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Abstract: This paper develops a simple general equilibrium model of global supply chains (GSCs) that jointly addresses three key decisions of firms forming GSCs, namely selection (whether to form a GSC), location (where to find GSC partners), and matching (with which firms to form a GSC). The model develops a Becker type assortative matching model of final producers and suppliers both of which are heterogeneous in capability (productivity/quality) of their tasks, and integrates it with a Melitz type model of selection and a Ricardian comparative advantage model of location. The model presents a new mechanism of gains from trade associated with firm heterogeneity. Namely, trade liberalization causes rematching of firms toward positive assortative matching at the world level as a recent empirical study on exporter-importer matching data observes.

JEL classification: F12.

Keywords: Global supply chains, firm heterogeneity, two-sided heterogeneity, matching, trade in intermediate goods, quality differentiation.

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1 Introduction

International trade has been traditionally modeled as cross-border transaction between consumers and firms representing industries, but the last decade has witnessed rises in two strands of literature challenging this traditional model. The first one is the "global supply chains" (GSCs) literature (e.g., Jones and Kierzkowski, 1990; Yi, 2003; Grossman and Rossi-Hansberg, 2007).[1] A production process in modern manufacturing is rarely finished within one country and often takes a form of team or "chain" of firms across different countries specializing in different tasks. The majority of international trade are nowadays firm-to-firm transactions of intermediate goods rather than firm-to-consumer transactions of final goods.[2] The second "heterogeneous firm trade" (HFT) literature emphasizes that a representative firm model may not fully capture the impact of trade liberalization on industries. A robust finding that only firms that have high productivity/quality engage in exporting outputs and/or importing inputs within industries has developed new theories that trade liberalization improves industrial performance by reallocating resources toward more capable firms within industries (e.g. Melitz, 2003; Bernard, Jensen, Eaton, and Kortum, 2003).[3] These advances suggest that to better understand modern international trade and its consequences, we need a model on how heterogeneous firms form GSCs.

This paper develops an integrated model of GSCs with firm heterogeneity. The model jointly addresses three key decisions of firms forming GSCs, namely "selection" (whether to form a GSC), "location" (where to find GSC partners), and "matching" (with which firms to form a GSC) in a simple general equilibrium framework. Among these three decisions, the GSC and HTF literatures have addressed selection and location. A major innovation of this paper is to model trade in intermediate goods in GSCs as two-sided matching of heterogeneous importers (final producers) and heterogeneous exporters (suppliers) and to integrate it with standard models of selection and location. The model demonstrates a new mechanism of gains from trade associated with firm heterogeneity and replicates a variety of facts on exporters and importers.

The empirical trade literature has recently started studying matching of exporters and importers using transaction level data. Though the literature is still in an early stage, several studies already found evidence that firms do not act in anonymous markets and that matching matters for firms. First, firms trade with few partners in contrast to the standard love of variety model where every exporter trades with every importer. In a narrowly defined product (e.g. HS 6 digit product) in one

[1] Global supply chains are called by many other names, e.g. fragmentation of production process, offshoring, outsourcing, and vertical integration.

[2] For instance, Bems, Johnson and Yi (2010) estimate that trade in intermediate goods account for 64% of the world trade.

[3] See, for example, Bernard and Jensen (1995, 1999) and Pavcnik (2002) for such findings that motivated the theories.

country, a typical exporter trades with only few importers, typically median one (Blum, Claro and Horstmann, 2010, 2012; Eaton, Eslava, Jinkins, Krizan, and Tybout, 2012; Carballo, Ottaviano, and Volpe Martincus, 2013; Sugita, Teshima, and Seira, 2015). Furthermore, a companion paper, Sugita et al. (2015), found that even firms trading one product with multiple partners in one country conduct most of trade (more than 80%) with the single main partners in Mexican textile/apparel exports to the US. Since firms trade with few firms, choosing right partners should be crucial for both exporters and importers. Second, matching seems systematically related with firm's characteristics instead of purely random matching. Sugita et al. (2015) investigated how matching of Mexican exporters and US importers changed when the US removed quota on textile/apparel products in 2005 and had a huge entry of Chinese exporters. They found US importers switched their Mexican partners to those making greater pre-shock exports whereas Mexican exporters switched their US partners to those making fewer pre-shock imports.

James Rauch (1996) has pioneered modeling firm-to-firm trade as two-sided matching and developed a series of models with his coauthors. Following Rauch, I consider matching of exporters and importers as an assignment model where firms can trade with a given number of partners for some exogenous reasons (e.g. transaction costs). To extend Rauch's models where firms are symmetric, the model incorporates firm heterogeneity in capability, which can be interpreted as productivity or quality, and the complementarity of firm capability within matches. That is, the model is the Becker (1973) marriage market model where final producers and suppliers match positive assortatively by capability.

I integrate the Beckerian matching model with standard models of selection and location: a Melitz (2003) type model of firm selection and a Ricardian comparative advantage model of GSC location. Each firm specializes in one task in a supply chain and becomes heterogeneous in capability by drawing random capability parameters at their entry as in Melitz (2003). After seeing their capabilities, final producers and suppliers match in one-to-one under perfect information. Thanks to complementarity within matches, a stable matching becomes positive assortative matching (PAM) by capability. Highly capable final producers match with highly capable suppliers while low capability final producers match with low capability suppliers.

I first consider international matching without any trade costs in a two country setting. Countries differ in entry costs into intermediate good sectors, which gives a rise to the Ricardian comparative advantage for international matching. Each country has relatively more high capability suppliers in those sectors with relatively low entry costs ("comparative advantage" sectors). Final producers are willing to match suppliers in foreign comparative advantage sectors (i.e. own country's comparative disadvantage sectors). The opening of international matching causes rematching of firms toward

PAM at the world level. High capable firms match with high capable firms in foreign countries. This rematching toward PAM leads to an efficient use of technology exhibiting complementarity and raises the world welfare. When production technology of a supply chain exhibits quasi-concavity in the capability of firms in each stage, this rematching improves technology for production of each variety of final goods, leading to higher quality or cheaper prices. Since trade volume is increasing in firm capability, these predicted rematching is consistent with the finding on Mexico-US rematching by Sugita et al. (2005).

The free trade model incorporates matching and location of GSCs, but not selection. To incorporate selection, I introduce fixed costs of international matching, which allow only high quality firms to trade. Some of high quality firms in the comparative advantage sectors form GSCs with foreign firms, while other firms form local supply chains (LSCs) with domestic firms. PAM naturally explains why exporters and importers are on average larger and more capable than nontrading firms. It is because firms trade with those with similar capability ranks. With fixed trade costs, the opening of international matching improves production technology of highly capable final producers, but worsen that of low capable final producers. Low capable final producers reduce product quality or increase prices. Some of the least capable final producers exit and the total number of varieties of final goods decrease. Despite of these negative effects on the consumer welfare, the model predicts that the opening of trade increases the aggregate welfare by improving matching among high capability firms.

Related Literature

The current paper is related to three strands of literature.

GSC Location First of all, the current model is related to the GSC models that emphasize comparative advantage as an important determinant of location of GSCs (e.g. Dixit and Grossman, 1982; Sanyal and Jones, 1982; Sanyal, 1983; Feenstra and Hanson, 1996; Yi, 2003; Grossman and Rossi-Hansberg, 2008; Costinot, Vogel, and Wang, 2013; Baldwin and Venables, 2013). These models do not analyze heterogeneous firms, but have richer structures such as multiple stages, multiple countries, and multiple factors than the current model. Therefore, the current model should be regarded as complement to these models rather than substitute.

Firm Selection Second, the current model is related to the HFT models that has mainly focused on selection of firms in exporting or importing. The literature has started from models of selection of exporters on productivity (Melitz, 2003; Bernard et al., 2003). Verhoogen (2008), Baldwin and Harrigan (2011) and Johnson (2012) developed models with selection of exporters on quality. Bernard, Redding and Schott (2007) and Okubo (2010) developed multi-industry versions of the Melitz models featuring a Heckscher-Ohlin type and a Ricardo type of comparative advantage, respectively. To address facts on selection of importers, Halpern, Koren, and Szeidl (2009) and Kasahara and Lapham (2013) introduced productivity heterogeneity of final producers in the love of variety model of trade in intermediates.

