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“Unified China and Divided Europe”

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Unified China and Divided Europe

Chiu Yu Ko, Mark Koyama, Tuan-Hwee Sng*

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Abstract

This paper studies the causes and consequences of political centralization and fragmentation in China and Europe. We argue that the severe and unidirectional threat of external invasion fostered political centralization in China while Europe faced a wider variety of moderate external threats and remained politically fragmented. Our model allows us to explore the economic consequences of political centralization and fragmentation. Political centralization in China led to lower taxation and hence faster population growth during peacetime than in Europe. But it also meant that China was relatively fragile in the event of an external invasion. Our results are consistent with historical evidence of violent conflicts, tax levels, and population growth in both China and Europe.

Keywords: China; Europe; Great Divergence; Political Fragmentation; Political Centralization

JEL Codes: H2, H4, H56; N30; N33; N35; N40; N43; N45

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1 Introduction

Since Montesquieu, scholars have attributed Europe’s success to its political fragmentation (Montesquieu, 1989; Jones, 2003; Mokyr, 1990; Diamond, 1997). Nevertheless, throughout most of history, the most economically developed region of the world was China, which was often a unified empire. This contrast poses a puzzle that has important implications for our understanding of the origins of modern economic growth: Why was Europe perennially fragmented after the collapse of Rome, whereas political centralization was an equilibrium for most of Chinese history? Can this fundamental difference in political institutions account for important differences in Chinese and European growth patterns?

This paper proposes an explanation for the different political equilibria in China and Europe. Our model predicts when and where empires are more likely to be viable based on the nature and intensity of the external threats that they face. We go on to develop conjectures about the consequences of political fragmentation and centralization for economic growth.

Historically, China faced a severe, unidirectional threat from the steppe. Europe confronted several moderate external threats from Scandinavia, Central Asia, the Middle East, and North Africa. We show that if multitasking was inefficient, empires would not be viable in Europe and political fragmentation would be the norm. On the other hand, empires were more likely to emerge and survive in China because the nomadic threat threatened the survival of small states more than larger ones. Political centralization allowed China to avoid wasteful interstate competition and thereby enjoy faster economic and population growth during peacetime. However, the presence of multiple states to protect different parts of the continent meant that Europe was relatively more robust to negative shocks.

To substantiate the mechanisms identified in our model, we show that its predictions are consistent with data on the frequency and intensity of internal and external wars and the level of taxation in Europe and China, and with evidence demonstrating the greater volatility of Chinese population growth.

This paper is related to a range of literature. Our theoretical framework builds on the literature on the size of nations originated by Friedman (1977) and Alesina and Spolaore (1997, 2003). In particular, our emphasis on the importance of external threats is related to the insights of Alesina and Spolaore (2005) who study the role of war in shaping political boundaries. In our model, rulers provide defense against external enemies, but due to costs of extending military power over large distances, a single ruler is inefficient in dealing with multiple threats.

Our main contribution is to clarify the emergence and stability of the European state system and its significance for the onset of sustained economic development. Numerous economists,
historians, political scientists, and sociologists have argued that political fragmentation in Europe led to the growth of economic and political freedom (Montesquieu, 1989); helped preserve the existence of independent city states and permitted the rise of a merchant class (Pirenne, 1925; Hicks, 1969; Jones, 2003; Hall, 1985; Rosenberg and Birdzell, 1986); encouraged experiments in political structures and investments in state capacity (Baechler, 1975; Cowen, 1990; Hoffman, 2012; Gennaioli and Voth, 2013);\(^1\) intensified warfare and therefore increased urbanization and incomes (Rosenthal and Wong, 2011; Voigtländer and Voth, 2013b);\(^2\) and fostered innovation and scientific development (Diamond, 1997; Mokyr, 2007; Lagerlof, 2014).\(^3\)

We propose a model which allows us to study both the causes and the consequences of political centralization and fragmentation in a single coherent framework that is consistent with the above observations. Our model shows that growth reversals are more likely in empires but between negative shocks, population in an empire may expand faster than in a system of competing states. Our approach also incorporates several existing explanations such as the importance of geographical differences between China and Europe as emphasized by Diamond (1997) and the importance of providing defense against external invasion by nomads in shaping the emergence of states in Eurasia (McNeil, 1964; Barfield, 1989; Lieberman, 2009).

The framework we introduce has implications for our understanding of the origins of economic growth. Growth theory often contains a scale-effect that implies that larger economies should be the first to experience modern economic growth (Kremer, 1993; Jones, 2001b). Our theory suggests that because it was more centralized, China was more vulnerable to negative shocks and therefore more likely to experience periodic growth reversals. As a steady increase in the stock

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\(^1\)Baechler observed that ‘political anarchy’ in Europe gave rise to experimentation in different state forms (Baechler, 1975, 74). Cowen (1990) argued that interstate competition in Europe provided an incentive for early modern states to develop capital markets and pro-market policies. Hoffman (2012) uses a tournament model to explain how interstate competition led to military innovation in the early modern Europe. Gennaioli and Voth (2013) show the military revolution induced investments in state capacity in some but not all European states.

\(^2\)Voigtländer and Voth (2013b) argue that political fragmentation interacted with the Black Death so as to shift Europe into a higher state-steady equilibrium. Rosenthal and Wong (2011) argue that political fragmentation led to more frequent warfare in medieval and early modern Europe. Continuous warfare imposed high costs. But political fragmentation also set in motion processes that would give Europe an advantage in producing an industrial revolution; in particular, it lent an urban bias to the development of manufacturing, which led to more capital intensive forms of production.

\(^3\)Diamond argued that ‘Europe’s geographic balkanization resulted in dozens or hundreds of independent, competing statelets and centers of innovation’ whereas in China ‘a decision by one despot could and repeatedly did halt innovation’ (Diamond, 1997, 414-415). Mokyr notes that ‘many of the most influential and innovative intellectuals took advantage of . . . the competitive “states system”. In different ways, Paracelsus, Comenius, Descartes, Hobbes, and Bayle, to name but a few, survived through strategic moves across national boundaries. They were able to flee persecutors, and while this imposed no-doubt considerable hardship, they survived and prospered’ (Mokyr, 2007, 24). Lagerlof (2014) develops a growth model that emphasizes the benefits to scale in innovation under political unification and a greater incentive to innovate under political fragmentation. He calibrates the model to the initial conditions of China and Europe and shows that there are parameter values in which political fragmentation can give rise to the emergence of sustained growth in Europe.
of population is important for cumulative innovation to occur, the start-stop nature of China’s growth diminished its chances of escaping the Malthusian trap, while the European economy was able to expand gradually to the point where the transition from stagnation to growth was triggered (as in theories of unified growth, for instance, Galor and Weil 2000; Galor 2011).

In this respect, our work complements research that emphasizes other aspects of Europe’s possible advantages in the Great Divergence such as the higher age of first marriage the rest of the world (Voigtländer and Voth, 2013a); public provision of poor relief verses reliance on clans as was the case in China (Greif et al., 2012); institutions that were less reliant on religion than in the Middle East (Rubin, 2011); greater human capital (Kelly et al., 2013), or higher social status for entrepreneurs and inventors (McCloskey, 2010). Finally, our analysis is related to the rise of state capacity in Europe (Dincecco, 2009; Dincecco and Katz, 2014; Johnson and Koyama, 2013, 2014a,b).

The rest of the paper is structured as follows. Section 2 provides historical evidence that characterizes (i) the extent to which China was politically unified and Europe fragmented throughout their respective histories, and (ii) the degree to which both China and Europe were threatened by external invasions. In Section 3 we introduce a formal model of political centralization and decentralization. We explore the implications in Section 4 by presenting evidence on wars, taxation, and population fluctuations in both ends of Eurasia. Section 5 concludes.

2 THE PUZZLE: UNIFIED CHINA AND DIVIDED EUROPE

Why was China politically unified for much of its history whereas Europe has been politically fragmented since the end of the Roman empire? Chinese historical records indicate that less than 80 states ruled over parts or all of China between AD 0 and 1800 (Wilkinson, 2012). Nussli (2011) provides data on the sovereign states in existence at hundred year intervals in Europe. Figure 1 plots the number of sovereign states in China and in Europe for the preindustrial period. Despite data limitations, it is incontrovertible that there have always been more states in Europe than in China; in fact since the Middle Ages there have been an order of magnitude more states in Europe than in China.⁴

At the beginning of the period, Europe was dominated by the Roman empire. With the

⁴The Nussli (2011) data does not capture all political entities in Europe since that number is unknown—there may have been as many as 1000 sovereign states within the Holy Roman Empire alone—but it does record the majority of large and small political entities (Abramson, 2013). By contrast, the Chinese dynastic tables are well known and the potential for disagreement is immaterial for our purpose. We count only sovereign states. Including vassal states would further strengthen the argument.
breakup of the Roman empire, the number of states in Europe increased from 37 in 600 A.D. to 61 in 900 and by 1300 there were 114 independent political entities. The level of political fragmentation in Europe remained high during the early modern period.

