Expected Inflation Regimes in Japan

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Abstract

This paper examines the dynamics of expected inflation regimes in Japan over the last three decades based on the smooth transition Phillips curve model. We find that there is a strong connection between the expected inflation and monetary policy regimes. The results also suggest that the introduction of the inflation targeting policy, and quantitative and qualitative easing in the beginning of 2013 have successfully escaped from the deflationary regime, but was not enough to achieve the 2% inflation target. Finally, our results indicate the significance of exchange rates in explaining the recent fluctuations of inflation, and the importance of oil and stock prices in maintaining the positive expected inflation regime.

JEL codes: C22, E31, E52
Key words: Hybrid Phillips curve, monetary policy, inflation targeting, qualitative and quantitative easing, smooth transition model

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1 Introduction

Inflation is obviously one of the most important variables for macroeconomists, practitioners, and policy makers, since inflation affects the value of money. As a result, inflation rates significantly influence the behaviors of individuals and firms. Therefore, the central banks and monetary authorities have paid special attention to the inflation rates recently, and inflation-targeting monetary policy has been adopted by more than 20 countries including US and UK. As for Japan, the first arrow in the first stage of Abenomics is to conduct an aggressive monetary policy; as part of this, the Bank of Japan (BOJ) introduced the inflation targeting policy in January 2013 by committing a 2% price stability target. Following this introduction, the BOJ has conducted innovative monetary policies, including the quantitative and qualitative easing (QQE) and negative interest policies. Nonetheless, it is still uncertain whether and when the 2% price stability target will be achieved.

To achieve this inflation target, one of the crucial indicators is expected inflation, which is expected inflation level for the future by individuals and firms.\textsuperscript{1} Since inflation is anticipated to approach the expected inflation in the future, aside from the short-term fluctuations along with the business cycles, it can provide a useful measure to evaluate the appropriateness of the current monetary policy as well as the necessity of additional monetary easing to achieve the inflation target. Therefore, it is extremely informative to examine the current expected inflation level and compare it with the inflation target set by the monetary authorities.

The purpose of this paper is to investigate the dynamics of expected inflation in Japan over the last three decades and their relationship with monetary policies. Over the last three decades, Japanese economy has experienced many significant events, such as the bursting of the bubble economy, the lost two decades, the Lehman and Euro financial crises, and the introduction of a zero interest rate and inflation targeting monetary policies. It is quite meaningful to empirically study how expected inflation rates in Japan have evolved through these events. In particular, the Japanese economy suffered from low expected inflation due to the deflationary mind formulated through what we call the lost two decades after the bursting of the bubble economy in 1990. It is important to examine whether the expected inflation during this period is indeed below zero or not and how the BOJ’s policies during this period, such as the zero interest and quantitative easing policies, affected the expected inflation. In addition, it is of great interest to analyze whether and how the expected inflation has become closer to the BOJ’s 2% target since the BOJ introduced the QQE.

\textsuperscript{1}Precisely speaking, there are many types of expected inflation depending on the definition of the future as well as the economic agent. The expected inflation analyzed in this paper is the expected inflation in the distant future by the mathematical definition. However, judging from the results of the analysis, there should be no substantial difference from the expected inflation over the next 5 or 10 years. In addition, since we use the inflation based on the consumer price index, the expected inflation for this paper is naturally considered to be the one by households.
One of the difficulties in assessing the dynamics of expected inflation is that expected inflation is unobservable. Although survey-based expected inflation rates have recently become available due to the development of survey data, it is often pointed out that surveyees are mostly limited to professionals, and survey expected inflation contains various possible biases. For example, Fuhrer, Olivei, and Tootell (2012) attempt to estimate the expected inflation rates in Japan based on the survey data, but their estimates uniformly exceed the actual inflation rates between the beginning of 1990 and 2010. Therefore, it is not straightforward to consider that the survey expected inflation to be unbiased estimates of expected inflation. In addition, it is not easy to estimate the expected inflation rates from the survey data, either.

To overcome the problem associated with the survey data, many recent studies have tried to estimate the expected inflation rates by modeling them as unobservable state variables and estimating them based on the macroeconometric models. In this vein, this paper estimates the expected inflation rates in Japan using the hybrid Phillips curve. More specifically, we introduce regime switching to the Phillips curve to capture the regime shifts in expected inflation rates, following Kaihatsu and Nakajima (2015). They employ the Markov-switching (MS) model to examine the possible regime changes in expected inflation, but their MS model is stationary. Therefore, the regime distribution will converge on the stationary distribution in the long-run, meaning that their MS model cannot capture a permanent regime change in expected inflation. However, it is likely that there are permanent regime changes in expected inflation given the experiences of the Japanese economy over the last three decades, including the bursting of the bubble economy and significant changes in monetary policies.

To capture the possible permanent regime changes in expected inflation rates, we apply the smooth-transition (ST) model to the hybrid Phillips curve. By doing so, we attempt to identify the number of regimes, characterize each regime, and estimate the dynamics of expected inflation. More specifically, we address the following questions: (i) How many expected inflation regimes were there over the last three decades in Japan? (ii) Is there any relationship between inflation regimes and monetary policy regimes? (iii) What were the expected inflation rates under each inflation regime? (iv) Has the expected inflation increased after the adoption of inflation targeting policy by BOJ? (v) Is the current expected inflation significantly different from the BOJ’s target of 2%? (vi) What are the effects of oil prices, stock prices, and exchange rates on Japanese inflation?

Our findings can be summarized as follows. First, there have been three regimes in expected inflation in Japan over the last three decades, and they reasonably corresponded to the traditional monetary policy regime between 1985 and 1994 (regime 1), the extremely low interest rate monetary policy regime between 1995 and 2012 (regime 2), and the inflation

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2In this paper, we use the regime to express the state, and both terms are used interchangeably.
targeting monetary policy regime since 2013 (regime 3).