Though these previous studies on firm selection only considered the heterogeneity of either exporters or importers, a recent study by Bernard, Moxnes, and Ulltveit-Moe (2013) developed models of selections of both heterogeneous exporters and importers based on productivity.[4] Their model and the current model consider different types of trade in intermediate goods: in Bernard et al. (2013), exporters produce horizontally differentiated goods, the reason for forming GSCs is the love of varieties of intermediate goods, while in the current model, exporters produce vertically (quality) differentiated goods and the reason for forming GSCs is comparative advantage.[5]

Matching and Trade James Rauch (1996) has proposed modeling firm-to-firm trade as two-sided matching and developed a series of models with his coauthors (Casella and Rauch, 2001; Rauch and Casella, 2003; Rauch and Trindade, 2003). Since these pioneering works were developed before the HFT literature has emerged, the early models analyze matching of *symmetric* and *horizontally differentiated* exporters and importers. These studies introduce random matching to express uncertainties in international trade and investigate how institutional and technological devices overcome uncertainties.[6]

The previous studies in the trade literature applied the assortative matching model mainly for matching of heterogeneous workers within/across countries.[7] Grossman and Maggi (2000), Ohnsorge and Trefler (2007), Costinot (2009), Costinot and Vogel (2010) and Grossman, Helpman, and Kircher (2103) considered local matching of workers and industries to generalize the comparative advantage theory into the case that worker's skills are of continuous type. Kremer and Maskin (2006) and Antras, Garicano, and Rossi-Hansberg (2006) developed international matching of Southern production workers and Northern managers by their skills. Costinot, Vogel, and Wang (2013) analyzed international matching of workers to form global supply chains in a multi-country model. There

[4]As a related study, Carballo et al. (2013) developed a model of trade between symmetrically heterogeneous consumers and heterogeneous exporters where consumers have different tastes as in ideal variety models.

[5]To see the difference between horizontal and vertical differentiations, suppose that goods were sold under an identical price and with no trade costs. Then, in Bernard et al. (2013), all importers would be willing to buy from all exporters, while in the current model, all importers would be willing to buy only from the highest quality exporters.

[6]Casella and Rauch (2001) and Rauch and Casella (2003) considered ethnic group ties, Rauch and Trindade (2003) information technology, and Grossman and Helpman (2005) internalization through vertical integration.

[7]An important exception is Nocke and Yeaple (2008) who analyze international M&As as matching of corporate assets and managers.

are two important differences between these worker-matching models and the current firm-matching model. First, in the former models, workers are heterogeneous in skills but all firms (or production teams of workers) use identical technology, while in the current model, workers are homogeneous but firms that are heterogeneous in technology (i.e. task quality). Second, in the former models, workers are heterogeneous in skills *ex ante* before they choose tasks, while in the current model, firms are homogeneous in capability *ex ante* and becomes heterogeneous after they enter.[8]

The rest of the paper is organized as follows. Section 2 sets up a model in a closed economy. Section 4 introduces free trade in intermediate goods in a two country setting. Section 4 introduces trade cost. Section 5 offers some concluding comments. Appendix presents calculations and proofs in more detail.

2 Closed Economy

2.1 Basic Setting

Consider an economy that produces differentiated final goods from the only production factor, labor. The labor endowment is given by L and the competitive wage is normalized to one.

Production of a final good requires three tasks, 1, 2, and X. Task $i = 1, 2$ produces different intermediate goods Z_i by using labor. Task X produces a final good by combining intermediate goods Z_1, Z_2 and labor. Each firm specializes in one task. I call three types of firms final producers, Z_1 suppliers, and Z_2 suppliers, respectively, according to their specializations.

The model has three stages. In stage 1, firms become *ex post* heterogeneous in capability in a Melitz (2003) style. Capability represents productivity and/or quality, depending on parameters specified below. There exist infinitely many potential entrants that are ex ante symmetric. Let x, z_1, and z_2 be the capability of final producers, Z_1 producers, and Z_2 producers, respectively. Each entrant decides a task to specialize in, pays fixed entry costs, and independently draws capability from an identical Pareto distribution. The density function is given by $g(s) \equiv k/s^{k+1}$ and the cumulative distribution function is by $G(s) \equiv 1 - (1/s)^k$ for $s \in [1, \infty)$, where $k > 3$ is assumed to ensure a finite GDP. While the probability of capability draws is symmetric across sectors, entry costs are asymmetric: entry requires f_{Xe} units of labor for final producers and f_{ie} units of labor for Z_i producers ($i = 1, 2$). I assume $f_{1e} < f_{Xe} < f_{2e}$ for expositional purposes, though all the main results depend only on the difference of f_{1e} and f_{2e}. Firms are risk neutral so they enter until their

[8]The two classes of models differ in the main predictions, e.g. on who receive a better match from the opening of international matching. For instance, in Antras et al. (2006), the productivity and output of the highest skill managers decreases in North, who could be interpreted as final producers importing intermediates, while in the current model, the product quality and output of the highest quality final producers always increase.

expected profits become zero.

In stage 2, after knowing their own capability, a final producer, a Z_1 supplier, and a Z_2 supplier form a team that produces one variety of final goods. For simplicity, matching is one-to-one(-to-one), so each firm joins only one team. Firms choose their partners and decide how to divide team's total profits in stage 3. Team capability θ depends on member's capability. Thus, matching endogenously determines the distribution of team capability θ across teams.

In stage 3, teams compete in a monopolistically competitive market as firms in Melitz (2003) do. The representative consumer maximizes the following CES utility function:

$$U = \left[\int_{\omega \in \Omega} \theta(\omega)^{\alpha} q(\omega)^{\rho} d\omega \right]^{1/\rho} \text{ subject to } \int_{\omega \in \Omega} p(\omega) q(\omega) d\omega = I$$

where Ω is the set of available varieties of final goods, $q(\omega)$ is the consumer's consumption of variety ω, $\theta(\omega)$ is the capability of the team producing ω, and I is the aggregate income. Consumer's demand for a variety with price p and capability θ is derived as:

$$q(p, \theta) = \frac{I \theta^{\alpha\sigma} p^{-\sigma}}{P^{1-\sigma}}, \tag{1}$$

where $\sigma \equiv 1/(1-\rho) > 1$ is the elasticity of substitution and $P \equiv \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} \theta(\omega)^{\alpha\sigma} d\omega \right]^{1/(1-\sigma)}$ is the price index.

Team capability depends on the capability of team members as follows:

$$\theta = x z_1 z_2. \tag{2}$$

Team capability (2) is increasing, supermodular, and quasi-concave.[9] The supermodularity captures complementarity among the quality of parts and components, following Kremer (1993) and Kugler and Verhoogen (2008). For instance, if a car producer upgrades the quality of engine, the supermodularity implies that the marginal quality improvement of the car is positively related to the quality of other components such as transmission, body, tires, etc. The quasi-concavity means that consumers prefer final goods with moderate combinations of parts quality. For instance, consumers may prefer a standard-class car with normal equipment to a luxury-class car with a poor air conditioner.[10]

Production technology is of Cobb-Douglass type. When a team produces q units of final goods, Z_i supplier in the team produces q_i units of intermediate good Z_i using l_i units of labor. Then, combining these Z_1 and Z_2 with l_x units of labor, the final producer assembles q units of final goods.

[9] An increasing twice-differentiable function with positive cross-derivatives is called (strictly) supermodular.

[10] An alternative interpretation of the quasi-concavity is that a moderate combination of the quality of intermediate goods allows final producers to develop high quality final goods.

The production functions of final producers and Z_i suppliers are given as:

$$q = x^{-\beta}(l_x - f_x)^{1/3} q_1^{1/3} q_2^{1/3} \text{ and } q_i = z_i^{-3\beta}\left(l_i - f_i\right),$$

The labor requirement for a team with capability θ producing q units of final goods becomes:

$$l(\theta, q) = \theta^{\beta} q + f, \tag{3}$$

where $f \equiv f_x + f_1 + f_2$.

