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The Measurement of Labour Content: A General Approach

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Abstract

This paper analyses the theoretical issues related to the measurement of labour content for general technologies with heterogeneous labour. A novel axiomatic framework is used in order to formulate the key properties of the notion of labour content and analyse its theoretical foundations. The main measures of labour content used in various strands of the literature are then characterised. Quite surprisingly, a unique axiomatic structure can be identified which underlies measures of labour aggregates used in such diverse fields as neoclassical growth theory, input-output approaches, productivity analysis, and classical political economy.

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1 Introduction

The measurement of the amount of labour used in the production of - or contained in - a bundle of goods plays a central role in many different fields and approaches in economics. The definition and measurement of labour aggregates (including human capital), for example, is crucial in debates on the determinants of growth and development,[1] in productivity analysis,[2] and in studies of the relation between technical change and profitability.[3]

In normative economics, the notion of labour content is fundamental in the theory of exploitation as the unequal exchange of labour,[4] but it also plays a pivotal - albeit often implicit - role in Kantian approaches to distributive justice.[5]

Last but not least, labour content is a critical concept in classical approaches. It is central, for example, in structural macrodynamic models in the Ricardian tradition;[6] and in classical price and value theory focusing on the notion of labour embodied.[7]

Outside of simple technologies with a single type of homogeneous labour, however, the concept of labour content is elusive and controversial, and there exists no widely accepted approach to aggregate heterogeneous labour inputs. In productivity analysis, for example, different indices of quality-adjusted labour inputs have been used to study total factor productivity (Jorgenson [24]). In neoclassical growth theory, the controversy on the determinants of growth crucially hinges upon different notions of labour input, or human capital (Jones [23]). In classical political economy, and in exploitation theory, many debates revolve precisely around the appropriate extension of the notion of embodied labour to economies with complex technologies and heterogeneous labour inputs.

Two main approaches have been proposed to the measurement of labour content. In growth theory, for example, "If we do not consider variations in worker quality or in effort, then labor input is the sum of hours worked in a given period" (Barro and Sala-i-Martin [1], p.348). This can be called the *simple additive approach*. Alternatively, if quality and effort are taken into account, then "The overall input is the weighted sum over all categories, where the weights are the relative wage rates" (Barro and Sala-i-Martin [1], p.349). This can be called the *wage-additive approach*.

Interestingly, despite significant differences between the various strands of the literature, these two approaches are also the main ones in input-output theory, and in productivity analysis where the wage additive approach is used to construct quality-adjusted indices of labour input. But also in classical political economy, and exploitation theory, where the wage-additive approach is often considered to reflect the classical economists' view on how to convert different types of labour into a single unit, whereby "the different kinds of labour are to be aggregated via the (gold) money wage rates" (Kurz and Salvadori [30], p.324). According to Smith, for example,

> "It is often difficult to ascertain the proportion between two different quantities of labour. The time spent in two different sorts of work will not always alone determine this proportion. The different degrees of hardship endured, and of ingenuity exercised, must likewise be taken into account. There may be more labour in an hour's hard

[1] See the classic papers by Mankiw et al. [32]; Klenow and Rodriguez-Clare [28]; Hall and Jones [22]; and the more recent contribution by Jones [23].

[2] See Denison [6]; Jorgenson and Griliches [25]; Chinloy [3]; Jorgenson et al [26]; Bureau of Labor Statistics [2]; and Jorgenson et al [27]. Gupta and Steedman [21]; Wolff and Howell [56]; and Flaschel et al. [14] analyse labour content and labour productivity within an input-output theoretic framework.

[3] See Roemer's [44, 45] analysis of technical change in classical linear models.

[4] See Roemer's classic contributions [46] and, more recently, Fleurbaey [15, 16]; Yoshihara [57, 58]; Veneziani [51, 52]; and Veneziani and Yoshihara [53, 54, 55].

[5] See the analysis of Kantian allocations and the *proportional solution* in Roemer [47, 48].

[6] The classic reference is Pasinetti [41, 42]. More recent contributions include Lavoie [31] and Trigg and Hartwig [50].

[7] For a thorough discussion, see Desai [7]; Kurz and Salvadori [30]; and Flaschel [13].

work, than in two hours easy business; or in an hour's application to a trade which it cost ten years labour to learn, than in a month's industry, at an ordinary and obvious employment. But it is not easy to find any accurate measure either of hardship or ingenuity. In exchanging, indeed, the different productions of different sorts of labour for one another, some allowance is commonly made for both. It is adjusted, however, not by any accurate measure, but by the higgling and bargaining of the market, according to that sort of rough equality which, though not exact, is sufficient for carrying on the business of common life" (Smith [49], ch. V, pp.34-35).

And one can similarly interpret Ricardo's arguments that "The estimation in which different quantities of labour are held, comes soon to be adjusted in the market with sufficient precision for all practical purposes, and depend much on the comparative skill of the labourer, and intensity of the labour performed" (Ricardo [43], ch. I, section II, p. 11).[8]

More generally, virtually all of the measures of labour input, or labour content proposed in the literature belong to the class of *linear aggregators*: labour aggregates are defined as the weighted sum of heterogeneous labour inputs, where different approaches advocate different weights. In the simple additive and in the wage-additive approaches, for example, the weights are assumed, respectively, to be equal and to coincide with relative wages. In development accounting, however, other proxies of workers' skills - such as schooling duration - are sometimes used to measure efficiency units and convert different types of labour into a single measure (Jones [23]). In productivity analysis, job-based measures of labour skill requirements have also been used (Wolff and Howell [56]). In classical-Marxian approaches, Krause [29] has suggested that the weights be given by the *reduction vector*, which is defined as the Frobenius eigenvector of the matrix $\mathbf{H} =< h_{ij} >$, where h_{ij} is the amount of type-i labour required directly or indirectly to reproduce one unit of type-j labour (e.g. in the household and education sectors).[9]

This paper tackles the issue of the appropriate *measure of labour content* (henceforth, MLC) for general convex production technologies with heterogeneous labour inputs (described in section 2), by rigorously stating and explicitly discussing some foundational properties that a MLC should satisfy. The purpose is not to adjudicate between alternative approaches and provide *the* unique appropriate index of labour content for *all* strands of the literature mentioned above. Rather, we aim to highlight the conceptual foundations of the main approaches and shed light on the implicit assumptions behind different measures. This is, in our view, a fundamental step in order to determine which measure is appropriate in which context.

One key, novel contribution of the paper is methodological: rather than proposing a MLC and comparing it with other measures in the literature, we adopt an axiomatic approach and discuss the appropriate way of measuring labour content starting from first principles. Although this approach is standard in theories of inequality and poverty measurement (Foster [19]), this paper provides the first application of axiomatic analysis to the measurement of labour content and to quality-adjusted indices of labour inputs, and one of the first applications to classical political economy.[10]

By adopting the axiomatic method, we are able to uniquely characterise the class of linear aggregators used in the literature: the *generalised additive MLC* measures the labour content of a bundle of goods as the weighted sum of the amounts of different types of labour used in production.

[8] Despite some debates on the concept of "abstract labour", the wage-additive measure is consistent also with Marx's ([35], pp.51-2) views on the conversion of complex labour into simple labour, although he refers to a social process, fixed by custom. See Morishima [38] and, especially, recent monetary approaches to classical value theory, such as the 'New Interpretation' (Duménil [9, 10]; Foley [17, 18]; Mohun [37]; Duménil et al. [11]) and the definition of 'actual labour values' by Flaschel [12, 13].

[9] But a definition of weights independent of price information has been proposed also by Okishio [39, 40] and Fujimori [20]. For a thorough discussion of the theoretical importance of additivity of labour measures in classical price and value theory, see Flaschel [13].

[10] Relevant exceptions include recent analyses of labour productivity (Flaschel et al. [14]) and of exploitation as the unequal exchange of labour (Yoshihara [57]; Veneziani and Yoshihara [53, 54, 55]).

This characterisation allows one to precisely identify the common theoretical foundations of *all* of the main measures proposed in the literature. Alternative approaches to the measurement of labour content can then be conceptualised as special cases of the general additive class of MLCs advocating different additional restrictions to determine the weights.

To be specific, in section 3, a MLC is conceptualised as a *binary relation* defined over pairs of bundles of goods, associated production activities, and price vectors such that it is possible and meaningful to say that a certain bundle produced with a certain activity at some prices contains more or less labour than another one.

In sections 4 and 5, we study MLCs that are transitive and complete when comparing the labour content of produced goods *at given prices* - called, (p, w)-labour orderings. Three axioms are analysed which capture important properties of (p, w)-labour orderings. *Dominance* says that if the production of a bundle of goods requires a strictly higher amount of each type of labour, then labour content is strictly higher. *Labour trade-offs* rules out the possibility that the labour content of any bundle of produced goods is determined only by a single type of labour. *Mixture invariance* restricts the way in which the measurement of labour content varies when different production techniques are combined.

The first substantive contribution of the paper is the proof that there is only one class of (p, w)-labour orderings that satisfies these three formally weak and theoretically robust properties (Theorem 1), namely the generalised additive MLC (formally defined in section 4). In other words, setting aside otherwise significant theoretical differences, the three axioms represent the theoretical core of all of the main approaches to labour measurement in the various strands of the literature cited above.

In section 5, we explore the main refinements of the linear approach, and provide two additional characterisations. First, we show that the simple additive MLC is the only measure satisfying Dominance, Mixture invariance, and a strengthening of Labour trade-offs - called *Labour Equivalence* - according to which all types of labour contribute equally to the determination of labour content. Then, we introduce a new axiom, called *Consistency with Progressive Technical Change* which incorporates a classical intuition that capital-using labour-saving technical change should increase labour productivity and decrease labour content. We show that, within the generalised additive class, the wage additive approach is the only one that satisfies this axiom. This confirms the intuition that quality-adjusted measures of labour content capture the relation between technical change and labour productivity in market economies.

Section 6 extends our analysis to comparisons of the labour content of produced goods *when prices may change* and generalises our previous results. Two additional axioms are introduced. One requires that no lexicographic priority rule is applied among different prices in the comparisons of the labour content across the different prices. The other requires that the comparisons of the labour content are invariant with respect to the scales of the bundles of produced goods and their associated production activities. Theorem 2 shows that suitably modified versions of *Dominance*, *Labour trade-offs*, and *Mixture invariance*, together with these mild conditions, uniquely characterise the generalised additive MLC even when prices may vary. We then provide characterisations of the simple additive and wage additive MLCs in this more general context.

Our results depend on the specific properties chosen, and alternative axioms would yield different MLCs. We think that the axioms formalised in this paper have robust theoretical foundations and impose rather weak restrictions on MLCs. Indeed, they incorporate properties often explicitly or implicitly advocated in the literature. But, perhaps more importantly, we see this inherent indeterminacy of the axiomatic approach as a virtue, rather than a shortcoming, for the explicit statement of the properties that a MLC does, or should satisfy helps to clarify the theoretical foundations and properties of different measures. We return to this issue in the concluding section.