There were fewer states in China than in Europe throughout the past two millennia. China’s first unification preceded Rome’s dominance of the Mediterranean. The Chinese built a unitary state as early as the third century BC under the Qin dynasty (Elvin, 1973; Fukuyama, 2011). Moreover, the Chinese empire was longer-lasting than Rome. Although dynasties rose and fell after the Qin unification, China as an empire survived until the early twentieth century. Between AD 1 and 1800 the landmass between the Mongol steppe and the South China Sea was ruled by one single authority for 1007 years (Ko and Sng, 2013).

**External threats** We argue that in order to understand why China has typically been unified whereas Europe has been fragmented, we need to assess the threats and challenges that they faced given their geography. Europe was threatened by Goths, Sarmatians, Vandals, Huns, Avars, Bulgars, Magyars, Vikings, Pechnegs, Cumans, Mongols, and Turks. Similarly, settled populations in China contended with a range of steppe nomads and semi-nomadic people: Xiongnu, Juanjuan, Uyghurs, Khitan, Jurchen, Mongols, and Manchus (Grousset, 1970; Barfield, 1989; Chaliand, 2005).

For largely geographical reasons, China faced more severe external threats than Europe.\(^5\)

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\(^5\)See Appendix A.7 for a list of all major nomadic invasions of both China and Europe. Lieberman (2009) distinguishes between China, which lies in the exposed zones of inner Asia, and the protected rimland of Europe and Southeast Asia. He notes that ‘For centuries nonpareil equestrian skills, an ethos focused on hunting and
While Eastern Europe, too, was vulnerable to incursions from the Eurasian steppe, Western Europe was relatively protected from nomadic invasions due to its forests and mountain ranges. China’s more compact geography meant that steppe invasions posed a more extensive threat to its settled agricultural communities and urban centers. Figure 2 illustrates the distance of cities in China and Europe from the Eurasian steppe. As it makes clear, Guangzhou, the southernmost major Chinese city, was almost as close to the steppe as Vienna, the easternmost major western European city.

The threat of steppe nomads played a decisive role in Chinese history. Despite the advantages that the Chinese enjoyed in terms of population and economic resources, before the development of effective gunpowder the steppe nomads often held the upper hand in military conflicts as their expertise in horses facilitated rapid mobilization and movement over long distances (Barfield, 1989). Furthermore, the Eurasian steppe constituted an undifferentiated ‘highway of grass’ (Frachetti, 2008) that allowed the nomads to move en masse from Mongolia to the Black Sea in a relatively short span of time. This gave the nomads an ‘indefinite margin of retreat’. No matter

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6 The Magyar invasions of the ninth century and the Mongol invasions of the thirteenth century pose partial exceptions to this.

7 Of the ten dynasties that ruled a unified China after 221 BC, three fell to nomadic invaders (Jin, Northern Song, and Ming) and two were set up by nomadic conquerors (Yuan and Qing). The steppe factor also featured prominently in the rise and fall of the other five dynasties (Qin, Former Han, Later Han, Sui, and Tang).
Figure 3: Although China was exposed to steppe invasion from its north, huge mountain ranges to its west, thick forests to its south, and the vast Pacific Ocean to its east meant that it was otherwise relatively isolated. By contrast, Europe was connected to the rest of Eurasia and Africa in multiple directions.

how badly they were defeated in battle, they could never be conquered in war (Lattimore, 1940).

Scholars have long recognized the importance of the steppe nomads to state formation in ancient China (Lattimore, 1940; Huang, 1988; Lieberman, 2009; Turchin, 2009; Ma, 2012; Deng, 2012). We build on this literature by highlighting another important element in the nature of this threat that has been overlooked so far: while—as the literature has pointed out—the severity of the nomadic problem provided the centripetal force that pushed the Chinese regions toward political centralization, it was also crucial that the external threats confronting China happened to be unidirectional and there were no major threats from other fronts that would have increased the appeal of a more flexible politically decentralized system.

Before 1800, all major invasions of China came from the north. We argue that this was geographically determined: as Figure 3 illustrates, China was shielded from the south and the west by the Himalayas, the Tibetan plateau, and the tropical rain forests of Indochina. By contrast, Europe was less isolated from other parts of Eurasia and consequently prospective

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8 A range of other factors also played an important role in fostering political unification in China including culture and ideology, a single (logographic) writing system, the imperial examination system, and topography (Elvin 1973, 20-22; Diamond 1997, 414; Lieberman 2009, 111). By focusing on the role of external shocks, our argument complements these existing explanations.

9 Lewis (1991) notes that the ‘high desiccated plateaus and deserts ... dominated the terrain of Mongolia and Turkestan ... formed barriers between it and the Indic and Islamic worlds to the west and south. While it was possible to reach Burma over difficult mountain passes leading there from the upper Yangtze valley, the most practicable routes to the west were by way of Kansu, Inner Mongolia, and eastern Turkestan and then on to Khorasan and southern Russia’ (Lewis, 1991, 4-5).
European empires typically faced enemies on multiple fronts: Vikings from the north, Muslims from the south, the Ottomans and others from the east. In the next section, we develop a theory that explains why a more severe but unidirectional threat gave rise to political centralization in China whereas political centralization in Europe was inherently transitory.

3 MODEL

3.1 Setup

We model a continent as a line $[0, 1]$ with a unit mass of individuals uniformly distributed along this line. An individual at $x \in [0, 1]$ is endowed with income $y + y$ where $y$ is taxed. The continent is divided into $s$ connected, mutually exclusive intervals each ruled by a separate political authority. For convenience, we restrict $s$ to $s \in \{1, 2\}$. Of interest is the comparison between $s = 1$, which corresponds to political centralization or empire, and $s = 2$, which we will refer to as interstate competition or political fragmentation. We use $e$ to label the single regime or empire when $s = 1$, and $l$ and $r$ to label the two regimes when $s = 2$. In the latter case, for convenience and because we are primarily interested in analyzing two comparable regimes, we will treat $l$ and $r$ as identical and focus on the symmetric equilibrium when deriving the results.

We do not model how regimes arise. Historically, the emergence of a regime is often fraught with stochastic elements—the birth of a military genius; policy errors made by the incumbent ruler; climatic change; and so on—which are difficult to capture in a model. Instead, we are concerned with the viability of a regime once it has emerged: we ask, for example, if the initial state is $s = 1$ (or $s = 2$), is this political configuration likely to persist given the continent’s external environment?

Regimes may invest in the military either (1) to compete against each other or (2) to resist exogenous threats. However, the strength of the military deteriorates over distance due to the cost involved in the movement of troops and supplies. As such, the location of a regime’s center of military deployment—referred to here as the capital city—is an important strategic variable.

For regime $i \in \{e, l, r\}$, $0 \leq G_i \leq 1$ denotes the location of its capital city and $M_i \geq 0$ the military investment. For notational convenience, $G_e$ and $G_l$ are measured from 0 while $G_r$ is measured from 1. As illustrated in Figure 4, for a location that is $t$ distance away from $G_i$.

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$^{10}$We will refer to both Europe and China as ‘continents’ for convenience.

$^{11}$If one of the two regimes rules a much larger interval than the other one, it may be more appropriate to use “empire” instead of “interstate competition” to describe the political reality of the continent.

$^{12}$The capital was known asjing-shi in Chinese, or literally the peak (jing) and the military (shi).
regime \(i\)'s level of **military strength** on that location is \(M_i - \beta t^2\), where \(\beta > 0\) captures the loss of military strength due to distance.\(^{13}\) The cost of providing \(M_i\) is \(\theta M_i^2\).

When \(s = 1\), the empire taxes the whole continent, so its **net revenue**, defined as tax revenue net of military investment, is \(V_e = y - \theta M_e^2\). When \(s = 2\), \(y\) is shared between the two regimes. Let \(b\) represent the **border** of the two regimes. Without loss of generality, we restrict the locations of capitals to \(G_l + G_r \leq 1\), that is, the capital city of regime \(l\) is always to the left of regime \(r\)'s capital city. The border \(b\) is determined by the condition \(M_l - \beta (b - G_l)^2 = M_r - \beta ((1 - b) - G_r)^2\). In other words, \(b\) is the location where the military strengths of both regimes are equal, as described in Figure 5.\(^{14}\) The net revenues for regimes \(l\) and \(r\) are \(V_l = b y - \theta M_l^2\) and \(V_r = (1 - b) y - \theta M_r^2\), respectively.

Besides determining the border, investing in the military also helps a regime to defend itself against threats from outside the continent. We model these external threats as emanating either from both frontiers of the continent (at \(x = 0\) and \(x = 1\)) or just from one frontier of the continent (at \(x = 0\) only without loss of generality). Whether the threat is one-sided or multi-sided depends on the continent’s geographical environment which is exogenously determined. An external threat, if realized, causes gross damage \(\Lambda > 0\) at the frontier(s) of the continent. The damage can spread further into the continent: if a point is \(t\) distance away from the frontier, the **gross damage** is \(\Lambda - \alpha t\) where \(\alpha\) captures the spillover strength of the threat.

