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Persistence and Snap Decision Making: Inefficient Decisions by a Reputation-Concerned Expert

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Abstract

Behaving consistently is widely observed, which implies that a person clings to his/her initial opinion and ignores future information that may be more accurate. We explain such behavior by proposing a model in which a reputation-concerned expert has two opportunities to recommend a choice to someone. Before making each recommendation, the expert receives a signal whose accuracy depends on his ability; the second signal is always more accurate. Since a high-ability expert is less likely to receive different signals, the expert has an incentive to pretend to have high ability by recommending the same choice throughout all opportunities. This fact results in the *persistency of the initial opinion* even when following the second signal is the efficient choice. Further, we consider the case that the expert has the option to remain silent at the first opportunity, which enables the sending of only the more accurate signal and concealing the receiving of different signals. Nevertheless, we find that the expert has an incentive to break silence at the first opportunity and also persists with the initial opinion, which is the driving force behind the expert's *snap decision*.

JEL classification: D82, D83, D90

Keywords: Reputation, herding, persistency of the initial opinion, snap decision

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1 Introduction

Behaving consistently is a widely observed and often valued phenomenon. Politicians who behave inconsistently are criticized for their behavior and economists who predict economic trends that oppose their previous predictions would not be trusted. Indeed, a preference for consistency is a key motivation behind an individual's behavior (Cialdini, 2006). However, such consistent behavior can lead to inefficient results. For example, consider the following situation. A politician decides to invest in a public project that is likely to generate a profit at first. Over time, however, the project turns out to be likely to make a loss. In this setting, the incentive to behave consistently thus prevents the politician from withdrawing from the investment, which results in an inefficient investment. One might come up with similar examples by replacing politicians with economists or consultants.

Many studies in psychology and behavioral economics have aimed to explain this behavior. In social psychology, for instance, Cialdini (2006) summarizes the evidence on and discusses consistent behaviors.

The present study adds to the body of knowledge on this topic by providing an economic explanation of such behavior. In particular, it proposes a model that comprises an expert (he) and an evaluator (she). The expert has two opportunities to recommend an alternative from two choices. If the last recommendation coincides with the unknown state of the world, he is (monetarily) awarded; otherwise, he is not. In addition to this standard monetary payoff, the expert is concerned about his reputation (i.e., how the evaluator will assess his ability). This assumption is standard in the literature and seems suitable in the situation that we consider. For example, politicians would be concerned about how voters would assess their image (so-called valence politics) and consultants would be concerned about how customers would rate their reputation. A higher reputation would lead to reelection for politicians and customer acquisition for consultants. Hence, assuming such so-called "reputation concern" is natural.

In our model, after the expert's recommendations, by observing the recommendations and the realized state of the world, the evaluator computes her belief regarding the expert's ability, which is also her assessment of his ability.

Before making each recommendation, the expert receives a state-contingent signal whose accuracy depends on his ability. The accuracy of this signal grows over time (e.g., the second signal is more accurate than the first one). Then, in the second period, recommending the choice based on the second signal is efficient. However, the expert is concerned about his reputation (i.e., the assessment of his ability). High-ability experts are less likely to receive inconsistent signals and thus the reputation-concerned expert tries to behave consistently to pretend to have high ability. We show that with a sufficiently small growth rate in signal accuracy, we can observe *persistency in the recommendation* at the perfect Bayesian equilibrium. That is, in the second period, the expert repeats his first recommendation regardless of the second signal.

One may believe that if persistency in the recommendations is observed and is inefficient, the expert would remain silent in the first period, which enables him to report only the second signal by concealing the inconsistency of the signals. Surprisingly, even when the expert has the right to silence in the first period, there is an equilibrium such that he voluntarily recommends his first signal in the first period and persists with his initial recommendation in the second period. We call the behavior of breaking silence in the first period, *snap decision making*. The intuition is as follows. By remaining silent in the first period, the evaluator believes that the expert's decision is based on his second period signal. On the contrary, if the expert employs the strategy of behaving consistently, the evaluator believes that the expert's decision is based on his first signal. Since the second signal is more accurate than the first, the evaluator's assessment of the expert's ability based on the decision using the second signal becomes severe regardless of whether the recommendation coincides with the state or not. Then, the incentive to avoid a severe assessment induces the expert to reveal only the less accurate information. In our model, the incentive to behave consistently works as a commitment device to send only the first signal to the receiver. Therefore, even when the expert can remain silent, there could be unnecessary snap decision making, which leads to an inefficient decision.

2 Related literature

Reputation concern distorts an expert's decision.[1] Falk and Zimmermann (2016) (hereafter, FZ), the closest work to our study, provide a model similar to ours and also show the persistence of decisions. Furthermore, FZ provide experimental evidence. In their model, which differs from ours, there are two types of signal accuracy: the high type learns the state perfectly, whereas the accuracy of the low-type signal grows over time. In our study, in contrast to FZ's model, the set of signal accuracy is an

[1] For example, see Ottaviani and Sørensen (2006a,b), who provide a model with a general signal structure.

arbitrary nonempty, non-singleton subset of the interval $[1/2, 1]$ and the probability distribution is also arbitrary. In addition, while FZ assume that the evaluator does not observe the realized state, we assume that the evaluator observes it. In the examples of politicians and economists, since we can observe the consequences of their behavior, our assumption is more suitable. More importantly, we also consider the case that the expert can remain silent in the first period and show a result called a snap decision, which is not shown by FZ.

This study also relates to the literature on herding behavior when several experts make sequential decisions, since persistency in the expert's recommendation is seen as herding to his initial recommendation. In a seminal work, Scharfstein and Stein (1990) develop a reputational herding model in which. each expert sequentially makes a decision and is concerned about his reputation. These authors show that in each equilibrium, the second expert follows the decision of the first expert regardless of the second expert's signal.

Among models of reputational herding and anti-herding behaviors, those of Levy (2004) and Sabourian and Sibert (2009) are also similar to ours. In both these models, in contrast to ours, while the expert's private signal is drawn once, the public signal is also drawn. In Levy's model, the public signal is drawn before the expert's decision, while in Sabourian and Sibert's model, the public signal is drawn after the expert's decision. Further, after observing the public signal, the expert has the right to change his decision. Levy shows anti-herding to the public signal regardless of the high-ability expert's signal. Related to our model, Sabourian and Sibert show the persistency of the expert's decision after observing the public signal regardless of the realization.

Note that the results of Falk and Zimmermann (2016); Levy (2004); Sabourian and Sibert (2009) show a (semi) separating equilibrium associated with the expert's ability. Therefore, the expert must know his ability in their models. However, in our model, most of the results are stated in a pooling equilibrium associated with the expert's ability. Therefore, it does not matter whether the expert knows his ability or not.[2][3] Indeed, our results can be generalized to the case of an arbitrary information structure about the expert's knowledge of his ability. This point is important since in an ordinary reputational herding model,

[2]Related to this point, as Levy does, Effinger and Polborn (2001) demonstrate that anti-herding behavior is realized in an equilibrium. In contrast to Levy's model, however, Effinger and Polborn assume that the expert(s) do not know their ability.

[3]In their Appendix, FZ also provide a continuous version of their model, which is a modification of Prendergast and Stole (1996). In that model, all states as well as the accuracy of the signals and actions are continuous. FZ show a pooling equilibrium at which decisions are distorted (but not perfectly consistent).

the result depends on whether the expert knows the information about his ability or not. Indeed, while in Scharfstein and Stein's work, the experts do not know their ability, Avery and Chevalier (1999) show that if experts obtain sufficient information about their ability, they anti-herd.

In contrast to Scharfstein and Stein's reputational herding model, Banerjee (1992); Bikhchandani, Hirshleifer and Welch (1992) develop statistical herding models in which experts are only interested in making the correct choice. Since the choices of previous experts also become a signal, the action taken by many experts becomes more attractive, which causes herding behaviors.

Belief persistence, a well-known confirmation bias, also relates to our study. This states that once a belief has been formed, people are reluctant to change it (Nickerson, 1998). Rabin and Schrag (1999), for example, provide a model that describes the preference for belief persistence, which also creates persistency in behavior. In our study, unlike their explanation, the belief is (Bayesian) updated, but only the behavior persists.

In contrast to the studies above, we also address another type of seemingly irrational behavior, namely snap decision making. The logic used to explain this behavior can be seen as a kind of *self-handicapping strategy*—taking a lower-performance action to maintain one's self-esteem (see Tirole, 2002). In our model, by reporting only the first signal, which yields lower performance, the expert can maintain his reputation related to his ability.

3 Model

The model comprises two players: an expert (sender) and an evaluator (receiver). The expert decides to recommend one of two alternatives $R = \{x, y\}$. The state of the world is $\omega \in X = \{x, y\}$. The prior belief that $\omega = x$ is true is $p = 1/2$. The expert has two opportunities to recommend. In opportunity $t \in \{1, 2\}$, he receives independent signal $s_t \in S = \{x, y\}$ with accuracy θ_t (i.e., $p(s_t = \omega \mid \omega) = \theta_t$). The accuracy of the signal grows in terms of the odds ratio; that is, the odds ratio of the signal received at the second opportunity is $\frac{\theta_2}{1-\theta_2} = (1+\alpha)\frac{\theta_1}{1-\theta_1}$, where $\alpha > 0$.[4] This assumption means that the expert receives a more accurate signal at his second opportunity.

The accuracy grows rate α is common and known to each player, while accuracy at the first opportunity

[4] Equivalently, $\theta_2 = \frac{(1+\alpha)\theta_1}{1+\alpha\theta_1}$.

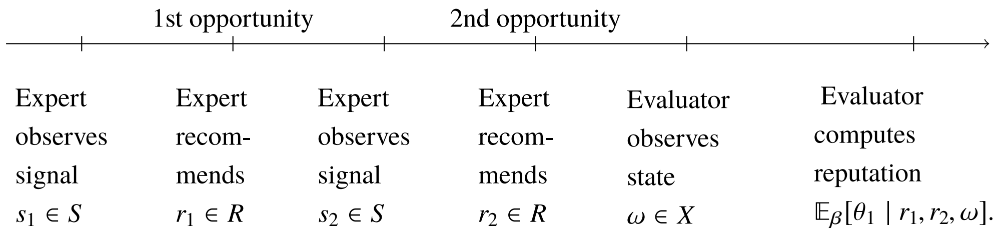

Figure 1: Timeline of the model

θ_1, also referred to as the expert's *ability*, is the expert's private information. We assume that θ_1 is distributed according to cumulative density function F on set $D \subseteq [1/2, 1]$ and that its associated probability density function is denoted by f.[5] Assume that the cardinality of D is no less than 2.[6] Let $\bar{\theta} := \sup D$ and $\underline{\theta} = \inf D$.

The state of the world is revealed to the evaluator after period 2 and she updates her belief regarding the expert's ability.

For each $\theta \in D$, let $r(\theta) = (r_1(\cdot : \theta), r_2(\cdot, \cdot, \cdot : \theta))$ denote the interim strategy, where $r_1(\cdot : \theta) : S \to R$ and $r_2(\cdot, \cdot, \cdot : \theta) : R \times S^2 \to R$. Let $\beta : R^2 \times X \to \Delta(D)$ denote a belief system regarding the expert's ability.

