Discussion Paper No. 2017-02

Inference on Local Average Treatment Effects for Misclassified Treatment

Takahide Yanagi*

This version: February 21, 2017 First version: January 27, 2017

Abstract

We develop point-identification and inference methods for the local average treatment effect when the binary treatment contains a measurement error. The standard instrumental variable estimator is inconsistent for the parameter since the measurement error is non-classical by construction. Our proposed analysis corrects the problem by identifying the distribution of the measurement error based on the use of an exogenous variable such as a covariate or instrument. The moment conditions derived from the identification lead to the generalized method of moments estimation with asymptotically valid inferences. Monte Carlo simulations demonstrate the desirable finite sample performance of the proposed procedure.

Keywords: misclassification; instrumental variable; non-differential measurement error; nonparametric method; causal inference.

JEL Classification: C14; C21; C25.

*Graduate School of Economics, Hitotsubashi University, 2-1 Naka, Kunitachi, Tokyo 186-8601, Japan. Email: t.yanagi@r.hit-u.ac.jp

1 Introduction

The local average treatment effect (LATE) is a popular causal parameter in the microeconometric literature (e.g., Angrist and Pischke, 2008, Chapter 4). It represents the average causal effect of binary endogenous treatment T^* on outcome Y for the unit whose treatment status changes depending on the value of binary instrument Z. As shown by Imbens and Angrist (1994), the instrumental variable (IV) estimator identifies the LATE under suitable identification conditions. While the identification of the LATE needs the precise measurement of *true* treatment T^* in addition to the identification conditions, in practice, observed binary treatment T may be a mismeasured variable of T^*. LATE applications may involve such a degree of misclassification that the actually treated unit can even be misrecorded as untreated and vice versa. While econometricians are not aware of the presence of measurement errors in their data, as we discuss below, ignoring such measurement errors may lead to drawing misleading economic implications.

As an illustrative application, let us consider the causal analysis of returns to schooling, one of the most important applications of LATE inferences. In this case, Y is an individual outcome such as wages, T^* is an indicator of educational attainment such as a high school, college, or doctoral degree, and Z is an IV based on some schooling systems such as variations in compulsory schooling laws (e.g., Acemoglu and Angrist, 2001; Lochner and Moretti, 2004). Educational attainment may be mismeasured for some reason such as random recording error or the provision of intentionally/unintentionally false statements. Indeed, several studies have pointed out the prevalence of measurement errors when reporting educational attainment (e.g., Kane and Rouse, 1995; Card, 1999; Black, Sanders, and Taylor, 2003; Battistin, De Nadai, and Sianesi, 2014). LATE inferences for returns to schooling may be contaminated by mismeasured educational attainment.

The present study contributes to the literature by proposing a novel inference procedure for the LATE with the mismeasured treatment. The measurement error brings out a bias such that the IV estimator under- or overestimates the LATE. While the IV estimation solves the problem of the *classical* measurement error that is independent of the true variable (e.g., Wooldridge, 2010, Chapters 4 and 5), the bias for the LATE is caused in our situation since the measurement error for the binary variable must be *non-classical*; in other words, the error is correlated with the true unobserved treatment because the support of the measurement error depends on the true variable (e.g., Aigner, 1973). Indeed, the bias for the LATE depends on the distribution of the measurement error, meaning that we cannot point-identify the LATE without identifying the distribution.

To correct the identification bias due to the measurement error, we develop a point-identification analysis for the LATE and the distribution of the measurement error based on the availability of an exogenous observable variable, say V. While many previous studies have corrected problems due to measurement errors by using exogenous variables (e.g., Hausman, Newey, Ichimura, and Powell, 1991; Lewbel, 1997, 1998, 2007; Mahajan, 2006; Schennach, 2007; Hu, 2008; Hu and Schennach, 2008), our identification analysis builds on this strand of the literature. In particular, we extend Lewbel's (2007) result for the average treatment effect (ATE) with the mismeasured exogenous treatment to the LATE inference with the mismeasured endogenous treatment.

The main idea behind our identification is, under a set of empirically plausible conditions, to derive moment conditions whose number is no fewer than the number of unknown parameters including the LATE, true first-stage regression, and distribution of the measurement error. The key identification conditions are threefold. First, the measurement error of the treatment is *non-differential* in the sense that mismeasured T does not affect the mean of outcome Y once true T^* is conditioned on. Second, V has to satisfy a rank condition that requires an effect of V on T^*. Finally, V has to satisfy an exclusion restriction under which V cannot affect the difference in the conditional means of Y and the distribution of the measurement error. These conditions may be satisfied when V is a covariate or instrument. Importantly, our exclusion restriction does not rule out the possibility that V directly affects Y. Our exclusion restriction may hold if V affects the potential outcomes with and without the true treatment equally.

To illustrate the intuitions behind our identification conditions, again, let us consider the returns to schooling analysis. To correct the problem of mismeasuring educational attainment, one may use the quarters of birth (QOB) as exogenous V (e.g., Angrist and

Keueger, 1991).[1] The non-differential error requires that the misreported educational degree in the data does not affect the mean wage once the true educational degree is conditioned on. Our rank condition is satisfied if the QOB affects the true degree. Our exclusion restriction allows the effect of the QOB on wages, but it requires that the effect of the QOB on potential wages with the degree is the same as that without the degree.

The moment conditions derived from the identification result lead to moment-based estimators for the LATE, true first-stage regression, and distribution of the measurement error. The present study proposes adopting Hansen's (1982) generalized method of moments (GMM) estimator because of its popularity in the econometric literature. Desirably, the GMM inference is easy to implement in practice, and its asymptotic properties are well understood. As usual, asymptotically valid inferences can be developed based on asymptotic normality or bootstrap procedures. In particular, the overidentification test allows us to examine the validity of the identification conditions.

Monte Carlo simulations illustrate the problem of the measurement error and evaluate the finite sample properties of the proposed GMM estimation based on the identification. These demonstrate that the IV estimator based on the mismeasured treatment exhibits significantly large bias for the LATE. On the contrary, the bias and standard deviation of the proposed GMM estimator are satisfactory with a sample size of 1,000.

Related literature To our knowledge, three studies have thus far examined LATE inferences where the binary endogenous treatment may contain a measurement error.

Battistin et al. (2014) use two repeated measurements for the possibly misclassified treatment to develop point-identification and semiparametric estimation for the LATE. The necessity of their analysis is the presence of multiple repeated measurements for the true treatment from resurvey data. While their approach is useful given the availability of resurvey data, econometricians often lack such data in practice. Instead of requiring resurvey data, the present study exploits the availability of an exogenous variable.

[1] For the returns to schooling analysis, the literature suggests many other potential exogenous variables such as proximity to college (e.g., Card, 1993), parental/sibling education (e.g., Altonji and Dunn, 1996), and the sex of siblings (e.g., Butcher and Case, 1994). They are candidates for the exogenous variable in order to overcome the measurement error problem. See, for example, Card (1999, 2001) for a survey on returns to schooling analyses based on such exogenous variables.

DiTraglia and Garcia-Jimeno (2016) develop identification for the LATE in a nonparametric model with an additively separable error under assumptions on its higher moments. Their approach has the advantage that it does not require the availability of an exogenous variable and/or resurvey data. However, their additively separable model does not allow the individual treatment effect to depend on the unobservables. Their approach cannot identify the LATE when, for example, returns to schooling depend on unobservables such as individual ability. On the contrary, the present study allows arbitrary heterogeneous treatment effects, although we still need the availability of an exogenous variable.

Ura (2016) proposes sharp partial-identification for the LATE with the mismeasured treatment in the general setting. His approach allows heterogeneous treatment effects in a nonparametric and nonseparable model in line with the present study. The advantage of his partial-identification approach is that it does not require resurvey data and/or the use of an exogenous variable. However, his analysis does not attain point-identification for the LATE in general, implying that one cannot point-estimate the LATE based on his result. Despite requiring the presence of an exogenous variable, our approach attains point-identification and point-estimation for the LATE.

The present study also relates to the literature on the misclassification problem. Bollinger (1996), Hausman, Abrevaya, and Scott-Morton (1998), and Frazis and Loewenstein (2003) consider the estimation of parametric and semiparametric models with possibly mismeasured discrete variables. Mahajan (2006) and Lewbel (2007) develop point-identification and estimation methods for the ATE in nonparametric models where the binary exogenous treatment may be misclassified with the non-differential error. In particular, the approach in the present study builds on Lewbel's (2007) result that allows the exogenous variable for correcting the misclassification problem to affect the outcome. As in the present paper, Yanagi (2017) extends Lewbel's (2007) result to the regression discontinuity design with the mismeasured treatment. Elsewhere, Imai and Yamamoto (2010) consider the partial-identification of the ATE when the binary treatment may contain a differential measurement error. Molinari (2008) and Hu (2008) propose inferences on the distribution of the measurement error for the discrete exogenous variable.

Finally, many econometric studies examine the non-classical measurement error and/or nonlinear errors-in-variables models, and our proposed inference also builds on this body of the literature. For example, Amemiya (1985), Hsiao (1989), Horowitz and Manski (1995), Hu and Schennach (2008), Hu and Sasaki (2015), and Song, Schennach, and White (2015) study problems and solutions for such measurement errors in microeconometric applications. See Bound, Brown, and Mathiowetz (2001), Chen, Hong, and Nekipelov (2011), and Schennach (2013) for excellent reviews of the literature in this regard.

Paper organization Section 2 introduces the setup, reviews the LATE inference, and explains the identification problem due to the measurement error. Section 3 develops a point-identification analysis. Section 4 proposes GMM estimation based on the identification. Section 5 presents the Monte Carlo simulations. Section 6 concludes. Appendices A, B, and C contain the proofs of all the theorems and some additional discussions.

2 Setting

This section explains the setting considered in this study. Section 2.1 introduces the econometric model and the LATE. Section 2.2 briefly reviews the LATE inference when the true treatment can be observed without a measurement error. Section 2.3 discusses the problem due to a measurement error for the treatment.

2.1 The model and the LATE

We have a random sample of outcome $Y \in \mathbb{R}$, possibly mismeasured binary treatment $T \in \{0,1\}$ for true unobservable treatment $T^* \in \{0,1\}$, binary IV $Z \in \{0,1\}$, and exogenous variable $V \in supp(V) \subset \mathbb{R}$ such as a covariate or instrument. While the standard LATE inference does not require the presence of exogenous variables such as V, our analysis needs V to correct the problem due to a measurement error for T.

True treatment T^* may be endogenous due to omitted variables in the sense that unobservables exist that affect both Y and T^*. Observed treatment T may contain a measurement error, meaning that $T \neq T^*$ in general. We discuss the features of the measurement error and the problem due to this error in Section 2.3.