Team capability θ may represent productivity and/or quality, depending on parameters α and β. When $\alpha = 0$ and $\beta < 0$, θ may be called productivity. As firms in the Melitz model, all teams face symmetric demand functions, while a team with high θ has lower marginal costs. When $\alpha > 0$ and $\beta > 0$, θ may be called quality. As firms in Baldwin and Harrigan (2011) and Johnson (2012), teams with high θ face a large demand at a given price but incur high marginal costs.

2.2 Autarky Equilibrium

This section solves for an autarky equilibrium by backward induction. Appendix presents calculations in more detail. I use subscripts i and j to denote variables and functions of Z_1 suppliers and Z_2 suppliers throughout the paper. They always mean that $i, j \in \{1, 2\}$ and that $i \neq j$ when i and j are used together.

Stage 3: Production Stage Since team's marginal costs is θ^{β}, team's optimal price is $p(\theta) = \theta^{\beta}/\rho$. Hence, team's revenue $R(\theta)$, total costs $C(\theta)$, and joint profits $\Pi\left(\theta\right)$ are

$$R(\theta) = \sigma A \theta^{\gamma},\ C(\theta) = \left(\sigma - 1\right) A \theta^{\gamma} + f,\ \text{and } \Pi\left(\theta\right) = A\theta^{\gamma} - f, \tag{4}$$

where $A \equiv \frac{I}{\sigma}\left(\rho P\right)^{\sigma - 1}$. Parameter $\gamma \equiv \alpha\sigma - \beta\left(\sigma - 1\right)$ summarizes how capability affects team profits. Since comparative statics on parameters α, β, and σ is not the main interest of the paper, I normalize $\gamma = 1$ by choosing the unit of θ. This normalization greatly simplifies calculation. Then, team profits become:

$$\Pi(x z_1 z_2) = A x z_1 z_2 - f. \tag{5}$$

Stage 2: Matching Stage Firms choose their partners and decide the distribution of team profits, taking A as given. Two types of functions, profit schedules, $\pi_X\left(x\right)$ and $\pi_i\left(z_i\right)$, and matching functions, $m_i(x)$, characterize equilibrium matching. A final producer with capability x matches with Z_i suppliers with capability $m_i(x)$ and receives the residual profit $\pi_X(x)$ after paying profits $\pi_i(m_i(x))$ for the partners. Following the matching literature, I focus on stable matching that satisfies two conditions: (i) (*individual rationality*) no firm is willing to deviate from the current team to exit; (ii) (*pair-wise stability*) no trio of a final producer, a Z_1 supplier, and a Z_2 supplier is willing to deviate from their teams to form a new team.[11] These two conditions are mathematically stated as follows: (i') all firms earn non-negative profits, $\pi_X(x) \geq 0$ and $\pi_i(z_i) \geq 0$ for all x and z_i; (ii') each firm is the optimal partner for the other team members:[12]

$$\begin{aligned}\pi_X(x) &= \Pi(xm_1(x)\,m_2(x)) - \pi_1(m_1(x)) - \pi_2(m_2(x)) \\ &= \max_{z_1,z_2} \Pi(xz_1z_2) - \pi_1(z_1) - \pi_2(z_2) \text{ and} \qquad (6)\\ \pi_i(m_i(x)) &= \Pi(xm_i(x)\,m_j(x)) - \pi_X(x) - \pi_j(m_j(x)) \\ &= \max_{x',z_j} \Pi\left(x'm_i(x)\,z_j\right) - \pi_X(x') - \pi_j(z_j). \qquad (7)\end{aligned}$$

It is known that stable matching is positive assortative matching (PAM) by capability, $m_i'(x) > 0$, (e.g., Becker, 1973; Sattinger, 1979).[13] Since team profits are supermodular, a high capability firm has a greater willingness to pay for the extra capability of its partners. Therefore, high capability firms outbid others for high capability firms, while low capability firms match with remaining low capability firms.

The first order conditions for the maximization in (6) and (7) are:

$$\pi_i'(m_i(x)) = Axm_j(x) > 0 \text{ and } \pi_x'(x) = Am_1(x)m_2(x) > 0. \qquad (8)$$

Conditions (8) imply that profit schedules $\pi_X(x)$ and $\pi_i(z_i)$ are increasing functions of own capabilities.

Because of fixed costs, there exists a cutoff of team capability θ_L such that only teams with capability $\theta \geq \theta_L$ produce on the market. PAM implies $\theta_L = x_L z_{1L} z_{2L}$ where x_L and z_{iL} are capability cutoffs for final producers and Z_i suppliers, respectively, such that only final producers with capability $x \geq x_L$ and Z_i suppliers with capability $z_i \geq z_{iL}$ participate in the matching market.

[11] Roth and Sotomayor (1990) is an excellent textbook on the matching literature.

[12] Although this definition of stability is commonly used in the literature, it should be noted that it implicitly assumes firms can write complete contracts on the distribution of team profits. Therefore, the model is silent on issues related with incomplete contracts, which include the choice of boundaries of firms.

[13] See e.g., Legros and Newman (2007) for the formal proof.

These capability cutoffs satisfy:

$$A\theta_L = Ax_L z_{1L} z_{2L} = f \text{ and } \pi_X(x_L) = \pi_1(z_{1L}) = \pi_2(z_{2L}) = 0. \tag{9}$$

Equilibrium matching functions satisfy *matching market clearing conditions*:

$$M_{Xe}\left[1 - G(x)\right] = M_{ie}\left[1 - G(m_i(x))\right] \text{ for all } x \geq x_L, \tag{10}$$

where M_{Xe}, M_{1e}, and M_{2e} are the mass of entrants of final producers, Z_1 suppliers, and Z_2 suppliers, respectively. The left-hand side of (10) is the mass of final producers whose capability is higher than x, while the right-hand side is the mass of Z_i suppliers whose capability is higher than $m_i(x)$. Because these two sets of firms match with each other under one-to-one PAM, the equality in (10) must hold for all $x \geq x_L$.

Equilibrium matching patterns reflect the relative mass of entrants across sectors. Figure 2.2 describes this relation. The width of each rectangle is equal to the mass of entrants in each sector under the assumption $M_{1e} > M_{2e}$, which will be proved later from $f_{1e} < f_{2e}$, while the vertical axis expresses the value of $G(s)$. The three gray areas express the mass of final producers with higher capability than x (the left area), that of Z_1 suppliers with higher capability than $m_1(x)$ (the center area), and that of Z_2 suppliers with higher capability than $m_2(x)$ (the right area), respectively. The matching market clearing condition (10) implies that all of the three areas must have the same size. The figure shows that a final producer matches with a higher capability Z_i supplier from the sector with more entrants (i.e. Z_1) and a lower capability one from the sector with fewer entrants (i.e. Z_2).

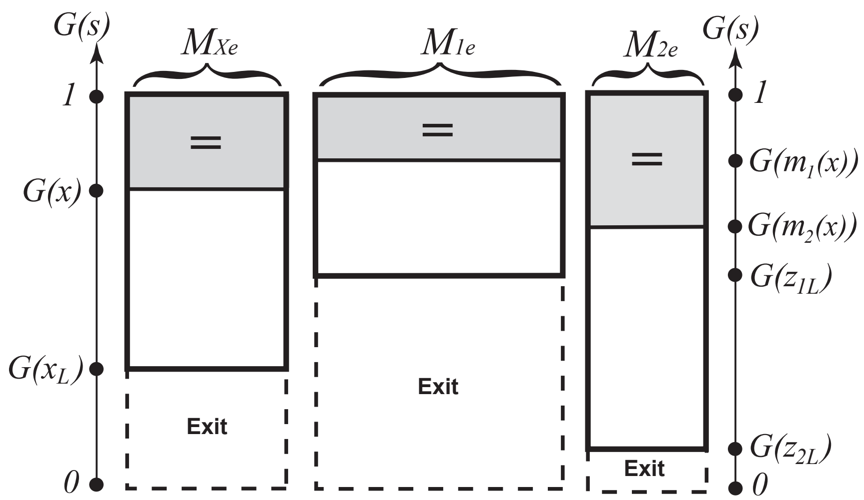

Figure 1: The autarky matching pattern reflects the relative size of entrants across sectors. A firm from a sector with more entrants has high capability in a team.

