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Asset Specificity, Human Capital Acquisition, and Labor Market Competition

Hodaka Morita
School of Economics, University of New South Wales
Hitotsubashi Institute for Advanced Study, Hitotsubashi University

Cheng-Tao Tang
Graduate School of International Relations, International University of Japan

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Asset Specificity, Human Capital Acquisition, and Labor Market Competition

Hodaka Morita* Cheng-Tao Tang†

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*School of Economics, University of New South Wales, and Hitotsubashi Institute for Advanced Study, Hitotsubashi University. Email: hodakamorita@gmail.com

†Graduate School of International Relations, International University of Japan. Email: chengtao.tang@gmail.com
Abstract

Firms let their employees operate assets to produce goods and services. Firm-specificity of asset and human capital, key concepts of transaction cost economics and labor economics respectively, play important roles in determining firms’ productivity and welfare consequences of their competition. How are the degrees of firm-specificity of asset and human capital determined? We address this question through exploring a new model that captures interconnections among asset specificity, human capital acquisition, managerial capability, and labor mobility. We consider a two-period model with two firms, where period 1 is the skill-acquisition period and period 2 is the output period. In the beginning of period 1, each firm chooses a level of its asset specificity and employs a certain number of workers from the labor market. The level of asset specificity is interpreted as the extent to which the firm tailors its asset to the unique features of the firm’s business strategies and products. A firm’s second-period productivity is determined by its managerial capability, the extent to which its asset is tailored, and its workers’ familiarity with its asset specificity. Managerial capability here means the capability of a firm’s top management to develop an effective strategy and create a unique competitive position.

We find that, as the importance of managerial capability increases, the labor mobility increases, and both the level of asset specificity and firm size decrease. When a firm chooses the specificity of its asset and the number of workers it employs in period 1, it estimates how many workers it will retain and how many workers it will hire from its rival in period 2. A higher importance of managerial capability increases the difference of period 2 productivity between a high-capability and a low-capability firm. Then, as the importance of managerial capability increases, each firm anticipates higher labor mobility, because a larger number of workers will move from a low-capability to a high-capability firm. Anticipation of higher labor mobility, in turn, reduces each firm’s incentives to hire more workers and increase the level of asset specificity in period 1.

We discuss implications of our model in the contexts of cross-industry and cross-country comparisons. In a newly emerging industry or in a business undergoing revolutionary technological changes, a business’s success critically depends on the quality of its strategic decision making because these industries face a high level of uncertainty. Whereas in industries facing lower levels of uncertainty, strategic decision making is less important. These arguments suggest that the importance of managerial capability is higher in the former types of industries, and the importance tends to be lower in the latter types of industries. Our model then predicts that labor mobility is higher, specificity of asset and human capital is lower, and average firm size is smaller in industries of the former type and vice-versa in industries of the latter type. Also, as the economy makes a transition from industrial capitalism to post-industrial capitalism, modern economies are becoming increasingly knowledge intensive which renders the disadvantage to the firms that heavily rely on physical assets. Our model yields new implications regarding the consequences of the transition.

Keywords: Asset specificity, competition, firm size, firm specificity, human capital, managerial capability

JEL: J24, L20, M50
1 Introduction

Firms let their employees operate their assets to produce goods and services. Firm-specificity of asset and human capital, key concepts of transaction cost economics (Williamson, 1975, 1979, 1985) and labor economics (Becker, 1962) respectively, play important roles in determining firms’ productivity and welfare consequences of their competition. The firm’s productivity becomes higher when the firm has its asset utilized by a worker who is familiar with that asset’s specific feature. Consequently, there exists important connection between asset specificity and labor mobility, because the acquired human capital of the workers who utilize the specificity of the asset loses its value if workers switch employers. In reality, there also exists connection between a firm’s managerial capability and labor mobility. A firm with high managerial capability is capable of expanding its business by hiring more workers, whereas a firm with low capability ends up shrinking its business by dismissing workers. This paper explores a new model that captures interconnections among asset specificity, acquisition of firm-specific human capital, firm size, managerial capability, and labor mobility.

We consider a two-period model with two firms, where period 1 is the skill-acquisition period and period 2 is the output period. Each firm $i$ ($i=1,2$) initially owns an asset, and in the beginning of period 1 it chooses a level of its asset specificity and employs a certain number of workers from the labor market. Workers can switch their employers between periods 1 and 2. The level of asset specificity is interpreted as the extent to which the firm tailors its asset to the unique features of the firm’s business strategies and products. As such, a firm incurs higher costs as it chooses a higher level of asset specificity. The idea that a firm chooses a level of its asset specificity is consistent with the insight put forth by Doeringer and Piore (1985), who discussed firm-specificity of technology where technology refers to the entire set of tasks that comprises a work process. They asserted that firm-specificity of technology can be chosen by a firm’s management.

In the current model, a firm’s second-period productivity is determined by its managerial capability, the extent to which its asset is tailored (i.e., asset specificity), and its workers’ familiarity with its asset specificity. Managerial capability here means the capability of a firm’s top management to develop an effective strategy and create a unique
competitive position. Let \( a_i \) denote the realization of firm \( i \)'s managerial capability. For simplicity, \( a_i \) is assumed to be a random draw from a symmetric binary distribution satisfying \( \Pr(a_i = a_H) = \Pr(a_i = a_L) = \frac{1}{2}, a_H > a_L, \) and \( a_H = -a_L = \alpha. \) Assume that \( a_i \) is ex-ante unknown to all agents including firm \( i \) itself and becomes common knowledge at the beginning of period 2 of firm \( i \)'s operation. A worker employed by firm \( i \) becomes familiar with the specificity of firm \( i \)'s asset. Hence, if the worker is retained by the same firm \( i, \) the worker’s second-period productivity is increasing in the level of the firm’s asset specificity. In contrast, a worker employed by firm \( j (\neq i) \) in period 1 is not familiar with the specificity of firm \( i \)'s asset so that, if firm \( j \)'s first-period employees move to firm \( i \) in period 2, their productivity is not improved by the level of firm \( i \)'s asset specificity. We introduce diseconomy of firm size in the model by assuming that a firm must incur per-worker supervision cost, and that the cost increases as the number of workers employed by the firm increases.

In our analysis of the model, we focus on the range of parameterizations in which, if one firm turns out to have a high managerial capability and the other has a low capability, some workers move from the low-capability firm to the high-capability one in equilibrium. A key comparative statics result is that, as the importance of managerial capability increases, the labor mobility increases, and both the level of asset specificity and firm size decrease.

The intuition behind this result can be explained as follows. When firm \( i \) chooses the specificity of its asset and the number of workers it employs in period 1, it estimates the expected number of its period 1 workers that the firm will retain for its period 2 operation, and the expected number of workers that the firm hires from its rival firm \( j \) in period 2. A higher importance of managerial capability increases the difference of period 2 productivity between a high-capability and a low-capability firm. Then, as the importance of managerial capability increases, each firm anticipates that a larger number of workers switch their employers in period 2 if the two firms' managerial capabilities turn out to be different. Hence higher importance of managerial capability decreases the expected number of retained workers, and it increases the expected number of workers who switch their employers. Holding the initial number of workers constant, this reduces each firm’s incentive to choose higher specificity of its asset, because the proportion of
retained workers decreases and the productivity gains over the investment cost for each retained worker becomes smaller.

Meanwhile, anticipating that the number of retained workers decreases while the supervision cost per worker in period 1 remains the same, each firm has a lower incentive to hire more workers in period 1. Because a larger number of period 1 workers increases a firm’s period 2 profit from its retained workers at the expense of increasing supervision cost per worker in period 1. Also, the reduction of each firm’s first-period workers (i.e., average firm size) decreases the proportion of retained worker in period 2, and in turn decreases the marginal benefit of choosing a higher asset specificity. Hence, the force of lowering average firm size further reduces each firm’s level of asset specificity. The result, then, is that the equilibrium level of asset specificity and average firm size decrease as the importance of managerial capability increases. And, higher labor mobility in period 2 and lower number of first-period workers both work in the direction of increasing the turnover rate. The model also yields similar comparative statics results concerning the importance of a firm’s asset specificity.