2 The basic framework

Consider general economies in which the production of commodities requires produced inputs and different types of labour. There are n produced goods, which may be consumed and/or used as inputs in different production activities. The set of types of labour inputs (potentially) used in production is $\mathcal{T} = \{1, ..., T\}$, with generic elements $\nu, \mu \in \mathcal{T}$.

For any integer $m > 0$, let $\mathbb{R}^m$ (resp., $\mathbb{R}^m_+$, $\mathbb{R}^m_{++}$, $\mathbb{R}^m_-$) denote the (resp., non-negative, strictly positive, non-positive) m-dimensional Euclidean space. Technology is described by a production set $P \subseteq \mathbb{R}^{2n+T}$ with elements - activities - of the form $a = (-a_l, -\underline{a}, \overline{a})$, where $a_l \equiv (a_{l\nu})_{\nu \in \mathcal{T}} \in \mathbb{R}^T_+$ is a profile of labour inputs measured in hours; $\underline{a} \in \mathbb{R}^n_+$ are the inputs of the produced goods; and $\overline{a} \in \mathbb{R}^n_+$ are the n outputs.

This modelling of production is quite general and it allows for any type of heterogeneity in labour inputs. The standard production technologies with homogeneous labour are contained as special cases. Different technologies requiring different types of heterogeneous labour can be represented by different production sets P. For instance, differences in *labour intensity* of each type of labour due to heterogeneous skills or human capital can be formalised as different production sets, since labour input vectors are measured in hours.[11]

In what follows, some economically meaningful and weak restrictions are imposed on the admissible class of production technologies.[12] Let $\mathbf{0} = (0, ..., 0)'$ denote the null vector.

Assumption 0 (A0). P is a closed convex cone in $\mathbb{R}^{2n+T}$ and $\mathbf{0} \in P$.

Assumption 1 (A1). For all $a \in P$, if $\overline{a} \geq \mathbf{0}$ then $a_l \geq \mathbf{0}$.

Assumption 2 (A2). For all $c \in \mathbb{R}^n_+$, there is $a \in P$ such that $\overline{a} - \underline{a} \geqq c$.

Assumption 3 (A3). For all $a \in P$, and for all $(-a'_l, -\underline{a}', \overline{a}') \in \mathbb{R}^T_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+$, if $(-a'_l, -\underline{a}', \overline{a}') \leqq a$ then $(-a'_l, -\underline{a}', \overline{a}') \in P$.

These assumptions are rather general and they are standard in all strands of the literature mentioned in the Introduction, including the canonical neoclassical growth model and standard input-output models. **A0** allows for general technologies with constant returns to scale. **A1** implies that *some* labour is indispensable to produce output. **A2** states that any non-negative commodity vector is producible as net output. **A3** is a standard *free disposal* condition. The set of all production sets that satisfy **A0-A3** is denoted by $\mathcal{P}$.

Let $p \in \mathbb{R}^n_+$ be the vector of prices of the n produced commodities and let $w \in \mathbb{R}^T_+$ be the vector of the wages of the T types of labour. At this stage, there is no reason to restrict $(p, w) \in \mathbb{R}^{n+T}_+$ to be an equilibrium price vector, but in what follows, we shall focus on the economically relevant allocations with a strictly positive wage vector w.

3 Comparing labour content

The main purpose of our analysis is to identify some theoretically relevant and widely shared intuitions about the measurement of labour content, and then analyse what they imply in terms of the appropriate MLC. Consequently, we aim to define axioms that impose restrictions on MLCs that are a priori as weak as possible, from both a formal and a theoretical viewpoint.

Thus, although one may think of many properties that a MLC should possess (including, for example, identifying a meaningful, cardinal amount of labour contained in a bundle), as a starting

[11] Alternatively, one may define activity vectors by measuring each type of labour input in efficiency units, so that the amount of type-ν labour $a_{l\nu}$ would be the product of labour hours times the *intensity* of this type of labour. All of our results would continue to hold under this approach after appropriate changes in the axiomatic system. A focus on labour time is, however, in line with the literature, and it allows us to clarify the conceptual foundations of existing measures.

[12] Vector inequalities: for all x, $y \in \mathbb{R}^m$, $x \geqq y$ if and only if $x_i \geqq y_i$ $(i = 1, \ldots, m)$; $x \geq y$ if and only if $x \geqq y$ and $x \neq y$; $x > y$ if and only if $x_i > y_i$ $(i = 1, \ldots, m)$.

point, and consistently with the literature, we simply require that a *MLC be able to compare the labour content of produced goods*. This choice has two important implications.

First, the existence of an appropriate definition of labour content for non-produced goods is set aside. This is an interesting theoretical question, for example, in environmental economics or in the economics of the household, but it is not the main focus of our analysis. As noted earlier, from an axiomatic perspective, in the first stage of the investigation it is appropriate to restrict the domain of the analysis in order to identify a set of theoretically robust properties and formally weak restrictions that are widely (albeit possibly implicitly) endorsed in the literature.

Second, if a key property of a MLC is to allow one to make meaningful statements of the form: "the bundle of produced goods c contains more labour than the bundle c'", then it can be conceptualised as *a binary relation*.

It is a priori unclear what type of information is necessary in order to make such comparisons. Should only observable variables matter, or rather should one focus on (possibly counterfactual) equilibrium allocations? Should prices enter the definition of labour content? And so on. We adopt the most general approach and allow the MLC to depend on *all* potentially relevant information.

Formally, we consider *profiles* (c, a, p, w), where $c \in \mathbb{R}^n_+$ is a non-negative bundle of goods producible as net output by using activity $a \in \phi^P(c) \equiv \{a' \in P \mid \overline{a}' - \underline{a}' \geqq c\}$ for some $P \in \mathcal{P}$ at the price vector $(p, w) \in \mathbb{R}^{n+T}_+$. Observe that this notation comprises all the information that is *potentially* relevant to the measurement of labour content, but it does *not* imply, for example, that price information must enter the definition of the MLC.

Observe further that very few restrictions are imposed on the variables in the admissible profiles. For example, they might be based purely on actual data, or they might be determined (possibly counterfactually) from optimal, equilibrium behaviour. Indeed, the only restriction imposed on two profiles $(c, a, p, w), (c', a', p, w)$ is that the vectors c and c' be productively feasible according to *some* technologies - a and a', respectively, - but a and a' are not even restricted to be *in the same production set*. In fact, it may be desirable in principle to compare the labour content of one (or more) vectors of net outputs, say, in nations with different technologies, or - in a dynamic perspective - as technology evolves over time.

Let the set of profiles (c, a, p, w) be denoted by $\mathcal{CP}$. Then:

Definition 1 *A measure of labour content is a binary relation $\succsim$ on $\mathcal{CP}$ such that for any (c, a, p, w), $(c', a', p, w) \in \mathcal{CP}$, vector c produced with a at (p, w)* contains at least as much labour *as vector c' produced with a' at (p', w') if and only if $(c, a, p, w) \succsim (c', a', p, w)$.*

Definition 1 provides a rigorous, general framework to study MLCs. For the specification of the desirable properties of a MLC can be seen as the identification of a set of axioms capturing different properties of the binary relation $\succsim$ on $\mathcal{CP}$. Note, for example, that Definition 1 imposes no restrictions on the transitivity and completeness of the relation $\succsim$.[13] This is important because different views can be expressed concerning the comparability of labour content when prices vary, especially if the analysis is not restricted to equilibrium allocations.

Similarly, Definition 1 imposes no restriction on the role of prices in the measurement of labour content. A central question concerns whether prices should enter the definition of labour content and, if so, whether only equilibrium prices should matter. This is a rather controversial issue and various views have been proposed in the literature, depending also on the focus of the analysis. Definition 1 is compatible with different views: at this stage, we simply allow for the *possibility* that the measurement of labour content depends on (equilibrium or disequilibrium) prices. Further, as noted above, by allowing the binary relation $\succsim$ to be potentially incomplete, Definition 1 allows for the possibility that the measurement of labour content be restricted to comparing bundle/technology pairs $(c, a), (c', a')$ only at given prices (p, w).

[13] Let $x \equiv (c, a; p, w)$. For any $x, x', x'' \in \mathcal{CP}$, $\succsim \subseteq \mathcal{CP} \times \mathcal{CP}$ is *reflexive* if and only if $x \succsim x$; *transitive* if and only if $x \succsim x'$ and $x' \succsim x''$ implies $x \succsim x''$; and *complete* if and only if $x \succsim x'$ or $x' \succsim x$.

The next sections analyse these issues and discuss the desirable properties that $\succsim$ should possess. In what follows, the asymmetric and the symmetric factors of $\succsim$ are denoted, respectively, as $\succ$ and $\sim$. They stand, respectively, for "contains strictly more labour than" and "contains the same amount of labour as".[14]

4 The foundations of labour measurement

In order to identify some basic, minimal properties that a MLC should satisfy, this section focuses on a subset of the set of possible MLCs by restricting attention to measures that can rank any profiles with the *same price vector*. Formally:

Definition 2 *For any (p, w), a measure of labour content $\succsim$ on $\mathcal{CP}$ is a (p, w)-*labour ordering *if there exists an ordering $\succsim_{(p,w)}$ on $\mathbb{R}^T_+$ such that for any $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$, $(c, a, p, w) \succsim (c', a', p, w)$ if and only if $a_l \succsim_{(p,w)} a'_l$.*

Observe that, because the binary relation $\succsim_{(p,w)}$ on $\mathbb{R}^T_+$ is an *ordering*, it is reflexive, transitive and complete. Therefore, Definition 2 implies that, *for any given price vector*, the MLC should be able to compare any two bundles and when several bundles of produced goods are considered, it should be possible to say which one contains more labour.[15] It may be argued that *in general* completeness and transitivity are desirable properties for any MLC, and may even be necessary for any consistent evaluation. Definition 2 is less demanding, and possibly less controversial, as it requires these properties to hold only in a given economic environment.[16]

In the rest of this section, we identify some theoretically relevant and formally weak restrictions on (p, w)-*labour orderings*. The first property is uncontroversial: it states that, given a price vector (p, w), if a bundle of produced goods c requires a strictly higher amount of *every* type of labour than a bundle c', then it contains more labour. Formally:

Dominance (D): For any $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$, if $a_l > a'_l$, then $a_l \succ_{(p,w)} a'_l$.

It might be argued that, for a given price vector (p, w), it should be sufficient for the amount of *one* type of labour to be strictly greater in a_l than in a'_l to conclude that c contains more labour than c'. This seems reasonable, for example, in an input-output analysis aimed at capturing labour multipliers. This view is not uncontroversial, though. Classical authors, for example, have long argued that some types of labour are inherently unproductive and do not affect the labour content of produced goods. We need not adjudicate this issue here. Given that we aim to lay out some *minimal* desirable properties of MLCs, it is theoretically appropriate to focus on the weaker, and less controversial, condition **D**.

The next property states that the MLC should allow for trade-offs between different types of labour used in production. To be precise, for a given price vector (p, w), for any pair of labour types ν and μ, there exist two production activities which only differ in the amount of labour of types ν and μ used and yield the same labour content, but one of them uses more of type-ν labour while the other uses more of type-μ labour.

