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NON-EQUIVALENCE OF SPECIFIC AND AD VALOREM TAXATION IN THE COMPETITIVE MARKET WITH TAX EVASION*

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Abstract

The well-known equivalence between specific and ad valorem taxation in competitive markets may not hold in the presence of tax evasion. Evading specific taxes has to take place via concealing quantities sold, whereas evading ad valorem taxes can take place via concealing selling prices as well as quantities sold. We show that in the competitive market (i) this difference could make the equivalence of these two taxes break down, and (ii) specific taxation may be superior to ad valorem taxation if it causes firms to channel fewer resources into tax evasion, given other things being equal.

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I. *Introduction*

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When comparing specific and ad valorem taxation on commodities, the focus of the literature has been on markets with imperfect competition at least since Wicksell (1896).[1] This focus may not be surprising. As Delipalla and Keen (1992, p. 351) say that their equivalence under perfect competition has long been recognized.[2]

Recently, to our knowledge, this equivalence has been challenged by a few studies. Delipalla and Keen (2006) consider endogenous product quality and, for the two tax types that have different incentives with regard to the choices of output quality and quantity, their equivalence no longer holds. On the other hand, there are a few studies explore the impact of evasion on these two tax types. Delipalla (2009) argues that the commodity tax structure will affect the firm's choice of informal activity. An increase in the specific tax rate, relative to an equivalent increase in ad valorem taxation, makes informality more attractive to firms. Hence, the ad valorem tax is superior to the specific tax in the presence of evasion. Goerke (2011) argues that if the market price is uncertain, these two tax types are not equivalent and, by some reasonable assumptions, marginally substituting a specific tax by an ad valorem tax can improve efficiency. In this paper, we focus on the difference of evading tax bases under these two regimes. Once there is evasion, evading specific taxes must take place via concealing quantities sold, whereas evading ad valorem taxes can take place via concealing selling prices as well as quantities sold. While this argument is not novel,[3] to our knowledge, it has not been formally analyzed in the literature. Although allowing more instruments to address a problem may be beneficial,[4] we show that with more instruments available for evasion causes a breakdown of the equivalence in competitive markets and, in particular, makes specific taxes more favorable in the welfare sense, as long as the firm implements evasion by expending more resources under ad valorem than under specific taxation.[5]

The defining feature of competitive markets, if there is any, is that firms are price takers, i.e., they take prices as given and exogenous to their profit-maximizing behavior. However, this feature does not necessarily imply that obtaining truly competitive prices is costless to the tax authorities. In our model, a firm may evade taxes by underreporting sales through price concealment in competitive markets when ad valorem taxation is in operation. Our assertion is that competitive firms (as well as consumers) know the relevant prices they face, but the tax authorities may not be well informed about these prices. This may not be unreasonable. Prices in a competitive market may fluctuate a great deal, commodities in question may be less

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[1] Wicksell (1896) considers the monopoly case. Under the assumption of a constant marginal cost, he shows that, given the same amount of revenue collected, ad valorem taxes will result in a lower consumer price and a higher level of quantity produced than specific taxes. Later work includes Suits and Musgrave (1953), Kay and Keen (1983), Delipalla and Keen (1992), Skeath and Trandel (1994), and Anderson et al. (2001).

[2] This recognition can at least date back to Cournot (1838), in which there was a realization that specific and ad valorem taxes require separate consideration under monopoly.

[3] Virmani (1989) mentioned: "In the sales tax context this is a very good assumption if evasion involves understatement of price at which sales were made. In discussion of the issue of ad valorem versus specific taxes the assertion is often made by policy-makers that revenue (price) is easier to conceal than output."

[4] See, for example, Kaplow (2006) on the undesirability of imposing commodity taxes when income taxation is available.

