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SPECIFIC VERSUS AD VALOREM TAXATION WITH TAX EVASION IN IMPERFECTLY COMPETITIVE MARKETS*

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Abstract

This paper calls into question the equivalence between specific and ad valorem taxation in the presence of tax evasion under imperfect competition. Once there is evasion, evading specific taxes has to take place via concealing quantities sold, whereas evading ad valorem taxes can take place via concealing selling prices as well as quantities sold. With this difference, we show that in imperfectly competitive markets (i) if per-unit taxes are the same, output will be larger under ad valorem taxation, and (ii) specific taxation may be superior to ad valorem taxation if it causes firms to channel fewer resources into tax evasion.

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I. *Introduction*

The relative efficiency of specific versus ad valorem taxes has been well explored in the

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literature since Wicksell (1896).[1] However, all literatures referring this issue neglected the effect of tax evasion, which is a prevailing phenomenon in the real world. Without evasion and with homogeneous goods, these two tax types are known to be equivalent in competitive markets, and the ad valorem taxation is superior to the specific taxation in imperfect competition markets [see Suits and Musgrave (1953), Kay and Keen (1983), Delipalla and Keen (1992), Skeath and Trandel (1994), Denicolo and Matteuzzi (2000), Anderson et al. (2001)].[2]

In this paper, we propose another difference between these two tax types as tax evasion is considered. Once there is evasion, evading specific taxes must take place via concealing quantities sold, whereas evading ad valorem taxes can take place via concealing selling prices as well as quantities sold. While this argument is not novel,[3] to our knowledge, it has not been formally analyzed in the literature. With more instruments available for taxpayers to evade taxes, ad valorem taxation may not be superior to specific taxation in imperfectly competitive markets, as far as the cost of evasion is concerned. In fact, the tax bases of ad valorem taxation and specific taxation are different: one is sales, and the other is the quantities sold. With different tax bases, it is natural for the evasion behavior under these two tax regimes to not be the same. The assumption of price concealment under ad valorem taxation is reasonable, since the transaction price, which is known to the participants in the market, may not be a price concerning which the tax authority is well informed. This is true because prices in a market may fluctuate significantly, and the commodities in question may be less familiar intermediate goods. In addition, and perhaps most importantly, while a firm may only need to face the prices of a single commodity, the tax authorities must deal with the prices of hundreds or even thousands of commodities with a limited size of enforcement staff. Even if the tax authorities are well informed regarding the prices, they have to audit the tax return filed by the taxpayers to detect evasion. In the real world, only a fraction of the returns will be audited, and tax authorities knowing prices alone cannot deter taxpayers from evasion and thereby price concealment is very likely to take place.

Empirical studies that refer to the evasion of sales taxes seldom distinguish price concealment from quantity concealment.[4] However, in the evasion of customs duties, it is commonly recognized that undervaluing imports is one of the methods used to reduce tariffs or VAT at the border.[5] Mishra et al. (2008) classify the measures of evasion into two categories: evasion in terms of import values and evasion in terms of import quantities. By investigating import and export statistics for India and comparing them with those of her trading partners during the period 1987-2003, they find a significant gap between the values imported and exported. Fisman and Wei (2004) conclude that in China there are widespread practices of

[1] Wicksell (1896) considers the monopoly case. Under the assumption of constant marginal cost, he shows that, given the same amount of revenue collected, the ad valorem tax results to a lower consumer price and a higher level of quantity produced than the specific tax does. Later work includes Suits and Musgrave (1953), Kay and Keen (1983), Delipalla and Keen (1992), Skeath and Trandel (1994), and Anderson et al. (2001).

[2] The only exception that ad valorem taxes may not be superior to specific taxes is in an oligopoly with heterogeneous products and Bertrand competition.

[3] Virmani (1989) mentioned: “In the sales tax context this is a very good assumption if evasion involves understatement of price at which sales were made. In discussion of the issue of ad valorem versus specific taxes the assertion is often made by policy-makers that revenue (price) is easier to conceal than output.”

[4] For a survey of empirical studies about the extent of tax evasion, see Goerke (2012).

[5] See Bhagwati (1964).

underreporting the unit values of imports and mislabeling higher-taxed products as lower-taxed varieties. The above papers provide strong evidence of the price concealment behavior of taxpayers.

