), and Dubey (2016)) equity notions.[1]

Even though monotonicity is a very weak efficiency axiom, it turns out to be strong enough to rule out the existence of representable SWOs for Y containing more than five distinct elements. We therefore explore the possibility of explicit representation of SWOs satisfying PD without insisting on monotonicity. It would bring out the conflict between the equity and explicit representation. Results are reported in Propositions 7-10. We show that the domain

[1] Similar correspondence principle holds for representation and construction of SWOs satisfying Strong Pareto which, in addition, do not display any impatience, see Banerjee and Dubey (2014, Theorem 2).

restrictions, for the SWOs satisfying PD (i) to be constructive; or (ii) to have an explicit representation, coincide. This leads to a variation of the correspondence principle (Theorem 2).

Rest of the paper is organized as follows. Section II contains the definitions of the concepts needed in the paper. In Section III, we consider monotone SWOs satisfying PD. Section IV deals with SWOs satisfying PD with no additional assumption. We conclude in Section V and the Appendix A contains all the proofs.

II. *Preliminaries*

1. Notation

Let $\mathbb{R}$ and $\mathbb{N}$ be the sets of real numbers and natural numbers respectively. For all $y, z \in \mathbb{R}^{\mathbb{N}}$, we write $y \geq z$ if $y_n \geq z_n$, for all $n \in \mathbb{N}$; we write $y > z$ if $y \geq z$ and $y \neq z$; and we write $y \gg z$ if $y_n > z_n$ for all $n \in \mathbb{N}$.

2. Definitions

Let Y, a non-empty subset of $\mathbb{R}$, be the set of all possible utilities that any generation can achieve. Then $X \equiv Y^{\mathbb{N}}$ is the set of all possible utility streams. If $x \in X$, then $x = (x_1, x_2, \cdots)$, where, for all $n \in \mathbb{N}$, $x_n \in Y$. We consider binary relations on X, denoted by $\succsim$, with symmetric and asymmetric parts denoted by $\sim$ and $\succ$ respectively, defined in the usual way. A *social welfare order* (SWO) is a complete and transitive binary relation. A *social welfare function* (SWF) is a mapping $W: X \rightarrow \mathbb{R}$. Given a SWO $\succsim$ on X, we say that $\succsim$ can be *represented* by a real-valued function if there is a mapping $W: X \rightarrow \mathbb{R}$ such that for all $x, y \in X$, we have $x \succsim y$ if and only if $W(x) \geq W(y)$.

1) Equity and Efficiency Axioms

The following consequentialist equity and efficiency axioms on social welfare orders are used in this paper.

Definition. Pigou-Dalton transfer principle (PD): If $x, y \in X$, and there exist $i, j \in \mathbb{N}$, such that $y_j > x_j \geq x_i > y_i$, and $y_j + y_i = x_j + x_i$ while $y_k = x_k$ for all $k \in \mathbb{N} \setminus \{i, j\}$, then $x \succ y$.

Definition. Monotonicity (M): If $x, y \in X$, with $x \geq y$, then $x \succsim y$.

2) Ramsey and Non-Ramsey Collections of Sets

Let T be an infinite subset of $\mathbb{N}$. We denote by $\Omega(T)$ the collection of all infinite subsets of T, and we denote $\Omega(\mathbb{N})$ by Ω. Thus, for any infinite subset T of $\mathbb{N}$, we have $T \subset \mathbb{N}$, and $T \in \Omega$. A collection of sets $\Gamma \subset \Omega$ is called *Ramsey* if there exists $T \in \Omega$ such that either $\Omega(T) \subset \Gamma$ or $\Omega(T) \subset \Omega \setminus \Gamma$. Next we define collection of sets known as non-Ramsey.

Definition. Non-Ramsey Sets: A collection of sets $\Gamma \subset \Omega$ is non-Ramsey if for every $T \in \Omega$, the collection $\Omega(T)$ intersects both Γ and its complement $\Omega \setminus \Gamma$.

3) Constructive Statements and Objects

Next we define statements and objects which are *constructive*. Consider the statement of the following form "There exists a SWO satisfying a given property.". This statement asserts the existence of an object (SWO, in this case) satisfying the given property. We call such statement to be constructive if it can be established in every model of Zermelo-Fraenkel (ZF) set theory, i.e., with AC (ZFC set theory); as well as without AC (ZF set theory). In this situation, we say that the object is constructive and that the object can be constructed.

However, if the statement (i) can be established in every model of ZFC set theory, but (ii) there is some model of ZF set theory (without AC) in which it cannot be established, we will say that the statement is *non-constructive*. Further, we will also say that the object is non-constructive, and that the object cannot be constructed.

For illustration, consider the statement, "There exists a non-Ramsey collection of sets $\Gamma\subset\Omega$.". This statement has been established under ZFC set theory in Erdós and Rado (1952). However, Mathias (1977) has shown that in Solovay's model $\mathfrak{m}_1$ (which satisfies ZF, but does not satisfy AC) every collection of sets $\Gamma\subset\Omega$ is Ramsey. Therefore, there is a model of ZF set theory in which the statement cannot be proved. Applying our definition, this statement is non-constructive. In addition, we say that the object "a non-Ramsey collection of sets" is a non-constructive object, and this object cannot be constructed.

We refer the reader to Dubey and Mitra (2014b, Section 2.2.5) for a detailed discussion drawn from the mathematics literature on the interpretation of "non-constructive statements and objects", which is relevant for this paper.

III. *Monotone SWF Satisfying PD*

Dubey and Mitra (2014a) have shown that SWF combining Strong Equity[2] with Monotonicity exists for Y containing not more than five distinct elements. Observe that for any pair of sequences x and y, comparing sequences using PD is a more restrictive exercise compared to using SE. Among all possible SE comparisons, only non-leaky transfers qualify for comparison using PD. Further non-rank switching condition is consistent with the possibility of the utilities of the rich and poor being equal post transfer. We continue with the domain $Y\equiv\{a, b, c, d, e\}$ with $a<b\leq c\leq d<e$ containing five (possibly distinct) elements. Observe that in order to perform any PD comparison, we need at least three distinct elements in Y. Also, elements in Y could be inconsistent for comparing utility streams using PD. In such case W: $X\rightarrow\mathbb{R}$ as $W(x)=0$ satisfies M and PD trivially.

(1) Y contains three distinct elements. Following proposition shows existence of a SWF satisfying PD and M.

Proposition 1. *If $Y=\{a, b, c\}$ is such that $a+c=2b$, then, there exists a SWF on $X\equiv Y^{\mathbb{N}}$ satisfying PD and M which is defined without using AC.*

(2) Y contains four distinct elements. Let $a+d=b+c$ and $2b=a+c$ (take $a=0$, $b=1$, $c=2$ and $d=3$). Following propositions deal with SWF and SWO in this case.

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[2] Strong Equity (SE) is defined as follows. "If $x, y\in X$, and there exist $i, j\in\mathbb{N}$, such that $y_j>x_j>x_i>y_i$ while $y_k=x_k$ for all $k\in\mathbb{N}\setminus\{i, j\}$, then $x>y$." It is a strong form of the *Equity* axiom of Hammond (1976) and was introduced by d'Aspremont and Gevers (1977) who referred to it as an *Extremist Equity Axiom*.

Proposition 2. *[Alcantud (2013, Theorem 1, footnote 3)] If $Y=\{a, b, c, d\}$ is such that $a+d=b+c$ and $2b=a+c$, then, there does not exist a representable SWO on $X\equiv Y^{\mathbb{N}}$ satisfying PD and M.*

Proposition 3. *If $Y=\{a, b, c, d\}$ is such that $a+d=b+c$ and $2b=a+c$, then, the existence of a SWO on $X\equiv Y^{\mathbb{N}}$, satisfying PD and M, entails the existence of a collection of sets $\Gamma\subset\Omega$, which is non-Ramsey.*

(3) Y contains five distinct elements, i.e. $Y\equiv\{a, b, c, d, e\}$.[3]

(a) Let $b+e=c+d$ and either (i) $2b=a+c$, or (ii) $2b=a+d$ or (iii) $2b=a+e$.[4] Following PD comparisons are possible for any pair of utility streams, with

(A) the pre-transfer rich and poor generation to have utility e and b respectively and the post-transfer rich and poor generation to have utility d and c respectively;

(B) the pre-transfer rich to have utility c, or d or e in case (i), (ii) or (iii) respectively, and the pre-transfer poor to have utility a; and the post-transfer rich and poor generation to have utility b and b respectively.

