THE RATIONALE BEHIND LÖSCHIAN TYPE OF CENTRAL-PLACE SYSTEM

FUJIO MIZUOKA

1. INTRODUCTION

Central-place theory (中心地理論) has been no doubt one of the most important theoretical contributions to the quantitative and theoretical revolution in geography, where setting up and testing theories on spatial patterns and processes are considered to be the task of geography rather than indulging in a mere description. Central-place theory originated in pre-War Germany by two different authors, Walter Christaller in 1933 (6), and August Lösch in 1939 (8), independently each other. These theories were then elaborated and modified mainly in the English-speaking countries by many quantitative-oriented geographers, among whom Brian J. L. Berry and William L. Garrison (2) must especially be noted.

Nowadays all of these three systems are known to provide urban geography with theoretical framework, although they have been treated largely in parallel. The writer has already discussed in another paper the underlying differences and the related policy implication between systems of Christaller and Berry-Garrison from this point of view (10). In this paper, referring to articles which give some insight into the point above, an attempt is made to compare Berry-Garrison’s ‘fixed k’ system with that of Lösch, in order to clarify through finding out the implied rationale the scope and limitation of Löschian type of central-place system.

II. THE ‘FIXED K=3’ SYSTEMS UNDER THRESHOLD CONCEPT

The most fundamental factor that leads to a hierarchical system of central places is the fact that every good or service (hereafter simply good) has its own and unique range (Reichweite, 達及範囲) of supply. Range can be broadly defined here as the outer boundary of an area in which a good in question may be supplied from its centre. This boundary occurs due to two alternate reasons: either 1. customers cannot afford to travel too far to obtain the good concerned, or 2. the suppliers of the good must obtain at least some appropriate (which is termed ‘normal’) profit. The former delineate ‘upper limit’ of range, or sometimes simply termed ‘range’, and the latter ‘lower limit’ or ‘threshold’ (閾値).

Berry and Garrison attempted in 1958 to relate central-place theory to some concepts of economics, and to show that ‘the theory may be formulated in terms of a simple concept of threshold’, (2, p. 107) in a bid to bring central-place theory more close to the reality and understandable more readily.

In the previous paper the writer has shown that the range-based system of Christaller can only come into existence when deliberate effort is made for planning or public intervention based on the inter-regional welfare policy to ascertain every person, no matter where he/she lives, to have an access to every variety of good; and that his system guarantee the suppliers profit only in case of the stretch of threshold being 0.5250 or less times as long as that of range. In a pure laissez-faire market economy the arrangement of supply functions after Christaller’s system will collapse when threshold stretches more than 0.5250 times longer than range, or the supply functions will pack together more closely each other to soak up all purchasing power economically viable when less than 0.5250 times as long, both eventually giving rise to the threshold-based arrangement of supply functions.

Berry and Garrison attempted in 1958 to relate central-place theory to some concepts of economics, and to show that ‘the theory may be formulated in terms of a simple concept of threshold’, (2, p. 107) in a bid to bring central-place theory more close to the reality and understandable more readily.

The starting point of Berry-Garrison’s formulation is to assume n types of central goods to be supplied, which are named in falling order of their threshold sales requirements, \( n, n - 1, \ldots, n - 1, n - 1, \ldots, n - 1, \ldots, 1 \). The suppliers of good with maximum threshold size \( n \) will then compete each other spatially to serve a portion of the area most efficiently, thus resulting in a hexagonal market area (complementary region) equal to the threshold requirement of good \( n \) in size, which earns the suppliers normal profit. These points where the suppliers of good \( n \) are located are to be called an A centre.\(^3\) Goods with threshold smaller than good \( n \), good \( n - 1 \) for example, will continue to be supplied from the A centres, due to ‘advantages from association with other establishments providing central goods.’ (2, p. 112) Goods with successively smaller threshold \( n - 2, n - 3 \) etc. are supplied from only the A centres as well. Excess profit is earned by the suppliers whose threshold sales requirement is smaller than the complementary region or tributary area of an A centre, which is identical with the threshold size of good \( n \). When ‘interstitial purchasing power’ (2, p. 112) between threshold market areas of