[14] Let $x \equiv (c, a; p, w)$. For all $x, x' \in \mathcal{CP}$, the asymmetric part $\succ$ of $\succsim$ is defined by $x \succ x'$ if and only if $x \succsim x'$ and not $x' \succsim x$; and the symmetric part $\sim$ of $\succcurlyeq$ is defined by $x \sim x'$ if and only if $x \succsim x'$ and $x' \succsim x$.

[15] It is worth emphasising, again, that Definition 2 does *not* imply that a MLC must incorporate price information, but only that it *can* do so.

[16] Another property of Definition 2 is worth noting in passing, which is relevant for input-output theory and classical political economy. Although Definition 2 states that the measurement of labour content is based on the *direct* labour inputs used in production, this does not imply that *indirect* labour - that is, the labour contained in produced inputs used in the production process - is irrelevant. Indeed, by **A0**-**A3**, focusing on the direct labour used to produce c as *net* output allows one to capture the *total* amount of labour contained in c, namely "the embodied labour - direct and indirect - in producing c from scratch" (Roemer [46], p.148).

Labour Trade-offs (LT): For all $\nu, \mu \in \mathcal{T}$, $\nu \neq \mu$, there exist $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$, such that $a_{l\nu} > a'_{l\nu}$, $a_{l\mu} < a'_{l\mu}$, and $a_{l\zeta} = a'_{l\zeta}$ for all $\zeta \neq \nu, \mu$, and $a_l \sim_{(p,w)} a'_l$.

Theoretically, axiom **LT** rules out the possibility that the labour content of produced goods is always determined by a single type of labour. This does not preclude the possibility that some types of labour have a (possibly much) bigger weight in the determination of labour content than others. Yet, intuitively, if the amount of type-ν labour decreases, there exists a sufficient increase in the amount of type-μ labour used in production that can conceivably compensate for it in the measurement of labour content. Formally, the axiom imposes a rather weak restriction in that it only requires that, for any pair of labour types $\nu, \mu \in \mathcal{T}$, there exists *one* pair of production activities in the set of all *conceivable* production techniques which yield the same amount of labour in producing *some* (possibly different) net output vectors.

The last axiom imposes a minimal requirement of consistency in labour measurement. It states that, for a given price vector (p, w), if two vectors of labour inputs dominate (in terms of corresponding labour content) another pair of vectors, then convex combinations of the former should dominate convex combinations of the latter.

Mixture Invariance (MI): Let $(c, a, p, w), (c', a', p, w), (\widetilde{c}, \widetilde{a}, p, w), (\widetilde{c}', \widetilde{a}', p, w) \in \mathcal{CP}$. Given $\tau \in (0, 1)$, let $a_l^\tau = \tau a_l + (1 - \tau)\widetilde{a}_l$ and $a_l'^\tau = \tau a'_l + (1 - \tau)\widetilde{a}'_l$. Then, $a_l^\tau \succ_{(p,w)} a_l'^\tau$, whenever $a_l \succ_{(p,w)} a'_l$ and $\widetilde{a}_l \succsim_{(p,w)} \widetilde{a}'_l$.

To see why **MI** is a desirable property, suppose that both a and $\widetilde{a}$ produce bundle c as net output, while a' and $\widetilde{a}'$ produce c'.[17] If **MI** were violated, then it would be possible to conclude that, overall, c' contains more labour than c when, say, a proportion $\tau \in (0, 1)$ of the firms use a and a' to produce, respectively, c and c' (and a proportion $(1 - \tau)$ use $\widetilde{a}$ and $\widetilde{a}'$ to produce, respectively, c and c'), even though for each individual activity a and a', c contains more labour than c', and the same holds for $\widetilde{a}$ and $\widetilde{a}'$. Or, consider firms 1 and 2 producing, respectively, c and c', and suppose that firm 1 (respectively, 2) uses technique a for a part $\tau \in (0, 1)$ of the year and $\widetilde{a}$ for the rest of the year (respectively, a' and $\widetilde{a}'$). Then it would be possible to conclude that, overall, the labour contained in 1's net output is lower than that contained in 2's, despite the fact that in each part of the production period the opposite holds.

Observe that **MI** restricts the way in which a MLC ranks mixtures, starting from original profiles. However, it does *not* require that the amount of labour in a bundle should remain the same, nor does it impose significant restrictions on the way in which such amount should vary.[18]

The three axioms are formally weak and the previous arguments suggest that they incorporate theoretically desirable properties. Indeed, all of the main approaches proposed in the literature satisfy them. It is immediate to see, for example, that the MLCs used in standard productivity analysis, or in the debates on the determinants of growth and development all satisfy **D**, **LT** and **MI**. Although it is less evident, the same holds for the standard definition of labour content in input-output theory. To see this, let the Leontief technology with a $n \times n$ non-negative and productive matrix, A, and a $1 \times n$ positive vector, L, of homogeneous labour requirements be represented by

$$P_{(A,L)} \equiv \left\{ a \in \mathbb{R}_- \times \mathbb{R}^n_- \times \mathbb{R}^n_+ \mid \exists x \in \mathbb{R}^n_+ : a \leqq (-Lx, -Ax, x) \right\},$$

and let $\mathcal{P}_{(\mathcal{A},\mathcal{L})} \subset \mathcal{P}$ be the set of all conceivable Leontief technologies.

In input-output theory and classical approaches, the vector of labour multipliers is defined as $v = L(I - A)^{-1}$ and, for any $(c, a, p, w) \in \mathcal{CP}_{(\mathcal{A},\mathcal{L})}$ such that $a = (-Lx, -Ax, x)$ and $c = (I - A)x$, the labour content of c is defined as $vc = Lx$. It is then possible to show that this

[17] A similar, albeit less transparent, argument holds if $c \neq \widetilde{c}$ and $c' \neq \widetilde{c}'$.

[18] Note also that, by the definition of the universal set $\mathcal{P}$, for all $a_l, \widetilde{a}_l$, such that $(c, a; p, w), (\widetilde{c}, \widetilde{a}; p, w) \in \mathcal{CP}$ and for all $\tau \in (0, 1)$, there exists a profile $(c^\tau, a^\tau; p, w) \in \mathcal{CP}$ such that $a_l^\tau = \tau a_l + (1 - \tau)\widetilde{a}_l$.

MLC satisfies the axioms. To see that **D** is met, note that for any $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}_{(\mathcal{A},\mathcal{L})}$, $Lx > L'x'$ immediately implies $a_l \succ_{(p,w)} a'_l$. To see that **MI** is satisfied, consider $(c,a,p,w),(c',a',p,w),(\widetilde{c},a,p,w),(\widetilde{c}',a',p,w) \in \mathcal{CP}_{(\mathcal{A},\mathcal{L})}$ such that $Lx > L'x'$ and $\widetilde{L}\widetilde{x} \geqq \widetilde{L}'\widetilde{x}'$. Then, for any $\tau \in (0,1)$, $a_l^{\tau} = \tau Lx + (1-\tau)\widetilde{L}\widetilde{x} > a_l'^{\tau} = \tau L'x' + (1-\tau)\widetilde{L}'\widetilde{x}'$, and so $a_l^{\tau} \succ_{(p,w)} a_l'^{\tau}$. Finally, because there is only one type of labour, **LT** is vacuously satisfied.

Our main result states that if one endorses **D**, **LT** and **MI**, then one must conclude that the labour content of a bundle of produced goods should be measured as the weighted sum of the amount of time of different types of labour spent in its production. Formally:

Definition 3 *For any (p,w), a (p,w)-labour ordering $\succsim$ on $\mathcal{CP}$ is* generalised additive *if there is some strictly positive vector $\sigma_{(p,w)} \in \mathbb{R}^T_{++}$ such that for all $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$, $a_l \succsim_{(p,w)} a'_l$ if and only if $\sigma_{(p,w)}a_l = \sum_{\nu \in \mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} \geqq \sum_{\nu \in \mathcal{T}} \sigma^{\nu}_{(p,w)} a'_{l\nu} = \sigma_{(p,w)}a'_l$.*

Theorem 1 demonstrates that the only measures that satisfy all axioms are indeed generalised additive.[19]

Theorem 1 *A (p,w)-labour ordering $\succsim$ on $\mathcal{CP}$ satisfies **Dominance**, **Labour Trade-offs**, and **Mixture Invariance** if and only if it is generalised additive.*

Although Theorem 1 does not uniquely characterise a MLC, it does identify a class of measures which share an important property: the labour content of a vector of net outputs is a weighted sum of the amounts of different types of labour used to produce them. This additive structure is often considered either as a fundamental property of a MLC and thus implicitly postulated *as an axiom*, or as the consequence of marginal product pricing in perfectly competitive markets. Instead, additivity is here derived as a *result* starting from more foundational principles that are directly related to the properties of labour measurement, without any assumptions on market structure, equilibrium pricing, or the existence of differentiable production functions.

Although the main contribution of this paper is conceptual, it is worth noting that, from a purely formal viewpoint, Theorem 1 provides an independent characterisation of the so-called *weak weighted utilitarian* ordering which is analysed in social choice theory in the context of evaluating welfare profiles.[20] Indeed, axioms **D**, **LT** and **MI** are analogous to well-known Paretian, anonymity and independence properties in social choice theory. However, the similarity is purely at the formal level: the interpretation and justification are completely different, and indeed some of the axioms are more defensible in the context of the measurement of labour content than in welfare economics. Diamond's [8] classic critique of utilitarianism, for example, is based on the rejection of independence (or 'sure thing') principles analogous to **MI**. For 'mixing' welfare across different individuals may produce ethically relevant effects.[21] Clearly, this normative argument does not apply to the measurement of labour content.

5 Labour content: refinements

Axioms **D**, **LT**, and **MI** are all independent of price information: for any given prices and wages (p,w), they focus on productive conditions and impose rather weak restrictions on the measurement of labour content. Nonetheless, they are sufficient to identify one class of MLCs which share an

[19] All formal proofs can be found in Appendix A.

[20] Actually, standard results in social choice theory highlight the robustness of the main conclusions of this paper. For it is well-known that weak weighted utilitarianism can be characterised based on various different sets of axioms, focusing for example on *invariance conditions*. See d'Aspremont ([4], Theorem 3.3.5, p.51), d'Aspremont and Gevers ([5], Theorem 4.2, p.509), Mitra and Ozbek ([36], Theorem 2, p.14). The axioms used in Theorem 1, however, are more intuitive and economically meaningful in the context of the measurement of labour content.