[5] A simple case is that if quantity concealment is too costly for the firm to implement, it may only engage in price concealment under ad valorem taxation and not evade any tax under specific taxation. In this circumstance, the concealment cost of the former is definitely higher than that of the latter and thereby the specific tax is superior to the ad valorem tax. See Proposition 3.

familiar intermediate goods to common people, and perhaps most importantly, while a firm may only need to face the prices of a single commodity, the tax authorities must deal with the prices of hundreds or even thousands of commodities with a limited size of enforcement staff.[6] Moreover, even though the tax authorities are well informed with regard to competitive market prices, the tax law, which imposes liabilities on the tax authorities to provide evidence to detect evasion, makes price concealment more plausible, for the evasion component of sales (either underreporting prices or quantities or both) cannot be identified without an audit. Therefore, knowing market prices by tax authorities alone cannot deter taxpayers from evasion, and price concealment is possible in competitive markets.

It is worth emphasizing at the outset that we do not claim that the equivalence of specific and ad valorem taxation in competitive markets will always break down in the presence of tax evasion. We only claim that, for some commodities or in some situations, this equivalence will no longer hold when there is tax evasion. Other things being equal, the resource costs of evasion under ad valorem and specific taxation are not the same. We will then formally show that specific and ad valorem taxes are not equivalent to each other even in competitive markets. In some cases the specific tax will be Pareto superior to the ad valorem tax in a competitive market.

In practice, for the same commodity some countries impose ad valorem taxation while others instead impose specific taxation. For instance, all countries in the European Union impose value added taxes and excise duty on tobacco, alcoholic drink, and gasoline. However, on alcoholic drink and gasoline most of the EU countries impose specific taxation while on tobacco countries use different types of taxation. This study can apply to explain such a phenomenon, since in the presence of tax evasion specific and ad valorem taxes are not equivalent to each other even in competitive markets.

The extent of evasion in regard to commodity taxes is significant. Silvani and Brondolo (1993) investigate noncompliance of value-added tax (VAT) for 19 mostly developing countries. By comparing the actual value-added tax revenue collected with the potential tax base, they find a median evasion rate of 31.5 percent, with New Zealand the lowest at 5.1 percent and Peru the highest at 68.2 percent. Slemrod (2007) cites a confidential study that was conducted in 2005 by the Forum on Tax Administration (a subsidiary body of the OECD's Committee on Fiscal Affairs). Estimated noncompliance rates in relation to VAT range from 4.0 to 17.5 percent for some countries in the study. In a recent study published by the European Commission (2009), the gap between the theoretical VAT liability and actual receipts in 2006 was estimated about 13 percent of VAT revenues in the European Union.[7]

Empirical studies refer to sales taxes evasion seldom discriminate price concealment from quantity concealment.[8] However, to customs evasion, it is commonly recognized that undervaluing imports is one of the methods to reduce tariff or VAT at the border. Mishra et al. (2008) classify the measure of evasions into two categories: evasion in import values and evasion in import quantities. By investigating the import and export data of India with trading

[6] A real case that occurred in Taiwan was where a firm underreported the price of imported seafood to evade custom duties (see United Daily News, July 7th, 2011, Taiwan).

[7] For more estimates on noncompliance with value-added tax, especially in European Union countries, see Keen and Smith (2007).

[8] This is because of the difficulty of data obtained.

partners during 1987-2003, they found a significant gap between the value imported and exported. Fisman and Wei (2004) investigate the “missing imports” in China and conclude that “there are widespread practices of underreporting the unit values of imports and mislabeling higher-taxed products as lower-taxed varieties.” These studies provide strong evidences about price concealment behaviors of the taxpayers.

The rest of this paper is organized as follows. In Section II, the model is briefly described. Section III derives the market equilibria under these two tax regime respectively. In Section IV, the welfare effects are compared, and Section V concludes.

II. *The Model*

Our model is based on Cremer and Gahvari (1993, hereafter CG), who analyze the evasion of specific taxes by competitive firms. We amend the CG model straightforwardly to accommodate competitive firms’ evasion of ad valorem taxes.

