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Organizational Concealment: An Incentive of Reducing the Responsibility

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We study workers' incentives of reporting problems within an OLG organization consisting of a subordinate and a manager. The subordinate is responsible for reporting a problem, and the manager is responsible for solving the reported problem. The subordinate has an incentive to conceal a detected problem since if he reports it but the manager is too lazy to solve the problem, the responsibility is transferred to the subordinate since he becomes a manager in the next period. We show that concealment is more likely either if subordinates are farsighted or the problem's growth rate increases over time.

Keywords: Concealment; overlapping generations; promotion; reducing the responsibility

JEL Classification: D23; D82; M51

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1. Introduction

Firms sometime have problems for their products and management systems. If these problems are ignored, they would cause accidents that harm consumers. Some problems grow over time and the harm from the accidents gets larger as time proceeds and then, procrastination of the solution leads to a catastrophe.

As an example, consider a mass food poisoning caused by a Japanese dairy company, Snow Brand Milk Products Co. in 2000. It left 14,780 people ill. As a cause, it is considered that the company's crisis management had some problems.[1] In this case, the harm from an accident could get larger over time since, as the scale of plants gets larger, the harm from a food poisoning also gets larger.

To prevent accidents, these problems should be solved in their early stages. Therefore workers should report problems as soon as they notice them. Do they report problems to their superior? In the above case, the problem in the company's crisis management was not resolved until the accident occurred. It is suspected that no one raised an issue. However, it is also doubtful that no one noticed the problem. Indeed, workers at the plant told that the plant's manual had been ignored for years.[2] Thus, it is also suspected that workers did notice the problem but not report it to the company. Such concealment of a problem is not specific to this case. Chernov and Sornette (2016) present many detailed case studies and say that long-time concealment of risk information is a crucial cause of catastrophic man-made disasters and accidents, such as Chernobyl nuclear disaster and many financial crises.

This paper investigates the incentive and cause of long-time concealment in such important problems. To see workers' incentives of concealing problems, suppose that a worker detects a problem and reports it to his manager. If the cost of solving the problem is too large for the manager, the manager may not solve the problem. In this case, the problem remains in the next period, when the worker may be promoted to a manager, in which case, he is in charge of dealing with the problem. Moreover, if he ignores the problem and an accident occurs, he receives harsh criticism since the problem has been reported in the firm (by himself) and he

[1] Failure Knowledge Database, http://www.sozogaku.com/fkd/en/cfen/CA1000622.html.
[2] "Snow Brand pays the price," The Japan Times, July 12, 2000.

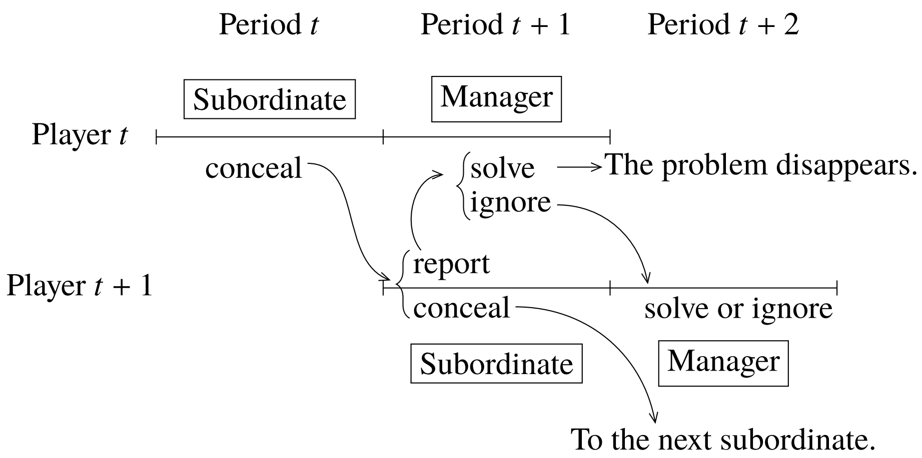

Figure 1: Timing of the game

knew the problem. This possibility gives him an incentive not to report the problem in the first place. By doing so, he can reduce the responsibility as a manager in the future. We call it *the incentive of reducing the responsibility*. This paper formalizes this idea.

We consider an overlapping-generation organization that consists of two kinds of workers: a subordinate and a manager. Each worker lives for two periods: he works as a subordinate in his first period and works as a manager in his second period. In each period, the subordinate is in charge of investigating whether there is a problem and reporting a problem to the manager. Reporting a problem is costless. The manager is in charge of solving the reported problem, which is costly for him. Figure 1 illustrates the timing of the game.

An unsolved problem may cause an accident, in which case, only the workers in that period may be punished or criticized. If the problem has been reported in the past and hence the manager knew the problem,[3] he is exposed to harsh criticism. On the other hand, the problem has not been reported, criticism against the manager is mild while the subordinate is punished for not detecting or reporting the problem.[4]

The size of punishments (or criticism) and the cost of solving a problem are proportional to

[3] In other case, the problem is detected by an outsider, which also causes blames for the workers.

[4] This assumption is justified by a behavioral bias that people tend to attribute heavier responsibility on the person who could make the decision. Bartling and Fischbacher (2012) show an experimental evidence. In their experiment, persons could mitigate harsh criticism to them by delegating their decision right to another person. In our model, if the problem has not been reported, the manager might not have a chance to solve and therefore, the criticism would be mitigated.

the scale of the problem. If the scale of the problem is large, solving the problem is more costly, and when it causes an accident, it brings more harm and arouses harsher criticism against the firm and the workers who are responsible. The problem's scale changes over time exogenously.

Under the above setting, the incentive of reducing the responsibility arises and it has a few interesting features. First, it has complementarity between subordinates in different generations. That is, a subordinate's incentive for reporting a problem is larger when the subordinate in the next generation reports. A subordinate conceals a problem mainly to leave the problem unknown to the firm, thereby reducing the criticism against him for not solving the problem when he is the manager. But if his subordinate reports it to him when he is the manager, his act of concealment is wasted and the problem is officially known in the firm and hence he will be criticized if he leaves the problem unsolved.

This complementarity creates multiple equilibria. Indeed, for some parameter values, multiple equilibria exist where no subordinate conceals in an equilibrium and all subordinate conceal in another equilibrium. In this case, as Kreps (1990) discusses, corporate culture has important role for determining which equilibrium is realized. That is, if "reporting" is a culture in the firm, each worker has an incentive to report because he believes that the other workers also report. The culture of "reporting" is realized as an equilibrium. In the same way, the culture of "concealment" is also realized as an equilibrium.

The present model also shows that subordinates' incentive to conceal is large when the scale of the problem grows fast. The reason is that the benefit of reporting a problem is to reduce punishment imposed when an accident occurs in the present period. On the other hand, the cost of reporting the problem is to increase punishment imposed when an accident occurs in the *next* period when he is the manager. Thus, if the scale of the problem grows at a higher rate, punishment in the next period is relatively larger, which makes reporting more costly.

A way to encourage the manager to solve reported problems is to increase punishment or criticism against the manager when an accident occurs due to a problem that has been reported in the firm but ignored. Indeed, the public's anger is usually stronger when the manager knew the problem. However, this additional criticism for ignoring the known problem may have adverse effects on subordinates' incentives to report. This is because, by reporting, the worker makes the problem officially known in the firm and hence increases criticism he receives when he is the manager if he does not solve the problem. The additional punishment for ignoring the problem does encourage the manager to solve problems that are reported, but it may discourage subordinates from reporting problems to the manager.

Another way to encourage the solution of problems is to give the manager a reward for solving problems. However, this also has adverse effects on subordinates' incentives to report: it gives a subordinate incentives to obtain rewards by hiding the problem and solving it himself after he becomes the manager. The problem is found and solved by the manager, but the solution is delayed.

1.1. Related Literature

Concealment of information is studied in the literature of disclosure games, which is developed by Grossman (1981); Milgrom (1981) and recent literature of information design (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011). These studies focus on the sender's disclosure strategy to control the receivers' belief. We rather focus on another function of information transmission; reporting a problem also transfers responsibility. Instead of simplifying the strategy to conceal, we discover a novel incentive of information concealment.

The incentive to report problems is also studied in the literature of whistleblowing.[5] For the costs of whistleblowing, many papers consider the revenge from the firm. For example, in the literature of corruption, a corrupting agent can threaten the monitor to retaliate, which discourages the monitor from reporting the corruption. In this context, Chassang and Padró i Miquel (2016) explore anti-corruption mechanisms. Heynes and Kapur (2009) provide a model where employees have several behavioral motivations of whistlebrowing, such as social welfare concern. Unlike these studies, the present paper assumes that neither workers are threatened retaliation from the firm nor they have behavioral incentives that Heynes and Kapur assume. One of our contributions is showing that workers may hide problems even when reporting them brings no friction between the company.

[5]For example, see Dasgupta and Kesharwani (2010) a survey of the literature and Bowen et al. (2010) and Dyck et al. (2010) for empirical studies.

This paper also relates to the studies of organizational corruption. Many papers investigate causes of persistent corruption and demonstrate the possibility of multiple equilibria. They consider the causes of the possibility of multiple equilibria as group reputation or social norms. For example, Tirole (1996) develops a model of group reputation in an OLG organization, in which a current member's incentives are affected not only by his past behavior but also by past members' behavior. Tirole shows the possibility of multiple steady states with and without corruptions. Several papers consider a corruptive situation where the cost of corrupting is cheaper when the others corrupt but expensive the others do not. This assumption is often justified by assuming that players take care of social norms. For example, players are reluctant to take actions that majority of the players do not.[6] This creates the situation of coordination games. Many papers investigate how coordination is maintained in coordination games,[7] which can apply to the theory of corruption.

This paper presents a motivation of concealment of a problem and also indicates a possibility of multiple equilibria by showing that the motivation has complementarity between subordinates in different generation. Interpreting concealment as a kind of corruption, the contribution of this paper is different from these papers in the following senses. First, in our model, players neither have reputation concern nor take care of social norms. Second, the many past studies assume the coordination game structure in the term of utility. In this paper, this coordination game situation is derived from the structure of the game: the future subordinate's concealment reduces the possibility of severe punishment, which strengthens the motive of reducing the responsibility. In this sense, our logic is similar to Tirole (1985)'s one who shows persistent bubble in OLG economies.

Our study also relates to the economics of crime (Becker, 1968), which shows that increasing punishments is less costly than increasing the probability of conviction. On the other hand, this study shows that increasing punishments against responsible managers distorts subordinates' incentives of reporting problems, thereby leaving problems unreported and raising the probability of accidents.

[6]For example, see Schneider and Bose (2017).

[7]In evolutionary games, see Young (1993); Kandori et al. (1993), and so on. In repeated games in an overlapping generation organization, see Kreps (1990); Acemoglu and Jackson (2015), and so on.

Our model also relates to dynamic contributions to a public good, since solving and reporting a problem can be considered as a public good. For example, Bliss and Nalebuff (1984) and Bilodeau and Slivinski (1996) consider dynamic contributions to a public good with continuous time and simultaneous decision making. Bolle (2011) and Bergstrom (2012) consider private provision of a public good with sequential decision making.

Unlike these papers, our model assumes that a providing a public good (i.e., solving a problem) needs two steps of contributions: reporting and solving. In the standard model of private provision of a public good, players' incentives of not contributing is to free-ride on others' contributions. In our model, a subordinate does not report since his manager does not solve and he has to complete the second step by himself when he becomes a manager.

2. Model

There are two players in each period. Player $t \in \mathbb{N} = \{0, 1, 2, \ldots\}$ lives for two periods: period t and $t + 1$. In period t, player t works as a subordinate and is promoted to a manager in period $t + 1$.

We assume that a single problem arises in period 0, and no other problem arises thereafter. A worker does not know what the problem is until he detects the problem himself or the problem is reported by someone else.

The sequence of events in each period is as follows. At the beginning of each period, the subordinate detects the problem with probability $q^S \in (0, 1)$ and the manager detects the problem with probability $q^M \in [0, 1)$. Then players make decisions in the following order:

1. If the problem is detected by the manager (possibly in the previous period) and the problem is not solved, the manager decides whether to solve the problem.

2. If the problem remains unsolved and the subordinate detects the problem, the subordinate decides whether to report the problem to the manager.

3. If the problem is reported (possibly in a previous period), the manager decides whether to solve the problem.

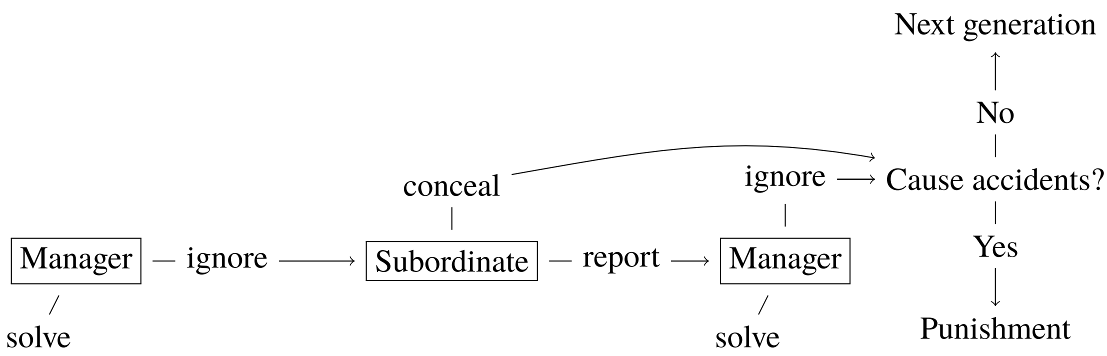

Figure 2: Flow of decisions in each period

The decision flow is illustrated in Figure 2. Once the problem is reported, the problem becomes known publicly in the firm and hence Stages 1 and 2 are skipped in subsequent periods. The game ends when the problem is solved.

Let s_t denote the scale of the problem at period t. The sequence of scales $(s_t)_{t\in\mathbb{N}}$ is exogenously given. The cost of solving the problem for manager in period t depends on the scale s_t and is given by $c_t s_t$, where c_t denotes the manager's problem-solving ability and is his private information. The manager learns the value of c_t after he is promoted to a manager in period t. For each t, c_t follows a distribution function F independently. We assume that the support of F is an interval in $\mathbb{R}_{++}$.

In each period, if the problem is unsolved, the problem causes an accident with probability $p \in (0, 1)$.[8] If an accident occurs in a period, only players who live in the period are punished (or criticized).

The amount of punishment (or criticism) against workers depends on whether the problem has been reported. If the problem has not been reported and hence the problem is not known to the firm, punishment (or disutility from it) is $d^{M,U} s_t$ to the manager and $d^S s_t$ to the subordinate. If the problem has been reported in the past, only the manager (in the current period) is punished and punishment is given by $d^{M,R} s_t$. We assume that $d^{M,R} > d^{M,U}$, i.e., punishment against the manager is harsher when the problem is known to the firm. Note that the manager in period t is punished if an accident occurs in period t even if the problem is reported in period $t' < t$.

We assume that when an accident occurs, the problem becomes known and the manager is

[8] An accident may include an event where the problem is detected by an outsider and becomes a scandal.

forced to solve the problem (and hence pay the cost c_t).

Let $b^M s_t$ be the amount of rewards to the manager for solving the problem, and $b^S s_t$ be the amount of rewards to the subordinate for reporting the problem.

Finally, let $\delta \in (0, 1)$ denote the common discount factor.

3. Behavior of managers and subordinates

3.1. Managers' behavior

To consider the manager's optimal action, first consider the case where the problem has been reported. The expected utility of solving the problem is $b^M s_t - c_t s_t$, while the expected utility of ignoring the problem is $-p(d^{M,R} s_t + c_t s_t)$. Therefore, the manager solves the reported problem if and only if

$$\frac{b^M + pd^{M,R}}{1-p} \geqslant c_t. \tag{1}$$

Next, we consider the case where the problem has not been reported but the manager knows the problem (i.e., the manager detected the problem in the current or preceding period). Then, before the subordinate acts, the manager decides whether to solve the problem. Let r_t be the probability that the subordinate in the current period t reports the problem when he detects the problem. Then, the manager's expected utility for ignoring the problem is given by

$$-(1 - q^S r_t)p(d^{M,U} s_t + c_t s_t) + q^S r_t \max\left\{b^M s_t - c_t s_t,\ -p(d^{M,R} s_t + c_t s_t)\right\}.$$

The expected utility of solving the problem is the same as above. The comparison of the expected utilities implies that the manager solves the problem if and only if

$$\frac{b^M + pd^{M,U}}{1-p} \geqslant c_t.$$

Since we assume $d^{M,R} > d^{M,U}$, the manager's incentive to solve the problem is stronger when the problem is reported.

3.2. Subordinates' behavior

We now consider the optimal behaviors of the subordinate. The subordinate in period t has nothing to do if the problem has been solved or reported in the past, or the subordinate fails to detect the problem. Thus consider the case when the problem has been neither solved nor reported, and the subordinate detects the problem in the current period.

Let $I(r_{t-1})$ denote the subordinate's belief that the manager will ignore the problem if the subordinate reports the problem. This is the subordinate's updated belief that the manager's cost c_t violates (1). The subordinate's belief on c_t is updated before his move since the manager could solve the problem at the beginning of this period if he knew the problem at that point, but he did not. Thus it is possible that the manager knew the problem at the beginning of this period and chose not to solve the problem, which implies that the manager's cost c_t is likely to be high. Formally, the subordinate's belief $I(r_{t-1})$ is given by

$$I(r_{t-1}) := \frac{1 - q^S r_{t-1}}{\{q^S(1 - r_{t-1}) + (1 - q^S)q^M\}\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1 - q^S)(1 - q^M)} \times \left[1 - F\left(\frac{b^M + pd^{M,R}}{1 - p}\right)\right]$$

Here $q^S(1 - r_{t-1}) + (1 - q^S)q^M$ is the posterior probability that the current manager knew the problem privately at the beginning of the current period.

The belief has to be modified for the manager in period 0 since he starts as a manager. To put it differently, since the problem arises in period 0, the problem is unknown to him when he is a subordinate. Thus the manager knows the problem in period 0 only if he detects it in this period. Therefore the belief of the subordinate in period 0 about his manager's cost c_0 is given by

$$I_0 := \frac{1}{q^M\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1 - q^M)} \times \left[1 - F\left(\frac{b^M + pd^{M,R}}{1 - p}\right)\right].$$

Since $I(1) = I_0$, we can assume, without loss of generality, that $r_{-1} = 1$.