V can be a binary, general discrete, or continuous variable. V may affect both T^* and Y. However, V has to satisfy an exclusion restriction and a rank condition to identify and estimate the LATE and the distribution of the measurement error for T. The conditions that V has to satisfy and their implications are discussed in Section 3.

The aim of the inference is to examine the causal relationship between T^* and Y. Let Y_0 and Y_1 be the potential outcomes when the unit is untreated ($T^* = 0$) and when it is treated ($T^* = 1$), respectively. Similarly, let T_0^* and T_1^* be the potential true treatment statuses when $Z = 0$ and $Z = 1$, respectively. We can write $Y = Y_0 + T^*(Y_1 - Y_0)$ and $T^* = T_0^* + Z(T_1^* - T_0^*)$. The individual causal effect of T^* on Y is $Y_1 - Y_0$, which may be heterogeneous across units depending on the observables and/or unobservables.

To define the causal parameter, we define the following subsets of the common probability space $(\Omega, \mathcal{F}, P)$ on which the random variables are defined:

$$\begin{aligned}
&\text{Always taker:} && A := \{\omega \in \Omega : T_0^*(\omega) = T_1^*(\omega) = 1\},\\
&\text{Complier:} && C := \{\omega \in \Omega : T_0^*(\omega) = 0,\ T_1^*(\omega) = 1\},\\
&\text{Defier:} && D := \{\omega \in \Omega : T_0^*(\omega) = 1,\ T_1^*(\omega) = 0\},\\
&\text{Never taker:} && N := \{\omega \in \Omega : T_0^*(\omega) = T_1^*(\omega) = 0\}.
\end{aligned}$$

Intuitively, A or D is the set of units that always take or deny, respectively, the treatment (in the sense of true T^*), C is that of units whose treatment statuses are positively affected by Z, and D is that of units whose treatment statuses are negatively affected by Z. The units belonging to C or D change their treatment statuses depending on the value of Z.

The parameter of interest is the LATE, which is defined as

$$E(Y_1 - Y_0 | C) = E(Y_1 - Y_0 | T_1^* > T_0^*). \tag{1}$$

The LATE, which is the ATE for compliers, captures the average causal effect for units whose treatment statuses are positively altered by their instrumental values.

For example, in a returns to schooling analysis, Y is an individual outcome such as wages, T^* is an indicator of whether the individual has a high school diploma, Z is the IV indicating whether the individual is subject to the compulsory schooling law that requires the minimum leaving age is 16, and V is a covariate such as parental or sibling education or an instrument such as proximity to college or the QOB. The LATE is the average wage returns of the high school diploma for individuals who graduate from high school if and only if they are subject to the compulsory schooling law.

Remark 1. In practice, outcome Y or instrument Z may also contain a measurement error. However, a non-differential measurement error for Y or Z may not contaminate the LATE inference (see Appendix C). For this reason, our analysis below presumes that Y and Z do not contain measurement errors.

2.2 Review of the LATE inference without a measurement error

This section reviews the standard inference on the LATE when true T^* can be observed without a measurement error. The analysis when we observe possibly mismeasured T as opposed to true T^* is developed in the subsequent sections.

Without the measurement error, the LATE in (1) is identified by the IV estimator based on (Y, T^*, Z) under the following conditions. These are essentially the same as the conditions in Imbens and Angrist (1994) and Angrist, Imbens, and Rubin (1996).

Assumption 2.1. (Y_1, Y_0, T_1^*, T_0^*) *is independent of* Z.

Assumption 2.2. $\Pr(T_1^* > T_0^*) = \Pr(C) > 0$ *and* $0 < \Pr(Z = 1) < 1$.

Assumption 2.3. $\Pr(T_1^* < T_0^*) = \Pr(D) = 0$.

Assumption 2.1 is an exclusion restriction that requires the instrument to be unrelated to the factors affecting the outcome and/or treatment. Intuitively, the assumption guarantees that the instrument is assigned as good as randomly. Assumption 2.2 requires the presence of compliers. This is a rank condition in the sense that it requires the positive effect of Z on T^*. Assumption 2.3 is known as a monotonicity condition in the literature on LATE inferences. This rules out the presence of defiers.

In the example of returns to schooling, Assumption 2.1 implies that the variation of the compulsory schooling law is unrelated to the factors affecting wages with and without a high school diploma and/or educational attainment with and without the compulsory schooling law. Assumption 2.2 requires the presence of individuals who graduate from high school if and only if the compulsory schooling law is enforced. Under Assumption 2.3, there are no individuals who do attain a high school diploma without the compulsory schooling law but do not attain the diploma with it.

Lemma 1. *Suppose that Assumptions 2.1, 2.2, and 2.3 are satisfied. It holds that*

$$E(Y_1 - Y_0|C) = \frac{\mu_1 - \mu_0}{p_1^* - p_0^*} = \frac{\Delta\mu}{\Delta p^*}, \tag{2}$$

where $\mu_z := E(Y|Z = z)$, $p_z^* := E(T^*|Z = z) = \Pr(T^* = 1|Z = z)$ *for* $z = 0, 1$, $\Delta\mu := \mu_1 - \mu_0$, *and* $\Delta p^* = p_1^* - p_0^*$.

Proof. The proof is in Imbens and Angrist (1994, Theorem 1) and we thus omit it. □

Without a measurement error for the treatment, the LATE is identified by the IV estimand based on the linear IV regression for (Y, T^*, Z) in the right-hand side in (2). The LATE is thus consistently estimated by the IV regression or the two-stage least squares (TSLS) estimation based on (Y, T^*, Z).

2.3 Identification problem due to measurement error

This section explores the identification problem for the LATE in (1) in the situation where observed treatment T may be a mismeasured variable of true treatment T^*.

The identification for the LATE in Lemma 1 implicitly requires the precise measurement of true T^* in addition to the identification conditions of Assumptions 2.1, 2.2, and 2.3. However, observed T may contain a measurement error in practice, of which there are a few types. For example, in the returns to schooling analysis, educational attainment may be misrecorded randomly during the process of correcting survey data. Such a measurement error may be independent of the factors affecting the outcome and/or true educational attainment. The inference procedure proposed in this study allows this type of measurement error. The other possibility for the presence of measurement errors is false reporting. For instance, individuals may misunderstand the question or have poor recall for their educational attainment when responding a survey. Individuals might also have an incentive to make false statements about their academic achievement to enhance their careers. Such measurement errors may be correlated with the observables and/or unobservables. Our identification can be developed even with false reporting if the measurement error does not depend on the unobservables affecting the outcome and/or the true treatment (see Remark 4 below).

To examine the misclassification problem, we define the misclassification probability:

$$m_t := \Pr(T \neq T^*|T^* = t) = \Pr(T = 1 - t|T^* = t) \qquad \text{for} \quad t = 0, 1.$$

In other words, m_0 is the probability that an individual who is actually untreated is misclassified as treated, while m_1 is analogous. The misclassification probability can also be regarded as the distribution of the measurement error.

Importantly, the measurement error for the treatment is non-classical in the sense that it is dependent on the true treatment. This is because the support of the measurement error for a discrete variable depends on the true variable. To see this, we denote the measurement error for T as $U_T := T - T^*$. By construction, $U_T \in \{0, 1\}$ given $T^* = 0$ but $U_T \in \{-1, 0\}$ given $T^* = 1$, meaning that U_T is dependent of T^*. Specifically, the correlation between the error and the true treatment is always negative, whereas its magnitude depends on the misclassification probabilities: $Cov(U_T, T^*) = -(m_0 + m_1)\Pr(T^* = 0)\Pr(T^* = 1)$. Also, the correlation between true T^* and mismeasured T depends on the misclassification probabilities:

$$Cov(T^*, T) = (1 - m_0 - m_1)\Pr(T^* = 0)\Pr(T^* = 1). \tag{3}$$

If the sum of the misclassification probabilities $m_0 + m_1$ is less than one, T is positively correlated with T^* and negatively correlated otherwise.

We now explore the identification problem for the LATE due to the measurement error for the treatment. Thanks to Lemma 1, it is sufficient to focus on the IV estimand based on true T^* to identify the LATE: $\beta^* = \Delta\mu/\Delta p^*$. Since the numerator can be identified by the error-free observables of Y and Z, we focus on the denominator, which is the first-stage regression in the TSLS estimation of T^*.

We examine the relationship between observable treatment probability $p_z := E(T|Z = z) = \Pr(T = 1|Z = z)$ and true treatment probability $p_z^* := E(T^*|Z = z) = \Pr(T^* = 1|Z = z)$. We have the following relationship according to the law of iterated expectations:

$$p_z := \Pr(T = 1|Z = z) = m_{0z}(1 - p_z^*) + (1 - m_{1z})p_z^* = m_{0z} + s_z p_z^*, \tag{4}$$

where $m_{tz} := \Pr(T \neq T^*|T^* = t, Z = z) = \Pr(T = 1 - t|T^* = t, Z = z)$ is the conditional misclassification probability for $t, z = 0, 1$ and $s_z := 1 - m_{0z} - m_{1z}$ for $z = 0, 1$. We note that $|s_z| \leq 1$ by definition. The above equation implies that

$$p_z^* = \frac{p_z - m_{0z}}{s_z}. \tag{5}$$

The true treatment probability thus depends on the conditional misclassification probabilities and the observable treatment probability.

The misclassification of the treatment variable brings out the serious problem that the LATE and the first-stage regression Δp^* cannot be point-identified based on the observables of (Y, T, Z). Equation (4) leads to the following system related to true treatment probability p_z^*, observed treatment probability p_z, and misclassification probability m_{tz}:

$$\begin{cases} p_0 = m_{00} + s_0 p_0^* \\ p_1 = m_{01} + s_1 p_1^* \end{cases}.$$

Even when Z is not related to the misclassification probability, namely $m_t = m_{tz}$ for $t, z = 0, 1$ (this implies $s = s_0 = s_1$ for $s := 1 - m_0 - m_1$), we cannot identify p_0^* and p_1^* because there are four unknown parameters (m_0, s, p_0^*, p_1^*) in the system of two equations.

As a result, the true first-stage regression Δp^* cannot be identified based on (Y, T, Z), implying that the true IV estimand β^* and the LATE are unable to be identified.

Importantly, the IV estimand based on the observables may over- or underestimate the true IV estimand β^* and the LATE. The IV estimand based on the observables is

$$\beta := \frac{\mu_1 - \mu_0}{p_1 - p_0}. \tag{6}$$

From the relationship in (4), the difference between the denominators of β and β^* (the difference between the observable and true first-stage regressions) is expanded as follows:

$$p_1 - p_0 - (p_1^* - p_0^*) = (m_{00} - m_{01}) + (1 + s_1)p_1^* - (1 + s_0)p_0^*,$$

which can be negative or positive depending on the misclassification probabilities. For example, if we assume that the misclassification probabilities for the truly untreated given $Z = 0$ and $Z = 1$ are the same so that $m_{00} = m_{01}$, the sign of the difference between the first-stage regressions depends on that of $(m_{10} + m_{11})/(m_{00} + m_{01}) - p_0^*/p_1^*$. As a result, observable β may over- or underestimate true β^* and the LATE.