Consider a competitive closed economy in which there are many industries. We focus on a representative industry in which there are many identical firms. Let x denote a firm’s output in the industry, p the price which the individual firm faces and $C(x)$ its cost function. The market demand of the industry is given by $X(p)=\Sigma x(p)$ with $X'(p)<0$.

The firm is obliged to pay taxes according to its sales, which are subject to either specific or ad valorem taxes. Under specific taxation, a fixed amount of tax $t>0$ is imposed on per unit output so that the firm’s net-of-tax sales revenue equals $(p-t)x$ if there is no evasion. Under ad valorem taxation, a fixed percentage of tax rate τ, $0<\tau<1$, is levied on the output price so that the firm’s net-of-tax sales revenue equals $(p-p\tau)x$ if there is no evasion.

The firm may attempt to evade taxes. As in the CG model, we assume that it is costly for the firm to conceal its sales information from the tax authorities (say, costs involved in falsifying invoices or fabricating accounts). Evading taxes must take place via concealing quantities sold in the CG model. By contrast, evading taxes may take place via concealing selling prices as well as quantities sold in our model. This is the main difference between these two models. Let α and β denote, respectively, the fractions of output and the selling price that the firm chooses to reveal in its accounting records. Since concealing activities are costly, it is plain that the firm will not understate its selling price under specific taxation; that is, $\beta=1$ will always hold with specific taxes. This is not necessarily true under ad valorem taxation. In addition to understating its output, the firm may understate its selling price for the revealed output quantity, αx, when ad valorem taxes are in operation. Since concealing activities are costly, and the firms do not have to conceal (and even report) the price of unrevealed output because the concealed outputs are not taxed, there is no need for the firm to carry out price concealment for the unreported output $(1-\alpha)x$.

We follow CG to model the firm’s cost of “quantity” concealment. That is to say, for the fraction of the output hidden from the tax authorities, each unit of output concealed entails a quantity concealing cost $G(1-\alpha)$, which is an increasing and convex function of $1-\alpha$ with $G(0)=0$. Thus, the total concealment cost associated with specific taxes is $G(1-\alpha)\cdot(1-\alpha)x$ when the firm conceals a $1-\alpha$ fraction of its output x.[9]

[9] We follow Cremer and Gahvari (1993) to assume a constant audit probability. Under such an assumption, the

We model the firm's cost of price concealment analogously. For the fraction of the selling price concealed to tax authorities, each unit of output *revealed* entails a price concealing cost $H(1-\beta)$, which is an increasing and convex function of $1-\beta$ with $H(0)=H'(0)=0$. Thus, the total concealment cost associated with ad valorem taxes is $G(1-\alpha)\cdot(1-\alpha)x+H(1-\beta)\cdot\alpha x$ when the firm conceals a $1-\alpha$ fraction of its output x and a $1-\beta$ fraction of its selling price for the revealed output αx. Of course, α and x under these two tax types may not be the same.

Suppose that the audit probability $A>0$ and a fine $F>1$ is levied on the amount of evaded tax if detected. As in CG, we assume that the firm's true sales will be detected accurately once the tax authorities carry out an audit, and $AF<1$ to make the existence of an interior solution conceivable.[10] For convenience, let $g(1-\alpha)\equiv(1-\alpha)G(1-\alpha)$ as in CG. The firm's expected profits $E(\pi)$ under these two tax types equal, respectively,

$$E(\pi^s)=(p-CC^s-Er^s(t))x-C(x), \tag{1}$$

$$E(\pi^a)=(p-CC^a-Er^a(p\tau))x-C(x), \tag{2}$$

where the superscript s denotes specific taxes and a denotes ad valorem taxes; meanwhile, $CC^s\equiv g(1-\alpha)$, $Er^s\equiv[\alpha+AF(1-\alpha)]t$, $CC^a\equiv g(1-\alpha)+\alpha H(1-\beta)$ and $Er^a\equiv[\alpha\beta+AF(1-\alpha\beta)]p\tau$. The firm's objective function under specific taxation, (1), is identical to that in the CG model (except for the cost function term). The term $\alpha\beta$ in (2) denotes the fraction of sales that the firm reports to the tax authorities. The firm's evasion rate equals $1-\alpha$ under specific taxation, while it equals $1-\alpha\beta$ under ad valorem taxation. This completes the description of our model.