The expert obtain profit $K \geqslant 0$ only when his final recommendation coincides with the state of the world. The expert incurs no cost to change his recommendation. More importantly, the expert also gains profit from the evaluator's assessment about the expert's ability θ_1 (reputation concern). Precisely, let r_t be the recommendation of period t. Then, his (vNM) utility is

$$I(r_2 = \omega)K + \mathbb{E}_\beta[\theta_1 \mid r_1, r_2, \omega],$$

where I is the indicator function: $I(E) = 1$ when the event E is true and $I(E) = 0$ otherwise. $\mathbb{E}_\beta[\theta_1 \mid r_1, r_2, \omega]$ is the expectation of θ_1 with the evaluator's updated belief regarding θ_1. Assume that the expert is risk neutral.

Figure 1 illustrates the timeline of this model.

[5] If D is a countable set, for each x, $f(x)$ is the probability that the expert's ability $\theta = x$.

[6] The set D is allowed to be discrete or continuous.

4 Equilibrium

Let $s_t \in \{x, y\}$ be the signal in period t. Let $p_{s_1 s_2}$ be the posterior belief of the expert regarding the event that $x = \omega$, which are calculated as

$$
\begin{aligned}
p_{xx} &= \frac{\theta\theta_2}{\theta\theta_2 + (1-\theta)(1-\theta_2)}, \quad p_{xy} = \frac{\theta(1-\theta_2)}{\theta(1-\theta_2) + \theta_2(1-\theta)},\\
p_{yx} &= \frac{(1-\theta)\theta_2}{\theta(1-\theta_2) + \theta_2(1-\theta)}, \quad p_{yy} = \frac{(1-\theta)(1-\theta_2)}{\theta\theta_2 + (1-\theta)(1-\theta_2)}.
\end{aligned}
$$

Note that $p_{xx} > p_{yx} > 1/2 > p_{xy} > p_{yy}$.

We focus on pure-strategy perfect Bayesian Nash equilibria (PBE) in which the expert recommends the signal that he receives at the first opportunity.

4.1 Truthful recommendation

Strategy $r = (r_1, r_2)$ is the *truthful recommendation* if the expert recommends the signal that he received in each period. Precisely, for each $\theta \in D$ and each history $(r_1, s_1, s_2) \in R \times S^2$, $r_1(s_1 : \theta) = s_1$ and $r_2(r_1, s_1, s_2) = s_2$. First, we verify whether this strategy is an equilibrium.

Let $\theta_{r_1 r_2 \omega}$ be the ex post expected value of the expert's ability θ_1 with respect to the evaluator's belief after observing recommendations r_1, r_2 and state realization ω. Under the truthful recommendation, these are calculated as

$$
\begin{aligned}
\theta^{Truth}_{xxx} &= \theta^{Truth}_{yyy} = \frac{\int \theta^2\theta_2 f(\theta)d\theta}{\int \theta\theta_2 f(\theta)d\theta} = \frac{\int \frac{\theta^3}{1+\alpha\theta} f(\theta)d\theta}{\int \frac{\theta^2}{1+\alpha\theta} f(\theta)d\theta}\\
\theta^{Truth}_{xxy} &= \theta^{Truth}_{yyx} = \frac{\int \theta \frac{(1-\theta)^2}{1+\alpha\theta} f(\theta)d\theta}{\int \frac{(1-\theta)^2}{1+\alpha\theta} f(\theta)d\theta}\\
\theta^{Truth}_{xyx} &= \theta^{Truth}_{xyy} = \theta^{Truth}_{yxx} = \theta^{Truth}_{yxy} = \frac{\int \frac{\theta^2(1-\theta)}{1+\alpha\theta} f(\theta)d\theta}{\int \frac{\theta(1-\theta)}{1+\alpha\theta} f(\theta)d\theta}.
\end{aligned}
\tag{1}
$$

Superscript *Truth* indicates that this is the assessment under the truthful recommendation.

Suppose that the expert receives signal $s_1 = x$ at his first opportunity and recommends it. At the second opportunity, if the truthful recommendation is the equilibrium, the following two inequalities hold:

$$\frac{\theta^2}{(1-\theta)^2}(1+\alpha) = \frac{p_{xx}}{1-p_{xx}} \geqslant \frac{\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy} + K}{\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} + K} \tag{2}$$

$$\frac{1}{1+\alpha} = \frac{p_{xy}}{1-p_{xy}} \leqslant \frac{\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy} + K}{\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} + K}. \tag{3}$$

The first inequality (2) is the condition that when the expert receives signal $s_2 = x$, $r_2 = x$ is optimal. The second inequality (3) is the condition that when he receives signal $s_2 = y$, $r_2 = y$ is optimal. Let $\varphi(\theta) = \frac{\frac{1}{1+\alpha\theta}f(\theta)}{\int \frac{1}{1+\alpha\theta'}f(\theta')d\theta'}$, $\tilde{\mu} = \int \theta\varphi(\theta)d\theta$, $\tilde{m}_2 = \int \theta^2\varphi(\theta)d\theta$ and $\tilde{m}_3 = \int \theta^3\varphi(\theta)d\theta$, which are respectively the first, second, and third moments of θ with density φ. Then, as long as $|D| \geqslant 2$, for each $\alpha \geqslant 0$, from the statistical facts, $\tilde{m}_3\tilde{\mu} \geqslant \tilde{m}_2^2$ and $\tilde{m}_2 > \tilde{\mu}^2$. Note also that since $\theta \in [1/2, 1]$, $\tilde{m}_2 \geqslant \tilde{m}_3$ and $\tilde{\mu} > 1/2$. Then, we have[7]

$$\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} - (\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy}) = \frac{\tilde{m}_3\tilde{\mu} - \tilde{m}_2^2 + (2\tilde{\mu}-1)\tilde{m}_2^2 + \tilde{m}_3(\tilde{m}_2 - \tilde{\mu}^2) + \tilde{\mu}^2(\tilde{m}_2 - \tilde{m}_3)}{(\tilde{\mu} - \tilde{m}_2)(1 - \tilde{\mu} - (\tilde{\mu} - \tilde{m}_2))\tilde{m}_2} > 0. \tag{4}$$

From this calculation, when $\alpha = 0$, the second inequality (3) becomes

$$1 = \frac{p_{xy}}{1-p_{xy}} \leqslant \frac{\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy} + K}{\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} + K},$$

which is violated since $\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} > \theta^{Truth}_{xyy} - \theta^{Truth}_{xxy}$. Therefore, in the neighborhood of $\alpha = 0$, the truthful recommendation is not an equilibrium.

Proposition 1. *For each $K > 0$, there exists $\bar{\alpha} > 0$ such that for each $\alpha < \bar{\alpha}$, the truthful recommendation is not a PBE.*

4.2 Consistent recommendation

We now consider another type of strategy, say a *consistent recommendation*. Under this strategy, while the expert truthfully reports his signal in the first period, in the second period, he follows the first period choice irrespective of his second signal. Precisely, for each $\theta \in D$, $r_1(s_1 : \theta) = s_1$ and $r_2(r_1, s_1, s_2 : \theta) = r_1$

[7]The calculation is given in Appendix B.1.

if $r_1(s_1 : \theta) = s_1$.[8] Out of the equilibrium path, that is, if $r_2 \neq r_1$, we assume that the evaluator believes that the expert recommended his signals truthfully at each opportunity. Let $\mu = \int \theta f(\theta)d\theta$, $m_2 = \int \theta^2 f(\theta)d\theta$, and $m_3 = \int \theta^3 f(\theta)d\theta$, which are the mean, second-order moments, and third-order moments, respectively. Under the consistent recommendation, the ex post expected values of the expert's ability are

$$\begin{aligned}
\theta_{xxx}^{Cons} &= \frac{\int \theta^2 f(\theta)d\theta}{\int \theta f(\theta)d\theta} = \frac{m_2}{\mu} \\
\theta_{xxy}^{Cons} &= \frac{\int \theta(1-\theta)f(\theta)d\theta}{\int (1-\theta)f(\theta)d\theta} = \frac{\mu-m_2}{1-\mu} \\
\theta_{xyx}^{Cons} &= \frac{\int \theta^2(1-\theta_2)f(\theta)d\theta}{\int \theta(1-\theta_2)f(\theta)d\theta} = \frac{\int \frac{\theta^2(1-\theta)}{1+\alpha\theta}f(\theta)d\theta}{\int \frac{\theta(1-\theta)}{1+\alpha\theta}f(\theta)d\theta} \\
\theta_{xyy}^{Cons} &= \frac{\int \theta\theta_2(1-\theta)f(\theta)d\theta}{\int \theta_2(1-\theta)f(\theta)d\theta} = \frac{\int \frac{\theta^2(1-\theta)}{1+\alpha\theta}f(\theta)d\theta}{\int \frac{\theta(1-\theta)}{1+\alpha\theta}f(\theta)d\theta} = \theta_{xyx}^{Cons} \\
\theta_{yyx}^{Cons} &= \theta_{xxy}^{Cons},\ \theta_{yyy}^{Cons} = \theta_{xxx}^{Cons},\ \theta_{yxx}^{Cons} = \theta_{xyy}^{Cons},\ \theta_{yxy}^{Cons} = \theta_{xyx}^{Cons}
\end{aligned} \tag{5}$$

Superscript *Cons* indicates that this is the assessment under the consistent recommendation.

Let us check whether the consistent recommendation is an equilibrium. Consider the second period. From the symmetry of the states, without loss of generality, the expert receives signal $s_1 = x$ and thus he chooses $r_1 = x$. Suppose that the expert receives signals $(s_1, s_2) = (x, x)$. Then, $r_2 = x$ is the best response if and only if[9]

$$\begin{aligned}
&p_{xx}(\theta_{xxx}^{Cons} + K) + (1 - p_{xx})\theta_{xxy}^{Cons} \geqslant p_{xx}\theta_{xyx}^{Cons} + (1 - p_{xx})(\theta_{xyy}^{Cons} + K) \\
\iff\ &\frac{\theta^2}{(1-\theta)^2}(1+\alpha) = \frac{p_{xx}}{1-p_{xx}} \geqslant \frac{\theta_{xyy}^{Cons} - \theta_{xxy}^{Cons} + K}{\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons} + K}.
\end{aligned} \tag{6}$$

Suppose that the expert observes signals $(s_1, s_2) = (x, y)$ and $r_1 = x$. Then, in the same way, $r_2 = x$ is

[8]Note that this strategy requires consistency only on the equilibrium path.