The connection between labor mobility and the levels of firms’ asset specificity and their employment sizes, uniquely explored in our model, yields empirical implications and predictions from a previously unexplored perspective, given that the importance of managerial capability and firm-specificity of asset can differ across industries and countries. In a newly emerging industry or in a business undergoing revolutionary technological changes, a business’s success critically depends on the quality of its strategic decision making because these industries face a high level of uncertainty. Whereas in industries facing lower levels of uncertainty, strategic decision making is less important. These arguments suggest that the importance of managerial capability is higher in the former types of industries, and the importance tends to be lower in the latter types of industries. Our model then predicts that labor mobility is higher, specificity of asset and human capital is lower, and average firm size is smaller in industries of the former type and vice-versa in industries of the latter type. Also, as the economy makes a transition from industrial capitalism to post-industrial capitalism, modern economies are becoming increasingly knowledge intensive which renders the disadvantage to the firms that heavily rely on physical assets. Our model yields new implications regarding
the consequences of the transition (see Section 4 for more discussions).

1.1 Related literature

This paper is related to two strands of literature: one on transaction cost economics and the other on human capital acquisition. The literature on the transaction cost economics, which centers on the view of Williamson (1975, 1979, 1985) and Klein, Crawford and Alchian (1978), now spans a large body of research. Whinston (2003) clearly indicates two starting point observations of the transaction cost economics. First, the content of contract between contracting parties is incomplete in many exchange relationship. Second, the investment of asset for a specific purpose often creates a surplus to be shared among contracting parties and makes those parties effectively “locked into” the transaction over the course of their relationship. The surplus, often referred to as “appropriable quasi-rents,” plays a critical role in the theory of the firm literature.\(^1\) The incompleteness nature of the contract and the presence of appropriable quasi-rents together open up possibilities for ex-post opportunistic behavior. As Masten (1984) points out, “The more specialized those assets, the larger will be the quasi-rents at stake over that period, and hence the greater the incentive for agents to attempt to influence the terms and trade through bargaining or other rent-seeking activities once the investments are in place.” This ex-post opportunistic behavior, regarded as socially inefficient, can be prevented by costly remedies such as vertical integration (Williamson, 1979, 1985; Klein et al., 1978).

Along this line of transaction cost economics literature, the concept of asset specificity is an important—perhaps the most important—building block in organizing some transactions one way and other transactions another.\(^2\) While the focus in this literature centers on the relationship between the structure of the internal organization and the asset specificity where the specificity of the asset is often treated as an exogenous variable. Attention has not been paid to the interlinkage among firms’ endogenous determination of asset specificity, employees’ human capital acquisition, and inter-firm competition in

\(^1\)See Gibbons (2005) for a review.

\(^2\)Williamson (1985, p.52) points out that “[asset specificity] is the most important and most distinguishes transaction cost economics from other treatment of economic organization, ...”
the labor market. The present study sheds new light on the transaction cost economics by studying this interlinkage.

Concerning human capital acquisition, a significant amount of literature grew out from Becker (1962)’s classical distinction between general and firm-specific human capital. One literature along this line focuses on general human capital and optimal investment decision (e.g. Ben-Porath, 1967; Wallace and Ihnen, 1975). Another literature that grew out of Becker’s work focuses on specific human capital. For example, Hashimoto (1981) formalizes Becker’s argument that the cost of and the return to specific human capital are shared by the worker and the firm. The approach following Becker’s analysis of human capital investment implicitly assumes that the pre- and post-training wages and investment levels in general and specific human capital are contractible. As pointed out by Gibbons and Waldman (1999), an equally useful approach to human capital investment is to assume investment levels are not contractible and that post-training wages are determined by bargaining. This approach introduces a number of results that do not arise in Becker’s formulation.

For example, Chang and Wang (1996) presents an asymmetric learning model, where human capital accumulation is not contractible. They show that the equilibrium training level is negatively related to the turnover rate and positively related to the specificity of training. This result is in contrast to Becker’s standard result where firms provide general training only if employees pay the cost (Becker, 1962).\textsuperscript{3} Acemoglu and Pischke (1999) generalize the argument where firms have incentives to invest in the general human capital of their workers. They show that several labor market imperfections (including asymmetric learning) that generate ex-post rent from bilateral monopoly can yield this conclusion. There also exists supporting evidence that firms share some of the costs and returns to general training (e.g. Acemoglu and Pischke, 1996). Along this line of literature, a degree of firm-specificity of human capital is often introduced and is typically treated as an exogenous variable.\textsuperscript{4}

\textsuperscript{3}Bai and Wang (2003) also show that, unlike the predictions from traditional human capital theory, the probability of separation has a positive relationship with the specific human capital investment if the uncertainty in labor productivity is either very high or very low.

\textsuperscript{4} introduce demand-side constraints in the labor-market behaviors where skilled and unskilled labors are not perfect substitutes in production and a firm incurs hiring cost when filling out its managerial position from the outside. They show that their model can generate various familiar practices...
This study follows the approach where the accumulation of human capital is not contractible but makes a departure in the following aspect. The firm-specificity of human capital in our model is endogenously determined by the degree of asset specificity. Because in reality there is important connection between firm-specificity of asset and human capital acquisition: it is the workers who utilize the asset to produce goods and services.

There are some other papers which depart from the standard labor-theoretical model but address different issues. Important examples include Morita (2001), Wang (2002), Mailath, Nocke and Postlewaite (2004), and Morita (2012). Morita (2001) studies the connection between continuous process improvement and the firm-specificity of training and explores its implications on the U.S.-Japanese differences. Wang (2002) analyzes the connection between product market conditions and job design and explores its implications on explanations for heterogeneity of human resource management practices across countries, industries, and firms. Mailath, Nocke and Postlewaite (2004) analyze the interaction between a firm's choice of business strategy and its manager's incentive for investing in "business-strategy-specific" human capital and explore its implications on the organization of business activities. Morita (2012) studies the interconnection between firm dynamics and specific human capital. He also shows that, through a different mechanism, as the importance of a firm’s managerial capability increases, the survival rate of firms decreases, and the level of firm-specific human capital investment decreases.5 Although this paper has many useful points of contact with this body of work, to our knowledge, our combination of asset specificity, acquisition of firm-specific human capital, firm size, managerial capability, and labor mobility is new, as is the attempt to explain cross-industry and cross-country facts in an integrated model.

The rest of the paper is organized as follows. Section 2 presents a model that each such as fast-track promotions, up-or-out rules, and promotion from within, which were previously inexplicable within a competitive, symmetric information, and complete-contracting framework.

5In Morita (2012)'s two-period model, each first-period entrant has an advantage over second-period entrants given that it has a worker who already accumulated a certain level of firm-specific human capital. This advantage associated with firm-specific human capital becomes smaller as the importance of managerial capability increases, resulting in a lower survival rate of the first-period entrant. The change of firm-specific human capital in his model directly comes from firm’s training investment; while in our model, it is determined by firms’ asset specificity choice and whether or not workers switch employers.
firm consists of the top management, assets, and workers who operate assets to produce goods and services. Section 3 analyzes the model, characterizes the equilibrium and presents comparative statics results. Section 4 discusses the implications of the model results. Section 5 concludes this paper.

2 The model

Consider an industry with two ex-ante identical firms in a two-period setting. Only one homogenous good is produced in this industry and the price is normalized to one. Labor is the only input, and a firm’s output is a summation of its employees’ outputs. There exists a large number of individuals, where each individual is of measure zero. In each period, labor supply is perfectly inelastic and fixed at one unit for each individual. To keep the analysis simple, firms and individuals do not discount the future, and they are risk-neutral.

At the beginning of period 1, each individual has the same general human capital and looks identical to the firms. Each firm $i$ simultaneously makes a first-period wage offer $\omega_{i,1}$ to $\tilde{n}_i \geq 0$ individuals. Let $n_i$ denote the number of firm $i$’s first-period employees. Individuals not employed by a firm become self-employed and remain so until the end of period 2, earning $\omega > 0$ per period.