Second, based on the three-regime ST Phillips curve model, each regime can be characterized as follows. The expected inflation in regime 1 was relatively high and stable at 1.5% for the core inflation and 2.0% for the core2 inflation. In regime 2, the expected inflation decreased considerably to $-0.20\%$ for the core inflation and to $-0.61\%$ for the core2 inflation. While the estimate of expected inflation was not significantly different from 0 for the core inflation, it was significantly negative for the core2 inflation. In addition, our results show that the expected inflation had been stable at below zero during this regime, although the BOJ enacted many pioneering monetary policies, such as a zero interest rate and quantitative easing monetary policies. Thus, our results suggest that those BOJ’s monetary policies were not enough to escape from the deflationary regime, although they most likely supported the Japanese economy and prevented the expected inflation from further decreases. Finally, the expected inflation in regime 3 showed a sizable increase to $0.34\%$ for the core inflation and $0.48\%$ for the core2 inflation. While the estimate of expected inflation was not significantly different from 0 for the core inflation, it was significantly positive for the core2 inflation. More importantly, our results clearly demonstrate that the expected inflation in regime 3 is significantly smaller than the BOJ’s 2% inflation target for both inflation measures. Thus, the QQE and the negative interest policies conducted in regime 3 were partially successful for escaping from the deflationary regime, but they were not sufficient to achieve the BOJ’s 2% inflation target.

Third, we extended the benchmark model so that the oil prices, stock prices, and exchange rates could affect inflation. Here, our results indicate that the oil price and exchange rate had a relatively large effect on the core inflation in regime 2 and regime 3, respectively. On the other hand, the oil prices, stock prices, and exchange rates had little effect on the core2 inflation in regimes 2, although the exchange rates became influential in regime 3. Thus, our results suggest that the depreciation of Japanese yen occurred with the introduction of the inflation targeting policy by the BOJ played an important role in the significant increase of inflation in 2013.

Finally, we consider another type of extended model in which the oil prices, stock prices, and exchange rates could affect the dynamics of the expected inflation regime. Our results show that the oil and stock prices had significant impacts on the expected inflation regime for the core inflation, while the stock price strongly affected the expected inflation regime for the core2 inflation. This greatly contrasts with the above results, showing the importance of the exchange rates to explain the realization of inflation. More specifically, our results demonstrate that, even after 2012, the expected inflation regime tended to move back to the

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3In this paper, we calculate two measures of inflations from the consumer price index (CPI). More precisely, the core inflation is calculated from the headline CPI, excluding fresh food; the core2 inflation is calculated from the headline CPI, excluding food and energy. See Section 3.1 for more details.
deflationary regime when the oil or stock prices decrease. Therefore, the BOJ’s purchase of Exchange Traded Funds (ETF) could be effective in preventing the expected inflation from going back to the deflationary regime, which was mainly observed before 2012.

The remainder of this paper is organized as follows. Section 2 introduces the ST Phillips curve model and explains how to choose the number of regimes. Section 3 summarizes the empirical results based on the benchmark model, while Section 4 provides the empirical results based on the extended models. Finally, Section 5 concludes the paper.

2 Methodology

This paper employs the hybrid Phillips curve model with regime switching to estimate the dynamics of the expected inflation rates in Japan, following Kaihatsu and Nakajima (2015). They apply the MS model to capture the possible regime changes in expected inflation. In the Markov-switching model’s framework, the expected inflation dynamics are typically modeled by a stationary Markov process; hence, the expected value of expected inflation is constant. Although it is possible for the expected inflation to deviate from its expected level temporarily, it has to revert to the stationary level eventually. In this sense, the MS Phillips curve model cannot capture permanent regime changes in the expected inflation. However, it is likely that there are permanent regime changes in the expected inflation, for example, after drastic changes to the monetary policies. Therefore, we adopt the ST Phillips curve model to capture the possible permanent regime changes in the expected inflation rates. By doing so, we try to find the answers to the questions addressed in Introduction.

2.1 ST Phillips Curve Model

The ST model is developed within the autoregressive model by, among others, Chan and Tong (1986), and Granger and Teräsvirta (1993); its statistical inference is established by Teräsvirta (1994). Since then, the ST model has been applied to many types of models. However, to our best knowledge, none of the studies use the ST Phillips curve model to estimate the regime shifts in expected inflation. In this subsection, we briefly discuss our model.

To examine the regime shifts in expected inflation in Japan, our benchmark model is a hybrid Phillips curve. One of the most important characteristics of the hybrid Phillips curve is that the current inflation rate depends not only on the current output gap and expected inflation but also the past inflation rates. Assuming that the expected inflation

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4 By imposing zero restrictions on the transition probabilities, the MS model can also capture permanent regime changes. However, we still prefer the ST model, since it allows regimes to change slowly from one to another while the MS model assumes the regime changes occur all of sudden in one period.

rate is constant, it is given as
\[ \pi_t = \sum_{k=1}^{K} \alpha_k \pi_{t-k} + \left( 1 - \sum_{k=1}^{K} \alpha_k \right) \mu + \beta x_t + \varepsilon_t. \] (1)

Here, \( \pi_t \) is an inflation rate at time \( t \), and \( x_t \) is an output gap. If the inflation is persistent partially due to the backward-looking price setting by firms, the past high inflation tends to be followed by relatively high current inflations. The hybrid Phillips curve (1) explicitly models this possibility and can be considered to be the autoregressive (AR) model with an exogenous variable, assuming that the output gap is exogenous. If the output gap stays stationary level at 0 and the AR coefficients in (1) satisfy the stationarity conditions,\(^6\) it is easy to confirm that
\[ \lim_{h \to \infty} E_t(\pi_{t+h}) = \mu. \]

Here, \( E_t(\cdot) \) is a conditional expectation given the information available at time \( t \). Therefore, we can interpret \( \mu \) as the expected inflation at time \( t \)

Over the past three decades, the Japanese economy has experienced many significant events, such as a bursting of the bubble economy, the lost two decades, the Lehman and Euro financial crises, and the introduction of a zero interest rate and inflation targeting monetary policies. As a result, it is likely that the expected inflation in Japan has had regime shifts over the last thirty years. In this paper, we investigate this possibility by modeling regime changes as permanent regime shifts. More specifically, we apply the ST model to the hybrid Phillips curve (1) so that the expected inflation has regime shifts as
\[ \mu_t = \mu^{(1)} + G(s_t; c, \gamma) \left( \mu^{(2)} - \mu^{(1)} \right). \] (2)

Here \( G(\cdot) \) is called a transition function taking the values between 0 and 1, and \( s_t \) is called a transition variable. If the current regime continues forever or the transition function \( G \) takes the same value forever, it can be easily shown that
\[ \lim_{h \to \infty} E_t(\pi_{t+h}) = \mu_t. \]

In this sense, \( \mu_t \) can be regarded as the expected inflation at time \( t \). If \( G(s_t) \) in (2) is equal to 0, \( \mu_t = \mu^{(1)} \), while \( G(s_t) = 1 \) implies \( \mu_t = \mu^{(2)} \). Thus, the expected inflation modeled as (2) is assumed to have two regimes characterized by \( \mu^{(1)} \) and \( \mu^{(2)} \), and it generally takes the value between them depending on the value of the transition function \( G \).