[21] For a discussion, see Mariotti and Veneziani [33, 34].

important and intuitive property – namely, additivity in labour amounts. Indeed, Theorem 1 highlights the common theoretical foundations and the shared intuitions behind all of the main approaches. In this section, we explore further restrictions that allow us to characterise the two main approaches in the literature – namely, the simple additive MLC and the wage-additive MLC – within the class identified by Theorem 1. Formally:

Definition 4 *For any (p, w), a (p, w)-labour ordering $\succsim$ on $\mathcal{CP}$ is* additive *if, for all (c, a, p, w), $(c', a', p, w) \in \mathcal{CP}$, $a_l \succsim_{(p,w)} a'_l$ if and only if $\sum_{\nu \in \mathcal{T}} a_{l\nu} \geqq \sum_{\nu \in \mathcal{T}} a'_{l\nu}$.*

Definition 5 *For any (p, w), a (p, w)-labour ordering $\succsim$ on $\mathcal{CP}$ is* wage additive *if, for all (c, a, p, w), $(c', a', p, w) \in \mathcal{CP}$, $a_l \succsim_{(p,w)} a'_l$ if and only if $\sum_{\nu \in \mathcal{T}} w_\nu a_{l\nu} \geqq \sum_{\nu \in \mathcal{T}} w_\nu a'_{l\nu}$.*

The key intuition behind the *additive* approach is that all types of labour contribute equally to the determination of labour content. This can be captured formally by the following strengthening of **LT**.

Labour Equivalence (LE): For all $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$ such that $a_{l\nu} = a'_{l\mu}$, $a_{l\mu} = a'_{l\nu}$, some $\nu, \mu \in \mathcal{T}$, and $a_{l\zeta} = a'_{l\zeta}$ for all $\zeta \neq \nu, \mu$, $a_l \sim_{(p,w)} a'_l$.

The next result states that the combination of **D**, **MI**, and **LE**, implies that the labour content of a bundle of produced goods should be measured as the total amount of hours of labour of different types spent in its production.

Corollary 1 *A (p, w)-labour ordering $\succsim$ on $\mathcal{CP}$ satisfies **Dominance**, **Labour Equivalence**, and **Mixture Invariance** if and only if it is additive.*

A characterisation of the wage-additive approach is less straightforward. Rather different arguments are used in various strands of the literature in order to justify the adoption of relative wage rates to aggregate heterogeneous labour, and so it is difficult to find a single axiom capturing the intuitions common to all theoretical approaches. In what follows, we analyse one such condition which aims to capture the relation between technical change and labour content in capitalist economies. For the relation between labour aggregates and productivity is central in all of the strands of the literature mentioned above, which emphasise the effect of capitalist behaviour and technological progress on labour productivity.

Our axiom is one - particularly clear and intuitive - way of formalising the relation between (cost reducing) technical change and labour content, which is specifically rooted in the classical tradition.[22] Thus, our analysis provides a different perspective on the intuitions behind the wage additive approach. For the latter is often justified using the standard production model and assuming marginal productivity pricing of labour in perfectly competitive markets. On the contrary, our axiomatic analysis is independent of any assumptions on market structure and on differentiability of production functions and provides an alternative justification focusing on the kind of information that the MLC should capture.

The axiom generalises an insight first proved rigorously by Roemer ([44]; see also Roemer [45] and Flaschel et al. [14]): any profitable (i.e., cost-reducing at current prices) technical change that is capital-using and labour-saving is *progressive*, that is, it leads to a decrease in labour content (and an increase in labour productivity). In the context of the standard Leontief models in which these results are derived, the definition of labour content is uncontroversial and so this

[22] The link between labour content and labour productivity, for example, is central in Marx's theory: "In general, the greater the productiveness of labour, the less is the labour-time required for the production of an article, the less is the amount of labour crystallised in that article, and the less is its value; and vice versa. The value of a commodity, therefore, varies directly as the quantity, and inversely as the productiveness, of the labour incorporated in it" (Marx [35], p.48).

insight is obtained as a result. However, the theoretical relevance of the link between technical change, productivity and labour content in the literature is arguably such that its epistemological status is as a postulate: the appropriate MLC is one which preserves the link between profitable innovations and labour productivity and labour content. Suppose that profitable capital-using and labour-saving technical changes – that are proved to be progressive in simple Leontief production economies with homogeneous labour – turn out to lose this property under a specific generalisation of the standard MLC in a broader class of economies. This would arguably suggest that the specific MLC adopted does not properly capture labour content, rather than proving a breakdown in the link between technical change and labour productivity in more general economies.

For all $\mathcal{P}' \subseteq \mathcal{P}$, and all $c \in \mathbb{R}^n_+$, let $\phi^{\mathcal{P}'}(c) \equiv \left\{a' \in \mathbb{R}^{T+n}_- \times \mathbb{R}^n_+ \mid \exists P' \in \mathcal{P}' : a' \in \phi^{P'}(c)\right\}$. The next axiom captures the labour-reducing effect of profitable capital-using technical change.

Consistency with Progressive Technical Change (CPTC): For any $(p, w) \in \mathbb{R}^{n+T}_+$, there exist a profile $(c, a, p, w) \in \mathcal{CP}$ with $c \geq \mathbf{0}$, $a \in P$, $P \in \mathcal{P}$, a neighbourhood $\mathcal{N}(P) \subseteq \mathcal{P}$, and a neighbourhood $\mathcal{N}(a) \subseteq \mathbb{R}^{T+n}_- \times \mathbb{R}^n_+$ of a such that $\mathcal{N}'(a) \equiv \left[\mathcal{N}(a) \cap \phi^{\mathcal{N}(P)}(c)\right] \setminus \{a\}$ is a neighbourhood of a excluding a itself and for all $a' \in \mathcal{N}'(a)$, if $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ and $\underline{a} \leq \underline{a}'$, then $a_l \succ_{(p,w)} a'_l$.

Several features of **CPTC** are worth noting. Firstly, the standard definition of labour content in Leontief models with homogeneous labour satisfies **CPTC** in $\mathcal{CP}_{(\mathcal{A},\mathcal{L})}$. To see this, given a price vector $(p, w) \in \mathbb{R}^{n+1}_+$, consider any $(c, a, p, w), (c, a', p, w) \in \mathcal{CP}_{(\mathcal{A},\mathcal{L})}$, such that $a = (-a_l, -Ax, x)$ and $a' = (-a'_l, -A'x', x')$, where $a \in P_{(A,L)}$ and $a' \in P_{(A',L')}$. Suppose that labour intensity is identical at a and at a'. Then, without loss of generality, we can set $Lx = a_l$ and $L'x' = a'_l$. In this setting, if $pAx + wLx > pA'x' + wL'x'$ and $Ax \leq A'x'$, then $Lx > L'x'$ and so $a_l \succ_{(p,w)} a'_l$.

Secondly, the axiom focuses exclusively on innovations that (weakly) increase the amount of *all* physical inputs used in a given process. As a general definition of profitable capital-using technical progress, this may be considered too restrictive. However, our aim is not to provide a general theory of technological change, and in the context of an axiomatic analysis of MLCs, focusing on a smaller set of innovations imposes weaker restrictions.

Thirdly, although no condition is explicitly imposed on labour inputs, the changes considered are, in a relevant sense, labour-saving. To see this, suppose that there is only one type of homogeneous labour. In this case, $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ and $\underline{a} \leq \underline{a}'$ imply that $a_l > a'_l$, and so technical change is labour-saving. In economies with heterogeneous labour, cost-reducing and capital-using technical changes are not necessarily labour-saving *for all types of labour*. In other words, $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ and $\underline{a} \leq \underline{a}'$ do not imply $a_l > a'_l$. However, the changes considered in **CPTC** do imply that the amount of at least one type of labour decreases, that is $a_{l\nu} > a'_{l\nu}$ for some $\nu \in \mathcal{T}$, and even if the amount of some labour input increases, this is more than outweighed by decreases in other types of labour. Thus, the axiom nicely captures, for example, some classic Marxian insights about the nature of technical change in market economies (Marx [35], chapter 23): capitalist dynamics always encourages capitalists to implement cost-reducing and capital-using technical change in order to reduce labour demand, which results in a reduction of labour costs.

Fourthly, the axiom focuses on innovations that change the technological conditions of the production of a *given* net output vector c. This is theoretically intuitive, and it makes the axiom weaker, but our key result remains valid even if **CPTC** is strengthened to hold for any $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$, and allowing for the possibility that $c \neq c'$.

Finally, **CPTC** focuses on innovations that are cost-reducing *at current prices*: the effect of technical change on prices and wages is ignored. This is standard in the literature on progressive technical change (e.g., Morishima [38]; Roemer [44, 45]; Flaschel et al. [14]).

The next result states that if one endorses **CPTC** together with **D**, **LT** and **MI**, then one must conclude that the labour content of a bundle of produced goods should be measured as the weighted sum of the different types of labour used in its production, with the weights given by relative wages.

Corollary 2 *A (p,w)-labour ordering $\succsim$ on $\mathcal{CP}$ satisfies* ***Dominance, Labour Trade-offs, Mixture Invariance,*** *and* ***Consistency with Progressive Technical Change*** *if and only if it is wage additive.*

By Corollary 2, if a small number of widely (albeit often implicitly) accepted principles of labour measurement with sound theoretical foundations are adopted, which impose rather weak formal restrictions on MLCs, then the vexed issue of how to convert different types of labour into a single measure has a unique, simple and intuitive answer: relative wages should be used to homogenise different types of labour.

Corollary 2 provides rigorous axiomatic foundations to the standard practice of measuring labour inputs based on wage costs in the input-output literature as well as in empirical studies on total factor productivity and growth. It is also consistent with the views of classical political economy on the conversion of complex labour into simple labour. Indeed, Corollary 2 suggests that the wage additive measure is the appropriate generalisation of the standard MLC universally used in linear economies with homogeneous labour. For the wage additive measure reduces to the standard MLC in those economies and, as shown above, the standard MLC satisfies all of the axioms on the set $\mathcal{CP}_{(\mathcal{A},\mathcal{L})} \subset \mathcal{CP}$, for a given price vector.

6 A generalisation

Theorem 1 characterises a measure that allows one to compare any pairs of produced bundles, *at given prices.* Formally, the MLC is transitive and complete over profiles $(c,a,p,w),(c',a',p',w') \in \mathcal{CP}$ such that $(p,w)=(p',w')$. However, it is silent whenever profiles with $(p,w) \neq (p',w')$ are considered. This section analyses whether our results can be extended to hold for *any* profiles $(c,a,p,w),(c',a',p',w') \in \mathcal{CP}$.

As a first step, we simply reformulate the three core axioms presented in section 4 as restrictions on the MLC $\succsim \subseteq \mathcal{CP} \times \mathcal{CP}$, without assuming the latter to be a (p,w)-labour ordering.

Dominance (D): For any $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$, if $a_l > a'_l$ then $(c,a,p,w) \succ (c',a',p,w)$.

Labour Trade-offs (LT): For all $\nu,\mu \in \mathcal{T}$, $\nu \neq \mu$, and all $(p,w) \in \mathbb{R}_+^{n+T}$, there are $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$, such that $a_{l\nu} > a'_{l\nu}$, $a_{l\mu} < a'_{l\mu}$, and $a_{l\zeta} = a'_{l\zeta}$ for each $\zeta \neq \nu,\mu$, and $(c,a,p,w) \sim (c',a',p,w)$.