Each firm’s production efficiency is determined by its managerial capability and the level of its asset specificity, where managerial capability is interpreted as representing the ability of a firm’s top management to develop an effective strategy and a unique competitive position. Let $a_i$ denote the realization of firm $i$’s managerial capability, which is a random draw from a symmetric binary distribution satisfying $\Pr(a_i = a_H) = \Pr(a_i = a_L) = \frac{1}{2}$, $a_H > a_L$, and $a_H = -a_L = \alpha$. Assume that $a_i$ is ex-ante unknown to all agents including firm $i$ itself and becomes common knowledge at the beginning of period 2 of firm $i$’s operation. This specification is consistent with the widely held view that ability of a firm’s top management is mostly innate, and difficult to observe or assess ex-ante.

Also at the beginning of period 1, each firm $i$ can invest $z_i \geq 0$ in its level of asset specificity at a cost of $c(z_i) \geq 0$, where $c(\cdot)$ is a convex function. The asset specificity
refers to how a firm tailors its physical asset for a particular use. To obtain closed-form solution in the analysis, let \( c(z) = \frac{1}{2}\theta z^2 \) and \( \theta > 0 \). Each firm chooses its level of asset specificity at the same time as making its first-period wage offer to individuals, and commits to the investment level. Each firm’s asset specificity affects each first-period worker’s production efficiency only in period 2. Formally, if a worker stays with the same employer \( i \) in both periods, his production efficiency associated with asset \( z_i \) is \( d(\geq 0) \) and \( f(z_i) \) for period 1 and 2 respectively, where \( f(0) > d \) and \( f'(z_i) > 0 \) for all \( z_i \geq 0 \). If a worker moves from firm \( i \) to the other firm \( j (j \neq i) \) in period 2, his second-period production efficiency associated with new firm’s asset is \( g(>0) \). That is, as the level of a firm’s asset specificity increases, the improved production efficiency only comes from the retained workers. If firm \( i \)’s asset is utilized by outside workers who are not familiar with the tailor-made feature of \( i \)’s asset, there is no production efficiency gain. Without loss of generality, we assume that \( d = 0 \). To obtain closed-form solution, let \( f(z_i) = 1 + sz_i \) where \( s > 0 \), and \( g = 1 \). Here \( s \) captures the importance of a firm’s asset specificity on its retained workers.

To produce the good, each worker also requires employer supervision. Assume that firm \( i \)’s per worker supervision cost is \( bn_i \) in each period, where \( b > 0 \). That is, as the number of workers increases, each worker’s net output declines because each worker receives less supervision from the employer.\(^6\) Each firm \( i \)’s period 1 profit is

\[
n_i xa_i - b n_i^2 - \frac{1}{2}\theta z_i^2 - W_{i,1},
\]

where \( x > 0 \) captures the importance of managerial capability and \( W_{i,1} \) denotes firm \( i \)’s first-period total wage bill.

At the beginning of period 2, each firm \( i \) simultaneously makes its second-period wage offer \( w_{i,2} \) to all workers, including workers employed at the other firm. Given the wage offers, each worker takes the highest one, and stays with his current employer if offers from both firms are the same. A firm expands in its firm size if, in addition to its first-period employees, it hires more workers from the other firm. A firm can also contract in its firm size if its first-period employees move to the other firm. A firm stays

\(^6\)See Zábojník and Bernhardt (2001) and DeVaro and Morita (2013) for a similar specification.
unchanged in its size only if it does not expand nor contract. Let $m_i (> -n_i)$ denote
the amount of firm $i$’s newly hired workers if $m_i \geq 0$ in the second period, or dismissed
workers if $m_i < 0$. Each firm $i$’s period 2 profit is

$$
\begin{cases}
(n_i + m_i)(xa_i) + n_i(1 + sz_i) + m_i - b(n_i + m_i)^2 - W_{i,2} & \text{if } m_i \geq 0 \\
(n_i + m_i)(xa_i + 1 + sz_i) - b(n_i + m_i)^2 - W_{i,2} & \text{if } m_i < 0
\end{cases},
$$

where $W_{i,2}$ denotes firm $i$’s second-period total wage bill. Firm $i$ expands in the second
period if $m_i > 0$; firm $i$ contracts if $m_i < 0$.

The timing of the game is as follows.

**Period 1**

**[Stage 1]** Each firm simultaneously makes first-period wage offers $w_i$ to $\tilde{n}_i$ individuals,
where $i = 1, 2$. At the same time, each firm chooses its level of asset specificity $z_i$ by
incuring a cost. Individuals choose between working for a firm and self-employment.
When indifferent, an individual chooses to work for a firm. When indifferent between
offers from both firms, an individual chooses randomly between them. Self-employed
workers remain so for both periods, earning a wage $\omega$ each period.

**[Stage 2]** Each firm $i$ that employed $n_i$ workers at Stage 1 produces $n_i$ units of the
good.

**Period 2**

**[Stage 3]** Firm $i$’s managerial capability $a_i$ is realized and becomes common knowledge.
Each firm $i$ then makes its second-period wage offer $w_{i,2}$ to all workers, including workers
employed at the other firm. Given the second-period wage offers, each worker takes the
highest one. A firm’s size expands if it hires workers from the rival firm, while a firm’s
size contracts if its first-period employees move to its rival firm. Otherwise, a firm stays
unchanged in its firm size.

**[Stage 4]** Each firm $i$ that employed $n_i + m_i$ workers produces $n_i + m_i$ units of the
good.
3 Analysis

In this section, we consider the symmetric Subgame Perfect Nash Equilibria (SPNE) in pure strategy, in which \( n_i = \tilde{n}_i \) in the equilibrium. That is, each firm \( i \) chooses the same level of asset specificity and the same number of first-period workers, and employs all individuals to whom it makes a first-period wage offer in equilibrium.

We focus on equilibria in which a strictly positive number (measure) of workers move from one firm to the other at the beginning of period 2 whenever the realizations of managerial capability are different in both firms, given that in reality expansion and contraction of firm size are common in most industries. Proposition 1 identifies necessary and sufficient conditions for such an equilibrium to exist, and characterizes the equilibrium. Proposition 2 and 3 then present comparative statics results on the level of asset specificity, the firm size and the expected labor turnover rate in the equilibrium. Note, all proofs are in the Appendix.

Suppose that there exists a symmetric equilibrium in which a strictly positive number of workers switch employers in period 2 when firms’ managerial capabilities are different. Consider a Stage 3 subgame where the managerial capability is realized. There are four cases, namely \((a_i, a_j) = (a_H, a_L)\), \((a_i, a_j) = (a_L, a_H)\), and \((a_i, a_j) = (a_L, a_L)\). First consider the case where \((a_i, a_j) = (a_H, a_L)\). If firm \( i \) expands in its firm size, firm \( i \) is willing to offer \( w_{i,2} = xa_H + 1 - 2b(n_i + m_i) \) where \( m_i > 0 \), which is equal to the expected productivity of firm \( j \)’s first-period employee at the margin; while firm \( j \) contracts in its firm size and is willing to offer \( w_{j,2} = xa_L + 1 + sz_j - 2b(n_j + m_j) \) where \( m_j < 0 \). In the equilibrium, both firms offer the same second-period wages \( w_{i,2} = w_{j,2} \). This condition, plus the condition that the number of workers from expansion equals that from contraction in equilibrium, we obtain \( m_i = -m_j = \frac{1}{2b}[2\alpha x - sz_j - 2b(n_i - n_j)] \equiv m_{E}^{E}(n_i, z_j, n_j) \). Then, the equilibrium second-period wage becomes \( \frac{1}{2}[2 + sz_j - 2b(n_i + n_j)] \equiv w_{E}^{E}(n_i, z_j, n_j) \).