Matching functions can be obtained from (10) in simple forms as:

$$m_i(x) = x\left(\frac{M_{ie}}{M_{Xe}}\right)^{1/k} = x\left(\frac{z_{iL}}{x_L}\right) \text{ for all } x \geq x_L. \tag{11}$$

As drawn in Figure 2.2, $m_i(x)$ is increasing and concave in M_{1e}/M_{Xe} when x is fixed. As the relative mass of entrants in the Z_1 sector to the final sector increases ($M_{1e}/M_{Xe}\uparrow$), a final producer with given capability x becomes able to match with a Z_1 supplier with higher capability, but final producer's gain in matching is diminishing because high capability firms are scarce. This concave relationship in Figure 5 depends on the shape of G, but is not specific to the Pareto distribution. It holds under a wide class of distributions including uniform, normal, exponential, and other frequently used distributions with the non-decreasing hazard rate $g(x)/(1-G(x))$.

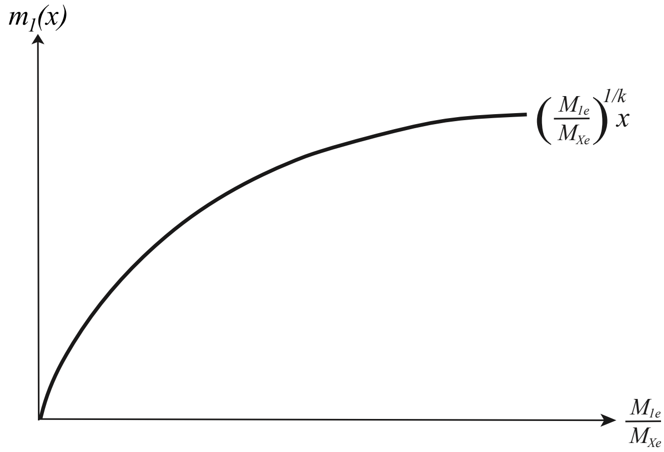

Figure 2: The capability $m_i(x)$ of a Z_i supplier matching with a final producer with capability x is increasing and concave in the mass of entrants in the Z_1 sector relative to the final sector, M_{ie}/M_{Xe}.

Profits of individual firms can be obtained by integrating the first order conditions (8) with initial conditions (9):

$$\pi_X(x) = A\int_{x_L}^{x} m_1(t)m_2(t)dt \text{ and } \pi_i(m_i(x)) = A\int_{x_L}^{x} tm_j(t)dt. \tag{12}$$

Firm's profits are increasing in the team capability and the firm's capability advantage over the least capable firms and increasing in the market condition A.[14] From (11) and (12), profit schedules in (12) are simplified as:

$$\pi_X(x) = \frac{f}{3}\left[\left(\frac{x}{x_L}\right)^3 - 1\right] \text{ and } \pi_i(z_i) = \frac{f}{3}\left[\left(\frac{z_i}{z_{iL}}\right)^3 - 1\right]. \tag{13}$$

[14]The stability condition alone determines the distribution of profits within teams. This is a virtue of this class of matching models with continuums of agents (Sattinger, 1979). We do not need to specify "bargaining power parameters" on how to split the matching surplus within matches.

From (11), the profit schedules in (13) also implies $\pi_X(x) = \pi_i(m_i(x))$, that is, each member receives exactly one third of the team profits, reflecting symmetry in production and capability draws.

Entry Stage Firms' entry conditions determine the capability cutoffs and the mass of entrants. Because risk-neutral firms enter until their expected profits become zero, free entry conditions are given by

$$[1 - G(x_L)]\,\bar{\pi}_X = f_{Xe} \text{ and } [1 - G(z_{iL})]\,\bar{\pi}_i = f_{ie}, \tag{14}$$

where $\bar{\pi}_X$ and $\bar{\pi}_i$ are the average profits of firms in the market, $\bar{\pi}_X = [1 - G(x_L)]^{-1} \int_{x_L}^{\infty} \pi_X(t)\, g(t)\, dt$ and $\bar{\pi}_i = [1 - G(z_{iL})]^{-1} \int_{z_{iL}}^{\infty} \pi_i(t)\, g(t)\, dt$. A manipulation from (13) [see Appendix] shows that the average profits are constant as follows[15]

$$\bar{\pi}_X = \bar{\pi}_i = \frac{f}{k-3}. \tag{15}$$

Because firms earn zero expected profits, the aggregate revenue of teams must be equal to the aggregate income, $M\bar{r} = L$, where $\bar{r}$ is the average revenue of surviving teams and M is the mass of surviving teams, i.e. the mass of varieties. From $\bar{r} = \sigma(\bar{\pi}_X + \bar{\pi}_1 + \bar{\pi}_2 + f)$ and (15), the mass of consumption varieties is proportional to the ratio of labor endowment to production fixed costs as in the standard Dixit-Stiglitz model:

$$M = \frac{(k-3)}{k\sigma}\left(\frac{L}{f}\right). \tag{16}$$

I assume $f/(k-3) \geq \max\{f_{Xe}, f_{ie}\}$ to ensure firms' entry. Then, the capability cutoffs are obtained from (14) and (15) as follows:

$$x_L = \left[\frac{f}{f_{Xe}(k-3)}\right]^{1/k} \text{ and } z_{iL} = \left[\frac{f}{f_{ie}(k-3)}\right]^{1/k}. \tag{17}$$

Since the mass of teams is equal to the mass of surviving firms in each sector, the mass of entrants are obtained as:

$$M_{Xe} = \frac{M}{1 - G(x_L)} = \frac{L}{f_{Xe}k\sigma} \text{ and } M_{ie} = \frac{M}{1 - G(z_{iL})} = \frac{L}{f_{ie}k\sigma}. \tag{18}$$

The relative size of labor endowment to entry costs determines the mass of entrants in each sector. From $f_{1e} < f_{Xe} < f_{2e}$, the mass of entrants follows $M_{1e} > M_{Xe} > M_{2e}$.

[15] The constant average profit also holds in the Melitz-type (2003) model with the Pareto distribution.

The capability cutoffs in (17) link the mass of consumption varieties and the mass of entrants. Lower entry costs attract more entrants, but the total mass of surviving firms (16) is independent of the size of entry costs. Therefore, the capability cutoffs are negatively related to entry costs and positively to production fixed costs.

2.3 Welfare Theorem

This section presents welfare properties of an autarky equilibrium. Suppose the social planner maximizes the welfare $W \equiv U^{\rho}$ by choosing output $q(\theta)$ of production teams, matching, capability cutoffs (x_L, z_{1L}, z_{2L}), and mass of entrants (M_{Xe}, M_{1e}, M_{2e}). Matching may be either deterministic or stochastic and determines the distribution function $H(\theta)$ of team capability θ and the capability cutoff θ_L. I call this problem the long run planner's problem and also consider the short run planner's problem where the mass of entrants are fixed. A long run decentralized equilibrium is the one considered in the previous section. I consider a short run decentralized equilibrium where the mass of entrants are fixed and the aggregate profits are lump sum transferred to the representative consumer.

In this setting, I establish the following welfare theorem.

Proposition 1. *A decentralized autarky equilibrium achieves the solution to the social planner's problem both in the short run and in the long run..*

The proof is given in Appendix. Proposition 1 is related to two known welfare theorems. The first theorem is a classic result of Koopmans and Beckmann (1957) and Shapley and Shubik (1972) that a stable matching in a frictionless market maximizes the total payoffs of agents when the distributions of agents are fixed. Gretsky, Ostroy and Zame (1992) prove the theorem for the case of continuums of agents. The second theorem is recently proved by Dhingra and Morrow (2014) that the monopolistic competition equilibrium maximizes the welfare in the Melitz model with a CES utility and a continuum of firms in autarky.

