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PARTIAL PRIVATIZATION AND SUBSIDIZATION IN A MIXED DUOPOLY: R&D VERSUS OUTPUT SUBSIDIES*

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Abstract

This study investigates R&D and output subsidies in a mixed duopoly with partial privatization. We show that an output subsidy is welfare-superior to an R&D subsidy policy, but the government has a higher incentive to privatize the public firm under the output subsidy than the R&D subsidy. We also show that when the government uses the policy mix of R&D and output subsidies together, it can achieve the first-best allocation, in which the degree of privatization does not influence output subsidies but influences R&D subsidies.

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I. *Introduction*

As globalization and innovation have intensified the market competition among innovative

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firms, policy makers have significantly recognized the importance of R&D activities and thus have enacted various policies to encourage them. Among the effective policy alternatives in the real world, governments are continuously increasing R&D subsidization toward public institutions and organizations, so that public firms are key players in R&D-intensive industries in contemporary economies, such as healthcare, medical, energy, and bio-agriculture.[1]

The policy consequences of R&D subsidies in mixed oligopolies, where public and private firms compete in R&D investments, are practical in both academic and political fields.[2] As such, the study of the relationship between R&D activity and subsidies in mixed oligopolies has clear policy importance regarding current economic issues on the development of a national innovation system.

Some contributors have studied cost-reducing R&D activities in the context of mixed oligopolies.[3] Regarding subsidy policies, recent studies have analyzed their effects on R&D activities and welfare. For instance, Zikos (2007) analyzed the policy mix of output and R&D subsidies in a mixed duopoly, and showed that the first-best can be obtained under full nationalization. Gil Molto, et al. (2011) examined an R&D subsidy, and showed that the subsidy leads to an increase total R&D and production, but not to an efficient distribution of production costs. They also found that full privatization of a public firm reduces R&D activities and welfare. Kesavayuth and Zikos (2013) also examined the relative welfare effects between R&D and output subsidies, and showed that an R&D subsidy is socially superior (inferior) to an output subsidy when R&D spillovers are high (low). On the other hand, Haruna and Goel (2015) compared two models with and without R&D under an output subsidy only, and found that output subsidy rankings are significantly affected by R&D spillovers, but the welfare ranking is not affected.

However, not all these studies considered the partial privatization of a public firm, which is a popular academic and realistic policy issue in mixed oligopolies.[4] In this study, we consider the optimal degree of privatization and compare the welfare consequences of output or R&D subsidies. We show that subsidy rate is always positive, irrespective of the degree of privatization, and that welfare is higher under output subsidy than that under R&D subsidy for any degree of privatization. This result is similar with Kesavayuth and Zikos's (2013), who only consider full nationalization. Further, we show that the government has a higher incentive to privatize the public firm under the output subsidy than the R&D subsidy. This is consistent with the result of Gil-Moltó et al. (2011), who showed that full privatization is not desirable, regardless of whether the government provides R&D subsidies to private and public firms.

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[1] Aanestad, et al. (2003) and Godø, et al. (2003) provided attentional case studies in the medical and energy sectors in European and OECD countries, and reported that public firms are key players in R&D-intensive industries. See also other interesting examples in Gil-Moltó, et al. (2011).

[2] The increasing interest of privatization policies in mixed oligopolies stems from their importance in regulatory reforms in the economies of developed regions, such as Western Europe, Canada, and Japan, and transitionary economies, such as those of China and Eastern Europe. See Bos (1986) and De Fraja and Delbono (1989) for early discussions, and Matsumura and Shimizu (2010) and Lee, et al. (2013) for recent developments.

[3] For example, Delbono and Denicolo (1993), Poyago-Theotoky (1998), Ishibashi and Matsumura (2006), and Heywood and Ye (2009) examined R&D competition in a mixed market, where patent races among firms are introduced. However, they did not incorporate R&D subsidies and their implications on the R&D policy.

[4] Since Matsumura (1998) examined partial privatization, studies on optimal privatization are increasingly popular and extensively used in many various contexts. For example, Ino and Matsumura (2010), Lee, et al. (2013) and Xu, et al. (2016) reviewed several research topics on optimal privatization.

We also consider the optimal policy mix of output and R&D subsidies, and show that the first-best allocation can be obtained irrespective of the degree of privatization policy. However, the rate of the output subsidy is constant, but the rate of the R&D subsidy is always negative, which is increasing in the degree of privatization. Therefore, the R&D subsidy should be used to discourage the over-investment when the output subsidy is already provided. It confirms the results of Zikos (2007) under full nationalization, but we show that the privatization policy does not influence welfare consequences although R&D stage is introduced. It is also consistent with Lee and Tomaru (2017), who introduced the approach of partial privatization with general demand and cost functions. We extend their analysis by different approaches in deriving the optimal policy mix of R&D and output subsidies. Further, we also explore which subsidization policy is more socially desirable and to what extent a public firm should be privatized when the policy mix is not available.