In each of these cases, we can show following Dubey and Mitra (2014a, Proposition 2) that no SWF satisfying PD and M exists. For illustration we consider the second case: $2b=a+d$.

Proposition 4. *If $Y=\{a, b, c, d, e\}$ is such that $b+e=c+d$, and $2b=a+d$, then, there does not exist a representable SWO on $X\equiv Y^{\mathbb{N}}$, satisfying PD and M.*

Following the approach in case (2), we show that the correspondence principle holds in this situation as well.

Proposition 5. *If $Y=\{a, b, c, d, e\}$ is such that $b+e=c+d$, and $2b=a+d$, then, the existence of a SWO on $X\equiv Y^{\mathbb{N}}$ satisfying PD and M, entails the existence of a collection of sets $\Gamma\subset\Omega$, which is non-Ramsey.*

(b) (i) Let $b+e=c+d$, with either $2c=a+d$ or $2c=a+e$. (take for instance $a=0$, $b=4$, $c=6$, $d=12$, and $e=14$; or $a=0$, $b=4$, $c=7$, $d=11$, and $e=14$). The utility streams can be compared using PD, with

(A) the pre-transfer rich and poor generation to have utility e and b respectively and the post-transfer rich and poor generation to have utility d and c respectively;

(B) the pre-transfer rich to have utility d or e and pre-transfer poor to have utility a and the post-transfer rich and poor generation to have utility c and c respectively;

[3] As in case (2), we exclude the case where Y contains an element which is not included in any PD transfer with the remaining four elements playing a role in a PD transfer (for example $Y=\{0, 1, 2, 3, 10\}$) where $e=10$ is not included in any PD comparison). We also exclude the following possibilities: (i) $\{a, b, c, d\}\in Y$ such that $a+d=b+c$, and $2b=a+c$; (ii) $\{a, b, c, e\}\in Y$ such that $a+e=b+c$, and $2b=a+c$; (iii) $\{a, b, d, e\}\in Y$ such that $a+e=b+d$, and $2b=a+d$; (iv) $\{a, c, d, e\}\in Y$ such that $a+e=c+d$, and $2c=a+d$; and (v) $\{b, c, d, e\}\in Y$ such that $b+e=c+d$, and $2c=b+d$, as these are already covered by negative result in Proposition 3.

[4] Take for instance (i) $a=0$, $b=4$, $c=8$, $d=10$, and $e=14$; (ii) $a=0$, $b=3$, $c=4$, $d=6$, and $e=7$; and (iii) $a=0$, $b=5$, $c=7$, $d=8$, and $e=10$ for the three cases.

(C) in case of $2c=a+e$, the pre-transfer rich and poor generation to have utility d and a respectively and the post-transfer rich and poor generation to have utility c and b respectively.

(ii) Let $a+e=c+d$, with either $2c=b+d$ or $2d=b+e$. (take for instance $a=0$, $b=4$, $c=5$, $d=6$, and $e=11$; or $a=0$, $b=1$, $c=5$, $d=6$, and $e=11$). Following PD comparisons are possible for any pair of utility streams, with
(A) the pre-transfer rich and poor generation to have utility e and a respectively and the post-transfer rich and poor generation to have utility d and c respectively;
(B) in case of $2c=b+d$, the pre-transfer rich to have utility d and pre-transfer poor to have utility b and the post-transfer rich and poor generation to have utility c and c respectively;
(C) in case of $2d=b+e$, the pre-transfer rich to have utility e and pre-transfer poor to have utility b and the post-transfer rich and poor generation to have utility d and d respectively;
(D) in case of $2d=b+e$, the pre-transfer rich and poor generation to have utility d and a respectively and the post-transfer rich and poor generation to have utility c and b respectively.

(iii) Let $a+e=b+d=2c$, $c+d\neq b+e$ and $2d\neq c+e$ (take $a=0$, $b=1$, $c=3$, $d=5$, and $e=6$). Following PD comparisons are possible for any pair of utility streams, with
(A) the pre-transfer rich and poor generation to have utility e and a respectively and the post-transfer rich and poor generation to have utility d and b respectively;
(B) the pre-transfer rich and poor generation to have utility d and b respectively and the post-transfer rich and poor generation to have utility c and c respectively;
(C) the pre-transfer rich and poor generation to have utility e and a respectively and the post-transfer rich and poor generation to have utility c and c respectively.
None of the utility levels b, c, and d can be assigned to both pre-transfer poor and post transfer rich generations.

In each of the cases (i), (ii) and (iii), we can show that the SWF in Dubey and Mitra (2014a, Proposition 1) satisfies PD and M (as well as WP).

(c) Let Y be such that $(a, c)\prec(b, b)$ and $(c, e)\prec(d, d)$ be the only two possible PD comparisons (i. e., $2b=a+c$, and $2d=c+e$ are such that no other PD comparison is feasible (take $a=0$, $b=1$, $c=2$, $d=5$, and $e=8$). Next proposition shows that there exists a SWF satisfying PD and M.

Proposition 6. *If $Y=\{a, b, c, d, e\}$ is such that $(a, c)\prec(b, b)$ and $(c, e)\prec(d, d)$ are the only feasible PD comparisons, then, there exists a SWF on $X\equiv Y^{\mathbb{N}}$ satisfying PD and M which is defined without using AC.*

The Propositions 1-6 exhaust all possible $Y\subset\mathbb{R}$ for which representation or explicit construction of SWO is possible. For remaining Y neither any SWF satisfying PD and M exists nor the SWO is constructive. Thus we are led to following theorem.

Theorem 1. *If SWO $\succsim$ on X satisfies PD and M, then the SWO admits a representation if and only if it is constructive.*

IV. *SWO Satisfying PD*

In this section we consider SWOs over infinite utility streams satisfying PD. It is known (see Alcantud (2010) and Sakamoto (2012)) that SWOs satisfying PD are representable when the domain set Y is the interval $[0, 1]$. However, the possibility result uses AC. In Dubey and Mitra (2014b) it has been shown that there is no SWO satisfying PD which can be constructed when $Y=[0, 1]$.

We take the subsets of $\mathbb{R}$ as our object of investigation with the aim to characterize the sets Y for which the SWFs can be described without relying on AC. Our result shows the restrictive effect of the weak inequality in the post transfer welfare of the two generations on the cardinality of domain set Y with regard to the construction possibility of the SWO. We show that for Y containing a minimal set of four distinct elements (appropriately chosen), an explicit formula of a SWF satisfying PD exists.

Proposition 7. *If $Y=\{a, b, c, d\}$ is such that $a+c=2b$ and $b+d=2c$, then, there exists a SWF on $X\equiv Y^{\mathbb{N}}$ satisfying PD which is defined without using AC.*

Next we show that for any Y consisting of five distinct elements (suitably chosen), every SWO satisfying Pigou-Dalton transfer principle must be non-constructive in nature.

Proposition 8. *If $Y=\{a, b, c, d, e\}$ is such that $a+c=2b$, $b+d=2c$, and $c+e=2d$, then, the existence of a SWO on $X\equiv Y^{\mathbb{N}}$ satisfying PD, entails the existence of a collection of sets $\Gamma\subset\Omega$, which is non-Ramsey.*

By Proposition 8 we have also been able to show that the SWF satisfying PD (which exist by Alcantud (2010, Proposition 5) and Sakamoto (2012, Proposition 3)) does not exhibit the property of explicit description. If it did, then it would be possible to construct the SWO in this case as "$x \succsim y$ if and only if $W(x)\geq W(y)$" contradicting the Proposition 8.

It is important to emphasize the role played by the choice of five elements in Proposition 8. For an appropriately chosen set Y consisting of six distinct elements, there exists a SWF satisfying PD.

Proposition 9. *If $Y=\{a, b, c, d, e, f\}$ is such that $a+d=b+c$, $b+e=c+d$, and $c+f=d+e$ and neither b nor e is the post transfer utility level of rich or poor, then, there exists a SWF on $X\equiv Y^{\mathbb{N}}$ satisfying PD which is defined without using AC.*

Mitra (2010) presents an example of a SWF satisfying SE for the domain Y containing seven distinct elements. It can be used to demonstrate that it is possible to describe explicitly a SWF satisfying PD for an appropriately chosen set Y consisting of seven distinct elements (for example, Y consisting of seven elements, $\{a, b, c, d, e, f, g\}$ with $a=1$, $b=2$, $c=6$, $d=7$, $e=11$, $f=12$ and $g=16$). Observe that the SWFs in Proposition 9 as well as in the seven element case violate M. However in Dubey and Mitra (2014b, Proposition 2), it has been shown that for Y with eight or more distinct elements, every SWO satisfying PD is necessarily non-constructive.