Mixture Invariance (MI): Let $(c,a,p,w),(c',a',p,w),(\widetilde{c},\widetilde{a},p,w),(\widetilde{c}',\widetilde{a}',p,w) \in \mathcal{CP}$. Given $\tau \in (0,1)$, let $a_l^\tau = \tau a_l + (1-\tau)\widetilde{a}_l$ and $a_l'^\tau = \tau a'_l + (1-\tau)\widetilde{a}'_l$. Then, $(c^\tau,a^\tau,p,w) \succ (c'^\tau,a'^\tau,p,w)$ holds, whenever $(c,a,p,w) \succ (c',a',p,w)$ and $(\widetilde{c},\widetilde{a},p,w) \succsim (\widetilde{c}',\widetilde{a}',p,w)$.

In order to generalise Theorem 1, we introduce some additional properties. The first one states that different profiles should not be ordered lexicographically focusing only on the prices of commodities, or on the vector of wages. The labour content of a bundle of goods c, produced as net output using activity a, at a price vector (p,w) should not be strictly more (or less) than the labour content of *all* other bundles c', produced as net output using *any* activity a', at a different price vector (p',w'). Formally:

Minimal Equivalence (ME): For any $(p,w),(p',w') \in \mathbb{R}_+^{n+T}$, there exist two profiles (c,a,p,w), $(c',a',p',w') \in \mathcal{CP}$ with $a_{l\nu} = a_{l\mu} > 0$ and $a'_{l\nu} = a'_{l\mu} > 0$ for any $\nu,\mu \in \mathcal{T}$, such that $(c,a,p,w) \sim (c',a',p',w')$.

Axiom **ME** imposes quite a weak restriction on the MLC as it only requires the existence of *one* pair of profiles that are indifferent for any two different price vectors. Moreover, the activity vector of each profile should use the *same* amount of time of every type of labour. This incorporates the intuition that different vectors of wage rates, w, w' may reflect different (unobserved) labour intensities, or skills, across the two profiles, but differences in labour intensity among different types of labour should be comparable and compensated by adjusting the amount of time (of all types of labour) spent in production.

The second property requires that the ranking of a pair of profiles is invariant to the scaling of the consumption bundle and the associated production activity. In other words, for any $k > 0$, if the labour content of a bundle of goods c, produced as net output of activity a at (p, w) is at least as much as the labour content of a bundle c', produced as net output of activity a' at (p', w') then the same is true for bundle kc, produced using ka at (p, w), when compared with kc' produced using ka' at (p', w'). Formally:

Scale Invariance (SI): For any $(c, a, p, w), (c', a', p', w') \in \mathcal{CP}$ such that for any $\nu, \mu \in \mathcal{T}$, $a_{l\nu} = a_{l\mu}$ and $a'_{l\nu} = a'_{l\mu}$, and for any positive real number $k > 0$, $(c, a, p, w) \succsim (c', a', p', w')$ holds if and only if $(kc, ka, p, w) \succsim (kc', ka', p', w')$ holds.

Scale invariance properties are standard in axiomatic analysis and incorporate the intuition that the comparison of two objects should not change if the objects compared are uniformly scaled up or down. Standard inequality measures and social welfare orderings, for example, typically satisfy scale invariance properties. **SI** seems particularly reasonable in the context of measuring labour content, especially given the convexity of production sets.

If one endorses **ME** and **SI**, together with **D**, **LT**, **MI**, then one must conclude that the labour content of a bundle of goods should be measured as the weighted sum of the different types of labour used in its production, with the weights depending on the price vector. Formally:

Definition 6 *A measure of labour content $\succsim$ on $\mathcal{CP}$ is* generalised additive *if, for all (c, a, p, w), $(c', a', p', w') \in \mathcal{CP}$, there exist some strictly positive vectors $\sigma_{(p,w)}, \sigma_{(p',w')} \in \mathbb{R}^{T}_{++}$ such that $(c, a, p, w) \succsim (c', a', p', w')$ if and only if $\sigma_{(p,w)} a_l = \sum_{\nu \in \mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} \geqq \sum_{\nu \in \mathcal{T}} \sigma^{\nu}_{(p',w')} a'_{l\nu} = \sigma_{(p',w')} a'_l$.*

The next result proves that the only reflexive, transitive and complete MLC that satisfies all axioms is indeed generalised additive.

Theorem 2 *A reflexive, transitive and complete MLC $\succsim$ on $\mathcal{CP}$ satisfies **Dominance**, **Labour Trade-offs**, **Mixture Invariance**, **Scale Invariance**, and **Minimal Equivalence** if and only if it is generalised additive.*

Next, we generalise the characterisations of the other two measures. Consider the simple additive measure first. Formally:

Definition 7 *A measure of labour content $\succsim$ on $\mathcal{CP}$ is* additive *if, for all $(c, a, p, w), (c', a', p', w') \in \mathcal{CP}$, $(c, a, p, w) \succsim (c', a', p', w')$ if and only if $\sum_{\nu \in \mathcal{T}} a_{l\nu} \geqq \sum_{\nu \in \mathcal{T}} a'_{l\nu}$.*

The next axiom is a straightforward extension of **LE** which allows the price vector to change.

Labour Equivalence (LE): For all $(c, a, p, w), (c', a', p', w') \in \mathcal{CP}$ such that $a_{l\nu} = a'_{l\mu}$, $a_{l\mu} = a'_{l\nu}$, some $\nu, \mu \in \mathcal{T}$, and $a_{l\zeta} = a'_{l\zeta}$ for all $\zeta \neq \nu, \mu$, $(c, a, p, w) \sim (c', a', p', w')$.

The next result states that the combination of **D**, **MI**, and **LE**, implies that the labour content of a bundle of goods should be measured as the total amount of hours of labour of different types spent in its production.

Corollary 3 *A reflexive, transitive and complete MLC $\succsim$ on $\mathcal{CP}$ satisfies **Dominance**, **Labour Equivalence**, and **Mixture Invariance** if and only if it is additive.*

Next, consider the wage additive MLC extended to hold for *any* pair of profiles:

Definition 8 *A measure of labour content* $\succsim$ *on* $\mathcal{CP}$ *is* wage additive *if, for all* $(c, a, p, w), (c', a', p', w') \in \mathcal{CP}$*,* $(c, a, p, w) \succsim (c', a', p', w')$ *if and only if* $wa_l = \sum_{\nu \in \mathcal{T}} w_\nu a_{l\nu} \geqq \sum_{\nu \in \mathcal{T}} w'_\nu a'_{l\nu} = w'a'_l$*.*

As a first step, we reformulate **CPTC** here as a restriction on the MLC $\succsim \subseteq \mathcal{CP} \times \mathcal{CP}$, without assuming the latter to be a (p, w)-labour ordering:

Consistency with Progressive Technical Change (CPTC): For any $(p, w) \in \mathbb{R}^{n+T}_+$, there exist a profile $(c, a, p, w) \in \mathcal{CP}$ with $c \geq \mathbf{0}$, $a \in P$, $P \in \mathcal{P}$, a neighbourhood $\mathcal{N}(P) \subseteq \mathcal{P}$, and a neighbourhood $\mathcal{N}(a) \subseteq \mathbb{R}^{T+n}_- \times \mathbb{R}^n_+$ of a such that $\mathcal{N}'(a) \equiv \left[\mathcal{N}(a) \cap \phi^{\mathcal{N}(P)}(c)\right] \setminus \{a\}$ is a neighbourhood of a excluding a itself and for all $a' \in \mathcal{N}'(a)$ with $(c, a', p, w) \in \mathcal{CP}$, if $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ and $\underline{a} \leq \underline{a}'$, then $(c, a, p, w) \succ (c, a', p, w)$.

In order to characterise the wage additive measure, we introduce two additional axioms. To begin with, note that if the price vector is allowed to vary, the wage additive *MLC* compares different profiles also based on information concerning the *absolute* level of wages, and not only about *relative* wages, unlike in the case of (p, w)-labour orderings. The first axiom captures the following intuition: in the universal domain $\mathcal{CP}$ of conceivable profiles, there exist two profiles $(c, a, p, w), (c, a', p, w')$ such that a uniform increase (resp., decrease) in wages can be interpreted as reflecting a generalised increase (resp., decrease) in labour productivity such that the amount of labour time necessary to produce a given bundle of goods, c, as net output decreases (resp., increases) proportionally – even if the vector of produced inputs used in production and the output vector remain the same – and the labour content of c remains unchanged. In other words, there exists at least one pair of profiles such that the labour content of a bundle of goods remains constant because a uniform increase in skills (reflected in the wages) compensates for a decrease in the amount of labour time spent in production. Formally:

Skill Substitutability (SS): For any $(p, w), (p, w') \in \mathbb{R}^{n+T}_+$ such that $w' = \lambda_{(w,w')} w$ for some $\lambda_{(w,w')} > 0$, there exist $(c, a, p, w), (c, a', p, w') \in \mathcal{CP}$ such that for any $\nu, \mu \in \mathcal{T}$, $a_{l\nu} = a_{l\mu}$ and $a'_{l\nu} = a'_{l\mu}$; $a_l = \lambda_{(w,w')} a'_l$; and $(\underline{a}, \overline{a}) = (\underline{a}', \overline{a}')$, and that $(c, a, p, w) \sim (c, a', p, w')$.

It is worth emphasising that **SS** imposes a very weak restriction on the MLS. For it applies only to a very small set of perturbations of a price vector (commodity prices must remain constant and wages must change by exactly the same factor) and it only requires the existence of *one* pair of profiles with the indifferent property.

Whereas axiom **SS** focuses on changes in the wage *level*, the next property constrains the effect of changes in *relative* wages on the labour content of a rather small subset of profiles. Consider any two price vectors $(p, w), (p', w')$ such that relative wage rates are different but the overall wage level is the same, in the sense that both wage vectors belong to the unit simplex ($\sum w_\nu = \sum w'_\nu = 1$). Then, in the universal domain $\mathcal{CP}$ of conceivable profiles, there exist two profiles $(c, a, p, w), (c, a, p', w')$ such that the constancy of the overall wage level in the sense specified can be interpreted as reflecting a constancy in overall labour productivity and therefore the amount of labour contained in the given bundle c produced with the given activity vector a is constant, assuming that the same amount of time of each type of labour is spent in production. For these profiles, labour content is independent of changes in relative wages. Formally:

Independence (I): For any $(p, w), (p', w') \in \mathbb{R}^{n+T}_+$ such that $w \neq w'$ and $\sum w_\nu = \sum w'_\nu = 1$, there exist $(c, a, p, w), (c, a, p', w') \in \mathcal{CP}$ such that for any $\nu, \mu \in \mathcal{T}$, $a_{l\nu} = a_{l\mu}$ and $(c, a, p, w) \sim (c, a, p', w')$.

Again, it is important to stress that **I** is a very weak property as it only applies to price vectors with wages belonging to the unit simplex and it only requires the existence of *one* pair of profiles with the desired property.