[9]Note that $\theta_{xxx}^{Cons} > \theta_{xyx}^{Cons}$. This is because

$$\theta_{xxx}^{Cons} > \theta_{xyx}^{Cons} \iff \int \frac{\theta(1-\theta)(m_2 - \theta\mu)}{1+\alpha\theta} df(\theta)d\theta > 0.$$

Since $m_2 - \theta\mu > 0$ if and only if $\theta < m_2/\mu$ and $\theta(1-\theta)/(1+\alpha\theta)$ is decreasing in $\theta \in (1/2, 1)$,

$$\int \frac{\theta(1-\theta)(m_2 - \theta\mu)}{1+\alpha\theta} df(\theta)d\theta \geqslant \int \frac{m_2/\mu(1 - m_2/\mu)(m_2 - \theta\mu)}{1+\alpha m_2/\mu} df(\theta)d\theta = \frac{m_2/\mu(1 - m_2/\mu)(m_2 - \mu^2)}{1+\alpha m_2/\mu} > 0.$$

Therefore, $\theta_{xxx}^{Cons} > \theta_{xyx}^{Cons}$.

the best response if and only if

$$p_{xy}(\theta^{Cons}_{xxx} + K) + (1 - p_{xy})\theta^{Cons}_{xxy} \geqslant p_{xx}\theta^{Cons}_{xyx} + (1 - p_{xx})(\theta^{Cons}_{xyy} + K)$$
$$\iff \frac{1}{1+\alpha} = \frac{p_{xy}}{1-p_{xy}} \geqslant \frac{\theta^{Cons}_{xyy} - \theta^{Cons}_{xxy} + K}{\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} + K}. \tag{7}$$

If $\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} > \theta^{Cons}_{xyy} - \theta^{Cons}_{xxy}$, (6) trivially holds. More importantly, when α is sufficiently small, inequality (7) also holds.[10] Thus, if $\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} > \theta^{Cons}_{xyy} - \theta^{Cons}_{xxy}$, the consistent recommendation is the optimal choice at the second opportunity. Indeed, the following lemma proves the inequality.

Lemma 1. *Under any consistent recommendation, $\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} > (\theta^{Cons}_{xyy} - \theta^{Cons}_{xxy})$ for each $\alpha \geqslant 0$.*

Proof. See Appendix A. □

Therefore, if inequality (7) holds, a consistent recommendation is optimal in the second period. We can also show that recommending s_1 is optimal at the first opportunity.

Proposition 2. *A consistent recommendation is a PBE if and only if inequality (7) holds.*

Proof. See Appendix A. □

The following is a sufficient condition for inequality (7), which is obvious from Lemma 1.

Lemma 2. *For each $K > 0$, there exists $\bar{\alpha} > 0$ such that for each $\alpha < \bar{\alpha}$, inequality (7) holds.*

According to Proposition 2 and Lemma 2, we have

Corollary 1. *For each $K > 0$, there exists $\bar{\alpha} > 0$ such that for each $\alpha < \bar{\alpha}$, a consistent recommendation is a PBE.*

Combining Proposition 1 and Corollary 1, with a sufficiently small α, among the equilibrium where the expert recommends truthfully at the first opportunity, the expert behaves consistently.

Example 1. Suppose that $D = \{1/2, 3/4, 1\}$ and $f(1/2) = a$, $f(1) = b$ and $f(3/4) = 1 - a - b$. Let $V(\alpha) := \frac{\theta^{Cons}_{xyy} - \theta^{Cons}_{xxy} + K}{\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} + K}$ and $W(\alpha) = \frac{\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy} + K}{\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} + K}$. Then, $V(\alpha)$ and $W(\alpha)$ are drawn in Figure 2 ($K = 1.2$, $a = b = 0.4$).

As depicted in Figure 2, when α is smaller than 0.2, inequality (7) is satisfied but (3) is violated. Then, only a consistent recommendation is a PBE.

[10] Note that θ^{Cons}_{xyx} varies by α. According to Lemma 1, $\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} > \theta^{Cons}_{xyy} - \theta^{Cons}_{xxy}$ holds for each $\alpha \geqslant 0$.

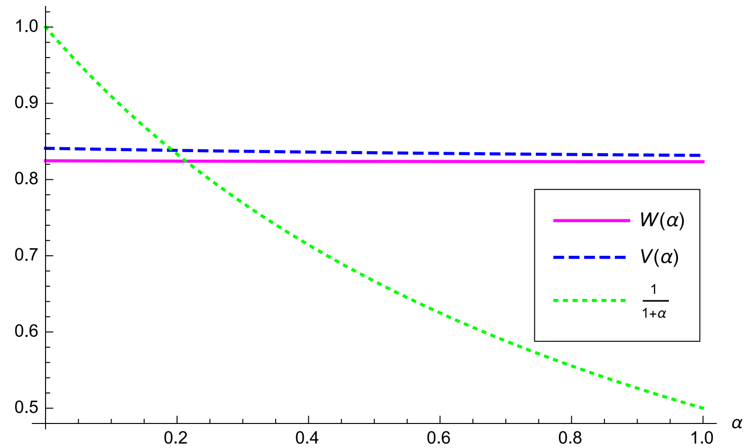

Figure 2: $V(\alpha)$, $W(\alpha)$ and $1/(1+\alpha)$

5 The right to silence

The previous section assumes that the expert necessarily recommends some alternative in all periods. However, as accuracy grows, only the second period signal is necessary for the efficient decision. Thus, there is no need for the first period recommendation. Our question is whether the expert remains silent in the first period if he is allowed. To formalize the question, we add the choice to remain silent in the first period. That is, $r_1 \in R_1\{x, y, \emptyset\}$, where $r_1 = \emptyset$ implies that the expert remains silent at the first opportunity. On the contrary, at the second opportunity, the expert must recommend one of the two alternatives, that is $r_2 \in R_2 = \{x, y\}$.

5.1 Waiting strategy and a snap decision

We focus on the *waiting strategy*, which requires that $r_1 = \emptyset$ and $r_2 = s_2$.[11] We assume that the evaluator has the following out-of-equilibrium belief: if the expert recommends something at the first opportunity, he employs a consistent recommendation regardless of his ability. That is, when $r_1 \neq \emptyset$, the evaluator does not update her belief regarding the expert's ability but rather believes that the expert receives signal $s_1 = r_1$. If inequality (7) holds, the strategy satisfies sequential rationality at the second opportunity.

In the second period, if $r_1 = \emptyset$, since the second signal is more accurate than the first, the evaluator has no information about the expert's first signal and believes that the expert recommends his second

[11]We can consider another type of waiting strategy such that $r_1 = \emptyset$ and $r_2 = s_1$. However, this strategy cannot be a PBE since deviating to $r_2 = s_2$ is necessarily profitable. Therefore, if $r_1 = \emptyset$, it must be $r_2 = s_2$ in the equilibrium.

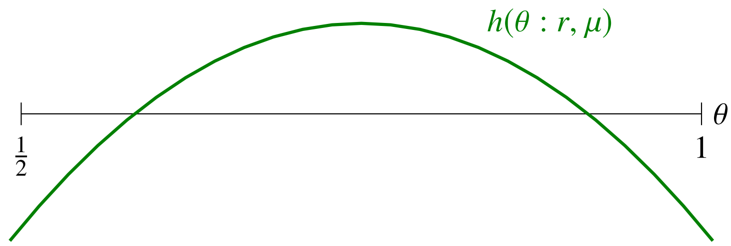

Figure 3: $h(\theta : r, \mu)$

signal; hence, $r_2 = s_2$ is optimal.

Then, the assessments are computed as

$$\begin{aligned} \theta_{xxx} &= \frac{\int \theta^2 f(\theta) d\theta}{\int \theta f(\theta) d\theta}, \quad \theta_{xxy} = \frac{\int \theta(1-\theta) f(\theta) d\theta}{\int (1-\theta) f(\theta) d\theta} \\ \theta_{\varnothing xx} &= \frac{\int \theta \frac{(1+\alpha)\theta}{1+\alpha\theta} f(\theta) d\theta}{\int \frac{(1+\alpha)\theta}{1+\alpha\theta} f(\theta) d\theta}, \quad \theta_{\varnothing xy} = \frac{\int \theta \frac{1-\theta}{1+\alpha\theta} f(\theta) d\theta}{\int \frac{1-\theta}{1+\alpha\theta} f(\theta) d.\theta} \end{aligned} \tag{8}$$

Consider the first period behavior. We compare two strategies: the waiting strategy and the consistent recommendation. Suppose that the expert observes signal $s_1 = x$. Then, the waiting strategy is preferred to consistent recommendations if and only if

$$\begin{aligned} &(\theta + \alpha\theta^2)(\theta_{\varnothing xx} - \theta_{xxx}) + \alpha\theta(1-\theta)(\theta_{\varnothing xx} - \theta_{\varnothing xy} + K) \\ &\geqslant (1 - \theta + \alpha\theta - \alpha\theta^2)(\theta_{xxy} - \theta_{\varnothing xy}). \end{aligned} \tag{9}$$

The derivation of inequality (9) is given in Appendix B.2.

First, we assume that the evaluator does not update her belief regarding the expert's ability when $r_1 \neq \varnothing$. Let $A(r, \beta) = \theta_{xxy} - \theta_{\varnothing xy}$, $B(r, \beta) = \theta_{\varnothing xx} - \theta_{xxx}$ and $C(r, \beta) = \theta_{\varnothing xx} - \theta_{\varnothing xy} + K$. Then, rearranging (9) yields[12]

$$\begin{aligned} h(\theta : r, \beta) := \alpha\theta^2 &(A(r, \beta) + B(r, \beta) - C(r, \beta)) \\ &+ \theta(B(r, \beta) + \alpha C(r, \beta) + (1 - \alpha)A(r, \beta)) - A(r, \beta) \geqslant 0. \end{aligned}$$

Note that since $A + B - C = \theta_{xxy} - \theta_{xxx} - K < 0$, $h(\theta : r, \beta)$ is a concave quadratic function (Figure 3).[13]

[12]When there is no room for confusion, we abuse the notation to drop (r, β).

[13]Note also that with any probability distribution, $\theta_{xxx} > \theta_{xxy}$ and $\theta_{\varnothing xx} > \theta_{\varnothing xy}$.

Under the waiting strategy and belief system β,

$$\begin{aligned}
A(r,\beta) &= \frac{\int \frac{1-\theta}{1+\alpha\theta}(\mu - m_2 - \theta(1-\mu))}{\int \frac{1-\theta}{1+\alpha\theta}(1-\mu)} > \frac{\frac{1}{1+\alpha\frac{\mu-m_2}{1-\mu}} \int (1-\theta)(\mu - m_2 - \theta(1-\mu))}{\int \frac{1-\theta}{1+\alpha\theta}(1-\mu)} = 0 \\
B(r,\beta) &= -\frac{\int \frac{\theta}{1+\alpha\theta}(m_2 - \mu\theta)}{\mu \int \frac{\theta}{1+\alpha\theta}} < -\frac{\frac{1}{1+\alpha m_2/\mu}\int \theta(m_2 - \mu\theta)}{\mu\int \frac{\theta}{1+\alpha\theta}} = 0 \\
C(r,\beta) &= \theta_{\emptyset xx} - \theta_{\emptyset xy} + K > 0.
\end{aligned}$$

When $\theta = 1$, $h(1 : r,\mu) = (1+\alpha)B < 0$. Therefore, under the belief, if $\bar{\theta} = 1$, the consistent recommendation is better than the waiting strategy for some $\theta \in D$ and thus the waiting strategy fails to be an equilibrium.