Proposition 10. *[Dubey and Mitra (2014b, Proposition 2)] If* $Y=\{a, b, c, d, e, f, g, h\}$ *is such that* $a+h=b+g$, $c+f=d+e$, $a+d=b+c$, $e+h=f+g$, *then, the existence of a SWO on* $X\equiv Y^{\mathbb{N}}$ *satisfying PD, entails the existence of a collection of sets* $\Gamma\subset\Omega$, *which is non-Ramsey.*

As in the case of Proposition 8, the Proposition 10 also entails that the SWF satisfying PD (which exists in this case, see Alcantud (2010, Proposition 5) and Sakamoto (2012, Proposition 3)) must be non-constructive. Thus we have covered all possible cardinality of set Y. For sets Y containing up to seven appropriately chosen distinct elements, there exists an explicitly constructed SWF (and SWO). For remaining Y neither any SWF satisfying PD has explicit description nor the SWO is constructive. The Propositions 7-10 lead to following theorem for SWO satisfying PD.

Theorem 2. *If SWO* $\succsim$ on *X satisfies PD, then the SWO admits a representation without using AC if and only if it is constructive.*

V. *Conclusions*

In this paper we have established the following version of correspondence principle. The domains Y, for which the SWO satisfying PD (with or without M) admits (a) explicit representation; or (b) is constructive, are identical. The cardinality of set Y is finite in each case. We present explicit functional forms of the social welfare functions when they exist. The second result pertains to restrictions of domain set Y for explicit representation and construction of social welfare orders satisfying PD. It explains the non-constructive nature of the possibility result in Alcantud (2010, Proposition 5) and Sakamoto (2012, Proposition 3).

Our paper has devised a convenient mechanism to examine the possibility of explicit description of SWF satisfying PD. If the SWO satisfying PD is non-constructive then the representation (the SWF) is also non- constructive.

We note that PD is a more restrictive equity condition compared to the Strong Equity axiom as PD requires transfers to be non-leaky and allows the post transfer rich and poor to have equal utility level. In section IV, we have examined PD without imposing any additional axiom. It is important to note that not all utility transfer satisfying SE would be comparable by PD.[5]

Although the set Y is finite in case the SWO is constructive, the cardinality of the set of utility streams which could be pairwise ranked is uncountable. For the population divided among rich, upper middle class, lower middle class, and poor (or ultra-rich, rich, median and poor on the basis of income), SWO satisfying PD could be useful tool for policy makers.

To further expand the cardinality of set Y admitting explicit representation of SWOs, we might consider weakening of PD. A weaker version of PD could be *no regressive transfer principle* which retains the strict ranking as obtained in PD but modifies the non-leaky transfer

[5] Consider $x=\{10, 1, 0, 0,\cdots\}$ and $y=\{9, 8, 0, 0,\cdots\}$. Then $y\succ x$ by SE but x and y are not comparable by PD.

condition to include the situations where the sum of post transfer welfare levels is higher than pre- transfer levels. Another version is the *weak Pigou Dalton transfer* which replaces the strict ranking in PD by weak ranking. We keep these two issues for future research.

Appendix A Proofs

Proof of Proposition 1. For $x \in X$, let, $N(x)=\{n: x_n=a\}$, $\alpha(n)=-\frac{1}{2^n}$, and $\delta(n)=-\alpha(n)$ for all $n \in \mathbb{N}$. We define $W: X \to \mathbb{R}$ as follows:

$$W(x)=\begin{cases} \sum_{n \in N(x)} \alpha(n) & \textit{if } N(x) \textit{ is non-empty}, \\ \sum_{n=1}^{\infty} \delta(n)(x_n-a) & \textit{otherwise}. \end{cases} \tag{1}$$

Brief intuition for showing that W satisfies PD and M is as follows. We partition the set of individuals $\mathbb{N}$ into poor $\{x_i=a\}$ and non-poor $\{x_i>a\}$. If no individual is poor in a utility stream, then W is a sum of discounted utilities (x_i-a) and is positive. Further, in case there is any poor individual, W ignores all non-poor individuals and assigns negative weight to each poor individual. To show that W satisfies PD, note that the utility a must be assigned to the pre-transfer poor generation. Since, W accounts for every improvement of the pre-transfer poor generation (having utility a), the above formula satisfies PD. It is easy to verify that W also satisfies WP. Let $x \gg y$. There are two possibilities. (i) $N(y) \neq \emptyset$. Then $W(x)>0>W(y)$. (ii) $N(y)=\emptyset$. Then $W(x)-W(y)=\sum_{n=1}^{\infty} \delta(n)(x_n-y_n)>0$.

□

Remark 1. This SWF also satisfies PD and M in case Y contains four ($a+d=b+c$, and $2b \neq a+c$, consider $a=0$, $b=1$, $c=3$, and $d=4$, where pre-transfer poor can be at utility level a only) or five (where b and c are never pre-transfer poor, consider $a=0$, $b=1$, $c=3$, $d=4$ and $e=8$ with only two PD redistributions, i. e., $(0, 4) \prec (1, 3)$ and $(0, 8) \prec (4, 4)$.) distinct elements.

Proof of Proposition 3. Let $N \equiv \{n_1, n_2, n_3, n_4, \cdots\}$ be an infinite subset of $\mathbb{N}$ such that $n_k < n_{k+1}$ for all $k \in \mathbb{N}$. Let $\overline{N}=\{1, 2, \cdots, 2(n_4-1)\}$. For any $T \in \Omega(N)$, $T \equiv \{t_1, t_2, t_3, t_4, \cdots\}$ with $t_k < t_{k+1}$ for all $k \in \mathbb{N}$, we partition the set of natural numbers $\mathbb{N}$ in $U=\{2t_1-1, 2t_1, \cdots, 2(t_2-1), 2t_3-1, \cdots, 2(t_4-1), \cdots\}$ and $L=\mathbb{N} \setminus U=\{1, 2, \cdots, 2(t_1-1), 2t_2-1, 2t_2, \cdots, 2(t_3-1), \cdots\}$. Let $\overline{LTE}=\{t \in L \cap \overline{N}: t \text{ is even}\}$ and $\overline{LTO}=L \cap \overline{N} \setminus \overline{LTE}$. Also, $\overline{UTE}=\{t \in U \cap \overline{N}: t \text{ is even}\}$, $\overline{UTO}=U \cap \overline{N} \setminus \overline{UTE}$, $\overline{LCN}=L \setminus \overline{N}$, and $\overline{UCN}=U \setminus \overline{N}$. We define the utility stream $x(T, \overline{N})$ whose components are,

$$x_t=\begin{cases} b & \text{if} \quad t \in \overline{LTO}, \quad d \quad \text{if} \quad t \in \overline{LTE}, \\ c & \text{if} \quad t \in \overline{UTO}, \quad c \quad \text{if} \quad t \in \overline{UTE}, \\ a & \text{if} \quad t \in \overline{LCN}, \quad b \quad \text{if} \quad t \in \overline{UCN}. \end{cases} \tag{2}$$

We also define the sequence $y(T, \overline{N})$ using the subset $T \setminus \{t_1\}$ in place of subset T, in the following fashion. The two partitions of the set of natural numbers $\mathbb{N}$ are $\widehat{U}=\{2t_2-1, 2t_2, \cdots, 2(t_3-1), 2t_4-1, \cdots, 2(t_5-1), \cdots\}$ and $\widehat{L}=\mathbb{N} \setminus U$. Let $\widehat{LTE}=\{t \in \widehat{L} \cap \overline{N}: t \text{ is even}\}$ and $\widehat{LTO}=\widehat{L} \cap \overline{N} \setminus \widehat{LTE}$. Also, $\widehat{UTE}=$

$\{t\in\widehat{U}\cap\overline{N}$: t is even$\}$, $\widehat{UTO}=\widehat{U}\cap\overline{N}\backslash\widehat{UTE}$, $\widehat{LCN}=\widehat{L}\backslash\overline{N}$, and $\widehat{UCN}=\widehat{U}\backslash\overline{N}$. We define the utility stream $y(T,\overline{N})$ whose components are,[6]

$$y_t=\begin{cases} b \text{ if } t\in\widehat{LTO}, & d \text{ if } t\in\widehat{LTE}, \\ c \text{ if } t\in\widehat{UTO}, & c \text{ if } t\in\widehat{UTE}, \\ a \text{ if } t\in\widehat{LCN}, & b \text{ if } t\in\widehat{UCN}. \end{cases} \tag{3}$$

As $\overline{N}$ is unique for any N, $x(S,\overline{N})$ and $y(S,\overline{N})$ are well-defined for any $S\in\Omega(N)$.