On the other hand, if firm \( i \) contracts and firm \( j \) expands, the equilibrium second-period wage and the size of expansion/contraction can be solved analogously; they are \( \frac{1}{2}[2 + sz_i - 2b(n_i - n_j)] \) and \( \frac{1}{2b}[-2\alpha x - sz_j - 2b(n_i - n_j)] \), respectively. Note that this latter case where firm \( i \) contracts and firm \( j \) expands can be ruled out since \(-2\alpha x - sz_j +
\[2b(n_i - n_j) < 0 \text{ for any symmetric equilibrium—a contradiction. As for the former case where firm } i \text{ expands and firm } j \text{ contracts in the equilibrium, the following condition must hold:}
\]
\[0 < \frac{1}{4b}[2ax - sz_j - 2b(n_i - n_j)] < n_j. \quad (1)\]

The right-hand side of condition (1) holds with strictly inequality; otherwise, firm \( j \) will shut down. Hence, in the SPNE outcome where both firms have different managerial capabilities, the firm with high managerial capability expands and the other contracts in their firm sizes. Firm \( i \)'s second-period profit conditional on \( a_i = a_H \) is given by
\[
\pi_i^E(z_i, n_i, z_j, n_j) \equiv (n_i + m_i^E(n_i, z_j, n_j))xa_H + n_i(1 + sz_i) + m_i^E(n_i, z_j, n_j) - b(n_i + m_i^E(n_i, z_j, n_j))^2 - (n_i + m_i^E(n_i, z_j, n_j))w_{i,2}^E(n_i, z_j, n_j) \quad (2)
\]

Next consider the case \((a_i, a_j) = (a_L, a_H)\). Let \( w_{i,2}^C(z_i, n_i, n_j) \) and \( m_i^C(z_i, n_i, n_j) \) be defined analogously given firm \( i \) contracts in its firm size. In the similar vein as the analysis in previous case, we obtain that firm \( i \)'s second-period profit conditional on \( a_i = a_L \) is given by\(^7\)
\[
\pi_i^C(z_i, n_i, n_j) \equiv (n_i - m_i^C(z_i, n_i, n_j))(xa_L + 1 + sz_i) - b(n_i - m_i^C(z_i, n_i, n_j))^2 - (n_i - m_i^C(z_i, n_i, n_j))w_{i,2}^C(n_i, z_i, n_j). \quad (3)
\]

Now consider a Stage 3 subgame where both firms have the same managerial capability, \( a_i = a_j \). In the equilibrium, each firm \( i \) offers the same second-period wage \( xa_i + 1 - 2bn_i \equiv w_{i,2}^S(a_i, n_i) \), and workers stay with their first-period employer so that there is no labor mobility. Then firm \( i \)'s second-period profit conditional on \( a_i = a_j \) is given by
\[
\pi_i^S(z_i, n_i) \equiv n_i(xa_i + 1 + sz_i) - bn_i^2 - n_iw_{i,2}^S(a_i, n_i). \quad (4)
\]

Note that \( \pi_i^S(z_i, n_i) \) is independent of managerial capability when it is the same for

\(^7\)The condition \( 0 < m_i^C(z_i, n_i, n_j) < n_i \) is the same as condition (1) in the equilibrium given that we focus on symmetric equilibrium.
both firms (see Claim 1 in the Appendix for details). Hence, given that the equilibrium is symmetric and strictly positive number of workers move across firms whenever both firms have different managerial capabilities, each firm $i$’s expected profit in period 2 is

$$\pi_{i,2}(z_i, n_i, z_j, n_j) = \Pr(a_H, a_L)\pi_{i}^{E}(z_i, n_i, z_j, n_j)$$

$$+ \Pr(a_L, a_H)\pi_{i}^{C}(z_i, n_i, n_j) + \Pr(a_i = a_j)\pi_{i}^{S}(z_i, n_i),$$

where the probability of each outcome $(a_i, a_j)$ is $\frac{1}{4}$.

At Stage 1, each firm $i$ offers the same first-period wage $w^*_i$ to $n_i = \tilde{n}^*$ individuals and employs $n^* = \tilde{n}^*$ workers in the equilibrium, given the level of asset specificity and first-period number of workers are symmetric. Also, every individual who receives an offer from firm $i$ is indifferent between taking and not taking the offer, anticipating that his/her second-period wage will be $w_{i,2}^{E}(n_i, z_j, n_j)$ if $(a_i, a_j) = (a_H, a_L)$, $w_{i,2}^{C}(z_i, n_i, n_j)$ if $(a_i, a_j) = (a_L, a_H)$, and $w_{i,2}^{S}(a_i, n_i)$ if $a_i = a_j$. Since $2\omega$ is the lifetime wage for self-employed individuals, $w^*_i + E[w_{i,2}(z_i, n_i, z_j, n_j)] = 2\omega$ holds for a worker employed by firm $i$ where

$$E[w_{i,2}(z_i, n_i, z_j, n_j)] = \Pr(a_H, a_L)w_{i,2}^{E}(n_i, z_j, n_j)$$

$$+ \Pr(a_L, a_H)w_{i,2}^{C}(z_i, n_i, n_j) + \Pr(a_i = a_j)w_{i,2}^{S}(a_i, n_i).$$

Hence $w^*_i = w^*_{i,1}(z_i, n_i, z_j, n_j)$ where

$$w^*_{i,1}(z_i, n_i, z_j, n_j) = 2\omega - E[w_{i,2}(z_i, n_i, z_j, n_j)].$$

Since firm $i$’s period 1 profit is $n_i x a_i - b n_i^2 - \frac{1}{2} a_i^2 - W_{i,1}$, firm $i$’s expected overall profit is $\Pi_i(z_i, n_i, z_j, n_j)$ in equilibrium, where

$$\Pi_i(z_i, n_i, z_j, n_j) = n_i[-b n_i - w^*_{i,1}(z_i, n_i, z_j, n_j)] - \frac{1}{2} a_i^2 + \pi_{i,2}(z_i, n_i, z_j, n_j).$$

In equilibrium, firm $i$ chooses $(z_i, n_i) = (z^*, n^*)$ maximizes the value of $\Pi_i(z_i, n_i, z^*, n^*)$, given $(z_j, n_j) = (z^*, n^*)$. Then the necessary first-order conditions that define the equilibrium level of asset specificity and the equilibrium number of first-period workers are
Given the symmetric level of asset specificity and the number of first-period workers in the equilibrium, we find (see Claim 1 in the Appendix for details) that

$$z^* = \frac{8s(1 - 2\omega) - 2\alpha xs}{32b\theta - 9s^2},$$

(9)

$$n^* = \frac{(1 - 2\omega)(32b\theta - s^2) - 2\alpha x s^2}{4b(32b\theta - 9s^2)}.$$  (10)

In addition, the necessary (and sufficient) second-order condition for the maximization problem of \(\Pi_i(z_i, n_i, z^*, n^*)\) implies \(32b\theta - 9s^2 > 0\) (see Claim 2 in the Appendix for details).

Given that \(z^* \geq 0\) and the denominator of \(z^*\) is positive, the following condition must hold in the equilibrium:

$$\alpha \leq \frac{8(1 - 2\omega)}{2x},$$

(11)

implying \(n^* \geq 0\) holds. Also, in equilibrium the condition \((1) - 0 < m^E_i(n^*, z^*, n^*) < n^*\)—must hold, where \(m^E_i(n^*, z^*, n^*) = \frac{1}{4s}(2\alpha x - sz^*)\). It further implies the following condition on \(\alpha\):

$$\frac{s^2(1 - 2\omega)}{2x(4b\theta - s^2)} < \alpha < \frac{(1 - 2\omega)(32b\theta + 7s^2)}{2x(32b\theta - 7s^2)}.$$  (12)

That is, conditions (11) and (12) are necessary for the existence of a unique and symmetric equilibrium, in which positive number of workers switch employers whenever firms’ managerial capabilities are different. Note that \(0 < \frac{s^2(1 - 2\omega)}{2x(4b\theta - s^2)} < \min\{\frac{8(1 - 2\omega)}{2x}, \frac{(1 - 2\omega)(32b\theta + 7s^2)}{2x(32b\theta - 7s^2)}\}\) holds such that the set of \(\alpha\) that satisfying conditions (11) and (12) is non-empty (see Claim 3 in the Appendix for details). That is, the necessary conditions (11) and (12) can be represented by an intermediate range of \(\alpha\). If the difference between high and low capability is too small, labor mobility will not occur since retained workers productivity is still high for each firm. If the difference between high and low capability is too large, a firm underwent contraction will completely shut down given that the productivity of its workers is relatively high in its rival firm. Also note that in equilibrium \(m^E_i(n^*, z^*, n^*) = m^C_i(z^*, n^*, n^*) \equiv m^*\) and \(w^E_{i, 2}(n^*, z^*, n^*) = w^C_{i, 2}(z^*, n^*, n^*) \equiv w^E_{2}/C\). Then, Proposition 1 below tells us that this condition is not only necessary but also
sufficient for the existence of a unique symmetric equilibrium.