Together with **D**, **LT**, **MI** and **CPTC**, if one endorses **SI**, **SS** and **I**, then one must conclude that the labour content of a bundle of produced goods should be measured as the weighted sum of the amount of time of different types of labour used in its production, with the weights given by the relevant wages, *even when the price vector changes.*

Theorem 3 *A reflexive, transitive and complete MLC $\succsim$ on $\mathcal{CP}$ satisfies* ***Dominance, Labour Trade-offs, Mixture Invariance, Scale Invariance, Skill Substitutability, Consistency with Progressive Technical Change*** *and* ***Independence*** *if and only if it is wage additive.*

7 Conclusion

This paper analyses the issue of the appropriate measurement of the labour content of produced goods. Measures of labour content are formally conceptualised as binary relations comparing bundles of goods produced with certain activities at a certain price vector. An axiomatic approach is adopted in order to identify some foundational properties that every MLC should satisfy. Strikingly, it is shown that a small number of axioms incorporating some foundational and widely shared technology-related intuitions uniquely identify the class of linear MLCs, according to which the labour content of a bundle of goods produced as net output is the weighted sum of the amount of time of different types of labour spent in its production. A linear aggregation of heterogeneous labour inputs is advocated in virtually *all* of the literature, and so our characterisation precisely identifies the theoretical foundations and intuitions shared in such diverse approaches and fields as input-output theory, productivity analysis, neoclassical growth theory, and classical political economy. We also characterise the two main measures used in the literature, namely the simple additive MLC, according to which the labour content of a bundle of produced goods corresponds to the total (unweighted) labour time spent in its production, and the wage additive MLC, which uses relative wages in order to convert different types of labour into a single measure.

The axiomatic analysis developed in this paper is motivated by the idea that the theoretical strength of a MLC depends - to a large extent - on the foundational principles that underlie it. There are two important caveats about this, which also suggest directions for further research.

First, although additive measures possess many desirable features from both the theoretical and the empirical viewpoint, alternative measures can certainly be proposed that capture different intuitions, and have different properties. From this perspective, the adoption of an axiomatic analysis aims precisely at making the relevant assumptions and intuitions explicit and open to discussion and criticism.

Second, it is certainly desirable for a MLC to have sound theoretical foundations. Yet one may argue that its cogency and usefulness ultimately rest on the insights that can be gained from using it. In this case, the fruitfulness of the additive measures considered in this paper, and in particular of the wage additive MLC, can only be judged when it is applied to economically relevant problems. From this perspective, too, this paper can be seen only as a first, and preliminary step in a wider research programme.

A Proofs

First of all, we prove two results that are of some interest in their own right. Lemma 1 derives some convexity properties of a (p, w)-*labour ordering* $\succsim$.

Lemma 1 *Let the ordering* $\succsim_{(p,w)}$ *on* $\mathbb{R}_+^T$ *satisfy **Mixture Invariance**. Consider any set* $\{a_l^1, ..., a_l^K\}$, $K > 1$, *such that* $(c^k, a^k, p, w) \in \mathcal{CP}$, *for all* $k = 1, ..., K$ *and* $a_l^i \sim_{(p,w)} a_l^j$, *for all* $i, j \in \{1, ..., K\}$. *Then, for all* $\{\tau_1, ..., \tau_K\}$ *such that* $\tau_i \in [0, 1]$ *all* $i \in \{1, ..., K\}$ *and* $\sum_{i=1}^K \tau_i = 1$, $\sum_{i=1}^K \tau_i a_l^i \sim_{(p,w)} a_l^j$, *for all* $j \in \{1, ..., K\}$.

Proof. 1. First of all, note that by the definition of the universal set $\mathcal{P}$, for all $\{a_l^1, ..., a_l^K\}$, such that $(c^k, a^k, p, w) \in \mathcal{CP}$, for all $k = 1, ..., K$, and for all $\{\tau_1, ..., \tau_K\}$ such that $\tau_i \in [0, 1]$ all $i \in \{1, ..., K\}$ and $\sum_{i=1}^K \tau_i = 1$, there exists a profile $(c^\tau, a^\tau, p, w) \in \mathcal{CP}$ such that $a_l^\tau = \sum_{i=1}^K \tau_i a_l^i$.

2. Note that if $\tau_i = 1$, some $i \in \{1, ..., K\}$, then the result holds by assumption. Therefore in what follows we focus on the case where $\tau_i \in [0, 1)$, all $i \in \{1, ..., K\}$.

3. We proceed by induction on K.

($K = 2$) Consider any pair $(c^1, a^1, p, w), (c^2, a^2, p, w) \in \mathcal{CP}$ such that $a_l^1 \sim_{(p,w)} a_l^2$. Suppose, by way of contradiction, that there exists some $\tau \in (0, 1)$, such that $\tau a_l^1 + (1-\tau) a_l^2 \nsim_{(p,w)} a_l^i$, for some $i \in \{1, 2\}$. Let $a_l^\tau \equiv \tau a_l^1 + (1-\tau) a_l^2$. By completeness, suppose that $a_l^\tau \succ_{(p,w)} a_l^i$, for some $i \in \{1, 2\}$, without loss of generality. By transitivity, $a_l^\tau \succ_{(p,w)} a_l^i$, for all $i \in \{1, 2\}$. But then **MI** implies $a_l^\tau \succ_{(p,w)} t a_l^1 + (1-t) a_l^2$ for all $t \in (0, 1)$. Setting $t = \tau$ yields the desired contradiction.

(Inductive step) Suppose that the result holds for all $K - 1 \geqq 2$. Consider $\{a_l^1, ..., a_l^K\}, K > 1$, such that $(c^k, a^k, p, w) \in \mathcal{CP}$, for all $k = 1, ..., K$, and $a_l^i \sim_{(p,w)} a_l^j$, for all $i, j \in \{1, ..., K\}$. Take any $\{\tau_1, ..., \tau_K\}$ such that $\tau_i \in [0, 1)$ all $i \in \{1, ..., K\}$ and $\sum_{i=1}^K \tau_i = 1$. We need to prove that $\sum_{i=1}^K \tau_i a_l^i \sim_{(p,w)} a_l^j$, for all $j \in \{1, ..., K\}$.

If $\tau_i = 0$, some $i \in \{1, ..., K\}$, then the result follows from the induction hypothesis and transitivity. So suppose that $\tau_i \in (0, 1)$, all $i \in \{1, ..., K\}$. Note that for any $k \in \{1, ..., K\}$, $\sum_{i=1}^K \tau_i a_l^i = \sum_{j \neq k} \tau_j \left(\sum_{i \neq k} \frac{\tau_i}{\sum_{j \neq k} \tau_j} a_l^i \right) + \tau_k a_l^k$ and by construction $\frac{\tau_i}{\sum_{j \neq k} \tau_j} \in (0, 1)$, all $i \in \{1, ..., K\} \backslash \{k\}$, and $\sum_{i \neq k} \frac{\tau_i}{\sum_{j \neq k} \tau_j} = 1$. Therefore by the induction hypothesis and transitivity, $\sum_{i \neq k} \frac{\tau_i}{\sum_{j \neq k} \tau_j} a_l^i \sim_{(p,w)} a_l^h$ for all $h \in \{1, ..., K\}$. Then the result follows by noting that $\sum_{j \neq k} \tau_j = 1 - \tau_k \in (0, 1)$ and by invoking the the induction hypothesis and transitivity again. ■

Remark: The restriction $K > 1$ in Lemma 1 is without loss of generality, as the result trivially holds in the case $K = 1$.

The next Lemma proves that any two vectors with the same amount of labour content actually identify a direction in the T-dimensional space along which all vectors have the same labour content.

Lemma 2 *Let the ordering* $\succsim_{(p,w)}$ *on* $\mathbb{R}_+^T$ *satisfy **Mixture Invariance**. Suppose* $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$ *and* $a_l \sim_{(p,w)} a_l'$. *If* $(c'', a'', p, w) \in \mathcal{CP}$ *and there exists* $t \in (0, 1)$ *such that* $a_l = t a_l'' + (1-t) a_l'$, *then* $a_l'' \sim_{(p,w)} a_l \sim_{(p,w)} a_l'$.

Proof. 1. Suppose that $(c, a, p, w), (c', a', p, w) \in \mathcal{CP}$ and $a_l \sim_{(p,w)} a_l'$. Suppose, by way of contradiction, that $(c'', a'', p, w) \in \mathcal{CP}$ and there exists $t \in (0, 1)$ such that $a_l = t a_l'' + (1-t) a_l'$, but $a_l'' \nsim_{(p,w)} a_l'$. By completeness, suppose $a_l'' \succ_{(p,w)} a_l'$, without loss of generality.

2. By **MI**, and noting that by the reflexivity of $\succsim_{(p,w)}$, $a_l'' \sim_{(p,w)} a_l''$ and $a_l' \sim_{(p,w)} a_l'$, it follows that $a_l'' \succ_{(p,w)} \tau a_l'' + (1-\tau) a_l' \succ_{(p,w)} a_l'$ holds for all $\tau \in (0, 1)$. The desired contradiction follows setting $\tau = t$. ■

We can now prove Theorem 1.[23]

Proof of Theorem 1. (*Necessity*) It is immediate that if a (p, w)-labour ordering $\succsim$ on $\mathcal{CP}$ is generalised additive, it satisfies the axioms.

[23] The properties in Theorem 1, and in the other characterisation results below, are independent.

(*Sufficiency*) Consider a (p,w)-labour ordering $\succsim$ on $\mathcal{CP}$ that satisfies **D**, **LT**, and **MI**. In order to show that $\succsim$ is generalised additive, we first show that any (p,w)-labour ordering $\succsim$ on $\mathcal{CP}$ that satisfies **D**, **LT**, and **MI** has an additive feature: that is, there is some $\sigma_{(p,w)} \in \mathbb{R}^T$, $\sigma_{(p,w)} > \mathbf{0}$, such that for all $(c,a,p,w), (c',a',p,w) \in \mathcal{CP}$, $a_l \succsim_{(p,w)} a'_l$ if and only if $\sigma_{(p,w)} a_l \geqq \sigma_{(p,w)} a'_l$.

Step 1. We prove that for any $(c,a,p,w), (c',a',p,w) \in \mathcal{CP}$, $a_l \succsim_{(p,w)} a'_l$ implies $a_l + y \succsim_{(p,w)} a'_l + y$, for all $y \in \mathbb{R}^T$ such that $a_l + y, a'_l + y \in \mathbb{R}^T_+$. To see this, suppose, by way of contradiction, that there exist $(c,a,p,w), (c',a',p,w) \in \mathcal{CP}$, and $y \in \mathbb{R}^T$ such that $a_l \succsim_{(p,w)} a'_l$ and $a_l + y, a'_l + y \in \mathbb{R}^T_+$, but $a_l + y \succsim_{(p,w)} a'_l + y$ dose not hold. By completeness, this implies $a'_l + y \succ_{(p,w)} a_l + y$. Then, by **MI**, for all $\tau \in (0,1)$, $\tau a_l + (1-\tau)(a'_l + y) \succ_{(p,w)} \tau a'_l + (1-\tau)(a_l + y)$. For $\tau = \frac{1}{2}$, the latter expression becomes

$$\frac{1}{2} a_l + \frac{1}{2}(a'_l + y) \succ_{(p,w)} \frac{1}{2} a'_l + \frac{1}{2}(a_l + y)$$

which violates reflexivity.