We now compute a worker's continuation utility in the second period when the problem remains unsolved. Let

$$D^R := \mathbb{E}_c\left[\max\{-p(d^{M,R}+c), b^M - c\}\right], \quad D^U := \mathbb{E}_c\left[\max\{-p(d^{M,U}+c), b^M - c\}\right].$$

Then $D^R s_{t+1}$ gives the expected utility in the second period when the problem is unsolved but reported, while $D^U s_{t+1}$ is the expected utility when the problem is unsolved and *un*reported.

Summing up these definitions, the subordinate's expected utility of reporting the problem is given by

$$b^S s_t + \delta(1-p)I(r_{t-1})D^R s_{t+1}.$$

If, on the other hand, the subordinate does not report the problem, his expected utility is

$$-pd^S s_t + \delta(1-p)\left[(1-q^S r_{t+1})D^U s_{t+1} + q^S r_{t+1} D^R s_{t+1}\right].$$

Thus the necessary and sufficient condition for the subordinate to report the problem is characterized as

Lemma 1. *The subordinate in period $t > 0$ reports the problem if and only if*

$$\varphi_{r_{t-1},r_{t+1}}(t) := b^S + pd^S + \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} \geqslant 0. \quad (2)$$

This value is the subordinate t's net payoff from reporting the problem, per unit of problem scale in period t. The first term b^S is the bonus for reporting. The second term pd^S is expected punishments he receives in the current period if he does not report. The term in the square bracket in (2) is the net gain from reporting in the second period, which consists of two terms. The first term shows that, by reporting, the subordinate loses the continuation payoff D^R (which may be negative) if the manager solves the problem right away (which occurs with probability $1 - I(r_{t-1})$). The next term shows that, by making the problem public, his payoff as a manager drops by $D^U - D^R > 0$ if his subordinate does not find or report the problem (which occurs with probability $1 - q^S r_{t+1}$).

We concentrate on the case that s_{t+1}/s_t is monotone in t. To guarantee the monotonicity of s_{t+1}/s_t, we prepare some sufficient conditions. At first, suppose that s_t has a continuous extension $s\colon \mathbb{R}_+ \to \mathbb{R}_+$. If $s(t)$ is log-concave (resp. log-convex) function of t, s_{t+1}/s_t is decreasing (resp. increasing) in t. For example, the density function of a normal distribution function is log-concave. The following fact gives other sufficient conditions.

Fact 1. *Consider an increasing function G such that $G(0) = 0$. Suppose that $s_{t+1} = G(s_t)$ for each $t \in \mathbb{N}$. Then, if G is (strictly) concave, $G(s)/s$ is (strictly) decreasing, and if G is (strictly) convex, $G(s)/s$ is (strictly) increasing. Thus, if $G(s) > s$ for each s and G is strictly concave, s_{t+1}/s_t is strictly decreasing.*

4. Equilibrium and welfare

This section characterizes pure-strategy perfect Bayesian equilibria (PBE). As a basic model, we assume that $b^M = 0$. This implies that $D^R < 0$ and $D^U < 0$. We first investigate the properties of incentives to report. Recall that $D^R < 0$, $D^U > D^R$, and $I(r_{t-1})$ is decreasing in r_{t-1}. These facts immediately implies that the subordinate's incentives to report satisfy a property of complementarity. Although the proof is evident, it gives an insight into incentives to report.

Lemma 2 (Complementarity of reporting)**.** *Suppose that $D^R < 0$. Then, for each t, $\varphi_{r,r'}(t)$ is increasing in r and r'.*

This implies that a subordinate has stronger incentives to report if the subordinates in contiguous generations report. Lemmas 1 and 2 yield the following necessary and sufficient conditions for the existence of two equilibria.

Lemma 3. *(1) There exists a PBE where the subordinate in each period reports the problem (if detected), if and only if $\varphi_{1,1}(t) \geqslant 0$ for each t.*

(2) There exists a PBE where the subordinate in each period conceals the problem, if and only if $\varphi_{1,0}(0) \leqslant 0$ and $\varphi_{0,0}(t) \leqslant 0$ for each $t \geqslant 1$.

Remark 1 (Multiple equilibria)**.** The conditions in the above lemmas are not mutually exclusive. Thus there exists a profile of parameter values for which both conditions are satisfied and hence multiple equilibria exist. As illustrated in Figure 3, if $\varphi_{1,0}(0) \leqslant 0$ and

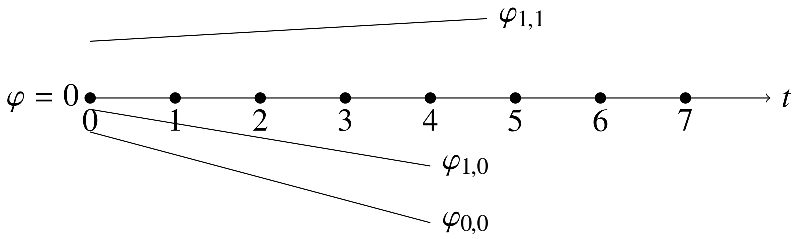

Figure 3: Multiple equilibria

$\varphi_{0,0}(t) \leqslant 0 \leqslant \varphi_{1,1}(t)$ for all t, then "all subordinates report" and "all subordinates conceal" are both equilibria. Thus, if everyone else reports, it is optimal to report. If everyone else hides, hiding is optimal. This suggests that corporate culture (Kreps, 1990) plays a critical role to determine workers' reporting behavior.

Note that while $\varphi_{1,1}$ is increasing in q^S, $\varphi_{0,0}$ is decreasing in q^S. Therefore, if q^S gets larger, since the difference $\varphi_{1,1} - \varphi_{0,0}$ becomes also larger, and then, the inequality $\varphi_{1,1} \geqslant 0 \geqslant \varphi_{0,0}$ becomes more likely to hold. Then, multiple equilibria are likely to occur. $\triangle$

We now explore the other equilibria. We focus on the case where the growth rate of the scale s_{t+1}/s_t is monotone.

First, we consider the case when s_{t+1}/s_t is nondecreasing in t, that is scale growth rate is nondecreasing over time. Let $t^{1,0} \in \mathbb{N}$ be such that $\varphi_{1,0}(t^{1,0}) > 0 > \varphi_{1,0}(t^{1,0} + 1)$. If $t^{1,0}$ exists, it is unique by the monotonicity of $\varphi_{1,0}$. If it exists, an additional equilibrium, as the following lemma shows.

Lemma 4. *Suppose that $b^M = 0$, s_{t+1}/s_t is nondecreasing in t, and $t^{1,0}$ exists. Then there exists a PBE where the subordinate at period t reports if and only if $t \leqslant t^{1,0}$.*

We have shown that there can be three PBEs with pure strategies. The following theorem shows that no other pure-strategy PBE exists.

Theorem 1. *Suppose that $b^M = 0$ and s_{t+1}/s_t is nondecreasing in t. Suppose also that $\varphi_{1,0}(t) \neq 0$ for all t. Then for each pure-strategy PBE, there exists $\hat{t} \in \{-1, t^{1,0}, +\infty\}$ such that the subordinate in period t reports if and only if $t \leqslant \hat{t}$.*

When s_{t+1}/s_t is nonincreasing, the set of equilibria is characterized similarly.

Corollary 1. *Suppose that $b^M = 0$, s_{t+1}/s_t is nonincreasing in t, and $\varphi_{0,1}(t) \neq 0$ for all t. Let $t^{0,1} \in \mathbb{N}$ be such that $\varphi_{0,1}(t^{0,1}) < 0 < \varphi_{0,1}(t^{0,1}+1)$, which is unique if exists. Then for each pure-strategy PBE, there exists $\hat{t} \in \{-1, t^{0,1}, +\infty\}$ such that the subordinate in period t conceals if and only if $t \leqslant \hat{t}$.*

In either case, the equilibrium may result in a tragedy. For example, when the growth rate of the problem is increasing (Theorem 1), if the problem is not detected before the threshold period $\hat{t}$, the problem will never be reported. Thus the problem continues to grow at an increasing rate until an accident occurs and causes large damage. On the other hand, when the growth rate of the problem is decreasing, the problem grows fast in the early stage but the problem is not reported until period $\hat{t}$.

4.1. Welfare

This section computes the social welfare to show the inefficiency of the equilibria. Let $L^R(s)$(resp. $L^U(s)$) be the total net loss from an accident for citizens excluding the workers of the firm, when the scale of the problem is s and the problem is reported (resp. unreported). Both $L^R(s)$ and $L^U(s)$ include compensation for the accident by the firm. The amounts $L^R(s)$ and $L^U(s)$ may differ since the amount of compensation may depend on whether the problem is reported. For example, if the workers carry liability insurance, the insurance payouts may depend on whether the worker is responsible for the accident. Typically, if the problem is reported, the worker has heavier responsibility and receives less from the insurance. Since the total loss includes the insurance firm's profits, $L^R(s) < L^U(s)$.

We now calculate the utilitarian social welfare. To simplify the notation, let

$$\bar{c}^R = \int^{\frac{pd^{M,R}}{1-p}} c dF(c), \quad \bar{c}^U = \int^{\frac{pd^{M,U}}{1-p}} c dF(c), \quad \bar{c} = \int c dF(c)$$
$$F^R = F\left(\frac{pd^{M,R}}{1-p}\right), \quad F^U = F\left(\frac{pd^{M,U}}{1-p}\right).$$

Consider an equilibrium where the subordinate in period t reports if and only if $t \leqslant \hat{t}$. The utilitarian social welfare $SW_{RC\hat{t}}$ evaluated at period 0 is given by

$$\begin{aligned} SW_{RC\hat{t}} = &-\sum_{t=0}^{\hat{t}} \left[(1-p)(1-q^S)(1-q^M F^U)\delta\right]^t \left[q^S SW^R(t) + (1-q^S)\widetilde{B}_t\right] \\ &- \left[(1-p)(1-q^S)(1-q^M F^U)\delta\right]^{\hat{t}+1} \Bigg[\widetilde{B}_{\hat{t}+1} \\ &+ (1-p)(1-q^M F^U)\delta \sum_{\tau=\hat{t}+2}^{\infty} [(1-p)(1-q^* F^U)\delta]^{\tau-\hat{t}-2} B_\tau \Bigg], \end{aligned}$$

where

$$\begin{aligned} SW^R(t) &= \sum_{\tau=t}^{\infty} \Big((1-F^R)(1-p)\delta\Big)^{\tau-t} A_\tau \\ A_\tau &= (1-p)\bar{c}^R s_\tau + p(1-F^R)\left(L^R(s_\tau) + d^{M,R} s_\tau\right) + p\bar{c}s_\tau, \\ \widetilde{B}_\tau &= q^M(1-p)\bar{c}^U s_\tau + p(1-q^M F^U)\left(L^U(s_\tau) + (d^{M,U} + d^S)s_\tau\right) + p\bar{c}s_\tau, \\ B_\tau &= q^*(1-p)\bar{c}^U s_\tau + p(1-q^* F^U)\left(L^U(s_\tau) + (d^{M,U} + d^S)s_\tau\right) + p\bar{c}s_\tau, \\ q^* &= 1-(1-q^M)(1-q^S). \end{aligned}$$

We consider the effect of delaying $\hat{t}$. That is, we compare the welfare when $\hat{t} = t^*$ and when $\hat{t} = t^* + 1$, which gives[9]

$$\begin{aligned} SW_{RCt^*+1} > SW_{RCt^*} \iff\ & q^S \widetilde{B}_{t^*+1} + (1-q^M F^U)(1-p)\delta(B_{t^*+2} - (1-q^S)\widetilde{B}_{t^*+2}) \\ &+ q^S[(1-q^M F^U)(1-p)\delta]^2 \sum_{\tau=t^*+3} [(1-q^* F^U)(1-p)\delta]^{\tau-t^*-3} B_\tau \\ &- q^S SW^R(t^*+1) > 0. \end{aligned}$$

Since $d^{M,R}$ appears only in $SW^R(t^*+1)$, a sufficiently large amount of $d^{M,R}$ will yield $SW_{RCt^*+1} < SW_{RCt^*}$. However, imposing an arbitrarily large amount of $d^{M,R}$ may be impossible in reality. The total amount of punishment is usually no more than the total social damage from the accident. To capture this idea, we impose the following assumption.

[9] The right hand side is $[(1-p)(1-q^S)(1-q^M F^U)\delta]^{-(t^*+1)}(SW_{RCt^*+1} - SW_{RCt^*})$, which is the difference in the social welfare evaluated at period $t^* + 1$.

Assumption 1. (1) For each s, $L^R(s) + d^{M,R}s = L^U(s) + (d^{M,U} + d^S)s = SD(s) > 0$.

(2) For each s, $L^R(s) \geqslant 0$ and $L^U(s) \geqslant 0$.

Assumption (1) says that the social damage from the accident is independent of whether the problem is reported, and $SD(s)$ denotes the social damage. If the punishment is compensation for the accident, this condition will be satisfied. Assumption (2) says that citizens do not benefit from accidents.

The assumption yields

Proposition 1. *Under Assumption 1, $SW_{RCt^*+1} > SW_{RCt^*}$ for all $t^* \in \mathbb{N}$.*

That is, the social welfare is increased if the last period of reporting is delayed. This also implies that the equilibrium is inefficient. Since the inequality holds for all t^*, the social optimum is no concealment (i.e., $t^* = \infty$).

Remark 2 (Private vs. social gain from concealing)**.** Suppose that L^U and L^R are linear in s_t, which implies that A_t, B_t, and $\widetilde{B}_t$ are also linear in s_t. Let $SW_{RCt^*}(t)$ denote the social welfare evaluated at period t. We now compare $SW_{RCt^*+1}(t^*+1) - SW_{RCt^*}(t^*+1)$ and $SW_{RCt^*}(t^*) - SW_{RCt^*-1}(t^*)$. The ratio between them is given by

$$\frac{SW_{RCt^*+1}(t^*+1) - SW_{RCt^*}(t^*+1)}{SW_{RCt^*}(t^*) - SW_{RCt^*-1}(t^*)} = \sum_{t=t^*}^{\infty} \gamma_t \frac{s_{t+1}}{s_t},$$

where $\sum_{t=t^*}^{\infty} \gamma_t = 1$ and $\gamma_t \geqslant 0$ for each $t \geqslant t^*$.[10] If the scale increases over time, i.e., $s_{t+1}/s_t > 1$ for each t, then we obtain $SW_{RCt^*+1}(t^*+1) - SW_{RCt^*}(t^*+1) > SW_{RCt^*}(t^*) - SW_{RCt^*-1}(t^*)$. Note that $SW_{RCt^*}(t^*) - SW_{RCt^*-1}(t^*)$ is the gain in social welfare evaluated at period t^* when the subordinate at t^* reports. Therefore, the social gain from the reporting of subordinate t^*, which is the social loss from his concealing, is larger for a larger t^*. Recall that if s_{t+1}/s_t is increasing, the incentive to conceal at period t^* is larger for a larger t^*. This implies that if the growth rate of the problem is positive and increasing, the private incentive to conceal is large when concealing causes a large social loss. $\triangle$

[10]This calculation uses the fact that $\frac{\sum_t a_t}{\sum_t b_t} = \sum_t \frac{b_t}{\sum_{t'} b_{t'}} \frac{a_t}{b_t}$ if $b_t > 0$ for each t. See also footnote 9 and the proof of Proposition 1.

5. Comparative statics

In this section, we assume that s_{t+1}/s_t is nondecreasing in t. We focus on the strategy profile in Lemma 4, where the subordinate in period t reports if and only if $t \leqslant t^{1,0}$. The equilibrium is characterized by the function $\varphi_{1,0}$ given by

$$\varphi_{1,0}(t) = \delta(1-p)\left[I(1)D^R - D^U\right]\frac{s_{t+1}}{s_t} + b^S + pd^S. \tag{3}$$

If $\varphi_{1,0}$ shifts upward, $t^{1,0}$ increases, which reduces concealment.

5.1. Discount factor

Consider the effect of discount factor δ. If $I(1)D^R - D^U \geqslant 0$, then $\varphi_{1,0}(t) > 0$ and therefore no one conceals. Thus if someone conceals, $I(1)D^R - D^U < 0$, in which case, increasing δ decreases $\varphi_{1,0}$ and hence increases concealment. Thus more farsighted subordinates are more likely to conceal. Since δ also captures the probability that a subordinate is promoted to a manager, it also suggests that subordinates who are more likely to be promoted are more likely to conceal.

5.2. Punishment and reward for subordinates

We now consider increasing punishment and reward for subordinates. Since they appear only in the last two terms in (3), increasing them directly increases $\varphi_{1,0}$. That is, increasing in punishment or reward for subordinates decreases concealment.

Remark 3 (Resolution of multiple equilibria by onetime punishment/reward)**.** Recall that for some parameter values, there exist multiple equilibria (Remark 1), where "all report" and "all conceal" are both equilibria. Their necessary and sufficient conditions are respectively, $\varphi_{1,1}(t) > 0$ and $\varphi_{0,0}(t) < 0$. The multiplicity of equilibria may be resolved by increasing punishment/reward for the subordinate at a given *single* period. To see this, let t be any period. Let $\Delta b^S > 0$ be a large additional reward given to the subordinate in period t if he reports. Choose a large amount to satisfy $\varphi_{1,1}(t) + \Delta b^S > 0$ and $\varphi_{0,0}(t) + \Delta b^S > 0$. Since the additional reward is give only in period t, $\varphi_{1,1}(t')$ and $\varphi_{0,0}(t')$ remain the same for all $t' \neq t$. Then, although "all report" remains an equilibrium, "all conceal" is no longer an equilibrium since the subordinate at period t wants to report even if the others conceal. For simplicity, suppose that the only equilibria in the original parameter profile are "all report" and "all conceal." With the additional reward, the set of equilibria depends on the values of $\varphi_{0,1}(t-1)$ and $\varphi_{1,0}(t+1)$. If they are both positive, the only remaining equilibrium is "all report," which resolves the multiplicity of equilibria.[11] △

5.3. Punishment for reported managers

We now consider increasing punishment $d^{M,R}$ for the manager when the problem is reported but he ignores it. Since $d^{M,R}$ appears in D^R and $I(1)$, the effect is not obvious. Differentiating $\varphi_{1,0}$ by $d^{M,R}$ gives

$$\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}} = \delta(1-p)\left[\frac{\partial I(1)}{\partial d^{M,R}} D^R + I(1)\frac{\partial D^R}{\partial d^{M,R}}\right]\frac{s_{t+1}}{s_t},$$

where

$$\begin{aligned}
\frac{\partial I(1)}{\partial d^{M,R}} &= -\frac{p}{1-p}\left[\frac{f\left(\frac{pd^{M,R}}{1-p}\right)}{q^M\left[1-F\left(\frac{pd^{M,U}}{1-p}\right)\right]+(1-q^M)}\right] < 0,\\
\frac{\partial D^R}{\partial d^{M,R}} &= -p[1-F(pd^{M,R}/(1-p))] < 0.
\end{aligned}$$

Since $D^R < 0$, we have the following observation.