**Proposition 1.** There exists a unique symmetric equilibrium in which a strictly positive number of workers move from one firm to the other whenever the realizations of both firms’ managerial capabilities are different in period 2, if and only if \( \alpha \in (\bar{\alpha}, \bar{\alpha}) \) where \( 0 < \alpha < \bar{\alpha} \). The equilibrium is characterized by each firm \( i \)'s level of asset specificity and number of first-period workers \((z_i, n_i) = (z^*, n^*) = \left( \frac{8\alpha(1-2\omega)-2\alpha xx}{\alpha(32\theta-9x^2)}, \frac{(1-2\omega)(32\theta-9x^2)-2\alpha xx^2}{4(32\theta-9x^2)} \right) \).

In the equilibrium, each firm chooses the level of asset specificity \( z^* \), makes first-period wage offer \( w_1^* \) to \( n^* \) individuals, and employs \( n^* \) workers at Stage 1. At Stage 3 if both firms have different managerial capabilities, firm \( i \) whose managerial capability \( a_i \) turns out to be higher retains all its first-period employees and expands in its firm size by hiring \( m^* \) workers from firm \( j \) \((j \neq i)\) at the second-period wage \( w_2^{E/C} \) whereas firm \( j \) contracts in its firm size and retains \( n^* - m^* \) workers at the same second-period wage \( w_2^{E/C} \). Hence if \( a_i \neq a_j \), there exists a strictly positive number of workers, \( m^* \), moving from the low-capability firm to the high-capability firm in the equilibrium. At Stage 3 if \( a_i = a_j \), each firm \( i \) retains all its first-period workers at the second-period wage \( w^S(a_i, n^*) \) and stays unchanged in its firm size (neither expansion nor contraction); thus there is no labor mobility. Then, the expected number of workers who switch their employers at the beginning of period 2 is \( \frac{1}{4} (2m^*) \), and each firm \( i \)'s expected labor turnover rate is

\[
\frac{1}{4} \left( \frac{2m^*}{n^*} \right)
\]

We will now turn to comparative statics on the equilibrium level of the asset specificity \( z^* \) and the firm size \( n^* \) (the period 1 number of workers), and the expected equilibrium turnover rate \( \frac{1}{4} \left( \frac{2m^*}{n^*} \right) \). Note that the period 1 number of workers \( n^* \) can be interpreted as average firm size measured by employment since \( n^* \) determines each firm’s expected number of workers over two periods.

**Proposition 2.** As the importance of managerial capability (captured by \( x \)) increases, the level of asset specificity decreases, the average firm size decreases, and the expected labor turnover rate increases in the equilibrium.

The key result here is that, as the importance of managerial capability increases, the expected number of workers who switch their employers (expected labor mobility)
in period 2 becomes larger, and each firm’s asset specificity and average employment size decreases. The logic behind this result can be explained as follows. When firm $i$ chooses the level of its asset specificity and the number of workers it employs in period 1, it estimates the expected number of its first-period employees that the firm will retain for second-period operation, and the expected number of workers that the firm hires from its rival firm $j$ in period 2. A higher importance of managerial capability increases the difference of period 2 productivity between a high-capability and a low-capability firm. Then, as the importance of managerial capability increases, each firm anticipates a larger number of workers switch their employers in period 2 if both firms’ managerial capabilities turn out to be different. Hence a higher importance of managerial capability decreases the expected number of retained workers, and it increases the expected number of workers who switch employers. Holding the initial number of workers constant, this reduces each firm’s incentive to choose higher specificity of its asset, because the proportion of retained workers decreases and the productivity gains over the investment cost for each retained worker becomes smaller.

Meanwhile, anticipating that the number of retained workers decreases while the supervision cost per worker in period 1 remains the same, each firm has a lower incentive to hire more workers in period 1. Because a larger number of period 1 workers increases a firm’s period 2 profit from its retained workers at the expense of increasing supervision cost per worker in period 1. Also, the reduction of each firm’s first-period workers decreases the proportion of retained worker in period 2, and in turn decreases the marginal benefit of choosing a higher asset specificity. Hence, the force of lowering average firm size further reduces each firm’s level of asset specificity. Taken together, the result is that, the equilibrium level of asset specificity and the average firm size decrease as the importance of managerial capability increases. The last result of Proposition 2 naturally follows from the key result mentioned above. As the importance of managerial capability increases, the expected labor turnover rate, measured by the ratio of expected number of workers who switch their employers in period 2 to the number of first-period workers, increases unambiguously in the equilibrium.

**Proposition 3.** As the importance of a firm’s asset specificity (captured by $s$) increases,
the level of asset specificity increases, the average firm size increases, and the expected labor turnover rate decreases in the equilibrium.

Recall that in period 1 firm $i$ estimates the number of workers it will retain and hire from the rival firm in period 2, respectively. As the importance of firm $i$’s asset specificity increases, firm $i$ will retain more of its first-period employees during the contraction phase, while hire fewer workers from its rival during the expansion phase since new workers become relatively less productive in firm $i$. This implies that each firm anticipates a smaller number of workers switch their employers in period 2 if managerial capabilities are different. Lower expected labor mobility, along with the higher return from retained workers (captured by $s$), implies that a firm has higher incentive to choose a higher level of asset specificity. These two effects are mutually reinforcing because the higher level of asset specificity reduces the expected turnover rate by increasing the retained workers’ productivity. The results of the average firm size and expected turnover rate follow through the logic analogous to the one explained for Proposition 2.

4 Discussion

4.1 An application to cross-industry differences

The first application concerns the idea that the importance of managerial capability can differ across industries, as discussed in Morita (2012).\footnote{See also empirical studies by Ely (1991) and Hogan and Sigler (1998). They find that the sensitivity of CEO’s compensation to firm performance differs significantly across industries. Assuming that the sensitivity captures the importance of managerial capability for firm performance, these findings suggest that the importance of managerial capability differs across industries.} For instance, in a newly emerging industry or in a business undergoing revolutionary technological changes, a business’s success critically depends on the quality of its strategic decision making because these industries face a high level of uncertainty about the needs of customers, the products and services that will prove to be the most desired, and the best configuration of activities and technologies to deliver them. Whereas in industries facing lower levels of uncertainty, strategic decision making is less important. These arguments suggest that the importance of managerial capability is higher in the newly emerging industries with higher level of uncertainty or in the new business undergoing revolutionary technological
changes, while the importance tends to be lower in mature industries or business with lower level of uncertainty and less technological advancement. Our model then predicts that labor mobility is higher, specificity of asset and human capital is lower, and average firm size is smaller in industries of the former type and vice-versa in industries of the latter type.9

These predictions are consistent with empirical and observational evidence. Concerning firm size, it has been documented that firm size tends to be smaller in the emerging industries compared with mature industries (Kohn, 1997). Also, Dinlersoz and MacDonald (2009) show that the average firm size (measured by the employment) in U.S. manufacturing industries from 1963 until 1997 declines during the rapid entry phase of the industry life-cycle, where the rapid entry phase occurs when there exists greater technological improvement. Concerning labor mobility, Benner (2002) studies labor markets in Silicon Valley and points out that, “The rapid turnover and volatility in employment in Silicon Valley is integrally connected to the nature of competition in the region’s high-technology industries. In these industries, markets and technology change extremely rapidly and in unpredictable ways.”