I present a sketch of the proof here. Following Dhingra and Morrow (2014), for given matching, capability cutoffs (x_L, z_{1L}, z_{2L}) and mass of entrants (M_{Xe}, M_{1e}, M_{2e}), I first obtain optimal production $q(\theta)$, which corresponds to production in a decentralized equilibrium because markups are identical across varieties. Then, substituting optimal $q(\theta)$ to the welfare, I obtain:

$$W = \left[M \int_{\theta_L}^{\infty} \theta dH(\theta)\right]^{\frac{1}{\sigma}} \left[L - M_{Xe} f_{Xe} - \sum_{i=1,2} M_{ie} f_{ie} - M_{Xe}[1 - G(x_L)] f\right]^{\frac{\sigma-1}{\sigma}}. \quad (19)$$

Notice that the welfare (19) is increasing in the aggregate team capability $M \int_{\theta_L}^{\infty} \theta dH(\theta)$ when cutoffs and mass of entrants are given. From the Koopmans-Beckmann-Shapley-Shubik theorem, a stable matching, that is PAM, maximizes the aggregate profits and the aggregate team capability $M\int_{\theta_L}^{\infty}\theta dH(\theta)$. Once $H(\theta)$ is optimally chosen, it is possible to show cutoffs and mass of entrants are chosen optimally. This is because of similar logic behind the Dhingra-Morrow theorem: a continuum of firms and monopolistic competition eliminates the rent-steeling motive of entries to make entries at the socially optimal level.

3 Free Trade

This section introduces trade in intermediate goods, i.e. international matching of firms, in a two country setting. To focus on new aspects of the model, I assume final goods are nontradable. Section 3 considers free trade where international matching requires no costs. Section 4 introduces costs of international matching.

3.1 Comparative Advantage

I introduce another country, Foreign, as a mirror image of Home on entry costs for Z_i sectors:

$$f_{1e} = f_{2e}^* < f_{2e} = f_{1e}^*. \tag{20}$$

Foreign variables and functions are labeled by "$*$". Home and Foreign are identical in other aspects. Introducing differences in entry costs is a simple way to formulate technological differences across countries, which gives a rise to Ricardian comparative advantage.[16] I refer to Home Z_1 sector and Foreign Z_2 sector as the CA (comparative advantage) sectors and Home Z_2 sector and Foreign Z_1 sector as the CD (comparative disadvantage) sectors. This mirror-image structure greatly simplifies the analysis: equilibrium functions and variables in the Home Z_i sector are the same as those in the Foreign Z_j sector and other aspects are identical between Home and Foreign. Because the two countries have the same wage, I continue to normalize it to one.

It is informative to compare Home and Foreign in autarky. First, from (11) and (18), autarky matching differs between the two countries:

$$m_1^a(x) = m_2^{*a}(x) = \left(\frac{f_{Xe}}{f_{1e}}\right)^{1/k} x > \left(\frac{f_{Xe}}{f_{2e}}\right)^{1/k} x = m_2^a(x) = m_1^{*a}(x) \text{ for all } x \geq x_L^a.$$

Variables and functions of an autarky equilibrium are labeled by superscript "a". Figure 3 draws autarky matching of Home and Foreign with an iso-θ curve for a final producer with capability x.

[16] An alternative approach is to assume symmetric entry costs and asymmetric Pareto distributions $G_t(s) = 1 - (v_t/s)^k$ that are allowed to have different means across sector $t \in \{1, 2, X\}$.

This iso-θ curve describes a combination of capability of Z_1 and Z_2 suppliers that a final producer with capability x needs to match in order to achieve team capability θ. Points A and $A*$ represent capability of Z_1 and Z_2 suppliers in Home and Foreign that match with final producers with capability x in autarky, respectively. In autarky, the CA sectors have a relatively larger number of entrants than the CD sectors compared to the foreign country. Therefore, Home final producers match with relative more capable Z_1 suppliers and less capable Z_2 suppliers compared to Foreign final producers with the same capability.

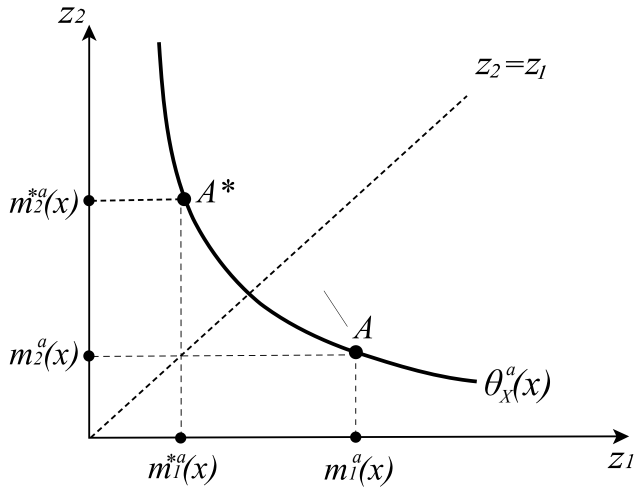

Figure 3: Capability of Z_1 and Z_2 suppliers matching with final producers with capability x in Home autarky (Point A) and Foreign autarky (Point $A*$)

When member capability within teams is interpreted as quality of components, or more generally product characteristics, Figure 3 predicts a cross-country difference in product characteristics. For example, consider car production requiring two input suppliers, designers (Z_1) and parts makers (Z_2), in Italy (Home) and Japan (Foreign). Suppose Italy has lower entry costs for designers and Japan has for parts makers. When cars sold in similar prices (i.e. similar θ) are compared, Italian cars have better design (Paint A) while Japanese cars have more durable parts (Point A^*).

Second, the profits of Z_i suppliers also differ between the two countries. From (13), Z_i suppliers in the CD sectors receive higher profits than foreign Z_i suppliers with the same capability:

$$\pi_2^a(z) - \pi_2^{*a}(z) = \pi_1^{*a}(z) - \pi_1^a(z) = \frac{f}{3}\left[\left(\frac{z}{z_{2L}^a}\right)^3 - \left(\frac{z}{z_{1L}^a}\right)^3\right] > 0. \tag{21}$$

The difference in autarky profits (21) implies that the autarky matching is unstable under free trade. Final producers in each country are willing to match with foreign Z_i suppliers from the foreign CA sectors. This international matching motivated by autarky profits differences is reminiscent of theories of comparative advantage where autarky prices differences leads to international trade. Notice that the profit difference (21) is increasing in capability x. Therefore, highly capable firms are more willing to participate in international trade than low capability firms.

The opening of trade causes two adjustments: (1) re-matching of existing firms and (2) new entry and exit of firms. To understand the role of each adjustment, section 3.2 analyzes a short-run equilibrium in which re-matching occurs only among firms that enter in an autarky equilibrium. For simplicity, I assume firms that entered but did not form any match in an autarky equilibrium can participate the matching market at no cost. Section 3.3 analyzes a long run equilibrium in which firms adjust entry and exit to satisfy the free entry conditions.

3.2 Short-run Equilibrium

After the opening of trade, matching remains unstable as long as the profit schedules of Z_i suppliers differ across countries. Therefore, in a new equilibrium, the profit schedules of Z_i suppliers must satisfy:

$$\pi_1(z) - \pi_1^*(z) = \pi_2(z) - \pi_2^*(z) = 0. \tag{22}$$

Under free trade, the matching market is globally integrated so that matching functions are equalized across countries. New matching functions satisfies the global matching market clearing condition:

$$\left(M_{Xe}^a + M_{Xe}^{*a}\right)[1 - G(x)] = \left(M_{ie}^a + M_{ie}^{*a}\right)[1 - G(m_i(x))] \text{ for all } x \geq x_L. \tag{23}$$

Notice that the mass of entrants are fixed at autarky levels in a short run equilibrium. Since the CA sectors have more entrants than the CD sectors, condition (23) implies that some suppliers in the CA sectors must match with foreign final producers. Let $s_X(x)$ be the share of importers among Home final producers with capability x and $s_i(z_i)$ be the share of exporters among Home Z_i suppliers with capability z_i. From (18) and (23), they are obtained as:

$$\begin{aligned} s_X(x) &= \frac{M_{1e}^a - M_{1e}^{*a}}{M_{1e}^a + M_{1e}^{*a}} = \frac{f_{1e}^* - f_{1e}}{f_{1e} + f_{1e}^*} \text{ for } x \geq x_L, \\ s_1(z_1) &= \frac{M_{1e}^a - M_{1e}^{*a}}{2M_{1e}^a} = \frac{f_{1e}^* - f_{1e}}{2f_{1e}^*} \text{ for } z_1 \geq z_{1L}, \text{ and } s_2(z_2) = 0 \text{ for } z_2 \geq z_{2L}. \end{aligned} \tag{24}$$

The share of trading firms increases in the extent of comparative advantage, $f_{1e}^* - f_{1e}$. Figure 4 describes the distributions of firms engaging in exporting and importing.