Observation 1. If $f(pd^{M,R}/(1-p))$ is sufficiently small, then increasing $d^{M,R}$ decreases $\varphi_{1,0}$.

Recall that I is the probability that the manager ignores the problem, which is lowered if the punishment $d^{M,R}$ is increased. However, if $f(pd^{M,R}/(1-p))$ is sufficiently small, the effect on I is negligible. On the other hand, if the value of I itself is not very small, i.e., the manager ignores the reported problem with a non-negligible probability, then if the subordinate reports

[11] If $\varphi_{0,1}(t-1)$ and $\varphi_{1,0}(t+1)$ are both negative, "all except for t conceal" is now equilibrium and hence multiple equilibria remain.

the problem, the subordinate has a non-negligible chance to face the problem when he is the manager. Since increasing $d^{M,R}$ lowers the reported manager's expected utility, it increases the subordinate's incentive to conceal.

Remark 4. The condition in Observation 1 may not hold if $d^{M,R}$ is sufficiently large. To see this, suppose that f is nondecreasing. Then, f has a support $[\underline{c}, \overline{c}]$ such that $\underline{c} < \overline{c} < \infty$ and $f(\overline{c}) > 0$. Let $d^{M,R} = \frac{1-p}{p}\overline{c}$. Then, since $\frac{\partial D^R}{\partial d^{M,R}} = 0$, $\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}} > 0$. This implies that $\varphi_{1,0}(t)$ is increasing in $d^{M,R}$ for sufficiently large $d^{M,R}$. Furthermore, note that

$$\frac{\partial^2 D^R}{\partial (d^{M,R})^2} = pf(pd^{M,R}/(1-p))] > 0.$$

Then, the second order derivative of $\varphi_{1,0}$ is

$$\frac{\partial^2 \varphi_{1,0}(t)}{\partial (d^{M,R})^2} = \delta(1-p)\left[\frac{\partial^2 I(1)}{\partial d^{M,R}} D^R + 2\frac{\partial I(1)}{\partial d^{M,R}}\frac{\partial D^R}{\partial d^{M,R}} + \frac{\partial^2 D^R}{\partial (d^{M,R})^2} I(1)\right]\frac{s_{t+1}}{s_t} > 0.$$

Thus, $\varphi_{1,0}(t)$ is a convex function of $d^{M,R}$ for each t. This implies that there exists $\bar{d}$ such that for each $d^{M,R} \leqslant \frac{1-p}{p}\overline{c}$, $\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}} > 0$ if and only if $d^{M,R} > \bar{d}$.

Intuition is the following. When $d^{M,R}$ is sufficiently large, the manager will solve the problem with probability close to 1, which gives the subordinate an incentive to report. This intuition depends on the assumption that each manager is able to solve the problem with probability 1. We revisit this problem in section 6.2. △

Example 1. Suppose that c is distributed uniformly on $(0, \bar{c})$. Then

$$\begin{aligned}\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}}\frac{s_t}{s_{t+1}}\frac{1}{\delta(1-p)} &= \frac{\partial I(1)}{\partial d^{M,R}} D^R + I(1)\frac{\partial D^R}{\partial d^{M,R}} \\ &= \begin{cases} \frac{3p^2 d^{M,R}}{(1-p)\bar{c}}\left(1 - \frac{1}{2}\frac{pd^{M,R}}{(1-p)\bar{c}}\right) - p\left(\frac{1-2p}{1-p}\right) & \text{if } \frac{pd^{M,R}}{1-p} \leqslant \bar{c}, \\ 0 & \text{otherwise.}\end{cases}\end{aligned}$$

This implies that if $p > 1/2$, $\varphi_{1,0}$ is increasing in $d^{M,R}$. For the case when $p < 1/2$, note first that $\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}} < 0$ if $d^{M,R} = 0$, and $\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}} > 0$ if $d^{M,R} = \bar{c}(1-p)/p$. By Remark 4, $\varphi_{1,0}$ is a convex function of $d^{M,R}$ since the uniform distribution has a nondecreasing density. Therefore there exists $\bar{d}$ such that $\varphi_{1,0}$ is decreasing in $d^{M,R} < \bar{d}$ and increasing in $d^{M,R} > \bar{d}$. △

5.4. Punishment for unreported managers

We here consider increasing $d^{M,U}$, punishment for the manager when the problem is not reported. Recall that the net benefit of reporting, φ, is given by

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Recall also that we have assumed $d^{M,U} < d^{M,R}$. By increasing $d^{M,U}$, the reduction of responsibility, $D^U - D^R$, gets close to 0. In the limit, where $d^{M,U} = d^{M,R}$, we have

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R\right]\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Since $D^R < 0$, we have $\varphi_{r_{t-1},r_{t+1}}(t) > 0$, which implies that there will be no concealment. The intuition is as follows. The subordinate has an incentive to conceal the problem if it reduces his responsibility when he is a manager. However, if $d^{M,R} = d^{M,U}$, the punishment for the manager is the same whether or not the problem is reported, and hence concealing the problem does not reduce his responsibility.

5.5. Scale growth rate

We consider two problems P and P', each of which is identified by a sequence of scales. That is, $P = (s_t)_{t=0}^{\infty}$ and $P' = (s'_t)_{t=0}^{\infty}$. Suppose that no other parameter differs between the two problems. Note that the corresponding φ and φ' are given by

$$\begin{aligned}
\varphi_{r_{t-1},r_{t+1}}(t) &= \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S,\\
\varphi'_{r_{t-1},r_{t+1}}(t) &= \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s'_{t+1}}{s'_t} + b^S + pd^S.
\end{aligned}$$

Assume that $(s_t)_t$ is a strictly increasing sequence and $(s'_t)_t$ is strictly decreasing. That is, $s_{t+1}/s_t > 1 > s'_{t'+1}/s'_{t'}$ for all $t, t' \in \mathbb{N}$. Assume that the coefficient of s_{t+1}/s_t in φ is negative.

Then $\varphi'_{r,r'}(t) > \varphi_{r,r'}(t')$ for all t, t' and all r, r'.

Now, suppose that problem P has no PBE where all subordinates conceal. Then by Proposition 3 (2), there exists $t' \in \mathbb{N}$ such that $\varphi_{0,0}(t') > 0$. This implies that $\varphi'_{0,0}(t) > 0$ for all t. Since $\varphi'_{r_{t-1},r_{t+1}}(t) > \varphi'_{0,0}(t) > 0$ for all t, the unique equilibrium in P' is that all subordinates report. To summarize,

Theorem 2. *Consider two problems P and P' where P has increasing scales and P' has decreasing scales. Then, if P has no equilibrium where everyone conceals, then a unique equilibrium of P' is that everyone reports. Conversely, if P' has no equilibrium where everyone reports, then a unique equilibrium of P is that everyone conceals.*

This result implies that concealment is more likely in problems with increasing scales. With increasing scales, the problem is relatively big in the next period, which gives subordinates strong incentives to reduce punishment in the next period.

6. Extensions

This section considers a few extensions of the model. We assume that s_{t+1}/s_t is strictly increasing, unless stated otherwise.

6.1. Introducing rewards for the managers

Here consider the case when $b^M > 0$, which increases D^R. If it remains $D^R < 0$, nothing changes. Thus we consider the case when $D^R > 0$. This creates another incentive to conceal: a subordinate may want to conceal the problem in order to solve it by himself in the next period as a manager since it is rewarded handsomely. We now consider the features of this additional incentive.

6.1.1. Equilibria

First, we characterize equilibria. Recall that $\varphi_{r_{t-1},r_{t+1}}$ is written as

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Since $I(r_{t-1})$ is decreasing in r_{t-1}, it follows that $\varphi_{r_{t-1},r_{t+1}}(t)$ is decreasing in r_{t-1}. This implies that the complementarity of reporting proved in the basic model does not extend. Since $D^U > D^R$, $\varphi_{r_{t-1},r_{t+1}}(t)$ is increasing in r_{t+1}. Thus $\varphi_{0,1}(t) \geqslant \varphi_{r_{t-1},r_{t+1}}(t) \geqslant \varphi_{1,0}(t)$.

We consider two cases.

Case 1: $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t)$ for some t.

This implies that the inequality $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t)$ holds for all t. In this case, all pure-strategy equilibria satisfy the following feature.

Lemma 5. *Suppose that there exists t such that $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t) \leqslant 0$. Then, in any pure-strategy PBE, subordinate t conceals the problem for each t such that $\varphi_{0,0}(t) < 0$.*

Here is a condition for a unique equilibrium.

Proposition 2. *Suppose that there exists t such that $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t) \leqslant 0$, there exists no t such that $\varphi_{1,0}(t) = 0$, and there exists at most one t such that $\varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)$. Then there exists a unique pure-strategy PBE, where subordinate t reports if $\varphi_{1,0}(t) > 0$ and conceals if $\varphi_{1,0}(t) < 0$.*

On the other hand, a pure-strategy PBE may not exist.

Proposition 3. *Suppose that $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t)$ for each t, there exists t such that $\varphi_{0,1}(t) \leqslant 0$, there exists no t such that $\varphi_{1,0}(t) = 0$, and there exist at least two t such that $\varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)$. Then there exists no pure-strategy PBE.*

If mixed strategies are included, the following result gives a sufficient condition for the existence of an equilibrium.

Theorem 3. *Suppose that there exists t such that $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t) \leqslant 0$ and there exists no t such that $\varphi_{1,0}(t) = 0$. Then there exists a PBE such that subordinate t conceals the problem for each t such that $\varphi_{0,0}(t) < 0$.*

Case 2: $\varphi_{0,0}(t) \geqslant \varphi_{1,1}(t)$ for some t.

In this case, pure-strategy equilibria exist and have the following features.

Theorem 4. *Suppose that there exists t' such that $0 > \varphi_{0,0}(t') \geqslant \varphi_{1,1}(t')$. Then, a pure-strategy PBE exists and any pure-strategy PBE satisfies:*

1. *Subordinate t conceals the problem for all t such that $\varphi_{0,0}(t) < 0$.*

2. *(Action Alternation) Let $B = \{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}$ and suppose that $|B| \geqslant 2$. Then for all $t \in B$ such that $t > \min B$ (hence $t - 1 \in B$), if subordinate t reports, subordinate $t - 1$ conceals, while if subordinate t conceals, subordinate $t - 1$ reports.*

The first feature is the same as in the previous case, while the second feature of action alternation is specific to the present case.

The reason for the action alternation is that the action plans[12] of subordinates of two consecutive generations are strategic substitutes. That is, if the subordinate in the previous period conceals, the subordinate in the current period has less incentives to conceal, since the current manager is the subordinate in the previous period and hence may already know the problem. Then, the subordinate expects that the cost of the current manager is high and thus, even when the subordinate reports the problem, the current manager may not solve the problem, which weakens the probability that missing the opportunity to gain the reward when the subordinate reports. Then, the subordinate has an incentive to report the problem to avoid punishment that is imposed when an accident occurs in the current period.

6.1.2. Comparative statics

We now consider the effect of increasing punishment or reward. Recall that the start period of concealment is determined by $\varphi_{0,0}$, which is given by

$$\varphi_{0,0}(t) = \delta(1 - p)(I(0)D^R - D^U)\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

[12] Realized actions do not alternate: once someone reports the problem, there is nothing to report or conceal in the following periods.

The following proposition shows that $\varphi_{0,0}$ reduces if punishment is strengthened for the manager who neglects the reported problem.

Proposition 4. *If $D^R > 0$, then $\varphi_{0,0}(t)$ is decreasing in $d^{M,R}$.*

The intuition is as follows. With $D^R > 0$, the subordinate does not want the current manager to solve the problem. If the manager does not solve, the subordinate can earn a reward by solving the problem himself after he becomes a manager. Increasing punishment $d^{M,R}$ increases the probability that the manager solves the problem. This motivates the subordinate to conceal the problem.

Consider the effect of increasing b^M, a reward to a manager who solves the problem.

Proposition 5. *Suppose that $f(x)/(1-F(x))$ is increasing in x. Let $w^R = (b^M + pd^{M,R})/(1-p)$ and $w^U = (b^M + pd^{M,U})/(1-p)$. Then*
(1) if $F(w^U) > 1/2$ and $D^R > 0$, then $\varphi_{0,0}(t)$ is decreasing in b^M for all t;
(2) if $F(w^R)(1 - F(w^R)) > F(w^U)$ and $D^R < 0$, then $\varphi_{0,0}(t)$ is increasing in b^M for all t.

If b^M is sufficiently high, the conditions $F(w^U) > 1/2$ and $D^R > 0$ are both satisfied. Thus, when b^M is sufficiently high, $\varphi_{0,0}(t)$ is decreasing in b^M. The intuition is straightforward. With sufficiently large b^M, the benefit from solving a problem is large for the manager, which implies a low probability of facing the problem as a manager. Since the reward is large, the incentive for seeking the reward is also large. Both of the effects strengthen the incentive for the subordinate to conceal the problem. If $D^R < 0$, on the other hand, managers do not want to face the problem. Therefore, reducing the probability of facing the problem weakens the incentive to conceal.

We also perform comparative static for the scale of the problem. If $D^R < 0$, as in the basic model, since the inequalities $\varphi_{0,0} < \varphi_{r,r'} < \varphi_{1,1}$ continue to hold for all $r, r' \in [0, 1]$, the statement of Theorem 2 remains true. With $D^R > 0$, on the other hand, the inequalities $\varphi_{0,0} < \varphi_{r,r'} < \varphi_{1,1}$ do not hold, but if $\varphi_{0,0} \leqslant \varphi_{1,1}$, the statement of Theorem 2 continues to hold for pure-strategy equilibria, as the following result shows.

Theorem 5. *Suppose that $\varphi_{0,0} \leqslant \varphi_{1,1}$. Consider two problems $P = (s_t)_{t=0}^{\infty}$ and $P' = (s'_t)_{t=0}^{\infty}$ such that $s_{t+1}/s_t > 1 > s'_{t+1}/s'_t$ for all t. Then, if P has no PBE where everyone conceals, then*

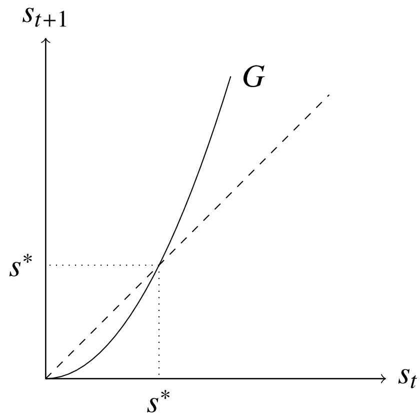

Figure 4: Illustration of function G in Example 2.

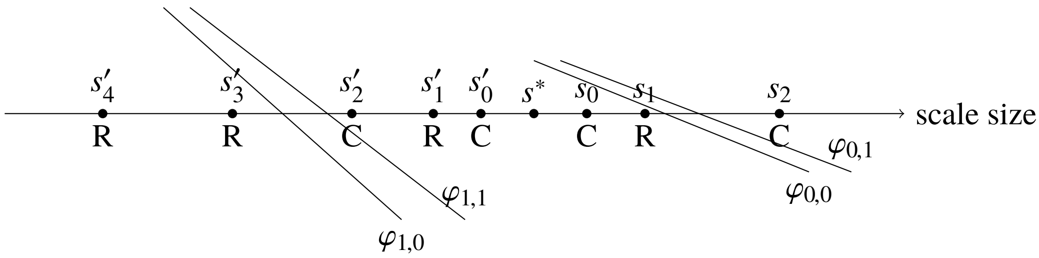

Figure 5: Illustration of Example 2

P' has a unique pure-strategy PBE, where everyone reports. Conversely, if P' has no PBE where everyone reports, then P has a unique pure-strategy PBE, where everyone conceals.

Theorem 5 does not extend to the case when $\varphi_{0,0} > \varphi_{1,1}$, as the following example shows.

Example 2. Suppose that there is an increasing function G as drawn in figure 4. Note that there exists s^* such that $G(s^*) = s^*$. Consider two initial values s_0 and s'_0 and let G generate two sequences $P = (s_t)_{t=0}^{\infty}$ and $P' = (s'_t)_{t=0}^{\infty}$ with $s_{t+1} = G(s_t)$ and $s'_{t+1} = G(s'_t)$. Let s_0 and s'_0 satisfy $s'_0 < s^* < s_0$, which implies that $P = (s_t)$ is increasing over time while $P' = (s'_t)$ is decreasing, as in Theorem 5. Since the scales are generated by G, the function φ can be rewritten as

$$\varphi_{r_{t-1},r_{t+1}}(s) \coloneqq \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{G(s)}{s} + b^S + pd^S,$$

as a function of scale s. Let the function $\varphi_{r_{t-1},r_{t+1}}(s)$ be as shown in figure 5, where s is denoted by the axis.

Figure 5 shows a PBE for both scale sequences, where "R" denotes the scale at which the subordinate reports and “C” denotes the scale where the subordinate conceals. The right half of the figure pertains to the sequence (s_t) while the left half pertains to (s'_t).

To see that this indeed gives PBEs, first consider the sequence $P = (s_t)$. For all $t \geqslant 2$, concealing is the dominant strategy since $\varphi_{r,r'}(s_t) < 0$ for all r, r'. For $t = 0$, since subordinate 0 is the first person who can detect the problem, it is as if $r_{-1} = 1$, as discussed early. Since $\varphi_{1,r}(s_0) < 0$ for all r, subordinate 0's best response is to conceal: $r_0 = 0$. For subordinate $t = 1$, since $r_0 = r_2 = 0$ and $\varphi_{0,0}(s_1) > 0$, the best response is to report. Thus, in the unique PBE with (s_t), not everyone conceals.