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certain type of good themselves reaches threshold size of the good, which shall be called good \( n - \frac{1}{2} \) and termed a hierarchical marginal good. This interstitial area will allow a new supplier to be established to supply good \( n - \frac{1}{2} \). In this case good \( n - \frac{1}{2} \) must be supplied from the new centres, named a B centre, in addition to the existing A centres, due to spatial competition to obtain more purchasing power, which also determines the location of the B centres so that distribution costs or consumer movements can be minimized. Berry and Garrison had postulated that hierarchical structure of central place can be developed ‘without the uniformity assumptions concerning purchasing power’ (2, p. 110), which has however later induced much criticisms and proved nowadays to be almost invalid. Sticking therefore to the Christaller’s original assumption of an unbound isotropic plane (等方性平面) which Lösch followed alike, the new B centres are obviously located at the centre of triangle connecting three A centres, just as is the case in Christaller.

In the same token successively lower order centres up to centre M will emerge, which will cater for goods with successively smaller threshold requirement, up to good 1. The ultimate picture shown in Table 1 has, if shown in a diagram, the well-known ‘k = 3’ central-place hierarchy, where numbers of lower order centres increase in geometrical progression in the multiple of three, i.e., 1, 3, 9, 27, 81, 273, ..., provided that higher order centres serving lower order functions alike are counted as many times as they serve the sets of functions proper to each order of centres.

This formulation of central-place hierarchy under threshold concept facilitates its comparison with Löschian system, which is from the beginning constructed upon the threshold concept.

III. CENTRAL-PLACE SYSTEM OF LÖSCHIAN TYPE

Lösch sets out from the case where one farm yields surplus farm product, beer in his example, which is beyond his own consumption and therefore designated for sale in the market immediately neighbouring him. Due to transport cost incurred, price of beer increases away from the farm. After the demand curve of beer FT, the sales amount PO at brewery price PO decreases as transport cost increases as shown in Fig. 1, eventually reaching to the point beyond which beer doesn’t sell at all. This distance PF is Christaller’s ‘range’ or ‘upper limit of range’ defined in Löschian way. ‘Threshold’ or ‘lower limit’ on the other hand, can be defined by means of Fig. 2, which is a modification by Parr and Denike (11) of Lösch’s original figure (Abb. 22 in B, S. 72, the English version, Fig. 22 in p. 106) in an attempt to make it theoretically more accurate and readily understandable, although in fact still certain misunderstandings exist. The curve \( \pi \), the envelope of the smallest average costs of farms producing beer in unit of various amount with best appropriate technology, indicates the trend of the minimum average cost as a function of quantity produced. Total demand per one producer, or volume of a demand cone calculated after the demand curve FT, is represented by the curve \( \Delta \). The intersection of the curves \( \Delta \) and \( \pi \) determines the equilibrium price and quantity, OM and OQ, respectively. Here however \( \Delta \) is still in part to the right of \( \pi \), indicating that there is still room for more potential producers to enter. It will mean smaller scale of production or increased cost at the farm. Total demand per producer will be reduced to reach the point where the two curves \( \pi \) and \( \Delta \) (now \( \Delta \) come to be tangent at a point \( N' \). This is the minimum viable scale of production \( M'N' \) at cost OM, which is identical with the volume of demand cone with centre at \( M' \) and radius \( R \) (Fig. 1).

Suppose there are many of these beer producers on an unbound isotropic plane, they will pack together to secure a hexagonal market area of their own, with radius of its inscribed circle being \( R \), which is considered to be the most efficient way to utilize the plane and to maximize number of suppliers. This regular distributional pattern of beer producers is termed the basic lattice (基础集落網), where purchasing power is assumed to cluster evenly at the points for beer production, or the basic settlements, to form discrete demand space.
Further goods with greater threshold requirement need more than one settlements to serve. A good with threshold requirement of three settlements can and will be located through spatial competition in the manner shown in Fig. 3. Here each settlement except that at the centre is shared among three market areas, thus having value of 1/3 to each supply centre. One centre can therefore dominate equivalent of 1/3 x 6 = 2 settlements of purchasing power plus one as the centre itself, three in all. The market area served by this type of centre is named Market Area No 1. In the same manner, a good requiring demand of four or seven settlements will be located as in Fig. 3, which shall be named Market Area No 2 or 3 respectively. Due to geometrical constraint it is not possible for goods with threshold requirements of two, five or six settlements to find their own supply centres. They must instead share centres with the nearest larger number of settlements, in which case excess profit unable to be eliminated will accrue to them. Table 2 shows the first ten possible market areas with number of entire settlements supplied.