Let $\succsim$ be a social welfare order satisfying M and PD. We claim that the collection of sets $\Gamma\equiv\{N\in\Omega\colon y(N)\succ x(N)\}$ is non-Ramsey. We need to show that for each $T\in\Omega$, the collection $\Omega(T)$ intersects both Γ and $\Omega\backslash\Gamma$. For this, it is sufficient to show that for each $T\in\Omega$, there exists $S\in\Omega(T)$ such that either $T\in\Gamma$ or $S\in\Gamma$, with the either/or being exclusive. Let $T\equiv\{t_1, t_2,\cdots\}$. In the remaining proof we are concerned with infinite utility sequences $x(T,\overline{T})$, $y(T,\overline{T})$ and $x(S,\overline{T})$, $y(S,\overline{T})$ where $S\in\Omega(T)$. For ease of notation, we omit reference to $\overline{T}$. As the binary relation is complete, one of the following cases must arise: (a) $y(T)\succ x(T)$; (b) $x(T)\succ y(T)$; (c) $x(T)\sim y(T)$. Accordingly, we now separate our analysis into three cases.

(a) Let $y(T)\succ x(T)$, i.e., $T\in\Gamma$. We drop t_1 from T to obtain $S=\{t_2, t_3, t_4,\cdots\}$. Hence $S\in\Omega(T)$. Let $T_1\equiv\{2t_1-1, 2t_1+1,\cdots, 2t_2-3\}$ and $T_2\equiv\{2t_1, 2t_1+2,\cdots, 2t_2-2\}$. Observe that

(A) for all $t\in\mathbb{N}$, $x_t(S)=y_t(T)$;
(B) for all $t\in T_1$, $x_t(T)=c>b=y_t(S)$; for all $t\in T_2$, $x_t(T)=c<d=y_t(S)$; and
(C) for all the remaining $t\in\mathbb{N}$, $x_t(T)=y_t(S)$.

Then for the generations $2t_1-1$ and $2t_1$, $y_{2t_1-1}(S)=b<c=x_{2t_1-1}(T)\le x_{2t_1}(T)=c<d=y_{2t_1}(S)$ and $y_{2t_1-1}(S)+y_{2t_1}(S)=x_{2t_1-1}(T)+x_{2t_1}(T)$ or $b+d=2c$. Similar inequalities hold for the pair of generations $\{2t_1+1, 2t_1+2\},\cdots,\{2t_2-3, 2t_2-2\}$. Each of these pairs leads to PD improvements in $x(T)$ compared to $y(S)$. Since these are finitely many PD improvements, $x(T)\succ y(S)$ by PD. Also, $x(S)\sim y(T)$. Since $y(T)\succ x(T)$, we get, $x(S)\sim y(T)\succ x(T)\succ y(S)$. Thus, $x(S)\succ y(S)$ by transitivity of $\succsim$, and so $S\notin\Gamma$.

(b) Let $x(T)\succ y(T)$, i.e., $T\notin\Gamma$. We drop t_1 and t_{4n}, t_{4n+1} for all $n\in\mathbb{N}$ from T to obtain $S=\{t_2, t_3, t_6, t_7,\cdots\}$. Hence $S\in\Omega(T)$. Let $T_1\equiv\{2t_1-1, 2t_1+1,\cdots, 2t_2-3\}$, $T_2\equiv\{2t_1, 2t_1+2,\cdots, 2t_2-2\}$, and $\widehat{T}\equiv\{2t_{4n}-1, 2t_{4n},\cdots, 2t_{4n+1}-2\colon n\in\mathbb{N}\}$. Observe that,

(A) for all $t\in T_1$, $x_t(T)=c>b=y_t(S)$; for all $t\in T_2$, $x_t(T)=c<d=y_t(S)$;
(B) for all $t\in\widehat{T}$, $x_t(T)=x_t(S)=a<b=y_t(T)=y_t(S)$; and
(C) for all the remaining coordinates, $x_t(T)=y_t(S)$ and $x_t(S)=y_t(T)$.

Then for the generations $2t_1-1$ and $2t_4-1$, $x_{2t_4-1}(T)=a<b=y_{2t_4-1}(S)\le y_{2t_1-1}(S)=b<c=x_{2t_1-1}(T)$ and $y_{2t_4-1}(S)+y_{2t_1-1}(S)=x_{2t_4-1}(T)+x_{2t_1-1}(T)$ or $a+c=2b$. There are finitely many generations in T_1 and infinitely many generations in $\widehat{T}$. Let the cardinality of set T_1 be K. Thus it is possible to choose

[6] If $n_1=1$, then $\{1,\cdots, 2(n_1-1)\}=\emptyset$. For illustration, for $N=\{1, 2, 3, 4,\cdots\}$, $\overline{N}=\{1, 2,\cdots, 6\}$ and the two utility streams are $x(N,\overline{N})=\{c, c, b, d, c, c, a, a, b, b,\cdots\}$ and $y(N,\overline{N})=\{b, d, c, c, b, d, b, b, a, a,\cdots\}$.

generations $l_1=2t_4-1$, $l_2,\cdots,l_K$ from $\widehat{T}$ such that similar inequalities hold for the pair of generations $\{2t_1+1, l_2\}$, $\cdots$, $\{2t_2-3, l_K\}$. Each of these pairs leads to PD improvements in $y(S)$ compared to $x(T)$. Since these are finitely many PD improvements, and also by comparing remaining generations $t\in T_2\cup\widehat{T}\backslash\{l_1,\cdots,l_K\}$, $y(S)\succ x(T)$ by PD and M. Also, $y(T)\succsim x(S)$ by M. Since $x(T)\succ y(T)$, we get $y(S)\succ x(T)\succ y(T)\succsim x(S)$. Thus, $y(S)\succ x(S)$ by transitivity of $\succsim$, and so $S\in\Gamma$.

(c) Let $x(T)\sim y(T)$, i.e., $T\notin\Gamma$. We drop t_1, t_2, t_3 and t_{4n+2}, t_{4n+3} for all $n\in\mathbb{N}$ from T to obtain $S=\{t_4, t_5, t_8, t_9,\cdots\}$. Hence $S\in\Omega(T)$. Denote the set of coordinates $T_1\equiv\{2t_2-1, 2t_2+1,\cdots, 2t_3-3\}$, $T_2\equiv\{2t_2, 2t_2+2,\cdots, 2t_3-2\}$, $T_3\equiv\{2t_1-1, 2t_1+1,\cdots, 2t_2-3\}\cup\{2t_3-1, 2t_3+1,\cdots, 2t_4-3\}$, $T_4\equiv\{2t_1, 2t_1+2,\cdots, 2t_2-2\}\cup\{2t_3, 2t_3+2,\cdots, 2t_4-2\}$, and $\widehat{T}\equiv\{2t_{4n+2}-1,\cdots, 2t_{4n+3}-2: n\in\mathbb{N}\}$.

(i) For $x(S)$ and $y(T)$,

(A) for all $t\in T_1$, $y_t(T)=c>b=x_t(S)$; for all $t\in T_2$, $y_t(T)=c<d=x_t(S)$;

(B) for all $t\in\widehat{T}$, $x_t(S)=a<b=y_t(T)$; and

(C) for all the remaining coordinates, $y_t(T)=x_t(S)$.

Then for the generations $2t_2-1$ and $2t_2$, $x_{2t_2-1}(S)=b<c=y_{2t_2-1}(T)\leq y_{2t_2}(T)=c<d=x_{2t_2}(S)$ and $x_{2t_2-1}(S)+x_{2t_2}(S)=y_{2t_2-1}(T)+y_{2t_2}(T)$ or $b+d=2c$. Similar inequalities hold for the pair of generations $\{2t_2+1, 2t_2+2\},\cdots,\{2t_3-3, 2t_3-2\}$. Each of these pairs leads to PD improvements in $y(T)$ compared to $x(S)$. Since these are finitely many pairs of PD improvements, and also by comparing generations $t\in\widehat{T}$, $x(S)\prec y(T)$ by PD and M.