Remark 2. While β exhibits a bias for β^* and the LATE in general, it is sufficient to estimate β if one is interested in testing the hypothesis of whether $\beta^* = 0$. Since $\beta^* = 0$ if and only if $\beta = 0$, the standard IV inference based on observed (Y, T, Z) allows us to examine whether the true treatment has a causal effect on the outcome.

Remark 3. When the misclassification probabilities do not depend on Z, namely when $m_t = m_{tz}$ for $t, z = 0, 1$ so that $s = s_0 = s_1$, it holds that

$$\beta := \frac{\mu_1 - \mu_0}{p_1 - p_0} = \frac{\mu_1 - \mu_0}{m_{01} - m_{00} + s_1 p_1^* - s_0 p_0^*} = \frac{\beta^*}{s}. \tag{7}$$

Since $|s| \leq 1$, $|\beta|$ is an upper bound of $|\beta^*|$, but it is not the sharp bound (see Ura, 2016).

3 Identification analysis

This section develops an identification analysis for the LATE in the situation introduced in Section 2. Section 3.1 shows the identification for the LATE and the distribution of the measurement error based on exogenous variable V. Section 3.2 presents the moment conditions derived from the identification result.

3.1 Identification for the LATE with the measurement error

As well as Lewbel (2007), we need four assumptions for our identification of the LATE and the distribution of the measurement error based on the use of exogenous variable V.

Assumption 3.1 (non-differential error)**.** $E(Y|T^*, T, Z, V) = E(Y|T^*, Z, V)$.

Assumption 3.1 implies that mismeasured T has no information on the mean of Y once true T^*, Z, and V are conditioned on. With measurement error $U_T := T - T^*$, the assumption means that $E(Y|T^*, U_T, Z, V) = E(Y|T^*, Z, V)$, implying that U_T is the *non-differential* measurement error. The assumption of a non-differential error is popular in the misclassification literature (e.g., Mahajan, 2006; Lewbel, 2007; Battistin et al., 2014; DiTraglia and Garcia-Jimeno, 2016).

The non-differential error requires that the error does not depend on the observables and unobservables affecting outcome Y and/or true treatment T^*. However, we note that with observable control variables, we can allow dependence between the measurement error and the observed variables (see Remark 4). The non-differential error also rules out placebo effects such that misclassified treatments affect the outcomes of individuals who do not actually receive the treatment.

For example, consider the returns to schooling analysis. Under Assumption 3.1, the mean wage for individuals who do report high school diplomas and, indeed, did not graduate from high school is identical to that for individuals who did not graduate from high school with precise reports. This could be satisfied if the misclassification is caused by accident or by false statements depending observable control variables such as age and sex. On the contrary, the assumption might be violated if the misclassification depends on the causal effect of the diploma $Y_1 - Y_0$ and/or unobservables affecting wages.

Assumption 3.2 (monotonicity)**.** $1 - m_{0z} - m_{1z} > 0$ *for* $z = 0, 1$.

Assumption 3.2 means that the sum of the misclassification probabilities is less than one. The assumption is known as a monotonicity condition in the misclassification literature (e.g., Bollinger, 1996; Mahajan, 2006; Lewbel, 2007). This is satisfied when the misclassification is better than the genuine random report for the treatment status in which the misclassification probabilities m_{0z} and m_{1z} are equal to half. Under this assumption, T^* is positively correlated with T according to (3), implying that the observed treatment has positive information on the true unobserved treatment. It may be satisfied when the misclassification is caused by accidental recording error and/or false reporting.

To introduce the next assumption, we define the shorthand notations:

$$\begin{aligned} m_{tzv} &\coloneqq \Pr(T \neq T^* | T^* = t, Z = z, V = v), \\ p^*_{zv} &\coloneqq E(T^* | Z = z, V = v) = \Pr(T^* = 1 | Z = z, V = v), \\ \tau^*_{zv} &\coloneqq E(Y | T^* = 1, Z = z, V = v) - E(Y | T^* = 0, Z = z, V = v), \\ \tau^*_{z} &\coloneqq E(Y | T^* = 1, Z = z) - E(Y | T^* = 0, Z = z), \end{aligned}$$

for $t, z = 0, 1$ and $v \in supp(V) \subset \mathbb{R}$. Here, m_{tzv} is the conditional misclassification probability, p^*_{zv} is the conditional true treatment probability, and τ^*_{zv} and τ^*_{z} are the differences in the conditional outcome means.

Assumption 3.3 (exclusion restriction and rank condition)**.** *For each* $z = 0, 1$, *there exist a subset* $\Omega_z \subset supp(V)$ *such that*

$$\tau^*_z = \tau^*_{zv}, \qquad m_{0z} = m_{0zv}, \qquad m_{1z} = m_{1zv},$$

for any $v \in \Omega_z$ *and*

$$p^*_{zv} \neq p^*_{zv'},$$

for any $v, v' \in \Omega_z$ *such that* $v \neq v'$*.*

Assumption 3.3 requires a set of exclusion restrictions in the sense that exogenous variable V does not affect τ^*_{zv} and the distribution of the measurement error. The assumption of τ^*_{zv} rules out the effect of V on the difference in the conditional means of Y. A sufficient condition of the assumption is that the functional form of $E(Y|T^*, Z, V)$ is given by $E(Y|T^*, Z, V) = h_1(T^*, Z) + h_2(Z, V)$ for functions h_1 and h_2. The assumption could hold especially when the effect of V on Y_1 is the same as that on Y_0. Remarkably, Assumption 3.3 allows the direct effect of V on Y. For example, even in the linear model of $E(Y|T^*, Z, V) = \gamma_0 + \gamma_1 T^* + \gamma_2 Z + \gamma_3 V + \gamma_4 ZV$ with coefficients γs, the assumption is satisfied. The other special case in which the assumption holds is when V is randomly assigned. Indeed, if V is independent of $(Y_1, Y_0, T^*_1, T^*_0, Z)$, then it holds that

$$\begin{aligned}
\tau^*_{zv} &= E(Y|T^* = 1, Z = z, V = v) - E(Y|T^* = 0, Z = z, V = v) \\
&= E(Y_1|T^*_z = 1, Z = z, V = v) - E(Y_0|T^*_z = 0, Z = z, V = v) \\
&= E(Y_1|T^*_z = 1, Z = z) - E(Y_0|T^*_z = 0, Z = z) = \tau^*_z.
\end{aligned}$$

Under Assumption 3.3, the generating process of the misclassification also does not depend on V. However, it allows the misclassification probabilities to depend on Z.

Assumption 3.3 further includes a rank condition under which V affects true treatment probability p^*_{zv}. It holds especially when V has a direct effect on T^*. Importantly, the rank condition is testable under the exclusion restriction that $m_{tz} = m_{tzv}$ in Assumption 3.3. Indeed, from the definition of the observable conditional treatment probability,

$$p_{zv} := E(T|Z = z, V = v) = \Pr(T = 1|Z = z, V = v),$$

the same procedure for showing (4) leads to $p_{zv} = m_{0zv} + (1 - m_{0zv} - m_{1zv})p^*_{zv}$. It implies that $p^*_{zv} \neq p^*_{zv'}$ if and only if $p_{zv} \neq p_{zv'}$ under exclusion restriction $m_{tz} = m_{tzv}$.

To understand the practical implication of Assumption 3.3 in empirical applications, let us consider the returns to schooling analysis. Suppose that Z is the indicator of the compulsory schooling law and that V indicates the QOB for the individual. The exclusion restriction for τ^*_{zv} is satisfied if the mean effect of the high school graduation on wages for individuals born in a quarter are identical to that for individuals born in the other quarters. The condition for m_{tzv} holds when the QOB do not determine the generation of the measurement error. The rank condition for p^*_{zv} is satisfied when the QOB are related to true educational attainment.

For the next assumption, we define the following observable parameter:

$$\tau_{zv} := E(Y|T=1, Z=z, V=v) - E(Y|T=0, Z=z, V=v),$$

for $z = 0, 1$ and $v \in supp(V) \subset \mathbb{R}$. Here, τ_{zv} is the difference in the conditional outcome means, which is identified from the observable data.

The following assumption includes the conditions to solve the systems of linear equations for the identification of the misclassification probabilities. Condition (i) is for the case where V takes at least three values. On the contrary, condition (ii) allows the situation where V is binary. We remember set $\Omega_z \subset supp(V)$ in Assumption 3.3.

Assumption 3.4 (nonsingularity)**.** *One of the following assumptions holds for each $z = 0, 1$. (i) There are at least three elements $\{v_1, v_2, v_3\} \subset \Omega_z$ such that*

$$\left(\frac{\tau_{zv_1}}{p_{zv_2}} - \frac{\tau_{zv_2}}{p_{zv_1}}\right)\left(\frac{\tau_{zv_1}}{1-p_{zv_3}} - \frac{\tau_{zv_3}}{1-p_{zv_1}}\right) \neq \left(\frac{\tau_{zv_1}}{1-p_{zv_2}} - \frac{\tau_{zv_2}}{1-p_{zv_1}}\right)\left(\frac{\tau_{zv_1}}{p_{zv_3}} - \frac{\tau_{zv_3}}{p_{zv_1}}\right).$$

(ii) It holds that $m_t = m_{tz}$ for each $t = 0, 1$ and $z = 0, 1$ and there are at least two elements $\{v_1, v_2\} \subset \Omega_z$ such that

$$\left(\frac{\tau_{0v_1}}{p_{0v_2}} - \frac{\tau_{0v_2}}{p_{0v_1}}\right)\left(\frac{\tau_{1v_1}}{1-p_{1v_2}} - \frac{\tau_{1v_2}}{1-p_{1v_1}}\right) \neq \left(\frac{\tau_{0v_1}}{1-p_{0v_2}} - \frac{\tau_{0v_2}}{1-p_{0v_1}}\right)\left(\frac{\tau_{1v_1}}{p_{1v_2}} - \frac{\tau_{1v_2}}{p_{1v_1}}\right).$$

Assumption 3.4 is a set of somewhat technical conditions that guarantees the unique solutions for the systems of linear equations. Conditions (i) and (ii) both require some inequality, while condition (ii) requires the exclusion restriction that the distribution of the measurement error does not depend on Z. The same exclusion restriction is assumed in DiTraglia and Garcia-Jimeno (2016) and Ura (2016). The inequalities are testable in principle since their components are identified by the data. The necessary and sufficient conditions of the inequalities are $\tau_z^* \neq 0$ and $m_{0z} + m_{1z} \neq 1$ (see Appendix B). In the returns to schooling example, the inequalities hold if and only if average wages for high school graduates are different from those for individuals without high school diplomas.