Huang et al. (2017) show that in the presence of tax evasion, the equivalence of specific and ad valorem taxes in the competitive market will break down. Moreover, the specific taxation may be superior to ad valorem taxation if it causes firms to channel fewer resources into tax evasion, given other things being equal. This paper will then further take into account the market structure of imperfect competition, which is widely seen in the real world. In the presence of tax evasion, when the same tax amount is levied per unit of output, the following interesting outcomes are obtained: (i) Either these two taxes make no difference, or ad valorem taxation induces less quantity concealment and more price concealment than specific taxation. (ii) The market price will be smaller and the output quantity will be larger under the ad valorem taxation if the demand function is not too convex. (iii) The marginal cost of tax evasion under ad valorem taxation is lower than that under specific taxation, and hence may induce more taxes evaded under ad valorem taxation.

Note that the third interesting outcome above makes taxations the specific taxation maybe superior to the ad valorem taxation from the view of social welfare in the presence of tax evasion. However, this cannot happen without tax evasion. Under heterogeneous production costs, the ad valorem taxation enlarges the firms' cost difference. When the cost difference is enlarged, the concentration ratio in this industry increases and hence the allocative production efficiency goes up (see Long and Soubeyran, 1997), hence making the ad valorem taxation superior to the specific taxation from the viewpoint of social welfare. Therefore, under imperfect competition it is likely for the specific taxation to be superior to the ad valorem taxation only if the cost difference is not large enough (or the production costs are the same).

The remainder of this paper is organized as follows. Section II analyzes the evasion behavior under the ad valorem taxation and specific taxation of firms. Section III compares the welfare under both tax regimes. Section IV concludes this paper.

II. *The Model*

Following Cremer and Gahvari (1993, hereafter CG), who analyze competitive firms' evasion of specific taxes, and Myles (1995), who amends CG to analyze the imperfect competition market structure,[6] consider an industry of n firms that have identical or differential constant marginal costs. Let c_i and x_i, $i=1,\dots,n$ denote the marginal cost and the output quantities of firm i, respectively. The market inverse demand function is $p=p(X)$, where $X=\sum_{i=1}^{n} x_i$ is the aggregate output quantity of firms, and $p'(X)<0$. The firms are obliged to pay taxes according to their sales. Under specific taxation, a fixed amount of tax where $t>0$ is imposed on per unit output so that a firm's net-of-tax sales revenue equals $(p-t)x$ if there is no evasion. Under ad valorem taxation, a fixed percentage tax of $0<\tau<1$ is levied on the output price so that a firm's net-of-tax sales revenue equals $(p-p\tau)x$ if there is no evasion.

[6] See Myles (1995) page 407.

1. Evasion under Different Tax Regimes

The firm may attempt to evade taxes. As in the CG model, we assume that it is costly for the firm to conceal its sales information from the tax authorities (say, costs involved in falsifying invoices or fabricating accounts).[7] Evading taxes must take place via concealing quantities sold under specific taxation. By contrast, evading taxes may take place via concealing selling prices as well as quantities sold under ad valorem taxation. This is the main difference between these two tax regimes in our model.

Let α_i and β_i denote, respectively, the fractions of output and the selling price the firm chooses to disclose to the tax authority. Since concealing activities are costly, it is clear that the firm will not understate its selling price under specific taxation; that is, $\beta_i=1, \forall i$. This is not necessarily true under ad valorem taxation. In addition to understating its output quantity, the firm may understate its selling price for the disclosed output, $\alpha_i x_i$, when ad valorem taxes are in operation, thus $0 \leq \beta_i \leq 1$. It is worth noting that there is no need for the firm to carry out "price" concealment for the concealed output $(1-\alpha_i)x_i$.

We follow CG to model the firm's cost of "quantity" concealment. That is, for the fraction of the output concealed from the tax authorities, each unit of output concealed entails a quantity concealing cost $G(1-\alpha_i)$, which is an increasing and convex function of $1-\alpha_i$, with $G(0)=G'(0)=0$, $G'(1)=\infty$ and $G''>0$. Hence, the total concealment cost associated with specific taxes is $G(1-\alpha_i)\cdot(1-\alpha_i)x_i$ when the firm conceals a $1-\alpha_i$ fraction of its output x_i.[8]

We model the firm's cost of "price" concealment analogously. That is, for the fraction of the selling price concealed from the tax authorities, each unit of output *disclosed* entails a price concealing cost $H(1-\beta_i)$, which is an increasing and convex function of $1-\beta_i$ with $H(0)=0$ and $H'(1)=\infty$. Thus, the total concealment cost associated with ad valorem taxes is $G(1-\alpha_i)\cdot(1-\alpha_i)x_i+H(1-\beta_i)\cdot\alpha_i x_i$ when the firm conceals a $1-\alpha_i$ fraction of its output x_i and a $1-\beta_i$ fraction of its selling price for the *disclosed* output $\alpha_i x_i$. Of course α_i and x_i under these two tax types may not be the same.