However, the presence of a military mitigates the damage caused by the threat and may even stop the threat from spreading into the rest of the continent. Under a one-sided threat (initiated at \(x = 0\)), if regime \(i\)'s military strength at \(x\) is no less than the gross damage

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\(^{13}\)We adopt this functional form to capture the idea that in the absence of modern transportation technologies, military strength falls off relatively rapidly with distance. Historically, a key factor constraining the projection of military power was logistics. Shen Kuo (1031-1095), a Chinese statesman and polymath, estimated that a soldier would need one laborer to carry his supplies to march 18 days. Extending the campaign from 18 to 31 days would involve a tripling of laborers as additional laborers would be required to carry the supplies of existing ones (Shen, 2011).

\(^{14}\)For a complete treatment of the determination of \(b\) in all cases, see Appendix A1.
of the external threat at that location, then \( x \) and any location to its right is said to be **adequately protected**, that is, individuals at these locations suffers zero damage from the threat. Formally, a location \( x \) is adequately protected by regime \( i \) if there exists \( 0 \leq x' \leq x \) such that \( M_i - \beta(G_i - x')^2 - (\Lambda - \alpha x') \geq 0 \). Let \( D_i \) denote the set of locations that is adequately protected by regime \( i \), or \( D_i \equiv \{ x \in [0, 1] : x \text{ is adequately protected by regime } i \} \). If \( x \) is not adequately protected by regime \( i \), or \( x \not\in D_i \), then \( \kappa_i \), the net damage at \( x \), is the gross damage caused by the threat minus the strength of regime \( i \) at \( x \), i.e., \( \kappa_i(x) \equiv \Lambda - \alpha x - M_i + \beta(G_i - x)^2 \).

For threats initiated at \( x = 1 \), we define adequate protection, the set \( D_i \), and the net damage \( \kappa_i \) in a similar fashion.

If a regime fails to provide adequate protection to \( \delta > 0 \) fraction of its population, then a revolution occurs and this yields a negative payoff for the regime.

### 3.2 Equilibrium

Consider the optimization problem facing a single regime or empire (\( e \)). Regime \( e \) first decides the location of capital \( G_e \in [0, 1] \) and then decides military investment \( M_e \geq 0 \) to maximize the net revenue \( V_e = y - \theta M_e^2 \). Since this is a two-stage decision process, we employ backward induction to solve the model.\(^{15}\)

**Proposition 1** (Empire). Let \( \Lambda_I = \frac{\alpha(1-\delta)}{2} \) and \( \Lambda_{II} = \frac{1}{8}\beta\delta^2 + \frac{1}{4}\alpha(3-\delta) \). Let \( \hat{\delta} \) denote the fraction of the continent that is adequately protected from the external threat in equilibrium (i.e. \( \hat{\delta} = |D_e| \)). When the threat is multi-sided:

1. If \( \Lambda \leq \Lambda_I \), the regime locates the capital city at \( G_e \in [0, 1] \), makes zero military investment, and \( \hat{\delta} \geq \delta \);

2. If \( \Lambda_I < \Lambda \leq \Lambda_{II} \), the regime locates the capital city closer to one frontier than the other, invests a non-zero amount on the military to confront the threat emanating from the frontier that its capital city is closer to, and \( \hat{\delta} = \delta \);

3. If \( \Lambda > \Lambda_{II} \), the regime locates the capital city at the center of the continent, spends a non-zero amount on the military to confront the threat emanating from both frontiers, and \( \hat{\delta} = \delta \);

When the threat is one-sided:

4. If \( \Lambda \leq 2\Lambda_I \), the regime locates the capital city at \( G_e \in [0, 1] \), makes zero military investment, and \( \hat{\delta} \geq \delta \);

\(^{15}\)Proofs of the propositions are provided in Appendix A2-A4.
5. If $\Lambda > 2\Lambda_I$, the regime chooses the capital city and spends a non-zero amount on the military to confront the threat emanating from $x = 0$, and $\hat{\delta} = \delta$.

In Cases 1, 2, and 4 above, the empire ignores the threat, either in whole or in part. This is because the sole motivation for it to invest in the military is to keep $\delta$ fraction of its population adequately protected (so as to prevent a revolution). Hence, if the threat is weak and does not affect more than $1 - \delta$ of the population (Cases 1 and 4), the regime merely ignores it. If instead both ends of the continent are under meaningful threat but dealing with the threat emanating from one frontier is sufficient to meet the threshold of adequately protecting $\delta$ of the population, the empire will ignore the threat emanating from the other frontier (Case 2).

Now consider the two-stage game with interstate competition ($s = 2$). Regimes $l$ and $r$ first simultaneously choose the location of their capital cities $G_l \in [0, 1]$ and $G_r \in [0, 1]$. After knowing the locations of each other, the two regimes simultaneously make military investments $M_l \geq 0$ and $M_r \geq 0$. This is a complete information game and we employ subgame-perfect equilibrium as the solution concept.

**Proposition 2** (Political Fragmentation). Let $\Lambda_{III} = \frac{\alpha(1-\delta)}{2} - \frac{1}{4}\beta\delta^2 + \max\{\frac{\delta\delta}{2}(\frac{y}{4\beta^2})^{1/3} + 1 - \frac{\delta}{4}, \frac{\delta\delta}{4} + \frac{3}{4}(\frac{y^2}{4\beta^2})^{1/3}\}$. $\hat{\delta}$ continues to denote the fraction of the continent that is adequately protected from the external threat in equilibrium (i.e. $\hat{\delta} = \left|D_l\right| + \left|D_r\right|$). When the threat is multi-sided:

1. If $\Lambda \leq \Lambda_{III}$, the revolution constraints do not bind and $\hat{\delta} \geq \delta$. The equilibrium military investments and location of capitals are the same as in the case when $\Lambda = 0$.

2. Otherwise, the revolution constraints bind and $\hat{\delta} = \delta$.

With or without the external threat, regimes in a competitive state system have to invest in the military to prevent being overrun by their counterparts. Case 1 of Proposition 2 states that if the external threat is mild or moderate (i.e. $\Lambda \leq \Lambda_{III}$), they do not have to make additional military investments to protect their populations as their existing military capacity—built up as a result of competition among themselves—already meets the requirement.

### 3.3 Implications for Political Centralization or Fragmentation

We now use this simple setup to show that whether or not a continent is politically centralized or fragmented is shaped by its geographical characteristics and hence the nature of the external threats that it faces.

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16In practice, this implies that the regime will show regional favoritism and will use the support it receives from the region that benefits from its rule to keep other regions of the empire subdued.
Since there are many permutations of the external threat based on its strength (the value of $\Lambda$) and origins (one-sided or multi-sided), we will only focus on two particular scenarios that are directly relevant to this paper: (1) the case of a severe ($\Lambda > 2\Lambda_I$) and one-sided threat; and (2) the case of a moderate ($\Lambda_{II} < \Lambda \leq \Lambda_{III}$) but multi-sided threat. They are analogous to the Chinese and the European cases respectively. Figures 6 and 7 illustrate them graphically.

From Propositions 1 and 2, we know that while an empire invests in the military only to protect itself against external threats, in a competitive state system regimes will invest in the military even in the absence of external threats so as to gain and maintain territorial control. Hence:

**Corollary 1 (Wastefulness of interstate competition).** In the absence of external threats, military investment is zero under an empire but strictly positive under interstate competition.

Propositions 1 and 2 also imply that if the external threat is significant enough for an empire to spend a non-zero amount on the military, it will only provide adequate protection to a fraction $\delta$ of the population, so as to satisfy the revolution constraint, but no more than that (Figure 8). By contrast, in a competitive state system ($s = 2$), the competition-induced over-investment in the military may result in a larger-than-$\delta$ fraction of the continent being defended from external threats (Figure 9). Hence:

**Corollary 2 (Robustness of interstate competition).** When external threats are significant, interstate competition adequately protects a weakly bigger interval of the continent than an empire does.

Next, we define a regime as *viable* if its equilibrium net revenue is weakly positive. When $s = 1$ and regime $e$ is not viable, then political centralization or empire cannot be viable for the continent, that is, even if an empire emerges, it is not sustainable in the long run. Likewise, when $s = 2$ and if one of the two regimes is not viable, political fragmentation is not viable for the continent.
Proposition 3 (Net revenue comparison). Under a one-sided threat, the net revenue of regime $e$ is always larger than the sum of net revenues of regimes $l$ and $r$. Under a moderate and multi-sided threat, the net revenue of regime $e$ is decreasing in $\theta$ and $\beta$ but the sum of net revenues for regimes $l$ and $r$ are increasing in $\theta$ and $\beta$.

Under a one-sided threat, as the strength of the threat ($\Lambda$) increases, the net revenue of regime $e$ and the sum of net revenues of regimes $l$ and $r$ both decrease. According to Proposition 3, there exists some threshold level $\Lambda$ such that when $\Lambda = \Lambda$, the sum of net revenues of regimes $l$ and $r$ is zero while the net revenue of regime $e$ is still strictly positive. Meanwhile, under a moderate, multi-sided threat, as the cost of military investment ($\theta$) or the loss of military strength due to distance ($\beta$) increases, the net revenue of $e$ will decrease relatively rapidly compared to the sum of net revenues of $l$ and $r$. Hence, Proposition 3 gives rise to Corollaries 3 and 4 below:

Corollary 3 (Viability under one-sided, severe threat). When the external threat is one-sided and severe, political centralization is more likely to be viable than political fragmentation.