The transition function and transition variable are determined according to the purpose of analysis. For example, Lin and Teräsvirta (1994) investigate a continuous permanent regime

\(^6\)Stationary conditions for AR models are that the absolute values of all roots of the AR characteristic equation
\[ 1 - \alpha_1 z - \cdots - \alpha_K z^K = 0 \]
are larger than 1.
change using the logistic transition function with a time-trend transition variable. Since the purpose of this paper is to detect the possible permanent regime shifts in expected inflation in Japan over the last three decades, we follow their research, and use a logistic transition function given as

$$G(s_t; c, \gamma) = \frac{1}{1 + \exp(-\gamma(s_t - c))}, \quad \gamma > 0.$$  \hfill (3)

Here, $c$ is a location parameter determining the timing of the transition, and $\gamma$ is a smoothness parameter capturing the speed of the transition. One of the advantages of the logistic transition function is that it can express various forms of transitions depending on the values of $c$ and $\gamma$. Additionally, $c$ and $\gamma$ can be estimated from the data, enabling the selection of the best transition patterns based on data, which is very attractive for the purposes of this paper, particularly examining the regimes shifts in expected inflation.

Finally, we use a time trend $s_t = t/T$ as the transition variable, following Lin and Ter"asvirta (1994), where $T$ is the sample size. In addition, we assume $0.05 < c < 0.95$ so that we can detect the regime shifts in expected inflation within the sample period. In this setting, $G(s_t)$ takes the value close to zero with smaller $s_t$ around the beginning of the sample period, making $\mu_t$ close to $\mu^{(1)}$. Therefore, $\mu^{(1)}$ can be considered to be the expected inflation around the beginning of the sample. In contrast, around the end of the sample, $\mu_t$ approaches $\mu^{(2)}$ as $G(s_t)$ approaches one with a larger $s_t$. Thus, $\mu^{(2)}$ can be considered to be the expected inflation around the end of the sample. Specifically, under these assumptions, the expected inflation $\mu_t$ changes from $\mu^{(1)}$ to $\mu^{(2)}$ with time as $G(s_t)$ changes from zero to one. By estimating the model, we can identify not only the expected inflation levels of two regimes, $\mu^{(1)}$ and $\mu^{(2)}$, but we can also identify the timing and speed of the change from $\mu^{(1)}$ to $\mu^{(2)}$.

Furthermore, we can extend the two-regime model (2) to models with three or more regimes. For example, the three-regime model can be written as

$$\mu_t = \mu^{(1)} + G_1(s_t; c_1, \gamma_1)(\mu^{(2)} - \mu^{(1)}) + G_2(s_t; c_2, \gamma_2)(\mu^{(3)} - \mu^{(2)}),$$  \hfill (4)

Here, $G_i$, $i = 1, 2$ is the logistic transition function (2), and we assume $c_1 < c_2$. In this model, when $s_t$ is near 0 or around the beginning of the sample, $\mu_t$ approaches the expected inflation of the first regime, $\mu^{(1)}$, because both $G_1(s_t)$ and $G_2(s_t)$ take small values near 0. When $c_1 < s_t < c_2$, $\mu_t$ becomes close to the expected inflation of the second regime, $\mu^{(2)}$; this is because $G_1(s_t)$ tends to be near one, but $G_2(s_t)$ tends to be near zero. Finally, when $c_2 < s_t$, $\mu_t$ is close to $\mu^{(3)}$ with $G_1(s_t)$ and $G_2(s_t)$ taking values near one. Thus, under this specification, the expected inflation $\mu_t$ changes from $\mu^{(1)}$ via $\mu^{(2)}$ to $\mu^{(3)}$ with time as first $G_1(s_t)$ changes from zero to one; this is followed by a similar change in $G_2(s_t)$. The extent to which the expected inflation is close to each regime's expected inflation, and the way in
which it becomes close, depends on the values of $c_i$ and $\gamma_i$, which can be estimated from data. This makes the multiple-regime ST model attractive for the examination of the dynamics of expected inflation in Japan.

A number of recent studies have pointed out that the flattening of the Phillips curve (1) has been promoted in advanced countries. For instance, Roberts (2006) reports that the US Phillips curve has been flattened since 1980s and that the changes in monetary policy have played an important role in this flattening. Similarly, De Veirman (2009) and Fuhrer, Olivei, and Tootell (2012) document that the Phillips curve has been flattened in Japan since the early 1990s. Therefore, it is quite relevant to consider the time variation in the slope of the Phillips curve expressed by $\beta$. Similar to $\mu_t$, we model $\beta_t$ using the ST model. For example, the three-regime model is specifically given as

$$\beta_t = \beta^{(1)} + G_1(s_t; c_1, \gamma_1)(\beta^{(2)} - \beta^{(1)}) + G_2(s_t; c_2, \gamma_2)(\beta^{(3)} - \beta^{(2)})$$

(5)

Here, we assume that the transition functions, $G_1$ and $G_2$, in (4) and (5) share the same parameters of $c_i$ and $\gamma_i$. In other words, it is assumed that the timing and speed of regime transitions in $\mu$ and $\beta$ are the same. However, note that this assumption is not that restrictive, since the actual dynamics could be very different due to the different estimates of $\mu^{(i)}$ and $\beta^{(i)}$ for each regime.

In sum, the ST Phillips curve model in this paper is given by

$$\pi_t = \sum_{k=1}^{K} \alpha_k \pi_{t-k} + \left(1 - \sum_{k=1}^{K} \alpha_k \right) \mu_t + \beta_t x_t + \varepsilon_t.$$  

In addition, $\mu_t$ and $\beta_t$ are modeled with the ST model such as (4) and (5).