The following theorem states the main identification result of this study.

Theorem 1. *Suppose that Assumptions 3.1, 3.2, 3.3, and 3.4 are satisfied. The misclassification probabilities m_{0z} and m_{1z}, true treatment probability p_z^* for each $z = 0, 1$, and IV estimand β^* are identified. As a result, the LATE in (1) is identified when, in addition, Assumptions 2.1, 2.2, and 2.3 are satisfied.*

The main idea behind the identification is, similar to Lewbel (2007), to construct moment conditions whose number is no fewer than the unknown parameters. The idea can be understood by a simple sketch of the proof of Theorem 1. This proof depends on whether we assume Assumption 3.4 (i) or (ii). Under Assumptions 3.1, 3.2, 3.3, and 3.4 (i), we can show the following system of equations for each $z = 0, 1$:

$$\begin{cases} B_{0z} w_{0zv_1v_2} + B_{1z} w_{1zv_1v_2} + w_{2zv_1v_2} = 0 \\ B_{0z} w_{0zv_1v_3} + B_{1z} w_{1zv_1v_3} + w_{2zv_1v_3} = 0 \end{cases}, \tag{8}$$

where the Bs and ws are parameters related to the unobservables and observables, respectively. The Bs and ws are defined by equation (19) in the proof of Theorem 1. We note that the ws are identified from the observable data. Assumption 3.4 (i) guarantees the existence of the unique solutions of Bs, implying that the Bs are identified. Similarly, under the assumption of $m_{tz} = m_t$ for $t, z = 0, 1$ in Assumption 3.4 (ii), we instead have

$$\begin{cases} B_0 w_{00v_1v_2} + B_1 w_{10v_1v_2} + w_{20v_1v_2} = 0 \\ B_0 w_{01v_1v_2} + B_1 w_{11v_1v_2} + w_{21v_1v_2} = 0 \end{cases}, \tag{9}$$

where the Bs and ws are the unobservable and observable parameters, respectively. Note that the Bs in (9) do not depend on z unlike the Bs in (8). In this case, Assumption

3.4 (ii) guarantees that the Bs are identified as the solution of simultaneous equations (9). As a next step, the identification of the Bs leads to identifying the misclassification probabilities. Specifically, under Assumption 3.4 (i), we have

$$s_z = \sqrt{(B_{0z} - B_{1z} + 1)^2 - 4B_{0z}}, \qquad m_{0z} = (B_{0z} - B_{1z} + 1 - s_z)/2, \qquad m_{1z} = 1 - m_{0z} - s_z$$

for each $z = 0, 1$. On the contrary, under Assumption 3.4 (ii), we have

$$s = \sqrt{(B_0 - B_1 + 1)^2 - 4B_0}, \qquad m_0 = (B_0 - B_1 + 1 - s)/2, \qquad m_1 = 1 - m_0 - s.$$

These mean that the distribution of the measurement error is identified. Finally, the true first-stage regression is identified based on the information on the misclassification probability and the observable treatment probability. Under Assumption 3.4 (i), we have

$$\Delta p^* = \frac{p_1 - m_{01}}{1 - m_{01} - m_{11}} - \frac{p_0 - m_{00}}{1 - m_{00} - m_{10}},$$

and, under Assumption 3.4 (ii), we have

$$\Delta p^* = \frac{p_1 - p_0}{1 - m_0 - m_1}.$$

As a result, IV estimand $\beta^* := \Delta\mu/\Delta p^*$ and the LATE are identified by equation (2).

Remark 4. The proposed analysis can be extended to situations in which other control variables are observed. Suppose that we observe a vector of control variables $S \in \mathbb{R}^d$ in addition to (Y, T, Z, V). Let the parameter of interest be the LATE conditional on the control variables: $E(Y_1 - Y_0|C, S = s) = E(Y_1 - Y_0|T_1^* > T_0^*, S = s)$ where $s \in supp(S)$. If Assumptions 2.1, 2.2, and 2.3 are satisfied conditional on S, it holds that

$$E(Y_1 - Y_0|T_1^* > T_0^*, S = s) = \frac{E(Y|Z = 1, S = s) - E(Y|Z = 0, S = s)}{E(T^*|Z = 1, S = s) - E(T^*|Z = 0, S = s)}.$$

The right-hand side is the IV estimand conditional on $S = s$, which is identified if (Y, T^*, Z, S) is observed without a measurement error. Also, if Assumptions 3.1, 3.2, 3.3, and 3.4 hold conditional on $S = s$, the identification result in Theorem 1 holds conditional on $S = s$. Importantly, owing to the presence of control variables S, we can allow the generation of the measurement error to depend on the observables. Specifically, Assumption 3.1 may be replaced with the non-differential error conditional on S: $E(Y|T^*, T, Z, V, S) = E(Y|T^*, Z, V, S)$. It implies that the measurement error for the treatment does not affect the mean of Y once T^* and S are conditioned on.

Remark 5. When V is binary as well as Z, one may consider the analysis in which the role of V is replaced with that of Z. Suppose that $\tilde{T}_0^*$ and $\tilde{T}_1^*$ are the potential true treatment statuses when $V = 0$ and $V = 1$, respectively. Then, the LATE based on V is

$$E(Y_1 - Y_0|\tilde{T}_1^* > \tilde{T}_0^*), \tag{10}$$

which is the ATE for compliers whose treatment statuses are positively altered with the value of V. The LATE in (10) may be different from the LATE in (1) since the definition of compliers depends on which of V and Z is the instrument. Under Assumptions 2.1, 2.2, and 2.3 with V in place of Z, the LATE in (10) satisfies the following equation:

$$E(Y_1 - Y_0|\tilde{T}_1 > \tilde{T}_0) = \frac{E(Y|V=1) - E(Y|V=0)}{E(T^*|V=1) - E(T^*|V=0)},$$

as well as Lemma 1. Further, if Assumptions 3.1, 3.2, 3.3, and 3.4 are satisfied with Z and V in place of V and Z, respectively, the LATE and the distribution of the measurement error are identified by adopting the same procedure as in Theorem 1. In practice, the LATEs in both (1) and (10) may be identified based on the information on (Y, T, Z, V) from Theorem 1 and the above discussion. In such a case, which of the LATEs in (1) and (10) should be the main parameter of interest would depend on the objective of the empirical application. Since the definition of the LATE depends on the members of the complier group, the main parameter of interest should be determined by assessing which of the compliers based on V and Z is of more interest.

3.2 Moment conditions

We derive the moment conditions based on the identification result in Theorem 1. The moment conditions lead to estimation procedures such as the nonlinear GMM estimation and the empirical likelihood estimation, which allow us to estimate the parameters. Here, we focus on the moment conditions under Assumption 3.4 (i) since that under Assumption 3.4 (ii) is simpler. We assume that V is a discrete random variable since the instruments or covariates in the LATE inference are often discrete. We can easily derive similar moment conditions when V is continuously distributed and/or when other control variables exist.

Define the vector of observable variables $X := (Y, T, Z, V)'$. For notational simplicity, suppose that discrete V takes K values in $supp(V) = \Omega_0 = \Omega_1 = \{v_1, v_2, \ldots, v_K\}$, where Ω_z is introduced in Assumptions 3.3 and 3.4. Define a vector of the $K + 3$ parameters:

$$\theta^{(z)} := (m_{0z}, m_{1z}, p^*_{zv_1}, p^*_{zv_2}, \ldots, p^*_{zv_K}, \tau^*_z)'.$$

Vector $\theta^{(z)}$ for $z = 0, 1$ contains the parameters conditional on $Z = z$ introduced in Section 3.1. The vector of the $2K + 9$ parameters to be estimated is

$$\theta := (\beta^*, \Delta p^*, r, \theta^{(0)'}, \theta^{(1)'})', \tag{11}$$

where $\Delta p^* := p^*_1 - p^*_0 = E(T^*|Z = 1) - E(T^*|Z = 0)$ is the true first-stage regression and $r := E(Z) = \Pr(Z = 1)$. The parameters except for IV estimand β^* and true first-stage regression Δp^* are nuisance parameters to overcome the misclassification problem. Nonetheless, as well as β^* and Δp^*, misclassification probability m_{tz} for $t = 0, 1$ and $z = 0, 1$ could be of interest in empirical applications. Suppose that θ_0 is the true value of θ in the parameter space $\Theta \subset \mathbb{R}^{2K+9}$.

Let $g(X, \theta)$ be the vector valued function with $4K+3$ elements, which are the following components: for $k = 1, 2, \ldots, K$ and $z = 0, 1$,

$$
\begin{aligned}
&r - Z, \\
&\Big(m_{0z} + (1 - m_{0z} - m_{1z})p^*_{zv_k} - T\Big) I_{zv_k}, \\
&\left(\tau^*_z + \frac{YT - (1 - m_{1z})p^*_{zv_k}\tau^*_z}{m_{0z} + (1 - m_{0z} - m_{1z})p^*_{zv_k}} - \frac{Y(1 - T) + (1 - m_{0z})(1 - p^*_{zv_k})\tau^*_z}{1 - (m_{0z} + (1 - m_{0z} - m_{1z})p^*_{zv_k})}\right) I_{zv_k}, \\
&\Delta p^* - \left(\frac{TZr^{-1} - m_{01}}{1 - m_{01} - m_{11}} - \frac{T(1 - Z)(1 - r)^{-1} - m_{00}}{1 - m_{00} - m_{10}}\right), \\
&\beta^* - \frac{YZr^{-1} - Y(1 - Z)(1 - r)^{-1}}{\Delta p^*},
\end{aligned}
\tag{12}
$$

where $I_{zv_k} \coloneqq \mathbf{1}(Z = z, V = v_k)$ is the shorthand notation for the indicator.

The following theorem shows that the moment condition is $E[g(X, \theta_0)] = 0$ with unique solution $\theta_0 \in \Theta$. This is directly shown by the identification result in Theorem 1.

Theorem 2. *Suppose that Assumptions 3.1, 3.2, 3.3, and 3.4 (i) hold with* $\Omega_0 = \Omega_1 = \{v_1, v_2, \ldots, v_K\}$. *Then, it holds that* $E[g(X, \theta_0)] = 0$ *and* θ_0 *is the unique solution of the moment condition in the sense that* $E[g(X, \theta)] \neq 0$ *for any* $\theta \in \Theta$ *such that* $\theta \neq \theta_0$.