Now, consider $P' = (s'_t)$. For all $t \geqslant 3$, reporting is the dominant strategy since $\varphi_{r,r'}(s'_t) > 0$ for all r, r'. For subordinate $t = 0$, since $\varphi_{1,r}(s'_0) < 0$ for all r, the best response is to conceal: $r_0 = 0$. For subordinate $t = 1$, since $r_0 = 0$ and $\varphi_{0,r}(s'_1) > 0$ for all r, the best response is to report: $r_1 = 1$. For subordinate $t = 2$, since $r_1 = r_3 = 1$ and $\varphi_{1,1}(s'_2) < 0$, the best response is to conceal: $r_2 = 0$. Thus, in the unique PBE with (s'_t), not everyone reports. This shows that Theorem 5 does not generalize when $\varphi_{0,0} > \varphi_{1,1}$. △

6.2. Possibility of failure in solving the problem

We have so far assumed that managers who face the reported problem have only two choices: whether or not to solve the problem. Here this section considers the case when there are infinite choices: the manager can choose the probability that the problem is solved, and a higher probability costs more.

For simplification, we assume that if the problem is not solved in the current period, the problem is handed over to the next manager, and the cost function for the next manager is not influenced by the amount of cost paid by the previous manager. We also assume that even when an accident occurs, the manager in the current period is not forced to solve the problem: if the current manager does not solve, the problem is handed over to the next manager.

The probability that the problem is solved is denoted by $\rho \in [0, \bar{\rho}]$, which is chosen by the manager. The cost function for manager t is given by $c_t \chi(\rho) s$, where the function $\chi \colon [0, \bar{\rho}) \to \mathbb{R}_+$ is strictly convex, continuously differentiable, and strictly increasing, and satisfies $\chi(0) = 0$ and $\lim_{\rho \to \bar{\rho}} \chi(\rho) = \lim_{\rho \to \bar{\rho}} \chi'(\rho) = \infty$. The value $\bar{\rho} \leqslant 1$ is the supremum of achievable probability. The assumptions imply that χ' is strictly increasing, thus having a strictly increasing inverse function.

As in the basic model, c_t is independently and identically distributed by a cumulative distribution function F on $(0, \bar{c})$, where $\bar{c} \in \mathbb{R}_{++}$. The value c_t is private information of manager t and he learns it only after he becomes a manager.

We assume that if the manager chooses a positive $\rho > 0$, it becomes known publicly that the manager knows the problem, although the exact value of ρ remains private. On the other hand, we also assume that the value of ρ does not affect the level of punishment for the manager when an accident occurs.

Let $\widetilde{D}^R$ be the expected utility of the reported manager, $\widetilde{D}^U$ be the expected utility of the unreported manager, and $\widetilde{I}$ be the probability that the reported manager ignores the problem.[13] Then, as in the basic model, we show

Lemma 6. *There exist a nonincreasing function $\widetilde{I}\colon [0,1] \to [0,1]$ and constants $\widetilde{D}^U$ and $\widetilde{D}^R$ such that subordinate t reports if and only if*

$$\widetilde{\varphi}_{r_{t-1},r_{t+1}}(t) := \delta\left[-(1-\widetilde{I}(r_{t-1}))\widetilde{D}^R - (1-p)(1-q^S r_{t+1})(\widetilde{D}^U - \widetilde{D}^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S \geqslant 0.$$

We consider the effect of $d^{M,R}$. As in the basic model, the equilibrium when $\widetilde{D}^R < 0$ is determined by $\widetilde{\varphi}_{1,0}$. In the basic model, example 1 shows that $\widetilde{\varphi}_{1,0}$ is increasing in $d^{M,R}$ if $d^{M,R}$ is sufficiently large. However, this may not hold in the current model, as the following result shows.

Theorem 6. *Suppose that the support of F is $(0, \bar{c})$. If $\bar{\rho} < 1-p$, there exists $\bar{d}$ such that for each $d^{M,R} > \bar{d}$, $\widetilde{\varphi}_{1,0}(t)$ is decreasing in $d^{M,R}$. If $\bar{\rho} > 1-p$, then there exists $\bar{d}$ such that for each $d^{M,R} > \bar{d}$, $\widetilde{\varphi}_{1,0}(t)$ is increasing in $d^{M,R}$.*

The intuition is as follows. Note that

$$\widetilde{\varphi}_{1,0}(t) := \delta\left[(\widetilde{I}(1) - p)\widetilde{D}^R - (1-p)\widetilde{D}^U\right]\frac{s_{t+1}}{s_t} + b^S + pd^S \geqslant 0.$$

[13] See the appendix for details.

As $d^{M,R}$ increases, $\widetilde{D}^R$ increases, which increases $\widetilde{\varphi}_{1,0}$ if and only if $\widetilde{I}(1) - p > 0$. If the subordinate reports, he will get $\widetilde{D}^R$ if the manager does not solve the problem, which occurs with probability $\widetilde{I}(1)$. On the other hand, if the subordinate conceals, he will get $\widetilde{D}^R$ if an accident occurs, which occurs with probability p. Since the probability that the manager solves the problem, $1 - \widetilde{I}(1)$, converges to $\bar{\rho}$ as $d^{M,R}$ goes to infinity, $\widetilde{I}(1) - p$ converges to $1 - \bar{\rho} - p$. Thus if $1 - \bar{\rho} - p > 0$, $\widetilde{\varphi}_{1,0}$ is increasing in $\widetilde{D}^R$ and hence in $d^{M,R}$ if $d^{M,R}$ is sufficiently large.

The following proposition gives a sufficient condition for $\widetilde{\varphi}_{1,0}$ to be decreasing in $d^{M,R}$.

Proposition 6. *If $q^M = d^{M,U} = 0$, $\chi(\rho) \equiv \rho/(\bar{\rho} - \rho)$, and $\bar{\rho} < 1 - p$, then $\widetilde{\varphi}_{1,0}$ is decreasing in $d^{M,R}$.*

6.3. After retirement blames

The previous sections assume that managers are not punished after their retirement. We here relax this assumption. Suppose that workers live for at most three periods. They work as a subordinate in their first period, work as a manager in the second period, and are retired in the third period. Workers may die after the second period, and let μ be the probability that a worker lives after his retirement.

If a manager neglects the reported problem, his negligence is noticed and punished when the problem causes an accident, if he is still alive, even if he is retired then. Let $d^R s_t$ be the disutility of punishment for the retired manager in period t.

We keep the assumption that the scale s_t is determined by a transition function G: $s_t = G(s_{t-1})$ for all t. Let $\hat{D}^R s_t$ be the reported manager's utility, $\hat{D}^U s_t$ be the unreported manager's utility, and $\hat{I}$ be the probability that the reported manager ignores the problem.[14] Then the per-scale net benefit of reporting, $\hat{\varphi}$, is defined as in the basic model. However, since a manager may be punished after retirement, the per-scale net benefit is affected by the scale in his after-retirement period. This makes $\hat{D}^R$ and $\hat{I}$ dependent on s_t and thus we write $\hat{D}^R(s_t)$ and $\hat{I}(r_{t-1}, s_t)$.[15]

We also suppose that the support of F is $(0, \bar{c})$ and F has differentiable density.

[14] See the appendix for details.
[15] For details, see the proof of the lemma.

Then, we can show

Lemma 7. *Suppose that $G(s)/s$ and f are well defined on $\mathbb{R}_+$ and $\sup_{s\in\mathbb{R}_+} G(s)/s < \infty$. Then, there exist functions $\hat{I}\colon [0,1]\times\mathbb{R} \to \mathbb{R}$ and $\hat{D}^R\colon \mathbb{R}\to\mathbb{R}$ and a constant $\hat{D}^U$ such that the subordinate in period t reports the problem if and only if $\hat{\varphi}_{r_{t-1},r_{t+1}}(s_t) \geqslant 0$, where*

$$\hat{\varphi}_{r_{t-1},r_{t+1}}(s_t) := \delta(1-p)\left[-\{1-\hat{I}(r_{t-1},s_t)\}\hat{D}^R(s_t) - (1-q^S r_{t+1})(\hat{D}^U - \hat{D}^R(s_t))\right]\frac{G(s_t)}{s_t} + b^S + pd^S.$$

Moreover, $\hat{I}$ is decreasing in the first argument.

Increasing the after-retirement punishment d^R has similar effects as increasing $d^{M,R}$. If $G(s)/s$ is increasing, it is the same as increasing $d^{M,R}$ for every period. However, the effect of increasing $d^{M,R}$ depends on the level of $d^{M,R}$ and the distribution function, making it difficult to determine the shape of $\hat{\varphi}$.

To simplify the analysis, we consider the case when $G(s) = \alpha s$, where $\alpha \in \mathbb{R}_{++}$ is given. Thus $\alpha > 1$ implies that the scale is increasing over time. Then, $\hat{\varphi}$, $\hat{D}$, and $\hat{I}$ are constant across periods since $G(s)/s = G^2(s)/G(s) = \alpha$. Thus these variables can be written as a function of α.

A pure-strategy equilibrium is either "all conceal" or "all report" since $\hat{\varphi}$ is constant. Thus we consider $\hat{\varphi}_{0,0}$ and $\hat{\varphi}_{1,1}$. For simplicity, we consider the case when $\hat{D}^U < 0$ (and thus $\hat{D}^R < 0$ for each α). As shown in Remark 4, if f is nondecreasing, $\hat{\varphi}$ is a convex function of punishment. Since increasing the scale has similar effects as increasing punishment, $\hat{\varphi}$ is high when α is either large enough or small enough, as the following results show.

Proposition 7. *If $\hat{D}^R < 0$, there exists $\bar{\alpha}$ such that for each $\alpha > \bar{\alpha}$, $\hat{\varphi}_{r_{t-1},r_{t+1}} > 0$ and $\hat{\varphi}_{r_{t-1},r_{t+1}}$ is increasing in α.*

Proposition 8. *Suppose that $\hat{D}^R < 0$ and F is a uniform distribution on $(0,\bar{c})$. Suppose also that p, b^M and $d^{M,U}$ are sufficiently small, $\bar{c}$ is sufficiently large, and $q^S < 1$. Then, there exists $\bar{\alpha}$ such that for each $\alpha < \bar{\alpha}$, $\hat{\varphi}_{r_{t-1},r_{t+1}}$ is decreasing in α.*

7. Discussion and Conclusion

This paper studies subordinates' incentives to conceal problems, showing that the main reason to conceal problems is to reduce responsibility to solve them later. Imposing strong punishment for managers who ignore reported problems may give subordinates incentives to avoid the punishment in the future by not reporting problems. We also show that concealment is likely when the problem grows over time. If the problem grows, the expected damage it may cause also grows and hence efficiency requires a speedy solution of the problem. However, in equilibrium, the problem is not even reported until the threshold period.

Our analysis surely depends on our assumptions. One of them is the assumption that only one subordinate exists in each period and he becomes a manager in his second period with probability one. If this assumption is relaxed to have multiple subordinates, then not all subordinates become a manager and therefore the incentive to conceal problems to avoid responsibility may be weakened. However, having multiple subordinates may also weaken each subordinate's responsibility to detect and report problems. Thus the overall effect of relaxing the assumption may be ambiguous.

Let us point out a few questions that are left for future research. The first is punishment and reward that maximize social welfare. In this paper, we omit the discussion about social welfare except for showing that the equilibrium is inefficient, but optimal incentive scheme is important for public policy. The second is to generalize the model to have more players in the firm. The third is to generalize the model to allow the problem scale to have nonlinear effect on punishment, reward, and problem-solving cost. In these generalizations, the incentive to reduce future responsibility will remain, but some of the properties of equilibria may not.

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A. Omitted Proofs

A.1. Proof in Section 3

Proof of Fact 1. Differentiating $G(s)/s$ gives

$$\left(\frac{G(s)}{s}\right)' = \frac{G'(s)s - G(s)}{s^2} = \frac{\frac{s^2}{2}G''(\lambda s) - G(0)}{s^2}.$$

The second equality is by the Taylor expansion of G, that is, $G(0) = G(s) - sG'(s) + \frac{s^2}{2}G''(\lambda s)$ for some $\lambda \in (0, 1)$. Since $G(0) = 0$, then $G(s)/s$ is increasing if G is convex and is decreasing if G is concave. □

A.2. Proofs in Section 4

Proof of Lemma 2. Note that $I(r_{t-1})$ is decreasing in r_{t-1}. Since $D^R < 0$, $\varphi_{r_{t-1},r_{t+1}}(t) > \varphi_{r'_{t-1},r_{t+1}}(t)$ for each $r_{t-1} > r'_{t-1}$. Since $D^R < D^U$, $\varphi_{r_{t-1},r_{t+1}}(t) > \varphi_{r_{t-1},r'_{t+1}}(t)$ for each $r_{t+1} > r'_{t+1}$. Therefore, for each $r_{t-1}, r_{t+1} \in [0, 1]$, $\varphi_{1,1}(s) \geqslant \varphi_{r_{t-1},r_{t+1}}(t) \geqslant \varphi_{0,0}(t)$. □

Proof of Lemma 3. (1) Suppose that for each t, $\varphi_{1,1}(t) \geqslant 0$. Consider the behavior of subordinate at period t. Suppose that the other player takes the action such that he reports the problem when he detects it. Then, $r_{t-1} = r_{t+1} = 1$ and thus, reporting the problem is a best response.

Suppose that $\varphi_{1,1}(t^*) < 0$ for some $t^* \in \mathbb{N}$. Then, since $\varphi_{r,r'}(t^*)$ is increasing in r, r', $\varphi_{r,r'}(t^*) \leqslant \varphi_{1,1}(t^*) < 0$ for each $r, r' \in [0, 1]$. Therefore, for subordinate t^*, concealing the problem is a strict dominant strategy. Therefore, reporting for each period is not an equilibrium.

(2) Suppose that for each t, $\varphi_{0,0}(t) \leqslant 0$ and $\varphi_{1,0}(0) \leqslant 0$. Then, for each subordinate $t > 0$, as in the proof of Lemma 3 (1), we can show that concealing is the best response. Then, since subordinate 1 conceals, $r_{-1} = 1$, and $\varphi_{1,0}(0) \leqslant 0$, for subordinate 0, concealing is the best response.

To prove contraposition, suppose that $\varphi_{0,0}(t) > 0$ for some t or $\varphi_{1,0}(0) > 0$. If $\varphi_{0,0}(t) > 0$ for some t, since $\varphi_{0,0}(t) < \varphi_{r,r'}(t)$ for each $r, r' \in [0, 1]$, for subordinate t, reporting is a strict dominant strategy.

Consider the latter case, $\varphi_{1,0}(0) > 0$. Then, since subordinate 1 conceals and $r_{-1} = 1$, reporting is the best response. In each case, the strategy profile where each subordinate reports is not an equilibrium. □

Proof of Lemma 4. Suppose that $t^{1,0}$ exists. Therefore, for some t, $\varphi_{1,0}(t) < 0$. Then, since $b^S + pd^S > 0$ and $s_t > 0$, $\delta(1-p)[I(1)D^R - D^U] < 0$. Since s_{t+1}/s_t is increasing over time t, $\varphi_{1,0}(t)$ is decreasing in t.

Consider the behavior of subordinate $t^* \in \mathbb{N}$. Suppose that the subordinate in each $t \neq t^*$ follows the strategy of the statement. Suppose also that $t^{1,0} > 0$.

Consider the case that $t^* > t^{1,0}$. We show that subordinate t^* conceals. Note that each subordinate $t > t^*$ conceals. Thus, $r_{t^*+1} = 0$. Since s_{t+1}/s_t is increasing in t, $\varphi_{1,0}(t^*) < 0$. Then, since $\varphi_{r_{t-1},r_{t+1}}$ is increasing in r_{t-1}, $\varphi_{r_{t-1},0}(t^*) < 0$. Therefore, for subordinate t^*, concealing is the best response.

Consider the case that $t^* \leqslant t^{1,0}$. We show that subordinate t^* reports. Note that each subordinate $t < t^*$ reports. Thus, $r_{t^*-1} = 1$. Since s_{t+1}/s_t is increasing in t, $\varphi_{1,0}(t^*) > 0$. Then, $\varphi_{r_{t-1},r_{t+1}}$ is increasing in r_{t+1} since $\varphi_{1,r_{t+1}}(t^*) > 0$. Therefore, for subordinate t^*, reporting is the best response.

Suppose that $t^* = 0$. Then, since $r_{-1} = 1$ and $0 \leqslant t^{1,0}$, as shown above, reporting is the best response. □

Proof of Theorem 1. By Lemmas 3 and 4, each of the strategies in the statement is a PBE under some condition.

Consider a pure-strategy equilibrium except for strategies "all report" and "all conceal". We show that in this strategy, the subordinate t reports if and only if $t \leqslant t^{1,0}$. Since we consider a strategy where there exist reporting subordinate and concealing subordinate, there exists $t^R \in \mathbb{N}$ such that the subordinate in period t^R reports the problem if he detects it and there also exists t^C such that the subordinate in period t^C does not report the problem even if he detects it.

To show the theorem, we prove

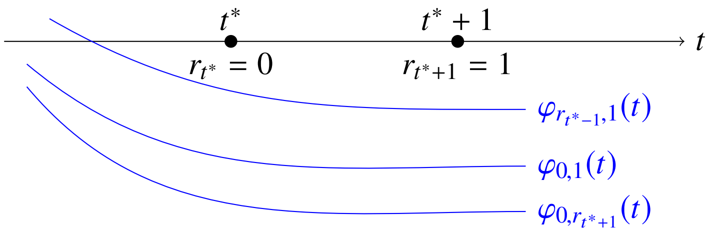

Figure 6: Illustration of Claim 1

Claim 1. *Suppose that the hypothesis of Theorem 1 holds. Let t^R be a period at which the subordinate reports and t^C a period at which the subordinate conceals. Then,*

(i) In each PBE, for each $t \leqslant t^R$, the subordinate in period $t \geqslant 0$ reports the problem.

(ii) In each PBE, for each $t \geqslant t^C$, the subordinate in period $t \geqslant 0$ does not report the problem.

Proof of Claim 1. We show the first part of the claim. Let $t \leqslant t^R$.