Table 2: The comparison as to number of settlements supplied between Berry-Garrison and Lösch.

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<th>Order of centre</th>
<th>Berry-Garrison</th>
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<th>Lösch</th>
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<td>Nomenclature of place</td>
<td>Number of basic settlements as a whole supplied</td>
<td>Nomenclature of place</td>
<td>Number of basic settlements as a whole supplied</td>
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<td>1</td>
<td>Centres</td>
<td>M</td>
<td>(beer-supplying)</td>
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<td>L</td>
<td>3</td>
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<td>3</td>
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<td>4</td>
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<td>H</td>
<td>243</td>
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<td>G</td>
<td>729</td>
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<td>F</td>
<td>2,187</td>
<td>7</td>
<td>16</td>
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<td>9</td>
<td>E</td>
<td>6,561</td>
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<td>19</td>
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<td>D</td>
<td>19,683</td>
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<td>C</td>
<td>59,049</td>
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Remarks: The lowest order, component of the basic lattice.
Fig. 3: The comparison between Berry-Garrison and Loschian systems.
These market areas with various size and distribution of its supply centre are then all superposed to obtain a single economic landscape. In so doing Lösch assumed one ‘metropolitan centre’ at the centre of the plane where every kind of goods is offered, and the maximum coincidence or proximity of supply centres whenever possible. The outcome is the well-known ‘economic landscape’. There exist two alternatives for location of the centre of market areas with asymmetrical position to the basic lattice. Choosing either determines the location of all subsequently higher order supply centres, which results in the sectors with more supply centres (‘city rich’) and less (‘city poor’). When the landscape is extended to include Market Area No. up to 150, Fig. 4 is obtained.

This is the essentials of the Löschian system.

IV. COMPARISON BETWEEN BERRY-GARRISON’S AND LÖSCHIAN SYSTEMS

Differences in theoretical framework of these two systems have been pointed out already by various authors.

Peter Haggett, for example, comments (7, pp. 146–150): firstly, in Christaller k-value is fixed to either 3, 4 or 7 as opposed to variable k value with all possible hexagonal solutions adopted by Lösch; secondly, size and functions of the centres in a particular tier in Christaller’s system are identical whereas in Lösch ‘settlements of the same size need not have the same function’ (7, p. 150), thirdly, all higher order places always contain functions of the smaller places in case of Christaller, which is not a requirement in Löschian system.

After Böwenter, differences are stated as follows (4, pp. 168–173): firstly, the deviations from the optimal spatial layout for the individual goods are much smaller in Löschian than in Christaller’s system due to greater number of possibilities for individual market areas; secondly, partial specialization of production between different places exists in Löschian system; thirdly, Löschian system does not make any statements about the sizes of the places; fourthly, the top to the bottom procedure is adopted by Christaller, as opposed to ‘the bottom to the top’ by Lösch.

These comparisons, although apparent discrepancies have been made clear, are still in want of deeper consideration as to the implied or expressed differences in assumptions and procedures that eventually lead to the two alternate central-place systems. A statement quite suggestive to this point is that made by Parr (12). He stated that in Löschian system ‘the possibility of localization economies (the advantages to like firms which are derived from clustering) is completely absent from the scheme’ (12, p. 190), while Christaller ‘is able to accommodate the influence of revenue-based agglomeration economies, the bases of which stem from the propensity of consumers to engage in multi-purpose trip-making’ (12, p. 191). He hinted here, economies of agglomeration implied behind both systems might play an decisive role for determining these two different arrangement of supplying functions.