III. *Market Equilibrium*

Consider the short-run equilibrium first. In the short run, there are a fixed number m of firms in the competitive market. Suppose the cost function $C(x)$ is convex in x, hence $C'(x)>0$ and $C''(x)>0$. In what follows, we denote (x^a, α^a, β^*) and (x^s, α^s) as the optimal choices of the firm under ad valorem and specific taxation, respectively.

1. Specific Taxation

The first- and second-order conditions for interior α^s and x^s from (1) are

$$g'(1-\alpha^s)=(1-AF)t, \tag{3}$$

$$p-CC^s-Er^s=C'(x^s), \tag{4}$$

concealing cost is positively related to the concealed amount. That is, under this setup the cost to conceal a higher proportion of a firm's price will also incur a higher concealing cost. In reality the audit probability may increase, hence making the concealment cost get higher. However, the qualitative results of this alternative setup will still be consistent with ours. Both setups will generate the necessary conditions for an interior solution. In other words, an increasing audit probability make it costlier to evade taxes, and this effect can also be caught up by an increasing concealment cost. Similarly, the setup of price-concealing cost function is later sections also follows this principle.

[10] It should be noted that $AF<1$ is necessary but not sufficient for tax evasion, since in this model evasion is costly.

$$g''(1-\alpha^s)>0,\ C''(x^s)>0, \tag{5}$$

where (5) is satisfied because G and C are increasing and convex functions of $1-\alpha$ and x, respectively. From (3) we can see that the tax evasion decision is independent of the output decision. Hence, the firm's optimization problem can be divided into two stages. In the first stage, the firm makes its evasion decision on

$$\theta^s(t)\equiv\min_{\alpha}\ CC^s(t)+Er^s(t),^{11} \tag{6}$$

where $\theta^s(t)$ is regarded as the effective tax burden (including evasion cost and expected tax payment) entailed on per unit of output. In the second stage, the firm makes its production decision.[12] Therefore, we can derive the individual firm's supply function$x^s=x^s(p,\ t)$ as well as the supply function of this industry by aggregating. Next, combining the market supply function with market demand function yields the short-run equilibrium price p^s, which is a function of m and t.

2. Ad Valorem Taxation

Similar to specific taxation, the firm's evasion choice of α and β is independent of its output choice of x under ad valorem taxation. The first- and second-order conditions for interior solutions (α^a, β^*, x^a) from (2) are, respectively,

$$g'(1-\alpha^a)=H(1-\beta^*)+(1-AF)\beta^* p\tau, \tag{7}$$

$$H'(1-\beta^*)=(1-AF)p\tau, \tag{8}$$

$$p-CC^a-Er^a=C'(x^a), \tag{9}$$

$$g''(1-\alpha^s)>0,\ H''(1-\beta^*)>0,\ C''(x^a)>0, \tag{10}$$

where (10) is satisfied because of the convexity assumption about G, H and C. The logic governing the firm's price evasion behind (8) is not different from the logic governing the firm's "quantity" evasion behind (3). Indeed, (8) and (3) resemble each other in the form of the formula. A more interesting result is (7), which shows that the firm's concealing its output has the additional benefit of saving the cost of price concealment (the first term on R.H.S.). The reason for this additional benefit is obvious: firms will not engage in costly price concealments for the quantities that they do not intend to reveal to the tax authorities. From (7) and (8) we can obtain (α^a, β^*). By the same procedures as in specific taxation, the effective tax burden entailed on per unit of output is

$$\theta^a(p\tau)\equiv\min_{\alpha,\,\beta}\ CC^a(p\tau)+Er^a(p\tau), \tag{11}$$

and by (9) we can derive the individual firm's supply function $x^a=x^a(p,\ \tau)$ as well as the supply function of this industry. Again, combining the market supply function with market

[11] From (3) we can derive $\alpha^s(t)$, and then substitute it into (4s) to yield $\theta^s(t)$.