Lemma 3. *Suppose that inequality (7) holds and $\bar{\theta} = 1$. Suppose also that the evaluator does not update her belief about whether the expert remains silent or not. Then, the waiting strategy fails to be a PBE.*

The intuition is as follows. If the expert remains silent in the first period, the expert's ability is assessed based on only his second period, while when he speaks in the first period, since the consistent recommendation is employed, his ability is assessed based only on his first signal. With a more accurate signal, if the recommendation is accurate, it is attributed to the growth of the signal but less to ability. On the contrary, if the recommendation is inaccurate, it is attributed to ability more than the case with a less accurate signal. Therefore, if the expert remains silent, his ability is discounted regardless of whether his recommendation is accurate or not. Thus, even when the expert has the right to silence in the first period, he has an incentive to employ a consistent recommendation. We call this situation a *snap decision* in the sense that the expert's decision is based on the immediate (and less accurate) signal. We then verify whether the consistent recommendation strategy is a PBE.

Unfortunately, under a certain condition, the consistent recommendation strategy also fails to be a PBE.

Lemma 4. *Suppose that $\underline{\theta} = 1/2$ and $K > 3$. Suppose also that the evaluator does not update her belief about whether the expert remains silent or not. Then, there exists $\bar{\alpha}$ such that for each $\alpha < \bar{\alpha}$, the consistent recommendation fails to be a PBE.*

Proof. See Appendix A. □

In the following subsection, by selecting the out-of-equilibrium belief adequately, we show the existence of a PBE that entails a snap decision.

5.2 Updating the expert's ability according to the first period behavior

Recall that the net payoff of employing the waiting strategy, $h(\theta : r, \beta)$, is a concave quadratic function of θ. Therefore, waiting is less profitable for the high-ability expert and for the low-ability expert, but not for the medium-ability expert.[14]

To consider the case that the evaluator updates his belief about the expert's ability according to the first period behavior, consider the following strategy and belief.

- If $\theta \notin [\theta_*, \theta^*]$: $r_1 = s_1$ and $r_2 = r_1$ when $r_1 = s_1$.
- If $\theta \in [\theta_*, \theta^*]$: $r_1 = \emptyset$ and $r_2 = s_2$.
- The evaluator believes that if $r_1 = \emptyset$, the expert employs the waiting strategy and if $r_1 \neq \emptyset$, the expert employs the consistent recommendation strategy.

With a sufficiently small α, since the truthful recommendation cannot be a PBE, a possible separating equilibrium such that $r_1 \in \{s_1, \emptyset\}$ must be the above strategy.[15] Therefore, a separating equilibrium associated with ability indicates the pair of the above strategy and belief.

Since the difference in the expected utilities between waiting and behaving consistently, denoted by h, is a concave quadratic function, if inequality (7) holds and the waiting strategy is employed by some experts, only this type of strategy is an equilibrium. This type of equilibrium is called *tripartite* (Figure 4). When $\theta_* = 1/2$ or $\theta^* = 1$, the tripartite equilibrium degenerates into a bipartite equilibrium (Figure 5). Possible separating equilibria associated with ability must be tripartite or bipartite.

[14]This result relates to Chung and Eső (2009), who consider a signaling model of a reputation-concerned expert. In their model, the expert chooses between informative and uninformative actions and they show that high- and low-ability experts prefer uninformative action. In our model, choosing to wait is a more informative action than not waiting since the expert reveals the second signal, which is more accurate than the first. This may enable the evaluator to estimate the expert's ability more accurately.

[15]For the uniqueness of the PBE such that $r_1 = \emptyset$, see also footnote 11.

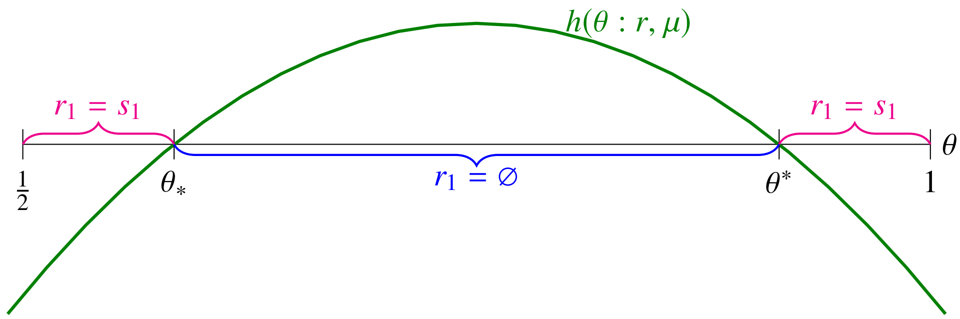

Figure 4: A tripartite equilibrium

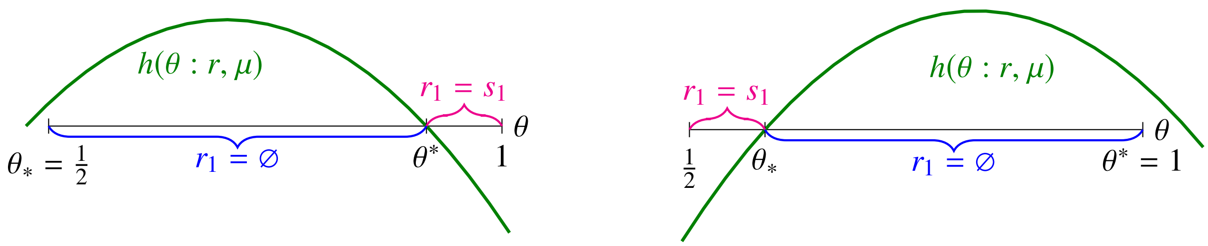

Figure 5: Bipartite equilibria

Note that in the second period, for each type that chooses $r_1 = \varnothing$, $r_2 = s_2$ is the best response. Suppose that (7) holds. Then, the assessments are

$$
\begin{aligned}
\theta_{xxx}(\theta_*, \theta^*) &= \frac{\int_{\theta \notin [\theta_*, \theta^*]} \theta^2 f(\theta) d\theta}{\int_{\theta \notin [\theta_*, \theta^*]} \theta f(\theta) d\theta} \text{ if } [1/2, 1] \setminus [\theta_*, \theta^*] \neq \varnothing \\
\theta_{xxy}(\theta_*, \theta^*) &= \frac{\int_{\theta \notin [\theta_*, \theta^*]} \theta(1-\theta) f(\theta) d\theta}{\int_{\theta \notin [\theta_*, \theta^*]} (1-\theta) f(\theta) d\theta} \text{ if } [1/2, 1] \setminus [\theta_*, \theta^*] \neq \varnothing \\
\theta_{\varnothing xx}(\theta_*, \theta^*) &= \frac{\int_{\theta \in [\theta_*, \theta^*]} \theta^2 \frac{(1+\alpha)}{1+\alpha\theta} f(\theta) d\theta}{\int_{\theta \in [\theta_*, \theta^*]} \frac{(1+\alpha)\theta}{1+\alpha\theta} f(\theta) d\theta} \text{ if } [\theta_*, \theta^*] \cap [1/2, 1] \neq \varnothing \\
\theta_{\varnothing xy}(\theta_*, \theta^*) &= \frac{\int_{\theta \in [\theta_*, \theta^*]} \theta \frac{(1-\theta)}{1+\alpha\theta} f(\theta) d\theta}{\int_{\theta \in [\theta_*, \theta^*]} \frac{(1-\theta)}{1+\alpha\theta} f(\theta) d\theta} \text{ if } [\theta_*, \theta^*] \cap [1/2, 1] \neq \varnothing.
\end{aligned}
$$

First, note that by selecting the out-of-equilibrium belief accordingly, both pooling to wait and pooling not to wait (therefore, a consistent recommendation) are equilibria.

Proposition 3. *(i) The waiting strategy is a PBE. The out-of-equilibrium belief places probability* 1 *on* $\theta = \underline{\theta}$.

(ii) Suppose that inequality (7) *holds and* α *is sufficiently small. Then, the consistent recommendation is a PBE. The out-of-equilibrium belief places probability* 1 *on* $\theta = \underline{\theta}$.

Proof. (i) Consider the waiting strategy. Suppose that $r_1(\theta) = \emptyset$ for each θ and the evaluator imposes the out-of-equilibrium belief that if $r_1 \neq \emptyset$, this places probability 1 on $\theta = \underline{\theta}$. Then, since $\theta_{xxx} = \theta_{xxy} = \underline{\theta}$, when the expert recommends in the first period, there is no difference in reputation whichever the expert recommends in the second period and the reputation is below that when the expert remains silent. Therefore, we need only to consider the monetary reward. However, since the waiting strategy recommends the best choice with respect to the second signal, the other strategies are never better than the waiting strategy. Therefore, the waiting strategy is optimal for each θ, and thus the pair of the strategy and belief constructs an equilibrium.

(ii) Consider the consistent recommendation. Suppose that the expert employs a consistent recommendation irrespective of his ability and the evaluator also has the out-of-equilibrium belief that if $r_1 = \emptyset$, this places probability 1 on $\theta = \underline{\theta}$. Then, since $\theta_{\emptyset xx} = \theta_{\emptyset xy} = \underline{\theta}$, $h_\theta(1/2 : r, \beta) = \alpha(\underline{\theta} - \theta_{xxy}) + (1+\alpha)(\theta_{xxy} - \theta_{xxx}) < 0$.[16] Since h is single-peaked, $h(\theta : r, \beta)$ is decreasing in $\theta \in [1/2, 1]$. We also have that

$$h(1/2 : r, \beta) = \frac{1}{4}\alpha(\theta_{xxy} - \theta_{xxx} - K) + \frac{1}{2}(\theta_{xxy} - \theta_{xxx} + \alpha(K + \theta_{xxy} - \underline{\theta})) - (\theta_{xxy} - \underline{\theta}).$$

Note that $\theta_{xxy} - \theta_{xxx} < 0$ and this does not depend on α. Then, for a small α, $h(1/2 : r, \beta) < 0$ and thus $h(\theta : r, \beta) < 0$ for each $\theta \in D$. Hence, the consistent recommendation is better than waiting for each θ. When inequality (7) holds, as shown in Proposition 2, the consistent recommendation is a PBE. □

In each equilibrium, the out-of-equilibrium belief is natural in the following sense. Consider the case that $K = 0$ and also consider the belief that if $r_1 \neq \emptyset$, $\theta = \underline{\theta}$. Under the belief, $h_\theta(1 : r, \beta) = (1+\alpha)(\theta_{\emptyset xx} - \theta_{\emptyset xy}) + \alpha(\theta_{\emptyset xx} - \underline{\theta}) > 0$. Then, since h is single-peaked, the peak is greater than 1 and thus for each θ, $h(\theta : r, \beta) > h(\underline{\theta} : r, \beta) > 0$. Under the strategy, the expert with $\theta = \underline{\theta}$ is most likely to deviate to $r_1 \neq \emptyset$.

Also consider the latter case. That is, if $r_1 = \emptyset$, the evaluator believes that $\theta = \underline{\theta}$. Under this belief, $h_\theta(1/2 : r, \beta) = \alpha(\underline{\theta} - \theta_{xxx}) + \theta_{xxy} - \theta_{xxx} < 0$. Since h is single-peaked, the peak is less than $1/2$ and thus $0 > h(\underline{\theta} : r, \beta) > h(\theta : r, \beta)$ for each θ. Therefore, the expert with $\theta = \underline{\theta}$ is most likely to deviate to $r_1 = \emptyset$.