(For fig. 3)

<table>
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<tr>
<th>Sector</th>
<th>Size of centres (number of coincident supply functions) indicated after:</th>
<th>The numerals indicate:</th>
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<tr>
<td>1 Berry-Garrison</td>
<td>threshold requirement, in number of settlements, of functions available at each centre ('1' is not shown as it is obvious to be available at all the centres).</td>
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<tr>
<td>2 Berry-Garrison</td>
<td>number of coincident supply functions at each centre after Lösch.</td>
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<td>3 Berry-Garrison</td>
<td>number of settlements as a whole in various market areas with the boundary on which number of settlement required appears (Nos. 3, 9, 27 and 81 are identical with those of Berry-Garrison system).</td>
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<td>4 Lösch</td>
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<tr>
<td>5 Berry-Garrison</td>
<td>number of coincident centres for various market areas after Lösch.</td>
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Parr with Denike (11) also suggested a way to compare these two systems, in that Lüscher basic lattice is taken as the 'locational reference points' (11, p. 573) for Christaller's as well. Later Marshall (9) practised this, where the \( k = 3 \) Christaller system is derived from Lüscher lattice as in Fig. 5. In fact this is the best way to discover implied rationales behind these systems, since in this manner the differences which purely contribute toward formation of the two different systems will clearly be left obvious.

Now let us examine in detail how the \( k = 3 \) Berry-Garrison system\(^8\) can be constructed on Lüscher lattice, to elaborate Parr’s statement above. The lowest-order place \( M \) in Berry-Garrison is identified with the location of beer suppliers in Lüscher, which only supplies to their own complementary region. The second lowest-order supply centre of Berry-Garrison \( L \) will cater for demand equivalent to three complementary regions of basic settlement according to the \( k = 3 \) system. The third lowest order centre of Berry-Garrison \( K \) is equally under constraint of the \( k = 3 \) system, which therefore must cater for nine complementary regions of basic settlement, or three of centre just one step below, \( L \). In the same manner, the fourth lowest \( J \) will include 27 complementary regions or three of centre \( K \), the fifth \( 81 \) regions or three of centre \( J \), etc, to maintain the \( k = 3 \) principle for all successively higher order centres alike.

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\(^8\) The \( k = 3 \) Berry-Garrison system refers to a hierarchical model of urban settlements where each higher-order centre serves a region that is three times the area of its lower-order counterparts, maintaining a constant density of population or activity.
In Lőschian system on the contrary, although the first and second lowest order centres are identical in arrangement, the third lowest order centre Market Area No. 2 caters for only four basic settlements in the manner similar to Christaller's \( k = 4 \) transport principle, as opposed to nine in Berry-Garrison, and the fourth lowest Market Area No. 3 for seven basic settlements like Christaller's \( k = 7 \) administrative principle, as opposed to 27 in Berry-Garrison system, etc. Table 2 compares this difference for eleven lowest order centres.9

A central place is however merely a spatial clustering of one or various suppliers. It is these suppliers that carry out actual supply functions, but not a centre itself. Since each supply function has its own threshold requirement, goods supplied from two centres of the same order both to Berry-Garrison and to Lősch, may differ greatly due to different size of complementary regions. To take the same order of centre in Table 2, it is clear that in Berry-Garrison it supplies far more goods of higher-order than in Lősch, except for two lowest-order centres. In Fig. 3, it is quite clear that centres spread out more scattered in Lőschian system, where 29 centres out of 59 in a 60° sector shown in the Figure belong to 'K' order in terms of number of functions supplied, as opposed to only 5 in Berry-Garrison.10

Table 3: The comparison as to variety of goods supplied from centres of different order between Berry-Garrison and (Lősch).