Suppose by contradiction that t does not report the problem. Then, there is $t^* \in \{t, t+1, \ldots, t^R - 1\}$ such that subordinate t^* does not report and subordinate t^*+1 reports.

Then, since subordinate t^* does not report at an equilibrium and $r_{t^*+1} = 1$, $\varphi_{r_{t^*-1},1}(t^*) < 0$. Since $\varphi_{r_{t^*-1},1}$ is increasing in r_{t^*-1} and there is no t such that $\varphi_{0,1}(t^*) = 0$, $\varphi_{0,1}(t^*) < 0$. Thus, in $\varphi_{0,1}$, the coefficient of s_{t+1}/s_t is negative. Note that s_{t+1}/s_t is nondecreasing in t and thus, $\varphi_{0,1}(t^*+1) < 0$. Since $\varphi_{0,r_{t^*+1}}(t^*+1)$ is increasing in r_{t^*+1}, $\varphi_{0,r_{t^*+1}}(t^*+1) < 0$ (Figure 6). Therefore, for subordinate t^*+1, not reporting is the unique best response, which is in contradiction with the fact that subordinate t^*+1 reports at equilibrium.

In the same way, we can show that in equilibrium, for each $t \geqslant t^C$, the subordinate in period t does not report the problem even if he detects it. □

By Claim 1, there exists $\hat{t}$ such that for each $t \geqslant \hat{t}$, $r_t = 0$ and for each $t < \hat{t}$, $r_t = 1$.

Suppose that $\varphi_{1,0}(\hat{t}) > 0$. Then, since $r_{\hat{t}+1} = 0$ and $r_{\hat{t}-1} = 1$, reporting is the best response, which is a contradiction. Therefore, $\varphi_{1,0}(\hat{t}) < 0$. Suppose that $\varphi_{1,0}(\hat{t}-1) < 0$. Then, since $r_{\hat{t}} = 0$ and $r_{\hat{t}-2} = 1$, not reporting is the best response, a contradiction. Therefore, $\varphi_{1,0}(\hat{t}-1) > 0 > \varphi_{1,0}(\hat{t})$. Thus, $\hat{t} - 1 = t^{1,0}$. See Figure 7 for the illustration. □

Proof of Proposition 1. Under Assumption 1, first note that for each t, $B_t > A_t$ and $B_t \geqslant$

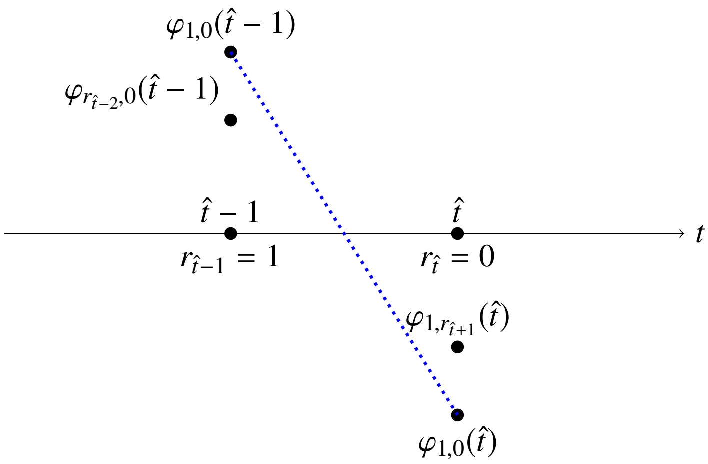

Figure 7: Illustration of Theorem 1.

$(1-q^S)\widetilde{B}_t + q^S A_t$. This is because $B_\tau - A_\tau$ is given by

$$B_\tau - A_\tau = (1-p)(\bar{c}^U q^* - \bar{c}^R)s_t + p(F^R - q^* F^U)SD(s_t).$$

When $q^* = 1$,

$$B_t - A_t = (1-p)(\bar{c}^U - \bar{c}^R)s_t + p(F^R - F^U)SD(s_t) \geqslant 0,$$

since $SD(s_t) \geqslant d^{M,R}s_t$ and $\bar{c}^R - \bar{c}^U = \int_{\frac{pd^{M,U}}{1-p}}^{\frac{pd^{M,R}}{1-p}} c dF(c)$. When $q^* = 0$,

$$B_t - A_t = -(1-p)(\bar{c}^R)s_t + pF^R SD(s_t) \geqslant 0.$$

Since $B_t - A_t$ is a linear function of q^*, $B_t - A_t \geqslant 0$.

Similarly,

$$B_t - q^S A_t - (1-q^S)\widetilde{B}_t = (1-p)q^S(\widetilde{c}^U - \bar{c}^R)s_t + pSD(s_t)q^S(F^R - F^U) \geqslant 0.$$

In the same way, we can show that $\widetilde{B}_t \geqslant B_t$. Then, since $1 - q^* F^U > 1 - F^R$ and $1 - q^M F^U >$

$1 - F^R$,

$$SW_{RCt^*+1} - SW_{RCt^*} > [(1-q^M F^U)(1-q^S)(1-p)\delta]^{t^*+1}$$
$$\times \Bigg[q^S(\widetilde{B}_{t^*+1} - A_{t^*+1}) + (1-q^M F^U)(1-p)\delta(B_{t^*+2} - (1-q^S)\widetilde{B}_{t^*+2} - q^S A_{t^*+2})$$
$$+ \sum_{\tau=t^*+3} q^S((1-q^* F^U)(1-p)\delta)^{\tau-t^*-1}(B_\tau - A_\tau) \Bigg] > 0.$$

□

Proof of Theorem 2. (1) Suppose that there is no equilibrium such that no one reports in problem P. Then, by Lemma 3 (2), $\varphi_{0,0}(t) > 0$ for some $t \in \mathbb{N}$ or $\varphi_{1,0}(0) > 0$.

Case 1. Suppose that $\varphi_{0,0}(t) > 0$ for some $t \in \mathbb{N}$. Then, we first show that $\varphi'_{0,0}(t) > 0$ for each $t \in \mathbb{N}$. Consider the case that $I(0)D^R - D^U \geqslant 0$. Then $\varphi'_{0,0}(t) > 0$ for each $t \in \mathbb{N}$.

Consider the case that $I(0)D^R - D^U < 0$. Then, we have $\widetilde{\varphi}_{0,0}(t) > \varphi_{0,0}(t)$. This implies that $\widetilde{\varphi}_{0,0}(t') > 0$ for each $t' \in \mathbb{N}$.

Since $\varphi'_{r_{t-1},r_{t+1}}(t') > \varphi'_{0,0}(t') > 0$ for each $t' \in \mathbb{N}$. Then, reporting is the dominant strategy for each subordinate t. Therefore, the unique PBE is the strategy that each player reports in problem P'.

Case 2. Suppose that $\varphi_{1,0}(0) > 0$. Then, as in the previous case, we have that $\varphi'_{1,0}(t) > 0$ for each $t \in \mathbb{N}$. Then, for subordinate 0, reporting is a strict dominant strategy. Since $\varphi'_{1,0}(s'_1) > 0$ and $\varphi'_{1,0}(t) < \varphi'_{1,r}(t)$ for each r, for subordinate 1, reporting is the unique best response. Continuing this process shows that reporting is the best response for each subordinate t.

(2) Suppose that there is no equilibrium such that no one conceals in problem P'. Then, by Lemma 3 (1), $\varphi'_{1,1}(t) < 0$ for some t. This implies that $-(1 - I(1))D^R - (1-q)(D^U - D^R) < 0$. Therefore, we have $\varphi'_{1,1} > \varphi_{1,1}$, which implies that $\varphi_{1,1}(t) < 0$ for each $t \in \mathbb{N}$. Then, $\varphi_{r_{t-1},r_{t+1}}(t') < 0$ for each $t' \in \mathbb{N}, r_{t-1}, r_{t+1} \in [0,1]$. This implies that not reporting is the dominant strategy, and thus the unique equilibrium is the strategy such that each player does not report in problem P. □

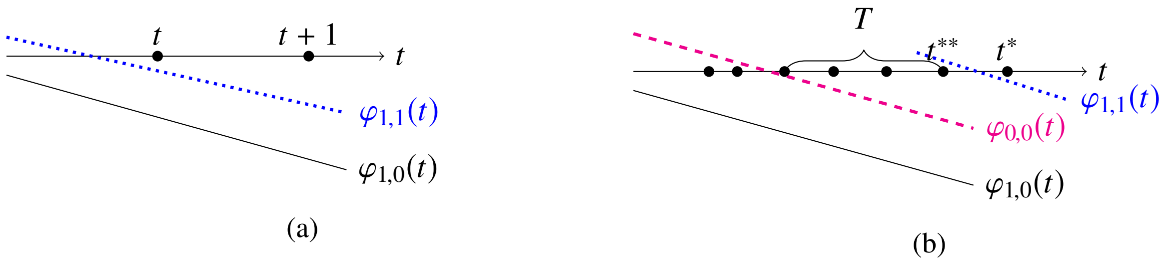

Figure 8: Illustration of Lemma 5

A.3. Proofs in Section 6.1

Proof of Lemma 5. Let $(r_t)_{t\in\mathbb{N}} \in \{0,1\}^{\mathbb{N}}$ be the profile of reporting probabilities in an equilibrium. By assumption, there exists t such that $\varphi_{1,1}(t) < 0$. Suppose that $r_t = 1$ (Figure 8 (a)). Note that $\varphi_{1,1}(t) < 0$ implies that $\varphi_{1,1}(t+1) < 0$ and $\varphi_{1,0}(t+1) < 0$. Therefore, we have $r_{t+1} = 0$. On the other hand, $\varphi_{1,1}(t) < 0$ also implies that $\varphi_{1,0}(t) < 0$ and $\varphi_{0,0}(t) < 0$, we have $r_t = 0$, a contradiction. Therefore, $r_t = 0$. Let $t^* = \min\{t : \varphi_{1,1}(t) < 0\}$. Since $\varphi_{1,1}(t)$ is decreasing in t, $r_t = 0$ for each $t \geqslant t^*$.

Let $T = \{t < t^* : \varphi_{0,0}(t) < 0\}$. If $T = \varnothing$, we are done. Suppose that $T \neq \varnothing$. Let $t^{**} = \max T$ (Figure 8 (b)). Then, since $\varphi_{0,0} < \varphi_{1,1}$, we have $t^{**} = t^* - 1$. Since $\varphi_{1,0}(t^{**}) < 0$, $\varphi_{0,0}(t^{**}) < 0$, and $r_{t^*} = 0$, we have $r_{t^{**}} = 0$. Note that $\varphi_{1,0}(t) < 0$ and $\varphi_{0,0}(t) < 0$ for each $t \in T$. Therefore, $r_{t^{**}-1} = 0$ if $t^{**} - 1 \in T$. Continuing this process yields $r_t = 0$ for each $t \in T$.

Let $t^{***} = \min T$. Since $\varphi_{0,0}(t) < 0$ if and only if $t \geqslant t^{***}$, the above discussion completes the proof. □

Proof of Proposition 2. Note that if $\varphi_{1,0}(t) > 0$, reporting is a strict dominant strategy. Thus, in equilibrium, $r_t = 1$. By Lemma 5, for each t such that $\varphi_{0,0}(t) < 0$, subordinate t does not report. For each t, such that $\varphi_{0,0}(t) < 0$, since $\varphi_{1,0}(t) < 0$, no one has an incentive to deviate.

If $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| = 0$, we are done. Consider the case that $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| = 1$. Let t be an element. Then, $r_{t-1} = 1$ and $r_{t+1} = 0$. Since $\varphi_{1,0}(t) < 0$, not reporting is the best response, which concludes the proof. □

Proof of Proposition 3. See Figure 9 for the illustration. Let $t^* := \max\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}$. By Lemma 5, $r_{t^*+1} = 0$.

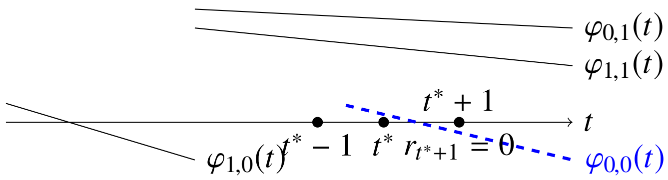

Figure 9: Illustration of Proposition 3.

Suppose that $r_{t^*}=0$. Then, since $\varphi_{0,0}(t^*)>0$, $r_{t^*-1}=1$. Since $|\{t:\varphi_{1,0}(t)<0<\varphi_{0,0}(t)\}|\geqslant 2$, $t^*-1\in\{t:\varphi_{1,0}(t)<0<\varphi_{0,0}(t)\}$. Then, it must be that $r_{t^*-2}=0$ since $\varphi_{1,0}(t^*-1)<0$. However, since $\varphi_{0,1}(t^*-2)>0$ and $\varphi_{1,1}(t^*-2)>0$, the best response for subordinate t^*-2 is to report, which is a contradiction. Suppose that $r_{t^*}=1$. Then, since $\varphi_{1,0}(t^*)<0$, $r_{t^*-1}=0$. However, since $\varphi_{1,1}(t^*-1)>0$ and $\varphi_{0,1}(t^*-1)$, reporting is the best response for subordinate t^*-1, a contradiction. □

Proof of Theorem 4. Let $(r_t)_{t\in\mathbb{N}}$ be the probability to report in an equilibrium. We will prove that each PBE satisfies the properties 1. and 2.

1. Suppose that $\varphi_{0,0}(t)<0$ for some $t\in\mathbb{N}$. Let $\underline{t}:=\min\{t:\varphi_{0,0}(t)<0\}$. We will prove that $r_{\underline{t}}=0$ (See Figure 10 (a)). To do this, suppose that $r_{\underline{t}}=1$. Then, since for each $t>\underline{t}$, $\varphi_{1,1}(t)<0$ and $\varphi_{1,0}(t)<0$, $r_{\underline{t}+1}=0$.

   Then, suppose that $r_{\underline{t}-1}=1$. Since $\varphi_{1,0}(\underline{t})<0$, it contradicts to the supposition that $r_{\underline{t}}=1$ and $r_{\underline{t}+1}=0$. Now suppose that $r_{\underline{t}-1}=0$. Since $\varphi_{0,0}(\underline{t})<0$, it is also a contradiction. Therefore, $r_{\underline{t}}=0$. In the same fashion, we can show that $r_{\underline{t}+j}=0$ for each $j>0$.

2. Let $T=\{t\in\mathbb{N}:\varphi_{1,1}(t)<0<\varphi_{0,0}(t)\}$. Suppose that $|T|\geqslant 2$ (See Figure 10 (b)). Let $t^*=\max T$. Then, $t^*+1=\underline{t}$. We now show that if $r_{t^*-j}=1$, $r_{t^*-j-1}=0$ and if $r_{t^*-j}=0$, $r_{t^*-j-1}=1$ for each $j=0,1,\ldots,|T|-1$.

   We consider the case for $j=0$. Suppose by contradiction that $r_{t^*}=1$ and $r_{t^*-1}=1$. Then, since $\varphi_{1,0}(t^*)<0$ and $r_{t^*+1}=0$, it follows that $r_{t^*}=0$ is the best response, a contradiction.

   Suppose by contradiction that $r_{t^*}=0$ and $r_{t^*-1}=0$. Then, since $\varphi_{0,0}(t^*)>0$ and $r_{t^*+1}=0$, $r_{t^*}=1$ is the best response, a contradiction.

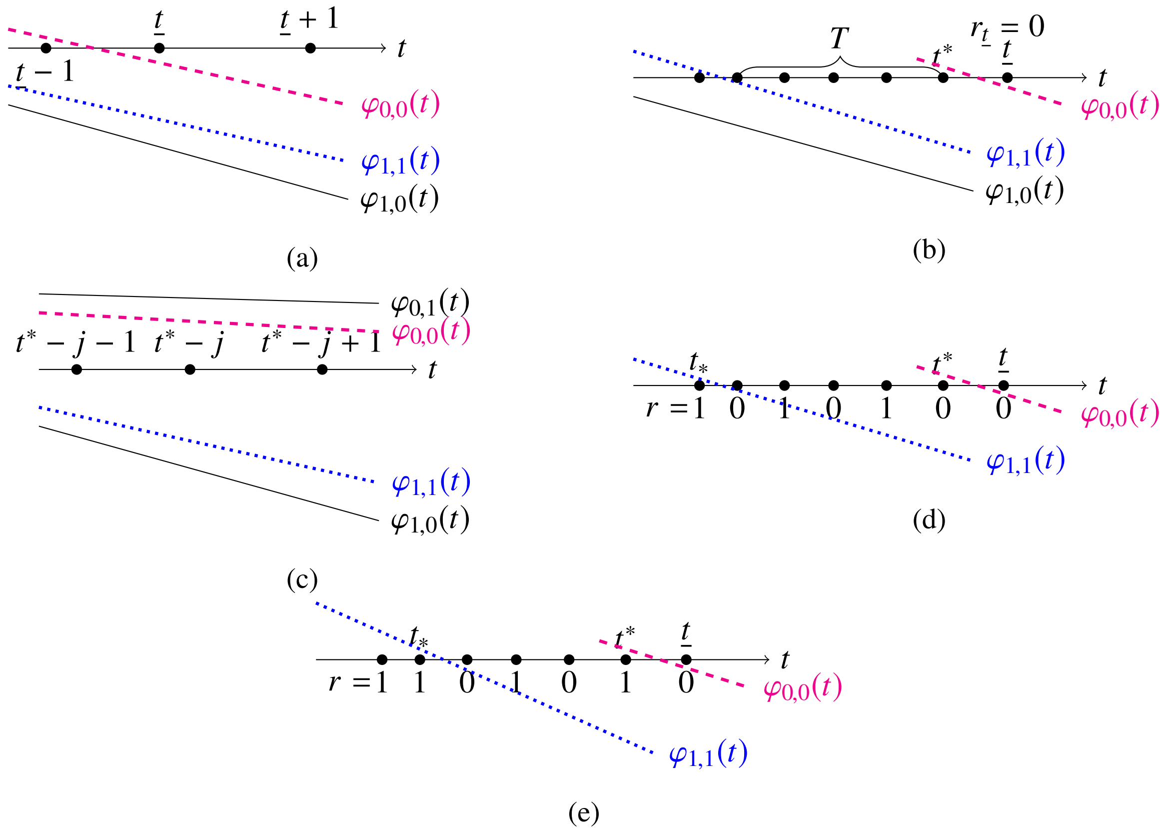

Figure 10: Illustration of Theorem 4.