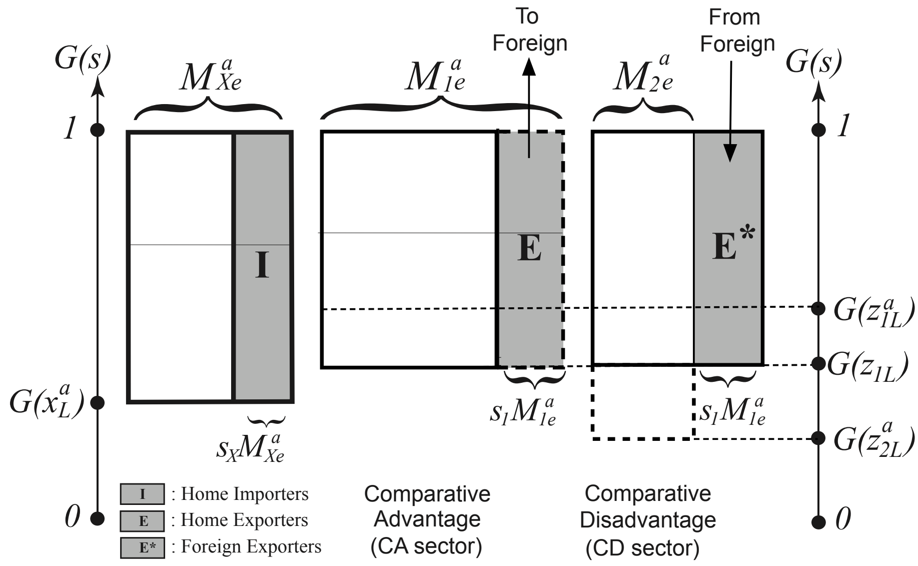

Figure 4: The distribution of exporters and importers in Home in a short run free trade equilibrium

From condition (23), Home matching functions are obtained as follows:

$$m_1(x) = m_2(x) = x\left(\frac{M^a_{1e} + M^{*a}_{1e}}{M^a_{Xe} + M^{*a}_{Xe}}\right)^{1/k} = x\left(\frac{z_{iL}}{x_L}\right) \text{ for } x \geq x_T. \tag{25}$$

Matching functions (25) are similar to autarky matching functions (11). Both functions are increasing and concave in the relative mass of entrants in the Z_i sector to the final sector. Figure 5 draws this relationship for $m_1(x)$. As Figure 5 shows, the capability of partner Z_i suppliers $m_i(x)$ under trade is higher than the average of the two partners in autarky, $m^a_1(x)$ and $m^a_2(x)$ $(= m^{*a}_1(x))$.

The concave curve in Figure 5 represents a source of product-level gains from international matching. Rather than having many entrants in one sector and few in other sectors, having a moderate combination of entrants allow final producers to match with higher capability suppliers on average. Thus, since team capability θ is quasi-concave, final producers increase team capability. Figure 6 draws iso-$\theta_X(x)$ curves for a Home final producer with x. Point A expresses the capability of autarky partners, $m^a_i(x)$, and Point S expresses the capability of new partners in a short run trade equilibrium, $m_i(x)$. From Figure 5, the capabilities of new partners are higher than the average capability of its autarky partners. Therefore, final producers increases team capability.

The new capability cutoffs are obtained from the labor market clearing condition:

$$L = M^a_{Xe}f_{Xe} + \sum_{i=1,2} M^a_{ie}f_{ie} + M^a_{Xe}\int_{x_L}^{\infty} C(\theta_X(t))g(t)dt. \tag{26}$$

where $\theta_X(x) \equiv xm_1(x)m_2(x)$ and $C(\theta_X(x))$ are team capability and team labor costs, respectively,

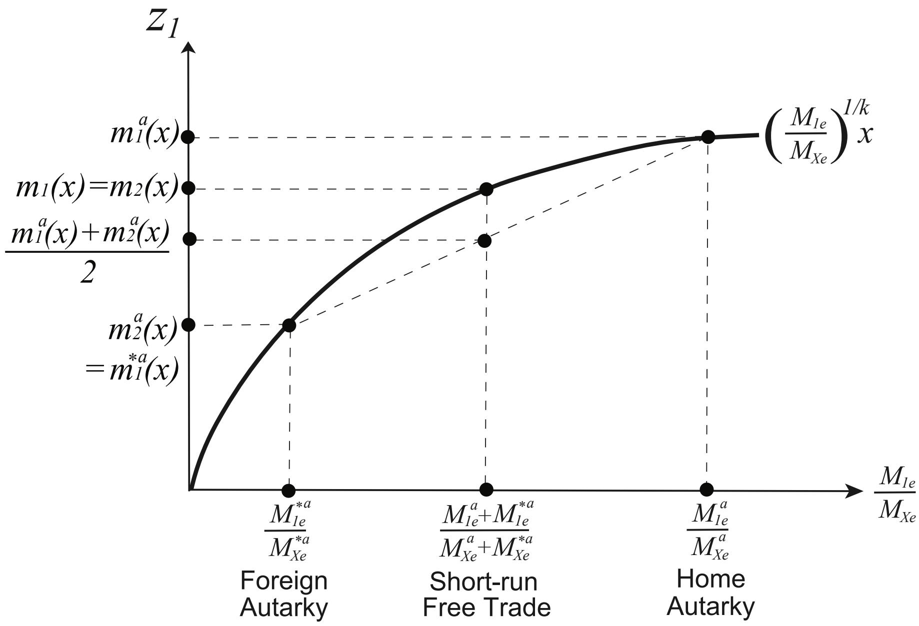

Figure 5: Under trade a final producer with x matches Z_1 and Z_2 suppliers that have higher capability than the average of the two autarky partners.

for a final producer with capability x. The first two terms in the right hand side of (26) represent labor for entry, while the last term represents labor for production. Since $C(\theta_X(x)) = (\sigma - 1)\, A\theta_X(x) + f$ from (4), a final producer with capability x increases its labor inputs from autarky if and only if $A\theta_X(x)$ increases from autarky. Since $A\theta_X(x) = \theta_X(x)/\theta_X(x_L) = (x/x_L)^3$ both in autarky and in a short run free trade equilibrium, if $x_L = x_L^a$, then $A\theta_X(x)$ does not change from autarky. This means $x_L = x_L^a$ since if $x_L = x_L^a$, then all teams use the same amount of labor and the labor market clearing condition (6) holds. Therefore, the cutoff for final producers does not change. On the other hand, the capability cutoffs change for Z_i suppliers. They fall in the CA sectors and rise in the CD sectors: $z_{1L}^a > z_{1L} = z_{2L} > z_{2L}^a$.

The change in the cutoffs leads to the change in profits. From (15) and (25), profit functions continue to be (13). From the changes in the capability cutoffs, final producers receive the same profits as in autarky, Z_i suppliers in the CA sectors increase profits, and Z_i suppliers in the CD sectors decrease profits.