(ii) For $x(T)$ and $y(S)$,

(A) for all $t\in T_3$, $x_t(T)=c>b=y_t(S)$; for all $t\in T_4$, $x_t(T)=c<d=y_t(S)$;

(B) for all $t\in\widehat{T}$, $x_t(T)=a<b=y_t(S)$; and

(C) for all the remaining coordinates, $x_t(T)=y_t(S)$.

Then for the generations $2t_1-1$ and $2t_6-1$, $x_{2t_6-1}(T)=a<b=y_{2t_6-1}(S)\leq y_{2t_1-1}(S)=b<c=x_{2t_1-1}(T)$ and $x_{2t_6-1}(T)+x_{2t_1-1}(T)=y_{2t_6-1}(S)+y_{2t_1-1}(S)$ or $a+c=2b$. There are finitely many generations in T_3 and infinitely many generations in $\widehat{T}$. Let the cardinality of set T_3 be K. Thus it is possible to choose generations $l_1=2t_6-1$, $l_2,\cdots,l_K$ from $\widehat{T}$ such that similar inequalities hold for the pair of generations $\{2t_1+1, l_2\},\cdots,\{2t_4-3, l_K\}$. Each of these pairs leads to PD improvements in $y(S)$ compared to $x(T)$. Since these are finitely many pairs of PD improvements, and also by comparing remaining generations $t\in T_4\cup\widehat{T}\backslash\{l_1,\cdots,l_K\}$, $y(S)\succ x(T)$ by PD and M.

Since $x(T)\sim y(T)$, we get $y(S)\succ x(T)\sim y(T)\succ x(S)$. Thus, $y(S)\succ x(S)$ by transitivity of $\succsim$, and so $S\in\Gamma$.

□

Proof of Proposition 4. If not, then there is $Y=\{a, b, c, d, e\}$, where $a<b<c<d<e$, and $\succsim$ is a representable social welfare order on $X=Y^{\mathbb{N}}$ satisfying the PD and M. Let W: $X\to\mathbb{R}$ be a function which represents $\succsim$ on X.

Let $I\equiv(0, 1)$ and $\{r_1, r_2,\cdots\}$ be a given enumeration of the rational numbers in I. For each real number $p\in I$, define $N(p)=\{n: n\in\mathbb{N}; n>2: r_n\in(0,p)\}$ and $M(p)=\mathbb{N}\backslash\{N(p)\cup\{1,2\}\}$. Define following pair of sequences $x(p)\in X$ and $y(p)\in X$ as:

$$x_n(p)=\begin{cases} e & \text{if } n=1, \\ b & \text{if } n=2, \\ b & \text{if } n\in N(p), \\ a & \text{otherwise,} \end{cases} \quad y_n(p)=\begin{cases} d & \text{if } n=1, \\ c & \text{if } n=2, \\ b & \text{if } n\in N(p), \\ a & \text{otherwise,} \end{cases} \tag{4}$$

Note that $x_2(p)=b<c=y_2(p)<y_1(p)=d<e=x_1(p)$ and $x_n(p)=y_n(p)$ for all $n>2$. Hence by PD, $y(p)\succ x(p)$, and $W(y(p))>W(x(p))$. Now let $q\in(p,1)$. Observe that $N(p)\subset N(q)$ and $M(q)\subset M(p)$. There are infinitely many elements in $N(q)\cap M(p)$. Let $j(p,q)\equiv\min\{N(q)\cap M(p)\}$ for which $y_{j(p,q)}(p)=a<b=x_{j(p,q)}(q)$ holds. Define

$$z_n=\begin{cases} d & \text{if } n=2, \\ y_n(p) & \text{otherwise,} \end{cases} \tag{5}$$

Since $z_2=d>c=y_2(p)$ and $z\geq y(p)$, $z\succcurlyeq y(p)$ by M. Also, $z_{j(p,q)}=a<b=x_{j(p,q)}(q)=b=x_2(q)<d=z_2$; and $x_n(q)\geq z_n$ for all other $n\in\mathbb{N}$. This implies $x(q)\succ z$ by PD and M. Combining it with $z\succcurlyeq y(p)$ we get $x(q)\succ y(p)$ and so $W(x(q))>W(y(p))$. This leads to a contradiction, by using the arguments in Basu and Mitra (2003, Theorem 1). □

Proof of Proposition 5. Define $Y\equiv\{a, b, c, d, e\}$, with $a<b<c<d<e<f$, $b+e=c+d$, and $a+d=2b$. We use the technique of proof used in Proposition 3 with the following modified sequence $x(T, \overline{N})$,

$$x_t=\begin{cases} b \text{ if } t\in\overline{LTO}, & e \text{ if } t\in\overline{LTE}, \\ c \text{ if } t\in\overline{UTO}, & d \text{ if } t\in\overline{UTE}, \\ a \text{ if } t\in\overline{LCN}, & b \text{ if } t\in\overline{UCN}. \end{cases} \tag{6}$$

and $y(T, \overline{N})$,

$$y_t=\begin{cases} b \text{ if } t\in\widehat{LTO}, & e \text{ if } t\in\widehat{LTE}, \\ c \text{ if } t\in\widehat{UTO}, & d \text{ if } t\in\widehat{UTE}, \\ a \text{ if } t\in\widehat{LCN}, & b \text{ if } t\in\widehat{UCN}. \end{cases} \tag{7}$$

□

Proof of Proposition 6. Given any sequence $x\in X$, define $N(x)=\{n: x_n=a\}$, and $M(x)=\{m: x_m=b$ or $x_m=c\}$. Let $\alpha(n)=-\frac{1}{2^n}$, $\beta(n)=-\frac{1}{3^n}$ and $\delta(n)=-\alpha(n)$ for all $n\in\mathbb{N}$. We define $W: X\rightarrow\mathbb{R}$ by

$$W(x)=\begin{cases} \sum_{n\in N(x)}\alpha(n)+\sum_{m\in M(x)}\beta(m) & \textit{if } N(x) \textit{ or } M(x) \textit{ is non-empty}, \\ \sum_{n=1}^{\infty}\delta(n)(x_n-a) & \textit{otherwise}. \end{cases} \tag{8}$$

Then, it is easy to show that $W(x)$ satisfies PD and M. □

Remark 2. Unlike in the previous cases, W does not satisfy WP. Consider $y=\{b, b,\cdots\}$ and $x=\{c, c,\cdots\}$.

Then, $x \succ y$ by WP, whereas $W(x)=W(y)=-\frac{1}{2}$.

Proof of Proposition 7. Note that Y with $a=0$, $b=1$, $c=2$ and $d=3$ satisfies the specified conditions. Let $\alpha(n)=-\frac{1}{2^n}$, and $\delta(n)=-\alpha(n)$. For $x \in X$, let $N(x)=\{n\colon\ x_n=a\}$, and $M(x)=\{n\colon\ x_n=d\}$. We define the SWF as follows.

$$W(x)=\begin{cases}\sum_{n\in N(x)}\alpha(n)+\sum_{m\in M(x)}\alpha(m) & \text{if } N(x) \text{ or } M(x) \text{ is non-empty,}\\ \sum_{n=1}^{\infty}\delta(n)(x_n-a) & \text{otherwise.}\end{cases} \tag{9}$$

Now we show that W satisfies PD. There are three possible situations in which we can compare pair of sequences x and y using PD.

(a) For $x, y \in X$ let there exists $i, j \in \mathbb{N}$ such that

$$y_i=a<b=x_i \leq x_j=c<d=y_j, \text{ and } x_k=y_k \text{ for all } k\in\mathbb{N}\setminus\{i,j\},$$

then we need to show that $W(x)>W(y)$. Note $N(x)\cup i=N(y)$ and $M(x)\cup j=M(y)$.

(i) If $N(x)=M(x)=\emptyset$, then, $W(x)>0>W(y)$ since $N(y)\neq\emptyset$ and $M(y)\neq\emptyset$.
(ii) If $N(x)\neq\emptyset$ or $M(x)\neq\emptyset$, then:

$$\begin{aligned}W(x)-W(y)&=\sum_{n\in N(x)}\alpha(n)-\sum_{n\in N(y)}\alpha(n)+\sum_{m\in M(x)}\alpha(m)-\sum_{m\in M(y)}\alpha(m)\\ &=\sum_{n\in N(x)}\alpha(n)-\sum_{n\in N(x)\cup i}\alpha(n)+\sum_{m\in M(x)}\alpha(m)-\sum_{m\in M(x)\cup j}\alpha(m)\\ &=-\alpha(i)-\alpha(j)=\frac{1}{2^i}+\frac{1}{2^j}>0.\end{aligned}$$

(b) For $x, y \in X$ let there exists $i, j \in \mathbb{N}$ such that

$$y_i=a<b=x_i \leq x_j=b<c=y_j, \text{ and } x_k=y_k \text{ for all } k\in\mathbb{N}\setminus\{i,j\},$$

then we need to show that $W(x)>W(y)$. Note $N(x)\cup i=N(y)$ and $M(x)=M(y)$.