The number of overidentification restrictions depends on the number of the elements in support of V, K. As stated above, the numbers of parameters and moment equations are $2K + 9$ and $4K + 3$, respectively, according to Assumption 3.4 (i). Hence, there are $2K - 6$ overidentification restrictions and θ_0 is just-identified when $K = 3$. Similarly, we can show that there are $2K - 4$ overidentification restrictions under Assumption 3.4 (ii), and the just-identification is achieved when $K = 2$.

4 GMM estimation

This section briefly discusses the GMM estimation based on $E[g(X, \theta_0)] = 0$ in Theorem 2. We develop the nonparametric estimation here, although we may also consider the semiparametric estimation by specifying the functional forms of the functions in θ.

We have a random sample $\mathbf{X} \coloneqq \{X_i\}_{i=1}^n = \{(Y_i, T_i, Z_i, V_i)\}_{i=1}^n$ of $X \coloneqq (Y, T, Z, V)$. Let $\hat{\Lambda}$ be a $(4K + 6) \times (4K + 6)$ positive semi-definite weighting matrix. The GMM estimator for θ is defined as

$$\hat{\theta} := \arg\min_{\theta\in\Theta} \left(\frac{1}{n}\sum_{i=1}^{n} g(X_i,\theta)\right)' \hat{\Lambda} \left(\frac{1}{n}\sum_{i=1}^{n} g(X_i,\theta)\right) = \arg\min_{\theta\in\Theta} \hat{g}_n(\mathbf{X},\theta)'\hat{\Lambda}\hat{g}_n(\mathbf{X},\theta),$$

where $\hat{g}_n(\mathbf{X},\theta) := n^{-1}\sum_{i=1}^{n} g(X_i,\theta)$ is a shorthand notation.

The $\sqrt{n}$-consistency and the asymptotic normality of $\hat{\theta}$ are followed by the standard arguments for the asymptotics on the M-estimator. Specifically, with $\hat{\Lambda} \xrightarrow{p} \Lambda$ for a positive definite matrix Λ, $G := E[\nabla_\theta g(X,\theta_0)]$, and $\Gamma := E[g(X,\theta)g(X,\theta)']$, Theorem 3.2 in Newey and McFadden (1994) means that

$$\sqrt{n}(\hat{\theta}-\theta_0) \xrightarrow{d} N\big(0,(G'\Lambda G)^{-1}G'\Lambda\Gamma\Lambda G(G'\Lambda G)^{-1}\big). \tag{13}$$

We can obtain the optimal GMM estimator based on the weighting matrix satisfying $\hat{\Lambda} \xrightarrow{p} \Gamma^{-1}$ (see Theorem 5.2 in Newey and McFadden, 1994) with asymptotic variance $(G'\Gamma^{-1}G)^{-1}$ that is the lower bound of the asymptotic variances of the GMM estimators based on $E[g(X,\theta_0)] = 0$. In practice, we first estimate Γ by $\hat{\Gamma} := n^{-1}\sum_{i=1}^{n} g(X_i,\tilde{\theta})g(X_i,\tilde{\theta})'$ with pilot GMM estimator $\tilde{\theta}$. Then, the optimal GMM estimator, say $\hat{\theta}_{opt}$, is based on the weighting matrix $\hat{\Lambda}_{opt} := \hat{\Gamma}^{-1}$. The asymptotic variance may be estimated by $(\hat{G}'\hat{\Gamma}\hat{G})^{-1}$ where $\hat{G} := n^{-1}\sum_{i=1}^{n} \nabla_\theta g(X_i,\hat{\theta})$ and $\hat{\Gamma} := n^{-1}\sum_{i=1}^{n} g(X_i,\hat{\theta})g(X_i,\hat{\theta})'$.

The confidence interval estimation and hypothesis testing for θ_0 may be developed based on the asymptotic normality or bootstrap procedures such as the nonparametric bootstrap (Hall and Horowitz, 1996) or the k-step bootstrap (Andrews, 2002). For example, the $1-\alpha$ confidence interval for the LATE may be given by $[\hat{\beta}^* - z_{\alpha/2}\cdot se(\hat{\beta}^*), \hat{\beta}^* + z_{\alpha/2}\cdot se(\hat{\beta}^*)]$ where $\hat{\beta}^*$ is the GMM estimator for the LATE, $se(\hat{\beta}^*)$ is the asymptotic standard error, and $z_{\alpha/2}$ is the $1-\alpha/2$ quantile of the standard normal distribution.

Importantly, the overidentification restrictions allow us to examine the validity of Assumptions 3.1, 3.2, 3.3, and 3.4 based on the overidentification test. Under H_0 : $E[g(X,\theta_0)] = 0$ with $2K-6$ overidentification restrictions, $n\hat{g}_n(\mathbf{X},\hat{\theta}_{opt})'\hat{\Lambda}_{opt}\hat{g}_n(\mathbf{X},\hat{\theta}_{opt}) \xrightarrow{d} \chi^2_{2K-6}$. If the value of the test statistic exceeds the $1-\alpha$ quantile of χ^2_{2K-6}, we reject $H_0 : E[g(X,\theta_0)] = 0$ with the significance of α. Since $E[g(X,\theta_0)] \neq 0$ implies the violation of identification conditions, we can examine their validity based on this test.

5 Monte Carlo simulations

This section reports the results of the Monte Carlo simulations. The aim of the simulations is to demonstrate the contamination of the LATE inference due to the measurement error and examine the finite sample performance of the proposed GMM estimation. The simulations are conducted with `R 3.3.2`. The number of simulation replications is 5,000.

DGP By using a sample size of 1,000, we generate the random variables by adopting the following data-generating processes. Instrument Z is generated with probabilities $\Pr(Z=0)=\Pr(Z=1)=0.5$. Assuming Assumption 3.4 (ii), we generate binary V independently of Z with probabilities $\Pr(V=0)=\Pr(V=1)=0.5$. We generate the unobservables affecting the outcome and the true treatment.

$$\begin{pmatrix} U_1 \\ U_2 \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & 0.1 \\ 0.1 & 1 \end{pmatrix}\right),$$

where (U_1,U_2) is independent of Z and V. The true treatment is generated by the probit model $T^*=\mathbf{1}(-0.5+Z-0.5V+ZV-U_1>0)$. Observed treatment T is misclassified independently of the other variables with misclassification probability $m_t \coloneqq \Pr(T\neq T^*|T^*=t)=0.2$ for each $t=0,1$. Two designs are considered for the outcome variable:

$$\text{Design 1:}\quad Y=1+2T^*+U_2, \qquad \text{Design 2:}\quad Y=U_2^3+T^*U_2^2(1-U_2).$$

Design 1 considers the homogeneous causal effect, whereas the causal effect in design 2 is heterogeneous depending on unobserved U_2.

Estimator We consider two estimators. The first is the GMM estimator based on the identification result developed in this study. The second is the naive IV estimator β in (6) based on observables (Y,T,Z), which is a benchmark estimator.

In this simulation, the GMM estimation involves 11 parameters to be estimated: $\theta = (\beta^*, \Delta p^*, r, m_0, p^*_{00}, p^*_{01}, \tau^*_0, m_1, p^*_{10}, p^*_{11}, \tau^*_1)'$. The parameters of interest are the LATE identical to IV estimand β^*, first-stage regression Δp^*, and misclassification probabilities m_0 and m_1. The moment condition $E[g(X, \theta_0)] = 0$ is composed of 11 elements. Since θ_0 is just-identified here, we select the identity matrix as the weighting matrix.

Result Tables 1 and 2 summarize the Monte Carlo simulation results with designs 1 and 2, respectively. The simulation results for the proposed GMM estimation and the naive IV estimation are reported in the columns labeled "GMM" and "IV," respectively. The rows in each table report the true value, bias, standard deviation (std), root mean squared error (RMSE), and 25%, 50%, and 75% quantiles of the 5,000 simulation estimates (LQ, MED, and UQ) for each parameter of β^*, Δp^*, m_0, and m_1.

Naive IV estimator β based on the observables exhibits large biases in both designs 1 and 2. In each design, the bias is over 60% of the true value of the LATE, and the naive IV estimator overestimates the LATE. The result is driven by the identification failure of the naive IV estimand for the LATE as we discussed in Section 2.3. The magnitude of the bias is also consistent with our theoretical investigation in (7). These simulation results demonstrate that the measurement error for the treatment variable significantly contaminates the LATE inferences. Hence, it is important to develop inference procedures for the LATE that incorporate the presence of the measurement error.

The performance of the proposed GMM estimation based on the identification result in Theorems 1 and 2 is successful. The bias of the GMM estimator for each parameter is satisfactory in each design. For example, the biases of the GMM estimator for the LATE in designs 1 and 2 are about 9% and 4% of the true value, respectively. The standard deviation and the root mean squared error of the GMM estimator are also moderate in each design for each parameter. With the quantiles of the estimates, we observe that the distribution of the estimates is centered close to the true value, which can be expected owing to the asymptotic normality of the GMM estimator. In short, our proposed GMM estimation can successfully infer the LATE and the true first-stage regression even when the treatment contains a measurement error.

Table 1: Monte Carlo simulation results with design 1

	GMM				IV
parameter of interest	β^*	Δp^*	m_0	m_1	β^*
true	2.000	0.532	0.200	0.200	2.000
bias	0.186	-0.026	-0.014	-0.039	1.356
std	0.454	0.100	0.077	0.084	0.346
RMSE	0.491	0.104	0.078	0.093	1.399
LQ	1.841	0.430	0.143	0.099	3.117
MED	2.145	0.497	0.196	0.173	3.334
UQ	2.479	0.574	0.241	0.226	3.572

Table 2: Monte Carlo simulation results with design 2

	GMM				IV
parameter of interest	β^*	Δp^*	m_0	m_1	β^*
true	0.996	0.532	0.200	0.200	0.996
bias	0.044	0.001	0.036	-0.071	0.661
std	0.436	0.146	0.113	0.122	0.599
RMSE	0.439	0.146	0.119	0.141	0.892
LQ	0.733	0.431	0.181	0.010	1.240
MED	0.996	0.494	0.270	0.096	1.661
UQ	1.301	0.620	0.321	0.246	2.035

6 Conclusion

This study presents novel point-identification and estimation methods for the LATE when the binary endogenous treatment may contain a measurement error. Since the measurement error must be non-classical by construction, the standard IV estimator cannot consistently estimate the LATE. To correct the bias due to the measurement error, we build on Lewbel's (2007) result to point-identify the LATE, true first-stage regression, and distribution of the measurement error based on the use of an exogenous variable such as a covariate or instrument. The moment conditions derived from the identification result lead to the GMM estimator for the parameters with asymptotically valid inferences. The Monte Carlo simulations demonstrate the successful performance of the proposed GMM estimation in finite samples.