Step 2. By **LT,** for all $\nu, \mu \in \mathcal{T}$, there are $(c,a,p,w), (c',a',p,w) \in \mathcal{CP}$ such that $a_{l\nu} > a'_{l\nu}$, $a_{l\mu} < a'_{l\mu}$, and $a_{l\zeta} = a'_{l\zeta}$, $\zeta \neq \nu, \mu$, and $a_l \sim_{(p,w)} a'_l$. Take $\nu = 1$: by **LT** for all $\mu \in \mathcal{T} \backslash \{1\}$, there exist $(c^\mu, a^\mu, p, w), (c'^\mu, a'^\mu, p, w) \in \mathcal{CP}$ such that $a^\mu_{l1} > a'^\mu_{l1}$, $a^\mu_{l\mu} < a'^\mu_{l\mu}$, and $a^\mu_{l\zeta} = a'^\mu_{l\zeta}$, $\zeta \neq 1, \mu$, and $a^\mu_l \sim_{(p,w)} a'^\mu_l$. Let the set of all $2(T-1)$ vectors $\{a^\mu_l, a'^\mu_l\}_{\mu \in \mathcal{T} \backslash \{1\}}$ be denoted as I^1. Construct $\sigma_{(p,w)} = \left(\sigma^1_{(p,w)}, ..., \sigma^T_{(p,w)}\right)$ as follows: for all $\mu \in \mathcal{T} \backslash \{1\}$, $\frac{\sigma^1_{(p,w)}}{\sigma^\mu_{(p,w)}} = \frac{a'^\mu_{l\mu} - a^\mu_{l\mu}}{a^\mu_{l1} - a'^\mu_{l1}}$ and $\sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} = 1$. By construction $\sigma_{(p,w)} > \mathbf{0}$ and, for all $\mu \in \mathcal{T} \backslash \{1\}$, $\sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} a^\mu_{l\nu} = \sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} a'^\mu_{l\nu}$. We show that, starting from I^1, one iso-labour surface can be constructed such that for all $(c,a,p,w), (c',a',p,w) \in \mathcal{CP}$ with $\sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} a_{l\nu} = \sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} a'_{l\nu} = k$, we have $a_l \sim_{(p,w)} a'_l$.

Step 3. Consider $a^2_l, a'^2_l \in I^1$: by construction $(c^2, a^2, p, w), (c'^2, a'^2, p, w) \in \mathcal{CP}$ are such that $a^2_{l1} > a'^2_{l1}$, $a^2_{l2} < a'^2_{l2}$, and $a^2_{l\zeta} = a'^2_{l\zeta}$, $\zeta \neq 1, 2$, and $a^2_l \sim_{(p,w)} a'^2_l$. Choose $y^2 \in \mathbb{R}^T_+$ such that $a^{\max}_l \equiv a^2_l + y^2 \geqq a^\mu_l$, for all $a^\mu_l \in I^1$. [If $a^2_l \geqq a^\mu_l$ for all $\mu \in \mathcal{T} \backslash \{1\}$, then $y^2 = \mathbf{0}$ can be chosen.] By Step 1, $a^2_l \sim_{(p,w)} a'^2_l$ implies $a^{\max}_l \equiv a^2_l + y^2 \sim_{(p,w)} a'^2_l + y^2$.

Similarly, consider any $a^\mu_l, a'^\mu_l \in I^1$, $\mu \in \mathcal{T} \backslash \{1,2\}$. By construction $(c^\mu, a^\mu, p, w), (c'^\mu, a'^\mu, p, w) \in \mathcal{CP}$ are such that $a^\mu_{l1} > a'^\mu_{l1}$, $a^\mu_{l\mu} < a'^\mu_{l\mu}$, and $a^\mu_{l\zeta} = a'^\mu_{l\zeta}$, $\zeta \neq 1, \mu$, and $a^\mu_l \sim_{(p,w)} a'^\mu_l$. For all $\mu \in \mathcal{T} \backslash \{1,2\}$, define $y^\mu \in \mathbb{R}^T_+$ such that for any $a^\mu_l, a'^\mu_l \in I^1$: $a^\mu_l + y^\mu = a^{\max}_l$. By Step 1, $a^\mu_l \sim_{(p,w)} a'^\mu_l$ implies $a^{\max}_l = a^\mu_l + y^\mu \sim_{(p,w)} a'^\mu_l + y^\mu$, for all $\mu \in \mathcal{T} \backslash \{1,2\}$.

Therefore, we obtain a set of T vectors $\left\{a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right\} \subset \mathbb{R}^T_+$ such that $a^{\max}_l \sim_{(p,w)} a'^\mu_l + y^\mu$, for all $\mu \in \mathcal{T} \backslash \{1\}$, and by transitivity, $a'^\eta_l + y^\eta \sim_{(p,w)} a'^\mu_l + y^\mu$, for all $\mu, \eta \in \mathcal{T} \backslash \{1\}$. Moreover, by the construction of $\sigma_{(p,w)}$ in Step 2, and noting that $a^{\max}_{l\nu} \geq \mathbf{0}$, $\sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} (a'^\mu_{l\nu} + y^\mu) = \sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} a^{\max}_{l\nu} = k > 0$, for all $\mu \in \mathcal{T} \backslash \{1\}$. Finally, noting that the addition of y^μ to each pair of vectors preserves the original inequalities, the T vectors are easily shown to be affinely independent.

Step 4. Let $\Delta\left(a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right)$ be the closed $T-1$ simplex defined by $\left\{a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right\} \subset \mathbb{R}^T_+$. Next, let $\Delta(e^1, ..., e^T)$ be the closed $T-1$ simplex defined by $\{e^1, ..., e^T\} \subset \mathbb{R}^T_+$, where for all $\nu \in \mathcal{T}$, $e^\nu \equiv \left(0, ..., \frac{k}{\sigma^\nu_{(p,w)}}, ..., 0\right)$. By construction, $\Delta\left(a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right) \subseteq \Delta(e^1, ..., e^T) = \left\{a_l \in \mathbb{R}^T_+ : \sum_{\nu \in \mathcal{T}} \sigma^\nu_{(p,w)} a_{l\nu} = k\right\}$.

Step 5. For all $(c,a,p,w) \in \mathcal{CP}$ such that $a_l \in \Delta\left(a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right)$, Lemma 1 implies $a_l \sim_{(p,w)} a^{\max}_l$. For all $(c,a,p,w) \in \mathcal{CP}$ such that $a_l \in \Delta(e^1, ..., e^T) \backslash \Delta\left(a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right)$, there exist $(\widetilde{c}, \widetilde{a}, p, w), (\widetilde{c}', \widetilde{a}', p, w) \in \mathcal{CP}$ and $t \in (0,1)$ such that $\widetilde{a}_l, \widetilde{a}'_l \in \Delta\left(a^{\max}_l, (a'^\mu_l + y^\mu)_{\mu \in \mathcal{T} \backslash \{1\}}\right)$ and $\widetilde{a}_l = t a_l + (1-t) \widetilde{a}'_l$. Then, noting that by the previous argument (together with transitivity) $\widetilde{a}_l \sim_{(p,w)} \widetilde{a}'_l$, by Lemma 2 it follows that $a_l \sim_{(p,w)} \widetilde{a}_l \sim_{(p,w)} \widetilde{a}'_l$.

Therefore by transitivity, we conclude that for all $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$ such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} = \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a'_{l\nu} = k$, we have $a_l \sim_{(p,w)} a'_l$.

Step 6. Next, we show that for all $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$ such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} = \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a'_{l\nu} = k' \neq k$, we have $a_l \sim_{(p,w)} a'_l$. Suppose first that $k' > k$. By Step 3, consider any $\left\{\left(c^i, a^i, p, w\right)\right\}_{i=1,\dots,T} \subset \mathcal{CP}$ such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a^i_{l\nu} = k$ for all $i = 1, \dots, T$, and $\left\{a^i_l\right\}_{i=1,\dots,T} \subset \mathbb{R}^T_+$ is a set of T affinely independent vectors. By Step 5, we have $a^i_l \sim_{(p,w)} a^j_l$, for all $i,j \in \{1,\dots,T\}$. Let $y = (k'-k, k'-k, \dots, k'-k) > \mathbf{0}$. Then $\left\{a^i_l + y\right\}_{i=1,\dots,T} \subset \mathbb{R}^T_+$ is a set of T affinely independent vectors such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} \left(a^i_{l\nu} + y_\nu\right) = k'$, for all $i = 1, \dots, T$, and by Step 1, $a^i_l + y \sim_{(p,w)} a^j_l + y$, for all $i,j \in \{1,\dots,T\}$. Therefore the argument in Steps 4 and 5 can be applied to conclude that for all $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$ such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} = \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a'_{l\nu} = k$, we have $a_l \sim_{(p,w)} a'_l$.

A similar argument holds for the case $k' < k$, restricting attention to the profiles $\left(c^i, a^i, p, w\right) \in \mathcal{CP}$ such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a^i_{l\nu} = k$ and such that if $y = (k'-k, k'-k, \dots, k'-k)$ then $a^i_l + y \in \mathbb{R}^T_+$.

Step 7. The previous arguments prove that if $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$ are such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} = \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a'_{l\nu}$, then $a_l \sim_{(p,w)} a'_l$. Then, by **D** and transitivity, it follows that for all $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$ such that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} > \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a'_{l\nu}$, it must be $a_l \succ_{(p,w)} a'_l$. ■

Proof of Corollary 1. Straightforward and therefore omitted. ■

Proof of Corollary 2.

(*Necessity*) To see that **CPTC** is satisfied, take any $(c,a,p,w),(c,a',p,w) \in \mathcal{CP}$ such that $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ and $\underline{a} \leq \underline{a}'$. For any $(p,w) \in \mathbb{R}^{n+T}_+$, $\underline{a} \leq \underline{a}'$ implies $p\underline{a} \leqq p\underline{a}'$. Therefore, given $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ it follows that $wa_l > wa'_l$, and so $a_l \succ_{(p,w)} a'_l$, as sought.