Corollary 4 (Viability under multi-sided, moderate threat). When the external threat is moderate and multi-sided, political centralization is less likely to be viable than political fragmentation if $\theta$ or $\beta$ is high.

3.4 Taxation and Public Goods Provision

Now let us endogenize taxation. Previously, the amount of taxes paid in the continent was always equal to per capita income $y$. Suppose regime $i$ has the option of reducing the tax burden of its people by $R_i \geq 0$. Lowering taxes (which is equivalent to providing a non-military public good that has a constant unit cost) eases the revolution constraint (as it helps keep the population contented) so that an individual at $x$ does not engage in revolution if:

$$R_i + M_i - \beta(G_i - x)^2 - (\Lambda - \alpha x) \geq 0.$$
When \( s = 1 \), as long as the threat level is sufficiently significant for the empire to spend a non-zero amount on the military, the revolution constraint will always bind in equilibrium regardless of whether the threats are one-sided or multi-sided. We show in Appendix A5 that if \( \theta \) is sufficiently high, that is, if building a military is costly, the empire will opt to provide some tax reimbursement instead of relying solely on building the military to satisfy the revolution constraint.

By contrast, when \( s = 2 \), the revolution constraint will not bind in equilibrium unless the threat is severe. Consequently, regimes \( l \) and \( r \) will set \( R_l = R_r = 0 \).

Consider the two scenarios that we are examining: in the case of a severe and one-sided threat, if an empire emerges, the effective level of taxation will be \( y - R_e \), where \( R_e \geq 0 \). In the case of a moderate and multi-sided threat, if interstate competition prevails, the level of taxation will remain at \( y \). More generally, we can state:

**Corollary 5 (Taxation).** Taxation is weakly lower and/or non-military public good provision weakly higher under political centralization than under political fragmentation.

### 3.5 Population Dynamics and Long-run Growth

Until now, we have assumed that external threats are always present. Consider a dynamic model where in each period, the external threat is realized with some positive probability. Each individual lives for one period and inelastically supplies labor to produce \( \overline{y} + y \), where \( \overline{y} \) is not taxable and \( y \) is taxed. For individual \( x \) under regime \( i \), the disposable income is \( \bar{y} = \overline{y} + R_i - \kappa_i(x) \) where \( R_i \) is the tax reimbursed by the regime \( i \) and \( \kappa_i(x) \) is the net damage caused by the stochastic shock. Each individual chooses between private consumption \( c \) and producing \( n \) offspring to maximize her utility \( c^{1-\gamma} n^\gamma \) subject to the budget constraint \( \rho n + c \leq \bar{y} \), where \( \rho \)
represents the cost of raising a child. Standard optimization implies that the optimal number of children is
\[ n = \frac{\gamma}{\rho} \cdot \bar{y}. \]
For simplicity, we assume that individuals redistribute themselves uniformly over the continent at the beginning of each period. Population growth is therefore given by:
\[ N = \int_0^1 ndx = \int_0^1 \frac{\gamma}{\rho} \cdot \bar{y} \, dx. \]

Let \( N_E \) and \( N_F \) denote population growths in continent \( E \) and continent \( F \) respectively. The two continents are identical except that continent \( E \) is ruled by an empire (\( s = 1 \)) and faces a severe one-sided threat of size \( \Lambda_E \), while continent \( F \) is politically fragmented (\( s = 2 \)) and faces a moderate multi-sided threat of size \( \Lambda_F \).

When the external threat is not realized, the populations in the two continents grow to
\[ N_E = \frac{\gamma}{\rho} \cdot (y + R_e) \quad \text{and} \quad N_F = \frac{\gamma}{\rho} \cdot y \]
respectively. Since \( N_E > N_F \), population grows faster under the empire.

However, the converse may be true if the external threat is realized. In this case, the realized population growths are given by:
\[ N_E = \frac{\gamma}{\rho} \cdot \left\{ (y + R_e) - \int_{x \notin D_e} \Lambda_E - \alpha x - [M_e - \beta (G_e - x)^2] \, dx \right\}, \]
\[ N_F = \frac{\gamma}{\rho} \cdot \left\{ y - 2 \cdot \int_{x < b, x \notin D_l} \Lambda_F - \alpha x - [M_l - \beta (G_l - x)^2] \, dx \right\}, \]
where \( \text{Area}(E) \) and \( \text{Area}(F) \) are illustrated in Figures 10 and 11.

Given the nature of the threats and the political configurations in the two continents, \( \text{Area}(E) \) is likely to be larger than \( \text{Area}(F) \) for two reasons: First, \( \Lambda_E > \Lambda_F \); Second, as we have shown in Corollary 2, the empire offers adequate protection to only \( \delta \) fraction of continent \( E \) (and less than \( \delta \) if tax reduction is offered), while the fraction of continent \( F \) that is adequately protected is always weakly larger than \( \delta \) due to the presence of interstate competition. If \( \text{Area}(F) < \text{Area}(E) - R_E \), it follows that \( N_E < N_F \):

**Corollary 6 (Population Change).** If the external threat is not realized, population grows faster under political centralization. If the external threat is realized, a population contraction is more likely under political centralization than under political fragmentation.
Corollary 6 suggests that population growth is usually faster under an empire. However, in the event of an exogenous shock, the fall in population may be less severe under political fragmentation. In other words, population growth is likely to be more volatile under political centralization relative to political fragmentation.\textsuperscript{17}

In interpreting our model, we have focused on external invasions. More generally, however, the shocks in our model can be seen as stemming from unforeseen political collapses and peasant rebellions in addition to invasions from outside. The central point we emphasize is that interstate competition results in a greater proportion of territory being protected than is the case under political centralization.

\section*{4 Predictions and Historical Evidence}

We are now in a position to apply our theory to explain the historical evolution of political institutions in China and Europe.

\textbf{The nature, frequency, and intensity of warfare in China and Europe} First, our model predicts that political fragmentation will lead to over-investment in the military (Corollary 1). This results in costly interstate competition. While we do not explicitly model interstate

\footnote{In Appendix A.6, we provide a numerical example of Corollary 6. The example illustrates that even if the severity of the threat ($A$) is the same in continents $E$ and $F$, in the event of an external shock the population may decline more under political centralization than under interstate competition because the latter offers adequate protection to a larger fraction of its population.}
warfare, our theory suggests that we should witness a greater frequency of interstate warfare in Europe relative to China. Second, the model predicts that Europe was less vulnerable than China to systematic shocks (Corollary 2). We show that this was indeed the case and that some of the most costly conflicts in the pre-industrial world took place in China.

Figure 12a plots the number of violent conflicts in Europe and China between 1400 and 1800 derived from Peter Brecke’s Conflict Catalog Dataset (Brecke, 1999). Violent conflicts were much more common in Europe. Additionally the majority of European wars were fought between European states whereas a significant share of China’s military conflicts between 1500 and 1800 were with nomads.

However, when major wars did occur in China they were more likely to be extraordinarily violent. Figure 12b plots some estimates of the most costly wars prior to 1750. It is clear that the most violent wars of the preindustrial period occurred in Asia and particularly in China. Only two wars with estimated death tolls in excess of 5 million are recorded for Europe compared with five for China. To be sure, all data on deaths from warfare in the preindustrial period are highly speculative, but for our purposes what is important is the order of magnitude rather than the precise numbers reported. Wars in China such as the An Lushan Rebellion, the Mongol invasions, and the Ming-Manchu transition were extremely costly conflicts. Warfare in Europe was endemic but they rarely resulted in large scale socio-economic collapse.

External threats, unification in China and the failure of European Empires Corollaries 3 and 4 are consistent with a history of fragmentation in Europe and centralization in China. China was first unified in 221 BC. Lattimore (1940) attributed China’s precocity in state-building to the relative proximity of the Eurasian steppes and the large river basins of China. His thesis, built upon in recent years by an increasing body of scholarship, highlights the existence of a natural and unbridgeable geographical divide between the component regions of China and the steppes to their north and west. In China, sufficient rainfall and favorable soil conditions made possible the early development of agriculture, especially along the major river basins. In the steppe, pastoralism prevailed given the limitations imposed by the arid climate. Persistent tensions between the steppe nomads and the sedentary farmers, especially during prolonged periods of cold temperature, provided the impetus that pushed the Chinese regions toward unification (Bai and Kung, 2011; Chen, 2012).

Europe has historically been politically fragmented; the closest Europe came to be ruled by a unified political system was under the Roman empire. The rise of Rome parallels the rise of the first empire in China (Scheidel, 2009). In terms of the model, one advantage Rome had over

18The majority of deaths in preindustrial wars did not occur in the battlefield but were the result of disease and pressure on food supplies (see Voigtländer and Voth 2013b, 781 for a discussion).
its rivals in the Hellenistic world was a lower $\theta$. Rome’s ability to project power and increase its resources of manpower was unequalled among European states in antiquity (Eckstein, 2011). Thus Rome was able to impose centralized rule upon much of Europe. Our model suggests that two factors can account for the decline of Rome: (1) over time, Rome’s military advantage ($\theta_{\text{Rome}}$) declined relative to the military capacities of its rivals such as the Persian empire or the Germanic confederacies; and (2) these rising threats came from multiple directions.\footnote{These claims are consistent with the vast historical scholarship on this topic (see Heather, 2006).} Like episodes of dynastic and imperial collapse in China, the fall of the western Empire was associated with political disintegration and economic collapse across Europe (Ward-Perkins, 2005). However, unlike in China, all subsequent attempts to rebuild the Roman empire failed.