Several important future research topics can be proposed on the basis of the findings presented herein. First, it is desirable to develop point-identification for the LATE with the *differential* measurement error for the treatment variable. The measurement error is differential when it is related with the outcome variable even conditional on the true variable. How to handle such a measurement error would be of interest but challenging. Second, it would be of interest to develop inference procedures for the LATE with a measurement error when the treatment is a general discrete variable. Without a measurement error for the discrete treatment, the IV estimand identifies the LATE with the variable treatment intensity (Angrist and Imbens, 1995). Our inference procedure proposed may be extended to such situations.

A Appendix: Proofs of the theorems

This appendix contains the proofs of Theorems 1 and 2. In the following, we write $W := (Z, V)'$ for notational convenience.

A.1 Proof of Theorem 1

The outline of the proof is an extension of the proof for Theorems 1 and 2 in Lewbel (2007). First, we examine the relationship between identified parameter $\tau_W := E(Y|T = 1, W) - E(Y|T = 0, W)$ and true version $\tau_W^* := E(Y|T^* = 1, W) - E(Y|T^* = 0, W)$ under Assumption 3.1. Second, we clarify that the relationship between τ_W and τ_W^* involves misclassification probabilities m_{0z}, m_{1z}, and $s_z := 1 - m_{0z} - m_{1z}$ under Assumptions 3.1, 3.2, and 3.3. Third, we show that m_{0z}, m_{1z}, and s_z are identified under Assumption 3.4 (i) or (ii). Finally, we argue that β^* is identified based on identified parameters m_{0z}, m_{1z}, and s_z, which implies the identification of the LATE thanks to Lemma 1.

Step 1: Assumption 3.1 means that

$$\begin{aligned}
E(Y|T^*, T, W) &= E(Y|T^*, W) \\
&= T^* E(Y|T^* = 1, W) + (1 - T^*) E(Y|T^* = 0, W) \\
&= E(Y|T^* = 0, W) + T^* \cdot \tau_W^*.
\end{aligned}$$

The law of iterated expectations leads to $E(Y|T, W) = E(Y|T^* = 0, W) + E(T^*|T, W)\tau_W^*$ so that we have

$$\begin{aligned}
\tau_W &:= E(Y|T = 1, W) - E(Y|T = 0, W) \\
&= [E(T^*|T = 1, W) - E(T^*|T = 0, W)]\tau_W^*.
\end{aligned} \tag{14}$$

Step 2: We next examine $E(T^*|T = t, W) = \Pr(T^* = 1|T = t, W)$ for $t = 0, 1$:

$$\Pr(T^* = 1|T = t, W) = \frac{\Pr(T = t|T^* = 1, W)\Pr(T^* = 1|W)}{\Pr(T = t|W)}.$$

Thus, we can write

$$E(T^*|T = 0, W) = \frac{m_{1W} \cdot p_W^*}{1 - p_W}, \qquad E(T^*|T = 1, W) = \frac{(1 - m_{1W})p_W^*}{p_W}. \tag{15}$$

Further, we have

$$p_W := \Pr(T = 1|W) = m_{0W}(1 - p_W^*) + (1 - m_{1W})p_W^* \\ = m_{0W} + (1 - m_{0W} - m_{1W})p_W^*. \tag{16}$$

This leads to

$$p_W^* = \frac{p_W - m_{0W}}{1 - m_{0W} - m_{1W}}, \tag{17}$$

under the assumption that $1 - m_{0W} - m_{1W} \neq 0$ (Assumption 3.2).

By substituting (15) and (17) into (14) and rearranging the equation, we get

$$\tau_W = M(m_{0W}, m_{1W}, p_W)\tau_W^*, \tag{18}$$

where we define

$$M(m_{0W}, m_{1W}, p_W) := \frac{1}{1 - m_{0W} - m_{1W}} \left(1 - \frac{m_{0W}(1 - m_{1W})}{p_W} - \frac{(1 - m_{0W})m_{1W}}{1 - p_W}\right).$$

Step 3: Under Assumption 3.3, (18) implies that for each $z \in \{0, 1\}$ and every $v, v' \in \Omega_z$

$$\tau_{zv} = M(m_{0z}, m_{1z}, p_{zv})\tau_z^*, \qquad \tau_{zv'} = M(m_{0z}, m_{1z}, p_{zv'})\tau_z^*.$$

This implies that $\tau_{zv}M(m_{0z}, m_{1z}, p_{zv'}) - \tau_{zv'}M(m_{0z}, m_{1z}, p_{zv}) = 0$ under Assumption 3.2. This equation can be rearranged as follows:

$$\begin{aligned}
&\tau_{zv}\left(1 - \frac{m_{0z}(1 - m_{1z})}{p_{zv'}} - \frac{(1 - m_{0z})m_{1z}}{1 - p_{zv'}}\right) - \tau_{zv'}\left(1 - \frac{m_{0z}(1 - m_{1z})}{p_{zv}} - \frac{(1 - m_{0z})m_{1z}}{1 - p_{zv}}\right) = 0 \\
&\iff m_{0z}(1 - m_{1z})\left(\frac{\tau_{zv}}{p_{zv'}} - \frac{\tau_{zv'}}{p_{zv}}\right) + (1 - m_{0z})m_{1z}\left(\frac{\tau_{zv}}{1 - p_{zv'}} - \frac{\tau_{zv'}}{1 - p_{zv}}\right) + (\tau_{zv'} - \tau_{zv}) = 0 \\
&\iff B_{0z}w_{0zvv'} + B_{1z}w_{1zvv'} + w_{2zvv'} = 0,
\end{aligned}$$

where we define

$$
\begin{aligned}
&B_{0z} := m_{0z}(1-m_{1z}), \qquad B_{1z} := (1-m_{0z})m_{1z}, \\
&w_{0zvv'} := \frac{\tau_{zv}}{p_{zv'}} - \frac{\tau_{zv'}}{p_{zv}}, \qquad w_{1zvv'} := \frac{\tau_{zv}}{1-p_{zv'}} - \frac{\tau_{zv'}}{1-p_{zv}}, \qquad w_{2zvv'} := \tau_{zv'} - \tau_{zv}.
\end{aligned} \tag{19}
$$

Note that ws are identified by the observables of (Y, T, Z, V).

The remaining proof depends on whether we assume Assumption 3.4 (i) or (ii).

Step 4 under Assumption 3.4 (i): We show the identification of B_{0z} and B_{1z} under Assumption 3.4 (i). Then Ω_z contains at least three elements (v_1, v_2, v_3) and we have the system of two linear equations:

$$
\begin{cases}
B_{0z}w_{0zv_1v_2} + B_{1z}w_{1zv_1v_2} + w_{2zv_1v_2} = 0 \\
B_{0z}w_{0zv_1v_3} + B_{1z}w_{1zv_1v_3} + w_{2zv_1v_3} = 0
\end{cases}
$$

The system can be uniquely solved for unknown parameter (B_{0z}, B_{1z}) as long as matrix $((w_{0zv_1v_2}, w_{0zv_1v_3})', (w_{1zv_1v_2}, w_{1zv_1v_3})')$ is nonsingular. The necessary and sufficient condition of the nonsingularity is the nonzero determinant of the matrix, i.e.,

$$
\left(\frac{\tau_{zv_1}}{p_{zv_2}} - \frac{\tau_{zv_2}}{p_{zv_1}}\right)\left(\frac{\tau_{zv_1}}{1-p_{zv_3}} - \frac{\tau_{zv_3}}{1-p_{zv_1}}\right) \neq \left(\frac{\tau_{zv_1}}{1-p_{zv_2}} - \frac{\tau_{zv_2}}{1-p_{zv_1}}\right)\left(\frac{\tau_{zv_1}}{p_{zv_3}} - \frac{\tau_{zv_3}}{p_{zv_1}}\right).
$$

Hence, (B_{0z}, B_{1z}) is identified under Assumption 3.4 (i).

We next show the identification of $s_z := 1 - m_{0z} - m_{1z}$. The equation $B_{tz} = m_{tz}(1 - m_{1-t,z})$ in (19) for $t = 0, 1$ implies that $(s_z + m_{0z})m_{0z} = B_{0z}$ and $2m_{0z} = B_{0z} - B_{1z} + 1 - s_z$. Substituting the second into the first provides

$$
\begin{aligned}
&\left(s_z + \frac{B_{0z} - B_{1z} + 1 - s_z}{2}\right)\frac{B_{0z} - B_{1z} + 1 - s_z}{2} = B_{0z} \\
\Longleftrightarrow\ & s_z = \sqrt{(B_{0z} - B_{1z} + 1)^2 - 4B_{0z}},
\end{aligned}
$$

under Assumptions 3.2 and 3.3. Hence, s_z is identified by the identification of the Bs. Since s_z, B_{0z}, and B_{1z} are identified, m_{0z} and m_{1z} are also identified by $m_{0z} = (B_{0z} - B_{1z} + 1 - s_z)/2$ and $s_z = 1 - m_{0z} - m_{1z}$.

Finally, we argue that IV estimand β^* is identified based on the above steps. Since it holds that $p_z = m_{0z} + (1 - m_{0z} - m_{1z})p_z^*$, the true treatment probability and the true first-stage regression are identified based on

$$p_z^* = \frac{p_z - m_{0z}}{1 - m_{0z} - m_{1z}}, \qquad p_1^* - p_0^* = \frac{p_1 - m_{01}}{1 - m_{01} - m_{11}} - \frac{p_0 - m_{00}}{1 - m_{00} - m_{10}}, \tag{20}$$

by identified parameters p_z, m_{0z}, and m_{1z}. Hence, we have shown that

$$\beta^* := \frac{\mu_1 - \mu_0}{p_1^* - p_0^*}, \tag{21}$$

is identified since the numerator is identified by the data.

Step 4 under Assumption 3.4 (ii): We show the identification of $B_0 := m_0(1 - m_1)$ and $B_1 := m_1(1 - m_0)$ under Assumption 3.4 (ii). With the exclusion restriction of $m_{tz} = m_t$ in Assumption 3.4 (ii), we have the system of two linear equations:

$$\begin{cases} B_0 w_{00v_1v_2} + B_1 w_{10v_1v_2} + w_{20v_1v_2} = 0 \\ B_0 w_{01v_1v_2} + B_1 w_{11v_1v_2} + w_{21v_1v_2} = 0 \end{cases}.$$

The system can be uniquely solved for unknown parameter (B_0, B_1) as long as matrix $((w_{00v_1v_2}, w_{01v_1v_2})', (w_{10v_1v_2}, w_{11v_1v_2})')$ is nonsingular. The necessary and sufficient condition of the nonsingularity is the nonzero determinant of the matrix, i.e.,

$$\left(\frac{\tau_{0v_1}}{p_{0v_2}} - \frac{\tau_{0v_2}}{p_{0v_1}}\right)\left(\frac{\tau_{1v_1}}{1 - p_{1v_2}} - \frac{\tau_{1v_2}}{1 - p_{1v_1}}\right) \neq \left(\frac{\tau_{0v_1}}{1 - p_{0v_2}} - \frac{\tau_{0v_2}}{1 - p_{0v_1}}\right)\left(\frac{\tau_{1v_1}}{p_{1v_2}} - \frac{\tau_{1v_2}}{p_{1v_1}}\right).$$

Hence, (B_0, B_1) is identified under Assumption 3.4 (ii).