Suppose that the audit probability $A>0$ and a fine $F>1$ is levied on the amount of evaded tax if detected, and $AF<1$ to make the existence of an interior solution conceivable.[9] As in CG, we assume that the firm's true sales will be detected accurately once the taxpayers are audited. For convenience, let $g(1-\alpha_i)\equiv(1-\alpha_i)G(1-\alpha_i)$ as in CG. Hence, firm i's expected profit $E(\pi)$ under specific taxation is

$$E(\pi_i^s)=[p^s-c_i-CC^s-ER^s]x_i, \tag{1}$$

where superscript s denotes the specific taxation and $CC^s\equiv g(1-\alpha_i)$ and $ER^s\equiv[\alpha_i+AF(1-\alpha_i)]t$ denote the evasion cost and expected tax payment per unit of output, respectively. Solving the necessary conditions for optimal choices, which are denoted as x_i^s and α_i^s, we have

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7 Some may argue that the cost of evasion is often interpreted as to proxy the expected cost of evasion in terms of penalty payments, but this is not the case in our model. The evasion costs in our model are the resources engaged in the evasion behavior of firms, while penalty payments are basically transfers analogous to taxes and regarded as a part of the tax burden in the presence of tax evasion (see later). The evasion costs can be interpreted as the effort of evaders to keep the audit probability within an acceptable level.

8 In order to simplify the analysis, we assume that each firm's tax evasion technology is identical.

9 It should be noted that $AF<1$ is necessary but not sufficient for tax evasion, since in this model evasion is costly.

$$(p^s)' \cdot x_i^s + p^s = c_i + CC^s + ER^s, \tag{2}$$

$$g'(1-\alpha_i^s) = (1-AF)t. \tag{3}$$

Equation (3) shows that the choice of α_i^s depends on the marginal expected gain $(1-AF)t$ from tax evasion only,[10] therefore $\alpha_i^s = \alpha^s$, for $i=1,\dots,n$.

Under ad valorem taxation, firm i's expected profit function is

$$E(\pi_i^a) = [p^a - c_i - CC^a - ER^a]x_i, \tag{4}$$

where the superscript a denotes ad valorem taxation, and $CC^a \equiv g(1-\alpha_i) + \alpha_i H(1-\beta_i)$, $ER^a \equiv [\alpha_i\beta_i + AF(1-\alpha_i\beta_i)]p^a\tau$ denote the evasion cost and expected tax payment per unit of output, respectively. The optimal choices of α_i, β_i and x_i are denoted as α_i^a, β_i^a and x_i^a, respectively. Thus we have

$$g'(1-\alpha_i^a) = H(1-\beta_i^a) + (1-AF)\beta_i^a p^a \tau, \tag{5}$$

$$H'(1-\beta_i^a) = (1-AF)p^a\tau, \tag{6}$$

$$(1-ER^a/p^a)(p^a)' x_i^a + p^a = c_i + CC^a + ER^a. \tag{7}$$

Solving (6) yields the optimal choice of $\beta_i^a = \beta_i^a(p^a\tau)$ and substituting it into (5) yields the optimal choice of $\alpha_i^a = \alpha_i^a(p^a\tau)$. Because p^a is the market price, each firm's evasion decision is identical, i.e., $\alpha_i^a = \alpha^a; \beta_i^a = \beta^a$, $\forall i=1,\dots,n$. This result indicates that the firm's evasion decisions (including price concealment and quantity concealment) merely depend on the per unit tax $p^a\tau$. Besides, the above equations show that the decision of price concealment is independent of the choice and cost of quantity concealment, but the choice as well as the cost of price concealment will affect the decision regarding quantity concealment.

From the analyses above, we can have the following lemmas:

Lemma 1. ***Given $t=p^a\tau$, $\alpha^a=\alpha^s<1$, if and only if $\beta^a=1$.***
Proof. If $\beta^a=1$, since $H(0)=0$, (5) becomes $g'(1-\alpha_i^a)=(1-AF)p^a\tau$. Given $t=p^a\tau$, by comparing (5) with (3), it is obvious that $\alpha^a=\alpha^s$.