(i) If $N(x)=M(x)=\emptyset$, then, $W(x)>0>W(y)$ since $N(y)\neq\emptyset$.
(ii) If $N(x)\neq\emptyset$ or $M(x)\neq\emptyset$, then:

$$\begin{aligned}W(x)-W(y)&=\sum_{n\in N(x)}\alpha(n)-\sum_{n\in N(y)}\alpha(n)\\ &=\sum_{n\in N(x)}\alpha(n)-\sum_{n\in N(x)\cup i}\alpha(n)=-\alpha(i)=\frac{1}{2^i}>0.\end{aligned}$$

(c) For $x, y \in X$ let there exists $i, j \in \mathbb{N}$ such that

$$y_i=b<c=x_i \leq x_j=c<d=y_j, \text{ and } x_k=y_k \text{ for all } k\in\mathbb{N}\setminus\{i,j\},$$

then we need to show that $W(x)>W(y)$. Note $N(x)=N(y)$ and $M(x)\cup j=M(y)$.

(i) If $N(x)=M(x)=\emptyset$, then, $W(x)>0>W(y)$ since $M(y)\neq\emptyset$.

(ii) If $N(x)\neq\emptyset$ or $M(x)\neq\emptyset$, then:

$$\begin{aligned} W(x)-W(y)&=\sum_{m\in M(x)}\alpha(m)-\sum_{m\in M(y)}\alpha(m)\\ &=\sum_{m\in M(x)}\alpha(m)-\sum_{m\in M(x)\cup j}\alpha(m)=-\alpha(j)=\frac{1}{2^j}>0. \end{aligned}$$

□

Remark 3. It is easy to see that the SWF violates M, as it should. Consider $x=\{b, b,\cdots\}$ and $y=\{d, d,\cdots\}$. Then $W(y)=-1$ and $W(x)=b-a>0$. Thus $W(x)>W(y)$ whereas $y\geqslant x$ by M.

Proof of Proposition 8. Define $Y\equiv\{a, b, c, d, e\}$ with $a<b<c<d<e$, $a+c=2b$, $b+d=2c$, and $c+e=2d$. Let $N\equiv\{n_1, n_2, n_3, n_4,\cdots\}$ be an infinite subset of $\mathbb{N}$ such that $n_k<n_{k+1}$ for all $k\in\mathbb{N}$. Let $\overline{N}=\{1, 2,\cdots, 2(n_4-1)\}$. For any $T\in\Omega(N)$, $T\equiv\{t_1, t_2, t_3, t_4,\cdots\}$ with $t_k<t_{k+1}$ for all $k\in\mathbb{N}$, we partition the set of natural numbers $\mathbb{N}$ in $U=\{2t_1-1, 2t_1,\cdots, 2(t_2-1), 2t_3-1,\cdots, 2(t_4-1),\cdots\}$ and $L=\mathbb{N}\setminus U=\{1,2,\cdots,2(t_1-1),2t_2-1,2t_2,\cdots,2(t_3-1),\cdots\}$. Let $\overline{LTE}=\{t\in L\cap\overline{N}$: t is even$\}$ and $\overline{LTO}=L\cap\overline{N}\setminus\overline{LTE}$. Also, $\overline{UTE}=\{t\in U\cap\overline{N}$: t is even$\}$ and $\overline{UTO}=U\cap\overline{N}\setminus\overline{UTE}$. Further, $\overline{LCTE}=\{t\in L\setminus\overline{N}$: t is even$\}$, $\overline{LCTO}=\{L\setminus\overline{N}\}\setminus\overline{LCTE}$, $\overline{UCTE}=\{t\in U\setminus\overline{N}$: t is even$\}$, $\overline{UCTO}=\{U\setminus\overline{N}\}\setminus\overline{UCTE}$. The utility stream $x(T,\overline{N})$ is,

$$x_t=\begin{cases} b \text{ if } t\in\overline{LTO}, & d \text{ if } t\in\overline{LTE},\\ c \text{ if } t\in\overline{UTO}, & c \text{ if } t\in\overline{UTE},\\ a \text{ if } t\in\overline{LCTO}, & e \text{ if } t\in\overline{LCTE},\\ b \text{ if } t\in\overline{UCTO}, & d \text{ if } t\in\overline{UCTE}. \end{cases} \tag{10}$$

The utility assigned to odd and even generations in $L\cap\overline{N}$ are b, and d, respectively. The utility assigned to each generation in $U\cap\overline{N}$ is c. Similarly the utility assigned to odd and even generations in $L\setminus\overline{N}$ are a, and e respectively. Lastly the utility assigned to odd and even generations in $U\setminus\overline{N}$ are b and d respectively.

The utility stream $y(T,\overline{N})$ is defined using the subset $T\setminus\{t_1\}$ in place of subset T, in identical fashion. The two partitions of the set of natural numbers $\mathbb{N}$ are $\widehat{U}=\{2t_2-1, 2t_2,\cdots, 2(t_3-1), 2t_4-1,\cdots,2(t_5-1),\cdots\}$ and $\widehat{L}=\mathbb{N}\setminus U$. Let $\widehat{LTE}=\{t\in\widehat{L}\cap\overline{N}$: t is even$\}$ and $\widehat{LTO}=\widehat{L}\cap\overline{N}\setminus\widehat{LTE}$. Also, $\widehat{UTE}=\{t\in\widehat{U}\cap\overline{N}$: t is even$\}$ and $\widehat{UTO}=\widehat{U}\cap\overline{N}\setminus\widehat{UTE}$. Further, $\widehat{LCTE}=\{t\in L\setminus\overline{N}$: t is even$\}$, $\widehat{LCTO}=\{L\setminus\overline{N}\}\setminus\widehat{LCTE}$, $\widehat{UCTE}=\{t\in U\setminus\overline{N}$: t is even$\}$, $\widehat{UCTO}=\{U\setminus\overline{N}\}\setminus\widehat{UCTE}$. The utility stream $y(T,\overline{N})$ is,[7]

$$y_t=\begin{cases} b \text{ if } t\in\widehat{LTO}, & d \text{ if } t\in\widehat{LTE},\\ c \text{ if } t\in\widehat{UTO}, & c \text{ if } t\in\widehat{UTE},\\ a \text{ if } t\in\widehat{LCTO}, & e \text{ if } t\in\widehat{LCTE},\\ b \text{ if } t\in\widehat{UCTO}, & d \text{ if } t\in\widehat{UCTE}. \end{cases} \tag{11}$$

[7] If $n_1=1$, then $\{1,\cdots, 2(n_1-1)\}=\emptyset$. For illustration, for $N=\{1, 2, 3, 4,\cdots\}$, $\overline{N}=\{1, 2, 3, 4, 5, 6\}$ and the two utility streams are $x(N, \overline{N})=\{c, c, b, d, c, c, a, e, b, d\cdots\}$ and $y(N, \overline{N})=\{b, d, c, c, b, d, c, c, a, e, b, d\cdots\}$.

As $\overline{N}$ is unique for any N, $x(S, \overline{N})$ and $y(S, \overline{N})$ are well-defined for any $S \in \Omega(N)$.

Let $\succsim$ be a social welfare order satisfying PD. We claim that the collection of sets $\Gamma \equiv \{N \in \Omega: y(N) \succ x(N)\}$ is non-Ramsey. We need to show that for each $T \in \Omega$, the collection $\Omega(T)$ intersects both Γ and $\Omega \backslash \Gamma$. For this, it is sufficient to show that for each $T \in \Omega$, there exists $S \in \Omega(T)$ such that either $T \in \Gamma$ or $S \in \Gamma$, with the either/or being exclusive. Let $T \equiv \{t_1, t_2, \cdots\}$. In the remaining proof we are concerned with infinite utility sequences $x(T, \overline{T})$, $y(T, \overline{T})$ and $x(S, \overline{T})$, $y(S, \overline{T})$ where $S \in \Omega(T)$. For ease of notation, we omit reference to $\overline{T}$. As the binary relation is complete, one of the following cases must arise: (a) $y(T) \succ x(T)$; (b) $x(T) \succ y(T)$; (c) $x(T) \sim y(T)$. Accordingly, we now separate our analysis into three cases.