We also show the identification of β^*. $B_t = m_t(1 - m_{1-t})$ for $t = 0, 1$ implies that $(s + m_0)m_0 = B_0$ and $2m_0 = B_0 - B_1 + 1 - s$. Substituting the second into the first provides

$$\left(s + \frac{B_0 - B_1 + 1 - s}{2}\right)\frac{B_0 - B_1 + 1 - s}{2} = B_0 \quad \Longleftrightarrow \quad s = \sqrt{(B_0 - B_1 + 1)^2 - 4B_0},$$

under Assumptions 3.2 and 3.3, meaning that s is identified. Since s, B_0, and B_1 are identified, m_0 and m_1 are also identified by $m_0 = (B_0 - B_1 + 1 - s)/2$ and $s = 1 - m_0 - m_1$. Hence, $p_z^* \coloneqq E(T^*|Z = z)$ for each $z = 0, 1$ is identified based on identified parameters p_z, m_0, and m_1, which implies the identification of β^*.

□

A.2 Proof of Theorem 2

To show the statement, it is sufficient to show that moment condition $E[g(X, \theta_0)] = 0$ with $g(X, \theta)$ defined in (12) is implied by the identification result in Theorem 1.

The first moment condition in $E[g(X, \theta_0)] = 0$ is $r = E(Z)$ by construction. The second moment condition in $E[g(X, \theta_0)] = 0$ is

$$
\begin{aligned}
& E\left[\left(m_{0z} + (1 - m_{0z} - m_{1z})p_{zv_k}^* - T\right)I_{zv_k}\right] = 0 \\
\iff & m_{0z} + (1 - m_{0z} - m_{1z})p_{zv_k}^* - p_{zv_k} = 0.
\end{aligned}
$$

This equation is equivalent to (16).

We examine the third moment restriction. By substituting the second moment condition into the third moment condition, we have

$$
\begin{aligned}
& E\left(\left(\tau_z^* + \frac{YT - (1 - m_{1z})p_{zv_k}^*\tau_z^*}{m_{0z} + (1 - m_{0z} - m_{1z})p_{zv_k}^*} - \frac{Y(1 - T) + (1 - m_{0z})(1 - p_{zv_k}^*)\tau_z^*}{1 - [m_{0z} + (1 - m_{0z} - m_{1z})p_{zv_k}^*]}\right) I_{zv_k}\right) = 0 \\
\iff & E\left(\left(\tau_z^* + \frac{YT}{p_{zv_k}} - \frac{(1 - m_{1z})p_{zv_k}^*\tau_z^*}{p_{zv_k}} - \frac{Y(1 - T)}{1 - p_{zv_k}} - \frac{(1 - m_{0z})(1 - p_{zv_k}^*)\tau_z^*}{1 - p_{zv_k}}\right) I_{zv_k}\right) = 0 \\
\iff & E\left(\left(\tau_z^* + \frac{YT}{p_{zv_k}} - \frac{(1 - m_{1z})\tau_z^*}{p_{zv_k}}\frac{p_{zv_k} - m_{0z}}{1 - m_{0z} - m_{1z}}\right.\right. \\
& \qquad \left.\left. - \frac{Y(1 - T)}{1 - p_{zv_k}} - \frac{(1 - m_{0z})\tau_z^*}{1 - p_{zv_k}}\frac{1 - m_{1z} - p_{zv_k}}{1 - m_{0z} - m_{1z}}\right) I_{zv_k}\right) = 0,
\end{aligned}
$$

where the last follows from (17). Since $p_{zv_k} = E(TI_{zv_k})/E(I_{zv_k})$, we have

$$\begin{aligned}
&\tau_z^* + \frac{E(YTI_{zv_k})}{E(TI_{zv_k})} - \frac{(1-m_{1z})\tau_z^*}{p_{zv_k}} \frac{p_{zv_k} - m_{0z}}{1 - m_{0z} - m_{1z}} \\
&\qquad - \frac{E(Y(1-T)I_{zv_k})}{E((1-T)I_{zv_k})} - \frac{(1-m_{0z})\tau_z^*}{1-p_{zv_k}} \frac{1 - m_{1z} - p_{zv_k}}{1 - m_{0z} - m_{1z}} = 0 \\
&\iff E(Y|T=1, Z=z, V=v_k) - E(Y|T=0, Z=z, V=v_k) \\
&\qquad = \left(\frac{1-m_{1z}}{p_{zv_k}} \frac{p_{zv_k} - m_{0z}}{1 - m_{0z} - m_{1z}} + \frac{1-m_{0z}}{1-p_{zv_k}} \frac{1 - m_{1z} - p_{zv_k}}{1 - m_{0z} - m_{1z}} - 1 \right) \tau_z^* \\
&\iff E(Y|T=1, Z=z, V=v_k) - E(Y|T=0, Z=z, V=v_k) \\
&\qquad = \frac{1}{1 - m_{0z} - m_{1z}} \left(1 - \frac{m_{0z}(1-m_{1z})}{p_{zv_k}} - \frac{(1-m_{0z})m_{1z}}{1-p_{zv_k}} \right) \tau_z^*,
\end{aligned}$$

which is identical to (18) under Assumption 3.3.

Noting that $E(TZ) = E(T|Z=1)E(Z)$, the fourth moment condition is rearranged as follows:

$$\begin{aligned}
&E\left[\Delta p^* - \left(\frac{TZr^{-1} - m_{01}}{1 - m_{01} - m_{11}} - \frac{T(1-Z)(1-r)^{-1} - m_{00}}{1 - m_{00} - m_{10}} \right) \right] = 0 \\
&\iff \Delta p^* - \left(\frac{p_1 - m_{01}}{1 - m_{01} - m_{11}} - \frac{p_0 - m_{00}}{1 - m_{00} - m_{10}} \right) = 0,
\end{aligned}$$

which is equal to (20). Similarly, noting that $E(YZ) = E(Y|Z=1)E(Z)$, the fifth moment condition is

$$E\left(\beta^* - \frac{YZr^{-1} - Y(1-Z)(1-r)^{-1}}{\Delta p^*} \right) = 0 \quad \iff \quad \beta^* = \frac{\mu_1 - \mu_0}{p_1^* - p_0^*},$$

which is identical to (21).

Therefore, from the identification result in Theorem 1, it holds that $E[g(X, \theta_0)] = 0$ with the unique solution θ_0.

□

B Appendix: Necessary and sufficient conditions of Assumption 3.4 (i) and (ii)

This appendix presents the necessary and sufficient conditions of Assumption 3.4 (i) and (ii). To this end, we first note that (14) and (15) lead to

$$\tau_W = \left(\frac{(1-m_1)p_W^*}{p_W} - \frac{m_1 p_W^*}{1-p_W} \right) \tau_Z^* \quad \Longleftrightarrow \quad \tau_W = \frac{p_W^*(1-m_1-p_W)}{p_W(1-p_W)} \tau_Z^*,$$

under Assumption 3.3. Here, from (17), we have $1-p_W^* = (1-m_{1Z}-p_W)/(1-m_{0Z}-m_{1Z})$. It thus holds that

$$\tau_W = \frac{p_W^*(1-p_W^*)}{p_W(1-p_W)}(1-m_{0Z}-m_{1Z})\tau_Z^*. \tag{22}$$

With the definition of $R_W := p_W^*(1-p_W^*)/[p_W(1-p_W)]$, we have $\tau_W = R_W(1-m_{0Z}-m_{1Z})\tau_Z^*$.

We next show that Assumption 3.4 (i) is identical to $\tau_z^* \neq 0$ and $m_{0z}+m_{1z} \neq 1$ for each $z = 0, 1$. According to (22), the equation in the assumption is rearranged as

$$\begin{aligned} &\left(\frac{\tau_{zv_1}}{p_{zv_2}} - \frac{\tau_{zv_2}}{p_{zv_1}} \right) \left(\frac{\tau_{zv_1}}{1-p_{zv_3}} - \frac{\tau_{zv_3}}{1-p_{zv_1}} \right) - \left(\frac{\tau_{zv_1}}{1-p_{zv_2}} - \frac{\tau_{zv_2}}{1-p_{zv_1}} \right) \left(\frac{\tau_{zv_1}}{p_{zv_3}} - \frac{\tau_{zv_3}}{p_{zv_1}} \right) \\ &= \left[\left(\frac{R_{zv_1}}{p_{zv_2}} - \frac{R_{zv_2}}{p_{zv_1}} \right) \left(\frac{R_{zv_1}}{1-p_{zv_3}} - \frac{R_{zv_3}}{1-p_{zv_1}} \right) - \left(\frac{R_{zv_1}}{1-p_{zv_2}} - \frac{R_{zv_2}}{1-p_{zv_1}} \right) \left(\frac{R_{zv_1}}{p_{zv_3}} - \frac{R_{zv_3}}{p_{zv_1}} \right) \right] \\ &\qquad \times (1-m_{0z}-m_{1z})\tau_z^*. \end{aligned}$$

Therefore, under Assumption 3.3, Assumption 3.4 (i) is satisfied if and only if $m_{0z}+m_{1z} \neq 1$ and $\tau_z^* \neq 0$ for each $z = 0, 1$.

We next show that the necessary and sufficient condition of Assumption 3.4 (ii) is also that $m_{0z}+m_{1z} \neq 1$ and $\tau_z^* \neq 0$ for each $z = 0, 1$. Under the condition that $m_0 = m_{00} = m_{01}$ and $m_1 = m_{10} = m_{11}$, the equation in the assumption is rewritten as

$$\left(\frac{\tau_{0v_1}}{p_{0v_2}} - \frac{\tau_{0v_2}}{p_{0v_1}} \right) \left(\frac{\tau_{1v_1}}{1-p_{1v_2}} - \frac{\tau_{1v_2}}{1-p_{1v_1}} \right) - \left(\frac{\tau_{0v_1}}{1-p_{0v_2}} - \frac{\tau_{0v_2}}{1-p_{0v_1}} \right) \left(\frac{\tau_{1v_1}}{p_{1v_2}} - \frac{\tau_{1v_2}}{p_{1v_1}} \right)$$

$$= \left(\frac{R_{0v_1}}{p_{0v_2}} - \frac{R_{0v_2}}{p_{0v_1}}\right)\left(\frac{R_{1v_0}}{1-p_{1v_2}} - \frac{R_{1v_2}}{1-p_{1v_1}}\right) - \left(\frac{R_{0v_1}}{1-p_{0v_2}} - \frac{R_{0v_2}}{1-p_{0v_1}}\right)\left(\frac{R_{1v_1}}{p_{1v_2}} - \frac{R_{1v_2}}{p_{1v_1}}\right)$$
$$\times (1-m_0-m_1)\tau_0^*\tau_1^*,$$

where we use (22). Hence, under Assumption 3.3, Assumption 3.4 (ii) is satisfied if and only if $\tau_z^* \neq 0$ and $m_0 + m_1 \neq 1$ for each $z = 0, 1$.