4.2 An application to U.S.-Japanese differences

The U.S. and Japan have been considered representing two contrasting employment systems, which attracts significant attention in the literature of international comparison in how internal labor markets operate. By capturing the interconnections among asset specificity, acquisition of firm-specific human capital, managerial capability, and labor mobility, our model offers new explanations for and predictions on the U.S.-Japanese differences based on the cross-country differences in the importance of both managerial capability and firm-specificity of asset.

Acemoglu, Aghion and Zilibotti (2006) argue in their analysis of technology frontiers and firm selection that managerial skill is more important for undertaking innovative activities than for adopting and imitating existing technologies from the world technology frontier. They then point out, based on their analysis of the correlation between distance

9In an alternative approach, Rossi-Hansberg, Sarte and Owens III (2009) also show that the average firm size decreases due to firm fragmentation in the evolution of urban structure.
to the frontier and R&D intensity using data from the OECD sectoral database, that innovation becomes more important as the economy approaches the world technology frontier and there remains less room for adoption and imitation. Following their analysis and argument, we argue that the importance of managerial capability was substantially lower in Japan than in the U.S. when most Japanese industries were catching up with the West in the postwar growth period (Okimoto, 1989).

On the other hand, as the economy makes the transition from industrial capitalism to post-industrial capitalism, modern economies are becoming increasingly knowledge intensive which renders the disadvantage to the firms that heavily rely on physical assets. For example, Iwai (2002) points out that, in the new era of post-industrial capitalism, the physical assets have surrendered their central position to the knowledge-based human assets that money can no longer buy and control. The advent of knowledge economy thus reduces the importance of firm-specificity of asset across the world. But different countries have different paths in making their transition to the knowledge economy. Based on the knowledge assessment methodology (KAM) developed by World Bank Institute that traces the challenges and opportunities of major economies’ transition to knowledge-based economy from 1995, the U.S. has a better shape than Japan in their development of knowledge economy (Shibata, 2006). Following this reasoning, we argue that the importance of firm-specificity of asset is higher in Japan than in the U.S.

Combining the above argument where Japan has lower importance of managerial capability and higher importance of firm-specificity of asset compared with U.S., the predictions of our model point out to the same direction. That is, labor mobility was lower, and specificity of asset and human capital was higher in Japan than in the U.S., especially in the catching up period.

These predictions are consistent with empirical and observational evidence. Concern-

10Also, Drucker (1993) for example, has argued that in the new economy, knowledge is not just another resource alongside the traditional factors of production—labor, capital, and land—but the only meaningful resource today.

11In an alternative approach where promotion serves as a signal of the worker’s ability, Owan (2004) shows that late-promotion practice which commonly observed in Japanese firms results in low turnover rate and long-term employment relationship, whereas early-promotion practice which is commonly observed in U.S. results in the opposite effects. Moreover, Chang and Wang (1995) explain the lower turnover rate and higher human capital accumulation in Japan than in the U.S. Their model is characterized by multiple equilibria and is analyzed in the framework where current employers have superior information on the abilities of their employees.
ing labor mobility, it was found that the labor turnover rate was much higher in U.S. than in Japan (see Hashimoto and Raisian, 1985; Mincer and Higuchi, 1988). Hashimoto and Raisian (1985), for instance, found that the 15-year job retention rates of the male population between the early 1960s and the late 1970s were much higher in Japan than in the United States across all age groups. Concerning firm-specific human capital, Koike (1977, 1988) found, in his comparative analysis of Japanese and U.S. industrial relations, that Japanese workers acquired more firm-specific human capital through rotation among related jobs (see also Dertouzos, Lester and Solow, 1989; Ito, 1992). Concerning specificity of asset, we are unaware of direct observations or evidence. However, it is well known that Japanese firms conducted continuous process improvement more than U.S. firms did in the postwar growth period.\textsuperscript{12} As argued by Morita (2001), if a firm conducts continuous process improvement, the technology is improved but a degree of specificity is introduced. This is because, in general, continuous process improvement involves a number of small changes and modifications, which lead to highly firm-specific technologies. This then suggests that specificity of asset was higher in Japanese firms than in the U.S. firms in the postwar growth period.

The Japanese economy has already caught up with the West, and most Japanese industries have got much closer to the world technology frontier. This increases the importance of managerial capability which is crucial for undertaking innovative activities. Our model then predicts that the degree of the U.S.-Japanese differences become smaller. That is, labor mobility had increased, and specificity of asset and human capital has decreased in Japan. In reality, however, such transitions in Japan are likely to take place rather slowly due to vested interests and institutional inertia embedded in the Japanese economic system (Lincoln, 2004). Several empirical studies have been undertaken recently to find out whether or not the Japanese employment system has exhibited significant changes, but findings are not clear-cut thus far.\textsuperscript{13}

\textsuperscript{12}See Morita (2001) for a review of evidence.

\textsuperscript{13}For example, Kambayashi and Kato (2011) study labor mobility and conduct cross-national analysis of micro data from Japan Employment Status Survey and U.S. Current Population Survey. They find that, on the one hand, core employees (age of 30-44 with at least five years of tenure) in Japan continued to enjoy much higher job stability than the U.S. counterparts consistently over the last twenty-five years. On the other hand, job stability for mid-career hires and youth employees deteriorated in Japan over the last twenty-five years, whereas there was no comparable decline in job stability in the U.S. counterparts.
5 Concluding remarks

This paper has explored a new model that in an environment of labor market competition, each firm consists of the top management, assets, and workers who operate assets to produce goods and services. Elaborating on the connection between labor mobility and the levels of firms’ asset specificity and their employment sizes, we have demonstrated that as the importance of managerial capability increases or the importance of a firm’s asset specificity decreases, the level of asset specificity decreases, the average firm size decreases, and the expected labor turnover rate increases in the equilibrium. This has yielded empirical implications and predictions from a previously unexplored perspective, given that the importances of both managerial capability and firm-specificity of asset can differ across industries and countries.

The current model can be enriched or extended to address several interesting questions. First, firms may want to collude in the labor market if it is profitable to reduce labor mobility and to increase asset specificity. Then there will be policy implication for government’s restriction on firms’ employment practices (e.g. no-poach agreement). Second, we can further assume that firm $i$’s choice of asset specificity affects both its first-period employees’ productivity in firm $j$ and the productivity of firm $j$’s first-period worker in firm $i$. Specifically, if a worker moves from firm $i$ to the other firm $j$ ($j \neq i$) in period 2, his second-period production efficiency associated with new firm’s asset is decreasing in both his first-period employer $i$’s and new employer $j$’s asset specificities. This reflects the idea that if firm $i$’s asset is utilized by outside workers who are not familiar with the tailor-made feature of $i$’s asset, the output becomes lower. The other is that if a worker already gains familiarity in utilizing firm $i$’s asset, his/her productivity in firm $j$ ($j \neq i$), holding $j$’s asset specificity level constant, becomes lower, perhaps arising because a worker’s prior experience from a different asset utilization can interfere with his/her learning efficiency in the current one. There may exist interesting implications from this distinction between the different sources of reduction in workers’ human capital acquisition. Third, it will be interesting to introduce downward-sloping demand schedule such that entry of firms is free. Then, there will be implications on firm dynamics and government’s policy on firm’s entry. We plan to fully explore these
extension/enrichment in the future works.
Appendix

Proof of Proposition 1:

Suppose that there exists a symmetric equilibrium characterized by \((z_i, n_i) = (z^*, n^*)\) for each \(i\), in which a strictly positive number of workers switch employers in period 2 whenever the realizations of managerial capability are different in both firms. In the text, it has been shown that conditions (11) and (12) are necessary for such an equilibrium to exist. Here, in Claim 1–3 below, we present proof of some computational details that have not been presented in the text.