(a) Let $y(T) \succ x(T)$; that is, $T \in \Gamma$. We drop t_1 from T to obtain $S = \{t_2, t_3, t_4, \cdots\}$. Hence $S \in \Omega(T)$. Let $T_1 \equiv \{2t_1-1, 2t_1+1, \cdots, 2t_2-3\}$ and $T_2 \equiv \{2t_1, 2t_1+2, \cdots, 2t_2-2\}$. Observe that

(A) for all $t \in T_1$, $x_t(T) = c > b = y_t(S)$;
(B) for all $t \in T_2$, $x_t(T) = c < d = y_t(S)$;
(C) for all the remaining $t \in \mathbb{N}$, $x_t(T) = y_t(S)$.
(D) for all $t \in \mathbb{N}$, $x_t(S) = y_t(T)$

Then for the generations $2t_1-1$ and $2t_1$,

$$y_{2t_1-1}(S) = b < c = x_{2t_1-1}(T) \leq x_{2t_1}(T) = c < d = y_{2t_1}(S), \text{ and } b+d = 2c.$$

Similar inequalities hold for the pair of generations $\{2t_1+1, 2t_1+2\}$, $\cdots$, $\{2t_2-3, 2t_2-2\}$. Each of these pairs leads to PD improvements in $x(T)$ compared to $y(S)$. Since these are finitely many PD improvements, $x(T) \succ y(S)$ by PD. Also, $x(S) \sim y(T)$. Since $y(T) \succ x(T)$, we get

$$x(S) \sim y(T) \succ x(T) \succ y(S).$$

Thus, $x(S) \succ y(S)$ by transitivity of $\succsim$, and so $S \notin \Gamma$.

(b) Let $x(T) \succ y(T)$; that is, $T \notin \Gamma$. We drop t_1 and minimum number of t_{4n}, t_{4n+1} such that

$$|\{2t_1-1, \cdots, 2t_2-2\}| \leq |\{2t_4-1, \cdots, 2t_5-2\} \cup \cdots \cup \{2t_{4k}-1, \cdots, 2t_{4k+1}-2\}|$$

from T to obtain $S = \{t_2, t_3, t_6, t_7, t_{10}, t_{11}, \cdots\}$. Hence $S \in \Omega(T)$. Denote the set of coordinates $\{2t_1-1, 2t_1+1, \cdots, 2t_2-3\}$ by T_1, $\{2t_1, 2t_1+2, \cdots, 2t_2-2\}$ by T_2, $\{2t_4-1, 2t_4+1, \cdots, 2t_5-3, \cdots, 2t_{4k}-1, \cdots, 2t_{4k+1}-3\}$ by T_3 and $\{2t_4, 2t_4+2, \cdots, 2t_5-2, \cdots, 2t_{4k}, \cdots, 2t_{4k+1}-2\}$ by T_4.

(i) For $x(T)$ and $y(S)$,

(A) for all $t \in T_1$, $x_t(T) = c > b = y_t(S)$;
(B) for all $t \in T_3$, $x_t(T) = a < b = y_t(S)$;
(C) for all $t \in T_2$, $x_t(T) = c < d = y_t(S)$;
(D) for all $t \in T_4$, $x_t(T) = e > d = y_t(S)$;
(E) for all the remaining coordinates, $x_t(T) = y_t(S)$.

Following cases arise.

(I) For the generations $2t_1-1$ and $2t_4-1$,

$$x_{2t_4-1}(T) = a < b = y_{2t_4-1}(S) \leq y_{2t_1-1}(S) = b < c = x_{2t_1-1}(T), \text{ and } a+c = 2b.$$

Similar inequalities hold for the pair of generations $\{2t_1+1, 2t_4+1\}$, $\cdots$, $\{2t_2-3, m\}$ where $m \in T_3$.

(II) For the generations $2t_1$ and $2t_4$,

$$x_{2t_1}(T)=c<d=y_{2t_1}(S)\leq y_{2t_4}(S)=d<e=x_{2t_4}(T), \text{ and } c+e=2d.$$

Similar inequalities hold for the pair of generations $\{2t_1+2, 2t_4+2\}$, $\cdots$, $\{2t_2-2, m+1\}$ where $m+1=m' \in T_4$.

(III) For the generations $m'+1$, $m'+2$, and remaining generations[8] in $T_3 \cup T_4$,

$$x_{m'+1}(T)=a<b=y_{m'+1}(S)<y_{m'+2}(S)=d<e=x_{m'+2}(T), \text{ and } a+e=b+d.$$

Each of these instances leads to PD improvements in $y(S)$ compared to $x(T)$ and there are finitely many of them. Hence, $y(S)\succ x(T)$ by PD.

(ii) For $x(S)$ and $y(T)$

(A) for all $t \in T_3$, $x_t(S)=a<b=y_t(S)$;
(B) for all $t \in T_4$, $x_t(S)=e>d=y_t(T)$;
(C) for all the remaining coordinates, $y_t(T)=x_t(S)$.

The case of generations in T_3 and T_4 is similar to (b)(i)(III) above. Since these are finitely many PD improvements, $y(T)\succ x(S)$ by PD. Since $x(T)\succ y(T)$, we get

$$y(S)\succ x(T)\succ y(T)\succ x(S).$$

Thus, $y(S)\succ x(S)$ by transitivity of $\succsim$, and so $S \in \Gamma$.

(c) Let $x(T)\sim y(T)$; that is, $T \notin \Gamma$. We drop t_1, t_2, t_3, and minimum number of t_{4n+2}, t_{4n+3} such that

$$|\{2t_1-1,\cdots, 2t_2-2\}\cup\{2t_3-1,\cdots, 2t_4-2\}|\leq|\{2t_6-1,\cdots, 2t_7-2\}\cup\cdots\cup\{2t_{4k+2}-1,\cdots, 2t_{4k+3}-2\}|$$

from T to obtain $S=\{t_4, t_5, t_8, t_9,\cdots\}$. Hence $S \in \Omega(T)$. Denote the set of coordinates $\{2t_2-1, 2t_2+1,\cdots, 2t_3-3\}$ by T_1, $\{2t_2, 2t_2+2,\cdots, 2t_3-2\}$ by T_2, $\{2t_1-1, 2t_1+1,\cdots, 2t_2-3\}\cup\{2t_3-1, 2t_3+1,\cdots, 2t_4-3\}$ by T_3, $\{2t_1, 2t_1+2,\cdots, 2t_2-2\}\cup\{2t_3, 2t_3+2,\cdots, 2t_4-2\}$ by T_4, $\{2t_6-1, 2t_6+1\cdots, 2t_7-3,\cdots, 2t_{4k+2}-1,\cdots, 2t_{4k+3}-3\}$ by $\widehat{T}_1$, and $\{2t_6,\cdots, 2t_7-2,\cdots, 2t_{4k+2},\cdots, 2t_{4k+3}-2\}$ by $\widehat{T}_2$.

(i) For $x(S)$ and $y(T)$,

(A) for all $t \in T_1$, $y_t(T)=c>b=x_t(S)$;
(B) for all $t \in T_2$, $y_t(T)=c<d=x_t(S)$;
(C) for all $t \in \widehat{T}_1$, $x_t(S)=a<b=y_t(T)$;
(D) for all $t \in \widehat{T}_2$, $x_t(S)=e>d=y_t(T)$;
(E) for all the remaining coordinates, $y_t(T)=x_t(S)$.

For $\widehat{T}_1$, $\widehat{T}_2$, PD improvements in $y(T)$ compared to $x(S)$ can be shown following the case (b)(i)(III) above. Also for T_1, T_2, PD improvements in $y(T)$ compared to $x(S)$ can be shown following the case (a) above. Since these are finitely many instances of PD improvements, $x(S)\prec y(T)$ by PD.

[8] The number of these generations is even.

(ii) For $x(T)$ and $y(S)$,

(A) for all $t \in T_3$, $x_t(T)=c>b=y_t(S)$;

(B) for all $t \in \widehat{T}_1$, $x_t(T)=a<b=y_t(S)$;

(C) for all $t \in T_4$, $x_t(T)=c<d=y_t(S)$;

(D) for all $t \in \widehat{T}_2$, $x_t(T)=e>d=y_t(S)$;

(E) for all the remaining coordinates, $y_t(S)=x_t(T)$.

Here, PD improvements in $y(T)$ compared to $x(S)$ can be shown following the case (b)(i) above. Since these are finitely many instances of PD improvements, $y(S) \succ x(T)$ by PD.