<table>
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<th>Threshold requirement of good (in unit of a basic settlement as a whole)</th>
<th>This good is supplied from centre(s):</th>
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<td>1*</td>
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<td>Metro-</td>
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**KEY:**
- **b or B** ... goods supplied in case of Berry Garrison's system.
- **L or L** ... goods supplied in case of Löschian system.
- **B or L** ... the lowest-order centre that supplies respective goods.
- **B** ... hierarchical marginal good of Berry-Garrison's system.
- ***** ... (from the top) order of centre, nomenclature by Berry-Garrison, that by Lösch.
- **** ... some inevitable clusterings
Looking into variety of functions itself supplied, a centre in Berry-Garrison system which supplies goods with threshold requirements ranging from 1 to an nth power of 3 also supplies in Löschian the good exactly with requirement of the nth power of 3 settlements (i.e. the hierarchical marginal good), quite often those immediately precede it and those requiring lower-than-the-nth powers of 3 settlements. For example, the centres of Lösch equivalent in position to Berry-Garrison’s 1st-order centre offering goods requiring 1–81 (= 3⁴) settlements also supply goods requiring 81 and 80 settlements, in addition to the lower order hierarchical marginal goods and some that precede immediately, or those requiring 27 (= 3³), 26, 9 (= 3²), 8, 3 (= 3¹), 2 and 1 settlements. The same principle applies to all other hierarchical systems with higher k values. a good requiring 64 (= 4³) settlements always comes together with those of 16 (= 4²) and 4 settlements, and 49 (7²) with 7 etc. Fig. 4 presents this point in wider scope. the centre of Market Area No. 77 (with 243 = 3⁵ settlements) cluster together with those of No. 30 (81 settlements), 11 (27 settlements), 4 (9 settlements) and 1 (3 settlements); that of Area No. 81 (256 = 4⁴ settlements) is always with No. 24 (64 settlements), 7 (16 settlements) and 2 (4 settlements); that of Area No. 106 (343 = 7³ settlements) is with No. 19 (49 settlements) and 3 (7 settlements), that of Area No. 55 (169 = 13² settlements) is with No. 6 (13 settlements) etc.¹¹ In short, Löschian economic landscape also displays infinite number of hierarchical arrangements of supply functions with various ‘fixed-k’ values, yet these centres concerned equipped only with supply of the respective hierarchical marginal good and the hierarchical marginal goods of all the centres lower than it in order.

Needless to say the functions are so arranged because it is geometrically most ideal also from the constraint of Lösch. In Löschian system, the location of each centre with varying size of threshold requirement is determined by the relative position of its market area to the basic lattice, or the arrangement of basic settlements within its market area. A ‘fixed-k’ arrangement on the other hand implies that a higher-order network of centres with a spatial arrangement of the component lower-order centres just one step below is always constructed upon the network of that component centres, each of which has again with exactly the same spatial arrangement of the component centres one step further below in order. It means in fixed-k hierarchy, the relative position of the market areas of centres is all the same to every different order. Thanks to this homogeneity, the superposition of different order centres with the same k-value can always effected successfully, by in the meantime satisfying the maximum-coincidence requirement, into a uniform ‘fixed-k’ hierarchical pattern of centres without displacements to supply at least all hierarchical marginal goods there.

V. RATIONALE OF LÖSCHIAN SYSTEM

The threshold requirement or minimum scale of supplying goods is determined through joint action of various economic factors by each goods supplied independently, without taking other supply functions into account at the initial stage. Hence, if there were no economic relations at all among these supply functions, each of them would determine their own location for supply centre by themselves to supply goods to its market area with appropriate size. Here would be no regular pattern of clusterings of supply centres nor of their complementary regions, and formation of hierarchical system of central place as a whole would not take place. Neither guarantees economic relation of any kind between supply functions automatically emergence of a central-place hierarchy, since such relation of dissociating nature would induce all supply functions to be located each other as far apart as possible, to result in the pattern as Fig. 6. Here all suppliers, even those with the identical threshold requirement, are located equidistant from each other, with no hierarchical arrangement.

Therefore it is quite obvious that there must be some common economies of agglomeration in order various supply functions to cluster together into a supply centre, rather to locate themselves apart each other. This can occur when consumers engage in ‘multi-purpose trip’ or take benefit mutually from a transportation route, network or other urban infrastructure.

To return to Lösch, there are three locational requirements for his system. one ‘metropolitan centre’ at the core of the landscape, all supply functions on one of the basic settlements and maximum coincidence or proximity of functions whenever (and only when) possible. Yet Lösch himself never stated any sufficient reasons for such requirements, especially for the ‘metropolitan centre’.¹² This clearly compares with the case of Berry-Garrison, where ‘multi-purpose trip’ is explicitly given as a reason of such clusterings.¹³

Should such economies of agglomeration be considered to the full extent, the picture might seem that all the supply functions to cluster at one and the same place, or no central-place system in its proper sense. This is of course ridiculous, and some dispersion of supply functions is more sensible to suppliers, since in such a manner each supplier may be able to exploit more purchasing power due to reduction in transport costs and inclusion of more consumers into its range limit, and to secure a spatially monopolized market area. The maximum possible dispersion of supply functions still compatible
with the requirement from economies of agglomeration can be attained when maximum number of functions clusters at a place while in the meantime new outlet for supply to be established whenever the minimum purchasing power enough to support it exists. Since the lower is the order of supply function the more the possible outlets in number, no larger degree of clustering is possible than all higher order supply functions to set up their outlets at places where lower-order goods are already offered. Partition into the largest number of outlets in relative terms is possible when area to justify new clustering of supply functions one step below in order is the largest under geometrical constraint of the basic lattice. $K = 2$ system would satisfy these conditions most, because $1/2$ in size of the complementary region of the centre one step above would be already small enough to support the supply of immediately lower-order goods, if it were geometrically viable. Doing it unrealistic, the second best has to be sought: the $k = 3$ system of Berry-Garrison. For this reason, Berry-Garrison's system has great logical consistency in it with respect to the economies of agglomeration.