### 2.2 Choice of number of regimes

The choice of the number of regimes is crucial for the ST model. As previously discussed, over the last three decades the Japanese economy has experienced many significant events, such as the bursting of the bubble economy, and the introduction of zero interest rate and inflation targeting monetary policies. As a result, it is likely for the expected inflation in Japan to have had several regime shifts over the last three decades, but there is no guidance about the number of regime. Hence, it is important to specify the number of regimes. In this paper, we choose the number of regimes based on hypothesis testing, which can compare the models with the $M$ and $M + 1$ regimes. In this subsection, we briefly discuss those tests.

First, we test the one-regime Phillips curve model (1) against the two-regime Phillips curve model, which is characterized by (2) and (6). In this case, the null hypothesis and the alternative hypothesis can be expressed as $H_0 : \gamma = 0$ and $H_1 : \gamma > 0$, respectively. However,
this test is not standard since parameters $\mu^{(i)}$ and $\beta^{(i)}, i = 1, 2$ cannot be identified under the $H_0$.\footnote{If we express the null hypotheses as $H_0: \mu^{(1)} = \mu^{(2)}, \beta^{(1)} = \beta^{(2)}$, then there is still an identification issue, since we cannot identify $\gamma$ and $c$.}

To deal with this identification problem, Luukkonen, Saikkonen, and Teräsvirta (1988) suggest approximating the logistic function with a first-order Taylor approximation around $\gamma = 0$. This results in the auxiliary regression of the form

$$
\pi_t = \sum_{k=1}^{K} a_k \pi_{t-k} + b_0 + b_1 x_t + b_2 s_t + b_3 x_t s_t + \epsilon_t.
$$

(7)

Luukkonen, Saikkonen, and Teräsvirta (1988) show that testing $H_0: \gamma = 0$ in the two-regime Phillips curve model (2), (6) is equivalent to testing $H'_0: b_2 = b_3 = 0$ in the auxiliary regression (7). Since there is no identification problem in this auxiliary regression, we can easily test the null hypothesis $H'_0: b_2 = b_3 = 0$. Specifically, Luukkonen, Saikkonen, and Teräsvirta (1988) demonstrate that the Lagrange-multiplier (LM) test statistic to test $H'_0$ asymptotically follow the Chi-squared distribution with 2 degree of freedom. Based on their results, we can test the one-regime model against the two-regime model.

Furthermore, Eitrheim and Teräsvirta (1996) develop the LM statistic to test the $M$-regime model against the alternative of the $M+1$ regime model based on the similar auxiliary regression model. Therefore, Based on their results, we can test, for example, the two-regime model against the three-regime model.

### 3 Empirical Analysis

In this and the following sections, we discuss our empirical results. Specifically, based on the empirical analysis we address the following questions: (i) How many expected inflation regimes were there over the last three decades in Japan? (ii) Is there any relationship between inflation regimes and monetary policy regimes? (iii) What were the expected inflation rates under each inflation regime? (iv) Has the expected inflation increased after the adoption of the inflation targeting policy by the BOJ? (v) Is the current expected inflation significantly different from the BOJ’s target of 2%? (vi) What are the effects of oil prices, stock prices, and exchange rates on Japanese inflation?

#### 3.1 Data

Our empirical analysis is based on the Japanese monthly data of CPI, industrial production index, nominal effective exchanger rate, oil prices, and stock prices, with the sample period lasting from January 1985 to July 2016. We use two measures for CPI, namely the CPI excluding fresh food and the CPI excluding food and energy. Both data are downloaded
from the Ministry of Internal Affairs and Communications, and they are seasonally adjusted by the X-12-ARIMA method. The seasonally adjusted industrial production is obtained from the Ministry of Economy, Trade, and Industry, while the nominal effective exchange rate is downloaded from the BOJ. For the oil price, we use the oil price index obtained from the International Monetary Fund. Finally we collect the TOKYO Price Index (TOPIX) from the Datastream for the stock price.

We calculate the inflation rate by taking the first difference of log of CPI measures and multiplying them 1200 to annualize. Thus, the inflation rate at time $t$, $\pi_t$, is calculated as

$$\pi_t = 1200(\log p_t - \log p_{t-1}),$$

where $p_t$ is a CPI measure at time $t$.\(^8\) We call the inflation based on the CPI excluding fresh food the core inflation, and the CPI excluding food and energy core\(^2\) inflation.\(^9\)

Finally, the output gap is calculated by taking the difference between the industrial production and its trend calculated by the Hodrick-Prescott filter.\(^10\)

### 3.2 Choice of Number of Regimes

The purpose of this paper is to investigate the dynamics of expected inflation in Japan over the last three decades and their relationship to monetary policy. To this end, we employ the ST Phillips curve (6) to detect the regime shifts in expected inflation. In this subsection, we select the number of regimes by the sequential testing based on Luukkonen, Saikkonen, and Teräsvirta (1988) and Eitrheim and Teräsvirta (1996), as discussed in the previous section.

The second and third rows of Table 1 summarize the results of hypothesis testing of the one-regime model against the two-regime model based on Luukkonen, Saikkonen, and Teräsvirta (1988).\(^11\) The second row shows the values of their proposed LM statistic, and the third row reports their $p$-values. As can be seen, the LM statistic takes large values with small $p$-values for both inflation measures. Thus, the one-regime model is obviously rejected against the two-regime model, meaning that there is at least one regime shift in Japanese inflation regimes.

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\(^8\)During the sample period, the consumption tax rate changed three times in April 1989, April 1997, and April 2014. In this paper, we assume that the consumption tax hikes are fully reflected within one month, and we adjust them by linearly interpolating the data of April from the data in March and May in the concerned year. Rigorously speaking, there are some goods, such as public utility charges, that reflect the consumption tax hike with more than a one-month delay. However, the proportion of those goods is relatively small and should not affect the estimation results.

\(^9\)Note that for many countries, including US, the core inflation typically means the core\(^2\) inflation in this paper. Note also that the core inflation is the BOJ’s official measure for the 2% inflation target.