The organization of this paper is as follows. In section II, we present a mixed duopoly model, in which output and R&D competition between public and private firms occurs. In section III, we consider a single subsidy policy and compare the welfare effects of output and R&D subsidy policies. In section IV, we discuss on the optimal degree of privatization and investigate the optimal policy mix of output and R&D subsidies. Finally, we conclude our analysis in section V.

II. *The Model*

Consider a duopoly market, where firms 0 and 1 produce homogeneous goods. Let the inverse demand function be $P(Q)=a-Q$, where P is the market price, $Q(=q_0+q_1)$ is the market output, and q_0 and q_1 are the outputs of firm *i,* respectively. Then, consumer surplus is $CS=Q^2/2$.

We assume that the cost of production and R&D are, respectively, $C(q_i, x_i)=(c-x_i)q_i+q_i^2$ and $\Gamma(x_i)=x_i^2$, where $a>c>0$ and x_i denote the amount of R&D investment for firm i. The production cost shows that a firm's R&D investment shifts its marginal cost function downwards, $\partial C/\partial q_i=c-x_i+2q_i$, but does not alter its slope.[5] Note that R&D activity is perfectly protected against imitation.[6] The firm has to spend x_i^2 to implement cost-reducing R&D, in which R&D investment can reduce its own cost by x_i per unit of output, but exhibits decreasing returns to scale. Finally, each firm receives an output or/and R&D subsidy, where $s_q q_i$ and $s_x x_i$ denote the per-unit subsidy to output quantity and R&D performance, respectively.

Then, the profit function of the firm is as follows:

$$\pi_i=(a-q_0-q_1)q_i-(c-x_i)q_i-q_i^2-x_i^2+s_q q_i+s_x x_i, \qquad i=0,1,$$

where s_q and s_x are, the output and R&D subsidy rates, respectively. Social welfare, defined as

[5] Following Zikos (2007), we assume a quadratic production cost function, which is standard in mixed market literature, for ruling out the uninteresting case of a public monopoly.

[6] We ignore R&D spillovers between the firms. However, part or all R&D results of a firm might spill over onto its rival in a mixed market. See Heywood and Ye (2009), Gil-Moltó, et al. (2011), Kesavayuth and Zikos (2013), and Haruna and Goel (2015).

the sum of consumer surplus, firms' profit and net subsidy, is given by

$$W=CS+\pi_0+\pi_1-s_q(q_0+q_1)-s_x(x_0+x_1).$$

Note that the subsidies are financed from taxpayers in a lump-sum manner, so that they do not directly influence welfare.

Firm 1 is a private firm that maximizes its own profit. On the other hand, firm 0 is a public firm owned by the welfare-maximizing government. We allow the government to sell its shares in firm 0 to profit-maximizing private investors. Let $\theta\in[0,1]$ be the shares in firm 0 that private investors hold. If $\theta\in(0,1)$, firm 0 becomes a partially privatized firm, which is jointly owned by the government and private investors. Following Matsumura (1998), we assume that firm 0 maximizes the convex combination of its profit and welfare:

$$V=(1-\theta)W+\theta\pi_0.$$

The mixed duopoly model with R&D is a three-stage game. In the first stage, the government selects the degree of privatization and either output or R&D subsidies to maximize welfare. Observing the government's decision, firms 0 and 1 independently and simultaneously choose their R&D investment levels in the second stage and their output levels in the third stage. We solve the subgame perfect Nash equilibrium of this game by backward induction.

III. *The Analysis*

1. Stage Three: Output Choice by Both Firms

In the third stage, the first-order conditions of the private firm and the public firm are as follows, respectively:

$$\frac{\partial V}{\partial q_0}=a-(q_0+q_1)-\theta q_0-(c-x_0)-2q_0+\theta s_q=0,$$

$$\frac{\partial \pi_1}{\partial q_1}=a-(q_0+q_1)-q_1-(c-x_1)-2q_1+s_q=0.$$

Rearranging these two equations yields the following reaction functions of the firms:

$$q_0=\frac{a-q_1-(c-x_0)+\theta s_q}{3+\theta} \text{ and } q_1=\frac{a-q_0-(c-x_1)+s_q}{4}.$$

As usual, outputs are strategic substitutes for both firms, but their magnitude depends on the degree of privatization and output subsidy rate. The equilibrium outputs of the third stage are:

$$q_0^*=\frac{3a-4(c-x_0)+(c-x_1)+(4\theta-1)s_q}{11+4\theta} \text{ and } q_1^*=\frac{(2+\theta)a-(3+\theta)(c-x_1)+(c-x_0)+3s_q}{11+4\theta}.$$

Then, we have the following:

$$\frac{\partial q_0^*}{\partial x_0}=\frac{4}{11+4\theta}>\frac{\partial q_1^*}{\partial x_1}=\frac{3+\theta}{11+4\theta}>0 \text{ and } \frac{\partial q_0^*}{\partial x_1}=\frac{\partial q_1^*}{\partial x_0}=\frac{-1}{11+4\theta}<0.$$

An increase in R&D by one firm increases the equilibrium output of the firm, but decreases that of the rival.