(*Sufficiency*) We only need to prove that for all $\nu,\mu \in \mathcal{T}$, $\frac{w_\nu}{w_\mu} = \frac{\sigma^{\nu}_{(p,w)}}{\sigma^{\mu}_{(p,w)}}$. Assume on the contrary that $\frac{w_\nu}{w_\mu} \neq \frac{\sigma^{\nu}_{(p,w)}}{\sigma^{\mu}_{(p,w)}}$ for some $\nu,\mu \in \mathcal{T}$. By **CPTC**, there exist a profile $(c,a,p,w) \in \mathcal{CP}$ with $c \geq \mathbf{0}$, $a \in P$, $P \in \mathcal{P}$, a neighbourhood $\mathcal{N}(P) \subseteq \mathcal{P}$, and a neighbourhood $\mathcal{N}(a) \subseteq \mathbb{R}^{T+n}_- \times \mathbb{R}^n_+$ of a such that $\mathcal{N}'(a) \equiv \left[\mathcal{N}(a) \cap \phi^{\mathcal{N}(P)}(c)\right] \setminus \{a\}$ is a neighbourhood of a excluding a itself and for all $a' \in \mathcal{N}'(a)$, if $p\underline{a} + wa_l > p\underline{a}' + wa'_l$ and $\underline{a} \leq \underline{a}'$, then $a_l \succ_{(p,w)} a'_l$. However, since $w \neq \sigma_{(p,w)}$, there exists $a'' \in \mathcal{N}'(a)$ such that $p\underline{a} + wa_l > p\underline{a}'' + wa''_l$ and $\underline{a} \leq \underline{a}''$, which implies $wa_l > wa''_l$, but $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} \leqq \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a''_{l\nu}$. Note that $a'' \in \phi^{\mathcal{N}(P)}(c)$ with the property of $p\underline{a} + wa_l > p\underline{a}'' + wa''_l$ and $\underline{a} \leq \underline{a}''$ is ensured by the universality of $\mathcal{P}$. Since $\succsim_{(p,w)}$ is generalised additive associated with a positive vector $\sigma_{(p,w)}$ by Theorem **1**, $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a_{l\nu} \leqq \sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} a''_{l\nu}$ implies $a_l \not\succ_{(p,w)} a'_l$, thus violating **CPTC**. ■

Proof of Theorem 2. (*Necessity*) It is immediate that if a labour ordering $\succsim$ on $\mathcal{CP}$ is generalised additive, it satisfies the axioms.

(*Sufficiency*) By Theorem **1**, for each $(p,w) \in \mathbb{R}^{n+T}_+$, and for any $(c,a,p,w),(c',a',p,w) \in \mathcal{CP}$, there exists $\sigma_{(p,w)} \in \mathbb{R}^T_{++}$ such that $(c,a,p,w) \succsim (c',a',p,w)$ if and only if $\sigma_{(p,w)} \cdot a_l \geqq \sigma_{(p,w)} \cdot a'_l$. Note that $\sum_{\nu\in\mathcal{T}} \sigma^{\nu}_{(p,w)} = 1$ holds by the construction in the proof of Theorem **1**.

By axiom **ME**, for any $(p,w),(p',w') \in \mathbb{R}^{n+T}_+$, there exist $(c,a,p,w),(c',a',p',w') \in \mathcal{CP}$ such that for any $\nu,\mu \in \mathcal{T}$, $a_{l\nu} = a_{l\mu} > 0$ and $a'_{l\nu} = a'_{l\mu} > 0$, and that $(c,a,p,w) \sim (c',a',p',w')$. Without loss of generality, let $\sigma_{(p,w)} \cdot a_l \neq \sigma_{(p',w')} \cdot a'_l$. Then, there exists $\lambda > 0$ such that $\sigma_{(p,w)} \cdot a_l = \lambda\sigma_{(p',w')} \cdot a'_l$. Let $\widetilde{\sigma}_{(p',w')} \equiv \lambda\sigma_{(p',w')}$, so that $\sigma_{(p,w)} \cdot a_l = \widetilde{\sigma}_{(p',w')} \cdot a'_l$. Then, by **SI** and the transitivity of $\succsim$, it follows that for any $(c'', a'', p, w), (c^*, a^*, p', w') \in \mathcal{CP}$, $(c'', a'', p, w) \succsim (c^*, a^*, p', w')$ if and only if $\sigma_{(p,w)} \cdot a_l'' \geqq \widetilde{\sigma}_{(p',w')} \cdot a_l^*$.

Consider any $(p, w), (p', w'), (p'', w'') \in \mathbb{R}_+^{n+T}$. Let $\lambda_{(p,w;p',w')} > 0$ be such that $\sigma_{(p,w)} \cdot a_l = \lambda_{(p,w;p',w')}\sigma_{(p',w')} \cdot a_l'$ for $(c, a, p, w), (c', a', p', w')$ with $(c, a, p, w) \sim (c', a', p', w')$; let $\lambda_{(p',w';p'',w'')} > 0$ be such that $\sigma_{(p',w')} \cdot a_l' = \lambda_{(p',w';p'',w'')}\sigma_{(p'',w'')} \cdot a_l''$ for $(c', a', p', w'), (c'', a'', p'', w'')$ with $(c', a', p', w') \sim (c'', a'', p'', w'')$; and let $\lambda_{(p,w;p'',w'')} > 0$ be such that $\sigma_{(p,w)} \cdot a_l = \lambda_{(p,w;p'',w'')}\sigma_{(p'',w'')} \cdot a_l''$ for $(c'', a'', p'', w''), (c, a, p, w)$. The proof is concluded by showing that $\lambda_{(p,w;p'',w'')} = \lambda_{(p,w;p',w')}\lambda_{(p',w';p'',w'')}$ holds.

Suppose, by way of contradiction, that $\lambda_{(p,w;p'',w'')} \neq \lambda_{(p,w;p',w')}\lambda_{(p',w';p'',w'')}$. By $\sigma_{(p,w)} \cdot a_l = \lambda_{(p,w;p',w')}\sigma_{(p',w')} \cdot a_l'$ and $\sigma_{(p',w')} \cdot a_l' = \lambda_{(p',w';p'',w'')}\sigma_{(p'',w'')} \cdot a_l''$, it follows that $(c, a, p, w) \sim (c', a', p', w') \sim (c'', a'', p'', w'')$, and $\sigma_{(p,w)} \cdot a_l = \lambda_{(p,w;p',w')}\sigma_{(p',w')} \cdot a_l' = \lambda_{(p,w;p',w')}\lambda_{(p',w';p'',w'')}\sigma_{(p'',w'')} \cdot a_l''$ holds. By the transitivity of $\succsim$, $(c, a, p, w) \sim (c'', a'', p'', w'')$ holds. Then, $\sigma_{(p,w)} \cdot a_l = \lambda_{(p,w;p'',w'')}\sigma_{(p'',w'')} \cdot a_l''$. However, $\sigma_{(p,w)} \cdot a_l = \lambda_{(p,w;p',w')}\lambda_{(p',w';p'',w'')}\sigma_{(p'',w'')} \cdot a_l''$ and $\lambda_{(p,w;p'',w'')} \neq \lambda_{(p,w;p',w')}\lambda_{(p',w';p'',w'')}$, which is a contradiction. Therefore, $\lambda_{(p,w;p'',w'')} = \lambda_{(p,w;p',w')}\lambda_{(p',w';p'',w'')}$ holds. ■

Proof of Corollary 3. Straightforward and therefore omitted. ■

Proof of Theorem 3. (*Necessity*) It is immediate that if a labour ordering $\succsim$ on $\mathcal{CP}$ is wage additive, it satisfies the axioms.

(*Sufficiency*) Take any pair of profiles $(c, a, p, w), (c', a', p', w') \in \mathcal{CP}$. Note that by the universality of $\mathcal{P}$, it is possible that $(c', a', p, w), (c, a, p', w') \in \mathcal{CP}$. Note that it follows from **CPTC** that $(c, a, p, w) \succsim (c', a', p, w)$ if and only if $w \cdot a_l \geqq w \cdot a_l'$. Likewise, $(c, a, p', w') \succsim (c', a', p', w')$ if and only if $w' \cdot a_l \geqq w' \cdot a_l'$.

Let $w^* > \mathbf{0}$ be such that $\sum w_\nu^* = 1$ and $w^* = \lambda_{(w,w^*)}w$ for some $\lambda_{(w,w^*)} > 0$. Also, let $w'^* > \mathbf{0}$ be such that $\sum w_\nu'^* = 1$ and $w'^* = \lambda_{(w',w'^*)}w'$ for some $\lambda_{(w',w'^*)} > 0$. Then, by **I**, there exist $(c^*, a^*, p, w^*), (c^*, a^*, p', w'^*) \in \mathcal{CP}$ such that for any $\nu, \mu \in \mathcal{T}$, $a_{l\nu}^* = a_{l\mu}^*$, and that $(c^*, a^*, p, w^*) \sim (c^*, a^*, p', w'^*)$ for $w^* a_l^* = w'^* a_l^*$. Moreover, by **SS**, there exist $(c^{**}, a^{**}, p, w^*), (c^{***}, a^{***}, p, w) \in \mathcal{CP}$ such that for any $\nu, \mu \in \mathcal{T}$, $a_{l\nu}^{**} = a_{l\mu}^{**}$ and $a_{l\nu}^{***} = a_{l\mu}^{***}$; $\lambda_{(w,w^*)}a_l^{**} = a_l^{***}$; and $(\underline{a}^{**}, \overline{a}^{**}) = (\underline{a}^{***}, \overline{a}^{***})$, and that $(c^{**}, a^{**}, p, w^*) \sim (c^{***}, a^{***}, p, w)$. By the same argument applying **SS**, there exist $(c'^{**}, a'^{**}, p', w'^*), (c'^{***}, a'^{***}, p', w') \in \mathcal{CP}$ such that $(c'^{**}, a'^{**}, p', w'^*) \sim (c'^{***}, a'^{***}, p', w')$.

Note that there exists $k > 0$ such that $a_l^* = k a_l^{**}$. Then, $(c^*, a^*, p, w^*) \sim (kc^{**}, ka^{**}, p, w^*)$ by $w^* a_l^* = w^* k a_l^{**}$ and **CPTC**. Then, $(kc^{**}, ka^{**}, p, w^*) \sim (kc^{***}, ka^{***}, p, w)$ by **SI**. Thus, $(c^*, a^*, p, w^*) \sim (kc^{***}, ka^{***}, p, w)$ by the transitivity of $\succsim$. Likeiwse, there exists $k' > 0$ such that $a_l^* = k' a_l'^{**}$. Then, $(c^*, a^*, p', w'^*) \sim (k'c'^{**}, k'a'^{**}, p', w'^*)$ by $w'^* a_l^* = w'^* k' a_l'^{**}$ and **CPTC**. Then, $(k'c'^{**}, k'a'^{**}, p', w'^*) \sim (k'c'^{***}, k'a'^{***}, p', w')$ by **SI**. Thus, $(c^*, a^*, p', w'^*) \sim (k'c'^{***}, k'a'^{***}, p', w')$ by the transitivity of $\succsim$.

In conclusion, by the transitivity of $\succsim$, $(kc^{***}, ka^{***}, p, w) \sim (k'c'^{***}, k'a'^{***}, p', w')$ holds for $wka_l^{***} = w'k'a_l'^{***}$. Then, by **D**, **SI**, and the transitivity of $\succsim$, we obtain for $(c, a, p, w), (c', a', p', w') \in \mathcal{CP}$, $(c, a, p, w) \succsim (c', a', p', w')$ holds if and only if $w \cdot a_l \geqq w' \cdot a_l'$ holds. ■

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