[16] By an abuse of the notation, we write $h_z = \frac{\partial h}{\partial z}$ for variable z.

We now discuss the separating equilibria. In each separating equilibrium, if a type employs a consistent recommendation, the type with sufficiently high ability necessarily reveals his signal to the evaluator in the first period.

Lemma 5. *There is no bipartite separation equilibrium associated with ability such that* $\theta^* = 1$.

Proof. Suppose that there is a PBE where $\theta^* = 1$. This implies that $\theta_{xxy} \leqslant \theta_{xxx} < \theta_* < \theta_{\varnothing xy} \leqslant \theta_{\varnothing xx}$. Therefore, $B > 0$ and $A < 0$ and thus $h(1 : r, \beta) > 0$ and $h(1/2 : r, \beta) > 0$. Since h is a concave quadratic function, $h(\theta : r, \beta) > 0$ for each $\theta \in [1/2, 1]$. This shows that the waiting strategy is preferred to any consistent recommendation for each $\theta \in [1/2, 1]$, which is a contradiction. □

Intuitively, when $\theta = 1$, the expert does not need the second period signal for his recommendation since he knows the true state in the first period with certainty. Hence, he is only concerned about his reputation. The advantage of recommending a choice in the first period is only reputation concern. Therefore, this advantage vanishes if and only if the expert with $\theta = 1$ prefers to remain silent. This lemma implies that at each separating equilibrium, the efficiency of the final decision is not increasing in ability in the sense that the high-ability expert makes a snap decision, while the medium-ability expert does not.

Finally in this section, let us mention the existence of a separating equilibrium associated with the expert's ability. In fact, in general, with a sufficiently small α, there is no pure-strategy separating equilibrium considered above.

Proposition 4. *Suppose that* $K > 3$. *Then, there exists* $\bar{\alpha}$ *such that for each* $\alpha < \bar{\alpha}$, *no separating equilibrium is associated with the expert's ability.*

Proof. See Appendix A. □

Although we assume a large K, this proposition is not trivial in the sense that no type makes a consistent recommendation. Indeed, note that for each $K > 0$, with a sufficiently small α, both the consistent recommendation and the waiting strategy are PBE.

By combining Lemmata 3 and 4, and Proposition 3 and 4, under certain conditions, only the strategies shown in Proposition 3 are PBE.

6 Extension

6.1 Unknown ability

In the basic model, we assumed that the expert knows his ability. In the literature on reputational herding, however, many studies assume that the expert does not know his ability. Our discussion is similar regardless of whether the expert knows his ability or not.

To provide a concise discussion, we modify our game as follows. The expert does not know his ability, but he receives a signal about his ability, denoted by τ. Moreover, under ability signal τ, the density function of θ is denoted by $f(\theta \mid \tau)$. Let T be denoted by the set of ability signals. We impose no restriction on T and $(f(\cdot \mid \tau))_{\tau \in T}$. Therefore, this includes the known ability case as a special case such that $T = D$ and for each $\theta \in D$, $f(\theta \mid \theta) = 1$ and $f(\theta' \mid \theta) = 0$ for each $\theta' \neq \theta$. The information structure also includes the case that the expert obtains no prior information about his ability by assuming that $|T| = 1$.

Then, we have the following corollaries, which are modifications of Proposition 1 and Corollary 1 respectively. The proofs are given in Appendix A.

Corollary 2. *Suppose that the expert receives a signal about his ability. Then, for each $K > 0$, there exists $\bar{\alpha} > 0$ such that for each $\alpha < \bar{\alpha}$, the truthful recommendation is not a PBE.*

Corollary 3. *Suppose that the expert receives a signal about his ability. Then, for each $K > 0$, there exists $\bar{\alpha} > 0$ such that for each $\alpha < \bar{\alpha}$, a consistent recommendation is a PBE.*

In addition, consider the case that the expert has the option to remain silent. Suppose that the expert receives no information about his ability in the interim stage. Then, in contrast to the known ability case, no snap decision is observed.

Proposition 5. *Suppose that the expert receives no information about his ability in the interim stage. Then, if $K > 0$, for a sufficiently small α, the waiting strategy is a PBE.*

Proof. See Appendix A. □

6.2 Different accuracy growth rates

This subsection considers different accuracy growth rates, which are assumed to be constant in the basic model. In this subsection, assume that the growth rate depends on ability θ. Then, as in the basic model, the consistent recommendation is at the equilibrium when

$$\frac{1}{1+\alpha(\theta)} \geqslant \frac{\theta_{xyy}^{Cons} - \theta_{xxy}^{Cons} + K}{\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons} + K}, \tag{10}$$

where

$$\theta_{xxx}^{Cons} = \frac{m_2}{\mu}, \quad \theta_{xxy}^{Cons} = \frac{\mu - m_2}{1-\mu}$$
$$\theta_{xyx}^{Cons} = \frac{\int \frac{\theta^2(1-\theta)}{1+\alpha(\theta)\theta} f(\theta)d\theta}{\int \frac{\theta(1-\theta)}{1+\alpha(\theta)\theta} f(\theta)d\theta}, \quad \theta_{xyy}^{Cons} = \frac{\int \frac{\theta^2(1-\theta)(1+\alpha(\theta))}{1+\alpha(\theta)\theta} f(\theta)d\theta}{\int \frac{\theta(1-\theta)(1+\alpha(\theta))}{1+\alpha(\theta)\theta} f(\theta)d\theta},$$

and thus

$$\begin{aligned}
\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons} - (\theta_{xyy}^{Cons} - \theta_{xxy}^{Cons}) &= \frac{1}{\mu(1-\mu)\int \frac{\theta(1-\theta)(1+\alpha(\theta))}{1+\theta\alpha(\theta)} f(\theta)d\theta \int \frac{\theta(1-\theta)}{1+\theta\alpha(\theta)} f(\theta)d\theta} \\
&\times \Bigg[\int \frac{\theta(1-\theta)(1+\alpha(\theta))}{1+\theta\alpha(\theta)} f(\theta)d\theta \int \frac{\theta(1-\theta)}{1+\theta\alpha(\theta)} \left(\frac{1}{2}M - \mu(1-\mu)\theta\right) f(\theta)d\theta \\
&+ \int \frac{\theta(1-\theta)}{1+\theta\alpha(\theta)} f(\theta)d\theta \int \frac{\theta(1-\theta)(1+\alpha(\theta))}{1+\theta\alpha(\theta)} \left(\frac{1}{2}M - \mu(1-\mu)\theta\right) f(\theta)d\theta \Bigg],
\end{aligned}$$

where $M = (1-2\mu)m_2 + \mu^2$. Assume that $\frac{1+\alpha(\theta)}{1+\theta\alpha(\theta)}$ and $\frac{1}{1+\theta\alpha(\theta)}$ are weakly decreasing in θ.[17] Then, as in the proof of Lemma 1, we can show that $\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons} - (\theta_{xyy}^{Cons} - \theta_{xxy}^{Cons}) > 0$. If $\max_{\theta \in D} \alpha(\theta)$ is sufficiently small, (10) holds for each $\theta \in D$. Therefore, the consistent recommendation can also be an equilibrium.

6.3 Competitive recommendation

This section extends our model to the two-expert setting. There are two experts $\{i, j\}$. These experts share the same accuracy growth rate α but have different abilities, denoted by θ_i, θ_j. Each of them simultaneously

[17] These are guaranteed by assuming that $-1 < \frac{\theta\alpha'(\theta)}{\alpha(\theta)} < 0$ for each $\theta \in D$. For example, $\alpha(\theta) = -a/\theta^2$ satisfies the assumption when $a \in (0, 1)$.

recommends at each opportunity $t \in \{1,2\}$. Each expert knows his ability but not the other's. The evaluator does not know the ability of either expert. Let $\theta_{kr_{i1}r_{j1}r_{i2}r_{j2}\omega} = \mathbb{E}_\beta[\theta_k \mid r_{i1}, r_{j1}, r_{i2}, r_{j2}, \omega]$ be the evaluator's assessment of expert $k \in \{i,j\}$'s ability after the state of the world has been revealed. The term r_{kt} is the recommendation of expert $k \in \{i,j\}$ in period $t \in \{1,2\}$. Expert $k \in \{i,j\}$ is only concerned about $\theta_{kr_{i1}r_{j1}r_{i2}r_{j2}\omega}$. Let μ_k, $k \in \{i,j\}$ be the mean and m_{2k} be the second-order moment of expert k's ability.

Now, we investigate the experts' incentives.

Case 1. $r_{i1} = r_{j1} = x$. Consider expert j's decision. Suppose that $s_{j2} = x$. Then, recommending x is dominant. Consider the case that $s_{j2} = y$. By updating j's belief about i's ability, the expectation of i's ability is

$$\mu_{ixxy} := \frac{\frac{\theta_j(1-\theta_j)(-\alpha m_{2i}+(1+\alpha)\mu_i)}{1+\alpha\theta_j}}{\frac{\theta_j(1-\theta_j)(-\alpha\mu_i+(1+\alpha))}{1+\alpha\theta_j}} = \frac{-\alpha m_{2i} + (1+\alpha)\mu_i}{-\alpha\mu_i + (1+\alpha)},$$

which is independent of θ_j. Then, the condition of recommending y is

$$\frac{\mu_{ixxy}}{1-\mu_{ixxy}} \frac{1}{1+\alpha} \leqslant \frac{\theta_{jxxr_{i2}yy} - \theta_{jxxr_{i2}xy} + K}{\theta_{jxxr_{i2}xx} - \theta_{jxxr_{i2}yx} + K}.$$

Since i recommends x, x is more likely. This incentive of statistical herding (i.e., term $\mu_{ixxy}/(1-\mu_{ixxy})$, Banerjee 1992; Bikhchandani et al. 1992) strengthens the incentive to make the recommendation consistent.

Case 2. $r_{i1} = y$, $r_{j1} = x$. Consider expert j's decision. Suppose that $s_{j2} = x$. The condition of recommending x is

$$\frac{1-\mu_{iyxx}}{\mu_{iyxx}} \left(\frac{\theta_j}{1-\theta_j}\right)^2 (1+\alpha) \geqslant \frac{\theta_{jyxr_{i2}yy} - \theta_{jyxr_{i2}xy} + K}{\theta_{jyxr_{i2}xx} - \theta_{jyxr_{i2}yx} + K},$$

where $\mu_{iyxx} = \mu_{ixxy}$.

When $s_{j2} = y$, the condition of recommending y is

$$\frac{1-\mu_{iyxy}}{\mu_{iyxy}} \frac{1}{1+\alpha} \leqslant \frac{\theta_{jyxr_{i2}yy} - \theta_{jyxr_{i2}xy} + K}{\theta_{jyxr_{i2}xx} - \theta_{jyxr_{i2}yx} + K},$$

where $\mu_{iyxy} = \frac{\alpha m_{2i}+\mu_i}{\alpha\mu_i+1}$.