2. Stage Two: R&D Choice by Both Firms

In the second stage, the first-order conditions of public and private firms are characterized by the following conditions, respectively:

$$\frac{\partial q_0^*}{\partial x_0}\times\frac{\partial V}{\partial q_0}+\frac{\partial q_1^*}{\partial x_0}\times\frac{\partial V}{\partial q_1}-\frac{\partial C}{\partial x_0}-\left(\frac{\partial \Gamma}{\partial x_0}-\theta s_x\right)=0,$$

$$\frac{\partial q_0^*}{\partial x_1}\times\frac{\partial \pi_1}{\partial q_0}+\frac{\partial q_1^*}{\partial x_1}\times\frac{\partial \pi_1}{\partial q_1}-\frac{\partial C}{\partial x_1}-\left(\frac{\partial \Gamma}{\partial x_1}-s_x\right)=0.$$

Using the envelope theorem and explicit outcomes, we have the following reaction functions, x_0 and x_1:

$$x_0=\frac{(31+16\theta+\theta^2)(a-c)-(14+3\theta-\theta^2)x_1-(3-35\theta-16\theta^2)s_q+\theta(11+4\theta)^2 s_x}{197+157\theta+32\theta^2},$$

$$x_1=\frac{4(6+5\theta+\theta^2)(a-c)-4(3+\theta)x_0+12(3+\theta)s_q+(11+4\theta)^2 s_x}{206+152\theta+28\theta^2}.$$

The reaction function of each firm declines with rival's R&D investment, but its magnitude depends on the degree of privatization and subsidy rates. This implies that R&D investments are also strategic substitutes for both firms. An increase in R&D investment by the firm leads to a decrease in the output by its rival firm, thereby reducing its incentives to conduct R&D. We have the equilibrium R&D investment of the second stage:

$$x_0^*=\frac{2(a-c)(275+248\theta+65\theta^2+4\theta^3)-2(51-313\theta-274\theta^2-56\theta^3)s_q-(11+4\theta)(14-203\theta-153\theta^2-28\theta^3)s_x}{3674+4318\theta+1700\theta^2+224\theta^3},$$

$$x_1^*=\frac{4(a-c)(3+\theta)(33+33\theta+8\theta^2)+8(3+\theta)(27+10\theta)s_q+(11+4\theta)(197+145\theta+28\theta^2)s_x}{3674+4318\theta+1700\theta^2+224\theta^3},$$

Then, we also have the followings:

$$\frac{\partial x_0}{\partial s_q}=\frac{-2(51-313\theta-274\theta^2-56\theta^3)}{3674+4318\theta+1700\theta^2+224\theta^3},\ \frac{\partial x_0}{\partial s_x}=\frac{-(11+4\theta)(14-203\theta-153\theta^2-28\theta^3)}{3674+4318\theta+1700\theta^2+224\theta^3},$$

$$\frac{\partial x_1}{\partial s_q}=\frac{8(3+\theta)(27+10\theta)}{3674+4318\theta+1700\theta^2+224\theta^3}>0,\ \text{and}\ \frac{\partial x_1}{\partial s_x}=\frac{(11+4\theta)(197+145\theta+28\theta^2)}{3674+4318\theta+1700\theta^2+224\theta^3}>0.$$

This shows that the private firm's R&D is increasing for both output and R&D subsidies, while the public firm's R&D is dependent upon the degree of privatization. Particularly, if θ is sufficiently small (large), the public firm's R&D is decreasing (increasing) for the output or R&D subsidies. However, the decrease in the public firm's R&D will be outweighed by the increase in the private firm's. Therefore, total R&D, $X^*=x_0^*+x_1^*$, is increasing for both output and R&D subsidies. However, the effects of the output subsidy on total R&D are lower than those of the R&D subsidy, that is, $\partial X^*/\partial s_x > \partial X^*/\partial s_q > 0$.

Finally, we have the following equilibrium outputs:

$$q_0^* = \frac{2(a-c)(583+443\theta+84\theta^2)-2(215-643\theta-570\theta^2-112\theta^3)s_q-(11+4\theta)(23-69\theta-28\theta^2)s_x}{2(1837+2159\theta+850\theta^2+112\theta^3)},$$

$$q_1^* = \frac{2(a-c)(11+4\theta)(33+33\theta+8\theta^2)+4(11+4\theta)(27+10\theta)s_q+(11+4\theta)(55+19\theta)s_x}{2(1837+2159\theta+850\theta^2+112\theta^3)}.$$

Note that both output and R&D subsidies induce the private firm to enlarge its output and R&D investment, but the effects on the public firm depend on the degree of privatization. Particularly, if θ is sufficiently small (large), the public firm's output is decreasing (increasing) for the output or R&D subsidies. However, the decrease in the public firm's output will be outweighed by the increase in the private firm's. Therefore, total industry outputs, $Q^* = q_0^* + q_1^*$, are increasing for both output and R&D subsidies. However, the effects of the output subsidy on total output are higher than of the R&D subsidy, that is, $\partial Q^*/\partial s_q > \partial Q^*/\partial s_x > 0$.