C Appendix: Mismeasured outcome or instrument

This appendix considers the situation in which $Y \in \mathbb{R}$ or $Z \in \{0,1\}$ may be a mismeasured variable of the true unobserved variable $Y^* \in \mathbb{R}$ or $Z^* \in \{0,1\}$, respectively. Here, we assume that treatment $T = T^*$ is observed without a measurement error for simplicity.

We first consider that continuous outcome Y may be a mismeasured variable of true continuous Y^*. By assuming that measurement error $U_Y := Y - Y^*$ satisfies $E(U_Y|Z = 1) = E(U_Y|Z = 0)$, it holds that

$$\frac{E(Y|Z=1)-E(Y|Z=0)}{E(T|Z=1)-E(T|Z=0)} = \frac{E(Y^*|Z=1)-E(Y^*|Z=0)}{E(T|Z=1)-E(T|Z=0)}.$$

Thus, the observable IV estimand identifies the LATE, meaning that the measurement error that is mean-independent of the instrument does not contaminate the inference.[2]

We then consider the situation under which Z may be a misclassified variable of true Z^*. The true parameter and the observable analogue are

$$\frac{E(Y|Z^*=1)-E(Y|Z^*=0)}{E(T|Z^*=1)-E(T|Z^*=0)}, \quad \text{and} \quad \frac{E(Y|Z=1)-E(Y|Z=0)}{E(T|Z=1)-E(T|Z=0)},$$

respectively. Note that, for each $z = 0, 1$, $E(Y|Z = z) = E(Y|Z^* = 1-z)\Pr(Z^* = 1-z|Z = z) + E(Y|Z^* = z)\Pr(Z^* = z|Z = z)$ under the non-differential measurement error $E(Y|Z^*, Z) = E(Y|Z^*)$. Hence, we have $E(Y|Z = 1) - E(Y|Z = 0) = [E(Y|Z^* = 1) - E(Y|Z^* = 0)][\Pr(Z^* = 1|Z = 1) - \Pr(Z^* = 1|Z = 0)]$. By using the same procedure,

[2] Note that the measurement error for discrete Y may not satisfy the condition of $E(U_Y|Z = 1) = E(U_Y|Z = 0)$ as in the analysis for the mismeasured binary treatment in this study. The measurement error for the discrete outcome should be analyzed based on other approaches.

we can show that $E(T|Z=1)-E(T|Z=0)=[E(T|Z^*=1)-E(T|Z^*=0)][\Pr(Z^*=1|Z=1)-\Pr(Z^*=1|Z=0)]$ under the non-differential measurement error $E(T|Z^*,Z)=E(T|Z^*)$. Therefore, we have

$$\frac{E(Y|Z=1)-E(Y|Z=0)}{E(T|Z=1)-E(T|Z=0)}=\frac{E(Y|Z^*=1)-E(Y|Z^*=0)}{E(T|Z^*=1)-E(T|Z^*=0)}.$$

Thus, the non-differential measurement error for Z does not contaminate the inference.

Acknowledgements

The author would like to thank Ryo Kambayashi, Yoshihiko Nishiyama, Ryo Okui, Yohei Yamamoto, and the seminar participants of the Economics and Statistics Workshop of Hitotsubashi University, 2016 Japan-Korea Allied Conference in Econometrics held in Tokyo, and the 2016 Kansai Econometrics Meeting held in Hiroshima for their helpful comments and discussions. The author acknowledges the financial support from the Japan Society for the Promotion of Science under KAKENHI Grant No. 15H06214.

References

D. Acemoglu and J. Angrist. How large are human-capital externalities? evidence from compulsory-schooling laws. *NBER Macroeconomics Annual 2000*, 15:9–74, 2001.

D. J. Aigner. Regression with a binary independent variable subject to errors of observation. *Journal of Econometrics*, 1(1):49–59, 1973.

J. G. Altonji and T. A. Dunn. The effects of family characteristics on the return to education. *The Review of Economics and Statistics*, 78(4):692–704, 1996.

Y. Amemiya. Instrumental variable estimator for the nonlinear errors-in-variables model. *Journal of Econometrics*, 28(3):273–289, 1985.

D. W. Andrews. Higher-order improvements of a computationally attractive k-step bootstrap for extremum estimators. *Econometrica*, 70(1):119–162, 2002.

J. D. Angrist and G. W. Imbens. Two-stage least squares estimation of average causal effects in models with variable treatment intensity. *Journal of the American statistical Association*, 90(430):431–442, 1995.

J. D. Angrist and A. B. Keueger. Does compulsory school attendance affect schooling and earnings? *The Quarterly Journal of Economics*, 106(4):979–1014, 1991.

J. D. Angrist and J.-S. Pischke. *Mostly harmless econometrics: An empiricist's companion*. Princeton university press, 2008.

J. D. Angrist, G. W. Imbens, and D. B. Rubin. Identification of causal effects using instrumental variables. *Journal of the American statistical Association*, 91(434):444–455, 1996.

E. Battistin, M. De Nadai, and B. Sianesi. Misreported schooling, multiple measures and returns to educational qualifications. *Journal of Econometrics*, 181(2):136–150, 2014.

D. Black, S. Sanders, and L. Taylor. Measurement of higher education in the census and current population survey. *Journal of the American Statistical Association*, (463): 545–554, 2003.

C. R. Bollinger. Bounding mean regressions when a binary regressor is mismeasured. *Journal of Econometrics*, 73(2):387–399, 1996.

J. Bound, C. Brown, and N. Mathiowetz. Measurement error in survey data. *Handbook of econometrics*, 5:3705–3843, 2001.

K. F. Butcher and A. Case. The effect of sibling sex composition on women's education and earnings. *The Quarterly Journal of Economics*, 109(3):531–563, 1994.

D. Card. Using geographic variation in college proximity to estimate the return to schooling. Technical report, National Bureau of Economic Research, 1993.

D. Card. The causal effect of education on earnings. *Handbook of labor economics*, 3: 1801–1863, 1999.

D. Card. Estimating the return to schooling: Progress on some persistent econometric problems. *Econometrica*, 69(5):1127–1160, 2001.

X. Chen, H. Hong, and D. Nekipelov. Nonlinear models of measurement errors. *Journal of Economic Literature*, 49(4):901–937, 2011.

F. DiTraglia and C. Garcia-Jimeno. On mis-measured binary regressors: new results and some comments on the literature. mimeo, 2016.

H. Frazis and M. A. Loewenstein. Estimating linear regressions with mismeasured, possibly endogenous, binary explanatory variables. *Journal of Econometrics*, 117(1):151–178, 2003.

P. Hall and J. L. Horowitz. Bootstrap critical values for tests based on generalized-method-of-moments estimators. *Econometrica: Journal of the Econometric Society*, pages 891–916, 1996.

L. P. Hansen. Large sample properties of generalized method of moments estimators. *Econometrica*, pages 1029–1054, 1982.

J. A. Hausman, W. K. Newey, H. Ichimura, and J. L. Powell. Identification and estimation of polynomial errors-in-variables models. *Journal of Econometrics*, 50(3):273–295, 1991.

J. A. Hausman, J. Abrevaya, and F. M. Scott-Morton. Misclassification of the dependent variable in a discrete-response setting. *Journal of Econometrics*, 87(2):239–269, 1998.

J. L. Horowitz and C. F. Manski. Identification and robustness with contaminated and corrupted data. *Econometrica*, 63(2):281–302, 1995.

C. Hsiao. Consistent estimation for some nonlinear errors-in-variables models. *Journal of econometrics*, 41(1):159–185, 1989.

Y. Hu. Identification and estimation of nonlinear models with misclassification error using instrumental variables: A general solution. *Journal of Econometrics*, 144(1): 27–61, 2008.

Y. Hu and Y. Sasaki. Closed-form estimation of nonparametric models with non-classical measurement errors. *Journal of Econometrics*, 185(2):392–408, 2015.

Y. Hu and S. M. Schennach. Instrumental variable treatment of nonclassical measurement error models. *Econometrica*, 76(1):195–216, 2008.

K. Imai and T. Yamamoto. Causal inference with differential measurement error: Nonparametric identification and sensitivity analysis. *American Journal of Political Science*, 54(2):543–560, 2010.

G. W. Imbens and J. D. Angrist. Identification and estimation of local average treatment effects. *Econometrica*, 62(2):467–475, 1994.

T. J. Kane and C. E. Rouse. Labor-market returns to two-and four-year college. *The American Economic Review*, 85(3):600–614, 1995.

A. Lewbel. Constructing instruments for regressions with measurement error when no additional data are available, with an application to patents and R&D. *Econometrica*, 65(5):1201–1213, 1997.

A. Lewbel. Semiparametric latent variable model estimation with endogenous or mismeasured regressors. *Econometrica*, 66(1):105–121, 1998.

A. Lewbel. Estimation of average treatment effects with misclassification. *Econometrica*, 75(2):537–551, 2007.

L. Lochner and E. Moretti. The effect of education on crime: Evidence from prison inmates, arrests, and self-reports. *The American Economic Review*, 94(1):155–189, 2004.

A. Mahajan. Identification and estimation of regression models with misclassification. *Econometrica*, 74(3):631–665, 2006.

F. Molinari. Partial identification of probability distributions with misclassified data. *Journal of Econometrics*, 144(1):81–117, 2008.

W. K. Newey and D. McFadden. Large sample estimation and hypothesis testing. *Handbook of econometrics*, 4:2111–2245, 1994.

S. M. Schennach. Instrumental variable estimation of nonlinear errors-in-variables models. *Econometrica*, 75(1):201–239, 2007.

S. M. Schennach. Measurement error in nonlinear models–a review. *Advances in Economics and Econometrics. Ed. by Daron Acemoglu, Manuel Arellano, and Eddie Dekel*, 3:296–337, 2013.

S. Song, S. M. Schennach, and H. White. Estimating nonseparable models with mismeasured endogenous variables. *Quantitative Economics*, 6(3):749–794, 2015.

T. Ura. Heterogeneous treatment effects with mismeasured endogenous treatment. mimeo, 2016.

J. M. Wooldridge. *Econometric analysis of cross section and panel data.* MIT press, 2010.

T. Yanagi. Regression discontinuity designs with nonclassical measurement error. mimeo, 2017.