**Taxation and public goods provision in China and Europe** Our model predicts that taxation is higher in Europe relative to China (Corollary 5). This contradicts traditional comparative accounts of Europe and China (e.g. Jones 2003), but it is consistent with recent scholarship in economic history. Taxes were high in early modern Europe; furthermore, European states did not in general provide non-military public goods. Tax revenues increased over time and were especially high in the Dutch Republic after 1600 and England after 1689 (Hoffman and Norberg, 1994; Bonney, 1999). Tax revenue increased in France as well (Johnson and Koyama, 2014a).\footnote{Johnson and Koyama (2014b) document the increase in tax revenues at a regional level in France throughout the seventeenth century.} In contrast, taxes were comparatively low in China. Karaman and Pamuk (2013) provide data on taxes revenues as a share of GDP for a range of European countries. Table 1 depicts this data in conjunction with estimates of per capita tax revenue from China from Sng (2014). The average European per capita level of taxation as measured in silver was roughly five times higher compared to China. As China was a net importer of silver, the value of silver in China might have been higher than in Europe. Following Ma (2013), we use the bare-bones subsistence basket constructed in Allen et al. (2011) to estimate the tax burden in Europe and China and obtain similar results. Clearly, as Corollary 5 suggests, taxation was lighter under politically centralized China than it was in fragmented Europe.

By and large, the taxes raised by European states were spent on warfare. In war years, over 75 percent of French revenue was spent on the military in the seventeenth century (Félix and Tallett, 2009, 155); in eighteenth century Britain, this figure varied between 61 and 74 per cent (Brewer, 1988, 32); while the peacetime military budget of Prussia during the eighteenth century accounted for 80 per cent of central government expenditure (Wilson, 2009, 119).

What about China? In the mid-eighteenth century, the Chinese state attempted to provide non-military public goods such as granaries (Will, 1990). Nevertheless, in China as in Europe,
Table 1: Per capita tax revenue in grams of silver. European average tax revenue includes Venice, Austria, Russia, Prussia, and Poland-Lithuania in addition to England, France, Dutch Republic and Spain. Sources: Karaman and Pamuk (2013) and Sng (2014). In parentheses we include a comparison of per capita tax revenue as a proportion of ‘bare-bones’ subsistence in 1750 as measured by Allen et al. (2011).

<table>
<thead>
<tr>
<th></th>
<th>Per Capita Tax Revenue in silver (grams)</th>
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<tbody>
<tr>
<td></td>
<td>1700</td>
</tr>
<tr>
<td>England</td>
<td>91.9</td>
</tr>
<tr>
<td>France</td>
<td>43.5</td>
</tr>
<tr>
<td>Dutch Republic</td>
<td>210.6</td>
</tr>
<tr>
<td>Spain</td>
<td>28.6</td>
</tr>
<tr>
<td>European average</td>
<td>52.1</td>
</tr>
<tr>
<td>China</td>
<td>10.4</td>
</tr>
</tbody>
</table>

the majority of government revenue was spent on the military (Vries, 2012). The main difference was that China’s total military spending was much lower:

‘with roughly twenty times as many inhabitants, China, in real terms, per year on average only spent roughly 1.8 times as much on the military as Britain did during the period from the 1760s to the 1820s. Per capita in real terms Britain thus spent more than ten times as much on its army and navy than China’ (Vries, 2012, 12).

This provides strong evidence in favor of Corollary 5.

Population cycles in China and Europe. Corollary 6 predicts that population growth should be more variable under political centralization because political centralization is associated with lower taxes during peacetime but also greater vulnerability to external shocks. We provide evidence in support of this proposition by drawing on population data from China and Europe.

McEvedy and Jones (1978) provides comparable population estimates for the past two thousand years. Figure 13(a) presents their population estimates for China and Europe. Panel (a) shows that the population growth of China was more variable than that of Europe. We fit the population estimates with polynomials up to the sixth order (see Appendix A.8 for details). We find that (i) it is easier to fit the European population estimates than it is to fit the Chinese population estimates because the latter are more scattered, and (ii) differences in the degree of goodness of fit aside, the fitted trend line of European population is smoother than that of Chinese population. Panel (b) confirms this finding by first differencing both time series. It is evident that the time series of Chinese population display greater variance.

We use McEvedy and Jones (1978) because they provide estimates for both China and Europe
Figure 13: Population Estimates in China and Europe (McEvedy and Jones, 1978)

(a) Estimated population levels

(b) First differences

Figure 14: Estimated population levels and major political crises in China (Cao, 2000)

over a long period of time. However, since they report data for every 50, 100, or 200 years, the resulting time series is necessarily smoother than would be the case if data was available at a higher frequency. In fact, this potential problem biases us finding a difference between the population fluctuations in China and Europe as there are several well-known sharp declines in Chinese population that are either absent or moderated in the McEvedy and Jones (1978) data.

Figure 14 displays a higher frequency population series from Cao (2000).\footnote{We use the population estimates provided in Cao (2000) because of its coverage and relative accuracy. The plunges in China’s population depicted in Figure 13 would appear even more severe if we had used official statistics drawn from Chinese historical records instead. For example, official historical records suggest that China’s population fell to 7 million in the third century after the collapse of the Han dynasty. A substantial amount of this population “loss” likely reflects the state’s inability to keep accurate records during times of crises instead of actual deaths. By contrast, Cao (2000) puts the estimate at 23 million.} This data series...

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is consistent with historical accounts which associate external invasions and political collapses with large declines in population. The fall of the Early and Later Han, Sui dynasties, the An Lushan Rebellion, the fall of the Northern Song dynasty, the Mongol invasions, and the end of the Yuan, and Ming dynasties are all visible in the figure.

For example, the Mongol invasions are associated with a sharp population collapse. Kuhn observes that ‘[p]opulation figures took another dramatic turn downward between 1223 and 1264, and by 1292 in the whole of China the population had decreased by roughly 30 million, or one third of the population, to 75 million. This was probably due to a combination of factors—warfare in north China, the Mongol invasions, and the bubonic plague or other epidemics. Whatever the causes, this was a decline in human population on a magnitude that the world has seldom seen’ (Kuhn, 2009, 75). The fall of the Yuan Dynasty is thought to have caused the population to fall again by approximately 23 percent. In contrast, historians of Europe report only one major Europe-wide collapse in population after the fall of the Roman Empire and this is the Black Death of the mid-fourteenth century.

The start-stop nature of population growth and economic development in China that is predicted in our model and witnessed in history is potentially important in helping to account for China’s failure to achieve modern economic growth before 1800. Growth theory suggests that innovation is more likely to occur in the largest economy since that is where factors such as learning-by-doing, technological diffusion, and the supply of innovative ideas should be largest (for an extended discussion see Kremer, 1993; Jones, 2001a).

By virtue of its population size, the Chinese economy was the largest in the world during much of the preindustrial period. However, sustained economic growth did not begin in China. Our framework provides a specific mechanism that can help explain why: as a unified empire, China was less robust and more vulnerable to negative shocks. As a result, its population and economy plunged during periods of crisis and this undermined the gradual accumulation of technological knowledge that plays such an important role in generating the transition to sustained growth in the theoretical growth literature (Galor and Weil, 2000; Galor, 2011).

For actual GDP estimates, see Maddison (2003) and Broadberry et al. (2012).

For example, during the Song period, the Chinese made extremely sophisticated water clocks but knowledge of this technology was completely lost as a result of conquest by the Jurchens. Mechanical clocks were only reintroduced into China by the Jesuits in the 17th century (Mokyr, 1990). Jack Goldstone (2002) calls episodes like this ‘growth efflorescences’ to distinguish them from sustained or modern economic growth. Theoretical and empirical work similarly suggests that what distinguishes modern developed economies is the ability to sustain positive GDP growth for long periods of time (see Jones and Olken, 2008; Che et al., 2013).
5 Conclusion

The idea that Europe’s political and economic success is related to its political fragmentation goes back to the Enlightenment. Montesquieu noted that in contrast to Asia where strong nations are able to conquer and subdue their neighbors, in ‘Europe on the contrary, strong nations are opposed to the strong; and those who join each other have nearly the same courage. This is the reason of the weakness of Asia and of the strength of Europe; of the liberty of Europe, and of the slavery of Asia’ (Montesquieu, 1989, 266).

In this paper we have proposed a novel theory of the origins, persistence, and consequences of political centralization and fragmentation in China and Europe. We build on the argument that external threats were a powerful force for political unification in China, but were less of a factor in Europe. Our theory suggests that political centralization should indeed be stable in China, but not in Europe, and that this centralization was beneficial from a static perspective as it minimized costly interstate competition. However, we show that in the event of an external invasion a centralized empire such as China was less robust than a decentralized state system.