[12] To solve this problem, putting $\theta^s(t)$ into (4) yields $p=\theta^s(t)+C'(x^s)$, and by this equation the output level x^s is determined.

demand function yields the short-run equilibrium price p^a, which is a function of m and τ.

IV. *Comparing Specific with Ad Valorem Taxation*

The comparison starts from the setting of equivalent per unit output tax under these two tax regimes, that is $p^a\tau=t$. In the absence of tax evasion, when $p^a\tau=t$, specific and ad valorem taxation in competitive markets can be shown to be equivalent in that both yield the same equilibrium price, output and tax revenue (both in the short run and long run). This result is one of the canons of undergraduate public finance textbooks, for example, Stiglitz (2000, pp. 488-490).

What would happen to this equivalence when there is tax evasion? First, considering the compliance rate under these two tax regimes, we derive two following lemmas.

Lemma 1. ***Given*** $t=p^a\tau$, $\alpha^a=\alpha^s<1$, ***if and only if*** $\beta^*=1$.

Proof. If $\beta^*=1$, since $H(0)=0$, (7) becomes $g'(1-\alpha^a)=(1-AF)p^a\tau$. Given $t=p^a\tau$, compare (7) with (3), and then it is obvious that $\alpha^a=\alpha^s$.

Suppose $\alpha^a=\alpha^s<1$ but $\beta^*<1$. Then based on (7) and (3), $(1-AF)t=H(1-\beta^*)+(1-AF)\beta^*p^a\tau$, or $(1-AF)t(1-\beta^*)=H(1-\beta^*)$ given $t=p^a\tau$. Moreover, when $\beta^*<1$, equation (8) holds, which follows that $H'(1-\beta^*)(1-\beta^*)=H(1-\beta^*)$ or $H'(1-\beta^*)=H(1-\beta^*)/(1-\beta^*)$. However, this is not possible as long as $H(\cdot)$ is strictly convex (where the marginal cost $H'(q)$ is always larger than the average cost $H(q)/q$ for any $q>0$). Therefore, $\beta^*=1$. Q.E.D.

Lemma 2. ***Given*** $t=p^a\tau$, $\alpha^a>\alpha^s$, ***if and only if*** $\beta^*<1$.

Proof: Based on (3) and (7), we have

$$\begin{aligned} g'(1-\alpha^s)-g'(1-\alpha^a) &= (1-AF)t-H(1-\beta^*)-(1-AF)\beta^*p^a\tau \\ &= (1-\beta^*)\left[H'(1-\beta^*)-\frac{H(1-\beta^*)}{(1-\beta^*)}\right]. \end{aligned}$$

Again, based on (8) when $\beta^*<1$,

$$g'(1-\alpha^s)-g'(1-\alpha^a)=(1-\beta^*)\left[H'(1-\beta^*)-\frac{H(1-\beta^*)}{(1-\beta^*)}\right]>0$$

for $H(\cdot)$ is strictly convex. Since $g(\cdot)$ is strictly convex, we have $\alpha^a>\alpha^s$. Q.E.D.