Suppose that when $j = k$, the statement is true and consider the case for $j = k + 1$ (See Figure 10 (c)). Suppose by contradiction that $r_{t^*-j} = r_{t^*-j-1} = 1$. By induction assumption, since $r_{t^*-j+1} = 0$ and $\varphi_{1,0}(t^* - j - 1) < 0$, $r_{t^*-j} = 0$ is the best response, a contradiction.

Suppose by contradiction that $r_{t^*-j} = r_{t^*-j-1} = 0$. By induction assumption, since $r_{t^*-j+1} = 1$ and $\varphi_{0,1}(t^* - j - 1) > 0$, $r_{t^*-j} = 1$ is the best response, a contradiction.

We now construct a PBE. Let $t_* = \min T - 1$. Consider the following strategy profile:

1. subordinate $t \leqslant t_*$ reports
2. subordinate t_*+2k reports and subordinate t_*+2k+1 conceals for each $0 \leqslant k \leqslant [t^*-t_*]/2$.
3. subordinate $t > t^*$ does not report the problem.

Consider $t > t^*$. Then, under the strategy profile, subordinate $t + 1$ conceals. Since for each $t > t^*$, $\varphi_{1,0} < \varphi_{0,0} < 0$, concealing is the best response.

Consider $t \in (t_*, t^*)$. Let $t = t_* + \ell$. If ℓ is an odd number, since $r_{t-1} = 1$, $r_{t+1} = 1$ and $\varphi_{1,1}(t) < 0$, concealing is the best response. If ℓ is an even number, since $\varphi_{0,0}(t) > 0$, reporting is the best response.

Then, for each t, except for t^* and $t \leqslant t_*$, this strategy profile is a best response against itself. Consider subordinate $t \leqslant t_*$. Since $\varphi_{1,1}(t) > 0$, $r_{-1} = 1$ and subordinate $t' \leqslant t_* + 1$ report, reporting is a best response.

For subordinate t^*, we have the following two cases:

Case 1. Suppose that $|T|$ is an odd number (See Figure 10 (d)). Note that $\varphi_{1,0}(t^*) < 0$. Since $|T|$ is an odd number, subordinate $t^* - 1$ reports and subordinate $t^* + 1$ does not report. Then, not reporting is a best response for subordinate t^*.

Case 2. Suppose that $|T|$ is an even number (See Figure 10 (e)). Note that $\varphi_{0,0}(t^*) > 0$. Since $|T|$ is an even number, subordinate $t^* - 1$ does not report and subordinate $t^* + 1$ does not report. Then, reporting is a best response for subordinate t^* .

Therefore, this strategy profile is a PBE. □

Proof of Proposition 4. Note that

$$\frac{\partial I(0)}{\partial d^{M,R}} = \frac{-\frac{p}{1-p} f\left(w^R\right)}{(q^S + (1-q^S)q^M)\left[1 - F\left(w^U\right)\right] + (1-q^S)(1-q^M)}$$

$$\frac{\partial D^R}{\partial d^{M,R}} = -p \int_{w^R} dF(c) < 0,$$

where $w^R = (b^M + pd^{M,R})/(1-p)$ and $w^U = (b^M + pd^{M,U})/(1-p)$. In this case, since $D^R > 0$, $\frac{\partial \varphi_{0,0}(s)}{\partial d^{M,R}} < 0$. □

Proof of Proposition 5. Differentiating $\varphi_{0,0}$ by b^M yields

$$\frac{\partial \varphi_{0,0}(t)}{\partial b^M} = \delta(1-p)\left(\frac{\partial I(0)}{\partial b^M} D^R + \frac{\partial D^R}{\partial b^M} I(0) - \frac{\partial D^U}{\partial b^M}\right)\frac{s_{t+1}}{s_t}.$$

Note that

$$\frac{\partial I(0)}{\partial b^M} = -\frac{1}{1-p}\frac{f\left(w^R\right)\left[Q\left[1-F\left(w^U\right)\right] + Q'\right] - Qf\left(w^U\right)\left[1 - F\left(w^R\right)\right]}{Q\left[1-F\left(w^U\right)\right] + Q'}$$

$$\frac{\partial D^R}{\partial b^M} = F(w^R), \quad \frac{\partial D^U}{\partial b^M} = F(w^U),$$

where $Q = (q^S + (1-q^S)q^M)$ and $Q' = (1-q^S)(1-q^M)$. Note that the probability $I(0)$ is maximized when $q^S = 1$ and minimized when $q^S = q^M = 0$. Then, we have

$$\frac{\partial I(0)}{\partial b^M} D^R + \frac{\partial D^R}{\partial b^M} I(0) - \frac{\partial D^U}{\partial b^M} < \frac{\partial I(0)}{\partial b^M} D^R + F(w^R)\frac{1-F(w^R)}{1-F(w^U)} - F(w^U), \quad \text{and}$$

$$\frac{\partial I(0)}{\partial b^M} D^R + \frac{\partial D^R}{\partial b^M} I(0) - \frac{\partial D^U}{\partial b^M} > \frac{\partial I(0)}{\partial b^M} D^R + F(w^R)(1-F(w^R)) - F(w^U).$$

Since $f(x)/(1-F(x))$ is increasing in x and $w^R > w^U$, $\frac{\partial I(0)}{\partial b^M}$ is negative. Then, if $D^R > 0$ and $F(w^R)(1-F(w^R)) < F(w^U)(1-F(w^U))$, $\frac{\partial \varphi_{0,0}(s)}{\partial b^M} < 0$. Conversely, if $D^R < 0$ and $F(w^R)(1-F(w^R)) > F(w^U)$, $\frac{\partial \varphi_{0,0}(s)}{\partial b^M} > 0$. □

Proof of Theorem 5. Suppose that there is no PBE such that no one reports in problem P. Thus, some subordinate reports. If the subordinate is subordinate 0, $\varphi_{1,0}(0) \geqslant 0$. Consider the case that $\varphi_{1,0}(0) < 0$. We now show that for some $t \in \mathbb{N}$, $\varphi_{0,0}(t) \geqslant 0$. Suppose by contradiction that for each t, $\varphi_{0,0}(t) < 0$. Then, since subordinate 0 conceals, there is a PBE such that each subordinate t conceals, a contradiction.

Therefore, for some $t \in \mathbb{N}$, $\varphi_{0,0}(t) > 0$ or $\varphi_{1,0}(0) > 0$.

Case 1. Suppose that for some $t \in \mathbb{N}$, $\varphi_{0,0}(t) \geqslant 0$. Then, as in the proof of Theorem 2, $\varphi'_{0,0}(t) > 0$ for each t. Then, we have $\varphi'_{0,1}(t) > 0$ for each t since $\varphi'_{0,1} > \varphi'_{0,0}$. Since $\varphi_{0,0} \leqslant \varphi_{1,1}$, $\varphi'_{1,1}(t) > 0$ for each t, which implies that the strategy profile that each player reports the problem is a PBE.

Suppose by contradiction that there is a pure-strategy PBE such that a subordinate, say subordinate t, conceals the problem. Then, it must be $\varphi'_{1,0}(t) < 0$. Let the equilibrium strategy profile of subordinates be denoted by $(r_t)_{t=0}^{\infty}$. Suppose that $r_{t+1} = 1$. Then, since $\varphi'_{0,1}(t) > 0$ and $\varphi'_{1,1}(t) > 0$, the best response is $r_t = 1$, a contradiction. Suppose that $r_{t+1} = 0$. Consider the case that $r_{t+2} = 1$. Then, since $\varphi'_{1,1}(t+1) > 0$, we also have a contradiction. Therefore, we consider the case that $r_{t+2} = 0$. However, in this case, since $\varphi'_{0,0}(t) > 0$ for each s, $\varphi'_{0,0}(t+1) > 0$. Thus, the best response for subordinate $t+1$ is $r_{t+1} = 1$, a contradiction.

Case 2. Suppose that $\varphi_{1,0}(0) \geqslant 0$. Then, since $\varphi'_{1,0}(t) < \varphi'_{1,r}(t)$ for each $r > 0$, as in the proof of Theorem 2, each subordinate reports the problem in PBE.

The latter case is shown in the same way. □

A.3.1. Equilibrium with mixed strategy

In this subsection, we consider equilibria with mixed strategy for the case that $\varphi_{1,1} > \varphi_{0,0}$. We first prove Theorem 3.

Proof of Theorem 3. Let $T_{10,00} = \{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}$. We show the case for $|T_{10,00}| \leqslant 1$ in Proposition 2. We first consider the case that $|T_{10,00}| = 2$ (See Figure 11 (a)). Let $T_{10,00} = \{t_*, t_* + 1\}$. Note that for each $t < t_*$, $r_t = 1$ is the dominant strategy. Now let r_{t_*}, r_{t_*+1} be the numbers that satisfy $\varphi_{1,r_{t_*+1}}(t_*) = 0$, $\varphi_{r_{t_*},0}(t_* + 1) = 0$. Since $\varphi_{r,r'}$ is continuous in r, r' and it holds that $t_*, t_* + 1 \in T_{10,00}$, such numbers exist. Let (r_t) be the strategy profile such that $r_t = 1$ for each $t < t_*$ and $r_t = 0$ for each $t > t_* + 1$. Then, (r_t) is an equilibrium.

As an induction assumption, we suppose that there is an equilibrium when $|T_{10,00}| = k$ such that for each $t < \min T_{10,00}$, $r_t = 1$ and $t > \max T_{10,00}$, $r_t = 0$. Consider the case that

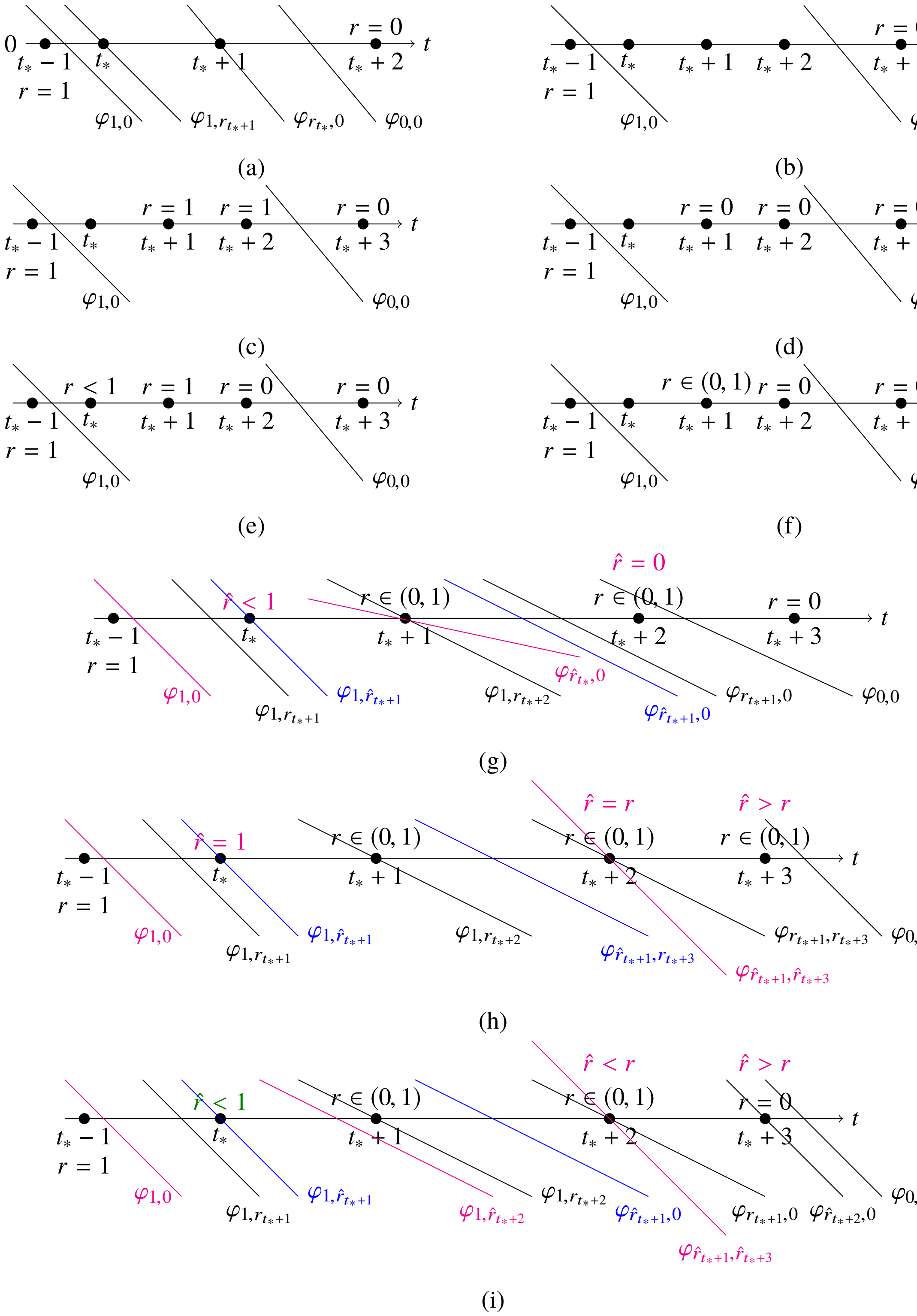

Figure 11: Illustration of Theorem 3.

$|T_{10,00}| = k + 1$.

Suppose by contradiction that there is no equilibria when $|T_{10,00}| = k+1$. Let $t_* = \min T_{10,00}$. Since $|T_{10,00} \setminus \{t_*\}| = k$, if $r_{t_*} = 1$, we can constructs an equilibrium $(r_t)_{t=t_*+1}^{\infty}$ after period t_* such that for each $t \geqslant t_* + k + 1$, $r_t = 0$. Let $M \neq \varnothing$ be the set of equilibrium strategy after r_{t_*} by assuming that $r_{t_*} = 1$. Therefore, $r_{t_*} = 1$ is not the best response for $(r_t)_{t=t_*+1}^{\infty}$ and $r_t = 1$ for each $t < t_*$, which implies that $\varphi_{1,r_{t_*+1}}(t_*) < 0$ for each $r \in M$.

Note that $t_* + k \in T_{10,00}$ but $t_* + k + 1 \notin T_{10,00}$. Recall that $r_{t_*+k+1} = 0$.

Case 1. Suppose that $r_{t_*+k} = 1$ (See Figure 11 (c)). Then, since $\varphi_{r',1}(t) > 0$ for each $t \in T_{10,00}$, $r_{t_*+k-1} = 1$. However, since $\varphi_{1,0}(t_* + k - 1) < 0$, the best response is $r_{t_*+k} = 0$, a contradiction.

Case 2. Suppose that $r_{t_*+k} = 0$. Suppose also that $r_{t_*+k-1} = 0$ (See Figure 11 (d)). Then, since $\varphi_{0,0}(t_* + k) > 0$, the best response is $r_{t_*+k} = 1$, a contradiction.

Suppose that $r_{t_*+k-1} = 1$ (See Figure 11 (e)). Then, since $\varphi_{1,0}(t_* + k - 1) < 0$, we must have $r_{t_*+k-2} < 1$. However, since $\varphi_{r,1}(t_* + k - 2) > 0$, it should be $r_{t_*+k-2} = 1$, a contradiction.

Therefore, we have $r_{t_*+k-1} \in (0, 1)$, that is, $\varphi_{r_{t_*+k-2},r_{t_*+k}}(t_* + k - 1) = \varphi_{r_{t_*+k-2},0}(t_* + k - 1) = 0$. Since there is no $t \in T_{10,00}$ such that $\varphi_{1,0}(t) = 0$, $\varphi_{0,0}(t) = 0$, $\varphi_{1,1}(t) = 0$, or $\varphi_{0,1}(t) = 0$, $r_{t_*+k-2} \in (0, 1)$ (See Figure 11 (f)).

Case 2-1. Suppose that $r_{t_*+k-3} = 1$. Then, for each $j < k - 2$, let $r_{t_*+j} = 1$. Then $r = (r_t)_{t=0}^{\infty}$ is an equilibrium, which contradicts the assumption that there is no equilibrium.

Case 2-2. Suppose that $r_{t_*+k-3} = 0$, then, since $\varphi_{0,r'}(t) > 0$ for each $r' \in [0, 1]$ and $t \in T_{10,00}$, $r_{t_*+k-2} = 1$ is the best response, a contradiction.

Therefore, we have $r_{t_*+k-3} \in (0, 1)$. Continuing this process, $r_{t_*+j} \in (0, 1)$ for each $j = 1, \ldots, k - 2$.

Case 3. Suppose that $r_{t_*+k} \in (0, 1)$, then, since $\varphi_{r_{t_*+k-1},0}(t_* + k - 1) = 0$, $r_{t_*+k-2} \in (0, 1)$ and as in the case 2, we have $r_{t_*+j} \in (0, 1)$ for each $j = 1, \ldots, k - 1$.

By cases 1,2 and 3, we have, $\varphi_{r_{t_*+j-1},r_{t_*+j+1}}(t_* + j) = 0$ for each $j = 1, \ldots, k - 1$. We also have that $\varphi_{r_{t_*+k-1},r_{t_*+k+1}}(t_* + k) \leqslant 0$.

Recall that $\varphi_{1,r_{t_*+1}}(t_*) < 0$ for each $r \in M$. Now let $\hat{r}_{t_*+1}$ be the number that satisfies $\varphi_{1,\hat{r}_{t_*+1}}(t_*) = 0$. Then, $\hat{r}_{t_*+1} > r_{t_*+1}$, and thus $\varphi_{\hat{r}_{t_*+1},r_{t_*+3}}(t_* + 2) < 0$. Then, since $\varphi_{r,1}(t_* +$

2) > 0 for each $r \in [0,1]$, there is $\hat{r}_{t_*+3} > r_{t_*+3}$ such that $\varphi_{\hat{r}_{t_*+1},\hat{r}_{t_*+3}}(t_*+2) = 0$. In turn, $\varphi_{\hat{r}_{t_*+3},r_{t_*+5}}(t_*+4) < 0$. Let k^* be the largest even number less than k. Continuing this process, for each $j = 2, 4, \ldots, k^*$ there exist $\hat{r}_{t_*+1}, \hat{r}_{t_*+3}, \ldots, \hat{r}_{t_*+k^*+1}$ such that $\varphi_{\hat{r}_{t_*+j-1},\hat{r}_{t_*+j+1}}(t_*+j) = 0$.