Scholars have argued that decentralization gave Europe an edge in the Great Divergence because it led to greater innovation (Mokyr, 1990; Diamond, 1997; Lagerlof, 2014); support for merchants (Rosenberg and Birdzell, 1986) or political freedoms and representation (Hall, 1985). Recent work has also shown how the consequences of political fragmentation interacted with the Black Death to raise incomes and urbanization in Europe (Voigtländer and Voth, 2013b). Our theory complements these existing arguments, but we emphasize the significance of one previously neglected consequence of political centralization in China. There were periods of economic expansion, innovation, and population growth in China, but these were brought to a halt by external invasions and political crises. It was these population crises, we conjecture, that help to explain why China did not enter a period of sustained economic growth in the preindustrial era. In contrast, Europe’s polycentric system of states gave it the institutional robustness that was one of the preconditions for modern economic growth to occur.

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A Online Appendix

A.1 Definition

Under non-negativity constraints, the gross damage $t$ distance away from the frontier is $\max\{\Lambda - \alpha t, 0\}$, and the military strength $t$ distance away from the capital city of regime $i$ is $\max\{M_i - \beta t^2\}$. A location $x$ is adequately protected by regime $i$ if there exists $0 \leq x' \leq x$ such that $\max\{M_i - \beta (G_i - x')^2, 0\} - \max\{\Lambda - \alpha x, 0\} \geq 0$.

Under interstate competition, we restrict our analysis to the case where $G_l + G_r \leq 1$. To formalize this, assume that the net revenues of regimes $l$ and $r$ are $V_l = V_r = -\infty$ when $G_l + G_r > 1$. Hence, we have

$$V_l = \begin{cases} 
by - \theta M_l^2 & \text{if } G_l \leq 1 - G_r \\
-\infty & \text{otherwise},
\end{cases}$$

and

$$V_r = \begin{cases} 
(1 - b)y - \theta M_r^2 & \text{if } G_l \leq 1 - G_r \\
-\infty & \text{otherwise},
\end{cases}$$

respectively.

The border $b$ between two regimes is defined as the point between their capital cities where the regimes are equal in military strength. Hence, $b$ is defined by the following equation:

$$M_l - (b - G_l)^2 \beta = M_r - ((1 - G_r) - b)^2 \beta.$$

The border $b$ follows the above definition even if the two regimes have zero military strength in the neighborhood of the border.

However, if $M_l > M_r + \beta (1 - (G_l + G_r))^2$, the effective military strength of regime $l$ exceeds that of regime $r$ in the capital city of regime $r$. We assume that in this case, regime $r$ collapses, regime $l$ rules over the whole continent, and the border $b$ is 1. Similarly, when $M_l < M_r - \beta (1 - (G_l + G_r))^2$, regime $l$ is overrun and $b = 0$. To summarize, we have

$$b = \begin{cases} 
0 & \text{if } M_l < M_r - \beta (1 - (G_l + G_r))^2, \\
\frac{M_l - M_r}{2\beta(1 - (G_r + G_l))} + \frac{(1 - G_r) + G_l}{2} & \text{if } |M_l - M_r| < \beta (1 - (G_l + G_r))^2, \\
1 & \text{if } M_l > M_r + \beta (1 - (G_l + G_r))^2.
\end{cases}$$
A.2 Proposition 1

We restate the Proposition 1 with technical details.

**Proposition 1.** When the threat is multi-sided:

1. If $\Lambda \leq \frac{\alpha(1-\delta)}{2}$, $M_e = 0$.

2. If $\frac{\alpha(1-\delta)}{2} < \Lambda \leq \frac{1}{8} \beta \delta^2 + \frac{1}{4} \alpha (3 - \delta)$, the regime spends a non-zero amount $M_e = \beta (\frac{\delta}{2})^2 + 2\Lambda - \alpha > 0$ on the military to confront the threat emanating from one frontier while ignoring the threat from the other frontier. The capital city is located at $G_e^* = 1 - (\delta - (1 - 2\Lambda\alpha))$ or $G_e^* = \delta - (1 - 2\Lambda\alpha)$; 

3. If $\Lambda > \frac{1}{8} \beta \delta^2 + \frac{1}{4} \alpha (3 - \delta)$, the regime locates the capital city at the center of the continent ($G_e^* = \frac{1}{2}$) and spends a non-zero amount $(M_e = \beta (\frac{\delta}{2})^2 + \Lambda - \frac{\alpha}{2} (1 - \delta))$ on the military to confront the threat emanating from both frontiers;

When the threat is one-sided:

4. If $\Lambda \leq \alpha (1 - \delta)$, equilibrium military investment is zero;

5. If $\Lambda > \alpha (1 - \delta)$, the regime spends a non-zero amount $(M_e = \Lambda - \alpha (1 - \delta))$ on the military to confront the threat emanating from $x = 0$.

**Proof.** First, it is clear that if $\Lambda < \alpha (1 - \delta)/2$, then the optimal military investment is zero if the capital is located at the center. Since the regime’s payoff is decreasing in its military investment, it is optimal to invest zero.

Second, if the threat is sufficiently high such that $\Lambda > \alpha$, then every location on the continent faces the same threat $2\Lambda - \alpha$. The optimal investment is $M_e = \beta (\frac{\delta}{2})^2 + 2\Lambda - \alpha$ and the optimal capital location can be anywhere in $[\delta/2, 1 - \delta/2]$, so that the center of the continent is still an optimal location.

Now consider the intermediate case when $\alpha (1 - \delta)/2 < \Lambda < \alpha$. We will compare two cases: (i) two sides are protected, and (ii) only one side is protected. Case (i): Suppose the regime is protecting both sides of the continent under threat. Let $x$ be the leftmost location of the regime where the threat-induced net damage is zero. Since military investment is costly, the optimal solution implies that revolutionary constraint is binding so that only $\delta$ fraction of people are adequately protected. Therefore, we have the following system of equations:

\[
M = \beta (G - x)^2 + \Lambda - \alpha x, \\
M = \beta (1 - G - (1 - \delta - x))^2 + \Lambda - \alpha (1 - \delta - x)
\]
Hence,
\[ x = \frac{1}{2\alpha + 2\beta\delta} \left( \alpha - \alpha \delta - \beta \delta^2 + 2\beta G \delta \right) . \]

Note that
\[ \frac{dx}{dG} = \frac{2\beta \delta}{2\alpha + 2\beta \delta} . \]

The centralized regime’s objective function is to choose the location of the capital city to minimize military investment:
\[ \min_G M = \beta (G - x)^2 + \Lambda - \alpha x , \]

such that
\[ x = \frac{1}{2\alpha + 2\beta\delta} \left( \alpha - \alpha \delta - \beta \delta^2 + 2\beta G \delta \right) . \]

We have the following FOC:
\[
\frac{dM}{dG} = 2\beta (G - x) \left( 1 - \frac{dx}{dG} \right) - \alpha \left( \frac{dx}{dG} \right) \\
= 2\beta \left( \gamma - \frac{1}{2\alpha + 2\beta \delta} \left( \alpha - \alpha \delta - \beta \delta^2 + 2\beta G \delta \right) \right) \left( \frac{2\alpha}{2\alpha + 2\beta \delta} \right) - \alpha \frac{2\beta \delta}{2\alpha + 2\beta \delta} \\
= \alpha^2 \frac{\beta}{(\alpha + \beta \delta)^2} (2G - 1) .
\]

The SOC is
\[
\frac{d^2M}{dG^2} = \frac{2\beta \alpha^2}{(\alpha + \beta \delta)^2} > 0 .
\]

Hence
\[ G^* = 1/2 . \]

This implies that
\[
x = \frac{1}{2\alpha + 2\beta\delta} \left( \alpha - \alpha \delta - \beta \delta^2 + 2\beta \left( \frac{1}{2} \right) \delta \right) , \\
= \frac{1}{2} (1 - \delta) ,
\]

and
\[
M^* = \beta (G - x)^2 + \Lambda - \alpha (x) \\
= \beta \left( \frac{\delta}{2} \right)^2 + \Lambda - \frac{\alpha}{2} (1 - \delta) .
\]
Now consider case (ii) when the regime protects only one side:

\[
G = 1 - \left( \delta - \left( 1 - 2\frac{\Lambda}{\alpha} \right) \right) \\
= 2 - \delta - 2\frac{\Lambda}{\alpha} \\
M = \Lambda - \alpha \left( 2 - \delta - 2\frac{\Lambda}{\alpha} \right) \\
= \Lambda - 2\alpha + \alpha\delta + 2\Lambda \\
= 3\Lambda - 2\alpha + \alpha\delta .
\]

Hence, the regime will be located at 1/2 if

\[
3\Lambda - 2\alpha + \alpha\delta > \beta \left( \frac{\delta}{2} \right)^2 + \Lambda - \frac{\alpha}{2} (1 - \delta)
\]
or

\[
0 < 3\Lambda - 2\alpha + \alpha\delta - \left( \beta \left( \frac{\delta}{2} \right)^2 + \Lambda - \frac{\alpha}{2} (1 - \delta) \right) \\
= 2\Lambda - \frac{1}{4} \beta\delta^2 + \frac{1}{2} \alpha \delta - \frac{3}{2} \alpha \\
= 2\Lambda - \frac{1}{4} \beta\delta^2 + \frac{1}{2} \alpha (\delta - 3) ,
\]

as desired.