3. Stage One: Subsidy Choice by Government

In the first stage, the government chooses either output or R&D subsidy to maximize welfare, given the degree of privatization. Consequently, social welfare can be rewritten as follows:

$$W^* = \frac{(Q^*)^2}{2} + \pi_0(x_0^*, x_1^*, q_0^*, q_1^*) + \pi_1(x_0^*, x_1^*, q_0^*, q_1^*) - s_q(q_0^* + q_1^*) - s_x(x_0^* + x_1^*).$$

From the first-order condition of $\partial W^*/\partial s_q = 0$ or $\partial W^*/\partial s_x = 0$, we have the following optimal output or R&D subsidy condition:

$$s_q(s_x,\theta) = \frac{\begin{array}{c}(2(a-c)(203247+3217769\theta+6385527\theta^2+7238119\theta^3+3849740\theta^4+952888\theta^5+89600\theta^6)-\\(851631+14771147\theta+26285547\theta^2+27051970\theta^3+13528620\theta^4+3196800\theta^5+288960\theta^6)s_x)\end{array}}{(1500150+23585160\theta+46700540\theta^2+52398000\theta^3+27548720\theta^4+6742400\theta^5+627200\theta^6)} \quad (1)$$

$$s_x(s_q,\theta) = \frac{\begin{array}{c}(2(a-c)(6501+84340\theta+195960\theta^2+162020\theta^3+42190\theta^4-800\theta^5-1120\theta^6)\\-(77421+1061330\theta+2003650\theta^2+1730670\theta^3+600540\theta^4+72240\theta^5)s_q)\end{array}}{(11+4\theta)(31914+470950\theta+631930\theta^2+596530\theta^3+305210\theta^4+77840\theta^5+7840\theta^6)} \quad (2)$$

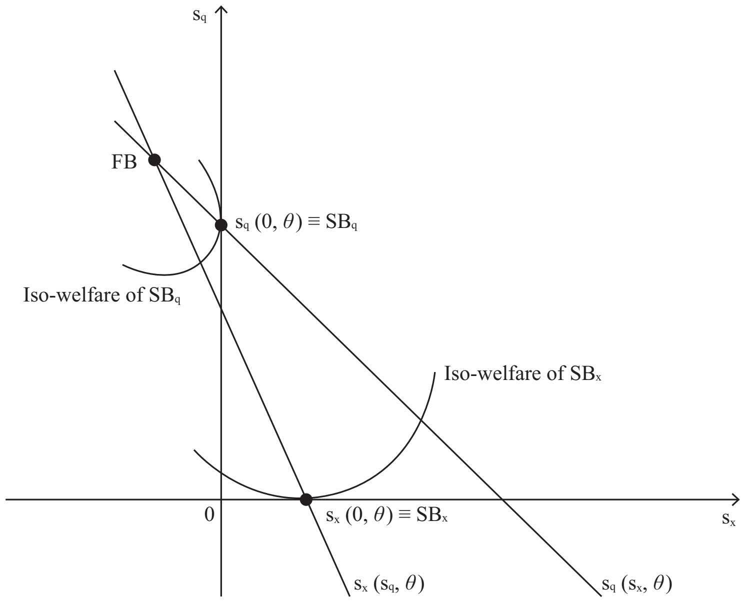
We now explore which subsidization policy between output or R&D subsidy is more socially desirable and to what extent a public firm should be privatized when a policy mix is not available.[7] Before proceeding, we need to examine the properties of optimal solutions in (1) and (2). Rearranging the two optimality equations provides the following:

$$s_q(s_x,\theta) = A_q - B_q s_x \quad (1')$$

$$s_q = A_x - B_x s_x(s_q,\theta) \quad (2')$$

[7] Gil-Moltó, et al. (2011) examined R&D subsidies, while Kesavayuth and Zikos (2013) investigated output subsidy in the presence of R&D spillovers in mixed markets.