\(^10\)More precisely, we normalize it to have zero mean and unit variance.

\(^11\)We set $K = 2$ in (6) for the rest of the analysis, since the higher order AR terms are not significant.
Next, we test the two-regime model against the three-regime model based on Eitrheim and Teräsvirta (1996). The results are shown in the fourth and fifth rows of Table 1, indicating the great contrast between the two inflation measures. For the core inflation, the two-regime model is preferred to the three-regime model with a large $p$-value of 0.46. However, for the core2 inflation, the two-regime model is rejected against the three-regime model with less than 5% $p$-value.

Finally, the last two columns of Table 1 report the results of the hypothesis testing of the three-regime model against the four-regime model. The relatively large $p$-values suggest that there is no evidence of the four-regime model over the three-regime model for both measures.

In sum, our results demonstrate that the two-regime model is preferred for core inflation, while the three-regime model is dominant for core2 inflation. In other words, it is enough to consider three regimes in expected inflation over the last three decades in Japan. For this reason and to make the results comparable between two inflation measures, we will use the three-regime ST Phillips curve characterized by (4)-(6) for both measures in the following analysis.

### 3.3 Inflation Regimes and Monetary Policy Regimes

In this subsection, we show the estimated regime transition dynamics to see when and how the regime shifts in the three-regime ST Phillips curve model (4)-(6) have occurred over the last three decades.

We estimate the three-regime ST Phillips curve model via the maximum likelihood estimation (MLE), assuming that $\varepsilon_t$ independently and identically follows the Normal distribution with the mean of 0 and variance of $\sigma^2$. We also impose the restrictions that the coefficients on the output gap are nonnegative in order to be consistent with the economic theory.\(^{12}\) The estimation results are documented in Table 2.\(^{13}\) For the core inflation, $c_1$ is estimated to be 0.29, meaning that the center of the transition from regime 1 to regime 2 is estimated to be around June 1994. In addition, the estimate of $\gamma_1$ of 200 indicates that the speed of the transition is very rapid. To see this point graphically, Figure 1 plots the estimated dynamics of the transition functions $G_1(s_t)$ and $G_2(s_t)$. As can be seen, for the core inflation, $G_1$ has been almost 0 until the end of 1993, but it increased rapidly to 1 during 1994 and has been 1 since the beginning of 1995. For the core2 inflation, $c_1$ is estimated as 0.35, suggesting that the center of transition is around March 2006. The speed of the transition is estimated to be relatively slow with a $\gamma_1$ estimate of 19.1. As a result, $G_1$ for the core2 inflation changes

\(^{12}\)If these restrictions are bound for some coefficients, we replace them with 0 and re-estimate the model.

\(^{13}\)If the transition function looks like a step function, the estimate of $\gamma_i$ becomes very large and is not well determined, since the log-likelihood becomes insensitive with $\gamma_i$. In these cases, we have fixed $\gamma_i$ at an upper bound equal to 200 and have re-estimated the model. In this case, the parameter’s standard errors are denoted by NA.
from 0 to 1 relatively smoothly between 1994 and 1998, as can be confirmed from Figure 1.

[Insert Table 2 and Figure 1 here]

Regarding $c_2$ and $\gamma_2$, both inflation measures show quite similar estimation results. $c_2$ is estimated to be 0.88 and 0.89 for the core and core2 inflation, respectively. In other words, the estimation results indicate that the center of the transition from regime 2 to regime 3 is around September 2012 and February 2013 for each measure, respectively. In addition, $\gamma_2$ is estimated to be 200 for both measures, meaning that the speed of transition is very rapid. These results can be also verified from the estimated dynamics of $G_2$, which are plotted in Figure 1.

In sum, our results suggest that the first regime shift occurred at around 1995, although the speed of transition depended on the measure of inflation. The BOJ had conducted the traditional monetary policy using the bank rate as a policy rate before 1995. However, the liberalization of interest rates was completed in October 1994, and the direct relationship between the bank rate and deposit interest rate has disappeared. Following these events, the BOJ adopted the call rate as a new policy rate in March 1995. Then, the BOJ started the extremely low interest rate policy with the 0.5% call rate in September 1995. Thus, there were significant changes to the BOJ’s monetary policy in 1995. Indeed, previous studies, such as Miyao (2000) and Inoue and Okimoto (2008), detect a regime shift in the effect of Japanese monetary policy around this period. Interestingly, the first regime shift in the three-regime Phillips curve seems to coincide with this timing.

The second regime shift took place at around the beginning of 2013, and it occurred very rapidly. The BOJ introduced the inflation targeting policy in January 2013 by committing to a 2% price stability target. Following the introduction, the BOJ launched the QQE in April 2013. Obviously, the second regime shift in the three-regime Phillips curve corresponds to these changes.

Thus, there seems to be strong relationship between the inflation regimes and monetary policy regimes in Japan over the last three decades. Specifically, the first inflation regime is characterized by the conventional monetary policy regime. The second inflation regime roughly corresponds to the low or zero interest rate monetary policy regime. The third inflation regime coincides with the inflation targeting monetary policy regime.

3.4 Expected Inflation Regimes

The previous subsection demonstrated the strong correspondence between the inflation regime and monetary policy regime. In this subsection, we investigate how the expected inflation and slope of the Phillips curve varied through the regime changes, focusing on the expected inflation.
Figure 2 plots the estimated dynamics of the expected inflation and slope of the Phillips curve based on the three-regime ST Phillips curve model (4)-(6). As can be seen, the expected inflation has changed significantly depending on the regimes. In particular, the expected inflation dropped considerably in regime 2 for both inflation measures with some rebound in regime 3. Similarly, the slope of the Phillips curve greatly decreased in regime 2, followed by no change in the core inflation and a further decline in the core2 inflation. This is generally consistent with the recent finding of flattening of the Phillips curve in Japan, which is documented by, among others, De Veirman (2009) and Fuhrer, Olivei, and Tootell (2012). The results also indicate that no shift in the slope of the Phillips curve in regime 3 is the main reason why the two-regime model is preferred for the core inflation. In other words, the results suggest that there were arguably three regimes in the expected inflation for both inflation measures.