Consider the case that k is an even number. Then, $k^* = k-2$. We now construct an equilibrium profile $(\hat{r})_{t=0}^{\infty}$. Let $\hat{r}_t = 1$ for each $t < t_*$ and $\hat{r}_t = 1$ for each $t > t_* + k$. Let $\hat{r}_k = r_k$. We have two cases.

(i) Suppose that $r_{t_*+k} > 0$ (See Figure 11 (g)). Then, $\varphi_{r_{t_*+k-1},0}(t_*+k) = 0$, and thus, $\varphi_{\hat{r}_{t_*+k-1},0}(t_*+k) < 0$. Then, let $\hat{r}_{t_*+k} = 0$. Since $\varphi_{r_{t_*+k-2},r_{t_*+k}}(t_*+k-1) = 0$, $\varphi_{r_{t_*+k-2},0}(t_*+k-1) < 0$. Then there exists $\hat{r}_{t_*+k-2} < r_{t_*+k-2}$ such that $\varphi_{\hat{r}_{t_*+k-2},0}(t_*+k-1) = 0$. Then, in turn, $\varphi_{r_{t_*+k-4},\hat{r}_{t_*+k-2}}(t_*+k-3) < 0$. Continuing this process, there exist $\hat{r}_{t_*}, \hat{r}_{t_*+2}, \ldots, \hat{r}_{t_*+k-1}$ for each $j = 2, 4, \ldots, k$ such that $\varphi_{\hat{r}_{t_*+j-2},\hat{r}_{t_*+j}}(t_*+j-1) = 0$. Then, $(\hat{r}_t)_{t=0}^{\infty}$ satisfies equilibrium conditions.

(ii) Suppose that $r_{t_*+k} = 0$. Since $\varphi_{r_{t_*+k-2},0}(t_*+k-1) < 0$, then $\varphi_{\hat{r}_{t_*+k-2},0}(t_*+k-1) < 0$. For each $j = 2, 4, \ldots, k$, let $\hat{r}_{t_*+j} = r_{t_*+j}$. Then, since $\varphi_{r_{t_*+j-1},r_{t_*+j+1}}(t_*+j) = 0$, we also have $\varphi_{\hat{r}_{t_*+j-1},\hat{r}_{t_*+j+1}}(t_*+j) = 0$. Then, $\hat{r}$ satisfies equilibrium conditions.

Consider the case that k is odd (See Figure 11 (h) for the case that $r_{t_*+k} \in (0,1)$ and (i) for the case of $r_{t_*+k} = 0$). Then, $k^* = k-1$. Then, since $\varphi_{r_{t_*+k-1},0}(t_*+k) \leqslant 0$, there is $\hat{r}_{t_*+k-1} \leqslant r_{t_*+k-1}$ such that $\varphi_{\hat{r}_{t_*+k-1},0}(t_*+k) = 0$. Then, in turn, $\varphi_{r_{t_*+k-3},\hat{r}_{t_*+k-1}}(t_*+k-2) \leqslant 0$. Since $\varphi_{0,\hat{r}_{t_*+k-1}}(t_*+k-2) > 0$, we can find $\hat{r}_{t_*+k-3} \leqslant r_{t_*+k-3}$ such that $\varphi_{\hat{r}_{t_*+k-3},\hat{r}_{t_*+k-1}}(t_*+k-2) = 0$. Continuing this process, there exist numbers $\hat{r}_{t_*}, \hat{r}_{t_*+2}, \ldots, \hat{r}_{t_*+k-2}$ such that $\varphi_{\hat{r}_{t_*+j-2},\hat{r}_{t_*+j}}(t_*+j+1) = 0$ for each $j = 2, 4, \ldots, k-1$. Then, $\hat{r}$ satisfies equilibrium conditions.

Thus, in each case, we can construct an equilibrium strategy, a contradiction. □

The following propositions show the properties of equilibria with mixed strategy. Combining Theorem 3 and the following proposition, it follows that if $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$,[16] we can characterize the equilibrium with mixed strategy.

Proposition 9. *Suppose that $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$. Then, in each equilibrium, for each t such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.*

[16] Note that $\varphi_{0,1}(t) \leqslant 0$ implies that $\varphi_{1,1}(t) \leqslant 0$.

Proof of Proposition 9. Suppose that $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$. Then, there exists t such that $\varphi_{0,1}(t) < 0$ and thus, for each r, r', $\varphi_{r,r'}(t) < 0$. This implies that for the subordinate in period t, not reporting is a strictly dominant strategy. Since $\varphi_{0,1}(t)$ is decreasing in t, for each $t' > t$, subordinate in period t' conceals in equilibrium.

Let $T := \{t' : \varphi_{0,1}(t') \geqslant 0 \text{ and } \varphi_{0,0}(t') < 0\}$. If $T = \emptyset$, we are done. Suppose that $T \neq \emptyset$. Consider $t^* := \max T$. Then, we have $\varphi_{0,1}(t^* + 1) < 0$. Thus, $r_{t^*+1} = 0$. Since $\varphi_{0,0}(t) \geqslant \varphi_{1,0}(t)$, $\varphi_{1,0}(t^*) < 0$. Therefore, not reporting is a best response for the subordinate in period t^*. Thus, we have $r_{t^*} = 0$. Continuing this process, we have that for each $t \in T$, subordinate t does not report in equilibrium. $\square$

The above proposition needs the assumption that $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$. If the assumption is violated, the equilibrium has the following properties.

Proposition 10. *Suppose that $\varphi_{1,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$ and $\varphi_{0,1}(t) \geqslant 0$ for each $t \in \mathbb{N}$. Let $t^* := \min\{t : \varphi_{1,1}(t) < 0\}$. Then, in each equilibrium, the following statements hold.*

(1) Suppose that $r_{t^*} = 0$. Then, for each $t < t^*$ such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.

(2) Suppose that $r_{t^*} \neq 0$. Then, for each t such that $\varphi_{1,1}(t) < 0$, subordinate t completely mixes reporting and concealing.

Proof of Proposition 10. (1) This claim is shown in the same way as Proposition 9.

(2) Let $t^* := \min\{t : \varphi_{1,1}(t) < 0\}$. Suppose that the subordinate in period t^* reports with probability 1, namely $r_{t^*} = 1$. Then, since $\varphi_{1,1}(t^*) < 0$, for each r and $t' > t^*$, $\varphi_{1,r}(t') < 0$. Therefore, $r_{t^*+1} = 0$. On the other hand, since $\varphi_{0,0}(t^*) < 0$, $\varphi_{r,0}(t^*) < 0$ for each $r \in [0, 1]$. Therefore, not reporting is the best response, a contradiction.

Suppose that the subordinate in period t^* reports with probability $r_{t^*} < 1$. This implies that $\varphi_{r_{t^*-1}, r_{t^*+1}}(t^*) = 0$. If $r_{t^*+1} = 0$, $\varphi_{r_{t^*-1}, r_{t^*+1}}(t^*) < 0$, a contradiction. If $r_{t^*-1} = 1$, it also yields a contradiction. Therefore, $r_{t^*+1} > 0$ and $r_{t^*-1} < 1$. We consider the following three cases:

1. $r_{t^*+1} = 1$ and $r_{t^*-1} = 0$,

2. $r_{t^*+1} = 1$ and $r_{t^*-1} > 0$,

3. $r_{t^*+1} < 1$.

Case 1. Suppose that $r_{t^*+1} = 1$ and $r_{t^*-1} = 0$, then, since $\varphi_{0,1}(t) > 0$ for each $t \in \mathbb{N}$, a contradiction.

Case 2. Suppose that $r_{t^*+1} = 1$ and $r_{t^*-1} > 0$. Then, $\varphi_{r_{t^*}, r_{t^*+2}}(t^* + 1) > 0$. Since $r_{t^*+1} = 1$ and $\varphi_{1,r}(t^* + 2) < 0$, $r_{t^*+2} = 0$. Thus, since $\varphi_{0,0}(t^* + 1) < 0$ and $\varphi_{1,0}(t^* + 1) < 0$, not reporting is the best response for subordinate $t^* + 1$, a contradiction.

Case 3. Suppose that $r_{t^*+1} < 1$. Since $r_{t^*+1} > 0$, $\varphi_{r_{t^*}, r_{t^*+2}}(t^* + 1) = 0$. Then, $r_{t^*+2} > 0$ since $\varphi_{r_{t^*},0}(t^* + 1) < 0$. If $r_{t^*+2} = 1$, as in case 2, we have a contradiction. Continuing this process, for each $t' > t^*$, $\varphi_{r_{t'-1}, r_{t'+1}}(t') = 0$. □

A.4. Proofs in Section 6.2

Proof of Lemma 6. When the problem is reported, the manager in period t's objective is

$$\max_{\rho} \rho b^M s_t + (1 - \rho)(-pd^{M,R})s_t - c\chi(\rho)s_t,$$

equivalently

$$\max_{\rho} \rho b^M + (1 - \rho)(-pd^{M,R}) - c\chi(\rho). \tag{4}$$

By the Karush–Kuhn–Tucker condition, the optimal probability $\rho^*(c)$ satisfies

$$b^M + pd^{M,R} - \chi'(\rho^*(c))c + \lambda = 0, \lambda\rho^*(c) = 0,$$

for some nonnegative real number λ. Since χ'^{-1} is increasing, if $b^M + pd^{M,R} < \chi'(0)c$, it must be $\lambda > 0$. Therefore, $\rho^*(c) = 0$. On the other hand, if $b^M + pd^{M,R} \geqslant \chi'(0)c$, $\lambda = 0$. Thus, problem (4) has an interior solution. Therefore,

$$\rho^*(c) = \begin{cases} 0 & \text{if} \quad b^M + pd^{M,R} < \chi'(0)c, \\ \chi'^{-1}((b^M + pd^{M,R})/c) & \text{if} \quad b^M + pd^{M,R} \geqslant \chi'(0)c. \end{cases}$$

Since χ'^{-1} is increasing, $\rho^*(c)$ is decreasing in c. Then, the expected utility of facing the reported problem as a manager is

$$\widetilde{D}^R = \int [\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - c\chi(\rho^*(c))]\, dF(c).$$

Consider the manager's problem when the manager knows the problem before the subordinate's report. If he ignores the problem and the problem is unreported, the expected utility is $-pd^{M,U}$. If he ignores the problem and the problem is reported, the expected utility is $\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c$. Let r be the probability that his subordinate reports the problem. Then, the expected utility of ignoring the problem is

$$r[\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - c\chi(\rho^*(c))] + (1-r)[-pd^{M,U}].$$

On the other hand, if he decides not to ignore the problem, that is, $\rho > 0$, the expected utility is $\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c$. Thus, the manager ignores the problem if and only if $\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c < -pd^{M,U}$.

As in the previous section, let $\widetilde{D}^U$ be the expected utility of facing the unreported problem, that is,

$$\widetilde{D}^U = \int [\max\{\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - c\chi(\rho^*(c)), -pd^{M,U}\}]\, dF(c).$$

It is easy to show that $\widetilde{D}^U \geqslant \widetilde{D}^R$. Let c^* be the number that solves $\rho^*(c^*)b^M + (1 - \rho^*(c^*))(-pd^{M,R}) - \chi(\rho^*(c^*))c^* = -pd^{M,U}$. Note that by the envelope theorem, $\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - c\chi(\rho^*(c))$ is nonincreasing in c. Then, c^* is uniquely determined and $\widetilde{D}^U$ is written as

$$\begin{aligned}\widetilde{D}^U = &\int^{c^*} [\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c]\, dF(c)\\ &-pd^{M,U}(1-F(c^*)).\end{aligned}$$

Therefore, the probability that the reported problem remains unsolved in the next period, which is denoted by $\widetilde{I}(r_{t-1})$ is

$$\widetilde{I}(r_{t-1}) := \Bigg((q^S(1-r_{t-1}) + (1-q^S)q^M) \int_{c^*} (1-\rho^*(c))\, dF(c) \\ + (1-q^S)(1-q^M)\left(\int (1-\rho^*(c))\, dF(c)\right)\Bigg) \\ \times \left((q^S(1-r_{t-1}) + (1-q^S)q^M)(1-F(c^*)) + (1-q^S)(1-q^M)\right)^{-1}$$

Note that $\widetilde{I}$ is nonincreasing in r_{t-1}.

Consider the subordinate's behavior. The expected utility of reporting is

$$\delta \widetilde{I}(r_{t-1})\widetilde{D}^R s_{t+1} + b^S s_t.$$

The expected utility of not reporting is

$$-pd^S s_t + \delta\left[p\widetilde{D}^R s_{t+1} + (1-p)(q^S r_{t+1}\widetilde{D}^R s_{t+1} + (1-q^S r_{t+1})\widetilde{D}^U s_{t+1})\right].$$

Then, calculating the difference of these equation yields $\widetilde{\varphi}_{r_{t-1},r_{t+1}}$. □

Proof of Theorem 6. Recall that

$$\widetilde{\varphi}_{1,0}(t) := \delta(1-p)\left[(\widetilde{I}(r_{t-1}) - 1)\widetilde{D}^R - (1-p)(\widetilde{D}^U - \widetilde{D}^R)\right]\frac{s_{t+1}}{s_t} + b^S + d^S.$$

Then,

$$\frac{\partial \widetilde{\varphi}_{1,0}(t)}{\partial d^{M,R}} = \delta(1-p)\left[\frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R + (\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} - \frac{\partial \widetilde{D}^U}{\partial d^{M,R}}\right]\frac{s_{t+1}}{s_t}.$$

Consider the case that $d^{M,R} \to \infty$. Then, for sufficiently large $d^{M,R}$, for each $c \in (0, \bar{c})$, $[b^M + pd^{M,R}]/c > \chi'(0)$. Therefore, the maximization problem (4) has an interior solution. Therefore by the first order condition, we have $\chi'(\rho^*(c)) = [b^M + pd^{M,R}]/c$. Then, since $\lim_{\rho\to\bar{\rho}} \chi'(\rho) = \infty$, as $d^{M,R} \to \infty$, $\rho^*(c) \to \bar{\rho}$. This implies that $\chi(\rho^*(c)) \to \infty$. Then, we have that for each $c > 0$, as $d^{M,R} \to \infty$, $\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c \to \infty$.

Then, for sufficiently large $d^{M,R}$, $c^* < 0$. This implies that for sufficiently large $d^{M,R}$, since $c^* < 0$ is not in the support of F, $f(c^*) = 0$. Therefore, for sufficiently large $d^{M,R}$,

$$\frac{\partial \widetilde{I}(r)}{\partial d^{M,R}} = -\int \frac{\partial \rho^*(c)}{\partial d^{M,R}}\, dF(c).$$

This also implies that $\partial \widetilde{D}^U / \partial d^{M,U} = 0$ for sufficiently large $d^{M,R}$. Therefore, to determine the sign of $\frac{\partial \widetilde{\varphi}_{1,0}(s)}{\partial d^{M,R}}$, we consider only

$$\frac{\partial \widetilde{I}(1)}{\partial d^{M,R}} \widetilde{D}^R + (I(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}}.$$

(1) We consider the case that $\bar{\rho} < 1$. We write $\widetilde{D}^R$ as a function of $d^{M,R}$ explicitly, that is $\widetilde{D}^R(d^{M,R})$. Since $\frac{\partial \widetilde{D}^R(d^{M,R})}{\partial d^{M,R}} = -p\int (1 - \rho^*(c))\, dF(c) < -p$, $|\widetilde{D}^R(d^{M,R}) - \widetilde{D}^R(0)| < pd^{M,R}$. Therefore, $|\widetilde{D}^R(d^{M,R})| < pd^{M,R} + |\widetilde{D}^R(0)|$.

To verify $\lim_{d^{M,R}\to\infty} \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}} \widetilde{D}^R(d^{M,R})$, we consider $\partial \rho^*(c)/\partial d^{M,R}$. Note that $\frac{\partial \rho^*(c)}{\partial d^{M,R}} > 0$. To show this, recall that $\rho^*(c) = \chi'^{-1}([b^M + pd^{M,R}]/c)$. Since χ'^{-1} is increasing, $\frac{\partial \rho^*(c)}{\partial d^{M,R}} > 0$. We also write $\rho^*(c)$ as a function of $d^{M,R}$, $\rho^*(c, d^{M,R})$. Note that

$$\begin{aligned}\rho^*(c, d^{M,R}) - \rho^*(c, d^{M,R}/2) &= \int_{d^{M,R}/2}^{d^{M,R}} (\partial \rho^*(c, d')/\partial d')\, dd' \\ &\geqslant \min_{d\in[d^{M,R}/2, d^{M,R}]} d^{M,R}(\partial \rho^*(c, d)/\partial d^{M,R})/2.\end{aligned}$$

Since $\rho^*(c, d^{M,R}) \to \bar{\rho}$ for each c as $d^{M,R} \to \infty$, $\rho^*(c, d^{M,R}) - \rho^*(c, d^{M,R}/2) \to 0$ as $d^{M,R} \to \infty$. Therefore,

$$\lim_{d^{M,R}\to\infty} \min_{d\in[d^{M,R}/2, d^{M,R}]} \frac{d^{M,R}}{2} \frac{\partial \rho^*(c, d)}{\partial d^{M,R}} = 0.$$

Since

$$\left| \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}} \widetilde{D}^R(d^{M,R}) \right| < \max_c \frac{\partial \rho^*(c, d)}{\partial d^{M,R}} (pd^{M,R} + |\widetilde{D}^R(0)|),$$

the RHS converges to 0 as $d^{M,R} \to \infty$.