FIGURE 1. ISO-WELFARES UNDER OUTPUT VS. R&D SUBSIDIES



where $A_q = \frac{2(a-c)(203247+321769\theta+638527\theta^2+723819\theta^3+384974\theta^4+95288\theta^5+8960\theta^6)}{(1500150+2358516\theta+4670054\theta^2+5239800\theta^3+2754872\theta^4+674240\theta^5+62720\theta^6)} > 0,$

$$B_q = \frac{(851631+1477147\theta+2628547\theta^2+2705197\theta^3+1352862\theta^4+319680\theta^5+28896\theta^6))}{(1500150+2358516\theta+4670054\theta^2+5239800\theta^3+2754872\theta^4+674240\theta^5+62720\theta^6)} > 0,$$

$$A_x = \frac{2(a-c)(6501+84340+19596\theta^2+16202\theta^3+4219\theta^4-80\theta^5-112\theta^6)}{77421+106133\theta+200365\theta^2+173067\theta^3+60054\theta^4+7224\theta^5} > 0 \text{ and}$$

$$B_x = \frac{(11+4\theta)(31914+47095\theta+63193\theta^2+59653\theta^3+30521\theta^4+7784\theta^5+784\theta^6)}{77421+106133\theta+200365\theta^2+173067\theta^3+60054\theta^4+7224\theta^5)} > 0.$$

We can show that $A_q > A_x > 0$ and $A_q/B_q > A_x/B_x > 0$ for $\theta \in [0,1]$. This implies that the optimal subsidies of s_q and s_x have a negative relationship, but the optimal output subsidy condition in (1') is flatter than the optimal R&D subsidy condition (2'), as shown in Fig.1. Note that FB in Fig.1 indicates the first-best policy mix of output and R&D subsidies. Here, if the government chooses either output or R&D subsidies, the optimal subsidy rate is indicated by SB_q or SB_x. This shows that there exists under-production and under-investment and, thus, the government should encourage production or/and R&D investment by setting a positive subsidy.

Now, we solve the optimal output or R&D subsidies. Using $s_x = 0$ or $s_q = 0$ in the optimal subsidy conditions into (1) and (2), we have the following output and R&D subsidies, respectively:

$$s_q^*(\theta) = s_q(0,\theta) = A_q, \tag{3}$$

$$s_x^*(\theta)=s_x(0,\theta)=A_x/B_x. \tag{4}$$

It is noteworthy that the government provides a positive R&D subsidy if there is no output subsidy. The importance of a positive R&D subsidy has already been shown in existing studies. For example, Gil-Moltó, et al. (2011) showed that a positive R&D subsidy resolves under-production by a private firm, even if there are R&D spillovers. In the analysis, we consider partial privatization and show that the optimal rate of the R&D subsidy is also positive, but dependent upon the degree of privatization. It implies that the effectiveness of the subsidy crucially depends on the degree of privatization and thus, the optimality of privatization should be carefully investigated when R&D stage is introduced.
Using the optimal output or R&D subsidies, we can show the following:

$$x_0^*(s_x^*(\theta))>x_1^*(s_x^*(\theta)) \text{ and } q_0^*(s_x^*(\theta))>q_1^*(s_x^*(\theta)) \tag{5}$$

$$x_0^*(s_q^*(\theta))<x_1^*(s_q^*(\theta)) \text{ and } q_0^*(s_q^*(\theta))>q_1^*(s_q^*(\theta)) \tag{6}$$

$$x_1^*(s_q^*(\theta))>x_1^*(s_x^*(\theta)) \text{ and } q_1^*(s_q^*(\theta))>q_1^*(s_x^*(\theta)) \tag{7}$$

$$x_0^*(s_q^*(\theta))<(>)x_0^*(s_x^*(\theta)) \text{ and } q_0^*(s_q^*(\theta))<(>)\ q_0^*(s_x^*(\theta)) \text{ when } \theta\rightarrow 0\ (1) \tag{8}$$

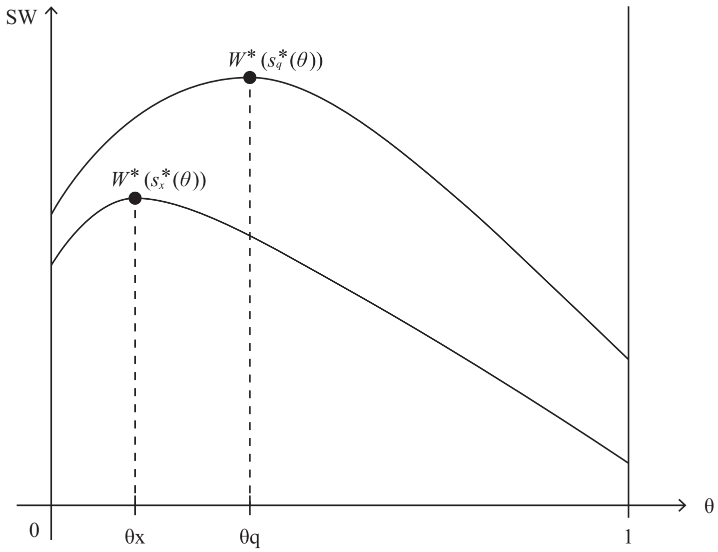
$$X^*(s_q^*(\theta))>X^*(s_x^*(\theta)) \text{ and } Q^*(s_q^*(\theta))>Q^*(s_x^*(\theta)) \tag{9}$$