Next, we characterize each expected inflation regime more precisely based on the estimates reported in Table 2. As can be seen in Table 2, the expected inflation in regime 1 is significantly positive with estimates of 1.5% and 1.9% for core and core2 inflations, respectively. In addition, as confirmed by Figure 2, the expected inflation had been stable until at least 1993. Thus, regime 1 is characterized by relatively high stable inflation for both inflation measures.

In contrast, the expected inflation in regime 2 is negatively estimated for both measures. More precisely, the expected core inflation in regime 2 is estimated to be $0.20$, but it is not significantly negative. As a result, the expected core inflation showed a significant decrease at around 1995; it was then stable until the end of 2012. On the other hand, the expected core2 inflation in regime 2 is estimated significantly negatively with an estimate of $-0.61$. As can be seen from Figure 2, the expected core2 inflation changed smoothly from 1.9% to $-0.61\%$ between 1994 and 1998, and it stayed the same until the end of 2012. In the 2000s, the BOJ enacted innovative monetary policies, such as the zero interest rate and quantitative easing monetary policies. However, our results indicate that these policies were not enough to keep or recover the positive expected inflation, although they might prevent the expected inflation from decreasing further.

Finally, the expected core inflation in regime 3 is estimated positively, but it is insignificantly different from 0 with an estimate of 0.34%. As a result, the core inflation jumped up from $-0.20\%$ to 0.34% in the beginning of 2013, as can be seen in Figure 2. In contrast, the estimate of the expected core2 inflation in regime 3 is significantly positive with an estimate of 0.47. As a result, the core2 inflation had a sizable rapid increase from $-0.61\%$ to 0.47% in the beginning of 2013, as confirmed in Figure 2. However, compared with the BOJ’s 2%
inflation target, the expected inflation in regime 3 is significantly smaller for both inflation measures. Thus, our results suggest that the BOJ’s policies after the introduction of the 2% inflation target, such as QQE and the negative interest policies, are partially successful for escaping from the deflationary regime, but they are not sufficient to achieve the BOJ’s 2% inflation target.

4 Results of the Extended Model

Analyses thus far have been based on the hybrid Phillips curve (6). In this model, the current inflation rate is determined by the past inflation rates and the current output gap. However, it is likely that inflation in Japan depends on other variables, such as oil prices, stock prices, and exchange rates. For example, a rise in inflation was observed after the introduction of the 2% inflation target by the BOJ, which is partly due to the increase in the expected inflation, as we confirmed the previous section. However, it is plausible that price increases in import goods due to the depreciation of yen also played some role. In addition, it is often argued that the recent Japanese inflation was suppressed by a decline in oil prices as a result of the stagnation of the emerging economy, particularly China, and the oversupply by oil producing countries. Indeed, previous studies, such as Hooker (2002) and Hara, Hiraki, and Ichise (2015), have proposed the Phillips curve, including the oil prices and exchange rates. Furthermore, it is conceivable that the stock prices could affect inflation, particularly expected inflation. Following these observations, in this section we extend the benchmark model of the previous section to accommodate the oil prices, stock prices, and exchange rates to see their effects on the inflation and check the robustness of the results of the previous section.

4.1 Extended Phillips Curve

In this subsection, we examine the effects of the oil prices, stock prices, and exchange rates by employing the extended three-state Phillips curve model including these variables. Specifically, following Hooker (2002) and Hara, Hiraki, and Ichise (2015), we consider the following extended Phillips curve:

\[
\pi_t = \sum_{k=1}^{K} \alpha_k \pi_{t-k} + (1 - \sum_{k=1}^{K} \alpha_k) \mu_t + \beta_t x_t + \delta_t \sum_{j=0}^{J} \Delta o_{t-j} \\
+ \xi_t \sum_{j=0}^{J} \Delta e_{t-j} + \theta_t \sum_{j=0}^{J} \Delta q_{t-j} + \epsilon_t. \tag{8}
\]

\[14\]Hooker (2002) uses \(J = 2\) without the contemporaneous term, and Hara, Hiraki, and Ichise (2015) set \(J = 3\). For our analysis, we employ \(J = 2\), which is preferred to \(J = 3\) by information criterion for the core inflation. However, the results with \(J = 3\) are qualitatively very similar.
Here, $\Delta o_t = \log(O_t) - \log(O_{t-1})$, $\Delta e_t = \log(E_t) - \log(E_{t-1})$, $\Delta q_t = \log(Q_t) - \log(Q_{t-1})$, and $O_t$, $E_t$, and $Q_t$ are the oil price index, nominal effective exchange rate, and TOPIX, respectively.\(^{15}\) One problem associated with this extended Phillips curve is that it is formidable, if not impossible, to consider the three-regime ST model due to the highly nonlinear structure with many parameters to estimate. Therefore, for this analysis, we restrict the data to between 1996 and 2016 and assume that each coefficient evolves the following two-regime ST model:\(^{16}\)

\[
\mu_t = \mu^{(2)} + G_2(s_t; c_2, \gamma_2) \left( \mu^{(3)} - \mu^{(2)} \right) \\
\beta_t = \beta^{(2)} + G_2(s_t; c_2, \gamma_2) \left( \beta^{(3)} - \beta^{(2)} \right) \\
\delta_t = \delta^{(2)} + G_2(s_t; c_2, \gamma_2) \left( \delta^{(3)} - \delta^{(2)} \right) \\
\xi_t = \xi^{(2)} + G_2(s_t; c_2, \gamma_2) \left( \xi^{(3)} - \xi^{(2)} \right) \\
\theta_t = \theta^{(2)} + G_2(s_t; c_2, \gamma_2) \left( \theta^{(3)} - \theta^{(2)} \right)
\]

Judging from the regime dynamics of the previous sections, the assumption of two regimes is not unreasonable when using the data after 1995. In addition, we assume that each coefficient shares the common transition parameters $c_2$ and $\gamma_2$, as the previous section, to restrain the number of parameters. Thus, we require the timing and speed of the regime transition of each parameter to be the same. However, note that this assumption is not restrictive since we still estimate the coefficient of each regime, for example $\mu^{(i)}$ and $\theta^{(i)}$, allowing for very different dynamics for each coefficient. Finally, like in the previous section, we impose the sign restrictions that the coefficient on output gap, oil price, exchange rate, and the stock price are consistent with the economic theory. Specifically, we assume that the coefficients on output gap, oil prices, and stock prices are nonnegative, while the coefficients on the exchange rate are nonpositive.\(^{17}\)