Suppose $\alpha^a=\alpha^s<1$ but $\beta^a<1$. Then, based on (3) and (5), $(1-AF)t = H(1-\beta^a) + (1-AF)\beta^a p^a\tau$, or $(1-AF)t(1-\beta^a) = H(1-\beta^a)$ given $t=p^a\tau$. Moreover, when $\beta^a<1$, equation (6) holds, and it follows that $H'(1-\beta^a)(1-\beta^a) = H(1-\beta^a)$ or $H'(1-\beta^a) = H(1-\beta^a)/(1-\beta^a)$. However, this is not possible as long as $H(\cdot)$ is strictly convex (where the marginal cost $H'(q)$ is always larger than the average cost $H(q)/q$ for any $q>0$). Therefore, $\beta^a=1$. Q.E.D.

Lemma 2. ***Given $t=p^a\tau$, $\alpha^a>\alpha^s$, if and only if $\beta^a<1$.***
Proof: Based on (3) and (5), we have

$$g'(1-\alpha^s) - g'(1-\alpha^a) = (1-AF)t - H(1-\beta^a) - (1-AF)\beta^a p^a \tau$$

[10] It is easy to verify that, as in the conventional literature on tax evasion, evasion decisions are independent of production decisions.

$$=(1-\beta^a)\left[(1-AF)t-\frac{H(1-\beta^a)}{(1-\beta^a)}\right].$$

Again, based on (6) when $\beta^a<1$,

$$g'(1-\alpha^s)-g'(1-\alpha^a)=(1-\beta^a)\left[H'(1-\beta^a)-\frac{H(1-\beta^a)}{(1-\beta^a)}\right]>0$$

for $H(\cdot)$ is strictly convex. Since $g(\cdot)$ is strictly convex, we have $\alpha^a>\alpha^s$. Q.E.D.

Based on the above two lemmas, we can conclude:

Proposition 1. ***Given $t=p^a\tau$, either these two taxes make no difference, or ad valorem taxation induces less quantity concealment and more price concealment than specific taxation.***

This result is intuitive for, with more instruments to conceal the tax base, the firm may use the price concealment to replace part of the quantity concealment, since the marginal cost of concealment is increasing with concealed quantities. Moreover, if $\beta^a<1$, the relative magnitude of the compliance rates under these two tax regimes (i.e., α^s, $\alpha^a\beta^a$) may not be equal. Therefore, these two types of taxes are not equivalent from the aspect of tax enforcement.

2. Market Equilibrium

To the taxpayers, the decision regarding α^s in Section II.1 is equivalent to that of minimizing the effective tax burden (including the evasion cost and expected tax payment) per unit of output, i.e.,

$$\underset{\alpha^s}{Min}\theta^s\equiv CC^s+ER^s.$$

Denoting the minimum of $\theta^s=\theta^s(t)$as the effective tax burden, and by the envelope theorem we can derive

$$\frac{\partial\theta^s(t)}{\partial t}=\alpha^s+AF(1-\alpha^s)>0. \tag{8}$$

Therefore, firm $i's$ expected profit function can be rewritten as

$$E(\pi_i^s)=[p^s-c_i-\theta^s(t)]x_i^s, \tag{1'}$$

and (2) can be simplified as

$$(p^S)'x_i+p^S = c_i+q^{\,S}(t), \tag{2'}$$

where $c_i+\theta^s(t)$ denotes the effective marginal cost of firm i.

Similarly, the choice of α^a and β^a under ad valorem taxation is equivalent to

$$\underset{\alpha^a,\,\beta^a}{Min}\theta^a\equiv CC^a+ER^a.$$

Denoting the minimum of $\theta^a=\theta^a(p^a\tau)$ as the effective tax burden and applying the Envelope Theorem, we derive

$$\frac{\partial\theta^a(p^a\tau)}{\partial p^a\tau}=\alpha^a\beta^a+AF(1-\alpha^a\beta^a)>0. \tag{9}$$