First, the public firm undertakes more R&D investments and produces more outputs than the private firm under the R&D subsidy, as shown in (5), while it produces more outputs but undertakes less R&D investments than the private firm under the output subsidy, as shown in (6). Second, the private firm produces more outputs and undertakes more R&D investments under the output subsidy rather than under the R&D subsidy, as shown in (7). Third, the comparisons of R&D investment and output of public firm between the output subsidy and the R&D subsidy are ambiguous, as shown in (8). In particular, as the degree of privatization increases, the R&D investment and output of public firm under the output subsidy are getting higher than those under the R&D subsidy. Finally, total industry outputs and total industry investments are higher under the output subsidy, as shown in (9). Therefore, the output subsidy is more effective to achieve the higher outputs and higher investments.

Regarding welfare ranks, Fig.1 also compares welfare under output and R&D subsidies. The iso-welfare curve of SB_q, which goes through $s_q(0, \theta)$, is closer to the first-best point FB, which maximizes social welfare in terms of output and R&D investment than the iso-welfare curve of SB_x, which goes through $s_x(0, \theta)$. This shows that the output subsidy yields a higher welfare than the R&D subsidy, regardless of the privatization degree. This is because the cost-saving effects under an R&D subsidy are smaller than the output-increasing effects under an output subsidy. This result also supports the analysis of Kesavayuth and Zikos (2013), who showed that an output subsidy yields a higher welfare than an R&D subsidy if R&D spillovers are sufficiently low. In our analysis, we obtained the same results under partial privatization, in that the welfare effect of the output subsidy, which enlarges total industry outputs, outweighs that of the R&D subsidy, which enlarges total R&D investments.

Now, we compare welfare under output or R&D subsidies. Replacing either $s_q^*(\theta)$ in (3) or $s_x^*(\theta)$ in (4) into the welfare function provides the following welfares under the optimal output or R&D subsidies, respectively:

FIGURE 2. THE WELFARES UNDER OUTPUT VS. R&D SUBSIDY



$$W(s_q^*(\theta))=\frac{(a-c)^2(428361+6736740\theta+13334436\theta^2+14964494\theta^3+7863950\theta^4+1922000\theta^5+178400\theta^6)}{1500150+23585160\theta+46700540\theta^2+52398000\theta^3+27548720\theta^4+6742400\theta^5+627200\theta^6} \quad (10)$$

$$W(s_x^*(\theta))=\frac{(a-c)^2(8865+132680\theta+171900\theta^2+164520\theta^3+80090\theta^4+17840\theta^5+1440\theta^6)}{31914+470950\theta+631930\theta^2+596530\theta^3+305210\theta^4+77840\theta^5+7840\theta^6} \quad (11)$$

Then, we can show that $\Delta W \equiv W(s_q^*(\theta))-W(s_x^*(\theta))>0$, for all $\theta\in[0,1]$.

Proposition 1. *Given the degree of privatization, social welfare is higher under the output subsidy than under the R&D subsidy.*

Without considering partial privatization, Kesavayuth and Zikos (2013) showed that the welfare effect of output and R&D subsidies crucially depends on the degree of R&D spillovers. Specifically, if the degree of R&D spillovers is sufficiently small, welfare is higher under an output subsidy than an R&D subsidy. In the absence of R&D spillovers, Proposition 1 further shows that an output subsidy always yields higher welfare than the R&D subsidy, regardless of the privatization degree, as shown in Fig.2. This is because cost savings under an R&D subsidy are not much larger and, thus, cannot offset the distortions associated with under-production. Therefore, the output subsidy is more effective in removing significant distortions due to under-production, which provides higher welfare.

IV. *Discussions*

1. Optimal Privatization Policy

We have shown that the output subsidy yields higher welfare than the R&D subsidy regardless of the degree of privatization. Now, it is important for the government to adjust the optimal degree of privatization to enhance welfare. Then, the first-order conditions for maximizing social welfare in (10) or (11) yield the optimal degree of privatization, i.e., $\theta_q \cong 0.367$ under the output subsidy, and $\theta_x \cong 0.175$ under the R&D subsidy.

Proposition 2. *Partial privatization is the optimal policy, but the optimal degree of privatization is greater under the output subsidy than under the R&D subsidy.*

Proposition 2 shows that partial privatization is the optimal policy, regardless of whether the government sets the optimal output or R&D subsidies. It also shows that the government has a greater incentive to privatize public firms under the output subsidy than under the R&D subsidy. (See Fig.2.)