Based on the above two lemmas, we can conclude:

Proposition 1. ***Given*** $t=p^a\tau$, ***either these two taxes make no difference, or ad valorem taxation induces less quantity concealment and more price concealment than specific taxation does.***

The intuition behind Proposition 1 is as follows: With more instruments to conceal tax base under ad valorem taxation, the firm may use the price concealment to replace part of the quantity concealment since the marginal cost of concealment is increasing with concealed quantities, hence $\beta^* < 1$ and $\alpha^a > \alpha^s$. Moreover, if $\beta^* < 1$, the relative magnitude of compliance rates under these two tax regimes (i.e., α^s, $\alpha^a\beta^*$) may not equal. Therefore, these two types of taxes are not equivalent from the aspect of tax enforcement.

Next, notice that the choice under specific taxation is also available under ad valorem tax (i.e., $\beta^* = 1$, $\alpha^a = \alpha^s$). Hence, as long as per unit taxes are equal, θ^s cannot be smaller than θ^a.

Lemma 3. ***Given*** $t = p^a\tau$, ***the "effective tax burden" cannot be lower under specific taxation, i.e.,*** $\theta^s(t) \geq \theta^a(t)$.

Since the effective tax burden is positively correlated with the firm's effective marginal cost, it is easy to know from Lemma 3 that $p^s \geq p^a$ in the short run. In this condition, consumers' surplus as well as producers' profit will be lower under specific taxation than those under ad valorem taxation. However, in the long run, the free entry and exit of firms will make the firm's expected profit vanish and each firm produces the output quantity at which the average cost is lowest, thereby the equilibrium prices must satisfy $p^s = \theta^s(t) + c_m \geq p^a = \theta^a(t) + c_m$, where c_m denotes the lowest average cost. In this condition, the number of firms under ad valorem taxation will be at least as large as that under specific taxation.

Proposition 2. ***Given*** $t = p^a\tau$, ***no matter in the long run or short run, the competitive equilibrium price under specific taxation cannot be lower than that under ad valorem taxation. Moreover, the number of firms under ad valorem taxation will be not smaller than that under specific taxation in the long run.***

Proposition 2 clearly shows that the equivalence between specific and ad valorem taxation in competitive markets will break down in the presence of tax evasion. Therefore, with tax evasion, the welfare effects under these two different taxes are not identical. To simplify the analyses, in what follows, we will demonstrate some cases in which specific taxation is Pareto superior to ad valorem taxation under long run equilibria. Before that, we prove a useful lemma.

Lemma 4. $\dfrac{\partial p^s}{\partial t} > 0$ *and* $\dfrac{\partial p^a}{\partial \tau} > 0$ *in competitive equilibrium.*

Proof: See the Appendix.

This lemma simply states that a higher tax will shift forward to consumers with the incidence of a higher consumer price in competitive markets. This is true, regardless of whether the tax imposed is specific or ad valorem. With above lemmas, we obtain the following proposition.

Proposition 3. *In confining the analysis to more concealment cost being incurred by a firm under ad valorem taxation than under specific taxation when the same amount of tax is levied per unit of output (i.e.,* $CC^a(p\tau) > CC^s(t)$ *when* $t = p\tau$*), the specific tax is Pareto superior to the ad valorem tax in a competitive market.*

Proof. Given $t = p^a\tau$, by Lemma 3, $p^s \geq p^a$. Since $CC^a(p^a\tau) > CC^s(t)$, we have $Er^a(p^a\tau) = p^a - CC^a(p^a\tau) - c_m < p^s - CC^s(t) - c_m = Er^s(t)$. By Lemma 4, there must exist a t' such that $t' \leq t$ and

$p^s(t')=p^a$. Besides, since $\partial\alpha^s/\partial t<0$ and CC^s is a decreasing function of α^s, $CC^s(t)\geq CC^s(t')$. These results yield $CC^a(p^a\tau)>CC^s(t)\geq CC^s(t')$, which implies that $Er^s(t')>Er^a(p^a\tau)$. Accordingly, $X^a(p^a)=X^s(p^s(t'))$ since $p^s(t')=p^a$, the government revenue with $ER^s\equiv Er^s(t')\cdot X^s(p(t'))>ER^a\equiv Er^a(p^a\tau)\cdot X^a(p^a)$. Q.E.D.