As in the previous case, the incentive of statistical herding (i.e., term $(1-\mu_{ir_is_{j1}s_{j2}})/\mu_{is_{j1}s_{j2}}$) mitigates the incentive to make the recommendations consistent. However, the incentive of statistical herding is also mitigated by reputation concern since the changing opinion inspires the evaluator because of the possibility that the expert has low ability (*anti*-herding incentive; Levy, 2004).[18] Therefore, the competition effect becomes ambiguous.

6.4 Reputation for the accuracy growth rate

Now, consider the case that while θ is fixed, α is unknown to the evaluator, and the expert is concerned about the reputation of α. Denote the probability density function of α by g on $\mathbb{R}_+$.

Under the truthful recommendation, let $\alpha_{r_1r_2\omega}$ be the assessment of α. Then,

$$\alpha_{xxx} = \frac{\int \alpha \frac{(1+\alpha)\theta^2}{1+\alpha\theta} g(\alpha)d\alpha}{\int \frac{(1+\alpha)\theta^2}{1+\alpha\theta} g(\alpha)d\alpha} = \frac{\int \alpha \frac{(1+\alpha)}{1+\alpha\theta} g(\alpha)d\alpha}{\int \frac{(1+\alpha)}{1+\alpha\theta} g(\alpha)d\alpha}$$
$$\alpha_{xxy} = \frac{\int \alpha \frac{(1-\theta)^2}{1+\alpha\theta} g(\alpha)d\alpha}{\int \frac{(1-\theta)^2}{1+\alpha\theta} g(\alpha)d\alpha} = \frac{\int \alpha \frac{1}{1+\alpha\theta} g(\alpha)d\alpha}{\int \frac{1}{1+\alpha\theta} g(\alpha)d\alpha}$$
$$\alpha_{xyx} = \frac{\int \alpha \frac{1}{1+\alpha\theta} g(\alpha)d\alpha}{\int \frac{1}{1+\alpha\theta} g(\alpha)d\alpha}, \quad \alpha_{xyy} = \frac{\int \alpha \frac{1+\alpha}{1+\alpha\theta} g(\alpha)d\alpha}{\int \frac{1+\alpha}{1+\alpha\theta} g(\alpha)d\alpha}.$$

Since $\alpha_{xxx} - \alpha_{xyx} = \alpha_{xyy} - \alpha_{xxy}$, the truthful recommendation is always optimal.

7 Conclusion

This study investigates the incentive to behave consistently. Although models of consistent behavior are proposed by authors such as Sabourian and Sibert (2009) and Falk and Zimmermann (2016), this study not only modifies their results, but also shows a new result, called a snap decision, which is induced from consistent behavior. The combination of consistent behavior and a snap decision makes the expert's decision inefficient. If the expert was a politician, such an inefficient decision could make the population suffer immense damage. The key finding of our study is that if the expert can obtain a more accurate

[18] On the contrary, if the expert does not know his own ability, this effect does not work.

signal in the future, the client should silence the expert at the first opportunity.

Let us point out the limitations of our study, which offer possibilities for future research. One is that the signals and actions are binary. As FZ do, future work must aim to extend our model to a continuous one and investigate the incentive behind making a snap decision. Another is that in this study, reputation concern is only the expectations of others. Ottaviani and Sørensen (2006a) consider a more general form of reputation concern. Lastly, our result only shows pooling equilibria associated with ability. The separating equilibrium might be more important since knowing which types of experts want to behave consistently may be useful for formulating public policies.

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A Omitted proofs

Proof of Lemma 1. Under the consistent recommendation, the difference becomes

$$\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons} - (\theta_{xyy}^{Cons} - \theta_{xxy}^{Cons}) = \frac{\int \frac{\theta(1-\theta)}{1+\alpha\theta}((1-2\mu)m_2 + \mu^2 - 2\mu(1-\mu)\theta) f(\theta) d\theta}{\mu(1-\mu)\int \frac{\theta(1-\theta)}{1+\alpha\theta} f(\theta) d\theta}.$$

To show that the above equation is positive, note that

$$
\begin{aligned}
&\int \theta(1-\theta)[(1-2\mu)m_2+\mu^2-2\mu(1-\mu)\theta]f(\theta)d\theta \\
&=(\mu-m_2)((1-2\mu)m_2+\mu^2)-2\mu(1-\mu)(m_2-m_3) \\
&=(1-2\mu)(m_3\mu-m_2^2)+\mu(\xi+(2\mu-1)\sigma) \\
&=(1-2\mu)(\mu\xi+\sigma\mu^2-\sigma^2)+(\mu\xi+(2\mu-1)\sigma\mu) \\
&=(2\mu-1)\sigma(\mu(1-\mu)+\sigma)+2(1-\mu)\mu\xi \\
&=(1-\mu)\mu((2\mu-1)\sigma+2\xi)+\sigma^2\underbrace{(2\mu-1)}_{\text{positive}}>0,
\end{aligned}
$$

where $\sigma=\int(\theta-\mu)^2f(\theta)d\theta=m_2-\mu^2$ and $\xi=\int(\theta-\mu)^3f(\theta)d\theta=m_3-3\mu m_2+2\mu^3=m_3-\mu m_2-2\mu\sigma$, which are the variance and skewness, respectively. The last inequality follows from the fact that $\mu>1/2$ and

$$
\begin{aligned}
(2\mu-1)\sigma+2\xi &= \int(\theta-\mu)^2(2\mu-1+2\theta-2\mu)f(\theta)d\theta \\
&= \int(\theta-\mu)^2\underbrace{(2\theta-1)}_{\text{positive}}f(\theta)d\theta>0.
\end{aligned}
$$

Let $\tilde{\theta}=\frac{(1-2\mu)m_2+\mu^2}{2\mu(1-\mu)}$. Note that $(1-2\mu)m_2+\mu^2-2\mu(1-\mu)\theta<0$ if and only if $\theta>\tilde{\theta}$. Since $\int\theta(1-\theta)[(1-2\mu)m_2+\mu^2-2\mu(1-\mu)\theta]>0$, $\tilde{\theta}>\underline{\theta}$. Then, since $1/(1+\alpha\theta)$ is nonincreasing in θ,

$$
\begin{aligned}
&\int\frac{\theta(1-\theta)}{1+\alpha\theta}((1-2\mu)m_2+\mu^2-2\mu(1-\mu)\theta)f(\theta)d\theta \\
&\geqslant \frac{\int(\theta(1-\theta))((1-2\mu)m_2+\mu^2-2\mu(1-\mu)\theta)f(\theta)d\theta}{1+\alpha\tilde{\theta}}>0,
\end{aligned}
$$

which shows that

$$
\theta^{Cons}_{xxx}-\theta^{Cons}_{xyx}-(\theta^{Cons}_{xyy}-\theta^{Cons}_{xxy})=\frac{\int\frac{\theta(1-\theta)}{1+\alpha\theta}((1-2\mu)m_2+\mu^2-2\mu(1-\mu)\theta)f(\theta)d\theta}{\mu(1-\mu)\int\frac{\theta(1-\theta)}{1+\alpha\theta}f(\theta)d\theta}>0.
$$

This inequality holds even when $\alpha=0$. □

Proof of Proposition 2. See the proof of Corollary 3. Proposition 2 is a special case of Corollary 3. □

Proof of Lemma 4. We prove the proposition by showing that for a sufficiently small α, $h(1/2; r, \mu) > 0$. Note that when $\alpha = 0$, $h(1/2 : r, \mu) = 0$. Then, it is sufficient to show that $\lim_{\alpha\to 0} h_\alpha(1/2 : r, \mu) > 0$, which is computed as

$$\lim_{\alpha\to 0} h_\alpha(1/2 : r, \beta) = \lim_{\alpha\to 0} \frac{1}{4}\left(C + 2\left(\frac{\partial B}{\partial \alpha} - \frac{\partial A}{\partial \alpha}\right)\right) < 0$$

Note that $\lim_{\alpha\to 0} \frac{\partial B}{\partial \alpha} = \frac{m_2^2 - \mu m_3}{\mu^2} = \left(\frac{m_2}{\mu}\right)^2 - \frac{m_3}{\mu}$, $\lim_{\alpha\to 0} \frac{\partial A}{\partial \alpha} = \frac{(m_2-m_3)(1-\mu)-(\mu-m_2)^2}{(1-\mu)^2} = \frac{(m_2-m_3)}{(1-\mu)} - \left(\frac{(\mu-m_2)}{(1-\mu)}\right)^2$ and $\lim_{\alpha\to 0} C = \frac{m_2-\mu^2}{\mu(1-\mu)} + K$. Note also that $|\frac{\partial A}{\partial \alpha}| < 3/4$ and $|\frac{\partial B}{\partial \alpha}| < 3/4$.[19]

Therefore, if $K > 3$, $\lim_{\alpha\to 0} h_\alpha(1/2 : r, \beta) > 0$ and thus the consistent recommendation fails to be a PBE. □

Proof of Proposition 4. Suppose by contradiction that for each $\bar{\alpha}$, there exists a separating equilibrium with some $\alpha < \bar{\alpha}$. Without loss of generality, we assume that there exists a separating equilibrium for each α.

Consider a sequence of separating equilibria such that $\lim_{\alpha\to 0} A + B \neq 0$. First, note that in this case, with a sufficiently small α, each separating equilibrium is bipartite, that is $\theta_* = 1/2$ according to Lemma 5. To see this, note that $\lim_{\alpha\to 0} h(\theta : r, \beta) = \theta(A + B) - A$. This fact implies that for each θ, the expert waits if and only if $\theta(A + B) > A$, which is a bipartite equilibrium. From Lemma 5, we state that $A + B < 0$. Then, in each separating equilibrium, the expert waits if and only if $\theta < \frac{A}{A+B}$. Note that for the existence of a separating equilibrium, $\lim_{\alpha\to 0} h(1 : r, \beta) = B < 0$. Furthermore, the threshold $\frac{A}{A+B} \in (1/2, 1)$, which implies that $A < 0$. However, at this separation equilibrium, the expert with lower ability remains silent, implying that $A = \theta_{xxy} - \theta_{\varnothing xy} > \theta^* - \theta_{\varnothing xy} > 0$, which is a contradiction.

Consider the case that $\lim_{\alpha\to 0} A + B = 0$. Then, $\lim_{\alpha\to 0} h(\theta : r, \beta) = -\lim_{\alpha\to 0} A$. Therefore, as long as $\lim_{\alpha\to 0} A \neq 0$, there can only be a pooling equilibrium for a sufficiently small α. Now, consider $\lim_{\alpha\to 0} A = 0$. From the assumption, this fact implies that $\lim_{\alpha\to 0} B = 0$.

If this is a sequence of bipartite separation equilibria, according to Lemma 5, $\theta^* < 1$ and $\theta_* = 1/2$.

[19] Since $\theta \in [1/2, 1]$, we can show that each of $\left(\frac{m_2}{\mu}\right)^2$, $\frac{m_3}{\mu}$, $\frac{m_2-m_3}{1-\mu}$ and $\left(\frac{\mu-m_2}{1-\mu}\right)^2$ is larger than 1/4 and less than 1. The difference is at most 3/4.