Since $x(T) \sim y(T)$, we get

$$y(S) \succ x(T) \sim y(T) \succ x(S)$$

Thus, $y(S) \succ x(S)$ by transitivity of $\succsim$, and so $S \in \Gamma$.

□

Proof of Proposition 9. Note that Y with $a=1$, $b=2$, $c=6$, $d=7$, $e=11$, and $f=12$ satisfies all conditions of the proposition. Let $N(x)=\{n\colon x_n=a \text{ or } x_n=f\}$, and $M(x)=\{n\colon x_n=b \text{ or } x_n=e\}$. Let $\alpha(n)=-\frac{1}{2^n}$, $\beta(n)=-\frac{1}{3^n}$ and $\delta(n)=-\alpha(n)$ for all $n \in \mathbb{N}$. We define the SWF as follows.

$$W(x)=\begin{cases} \sum_{n \in N(x)} \alpha(n)+\sum_{m \in M(x)} \beta(n) & \text{if } N(x) \text{ or } M(x) \text{ is non-empty,} \\ \sum_{n=1}^{\infty} \delta(n)(x_n-a) & \textit{otherwise.} \end{cases} \tag{12}$$

We show that W satisfies PD. Suppose $x,y \in X$ are such that (i) there exist $i,j \in \mathbb{N}$ with $y_i<x_i<x_j<y_j$, $y_i+y_j=x_i+x_j$, and (ii) $x_k=y_k$ for all $k \in \mathbb{N} \setminus \{i, j\}$. Observe that there are only six feasible PD transfers.

(a) $y_i=a<b=x_i<c=x_j<d=y_j$. Both $N(x)$ and $N(y)$ are non-empty in this case and $M(x)=M(y)$. Therefore, $W(x)-W(y)=\beta(i)-\alpha(i)=-\frac{1}{3^i}+\frac{1}{2^i}>0$.

(b) $y_i=a<c=x_i \leq c=x_j<e=y_j$. Then, $N(y) \neq \emptyset$. If $N(x)$ or $M(x)$ is non-empty, then $W(x)-W(y)=-\alpha(i)-\beta(j)>0$. If both $N(x)$ and $M(x)$ are empty sets, then $W(x)>0>W(y)$.

(c) $y_i=a<c=x_i<d=x_j<f=y_j$. Then, $N(y) \neq \emptyset$. If $N(x)$ or $M(x)$ is non-empty, then $W(x)-W(y)=-\alpha(i)-\alpha(j)>0$. If both $N(x)$ and $M(x)$ are empty sets, then $W(x)>0>W(y)$.

(d) $y_i=b<c=x_i<d=x_j<e=y_j$. Then, $M(y) \neq \emptyset$. If $N(x)$ or $M(x)$ is non-empty, then $W(x)-W(y)=-\beta(i)-\beta(j)>0$. If both $N(x)$ and $M(x)$ are empty sets, then $W(x)>0>W(y)$.

(e) $y_i=b<d=x_i \leq d=x_j<f=y_j$. Then, $N(y) \neq \emptyset$ and $M(y) \neq \emptyset$. If $N(x)$ or $M(x)$ is non-empty, then $W(x)-W(y)=-\beta(i)-\alpha(j)>0$. If both $N(x)$ and $M(x)$ are empty sets, then $W(x)>0>W(y)$.

(f) $y_i=c<d=x_i<e=x_j<f=y_j$. Both $M(x)$ and $M(y)$ are non-empty in this case and $N(x)=N(y)$.

Therefore, $W(x)-W(y)=\beta(j)-\alpha(j)=-\frac{1}{3^j}+\frac{1}{2^j}>0.$

$\square$

References

Adler, M.D. (2013), The Pigou-Dalton Principle and the Structure of Distributive Justice.

Alcantud, J.C.R. (2010), "The Compromise Efficiency vs. Egalitarianism among Generations with an Infinite Horizon," MPRA Working Paper 22467.

Alcantud, J.C.R. (2012), "Inequality Averse Criteria for Evaluating Infinite Utility Streams: The Impossibility of Weak Pareto," *Journal of Economic Theory* 147, pp.353–363.

Alcantud, J.C.R. (2013), "The Impossibility of Social Evaluations of Infinite Streams with Strict Inequality Aversion," *Economic Theory Bulletin* 1, pp.123–130.

Arrow, K.J. (1951), *Social Choice and Individual Values*, Wiley, New York.

Atkinson, A.B. (1970), "On the Measurement of Inequality," *Journal of Economic Theory* 2, pp.244–263.

Banerjee, K. and R.S. Dubey (2014), "Do All Constructive Strongly Monotone Inter-temporal Orders Exhibit Impatience?," *Journal of Mathematical Economics* 52, pp.66–69.

Basu, K. and T. Mitra (2003), "Aggregating Infinite Utility Streams with Intergenerational Equity: The Impossibility of Being Paretian," *Econometrica* 71, pp.1557–1563.

Basu, K. and T. Mitra (2007), "Possibility Theorems for Aggregating Infinite Utility Streams Equitably," in: J. Roemer and K. Suzumura, ed., *Intergenerational Equity and Sustainability*, Palgrave Macmillan, pp.69–74.

Bossert, W., Y. Sprumont and K. Suzumura (2007), "Ordering Infinite Utility Streams," *Journal of Economic Theory* 135, pp.579–589.

Dalton H. (1920), "The Measurement of the Inequality of Incomes," *The Economic Journal* 30, pp.348–361.

Dasgupta, P., A. Sen and D. Starrett (1973), "Notes on the Measurement of Inequality," *Journal of Economic Theory* 6, pp.180–187.

d'Aspremont, C. and L. Gevers (1977), "Equity and Informational Basis of Collective Choice," *Review of Economic Studies* 44, pp.199–209.

Diamond, P.A. (1965), "The Evaluation of Infinite Utility Streams," *Econometrica* 33, pp.170–177.

Dubey, R.S. (2011), "Fleurbaey-Michel Conjecture on Equitable Weak Paretian Social Welfare Order," *Journal of Mathematical Economics* 47, pp.434–439.

Dubey, R.S. (2016), "On Construction of Social Welfare Orders Satisfying Hammond Equity and Weak Pareto Axioms," SSRN Working paper No. 2731300.

Dubey, R.S. and T. Mitra (2014a), "Combining Monotonicity and Strong Equity: Construction and Representation of Orders on Infinite Utility Streams," *Social Choice and Welfare* 43, pp.591–602.

Dubey, R.S. and T. Mitra (2014b), "On Construction of Equitable Social Welfare Orders on Infinite Utility Streams." *Mathematical Social Sciences* 71, pp.53–60.

Erdós P. and R. Rado (1952), "Combinatorial Theorems on Classifications of Subsets of a

Given Set," *Proceedings of the London Mathematical Society* 3, pp.417–439.

Fleurbaey M. and P. Michel (2003), "Intertemporal Equity and the Extension of the Ramsey Criterion," *Journal of Mathematical Economics* 39, pp.777–802.

Hammond, P.J. (1976), "Equity, Arrows' Conditions, and Rawl's Difference Principle," *Econometrica* 44, pp.793–804.

Hara, C., T. Shinotsuka, K. Suzumura, and Y.S. Xu (2008), "Continuity and Egalitarianism in the Evaluation of Infinite Utility Streams," *Social Choice and Welfare* 31, pp.179–191.

Hardy, G.H., J.E. Littlewood and G. Polya (1952), *Inequalities,* Cambridge University Press, Cambridge, second edition.

Lauwers, S. (2010), "Ordering Infinite Utility Streams Comes at the Cost of a Non-Ramsey Set," *Journal of Mathematical Economics* 46, pp.32–37.

Mathias, A.R.D. (1977), "Happy Families," *Annals of Mathematical Logic* 12, pp.59–111.

Mitra, T. (2010), "On Representation of the Strong Equity Axiom," mimeo.

Pigou, A.C. (1912), *Wealth and Welfare*, Macmillan and Co., Ltd., London.

Sakai, T. (2006), "Equitable Intergenerational Preferences on Restricted Domains," *Social Choice and Welfare* 27, pp.41–54.

Sakamoto, N. (2012), "Impossibilities of Paretian Social Welfare Functions for Infinite Utility Streams with Distributive Equity," *Hitotsubashi Journal of Economics* 53, pp.121–130.

Svensson, L.G. (1980), "Equity among Generations," *Econometrica* 48, pp.1251–1256.

Zame, W.R. (2007), "Can Utilitarianism Be Operationalized?," *Theoretical Economics* 2, pp.187–202.