Finally, consider a one-sided threat. It is clear that if \( \Lambda \leq \alpha (1 - \delta) \), the optimal military investment is zero if the capital city is located at the center. Since the regime’s payoff is decreasing in its military investment, it is optimal to invest zero. If \( \Lambda > \alpha (1 - \delta) \), the regime now has to make a strictly positive military investment. Since military investment is costly, the optimal solution requires that the revolutionary constraint binds and only \( \delta \) fraction of the people are adequately protected. Therefore, we have the following:

\[
G = 1 - \delta , \\
M = \Lambda - \alpha (1 - \delta) ;
\]

as desired.

\[\square\]

A.3 Proposition 2

Before proving Proposition 2, it is useful to characterize the outcome of interstate competition in the absence of an external threat (\( \Lambda = 0 \)).

**Lemma 1.** When there is no external threat, then interstate competition between two regimes
implies that equilibrium military expenses are the same for both regimes:

\[ M_l^* = M_r^* = \frac{y}{4\beta\theta (1 - (G_r + G_l))}, \]

and the distance between the two capital cities is

\[ 1 - (G_r + G_l) = \left( \frac{y}{4\theta\beta^2} \right)^{1/3}. \]

Proof. We will solve the model by backward induction. We solve first the military investment problem given the locations of the two capital cities and then solve for the locations of the two capital cities.

Since regime \( l \) has surplus \( V_l \equiv by - \theta M_l^2 \), the optimization problem is

\[ \max_{M_l} V_l = by - \theta M_l^2 = \left( \frac{M_l - M_r}{2\beta (1 - (G_r + G_l))} + \frac{1 - G_r + G_l}{2} \right) y - \theta M_l^2. \]

Hence the FOC is

\[ \frac{y}{2\beta (1 - (G_r + G_l))} - 2\theta M_l^* = 0, \]

or

\[ M_l^* = \frac{y}{4\beta\theta (1 - (G_r + G_l))}. \]

Since \( V_l'' < 0 \), the SOC is satisfied. Regime \( r \)'s optimization problem and equilibrium solution are similar. Hence,

\[ M_r^* = \frac{y}{4\beta\theta (1 - (G_r + G_l))}. \]

By backward induction, regime \( l \)'s optimization problem is

\[ \max_{G_l} V_l = \left( \frac{M_l^* - M_r^*}{2\beta (1 - (G_r + G_l))} + \frac{1 - G_r + G_l}{2} \right) y - \theta (M_l^*)^2. \]

Hence

\[ \max_{G_l} \left( \frac{1 - G_r + G_l}{2} \right) y - \theta \left[ \frac{y}{4\beta\theta (1 - (G_r + G_l))} \right]^2. \]

The FOC is

\[ \frac{y}{2} + \frac{y^2}{16\beta^2c (1 - (G_r^* + G_l^*))^3} - 2 = 0. \]
where the SOC is satisfied given $V_i'' < 0$. Therefore, we have

$$1 - (G^*_r + G^*_l) = \left( \frac{y}{4\theta\beta^2} \right)^{1/3}.$$  

We obtain the same expression from solving the optimization problem of regime $r$. Finally, we need to check if the military strength of each regime is positive at the border. Without loss of generality, the military strength of regime $l$ at the border is

$$M^*_l - \left( b^* - G^*_l \right)^2 \beta = M^*_l - \left( \frac{M^*_l - M^*_r}{2\beta (1 - (G^*_r + G^*_l))} + \frac{1 - G^*_r + G^*_l}{2} - G^*_l \right)^2 \beta$$

$$= \frac{y}{4\beta\theta (1 - (G^*_r + G^*_l))} - \left( \frac{(1 - (G^*_r + G^*_l))}{2} \right)^2 \beta.$$  

This is positive if and only if

$$(1 - (G^*_r + G^*_l))^3 \leq \frac{y}{\theta\beta^2}.$$  

Since $1 - (G^*_r + G^*_l) = \left( \frac{y}{4\theta\beta^2} \right)^{1/3}$, the non-negativity constraint is satisfied. 

In the symmetric equilibrium where the continent is equally shared by the two regimes ($b = 1/2$), the location of the capital city and the level of military investment are

$$G^*_l = G^*_r = \frac{1}{2} \left( 1 - \left( \frac{y}{4\theta\beta^2} \right)^{1/3} \right), \text{ and}$$

$$M^*_l = M^*_r = \frac{y}{4\beta\theta (1 - (G^*_r + G^*_l))} = \frac{y}{4\beta\theta \left( \frac{y}{4\theta\beta^2} \right)^{1/3}} = \frac{y^{2/3}}{4^{2/3}\beta^{1/3}\theta^{2/3}}.$$  

Now consider interstate competition with a positive external threat ($\Lambda > 0$). Since regime $l$
needs to keep $\delta$ fraction of its people adequately protected, it must invest no less than $M_t^{**}$:

$$M_t^{**} = \Lambda - \frac{\alpha}{2} (1 - \delta) + \beta \left( G_i^* - \frac{1}{2} (1 - \delta) \right)^2$$

$$= \Lambda - \frac{\alpha}{2} (1 - \delta) + \beta \left( \frac{1}{2} \left( 1 - \left( \frac{y}{4\theta\beta^2} \right)^{1/3} \right) - \frac{1}{2} (1 - \delta) \right)^2$$

$$= \Lambda - \frac{\alpha}{2} (1 - \delta) + \frac{\beta}{4} \left( \left( \frac{y}{4\theta\beta^2} \right)^{1/3} - \delta \right)^2.$$

Now let us restate Proposition 2.

**Proposition 2 (Political Fragmentation).** When the threat is multi-sided:

1. If $\Lambda \leq \frac{\alpha(1-\delta)}{2} - \frac{1}{4} \beta \delta^2 + \frac{\beta}{2} \left( \frac{y}{4\theta\beta^2} \right)^{1/3} + \frac{3}{4} \left( \frac{y^2}{4\theta^2\beta^2} \right)^{1/3}$, the interval that is adequately protected is no less than $\delta$ so that the equilibrium outcome is the same as the case without any external threat ($\Lambda = 0$);

2. Otherwise, the revolution constraints are binding and the interval that is adequately protected is exactly $\delta$.

**Proof.** If $M_t^* = \frac{y^{2/3}}{4^{2/3}\beta^{1/3}\theta^{2/3}} > M_t^{**} = \Lambda - \frac{\alpha}{2} (1 - \delta) + \frac{\beta}{4} \left( \left( \frac{y}{4\theta\beta^2} \right)^{1/3} - \delta \right)^2$, then the revolution constraint is not binding. By rearranging the inequality, we obtain the desired condition 1. To avoid violating the revolution constraint, each regime has to ensure that at least $\delta$ fraction of its people are adequately protected. Suppose the revolution constraint is not binding. Recall that for all $i \in \{l, r\}$, $\partial V_i / \partial G_i > 0$ for all $G_i \leq G_i^*$. Hence, it must be the case that $G_i \geq G_i^*$ in equilibrium. However, since $\partial V_i / \partial M_i < 0$ when $G_i \geq G_i^*$, the revolution constraint must be binding, which is a contradiction. Therefore, it is optimal for each regime to adequately protect exactly $\delta$ fraction of its people.  

A.4 Proposition 3

**Proposition 3 (Viability).** Under a one-sided threat, the net tax revenue of regime $e$ is always larger than the sum of net tax revenues of regimes $l$ and $r$. If the threat is sufficiently large, regime $e$ is viable but regimes $l$ and $r$ are not. Under a moderate and multi-sided threat, the net tax revenue of regime $e$ is decreasing in $\theta$ and $\beta$ but the sum of net tax revenues for regimes $l$ and $r$ are increasing in $\theta$ and $\beta$.

**Proof.** First consider the case of a one-sided threat. Suppose to the contrary of Proposition 3,
\[ V_e^* < V_l^* + V_r^*. \]

Then regime \( e \) can mimic the choices of regime \( l \), set \( G_e = G_l^* \) and \( M_e = M_l^* \), and obtain a payoff that is weakly greater than the sum of the net tax revenues of regimes \( l \) and \( r \), which is a contradiction. Hence, it must be the case that

\[ V_e^* \geq V_l^* + V_r^*. \]

In fact the inequality has to be strict since regime \( r \) makes a non-zero military investment.

To show Corollary 3, first note that both \( V_e^* \) and \( V_l^* + V_r^* \) are decreasing in \( \Lambda \). As the threat increases steadily in severity (as \( \Lambda \) increases), beyond some value of \( \Lambda \), \( V_l^* + V_r^* \) will turn negative while \( V_e^* \) is still positive.