Welfare Theorem The welfare theorem continues to hold after the opening of trade. Consider the social planner who maximizes an increasing symmetric concave function of U and U^*. Then, the short run free trade equilibrium maximizes the world welfare. Since an autarky equilibrium allocation is feasible under free trade, a corollary is that the opening of free trade increases the world welfare in the short run. Since $U = U^*$ always holds from symmetry, this corollary also implies the opening

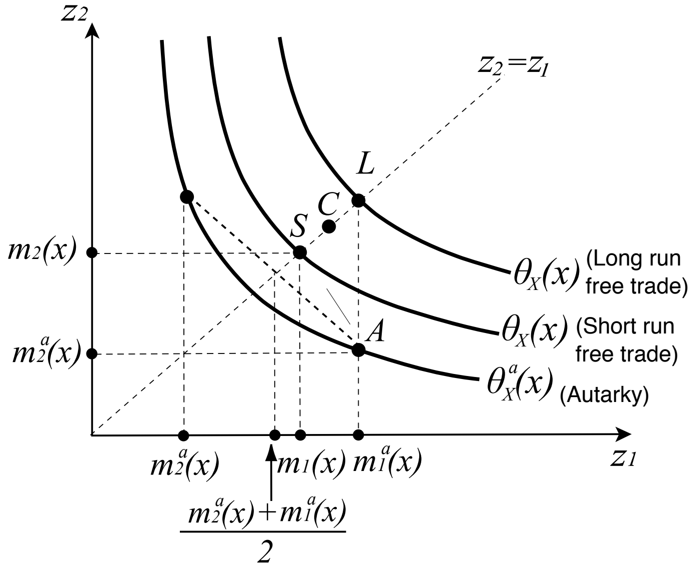

Figure 6: Final producers improve team capability under trade.

of free trade increases the welfare of each country in the short run.

Proposition 2. *(1) The short run free trade equilibrium maximizes the world welfare. (2) The opening of free trade increases the welfare of each country in the short run.*

The proof is in Appendix. The key intuition is again that the welfare function (1) is increasing in the aggregate team capability, which is maximized under stable matching. After the opening of trade, autarky matching becomes unstable since it is not PAM at the world level. International trade systematically changes matching so that PAM holds at the world level, raising the aggregate capability and the welfare.

Evidence for Welfare Improving Rematching The key welfare improving mechanism is that trade liberalization causes rematching toward PAM at the world level. Define that a firm upgrades *partner* if the firm switches its partner to the one with higher capability, and that a firm downgrades partner if the firm switches its partner to the one with lower capability. Then, the rematching toward PAM is summarized as follows. First, final producers have partner upgrading in the CD sectors and partner downgrading in the CA sectors. Second, Z_i suppliers in the CA sectors have partner upgrading. Third, Z_i suppliers in the CD sectors have partner downgrading.

The Model's prediction on rematching in trade liberalization is consistent with a recent empirical finding from exporter-importer matching data. Sugita, Teshima, and Seira (2015) investigate customs transaction records of Mexican textile/apparel exports to the US where they can observe identities of Mexican exporters and US importers and transaction volume for each hs 6 digit product in each year from 2004 to 2009. They find two pieces of evidence for PAM of exporters and importers by capability.

First, they found in each hs 6 digit product and year, matching of Mexican exporters and US importers is approximately one-to-one. Even though there are firms trading with multiple partners, these firms concentrate their product-level trade with the single main partners. They calculate trade volume of "main-to-main" transactions where the exporter and the importer are both the main partner of each other for the product and find that the share of "main-to-main" trade in the aggregate textile/apparel trade volume is more than 80%. This finding justifies the matching approach to modeling international trade.

Second, Sugita et al. (205) find that Mexican textile/apparel exports to the US offers a unique natural experiment for testing the key prediction of PAM, systematic rematching in trade liberalization. Before 2005, the US imposed import quota on some textile/apparel products under the Mutil-fibre arrangement (MFA). Since Mexican exports to the US enjoy quota-free access through the North American Free Trade Agreement (NAFTA), the MFA quota protected Mexican exports to the US as if they were domestic transactions. In 2005, the US removed the MFA quota, which results in a huge increase in Chinese exports and a corresponding drop in Mexican exports in the US market. They find the following systematic partner changes occur more often in products for which Chinese exports to the US were subject to binding quotas. These partner changes are that US importers switched their main Mexican partners to those making greater pre-shock exports; that Mexican exporters switched their main US partners to those making fewer pre-shock imports.

The second findings of Sugita et al. (2015) are consistent with the current model by interpreting trade between US firms and Mexican firms within NAFTA as if they were domestic matching. Since firm's trade volume is increasing in its own capability in the model, their findings can be interpreted as partner upgrading of US final producers and partner downgrading of Mexican suppliers in the CD sectors where China has comparative advantages.

3.3 Long-run Equilibrium

Free trade equalizes profits of Z_i suppliers between the CA and CD sectors, but entry costs differ between them. Therefore, in a long run free trade equilibrium, no firm enters in the CD sector. Countries completely specialized in the CA sectors.

A long run free equilibrium is obtained as follows. First, from complete specialization, matching market clearing conditions become

$$(M_{Xe} + M_{Xe}^{*})\,[1 - G(x)] = M_{1e}[1 - G(m_1(x))] = M_{2e}^{*}[1 - G(m_2(x))]$$

and matching functions become

$$m_1(x) = m_2(x) = \left(\frac{M_{1e}}{M_{Xe} + M_{Xe}^*}\right)^{1/k} x = \left(\frac{z_{1L}}{x_L}\right) x \text{ for all } x \geq x_L. \tag{27}$$

Since the From (12) and (27), profits functions continue to be (13). Furthermore, it is possible to show that expected profits also continue to be (15). Thus, the mass of varieties and the capability cutoffs continue to be (16) and (17), respectively. From (27), $2M = 2M_{Xe}[1 - G(x_L)] = M_{1e}[1 - G(z_{1L})] = M_{2e}^*[1 - G(z_{2L}^*)]$ hold. Then, the mass of firms are obtained as $M_{Xe} = M_{Xe}^a$, $M_{Z1e} = 2M_{Z1e}^*$, and $M_{Z2e} = 0$.

Notice that $M_{Z1e} > M_{Z1}^a + M_{Z2}^a$ implies that the relative mass of entrants in the Z_i sectors to the final sector in the world increases to M_{Z1e}^a / M_{Xe}^a for $i = 1, 2$. Point L in Figure 6 expresses matching in a long run free trade equilibrium, showing each final producer further improves team capability from a short run equilibrium.

4 Costly Trade

The heterogeneous firm trade literature has established two facts: (1) only large and highly capable firms engage in exporting and importing; (2) trade liberalization leads the least capable firms to exit. The model in section 3 fails to address these two facts, predicting no correlation of capability and participation in trade, and no exit of final producers after trade liberalization. This section shows that a simple extension of introducing fixed trade costs allows the model to predict these two facts.

International teams requires fixed trade costs: f_M units of local labor for an importing final producer and f_E units of local labor for an exporting Z_i supplier. For simplicity, I assume trade requires no variable trade cost. Let $f_T \equiv f_M + f_E$ be total fixed trade costs for an international team. Then, team profits are expressed as:

$$\Pi(x, z_1, z_2) - f_T I_I = Axz_1z_2 - f_T I_I, \text{ where } I_I = \begin{cases} 1 & \text{for an international team} \\ 0 & \text{for a domestic team} \end{cases}.$$

In the following, I consider a long run equilibrium where f_T is sufficiently high that the CD sector has positive entry.

Trade Pattern With fixed trade costs, only firms with high capability engage in international matching. Notice that the autarky profit difference in (21) increases in capability z. There exist thresholds for capability x_T and z_T such that only final producers with $x \geq x_T$ and Z_i suppliers in the CA sectors with $z \geq z_T$ are willing to form international teams. In equilibrium, the profit difference for highly capable Z_i suppliers between Home and Foreign must be equal to fixed trade costs.

Lemma 1. *There exists an export capability cutoff $z_T > z_{iL}$ for $i = 1, 2$ such that profit schedules for Z_i suppliers satisfy the following* no arbitrage condition*:*

$$\pi_1^* (z) - \pi_1 (z) = \pi_2 (z) - \pi_2^* (z) \begin{cases} = f_T & \text{if} \quad z \geq z_T \\ < f_T & \quad \text{otherwise.} \end{cases} \tag{28}$$

Proof. In Appendix. □

Since the no arbitrage condition (28) holds only for highly capable firms, the matching market is globally integrated for high capability firms, while it is segmented across countries for low capability firms.