Claim 1: In a symmetric equilibrium where a strictly positive number of workers switch employers in period 2 whenever the realizations of managerial capability are different in both firms, each firm \(i\) chooses \(z_i = \frac{8s(1-2\omega)-2a\omega s}{32\theta-9s^2} \equiv z^*\) and \(n_i = \frac{(1-2\omega)(32\theta-s^2)-2a\omega s^2}{4b(32\theta-9s^2)} \equiv n^*\).

Proof of Claim 1: Suppose that there exists a symmetric equilibrium in which a strictly positive number of workers switch employers in period 2 when firms’ managerial capabilities are different. First note that in the Stage 3 subgame, firm \(i\)'s second-period profit conditional on \(a_i = a_H\) and \(a_i \neq a_j\) is

\[
\pi_i^E(z_i, n_i, z_j, n_j) = (n_i + m_i^E(z_i, n_i, z_j, n_j))(xa_H + 1 + sz_i) + n_i(1 + sz_i) + m_i^E(z_i, n_i, z_j, n_j)b(n_i + m_i^E(z_i, n_i, z_j, n_j))^2 - w_{i,2}^E(z_i, n_i, z_j, n_j)(n_i + m_i^E(z_i, n_i, z_j, n_j))
\]

as analyzed in the text, implying that

\[
\pi_i^E(z_i, n_i, z_j, n_j) = \frac{1}{16b} \left\{ (2ax - sz_j)^2 + 4b^2(n_i + n_j)^2 \right\} + 4b \left\{ 2ax(n_i + n_j) + 4sn_i z_i - s(n_i + n_j)z_j \right\},
\]

given \(m_i^E(n_i, z_j, n_j) = \frac{1}{16b} [2ax - sz_j - 2b(n_i - n_j)]\) and \(w_{i,2}^E(n_i, z_j, n_j) = \frac{1}{2} \left[ 2 + sz_j - 2b(n_i + n_j) \right]\).

Consider the Stage 3 subgame where \((a_i, a_j) = (a_L, a_H)\). If firm \(i\) contracts and firm \(j\) expands in their firm sizes, firm \(i\) will offer \(w_{i,2} = xa_L + (1 + sz_i) - 2b(n_i + m_i)\) where \(m_i < 0\) and firm \(j\) will offer \(w_{j,2} = xa_H + 1 - 2b(n_j + m_j)\) where \(m_j > 0\), that equal to the expected productivity of each firm’s first-period employee at the margin, respectively. In the equilibrium, both firms offer the same second-period wages \(w_{i,2} = w_{j,2}\), and the
number of workers from contraction equals that from expansion, \(-m_i = m_j\). We obtain
\[ m_i^C(z_i, n_i, n_j) = \frac{1}{2b}[2ax - sz_i - 2b(n_j - n_i)]. \]
Then, the equilibrium second-period wage becomes
\[ w_{i,2}^C(z_i, n_i, n_j) = \frac{1}{2}[2 + sz_i - 2b(n_i + n_j)]. \]
If firm \(i\) expands and firm \(j\) contracts in the equilibrium, the expansion/contraction size is
\[ \frac{1}{2b}[-2ax - sz_i + 2b(n_j - n_i)]. \]
This equilibrium can be ruled out given \(-2ax - sz_i + 2b(n_j - n_i) < 0\) for any symmetric equilibrium—a contradiction. Also, \(0 < m_i^C(z_i, n_i, n_j) \leq n_i\) holds in the equilibrium, which suggests \(0 < \frac{1}{2b}[2ax - sz_i - 2b(n_j - n_i)] < n_i\). This condition is the same as the condition (1) in the equilibrium. Then, firm \(i\)'s second-period profit conditional on \(a_i = a_L\) is
\[ \pi_i^C(z_i, n_i, n_j) = (n_i - m_i^C(z_i, n_i, n_j))(xa_L + 1 + sz_i) - b(n_i - m_i^C(z_i, n_i, n_j))^2 - (n_i - m_i^C(z_i, n_i, n_j))w_{i,2}^C(z_i, n_i, n_j), \]
implying that
\[ \pi_i^C(z_i, n_i, n_j) = \frac{1}{16b}[2ax - sz_i - 2b(n_i + n_j)]^2. \] (14)

Next consider the the Stage 3 subgame where both firms have the same managerial capability, \(a_i = a_j\). First note that it cannot be the case that one firm expands and the other contracts in this case given that either \(-sz_j + 2b(n_i - n_j) \neq 0\) or \(-sz_i + 2b(n_j - n_i) \neq 0\) (the managerial capability gap is now 0) under symmetric equilibrium. Then, both firms must stay unchanged in their firm sizes in the equilibrium. That is, each firm \(i\) will offer the wage to the extent that the expected productivity from the outside workers are the same at the level \(xa_i + 1 - 2b(n_i + m_i)\) where \(m_i = 0\) holds in the equilibrium. That is, \(w_{i,2}^S = w_{i,2}^S(a_i, n_i) = xa_i + 1 - 2bn_i\). Then firm \(i\)'s second-period profit conditional on \(a_i = a_j\) is
\[ \pi_i^S(z_i, n_i) = n_i(xa_i + 1 + sz_i) - bn_i^2 - nw_{i,2}^S(a_i, n_i), \]
implying that
\[ \pi_i^S(z_i, n_i) = n_i(sz_i + bn_i). \] (15)

Consequently, in the equilibrium each firm \(i\)'s expected profit in period 2 is
\[ \pi_{i,2}(z_i, n_i, z_j, n_j) = \frac{1}{4}\pi_i^E(z_i, n_i, z_j, n_j) + \frac{1}{4}\pi_i^C(z_i, n_i, n_j) + \frac{1}{4}\pi_i^S(z_i, n_i), \]
where \(\Pr(a_H, a_L) = \Pr(a_L, a_H) = \frac{1}{4}\) and \(\Pr(a_i = a_j) = \frac{1}{2}\). Also note that \(w_{i,2}^S + E[w_{i,2}(z_i, n_i, z_j, n_j)] = 2\omega\) holds for a worker employed by firm \(i\) in the equilibrium, where
\[ E[w_{i,2}(z_i, n_i, z_j, n_j)] = 1 + \frac{1}{8}[s(z_i + z_j) - 4b(n_i + n_j)], \] (16)

25
the weighted average of \( w_{i,1}^E(z_i, n_i, z_j, n_j) \), \( w_{i,2}^C(z_i, n_i, z_j, n_j) \), and \( w_{i,2}^S(a_i, z_i, n_i, z_j) \). Then, we find \( w_i^* = w_{i,1}^*(z_i, n_i, z_j, n_j) \) where
\[
 w_{i,1}^*(z_i, n_i, z_j, n_j) = 2\omega - 1 - \frac{1}{8}[s(z_i + z_j) - 4b(3n_i + n_j)]. \tag{17}
\]

Hence, each firm \( i \)'s expected overall profit is \( \Pi_i(z_i, n_i, z_j, n_j) = n_i x a_i - b n_i^2 - \frac{1}{2} \theta z_i^2 - W_{i,1} + \pi_2(z_i, n_i, z_j, n_j) \), implying that
\[
 \Pi_i(z_i, n_i, z_j, n_j) = n_i[-b n_i - w_{i}^*(z_i, n_i, z_j, n_j)] - \frac{1}{2} \theta z_i^2 \\
+ \frac{1}{4} \pi_i^E(z_i, n_i, z_j, n_j) + \frac{1}{4} \pi_i^C(z_i, n_i, n_j) + \frac{1}{2} \pi_i^S(z_i, n_i),
\tag{18}
\]
where \( \pi_i^E(z_i, n_i, z_j, n_j) \), \( \pi_i^C(z_i, n_i, n_j) \), and \( \pi_i^S(z_i, n_i) \) are defined in the equations (13), (14), and (15) respectively. Let compute the partial derivatives of \( \Pi(z_i, n_i, z_j, n_j) \). We find
\[
(\partial/\partial z_i)\Pi_i(z_i, n_i, z_j, n_j) = \frac{1}{32b}[-2axs + s^2z_i + 2b(15sn_i + sn_j - 16z_i)],
\]
\[
(\partial/\partial n_i)\Pi_i(z_i, n_i, z_j, n_j) = 1 - 2\omega + 15sz_i + sz_j - 4b(15n_i + n_j)].
\]

Given that \( (z_i, n_i) = (z_j, n_j) = (z^*, n^*) \) in the equilibrium, the first-order conditions imply
\[
\begin{cases}
\left( \frac{1}{32b} \right) s(z^* - 2ax) + (sn^* - \theta z^*) = 0 \\
1 - 2\omega + sz^* - 4bn^* = 0
\end{cases}
\tag{19}
\]
Solving the simultaneous equations, we obtain \( z^* = \frac{8s(1 - 2\omega) - 2axs}{32b(9s - 9\omega)} \) and \( n^* = \frac{(1 - 2\omega)(32b - 9s^2) - 2axs^2}{4b(32b - 9\omega)} \) in the equilibrium. Then, if such a symmetric equilibrium exists, it must be unique. □

**Claim 2:** There exists a unique symmetric equilibrium in which workers switch employers whenever firms managerial capabilities are different, if and only if conditions (11) and (12) presented in the text hold.