This fact implies that $\theta_{xr_2\omega} - \theta_{\varnothing r_2\omega} > \varepsilon$ for each $r_2, \omega \in \{x, y\}$ and for some $\varepsilon > 0$. This is because while $\theta_{\varnothing r_2\omega}$ is on average below θ^*, $\theta_{xr_2\omega}$ is on average above θ^*. Then, it must hold that $\lim_{\alpha\to 0} B < 0$ and $\lim_{\alpha\to 0} A > 0$, which is a contradiction. Therefore, the separation equilibrium must be tripartite with a sufficiently small α, that is $1/2 < \theta_* < \theta^* < 1$. For the existence of a tripartite separation equilibrium for a sufficiently small α, since $\lim_{\alpha\to 0} h(\theta : r, \beta) = 0$, it must hold that $\lim_{\alpha\to 0} h_\alpha(1/2 : r, \beta) < 0$, $\lim_{\alpha\to 0} h_\alpha(1 : r, \beta) < 0$ and $\lim_{\alpha\to 0} h_\alpha(\theta : r, \beta) > 0$ for some $\theta \in [1/2, 1]$ (see Figure 4). Let $\hat{\mu} = \frac{\int_{\theta\in[\theta_*,\theta^*]} \theta f(\theta) d\theta}{\int_{\theta\in[\theta_*,\theta^*]} f(\theta) d\theta}$, $\hat{m}_k = \frac{\int_{\theta\in[\theta_*,\theta^*]} \theta^k f(\theta) d\theta}{\int_{\theta\in[\theta_*,\theta^*]} f(\theta) d\theta}$, $k \in \{2, 3\}$. Then, the following inequality is necessary:

$$\lim_{\alpha\to 0} h_\alpha(1/2 : r, \beta) = \lim_{\alpha\to 0} \frac{1}{4}\left(C + 2\left(\frac{\partial B}{\partial \alpha} - \frac{\partial A}{\partial \alpha}\right)\right) < 0$$

Note that $\lim_{\alpha\to 0} \frac{\partial B}{\partial \alpha} = \frac{\hat{m}_2^2 - \hat{\mu}\hat{m}_3}{\hat{\mu}^2} = \left(\frac{\hat{m}_2}{\hat{\mu}}\right)^2 - \frac{\hat{m}_3}{\hat{\mu}}$, $\lim_{\alpha\to 0} \frac{\partial A}{\partial \alpha} = \frac{(\hat{m}_2 - \hat{m}_3)(1-\hat{\mu}) - (\hat{\mu} - \hat{m}_2)^2}{(1-\hat{\mu})^2} = \frac{(\hat{m}_2 - \hat{m}_3)}{(1-\hat{\mu})} - \left(\frac{(\hat{\mu} - \hat{m}_2)}{(1-\hat{\mu})}\right)^2$ and $\lim_{\alpha\to 0} C = \frac{\hat{m}_2 - \hat{\mu}^2}{\hat{\mu}(1-\hat{\mu})} + K$. Note also that $|\frac{\partial A}{\partial \alpha}| < 3/4$ and $|\frac{\partial B}{\partial \alpha}| < 3/4$.[20]

Therefore, if $K > 3$, $\lim_{\alpha\to 0} h_\alpha(1/2 : r, \beta) > 0$. This fact implies that with a sufficiently small α, $h(1/2 : r, \beta) > 0$, which disproves the existence of a tripartite separating equilibrium. □

Proof of Corollary 2. Consider the second opportunity. Suppose that $r_1 = s_1 = x$ and $s_2 = y$. Then, the truthful recommendation (i.e., recommending y) is optimal when

$$p_{xy}\theta^{Truth}_{xyx} + (1 - p_{xy})(\theta^{Truth}_{xyy} + K) \geqslant p_{xy}(\theta^{Truth}_{xxx} + K) + (1 - p_{xy})\theta^{Truth}_{xxy}.$$

The left-hand side is the expected utility of recommending y and the right-hand side is that of recommending x. Therefore, recommending y is optimal if and only if

$$\frac{p_{xy}}{1 - p_{xy}} \leqslant \frac{\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy} + K}{\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} + K}.$$

Here, p_{xy} is computed as

$$p_{xy} = \int \frac{\theta(1 - \theta_2)}{\theta(1 - \theta_2) + \theta_2(1 - \theta)} f(\theta \mid \tau, s_1 = x, s_2 = y) d\theta = \frac{1}{2 + \alpha}.$$

[20] Since $\theta \in [1/2, 1]$, we can show that each of $\left(\frac{\hat{m}_2}{\hat{\mu}}\right)^2$, $\frac{\hat{m}_3}{\hat{\mu}}$, $\frac{\hat{m}_2 - \hat{m}_3}{1-\hat{\mu}}$ and $\left(\frac{\hat{\mu} - \hat{m}_2}{1-\hat{\mu}}\right)^2$ is larger than $1/4$ and less than 1. The difference is at most $3/4$.

Therefore,

$$\frac{1}{1+\alpha} \leqslant \frac{\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy} + K}{\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} + K}. \tag{11}$$

On the contrary, since the evaluator believes that the expert recommends truthfully, the assessments are given as (1). Therefore $\theta^{Truth}_{xxx} - \theta^{Truth}_{xyx} - (\theta^{Truth}_{xyy} - \theta^{Truth}_{xxy})$ is also calculated as (4). Then, with a sufficiently small α, inequality (11) is violated. □

Proof of Corollary 3. Consider the second opportunity. Suppose that $r_1 = s_1 = x$. As in the proof of Corollary 2, at the second opportunity, recommending x is optimal when

$$\begin{aligned} \frac{p_{xx}}{1-p_{xx}} &\geqslant \frac{\theta^{Cons}_{xyy} - \theta^{Cons}_{xxy} + K}{\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} + K} \quad \text{if } r_2 = x \\ \frac{p_{xy}}{1-p_{xy}} &\geqslant \frac{\theta^{Cons}_{xyy} - \theta^{Cons}_{xxy} + K}{\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} + K} \quad \text{if } r_2 = y. \end{aligned}$$

Here,

$$\begin{aligned} p_{xx} &= \int \frac{\theta\theta_2}{\theta\theta_2 + (1-\theta)(1-\theta_2)} f(\theta \mid \tau, s_1 = s_2 = x) d\theta \geqslant 1/2, \\ p_{xy} &= \int \frac{\theta(1-\theta_2)}{\theta(1-\theta_2) + \theta_2(1-\theta)} f(\theta \mid \tau, s_1 = x, s_2 = y) d\theta = \frac{1}{2+\alpha}. \end{aligned}$$

Note also that since the evaluator believes that the expert makes a consistent recommendation, the assessments are given by (5). Therefore, $\theta^{Cons}_{xxx} - \theta^{Cons}_{xyx} - (\theta^{Cons}_{xyy} - \theta^{Cons}_{xxy})$ is calculated as in the proof of Lemma 1. Since $p_{xx}/(1-p_{xx}) \geqslant 1$ and $p_{xy}/(1-p_{xy}) = 1/(1+\alpha)$ and from Lemma 1, recommending x is optimal if and only if (7) holds. According to Lemma 2, with a sufficiently small α, (7) holds.

Consider the first stage. Without loss of generality, we assume that $s_1 = x$. To show that recommending x is optimal at the first opportunity, let $r_2(y, x, s_2)$ be the continuation strategy when $r_1 = y$. Then, we have the following four cases:

1. $r_2(y, x, s_2) = x$. Let $S(\theta) = \theta\theta_2 + (1-\theta)(1-\theta_2)$, which is the probability that the expert draws the

same signals. Then, recommending x is optimal if and only if

$$\begin{aligned}
&\int \big[S(\theta)(r_{xx}(\theta_{xxx}+K)+(1-r_{xx})(\theta_{xxy})) \\
&+(1-S(\theta))(r_{xy}(\theta_{xxx}+K)+(1-r_{xy})(\theta_{xxy}))\big] f(\theta \mid \tau)d\theta \\
&> \int \big[S(\theta)(r_{xx}(\theta_{yxx}+K)+(1-r_{xx})(\theta_{yxy})) \\
&+(1-S(\theta))(r_{xy}(\theta_{yxx}+K)+(1-r_{xy})(\theta_{yxy}))\big] f(\theta \mid \tau)d\theta.
\end{aligned}$$

Equivalently, from (5),

$$\int [\theta(\theta^{Cons}_{xxx}-\theta^{Cons}_{yxx})+(1-\theta)(\theta^{Cons}_{xxy}-\theta^{Cons}_{yxy})] f(\theta \mid \tau)d\theta > 0.$$

Since $\theta \geqslant 1/2$ and $(\theta^{Cons}_{xxx}-\theta^{Cons}_{yxx}) > (\theta^{Cons}_{yxy}-\theta^{Cons}_{xxy})$ from Lemma 1,[21] the inequality holds.

2. $r_2(y,x,s_2)=y$. Recommending x is optimal if and only if

$$\begin{aligned}
&\int \big[S(\theta)(r_{xx}(\theta^{Cons}_{xxx}+K)+(1-r_{xx})(\theta^{Cons}_{xxy})) \\
&+(1-S(\theta))(r_{xy}(\theta^{Cons}_{xxx}+K)+(1-r_{xy})(\theta^{Cons}_{xxy}))\big] f(\theta \mid \tau)d\theta \\
&\geqslant \int \big[S(\theta)(r_{xx}(\theta^{Cons}_{yyx})+(1-r_{xx})(\theta^{Cons}_{yyy}+K)) \\
&+(1-S(\theta))(r_{xy}(\theta^{Cons}_{yyx})+(1-r_{xy})(\theta^{Cons}_{yyy})+K)\big] f(\theta \mid \tau)d\theta.
\end{aligned}$$

Equivalently, from (5),

$$\begin{aligned}
&\int \big[\theta(\theta^{Cons}_{xxx}-\theta^{Cons}_{yyx}+K)+(1-\theta)(\theta^{Cons}_{xxy}-\theta^{Cons}_{yyy}-K)\big] f(\theta \mid \tau) \\
&\qquad (\theta^{Cons}_{xxx}-\theta^{Cons}_{yyx}+K)\int(2\theta-1)f(\theta \mid \tau) > 0.
\end{aligned}$$

Since $\theta^{Cons}_{xxx} > \theta^{Cons}_{yyx}$ and $\theta > 1/2$, the above inequality holds.