Thus, under ad valorem taxation, the effective tax burden is monotonically increasing with the amount of tax per unit of output. Again, the effective marginal cost of firm i is $c_i+\theta^a(p^a\tau)$ and equations (4) and (7) can be rewritten as

$$E(\pi_i^a) = [p^a-c_i-\theta^a(p^a\tau)]x_i^a, \tag{4$'$}$$

$$(1-ER^a/p^a)(p^a)'x_i^a+p^a = c_i+\theta^a(p^a\tau). \tag{7$'$}$$

Given $t=t^s$ and $t^a\equiv p^a\tau$, under specific taxation, from (2$'$)

$$(p^s)'x_i^s+p^s-c_i-\theta^s(t^s) = 0, \quad \forall i=1,\dots,n. \tag{10}$$

Aggregating these n equations above yields

$$(p^s)'X^s+np^s-nc_M-n\theta^s(t^s) = 0, \tag{11}$$

where $c_M\equiv\sum c_i/n$ is the average production cost for the industry as a whole. Under ad valorem taxation, from (7$'$)

$$(p^a)'(1-ER^a/p^a)x_i^a+p^a-c_i-\theta^a(t^a) = 0, \quad \forall i=1,\dots,n. \tag{12}$$

Aggregating these n equations above yields

$$(p^a)'X^a(1-ER^a/p^a)+np^a-nc_M-n\theta^a(t^a) = 0. \tag{13}$$

Next, in order to derive an important result, we provide the following lemma.

Lemma 3. ***Given*** $t^s=t^a$, $\theta^a(t^a)\leq\theta^s(t^s)$.
Proof. Since $(\alpha, \beta)=(\alpha^s, 1)$ is always available for the firm, if $t^s=t^a$, θ^a cannot be larger than θ^s, thus $\theta^a(t^a)\leq\theta^s(t^s)$. Q.E.D.

With this lemma, we can propose the following proposition.

Proposition 2. ***Supposing the tax per unit of output under specific taxation is the same as that under ad valorem taxation, the market price will be smaller and the output quantity will be larger under the ad valorem taxation if the demand function is not too convex.***
Proof. Given that $t^s=t^a$, by Lemma 3 and combining Eqs. (11) and (13), we have

$$n(p^s-p^a)\geq(1-ER^a/p^a)(p^a)'X^a-(p^s)'X^s.$$

By apagoge, suppose that $p^s\leq p^a$, $n(p^s-p^a)\leq 0$ and $X^s\geq X^a$ since $p'(X)<0$. Next, $ER^a/p^a=[\alpha^a\beta^a+AF(1-\alpha^a\beta^a)]\tau$, since $AF<1$ and $0<\alpha^a\beta^a\leq 1$, $0<\alpha^a\beta^a+AF(1-\alpha^a\beta^a)<1$, thus

$0 < ER^a/p^a < 1$. Therefore, if the market demand curve is not too convex, $(1 - ER^a/p^a)(p^a)'X^a - (p^s)'X^s > 0$ and hence $n(p^s - p^a) \leq 0 < (1 - ER^a/p^a)(p^a)'X^a - (p^s)'X^s$, a contradiction. Q.E.D.

Proposition 2 shows that, with a lower effective tax burden per unit of output under ad valorem taxation, the firms are able to provide more output to the market at a lower price. Notice that the lower effective tax burden mentioned above is on a per unit of output basis. Whether the overall tax burden on taxpayers is lower under ad valorem taxation is uncertain because of the larger output quantity. Thus, even if the consumers benefit from a lower price with more surplus, we still cannot compare their welfare effects under this setting.

III. *Welfare Comparison*

Following Anderson et al. (2001), the comparison of welfare effects under these two tax regimes is built on the grounds of equivalent market equilibria. First, the following lemma is needed:

Lemma 4. ***Given*** τ ***and*** t^s ***such that*** $X^s = X^a$, $\theta^a(t^a) > \theta^s(t^s)$.

Proof. When $X^s = X^a \equiv X$, then $p^s = p^a = p$ and $(p^s)' = (p^a)' = p'$. Combining (11) and (13) results in $\theta^s - \theta^a = p'XER^a/(np) < 0$. Q.E.D.

Next, by utilizing lemmas 3 and 4, we can derive the following proposition:

Proposition 3. ***Under the market equilibria of*** $X^s(t^s) = X^a(t^a)$ *or* $p^s = p^a$, $t^a > t^s$.

Proof: Given $X^s(t^s) = X^a(t^a)$, by Lemma 4, $\theta^a(t^a) > \theta^s(t^s)$. Next, Lemma 3 indicates that $t^a > t^s$ since $\partial\theta^j/\partial t^j > 0$, $j = s, a$.