There are two instruments (price and quantity) available for the firm to conceal its sales from the tax authorities in ad valorem taxation, whereas there is only one instrument (quantity) available in specific taxation. Other things being equal, if the availability of the price as well as the quantity instruments under ad valorem taxation causes a firm to incur more cost for evasion, it must be the tax payment under ad valorem taxation is less than that under specific taxation. Therefore, we can substitute the ad valorem tax with a specific tax to reduce the concealment cost and increase tax revenue without altering the market price. This is the economic meaning of Proposition 3.

It is worth noting that there is no definite superiority between these two taxes under the situation of $CC^a(p\tau)<CC^s(t)$ when $t=p\tau$. That is, the ad valorem tax is not necessarily Pareto superior to the specific tax in this situation. Note also that there is no general rule about when the condition of Proposition 3 is fulfilled.

V. *Conclusion*

Tax evasion per se does no harm to an economy, as put by Slemrod (2007, p.41). The government must collect a fixed amount of revenue and, therefore, it can respond to evasion by raising tax rates appropriately in our model. However, Slemrod (2007, p.42) points out that resources taxpayers expend to implement and camouflage noncompliance.

In this paper we focus on the resources taxpayers expend. There are price and quantity instruments for the firm to implement evasion under ad valorem taxation, whereas there is only a quantity instrument for the firm to implement evasion under specific taxation. As a result, other things being equal, the resource costs of evasion under ad valorem and specific taxation are not the same. By building on this key insight, we formally show that specific and ad valorem taxes are not equivalent to each other even in competitive markets. Furthermore, in some cases the specific tax will be Pareto superior to the ad valorem tax in a competitive market. This paper only takes into account the concealing cost of tax payers. It could also be argued that the administration cost is usually smaller in the case of the specific tax because the tax authorities can detect tax evasion by only checking quantities. If the administrative cost is also considered, it is more likely that the specific tax will be Pareto superior to the ad valorem tax in a competitive market.

Appendix

Proof of Lemma 4.

Applying the envelope theorem to (6) and (11) yields, respectively,

$$\frac{\partial \theta^s}{\partial t} = \alpha^s + AF(1-\alpha^s), \tag{A1}$$

$$\frac{\partial \theta(p^a \tau)}{\partial p^a \tau} = \alpha^a \beta + AF(1-\alpha^a \beta). \tag{A2}$$

Utilizing $E(\pi^s)=0$ in equilibrium gives

$$p^s - c_m = \theta^s, \tag{A3}$$

$$p^a - c_m = \theta^a. \tag{A4}$$

Combining (A1) with (A3), (A2) with (A4) respectively, this leads to

$$\frac{\partial p^s}{\partial t} = \frac{\partial \theta^s}{\partial t} = \alpha^s + AF(1-\alpha^s), \tag{A5}$$

$$\frac{\partial p^a}{\partial (p^a \tau)} = \frac{\partial \theta^a(p^a \tau)}{\partial (p^a \tau)} = \alpha^a \beta + AF(1-\alpha^a \beta). \tag{A6}$$

Because $\partial p^a/\partial \tau = [\partial p^a/\partial(p^a\tau)][\partial(p^a\tau)/\partial\tau] = [\partial p^a/\partial(p^a\tau)][p^a + \tau(\partial p^a/\partial\tau)]$, we obtain from (A6)

$$\frac{\partial p^a}{\partial \tau} = \frac{[\alpha^a\beta + (1-\alpha^a\beta)AF]p^a}{1-\tau[\alpha^a\beta + (1-\alpha^a\beta)AF]}\ . \tag{A7}$$

(A5) is obviously positive. Since $AF < 1$ by assumption, (A7) is also positive. Q.E.D.

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