The economic explanations are as follows. Consider the nationalization case, where the public firm maximizes welfare rather than its own profit. Under the output subsidy, the public firm produces more output and invests less in R&D than the private firm, as shown in (6). The higher privatization has the effect of redistributing output from the higher-marginal-cost public firm to the lower-marginal-cost private firm. The resulting increase in the private firm's output lowers total industry costs, which induces the distribution of production costs across the firms to be more efficient. Further, due to the output substitution effect, the private firm enjoys an increase in its market share, which encourages it to engage in more cost-reducing R&D to earn higher profits. Again, the lower industry costs tend to increase total industry outputs. Therefore, non-nationalization is effective for obtaining higher welfare under the output subsidy. However, for a high degree of privatization, although it can remove cost inefficiency, under-production distortion is serious. Consequently, partial privatization is optimal under the output subsidy.

On the other hand, under the R&D subsidy, the nationalized public firm also produces more output and invests more in R&D than the private firm, as shown in (5). As such, a higher privatization will induce the private firm to enlarge its R&D investment and, thus, reduce its marginal cost. The resulting decrease in the public firm's output works toward lowering total industry costs, which induces the distribution of production costs across the firms to be more efficient. The lower industry costs also increase total industry outputs. Therefore, non-nationalization is also effective in obtaining higher welfare under the R&D subsidy. However, at the same degree of privatization under the output subsidy, the distortion of under-production will be more serious without an output subsidy, as shown in (9). As a result, partial privatization is optimal under the R&D subsidy and it should be lower than that under the output subsidy.

2. Optimal Subsidization Policy Mix

We consider the case where the government chooses the optimal policy mix of output and

R&D subsidies. Then, solving the first-order conditions of output and R&D subsidies in (1) and (2) together provides the following proposition.

Proposition 3. *The optimal policy mix of output subsidy, $s_q^F = 2(a-c)/7$, and R&D subsidy, $s_x^F(\theta) = -2(a-c)/7(11+4\theta)$, can achieve the first-best outcome.*

[Proof] We can define the first-best (FB) outcome, which maximizes social welfare in terms of output and R&D investment (q^f, x^f) from the first-order conditions, i.e., $\frac{\partial W}{\partial q} = 0$ and $\frac{\partial W}{\partial x} = 0$. Then, the first-best outcome provides $q^f = 2(a-c)/7$ and $x^f = (a-c)/7$, which is described as FB in Fig. 1. It also satisfies the second-order conditions. Then, it is easy to show that the optimal policy mix of output subsidy can attain this first-best outcome at the subgame perfect Nash equilibrium. Q.E.D.

Note that the first-best outcome requires the principles of marginal cost pricing and cost minimization, i.e., $P(Q^f) = C_q(q^f, x^f)$ and $-C_x(q^f, x^f) = \Gamma_x$. We can elicit several salient implications from this proposition. First, the positive rate of the output subsidy will induce firms with market power to produce more outputs. This is because oligopolistic firms produce less outputs than under perfect competition. Therefore, the positive output subsidy remedies the deviation from the market price of the firm's marginal revenue, $P - MR_i = -P' q_i > 0$, to make the firms behave in a perfectly competitive way.[8]

Second, the negative rate of the R&D subsidy is in fact R&D tax, which will remove the distortion of cost inefficiency due to firm over-investment, which is caused by the output subsidy.[9] The output subsidy encourages firms to overinvest because greater investments lead to higher production and, thus, higher market shares. Furthermore, the optimal rate of the R&D subsidy depends on the privatization degree. Particularly, the R&D tax rate is increasing in the degree of privatization, that is, $\partial s_x^* / \partial \theta > 0$, as a higher degree of privatization makes the public firm produce less for a given R&D profile, which enlarges private firm's outputs due to strategic substitution. Thus, the government should increase the R&D tax rate to make private firms lose their incentives to conduct R&D investment.

Third, the first-best outcomes can be achieved irrespective of the degree of privatization. For example, under the optimal policy mix, Zikos (2007) showed that the first-best can be achieved in a mixed duopoly under full nationalization ($\theta = 0$), while Lee and Tomaru (2017) showed that the first-best can be achieved in a mixed oligopoly under full privatization ($\theta = 1$). Therefore, our results confirm results in previous literature, but we show that the first-best can be achieved for any degree of privatization if the government uses the optimal policy mix of output and R&D subsidies. In fact, there are four different decisions of market failure because public and private firms have heterogeneous objectives: allocative inefficiencies from under-production and cost inefficiencies in the allocation of production costs across public and private firms. However, if the government sets full nationalization ($\theta = 0$), as assumed in Zikos (2007),

[8] In a private market, Lee (1999) compared the efficiency of output subsidy between blockaded and free entry equilibrium, while Lee (1998) discussed the efficiency of R&D subsidy on the regulated firm under asymmetric information.