Figure 3 plots the estimated dynamics of each coefficient based on the extended three-regime ST Phillips curve, which is characterized by (8)-(9), for the core inflation.\(^{18}\) As can be seen, the results demonstrate that the expected inflation experienced a noticeable increase around the beginning of 2013, which is consistent with the results of the previous section. The results also indicate that the coefficient on the output gap and oil price had been positive throughout the sample, although the oil price became less important in regime 3. However, the exchange rate turned into an important factor in regime 3 despite the fact that it had no effect in regime 2. Finally, the results clearly suggest that stock prices played a minor role throughout the period, particularly in regime 3.

\[\text{[Insert Figure 3 here]}\]

\(^{15}\)More precisely, we normalize each variable to have to have zero mean and unit variance as ab output gap to make each coefficient easy to compare.

\(^{16}\)We call these two regimes regime 2 and regime 3 to correspond with results of the previous section.

\(^{17}\)If these restrictions are bound for some coefficients, we replace them with 0 and re-estimate the model.

\(^{18}\)To save space, the estimates are not reported here, but they are available upon request from the author.
Figure 4 shows the results of the dynamics of each coefficient in the extended three-regime ST Phillips curve (8)-(9) for the core2 inflation. As can be seen, the results are qualitatively similar, except that the oil prices were not important throughout the sample. This is not surprising, since the core2 inflation is calculated from the CPI, excluding food and energy. More importantly, the results suggest that the exchange rate became influential in regime 3, although the effects of stock prices were completely negligible.

[Insert Figure 4 here]

To see these points more clearly, we have calculated the estimate for each factor component in the extended three-regime ST Phillips curve (8)-(9) and plotted them in Figure 5. As can be seen, the expected core inflation lowered the core inflation by −0.16% in regime 2, while it elevated the core inflation by 0.25% in regime 3. Similarly, the expected core2 inflation decreased the core2 inflation by −0.40% in regime 2, while it increased the core2 inflation by 0.46% in regime 3. Thus, the results demonstrate that the negative expected inflation was one of the main causes of the deflationary regime between 1996 and 2012; the result also indicate that changes in expected inflation were one of the key factors in the recent increase in inflation. Furthermore, the results illustrate that both the output gap and oil prices played a significant role in determining the core inflation in regime 2, but they became less important in regime 3. In particular, the results indicate that the recent decline in oil prices has lowered the core inflation at most 0.1%. Correspondingly, both output and oil prices had no effect on the core2 inflation in regime 3, although the output gap was relatively influential in regime 2. On the contrary, the exchange rate affected the core and core2 inflation more significantly in regime 3 than in regime 2, showing that the rapid depreciation of the yen after the introduction of the 2% inflation target increased the core (core2) inflation by up to 0.18% (0.24%), but the appreciation of the yen during the last one year of the sample lowered it by −0.27% (−0.35%). Finally, the stock prices had little effect on either the core or the core2 inflations in both regimes 2 and 3.

[Insert Figures 5 here]

In sum, our results are clear-cut. Even if we consider the effects of oil prices, stock prices, and exchange rates on inflation, expected inflation shows similar dynamics to the previous section. Specifically, expected inflation has been below 0% and stable until the end of 2012; it rapidly increased to more than 0% at around the beginning of 2013. Thus, our results demonstrate that the negative expected inflation was one of the main causes of the deflationary regime between 1996 and 2012; our results also indicate that changes in expected inflation were one of the key factors for the recent increase in inflation. In addition,

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19For this figure, we have taken the 12-month moving average to filter out the short-run fluctuation.
our results suggest that while the output gap and oil prices were relatively important in regime 2, the exchange rate played an essential role in explaining the rise and decline of both core and core2 inflations in regime 3. Finally, it was clear that stock prices were not a significant determinant of inflation in both regime 2 and regime 3.

4.2 ST Phillips Curve with Multiple Transition Variables

In the previous subsection, we extended the Phillips curve so that the oil prices, stock prices, and exchange rates could affect the realization of inflation. However, in that model, these variables could not influence the expected inflation directly, although it is of great interest to examine the effects of these variables on the expected inflation. Therefore, it is informative to investigate how the recent depreciation of the yen, decline in oil prices, and surge in stock prices affect the dynamics of expected inflation regimes. In this subsection, we extend the benchmark model so that the oil prices, stock prices, and exchange rates can influence the expected inflation regime.

To this end, we use the benchmark ST Phillips curve model (6) with the extended transition function (3), including oil prices, stock prices, and exchange rates. More specifically, we define a transition variable vector as

\[ s_t = \left( t/T, \sum_{j=0}^J \Delta q_{t-j-1}, \sum_{j=0}^J \Delta e_{t-j-1}, \sum_{j=0}^J \Delta e_{t-j-1} \right) \]

and assume that the transition function for the expected inflation can be written as

\[ G(s_t) = \frac{1}{1 + \exp[-\gamma_T(s_{1,t} - c_T) - \gamma_O(s_{2,t} - c_O) - \gamma_E(s_{3,t} - c_E) - \gamma_Q(s_{4,t} - c_Q)]}, \quad \gamma_T > 0 \]

(10)

Thus, in this model, we can capture the transition of expected inflation depending not only on time but also on the oil prices, stock prices, and exchange rates. Similar to the model in the previous subsection, this model may be too complicated to estimate the three-regime model. Therefore, we restrict the data to between 1996 and 2016, and we assume that there are only the two regimes, like in the previous subsection.\(^{20}\) In addition, we assume \( \gamma_O = 0 \) for the core2 inflation, since it is hard to expect that the oil price affects the expected inflation of core2 inflation by definition.

Table 3 summarizes the estimation results of the two-regime ST Phillips curve model (6) with (10) as a transition function for the expected inflation regime.\(^{21}\) As can be seen, the results of the expected inflation are consistent with the previous results. To be specific, for the core inflation, the expected inflation is estimated to be negative in regime 2 and positive in regime 3, although they are not significantly different from 0. In contrast, for the core2 inflation, the expected inflation was significantly negative in regime 2 and significantly

\(^{20}\)We again call these two regimes regime 2 and regime 3 to make them correspond with the results so far.