Proposition 3 shows that, with the same market equilibria, the tax per unit of output will be larger under ad valorem taxation than under specific taxation. Intuitively, replacing an ad valorem tax by a specific tax will reduce the magnitude of tax evasion, for the firm can no longer evade tax by price concealment. Thus a lighter tax per unit of output is needed under specific taxation to attain the same equilibrium market price as under ad valorem taxation.

Now we are ready to compare the relative welfare under these two tax regimes. The total welfare includes the consumer's surplus, producer's surplus and government revenue. Given that $X^s = X^a$ and $p^s = p^a$, the consumer's surplus is identical under these two tax regimes, so we only need to compare the sums of the producer's surplus and government revenue, which are denoted as:

$$PE^j \equiv \sum_{i=1}^{n} E(\pi_i^j) + \sum_{i=1}^{n} ER_i^j x_i^j = \sum_{i=1}^{n} [p^j - c_i - CC^j(t^j)]x_i^j, \quad j = a, s. \tag{14}$$

Utilizing (11) and (13), the terms $\sum_{i=1}^{n} c_i x_i^j$ in (14) under specific taxation and ad valorem taxation are

$$\sum_{i=1}^{n} c_i x_i^s = -\frac{(p-\theta^s)n\bar{c} - \sum_{i=1}^{n} c_i^2}{p'} \tag{15}$$

and

$$\sum_{i=1}^{n} c_i x_i^a = -\frac{(p-\theta^a)n\bar{c} - \sum_{i=1}^{n} c_i^2}{p'(1-ER^a/p)}, \tag{16}$$

respectively, where $\bar{c} = \sum_{i=1}^{n} c_i/n$. Hence,

$$\begin{aligned} PE^a - PE^s &= \sum_{i=1}^{n} [(c_i x_i^s - c_i x_i^a) + (CC^s x_i^s - CC^a x_i^a)] \\ &= \frac{-nV_M ER^a}{p'(p-ER^a)} + X(CC^s - CC^a). \end{aligned} \tag{17}$$

In (17), $V_M = \sum_i (c_i - \bar{c})^2/n$ is the variance of the cost distribution in the industry, and it also represents the allocative production efficiency effect in Long and Soubeyran (1997). Note that the first term on the R.H.S. of (17) is positive, and thus ad valorem taxes are more favorable as $V_M \neq 0$ (i.e., heterogeneous production costs) if there is no tax evasion.

The ad valorem taxation enlarges the firms' cost difference. When the cost difference is enlarged, in order to keep the industry's output the same, firms with relatively lower costs will increase their outputs while those with relatively higher costs will decrease their outputs, hence reducing the industry's total production cost. This is so-called 'allocative production efficiency effect'. The ad valorem taxation effectively enhances the industry's allocative production efficiency while the specific taxation does not have such an effect. In other words, under heterogeneous production costs, the ad valorem taxation is superior to the specific taxation from the viewpoint of social welfare.

By contrast, with tax evasion, the sign of the second term on the R.H.S. of (17), which represents the difference in evasion costs under specific taxation and ad valorem taxation, is ambiguous. Consequently, ad valorem taxation is not necessarily superior to specific taxation. This is because the firm may either conceal the quantity or the price or both under the ad valorem taxation, compared to that it can only conceal the quantity under the specific taxation. Given the same unit tax on the output, ad valorem taxation gives the firm one more instrument (i.e., quantity concealment) to evade taxes, making its marginal cost of tax evasion lower under the valorem taxation. Consequently, the amount taxes evaded is higher while the social welfare is lower under the ad valorem taxation in imperfect competition.

Proposition 4. ***In an oligopolistic market, ad valorem taxation may not be superior to specific taxation in the presence of tax evasion.***

When the firms are homogeneous, i.e., $c_i=c$, $\forall i=1,...,n$, the allocative production efficiency effect vanishes, hence the superiority of these two tax types rests on the comparison of $CC^a(t^a)$ and $CC^s(t^s)$. In general, the relative magnitude of $CC^a(t^a)$ and $CC^s(t^s)$ is uncertain. However, as long as the cost of quantity concealment is sufficiently high, $CC^a > CC^s$, specific taxation will be superior to ad valorem taxation in this situation.