On the contrary, in Löschian system at centres belonging to each order in a 'fixed-k' hierarchy cluster only supply of the hierarchical marginal goods plus some with immediately smaller threshold requirement. All other goods are supplied from various centres displaced differently from the centres following the rigid 'fixed-k' system. This arrangement is of course due to Lösch's original assumptions as to far larger variety in size of market areas and the three conditions for clusterings. Yet nowhere in his statements rationales why clusterings take place only in those cases are given, but they are simply speculated. It means that Lösch takes the requirement from economies of agglomeration into account only partially and arbitrarily without convincing economic backgrounds. Under different considerations of conditions for clusterings, other unlimited number of arrangements can claim their right as a central-place system, which can never be rejected since one of such claims, the Löschian, has been admitted.

In sum, the difference between these two systems is rooted in the extent how much the economies of agglomeration are taken into account of the systems, and in so doing, Berry-Garrison has full logical consistency while Lösch is quite irrational.

VI. EMPIRICAL VIABILITY OF LÖSCHIAN SYSTEM

The above findings give deeper insight into the empirical viability of these two alternate central-place systems.

As mentioned previously, 'multi-purpose trip' behaviour of consumers is one of the major factors contributing to economies of agglomeration for supply functions. Incentive of consumers to minimize transport cost and effort by purchasing
more items or settle more business at a time gets stronger, the longer a consumer has to travel to visit a supply centre. In this case a supplier of a particular good isolated from spatial association with other functions, or from a central place, would be greatly disadvantaged, thus eventually will lose in the competition among suppliers of the same good and forced to close down. Such a longer trip may occur in reality in case of rural-urban relations, especially when population is sparsely distributed. Incentive of suppliers to cluster together is under these circumstances stronger, which results in a more rigid k-system close to that postulated by Berry-Garrison on the isotropic plane.

On the contrary, if distance of travel required to obtain goods is rather short, or if goods needed are of such particular sort that demand for them arises infrequently or that they must satisfy special desire or taste of consumers, people would hesitate less to arrange a trip which is solely for obtaining one particular type of good. Yet this presupposes that the consumers supporting such centre must be within the range of the good supplied from there, if not within its threshold market area. Here concern of consumers may be to save transport cost less through 'multi-purpose trip' behaviour than through choosing an articles that exactly fits their desire as to quality, style or price etc. In this case suppliers will benefit more by clustering together with those offering the same sort of good than those different to attract more consumers. Such clustering inevitably accompanies enlargement of its market area if most of suppliers there are to survive, and keener competition among them for attracting larger share of consumers visiting the centre by offering goods with higher quality, better style or lower price etc. In another case this type of supplying-function may take place where dispersion of functions is not so wide apart that it allows consumers to hop around these functions with far less time and cost than those would be required to visit a centre from rural areas. This may occur in reality in case of higher population density, or especially of central-place arrangement within a city. The picture here will be somewhat closer to Löschian system, although existence of 'metropolitan centre' is still unjustifiable.

ACKNOWLEDGEMENTS

The points discussed here are a part of my talk 'Three Central Places: Christaller, Berry & Garrison and Lösch' presented in January, 1980 by the GGAS, HKU, and a topic treated in some of my tutorials during the academic year 1979–80. The writer appreciates all participating students in these occasions, who sometimes gave me quite stimulative suggestions to formulate this paper. The writer is also grateful for Mr H. K. Chiu, who kindly took great pains to prepare Figs. 1, 2 and 3.

NOTES

1. Although their outlooks seem somewhat similar, Lösch formulated his theory on central place without knowledge of the work by Christaller (see Lösch, S.93, the English version p. 133). Therefore it is simply wrong to attribute the origin of central-place theory solely to Christaller and to regard only improvements took place thereafter.