\(^{21}\)As in the previous subsection, we set \( J = 2 \) for this analysis. In addition, if the estimate of \( \gamma_i \) reaches the upper limit of 200, we set \( \gamma_i = 200 \) and re-estimate the model. In this case, the parameter’s standard errors are denoted by NA.
positive in regime 3. The results also indicate that the oil prices significantly affected the expected inflation regime for the core inflation. Specifically, the positive estimate suggests that if the oil price has increased over the last three months, then the expected inflation regime tends to shift from the deflationary regime (regime 2) to the inflationary regime (regime 3). Thus, our results indicate the possibility that the recent decline in the oil prices pulls back the current inflation regime to the deflationary regime. Finally, our results show that the exchange rate played an insignificant role in determining the expected inflation regime for both inflations, while stock prices had a significant positive impact on the expected inflation. This greatly contrasts with the results of the previous subsections. Thus, for the realization of the inflation, the exchange rate matters more; however, for the expected inflation regime, the stock prices are more important. This is relevant because the exchange rate directly affects inflation through the price of import goods, while the stock prices should have more of an effect on the people’s minds, altering the expected inflation.

[Insert Table 3 here]

To see the joint effects of these variables on the expected inflation, Figure 6 plots the estimated dynamics of expected inflation based on the two-regime ST Phillips curve model (6) with (10) as a transition function for the expected inflation regime. As can be seen, the results suggest that the expected inflation had been negative and stable almost all the time before 2013, but the expected inflation increased rapidly to more than 0.5% around the beginning of 2013 for both inflation measures. This regime shift was caused by the drastic changes in the BOJ’s monetary policy, which was captured by the time-trend variable and the corresponding surge in stock prices. However, the expected inflation occasionally went back to the deflationary regime even after that. For the core inflation, the oil and stock price decreases seemed to be equally responsible for this reversion, while the stock price decline was the main reason for the core2 inflation.

[Insert Figure 6 here]

In sum, the results of this subsection show the robustness of our findings of the shift in the expected inflation from the deflationary regime to the inflationary regime after the introduction of the 2% inflation target by the BOJ. In addition, our results indicate the importance of oil and stock prices in maintaining the inflationary regime. The BOJ tripled the purchase of ETF in October 2014 and almost doubled it again in July 2016. Our results suggest that this kind of policy could be effective in preventing the expected inflation regime from going back to the deflationary regime, which was mainly observed before 2013.
5 Conclusions

Over the last three decades, the Japanese economy has experienced many significant events, such as the bursting of the bubble economy, the lost two decades, the Lehman and Euro financial crises, and the introduction of a zero interest rate and inflation targeting monetary policies. It is quite meaningful to empirically study how expected inflation rates in Japan have evolved through these events. Along this vein, this paper investigated the possible regime shifts in expected inflation in Japan over the last three decades and their relation with monetary policy regimes.

We applied the ST Phillips curve model to identify the number of regimes, characterize each regime, and estimate the dynamics of expected inflation. Our results indicated that there were arguably three regimes in the expected inflation in Japan over the last three decades. In addition, our results suggested that these three regimes reasonably corresponded to the traditional monetary policy regime between 1985 and 1994 (regime 1), the extremely low interest rate monetary policy regime between 1995 and 2012 (regime 2), and the inflation targeting monetary policy regime since 2013 (regime 3).

Our three-regime ST Phillips curve model showed that the expected inflation in regime 1 was relatively high with more than 1.5%, and stable. Nonetheless, the expected inflation in regime 2 decreased considerably and stayed below 0% most of the time in this regime, although in the 2000s the BOJ enacted many pioneering monetary policies, such as a zero interest rate and quantitative easing monetary policies. Thus, those BOJ's monetary policies were not enough to escape from the deflationary regime, although they most likely supported the Japanese economy and prevented the expected inflation from decreasing further. Finally, the expected inflation in regime 3 revealed a sizable increase, recovering the inflationary regime. However, our results clearly demonstrated that the expected inflation in regime 3 was significantly smaller than the BOJ's 2% inflation target for both inflation measures. Thus, the QQE and the negative interest policies conducted in regime 3 were partially successful to escape from the deflationary regime, but they were not sufficient to achieve the BOJ's 2% inflation target.

In addition, we extended the benchmark model so that the oil prices, stock prices, and exchange rates could affect inflation. Here, our results indicated that the oil price and exchange rate had relatively large effects on the core inflation in regime 2 and regime 3, respectively. On the other hand, the oil prices, stock prices, and exchange rates had little effect on the core inflation in regime 2, although the exchange rates became influential in regime 3. Thus, our results suggested that the depreciation of the Japanese yen occurred due to the introduction of the inflation targeting policy by the BOJ played an important role on the significant increase of inflation in 2013. Furthermore, the recent rebound of the exchange
rates is an important cause of the recent decline in inflation.

Finally, we also considered another type of extended model in which the oil prices, stock prices, and exchange rates could affect the dynamics of the expected inflation regime. Our results showed that the oil and stock prices had a significant impact on the expected inflation regime for the core inflation, while the stock price strongly affected the expected inflation regime for the core2 inflation. This greatly contrasts with the above results, which show the importance of the exchange rates in explaining the realization of inflation. More specifically, our results demonstrated that, even after 2012, the expected inflation regime tended to move back to the deflationary regime when the oil prices or stock prices decreased markedly. Therefore, the BOJ’s purchase of ETF could be effective in preventing the expected inflation from going back to the deflationary regime that was mainly observed before 2013.

References


Table 1: Results of the hypothesis testing

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Table 2: Estimation results of bench mark model
Table 3: Estimation results of multiple transition variable model

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Figure 1: Dynamics of transition functions
Figure 2: Dynamics of expected inflation and the slope of the Phillips curve
Figure 3: Dynamics of each coefficient for the core inflation
Figure 4: Dynamics of each coefficient for the core2 inflation

- exinf
- output
- oil
- fx
- stock
Figure 5: Factor decomposition of the inflations
Figure 6: Dynamics of expected inflation based on the multiple transition variable model