Corollary 1. ***In an oligopolistic market with homogeneous firms, specific taxation is superior to ad valorem taxation as long as the cost of concealment is lower under the former.***

To demonstrate the above proposition, consider an example in which $H(1-\beta)$ and $g(1-\alpha)$ are quadratic functions as $H(1-\beta)=\frac{1}{2}(1-\beta)^2$ and $g(1-\alpha)=\frac{1}{2}\kappa(1-\alpha)^2$, where parameter κ reflects the level of concealment cost. The higher the value of κ, the more difficult it is to evade tax by concealing output quantities. Given $t^a=t^s=t$, the first-order conditions under specific and ad valorem taxation are, respectively,

$$1-\alpha^s = (1-AF)t/\kappa, \text{ where } \kappa \geq (1-AF)t;$$

$$1-\alpha^a=\frac{(1-AF)t}{\kappa}[1-(1-AF)t/2],$$

$$1-\beta^*=(1-AF)t.$$

In addition,

$$\theta^s=t-(1-AF)^2t^2/2\kappa,$$

$$\theta^a=\theta^s+\frac{(1-AF)^2t^2}{2}\left[\frac{(1-AF)t-(1-AF)^2t^2/4}{\kappa}-1\right].$$

Since $\kappa \geq (1-AF)t$, then $\theta^a < \theta^s$. Next, the evasion cost under specific taxation is

$$CC^s(t)=\frac{(1-AF)^2t^2}{2\kappa},$$

which is negatively related to the difficulty encountered in evading tax. On the other hand, the evasion cost under ad valorem taxation is

$$CC^a(t)=\frac{(1-AF)^2t^2}{2\kappa}\left\{\kappa+\left(1-\frac{3(1-AF)t}{2}\right)\left(1-\frac{(1-AF)t}{2}\right)\right\};$$

Thus,

$$CC^a(t)-CC^s(t)=\frac{(1-AF)^2t^2}{2}\left(1-\frac{[8-3(1-AF)t](1-AF)t}{4\kappa}\right).$$

With the generally accepted assumption of $(1-AF)t \leq 1$, the magnitude of $CC^a(t)-CC^s(t)$ is monotonously increasing with κ. Thus, there exists a threshold $\bar{\kappa} \equiv [8-3(1-AF)t](1-AF)t/4$

such that $CC^a(t) > CC^s(t)$ if $\kappa > \bar{\kappa}$. Next, because $\partial CC^s/\partial t^s = -g'(\partial\alpha^s/\partial t^s) = (1-AF)g'/g'' > 0$, we can obtain that, given τ and t^s such that $X^s = X^a$, if κ is large enough, then $CC^a > CC^s$.

To illustrate the possible superiority of specific taxation more specifically, consider an extreme case in which quantity concealment is implausible (for example, selling buildings or cars that have to be registered for ownership). The firm can evade taxes through price concealment only, thus $CC^s(t^s) = 0$ and thereby $CC^a(t^a)$ is definitely larger than $CC^s(t^s)$. By contrast, if price concealment is implausible, then $CC^a = CC^s$ and these two tax regimes are equivalent.

The above results can be easily extended to the analysis under monopoly and competitive markets. In both cases $V_M = 0$. However, whether price concealment is possible in the competitive markets remains questionable because the market price is fully known to the tax authority. Nevertheless, as mentioned in the Introduction, the authority's being informed of the market price alone cannot deter taxpayers from evasion since only a portion of the tax returns are audited. If this is real, specific taxation may not be equivalent to the ad valorem taxation even in the competitive market.

IV. *Conclusion*

It is traditionally recognized that, when the quality of products is ignored, ad valorem taxation will be superior to specific taxation. This result is robust and applies to long-term or short-term analysis, and also to price competition or quantity competition oligopolies (Anderson et al., 2001).

The government must collect a fixed amount of revenue and therefore it can respond to evasion by raising tax rates appropriately in our model. However, as Slemrod (2007, p. 42) points out, tax evasion imposes efficiency costs because taxpayers expend to implement and camouflage noncompliance and the tax authority also expends to address it.

This paper focuses on the resources taxpayers expend. There are price and quantity instruments for the firm to implement evasion under ad valorem taxation, whereas there is only a quantity instrument for the firm to implement evasion under specific taxation. As a result, other things being equal, the resource costs of evasion under ad valorem and specific taxation are not the same. With this non-trivial difference, we show that specific taxation may be superior to ad valorem taxation when the evasion costs under these two tax regimes are considered.

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