2. The nomenclature of 'range' is thus used in quite ambiguous manner by various authors, hence in this paper it shall always be taken to mean 'upper limit' of range.

3. To this point a confusion exists when Berry-Garrison states that 'market areas will be bounded by lower limits to the range of A centres' (Berry-Garrison, p. 112). Remember, goods with various threshold and range size are supplied from centres with various size of complementary region.

4. See Saey (11, pp. 186–190), for example.

5. Carter sees this point differently that 'MF (see Fig. 2) is the upper limit' (5, p. 83). However, referring back to Christaller's original definition (6, S. 59, the English version p. 60), the 'upper limit' is the concept to indicate the point beyond which consumers cannot obtain a good, NOT which suppliers will not supply it. In Fig. 2 it is quite indefinite whether consumers can or cannot obtain it beyond or within the area with radius MF, whereas in Fig. 1 consumers are clearly unable to afford it beyond PF due to excessively high price. Therefore the equivalent of Christaller's 'upper limit' or range in Lösch must be PF, not MF.

6. As stated by Lösch (see 8, S. 73–74, the English version p. 108), is the envelope of the minimum average costs of various cost schedules in different scale with different technologies, not a cost schedule of one single producer. For this reason, the equilibrium is reached not at the intersection of marginal revenue and cost curves as stipulated by Parr and Denike (11, p. 569), but at the point where one out of various average costs (a) and one out of many alternative price levels attained simultaneously at various distances from the supply centre (Δ) meet each other through perfect competition, i.e., the point N.
7. This derives from Lösch's ideal for independent existence in that men are 'to live by their own wills and on their own responsibility' (8, S. 47, the English version p. 67), 'in harmony with his nature' (8, S. 46, the English version p. 66). From this point he highly praised 'a new tendency toward the maximization of the number of economic units' (8, S. 47, the English version p. 67). However 'maximization of the number of economic units' under such ideal is never a concern of individual producer, whose main interest is always to cut their production cost inclusive of transport per unit to the lowest possible level. Such being the case, producers with higher production cost due to smaller scale to ensure the maximum entries as praised by Lösch shown in Fig. 2 cannot survive or even enter in reality. The actual size of an individual market area for the optimum scale must therefore be larger than that Lösch presumed. Yamana (15, pp. 138–138) has commented on this failure of Lösch by stating: 'positive factor determining the scale of production and price of commodity lies on the side of production conditions, while demand may restrict it only in passivity.'

8. Christaller's system adopted by Parr (and Denike) (11, 12) and Marshall (9) as the counterpart of Lösch is as mentioned previously based on range concept, hence in fact not suitable for such comparison. The 'fixed-k' system based on threshold by Berry-Garrison should instead be used.

9. It is sometimes claimed that the uniqueness of Christaller-type central-place system arises from his procedure to construct the system: the top to the bottom (see 4, p. 168–9). This is however merely a matter of convenience, and the reverse direction results in the same system as well if appropriate premises are still kept, as demonstrated here.

10. It deserves to note that there are sometimes great discrepancies between relative degrees in clustering of functions and that of centres. To take an example, number of functions of a place where 2 supply centres cluster together ranges from 3 to 10 (see Fig. 3, compare upper left with lower right sector). All previous discussions on Löschian central place system, solely based on the data how many centres cluster, may therefore be quite misleading in determining 'hierarchies' in Löschian system.

11. For the relation between Market Area Nos. and numbers of settlements served see Beavon (1, p. 92–6). Table 6.3)

12. This irrationality has already been pointed out by Stolper (14, p. 632).

13. Böventer is again incorrect in his statement: 'on each trip made by a household member only one purchase must be made, ... there must be no external economies in shopping' (4, p. 172). Existence of external economies is an essential condition implied for both system, if not explicitly stated.

14. In this sense, the alleged complete absence of agglomeration economies in Löschian system by Parr (12, p. 190) is not justifiable, although Parr's claim that Lösch fails to consider possible change in spatial demand curve in case of multi-purpose trip still holds true.

15. The empirical study on intra-urban supply-centre arrangements in Cape Town, South Africa by K. S. O. Beavon (1, pp. 62–79) is in favour of Löschian system rather than Christaller's.

REFERENCES