Consider $\widetilde{I}(1)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}}$. Since $c^* < 0$ for sufficiently large $d^{M,R}$, $\widetilde{I}(1) = \int (1 - \rho^*(c))\, dF(c)$. Then,

$$(\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} = -p\left[\int (1 - \rho^*(c, d^{M,R}))\, dF(c) - p\right]\left[\int (1 - \rho^*(c, d^{M,R}))\, dF(c)\right].$$

Therefore, if $\bar{\rho} < 1 - p$, since for each c, $\rho^*(c) \to \bar{\rho}$, $(\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} < 0$ and thus, $\frac{\partial \widetilde{\varphi}_{1,0}}{\partial d^{M,R}} < 0$. On the other hand, if $\bar{\rho} > 1 - p$, $(\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} > 0$ and thus, $\frac{\partial \widetilde{\varphi}_{1,0}}{\partial d^{M,R}} > 0$. □

Proof of Proposition 6. Under the assumption, $\widetilde{D}^U = 0$. Note that if $q^M = 0$, $\widetilde{I}(1) = \int (1 - \rho^*(c))\, dF(c)$. Then,

$$\widetilde{\varphi}_{1,0}(t) = \delta(\widetilde{I}(1) - p)\widetilde{D}^R \frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Note that since $\chi(\rho) = \rho/(\bar{\rho} - \rho)$, by the Karush–Kuhn–Tucker condition,

$$\rho^*(c) = \max\left\{\bar{\rho} - \sqrt{\bar{\rho}\frac{c}{b^M + pd^{M,R}}}, 0\right\}.$$

The first-order derivative is

$$\delta\left[(\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} + \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R\right]\frac{s_{t+1}}{s_t}.$$

Consider $(\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} + \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R$. This is calculated as

$$\begin{aligned}(\widetilde{I}(1) - p)&\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} + \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R \\ &= -p(1 - \bar{\rho})(1 - \bar{\rho} - p) - p\sqrt{\bar{\rho}}\tau A\left(\frac{\mu A^2}{2} + 1 - \bar{\rho} - p + 1 - \frac{pd^{M,R}}{2(b^M + pd^{M,R})} - \frac{\bar{\rho}}{2}\right), \qquad (5)\end{aligned}$$

where $A = 1/(b^M + pd^{M,R})^{1/2}$, $\tau = \int^{\bar{\rho}/A^2} \sqrt{c}\, dF(c)$ and $\mu = \int^{\bar{\rho}/A^2} c\, dF(c)$. Then, if $1 - p > \bar{\rho}$, the first-order derivative (5) is negative. □

A.5. Proofs in Section 6.3

Proof of Lemma 7. Consider a manager in period t who is reported a problem. Let $\xi(s)$ be the probability that the manager $t+1$ ignores the problem. Then, the manager's expected utility of ignoring the problem is

$$-p(d^{M,R}s_t + cs_t) - (1-p)p\delta\xi(s_t)(1-\mu)d^R G(s_t).$$

The manager in period t solves the problem if and only if

$$\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(s_t)(1-\mu)d^R[G(s_t)/s_t]}{1-p} \geqslant c$$

Therefore, the probability that a manager ignores, ξ, satisfies

$$\xi(s) = H(\xi)(s) := \left[1 - F\left(\frac{b^M + pd^{M,R} + \delta(1-p)\xi(G(s))(1-\mu)d^R \frac{G^2(s)}{G(s)}}{1-p}\right)\right]. \tag{6}$$

Then, if there is a ξ that satisfies the above condition, the managers' behavior is determined.

Claim 2. *Suppose that $G(s)/s$ and f are well defined on $\mathbb{R}_+$ and $\sup_{s\in\mathbb{R}_+} G(s)/s < \infty$. Then, there exists a function $\xi\colon \mathbb{R}_+ \to [0,1]$ that satisfies (6).*

Proof of Claim 2. To show the existence of ξ that satisfies (6), we show the existence of a fixed point of H.

Since each $[0,1]$ is a nonempty convex compact set, $[0,1]^{\mathbb{R}_+}$ is a convex set and by the Tychonoff theorem, $[0,1]^{\mathbb{R}_+}$ is a compact set under the product topology. Let $\mathcal{O}$ be the product topology of $\mathbb{R}^{\mathbb{R}_+}$. To show the existence of a fixed point of H, we use

Fact 2 (Aliprantis and Border 2006, p.206)**.** *$(\mathbb{R}^{\mathbb{R}_+}, \mathcal{O})$ is locally convex Hausdorff space.*

Fact 3 (Brouwer–Schauder–Tychonoff's fixed point theorem, Aliprantis and Border 2006, p.583)**.** *Let C be a nonempty compact convex subset of locally convex Hausdorff space, and let $f\colon C \to C$ be a continuous function. Then f has a fixed point.*

Therefore, we only to show that H is continuous on $[0,1]^{\mathbb{R}_+}$. To show this, let $\xi, \xi' \in [0,1]^{\mathbb{R}_+}$. Note that the product topology is generated by the family of seminorms $(|h(s)|)_{s\in\mathbb{R}_+}$ for each $h \in [0,1]^{\mathbb{R}_+}$. Note also that by the mean value theorem, there exists $\widetilde{\xi} \in [0,1]$ such that

$$\begin{aligned}
&|H(\xi)(s) - H(\xi')(s)| \\
&= \delta p(1-\mu)d^R \frac{G^2(s)}{G(s)} f\left(\frac{b^M + pd^{M,R} + \delta(1-p)p\widetilde{\xi}(1-\mu)d^R \frac{G^2(s)}{G(s)}}{1-p}\right) \\
&\quad \times |\xi(G(s)) - \xi'(G(s))|.
\end{aligned}$$

Then, since $G(s)/s$ is bounded above, when $\xi'(s) \to \xi(s)$ for each s, $H(\xi')(s) \to H(\xi)(s)$ for each s. Therefore, H is continuous. Then H has a fixed point, ξ. ◻

If the problem is unreported, the expected utility of ignoring the problem is

$$\begin{aligned}
(1-r)(-pd^{M,U}s_t + cs_t) + r\max\Big\{&b^M s_t - d^{M,R}s_t, \\
&- p(d^{M,R}s_t + cs_t) - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)\Big\},
\end{aligned}$$

where r is the probability that the problem is reported in the next period. Therefore, the manager solves if and only if

$$\begin{aligned}
b^M s_t - cs_t \geqslant -(1-r)p(d^{M,U} + c)s_t + r\max\Big\{&b^M s_t - cs_t, \\
&-pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)\Big\}
\end{aligned}$$

If $b^M s_t - cs_t \geqslant -pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)$, the condition is

$$b^M s_t - cs_t \geqslant -p(d^{M,U} + c)s_t.$$

If $b^M s_t - cs_t < -p(d^{M,R} + c)s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)$, the condition is

$$b^M s_t - cs_t \geqslant -(1-r)p(d^{M,U} + c)s_t - r(pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)).$$

However, since $d^{M,R} > d^{M,U}$, $-pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t) < -pd^{M,U}s_t$. Therefore, if $b^M s_t - cs_t < -p(d^{M,R}+c)s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)$, the above condition must not be satisfied. Therefore, the manager solves if and only if $\frac{b^M + pd^{M,U}}{1-p} \geqslant c$. Thus, the probability that the manager ignores problem is

$$\hat{I}(r_{t-1}, s) = \frac{(q^S(1-r_{t-1}) + q^M(1-q^S))\xi(s)}{[q^S(1-r_{t-1}) + q^M(1-q^S)]\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1-q^S)(1-q^M)}.$$

We can also show that $\hat{I}$ is decreasing in r_{t-1}. Let $\hat{D}^U := \mathbb{E}\left[\max\left\{b^M - c, -p(d^{M,U}+c)\right\}\right]$, and

$$\hat{D}^R(s) := \mathbb{E}\left[\max\left\{b^M - c, -p(d^{M,R}+c) - (1-p)\delta p\xi(G(s))(1-\mu)d^R \frac{G^2(s)}{G(s)}\right\}\right].$$

Then as in the basic model, we can define $\hat{\varphi}_{r,r'}$. □

Proof of Proposition 7. Note that when $s_{t+1} = \alpha s_t$, $\hat{D}^R$, ξ and $\hat{I}$ are no longer dependent on s but on α. Therefore, we write these variables as functions of α.

Note also that since ξ is independent of s, ξ is uniquely determined. This is because, (6) is written as

$$\xi(\alpha) = H_\alpha(\xi(\alpha)) := \left[1 - F\left(\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(\alpha)(1-\mu)d^R\alpha}{1-p}\right)\right]. \tag{7}$$

Then, since $H_\alpha(\xi)$ is decreasing in ξ, for each α, $\xi(\alpha)$ is uniquely determined.

Let $E(\alpha) := [-\{1 - \hat{I}(r_{t-1}, \alpha)\}\hat{D}^R(\alpha) - (1 - q^S r_{t+1})(\hat{D}^U - \hat{D}^R(\alpha))]$. The proof consists of the following six steps.

Step 1: *$\xi(\alpha)$ is decreasing in α.* First note the derivative of ξ. By the implicit function theorem, $\xi(G)$ is differentiable and by (7), the derivative is given by

$$\xi'(\alpha) = -\frac{A}{1 + A\alpha}\xi(\alpha) < 0,$$

where

$$A = p\delta(1-\mu)d^R f\left(\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(\alpha)(1-\mu)d^R\alpha}{1-p}\right).$$

Therefore, $\xi(\alpha)$ is decreasing in α. ■

Step 2: *$\xi(\alpha)\alpha$ is increasing in α.* By step 1, $\xi(\alpha)$ is decreasing in α. On the other hand, the RHS of (7) is a decreasing function of $\xi(\alpha)\alpha$. Therefore, $\xi(\alpha)\alpha$ is increasing in α. ■

Step 3: *$\xi(\alpha)\alpha$ converges to a real number as $\alpha \to \infty$.* Suppose by contradiction that $\xi(\alpha)\alpha$ does not converge. By step 2, since $\xi(\alpha)\alpha$ is increasing, $\xi(\alpha)\alpha \to \infty$. Then, there exists $\bar{\alpha}$ such that for each $\alpha > \bar{\alpha}$,

$$\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(\alpha)(1-\mu)d^R\alpha}{1-p} > \bar{c}.$$

Then, since the support of f is $(0, \bar{c})$, $\xi(\alpha) = 0$, which implies that $\xi(\alpha)\alpha = 0$, a contradiction.
■

Step 4: $\lim_{\alpha\to\infty} E(\alpha) > 0$. By step 3, since $\xi(\alpha)\alpha$ converges as $\alpha \to \infty$, $\xi(\alpha) \to 0$. This implies that $\hat{I}(r, \alpha) \to 0$. Then, since $\hat{D}^R(\alpha) < 0$ and $\hat{D}^U < 0$,

$$\lim_{\alpha\to\infty} E(\alpha) = -q^S r_{t+1}\hat{D}^R(\alpha) - (1 - q^S r_{t+1})\hat{D}^U > 0.$$

■

Step 5: $\lim_{G\to\infty} \frac{\partial E(\alpha)}{\partial\alpha}\alpha \geqslant 0$. The derivative of $E(\alpha)$ is given by

$$\frac{\partial E(\alpha)}{\partial\alpha} = \frac{\partial \hat{I}(r_{t-1}, \alpha)}{\partial\alpha}\hat{D}^R(\alpha) + (\hat{I}(r_{t-1}, \alpha) - q^S r_{t+1})\frac{\partial \hat{D}^R(\alpha)}{\partial\alpha},$$

where

$$\frac{\partial \hat{I}(r_{t-1},\alpha)}{\partial \alpha} = \frac{(1-q^S r_{t-1})\xi'(\alpha)}{[q^S(1-r_{t-1})+q^M(1-q^S)]\left[1-F\left(\frac{b^M+pd^{M,U}}{1-p}\right)\right]+(1-q^S)(1-q^M)} < 0,$$

$$\frac{\partial \hat{D}^R(\alpha)}{\partial \alpha} = -(1-p)p\delta(1-\mu)d^R(\xi(\alpha)\alpha)'\xi(\alpha) < 0.$$

Note that since $\xi(\alpha) \to 0$, $\xi'(\alpha) \to 0$. Therefore, $\frac{\partial \hat{I}(r_{t-1},\alpha)}{\partial \alpha} \to 0$ and $\frac{\partial \hat{D}^R(\alpha)}{\partial \alpha} \to 0$. If $r_{t+1} > 0$, since $\hat{I}(r_{t-1},\alpha) \to 0$ as $\alpha \to \infty$, $\frac{\partial E(\alpha)}{\partial \alpha} > 0$ for sufficiently large α. ■

Step 6: *Completing the proof.* Consider the case that $r_{t+1} = 0$. Then, since $\xi(\alpha)\alpha$ is bounded above, $\hat{I}(r_{t-1},\alpha)\alpha$ is bounded above. Since $\frac{\partial \hat{D}^R(\alpha)}{\partial \alpha} \to 0$, $(\hat{I}(r_{t-1},\alpha) - q^S r_{t+1})\frac{\partial \hat{D}^R(\alpha)}{\partial \alpha} \to 0$. Thus, since $\hat{D}^R(\alpha) < 0$ and $\frac{\partial \hat{I}(r_{t-1},\alpha)}{\partial \alpha} < 0$, $\lim_{\alpha\to\infty} \frac{\partial E(\alpha)}{\partial \alpha}\alpha \geqslant 0$.

Note that $\hat{\varphi}_{r_{t-1},r_{t+1}}(\alpha) = \delta(1-p)E(\alpha)\alpha + b^S + pd^S$. Then, if $E(\alpha) > 0$, $\hat{\varphi}_{r_{t-1},r_{t+1}}(\alpha) > 0$. Thus, by step 5, $\lim_{\alpha\to\infty} \hat{\varphi}_{r_{t-1},r_{t+1}}(\alpha) > 0$.

Note also that $\partial\hat{\varphi}_{r_{t-1},r_{t+1}}(\alpha)/\partial\alpha = \delta(1-p)\frac{\partial E(\alpha)}{\partial \alpha}\alpha + \delta(1-p)E(\alpha)$. Then by steps 4 and 5, $\partial\hat{\varphi}_{r_{t-1},r_{t+1}}(\alpha)/\partial\alpha > 0$. □

Proof of Proposition 8. As in the proof of Proposition 7, let

$$E(\alpha) = [-\{1-\hat{I}(r_{t-1},\alpha)\}\hat{D}^R(\alpha) - (1-q^S r_{t+1})(\hat{D}^U - \hat{D}^R(\alpha))]$$

Since $\partial\hat{\varphi}_{r_{t-1},r_{t+1}}(\alpha)/\partial\alpha = \delta(1-p)\frac{\partial E(\alpha)}{\partial \alpha}\alpha + \delta(1-p)E(\alpha)$, it is sufficient to show that $\frac{\partial E(0)}{\partial \alpha} < 0$ and $E(0) < 0$ for sufficiently large $\bar{c}$. Note that

$$\xi(0) = 1 - F\left(\frac{b^M + pd^{M,R}}{1-p}\right) = 1 - \frac{b^M + pd^{M,R}}{(1-p)\bar{c}}.$$

Then, we can write $\hat{I}(r_{t-1},0) = A(\bar{c})\xi(0)$. Note that $A(\bar{c}) \geqslant 1$ and $\lim_{\bar{c}\to\infty} A(\bar{c}) = 1$. On the other hand,

$$\hat{D}^R(0) = b^M \frac{b^M + pd^{M,R}}{(1-p)\bar{c}} - \left(\frac{b^M + pd^{M,R}}{(1-p)\bar{c}}\right)^2 \frac{1}{\bar{c}} - pd^{M,R}\left(1 - \frac{b^M + pd^{M,R}}{(1-p)\bar{c}}\right)$$
$$+ \frac{p}{\bar{c}}\left(\frac{b^M + pd^{M,R}}{(1-p)\bar{c}}\right)^2 - p\bar{c}$$
$$\hat{D}^U = b^M \frac{b^M + pd^{M,U}}{(1-p)\bar{c}} - \left(\frac{b^M + pd^{M,U}}{(1-p)\bar{c}}\right)^2 \frac{1}{\bar{c}} - pd^{M,U}\left(1 - \frac{b^M + pd^{M,U}}{(1-p)\bar{c}}\right)$$
$$+ \frac{p}{\bar{c}}\left(\frac{b^M + pd^{M,U}}{(1-p)\bar{c}}\right)^2 - p\bar{c}$$

Then, since $\hat{D}^R(0) < 0$,

$$E(0) = [-\{1 - I(r_{t-1}, 0)\}\hat{D}^R(0) - (1 - q^S r_{t+1})(\hat{D}^U - \hat{D}^R(0))]$$
$$< -\hat{D}^R(0)\frac{b^M + pd^{M,R}}{1-p} - (1 - q^S r_{t+1})(\hat{D}^U - \hat{D}^R(0)).$$

Letting $\bar{c} \to \infty$ yields that

$$\lim_{\bar{c}\to\infty} -\hat{D}^R(0)\frac{b^M + pd^{M,R}}{1-p} - (1 - q^S r_{t+1})(\hat{D}^U - \hat{D}^R(0))$$
$$= p\frac{b^M + pd^{M,R}}{1-p} - (1 - q^S r_{t+1})p(d^{M,R} - d^{M,U}).$$

Therefore, $\lim_{\bar{c}\to\infty} E(0) < 0$ if

$$p < \frac{(1 - q^S r_{t+1})(d^{M,R} - d^{M,U}) - b^M}{d^{M,R} - (1 - q^S r_{t+1})(d^{M,R} - d^{M,U})}.$$

The above condition holds when p, b^M and $d^{M,U}$ are sufficiently small.

Next, we consider $\frac{\partial E(\alpha)}{\partial \alpha}$, which is written as

$$\frac{\partial E(\alpha)}{\partial \alpha} = \frac{\partial I(r_{t-1}, \alpha)}{\partial \alpha}\hat{D}^R(\alpha) + (\hat{I}(r_{t-1}, \alpha) - q^S r_{t+1})\frac{\partial \hat{D}^R(\alpha)}{\partial \alpha}.$$

Note that

$$\xi'(0) = -A\xi(0) = -\frac{p\delta(1-\mu)d^R}{\bar{c}}\xi(0), \quad \frac{\partial \hat{D}^R(0)}{\partial \alpha} = -(1-p)\delta(1-\mu)d^R(\xi(0))^2.$$

Therefore,

$$\begin{aligned}\lim_{\bar{c}\to\infty} \frac{\partial E(0)}{\partial \alpha} &= p^2\delta(1-\mu)d^R - (1-q^S r_{t+1})(1-p)\delta(1-\mu)d^R \\ &= \delta(1-\mu)d^R(p^2 - (1-q^S r_{t+1})(1-p)).\end{aligned}$$

Thus, if $q^S < 1$ and p is sufficiently small, $\lim_{\bar{c}\to\infty} \frac{\partial E(0)}{\partial \alpha} < 0$. □