[9] Learhy and Neary (1997) provided the economic rationale on the negative R&D subsidy in a private market, while Gil-Moltó, et al. (2011) showed that the rate of the R&D subsidy in a mixed market will be positive in the absence of the output subsidy.

the public firm will maximize welfare, which is the objective of the government, and thus, the government controls decisions on both the output and R&D investment of the public firm. Therefore, the policy mix of two subsidies can work to remedy the four market failures. Additionally, if the government sets full privatization ($\theta=1$), as an example in Lee and Tomatu (2017), there exists symmetric equilibrium of outputs and R&D investments for both private firms, which have homogeneous objective functions. Thus, the policy mix of output and R&D subsidies can also achieve a first-best. In the case of partial privatization, where $0<\theta<1$, we can also show that three policy instruments are sufficient to treat these market failures, as long as the R&D subsidy adjusts the degree of privatization.

Fourth, our results show that the positive rate of output subsidy is independent of the degree of privatization. Without considering R&D investments in the model, it supports the well-known Privatization Neutrality Theorem (PNT) in literature on mixed markets. PNT states that, in the absence of R&D investment choices, the same output subsidy rate yields the first-best before and after privatization.[10] We show that the first-best outputs are chosen under the positive rate of output subsidy, s_q^*, irrespective of whether the public firm is privatized under the first-best R&D investment.

Fifth, the PNT does not hold once the R&D setting stage is introduced. That is, the PNT fails because the optimal rate of R&D subsidy is dependent of the degree of privatization and, thus, the first-best is affected by the degree of privatization. Some previous studies have already presented the failure of the PNT by showing that subsidies cannot achieve the first-best (see footnote 10). In contrast, we found that, while the first-best allocation is achievable, the degree of privatization does not influence the optimal rate of the output subsidy, but influences that of the R&D subsidy.

Finally, we can reevaluate the optimal degree of privatization when other economic or political conflicts are taken into consideration. Particularly, when the government must minimize payments for subsidies due to strict budget constraints or excess burden of taxation, for instance, full nationalization (i.e., $\theta=0$) would be desirable. Recall that the optimal rate of the output subsidy is a constant, while that of the R&D subsidy is increasing with the degree of privatization. Therefore, payment for total subsidies, $2s_q^F q^f+2s_x^F(\theta)x^f$, is minimized under full nationalization ($\theta=0$). This result is in sharp contrast with the results of previous studies on R&D investment in a mixed market. For example, Heywood and Ye (2009) considered the same model, wherein a partially privatized firm and a private firm compete in quantity and R&D in the absence of subsidies, and showed that the optimal policy is partial privatization. Gil-Moltó, et al. (2011) showed that full privatization is not desirable, regardless of whether the government provides R&D subsidies to private and public firms.

[10] PNT states that privatization does not affect welfare, regardless of time structure, competition mode, the number of firms, product differentiation, and the degree of privatization under the optimal output subsidy. This has been continuously discussed by White (1996), Pal and White (1998), Poyago-Theotoky (2001), Hashimzade, et al. (2007) and Matsumura and Okumura (2013). However, Matsumura and Tomaru (2013, 2015) showed that PNT failed under the existence of either foreign competitors or an excess burden of taxation.

V. *Conclusion*

The study of R&D activities and government's subsidies in mixed oligopolies has a significant relevance in current economic issues on the innovation system. Incorporating the partial privatization approach, we investigated the welfare consequences of output and R&D subsidies, and showed that welfare is higher under the output subsidy than under the R&D subsidy, regardless of the degree of privatization. Further, partial privatization is the optimal policy in both output and R&D subsidies, but the government has a higher incentive to privatize the public firm under the output subsidy than under the R&D subsidy. Finally, we showed that the optimal policy mix of output and R&D subsidies can attain the first-best allocation, but the degree of privatization does not influence the optimal rate of output subsidy, but influences that of R&D subsidy.

There remains future research. The simplified model with Cournot duopolistic competition with homogenous products should be further examined. The endogenous market structure, such as Cournot, Bertrand, and Stackelberg, under a differentiated products market is also a promising topic for future research.[11] Further, positive externalities such as strong R&D spillover effects or output network effects might change the results on the welfare consequences between output and R&D subsidies. Finally, uncertainty is one of elements in designing R&D strategies in complex environments, which has been extensively examined in the R&D literature.[12] Thus, high standard expertise is needed for the decisions on the different R&D programs, which implies that the owners (both government and the private investors) as the laymen may be not able to choose optimal R&D strategies. As a result, the owner might simply evaluate the efficiency of R&D performances as an R&D policy target[13] or devise managerial incentive schemes. These topics are challenging issues for future research.

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[11] In the endogenous timing game under mixed duopoly without considering R&D investments, Matsumura and Ogawa (2012) showed that price competition is an equilibrium while Scrimitore (2013) showed that quantity competition is an equilibrium under output subsidization. Tomaru, et al. (2011) considered strategic delegation game and analyzed the effect of privatization on the firm's delegation type.

[12] See, for example, Whalley (2011), Kesavayuth, et al. (2017) and references therein.

[13] Conte, et al. (2009) examined the efficiency of public R&D expenditures of European Commission, while Chen, et al. (2017) analyzed the R&D performances of Chinese high-tech firms in mixed markets.

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