Now consider the case of a multi-sided threat. If the threat is moderate, the surplus of the centralized regime is

\[ V_e = y - \theta \left( \beta \left( \frac{\delta}{2} \right)^2 + \Lambda - \frac{\alpha}{2} (1 - \delta) \right)^2 \]

and the surplus of regime \( l \) (or \( r \)) is \( V_r = V_l = \frac{y^{4/3}}{4^{4/3} \beta^{2/3} \delta^{1/3}} \). Let \( \Delta V \equiv V_e - (V_l + V_r) \). We have

\[ \Delta V = -\theta \left( \beta \left( \frac{\delta}{2} \right)^2 + \Lambda - \frac{\alpha}{2} (1 - \delta) \right)^2 + 2 \frac{y^{4/3}}{4^{4/3} \beta^{2/3} \delta^{1/3}}. \]

The comparative statics results are as follows:

\[ \frac{\partial \Delta V}{\partial \Lambda} < 0; \quad \frac{\partial \Delta V}{\partial \alpha} > 0; \]
\[ \frac{\partial \Delta V}{\partial \beta} < 0; \quad \frac{\partial \Delta V}{\partial \theta} < 0; \]
\[ \frac{\partial \Delta V}{\partial \delta} < 0; \quad \frac{\partial \Delta V}{\partial y} > 0. \]

A.5 Tax Reimbursement

Claim. If \( \Lambda - \alpha (1 - \delta) > \frac{1}{2\beta} \), the empire always provides a strictly positive amount of tax reimbursement \( (R > 0) \).

Proof. Consider the case of an empire facing a one-sided threat. In the second stage, the
optimization problem is

$$\max_{R,M} V_e = y - (R + \theta M^2)$$

such that

$$R + M - \beta (G - (1 - \delta))^2 - (\Lambda - \alpha (1 - \delta)) \geq 0$$

$$R \geq 0$$

$$M - \beta (G - x)^2 \geq \Lambda - \alpha x \text{ for some } x \leq \Lambda/\alpha$$

where the last constraint ensures that there exists some location that suffers zero net damage from the threat. Since the net revenue is decreasing in $M$ and $R$, the first constraint must be binding in equilibrium. Returning to the last constraint, let $x^* \in \text{argmax}(M - \beta (G - x)^2 - \Lambda + \alpha x)$, if $M - \beta (G - x)^2 \geq \Lambda - \alpha x \text{ for some } x \leq \Lambda/\alpha$, it must be the case that

$$M - \beta (G - x^*)^2 - \Lambda + \alpha x^* \geq 0$$

Therefore, we only need to consider $x$ such that

$$2\beta (G - x) + \alpha = 0 \text{ or } x = G + \frac{\alpha}{2\beta}.$$  

We ignore the non-negativity constraint of $R$ because we already know the solution when $R = 0$. Therefore, the problem becomes

$$\max_{R,M} y - (R + \theta M^2),$$

such that

$$R + M - \beta (G - (1 - \delta))^2 - (\Lambda - \alpha (1 - \delta)) = 0$$

$$M - \beta \left(\frac{\alpha}{2\beta}\right)^2 \geq \Lambda - \alpha \left(G + \frac{\alpha}{2\beta}\right)$$

The corresponding Lagrangian is

$$L = y - (R + \theta M^2) + \phi(R + M - \beta (G - (1 - \delta))^2 - (\Lambda - \alpha (1 - \delta)))$$

$$+ \lambda \left(M - \beta \left(\frac{\alpha}{2\beta}\right)^2 - \Lambda + \alpha \left(G + \frac{\alpha}{2\beta}\right)\right)$$
Hence

\[(M) : -2\theta M + \phi + \lambda = 0\]
\[(R) : -1 + \phi = 0\]
\[(\phi) : R + M - \beta (G - (1 - \delta))^2 - (\Lambda - \alpha (1 - \delta)) = 0\]
\[(\lambda) : \lambda \left( M - \beta \left( \frac{\alpha}{2\beta} \right)^2 - \Lambda + \alpha \left( G + \frac{\alpha}{2\beta} \right) \right) \geq 0\]

\[\lambda = 0 \text{ or } M - \beta \left( \frac{\alpha}{2\beta} \right)^2 = \Lambda - \alpha \left( G + \frac{\alpha}{2\beta} \right)\]

**Case 1:** \( \lambda = 0 \). Then we have

\[M = \frac{1}{2\theta}\]
\[R = -\frac{1}{2\theta} + \beta (G - (1 - \delta))^2 + (\Lambda - \alpha (1 - \delta))\]

**Case 2:** \( \lambda \neq 0 \). We have

\[M = \beta \left( \frac{\alpha}{2\beta} \right)^2 + \Lambda - \alpha \left( G + \frac{\alpha}{2\beta} \right)\]
\[R = -M + \beta (G - (1 - \delta))^2 + (\Lambda - \alpha (1 - \delta))\]
\[= \frac{1}{4\beta} (\alpha - 2\beta + 2G\beta + 2\beta\delta)^2 > 0 .\]

Since the tax rebate is always positive in Case 2, we only need to consider the choice of the capital city in Case 1:

\[
\max_{\tilde{G}} V_e = y - (R + \theta M^2)
\]
\[= y - \left( -\frac{1}{2\theta} + \beta (G - (1 - \delta))^2 + (\Lambda - \alpha (1 - \delta)) + \theta \left( \frac{1}{2\theta} \right)^2 \right).\]

The FOC is

\[-2\beta (G - (1 - \delta)) = 0 ,\]
where the SOC is negative (hence this is a maximization problem). We have
\[
G = 1 - \delta ,
R = -\frac{1}{2\theta} + (\Lambda - \alpha (1 - \delta)) ,
M = \frac{1}{2\theta} .
\]

Note that
\[
V_e = y - \left( -\frac{1}{2\theta} + (\Lambda - \alpha (1 - \delta)) + \frac{1}{4\theta} \right) = y - (\Lambda - \alpha (1 - \delta)) + \frac{1}{4\theta} .
\]

Recall that if tax reimbursement is zero:
\[
V_e (R = 0) = y - \theta (\Lambda - \alpha (1 - \delta))^2 .
\]

We have
\[
y - (\Lambda - \alpha (1 - \delta)) + \frac{1}{4\theta} - (y - \theta (\Lambda - \alpha (1 - \delta))^2)
= \theta \left( (\Lambda - \alpha (1 - \delta))^2 - \frac{1}{\theta} (\Lambda - \alpha (1 - \delta)) + \frac{1}{4\theta^2} \right)
= \theta \left( (\Lambda - \alpha (1 - \delta)) - \frac{1}{2\theta} \right)^2 \geq 0 .
\]

Since \((\Lambda - \alpha (1 - \delta)) > \frac{1}{2\theta}\), net revenue is higher when \(R\) is positive, as desired.

\[\square\]

A.6 A Numerical Example

We illustrate Corollary 6 using a simple numerical example. Let: \(\Lambda = 20\), \(\alpha = 35\), \(\beta = 100\), \(\delta = 0.45\), \(y = 1500\), and \(\theta = 1\). To show \(N_F > N_E\), it suffices to demonstrate Area \(\langle E \rangle + R_e < \text{Area } \langle F \rangle\).

For continent \(E\), the capital city is located at \(1 - \delta = 0.55\), military investment is \(M = 1/(2\theta) = 0.5\), and the tax rebate is \(R_e = -\frac{1}{2\theta} + (\Lambda_E - \alpha (1 - \delta)) = 0.25\). Subsequently, Area \(\langle E \rangle + R_e = -5.4327\).

For continent \(F\), the location of two capital cities are given by \(G_l = G_r = \frac{1}{2} - \frac{1}{2} \left( \frac{y}{4\theta^2} \right)^{1/3} = 0.33264\). Each regimes invests \(M = \frac{y^{2/3}}{4\theta^{3/2}} \approx 0.33264\). Each regime invests \(M = \frac{y^{2/3}}{4\theta^{3/2}} \approx 0.33264\). The tax rebate is \(R_e = -\frac{1}{2\theta} + (\Lambda_E - \alpha (1 - \delta)) = 0.25\). Subsequently, Area \(\langle F \rangle = 4.6374\).

Since Area \(\langle E \rangle + R_e - \text{Area } \langle F \rangle = -5.4327 + 4.6374 = -0.7953 < 0\), we show that in this case,
the population falls more sharply under political centralization than under political fragmentation when the shock is realized.

A.7 Location of Nomadic Invasions

<table>
<thead>
<tr>
<th>Phase</th>
<th>Century</th>
<th>Nomadic Peoples</th>
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<th>Russia</th>
<th>China</th>
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<td>Manchus</td>
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Table 2: Major waves of nomadic invasions. Sources: Chaliand (2005). It is evident that China faced a greater threat from the steppe invaders that did Europe.
A.8  Population Fluctuations

<table>
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<tr>
<th>Pop. ('000)</th>
<th>$t$</th>
<th>$t^2$</th>
<th>$t^3$</th>
<th>$t^4$</th>
<th>$t^5$</th>
<th>$t^6$</th>
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<th>Adj. $R^2$</th>
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<tr>
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<td>0.068***</td>
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</table>

Constant terms are not reported. * significant at 10%, ** significant at 5%, *** significant at 1%.

Table 3: Fitting Year Polynomials to Chinese and European Population Data. Adjusted $R^2$ is higher for Europe than for China in each case.