3. $r_2(y,x,x)=x$ and $r_2(y,x,y)=y$. Here, consider the truthful recommendation. Recommending x

[21] Note that $\theta^{Cons}_{yxx}=\theta^{Cons}_{xyx}=\theta^{Cons}_{xyy}=\theta^{Cons}_{yxy}$.

is optimal if and only if

$$\int \big[S(\theta)(r_{xx}(\theta_{xxx}^{Cons} + K) + (1 - r_{xx})(\theta_{xxy}^{Cons}))$$

$$+ (1 - S(\theta))(r_{xy}(\theta_{xyx}^{Cons}) + (1 - r_{xy})(\theta_{xyy}^{Cons}) + K)\big] f(\theta \mid \tau) d\theta$$

$$\geqslant \int \big[S(\theta)(r_{xx}(\theta_{yxx} + K) + (1 - r_{xx})(\theta_{yxy}^{Cons}))$$

$$+ (1 - S(\theta))(r_{xy}(\theta_{yyx}^{Cons}) + (1 - r_{xy})(\theta_{yyy}^{Cons}) + K)\big] f(\theta \mid \tau) d\theta.$$

Equivalently, from (5),

$$\int [(\theta_{xxx}^{Cons} - \theta_{yxx}^{Cons})\theta(2\theta_2 - 1) + (1 - \theta_2)(\theta_{yxy}^{Cons} - \theta_{xxy}^{Cons})(2\theta - 1)] f(\theta \mid \tau) d\theta > 0$$

This inequality holds since $\theta_{xxx}^{Cons} > \theta_{yxx}^{Cons}$, $\theta_{yxy}^{Cons} > \theta_{xxy}^{Cons}$, $\theta > 1/2$ and $\theta_2 > 1/2$.

If inequality (7) holds, the truthful recommendation is inferior to the consistent recommendation and thus recommending x is optimal.

4. $r_2(y, x, x) = y$ and $r_2(y, x, y) = x$. In addition, consider the truthful recommendation. Then, recommending x is optimal if and only if

$$\int \big[S(\theta)(r_{xx}(\theta_{xxx}^{Cons} + K) + (1 - r_{xx})(\theta_{xxy}^{Cons}))$$

$$+ (1 - S(\theta))(r_{xy}(\theta_{xyx}) + (1 - r_{xy})(\theta_{xyy}) + K)\big] f(\theta \mid \tau) d\theta$$

$$\geqslant \int \big[S(\theta)(r_{xx}(\theta_{yxy}^{Cons}) + (1 - r_{xx})(\theta_{yxy}^{Cons} + K))$$

$$+ (1 - S(\theta))(r_{xy}(\theta_{yxx}^{Cons} + K) + (1 - r_{xy})(\theta_{yyx}^{Cons}))\big] f(\theta \mid \tau) d\theta.$$

Equivalently, from (5),

$$\int \big[\theta\theta_2(\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons} + K) + (1-\theta)(1-\theta_2)(\theta_{xxy}^{Cons} - \theta_{yxy}^{Cons} - K)$$
$$+ (1-\theta_2)\theta(\theta_{xyx}^{Cons} - \theta_{yxx}^{Cons} - K) + \theta_2(1-\theta)(\theta_{xyy}^{Cons} - \theta_{yyx}^{Cons} + K)\big] f(\theta \mid \tau) \geqslant 0$$
$$\iff \int \big[\theta\theta_2(\theta_{xxx}^{Cons} - \theta_{xyx}^{Cons}) + K\theta(2\theta_2 - 1)$$
$$+ (1-\theta)(\theta_{xyy}^{Cons} - \theta_{yyx}^{Cons} + K)(2\theta_2 - 1)\big] f(\theta \mid \tau) d\theta \geqslant 0.$$

This inequality holds.

From the above results, recommending x is optimal and thus the consistent recommendation is an equilibrium. □

Proof of Proposition 5. As in the derivation of (9), the waiting strategy is better than the consistent recommendation if and only if

$$\int [\theta(\theta_{\varnothing xx} - \theta_{xxx}) + (\theta_2 - \theta)(\theta_{\varnothing xx} - \theta_{\varnothing xy} + K) - (1-\theta)(\theta_{xxy} - \theta_{\varnothing xy})] f(\theta) d\theta \geqslant 0$$
$$\iff H(\alpha, r, \beta) := \mu B + \int \frac{\alpha\theta(1-\theta)}{1+\alpha\theta} C - (1-\mu) A \geqslant 0.$$

Note that $H(0, r, \beta) = 0$ and

$$\frac{\partial H(0, r, \beta)}{\partial \alpha} = \lim_{\alpha \to 0} \mu \frac{\partial B}{\partial \alpha} - (1-\mu)\frac{\partial A}{\partial \alpha} + (\mu - m_2)C = (\mu - m_2)K > 0,$$

where

$$\lim_{\alpha \to 0} \frac{\partial A}{\partial \alpha} = \frac{(m_2 - m_3)(1-\mu) - (\mu - m_2)^2}{(1-\mu)^2}, \quad \lim_{\alpha \to 0} \frac{\partial B}{\partial \alpha} = \frac{m_2^2 - \mu m_3}{\mu^2},$$
$$\lim_{\alpha \to 0} C = \frac{m_2}{\mu} - \frac{\mu - m_2}{1-\mu} + K.$$

Then, for a sufficiently small α, $H(0, r, \beta) > 0$, and thus the waiting strategy is a PBE. □

B Omitted calculations

B.1 Derivation of inequality (4)

$$
\begin{aligned}
&\theta_{xxx}^{Truth} - \theta_{xyx}^{Truth} - (\theta_{xyy}^{Truth} - \theta_{xxy}^{Truth}) \\
&= \frac{(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))(\tilde{m}_3\tilde{\mu}-\tilde{m}_2^2) - \tilde{m}_2(1-\tilde{\mu})(\tilde{m}_2-\tilde{m}_3) + \tilde{m}_2(\tilde{\mu}-\tilde{m}_2)^2}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{(1-\tilde{\mu})(\tilde{m}_3\tilde{\mu}+\tilde{m}_2\tilde{m}_3-2\tilde{m}_2^2) + (\tilde{\mu}-\tilde{m}_2)[\tilde{m}_2(\tilde{\mu}-\tilde{m}_2) - (\tilde{m}_3\tilde{\mu}-\tilde{m}_2^2)]}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{(1-\tilde{\mu})(\tilde{m}_3\tilde{\mu}+\tilde{m}_2\tilde{m}_3-2\tilde{m}_2^2) + (\tilde{\mu}-\tilde{m}_2)[\tilde{m}_2\tilde{\mu}-\tilde{m}_3\tilde{\mu}]}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{(1-2\tilde{\mu}+\tilde{m}_2)\tilde{m}_3\tilde{\mu} + (1-\tilde{\mu})(\tilde{m}_2\tilde{m}_3-2\tilde{m}_2^2) + (\tilde{\mu}-\tilde{m}_2)\tilde{m}_2\tilde{\mu}}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{(1-2\tilde{\mu})\tilde{m}_3\tilde{\mu} + \tilde{m}_2\tilde{m}_3 - (2-\tilde{\mu})\tilde{m}_2^2 + \tilde{m}_2\tilde{\mu}^2}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{(1-2\tilde{\mu})(\tilde{m}_3\tilde{\mu}-\tilde{m}_2^2) + \tilde{m}_2\tilde{m}_3 - \tilde{m}_2^2 + \tilde{m}_2\tilde{\mu}^2}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{\tilde{m}_3\tilde{\mu} - \tilde{m}_2^2 - 2\tilde{m}_3\tilde{\mu}^2 + 2\tilde{\mu}\tilde{m}_2^2 + \tilde{m}_2\tilde{m}_3 - \tilde{m}_2^2 + \tilde{m}_2\tilde{\mu}^2}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{\tilde{m}_3\tilde{\mu} - \tilde{m}_2^2 + (2\tilde{\mu}-1)\tilde{m}_2^2 + \tilde{m}_2\tilde{m}_3 - \tilde{m}_3\tilde{\mu}^2 + \tilde{m}_2\tilde{\mu}^2 - \tilde{m}_3\tilde{\mu}^2}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2} \\
&= \frac{\tilde{m}_3\tilde{\mu} - \tilde{m}_2^2 + (2\tilde{\mu}-1)\tilde{m}_2^2 + \tilde{m}_3(\tilde{m}_2-\tilde{\mu}^2) + \tilde{\mu}^2(\tilde{m}_2-\tilde{m}_3)}{(\tilde{\mu}-\tilde{m}_2)(1-\tilde{\mu}-(\tilde{\mu}-\tilde{m}_2))\tilde{m}_2}.
\end{aligned}
$$

B.2 Derivation of inequality (9)

The waiting strategy is preferred to consistent recommendations if and only if

$$
\begin{aligned}
&(\theta\theta_2 + (1-\theta)(1-\theta_2))p_{xx}(\theta_{\emptyset xx} + K) + (\theta\theta_2 + (1-\theta)(1-\theta_2))(1-p_{xx})(\theta_{\emptyset xy}) \\
&+ (\theta(1-\theta_2) + (1-\theta)\theta_2)p_{xy}\theta_{\emptyset yx} + (\theta(1-\theta_2) + (1-\theta)\theta_2)(1-p_{xy})[\theta_{\emptyset yy} + K] \\
&\geqslant [(\theta\theta_2 + (1-\theta)(1-\theta_2))p_{xx} + (\theta(1-\theta_2) + (1-\theta)\theta_2)p_{xy}](\theta_{xxx} + K) \\
&+ [(\theta\theta_2 + (1-\theta)(1-\theta_2))(1-p_{xx}) + (\theta(1-\theta_2) + (1-\theta)\theta_2)(1-p_{xy})]\theta_{xxy} \\
&\iff \theta\theta_2(\theta_{\emptyset xx} + K) + (1-\theta)(1-\theta_2)\theta_{\emptyset xy} + \theta(1-\theta_2)\theta_{\emptyset yx} + (1-\theta)\theta_2(\theta_{\emptyset yy} + K) \\
&\geqslant \theta(\theta_{xxx} + K) + (1-\theta)\theta_{xxy} \\
&\iff \theta\theta_2(\theta_{\emptyset xx}) + (1-\theta)(1-\theta_2)\theta_{\emptyset xy} + \theta(1-\theta_2)\theta_{\emptyset yx} + (1-\theta)\theta_2(\theta_{\emptyset yy}) + (\theta_2 - \theta)K \\
&\geqslant \theta(\theta_{xxx}) + (1-\theta)\theta_{xxy} \\
&\iff \theta\theta_{\emptyset xx} + (1-\theta)\theta_{\emptyset xy} + (\theta_2 - \theta)(\theta_{\emptyset xx} - \theta_{\emptyset xy} + K) \geqslant \theta(\theta_{xxx}) + (1-\theta)\theta_{xxy} \\
&\iff \theta(\theta_{\emptyset xx} - \theta_{xxx}) + (\theta_2 - \theta)(\theta_{\emptyset xx} - \theta_{\emptyset xy} + K) \geqslant (1-\theta)(\theta_{xxy} - \theta_{\emptyset xy}) \\
&\iff (\theta + \alpha\theta^2)(\theta_{\emptyset xx} - \theta_{xxx}) + \alpha\theta(1-\theta)(\theta_{\emptyset xx} - \theta_{\emptyset xy} + K) \geqslant (1 - \theta + \alpha\theta - \alpha\theta^2)(\theta_{xxy} - \theta_{\emptyset xy}).
\end{aligned}
$$

The first and second lines are the expected utility of following the waiting strategy and the third and fourth lines are that of following the consistent recommendation. Note that $\theta_2 - \theta = \frac{\alpha\theta(1-\theta)}{1+\alpha\theta}$. The final line is derived by multiplying both sides by $1 + \alpha\theta$.