**Proof of Claim 2:** Given Claim 1, if firm \( i \)'s expected overall profit at Stage 1 is \( \Pi_i(z_i, n_i, z_j, n_j) \), it must be the case that in the subsequent subgame, firm \( i \)'s optimal de-
Proof of Proposition 2 and 3:  

Consider the maximization problem of firm $i$’s parametrization such that conditions (11) and (12), which holds given $\bar{a} = (\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4)$, and to stay unchanged in its firm size if $a_i = a_j$. That is, there exists a symmetric equilibrium with positive labor mobility whenever firms’ managerial capabilties are different, if and only if each firm $i$’s expected overall profit at Stage 1 is $\Pi_i(z_i, n_i, z_j, n_j)$. Then, given firm $j$ ($j \neq i$) chooses $(z_j, n_j) = (z^*, n^*)$, let us check the second-order condition of firm $i$’s maximization problem of $\Pi_i(z_i, n_i, z^*, n^*)$. We have $(\partial^2/\partial z_i^2)\Pi_i(z_i, n_i, z^*, n^*) = -\frac{1}{10}(16b\theta - s^2)$, $(\partial^2/\partial n_i^2)\Pi_i(z_i, n_i, z^*, n^*) = -\frac{7}{8}b$, and $(\partial^2/\partial z_i\partial n_i)\Pi_i(z_i, n_i, z^*, n^*) = (\partial^2/\partial n_i\partial z_i)\Pi_i(z_i, n_i, z^*, n^*) = \frac{5}{8}s$. Hence, the first-order leading principal minor of $D^2\Pi_i(z_i, n_i, z^*, n^*)$ is negative and the second-order leading principal minor of $D^2\Pi_i(z_i, n_i, z^*, n^*)$ is $\frac{7}{64}(32b\theta - 9s^2)$. That is, $D^2\Pi_i(z_i, n_i, z^*, n^*)$ is negative definite over all $z_i \geq 0$ and $n_i \geq 0$ if and only if $32b\theta - 9s^2 > 0$. Note that $0 < \frac{8(1-\omega)}{2x} < \frac{1}{2x}(32\theta + 7s^2) < \frac{1}{2x}(32\theta - 7s^2) < \frac{8(1-\omega)}{2x}$, which holds given $32b\theta > 9s^2$. Also, $\frac{8(1-\omega)(32\theta + 7s^2)}{2x(32\theta - 7s^2)} > \frac{s^2(1-\omega)}{2x(46\theta - s^2)} \Leftrightarrow 128(b\theta^2 - 8s^2b\theta - 7s^4) > 32\theta \bar{b}^2 - 7s^4 \Leftrightarrow 32b\theta > 9s^2$, which holds given $32b\theta > 9s^2$. Further note that $\frac{s^2(1-\omega)}{2x(46\theta - s^2)} > 0$ given $32b\theta - 8s^2 > 0$ and $1 - 2\omega > 0$. This implies the result. 

Claim 3: There exists a range of parametrization such that conditions (11) and (12) presented in the text hold.

Proof of Claim 3: We want to show that $0 < \frac{s^2(1-\omega)}{2x(46\theta - s^2)} < \frac{1}{2x}(32\theta + 7s^2) < \frac{8(1-\omega)}{2x}$.

First note that $\frac{8(1-\omega)}{2x} > \frac{(1-\omega)(32\theta + 7s^2)}{2x(32\theta - 7s^2)} \Leftrightarrow 32\theta \bar{b}^2 - 7s^4 \Leftrightarrow 32b\theta > 9s^2$. Also, $\frac{8(1-\omega)(32\theta + 7s^2)}{2x(32\theta - 7s^2)} > \frac{s^2(1-\omega)}{2x(46\theta - s^2)} \Leftrightarrow 128(b\theta^2 - 4s^2b\theta - 7s^4) > 32\theta \bar{b}^2 - 7s^4 \Leftrightarrow 32b\theta > 9s^2$, which holds given $32b\theta > 9s^2$. Further note that $\frac{s^2(1-\omega)}{2x(46\theta - s^2)} > 0$ given $32b\theta - 8s^2 > 0$ and $1 - 2\omega > 0$. This implies the result. 

Let $\bar{a} = \frac{(1-\omega)(32\theta + 7s^2)}{2x(32\theta - 7s^2)}$ and $\bar{a} = \frac{s^2(1-\omega)}{2x(46\theta - s^2)}$. Then all the necessary and sufficient conditions boil down to $\alpha \in (\bar{a}, \bar{a}]$. This completes the proof. 

Proof of Proposition 2 and 3: 

(i) Note $z^* = \frac{8s(1-2\omega)-2axs}{32b\theta - 9s^2}$, we find $\frac{\partial z^*}{\partial x} = \frac{-2axs}{32b\theta - 9s^2} < 0$. Also, $\frac{\partial z^*}{\partial s_1} = \frac{\frac{8s}{32b\theta - 9s^2} - \frac{2axs}{32b\theta - 9s^2}}{\frac{8s}{32b\theta - 9s^2}} > 0$ given $8s(1-2\omega) - 2axs > 0$. 


(ii) Note \( n^* = \frac{(1-2\omega)(32\theta - s^2) - 2\alpha s}{4\theta(4\theta^2 - 9s^2)} \), we find \( \frac{\partial n^*}{\partial x} = \frac{-2\alpha s}{36(32\theta - 9s^2)} < 0 \). Also, \( \frac{\partial n^*}{\partial s} = \frac{32s[4(1-2\omega)-\alpha]}{(32\theta - 9s^2)^2} > 0 \) given \( 8s(1 - 2\omega) - 2\alpha s > 0 \).

(iii) Note \( m^* = \frac{1}{4\theta}(2\alpha x - s z^*), \) we find \( \frac{\partial m^*}{\partial x} = \frac{1}{4\theta}(2\alpha - \frac{\partial z^*}{\partial x}) > 0 \) given \( \frac{\partial z^*}{\partial x} < 0 \). Then given \( \frac{\partial n^*}{\partial x} < 0 \) and \( \frac{\partial m^*}{\partial x} > 0 \), we find \( \partial \left( \frac{1}{4} m^* \right)/\partial x = \frac{1}{(2n^*)^2} \left[ \frac{\partial m^*}{\partial x}(2n^*) - m^* \left( \frac{\partial m^*}{\partial x} \right) \right] > 0 \). Also, we find \( \frac{\partial m^*}{\partial s} = -\frac{1}{4\theta} \frac{\partial z^*}{\partial s} < 0 \) given \( \frac{\partial z^*}{\partial s} > 0 \). Then given \( \frac{\partial n^*}{\partial s} > 0 \) and \( \frac{\partial m^*}{\partial s} < 0 \), we find \( \partial \left( \frac{1}{4} m^* \right)/\partial s = \frac{1}{(2n^*)^2} \left[ \frac{\partial m^*}{\partial s}(2n^*) - m^* \left( \frac{\partial m^*}{\partial s} \right) \right] < 0 \).
References