Lemma 2. *There exist* an import cutoff *for capability $x_T > x_L$ such that Home matching functions $m_i(x)$ satisfy (i) $z_T = m_i (x_T)$, (ii)* global matching market clearing conditions*:*

$$\left(M_{Xe} + M_{Xe}^*\right) [1 - G (x)] = \left(M_{ie} + M_{ie}^*\right) [1 - G (m_i (x))] \tag{29}$$

for all $x \in [x_T, \infty)$ *and (iii)* local matching market clearing conditions*:*

$$M_{Xe} [1 - G (x)] = M_{1e} [1 - G(m_1 (x))] - M_T = M_{2e} [1 - G(m_2 (x))] + M_T \tag{30}$$

for all $x \in [x_L, x_T)$, where *M_T is the mass of Home Z_1 suppliers (Foreign Z_2 suppliers) engaging in international matching. Foreign matching functions are obtained from $m_i^* (x) = m_j (x)$.*

Proof. In Appendix. □

Because $M_{Xe} = M_{Xe}^*$ and $M_{ie} = M_{je}^*$ under the mirror image assumption, the global matching market clearing condition (29) implies that final producers with $x \geq x_T$ in each country as a whole match with exactly a half of Z_i suppliers with $z \geq z_T$ in the world. Therefore, Home and Foreign have identical matching functions, $m_i (x) = m_i^*(x)$ for $x \geq x_T$. From these properties, I obtain the next lemma on the mass of firms that engage in international matching.

Lemma 3. *The share of importers among Home final producers and the share of exporters among*

Home Z_i suppliers satisfy

$$s_X(x)=\begin{cases}\frac{M_{1e}-M_{1e}^*}{M_{1e}+M_{1e}^*}\equiv s_X & \text{for } x\geq x_T\\ 0 & \text{otherwise}\end{cases};\ s_1(z_1)=\begin{cases}\frac{M_{1e}-M_{1e}^*}{2M_{1e}}\equiv s_1 & \text{for } z_1\geq z_T\\ 0 & \text{otherwise}\end{cases} \tag{31}$$

and $s_2(z_2)=0$ for all z_2. The corresponding shares of Foreign firms are given by $s_X^*(x)=s_X(x)$, $s_1^*(z)=s_2(z)$, and $s_2^*(z)=s_1(z)$. The mass of exporters and the mass of importers are equal to $M_T=s_X M_{Xe}[1-G(x_T)]=s_1 M_{1e}[1-G(z_T)]$.

Proof. In Appendix. □

Figure 7 expresses the distribution of capability of Home exporters and importers. Area E expresses Home exporters in the Z_1 sector that match with Foreign final producers; Area E^* expresses Foreign exporters in the Z_2 sector that match with Home importers in the final sector expressed in Area I. The size of each of Areas E, E^*, and I is equal to M_T.

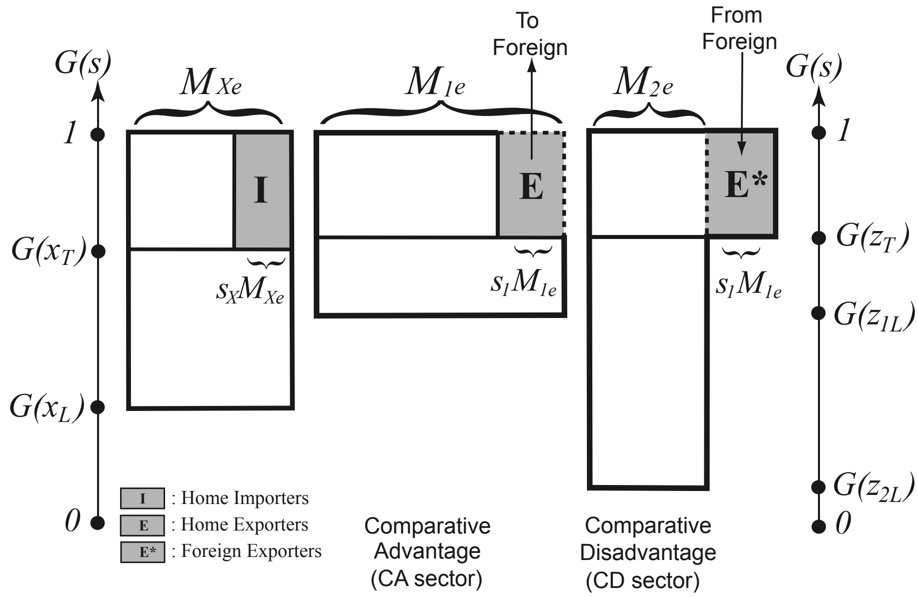

Figure 7: International matching occurs among high capability firms

As Figure 7 shows, only highly capable firms engage in exporting and importing. Since high capable firms have larger revenue and employment than low capable firms, the model predicts a well established fact on firms engaging in international trade. That is, exporters and importers are on average more capable and larger than non-trading firms.

Proposition 3. *In each sector, (i) exporters have on average higher capability and larger revenue than non-exporters; (ii) importers have on average higher capability and larger revenue than non-importers.*

After liberalization, Z_i suppliers in the CA sectors gain in matching and Z_i suppliers in in the CD sectors lose. Therefore, in the long run, entry increases in the CD sectors relative to the CA sectors. Thanks to this specialization into sectors with lower entry costs, the total mass of entrants of Z_i suppliers to final producers in the world increase.

Lemma 4. *In the long run after the opening of trade from autarky: (i) the mass of entrants in the final sector does not change $M_{Xe} = M^a_{Xe}$; (ii) the mass of entrants in the Z_i sectors increases in the CA sectors, but decreases in the CD sectors, $M_{1e} > M^a_{1e} > M^a_{2e} > M_{2e}$.*

Proof. In Appendix □

The opening of trade with fixed trade costs improves team capability of highly capable final producers, but at the same time, it decreases team capability of the lowest capable final producers. From (29), matching functions for high capability final producers take a form similar to (25) as follows:

$$m_1(x) = m_2(x) = \left(\frac{M_{ie} + M^*_{ie}}{M_{Xe} + M^*_{Xe}}\right)^{1/k} x \text{ for } x \geq x_T. \tag{32}$$

Lemma 4 implies that the mass of entrants in the Z_i sectors relative to the final sector increases from autarky, $\frac{M_{ie}+M^*_{ie}}{M_{Xe}+M^*_{Xe}} > \frac{M^a_{ie}+M^{a*}_{ie}}{M^a_{Xe}+M^{a*}_{Xe}}$. Therefore, a comparison of (25) and (32) shows that a final producer with higher capability than x_T matches suppliers with higher capability than in a short run free trade equilibrium. Figure 6 describes new matching as Point LC and shows team capability increases from autarky.

The specialization toward the CA sectors, however, hurts final producers with low capability. Because the CA sector already has more entrants than the CD sector, further entry into the CA sector widens the imbalance of entrants in the two intermediate good sectors. The next lemma shows that final producers with the lowest capability reduce their team capability below the autarky level since they are forced to match with suppliers having more extreme combinations of capability than in autarky.

Lemma 5. *In a long run costly trade equilibrium, there exists a threshold $x' > x_L$: (i) final producers with capability x increase team capability compared to autarky, $\theta_X(x) > \theta^a_X(x)$, if $x > x'$ and decrease it, $\theta_X(x) < \theta^a_X(x)$, if $x < x'$; (ii) the capability of Z_i suppliers in a team converges compared to autarky, $m_1(x) < m^a_1(x)$ and $m_2(x) > m^a_2(x)$, if $x > x'$, while they diverge, $m_1(x) > m^a_1(x)$ and $m_2(x) < m^a_2(x)$, if $x < x'$.*

Proof. In Appendix. □

The free entry conditions (14) determine the capability cutoffs in a long run equilibrium. The opening of trade intermediate goods changes expected profits across sectors for given domestic cutoffs. Expected profits increase more strongly in the CA sectors than in the final sectors and more strongly in the final sectors than in the CD sectors. To satisfy to satisfy the free entry conditions, the domestic cutoffs increase more strongly in the CA sector than in final sector and more strongly in the final sector than in the CD sector. [17]

Lemma 6. *In the long run after the opening of trade in intermediate goods from autarky, the capability cutoffs increase more strongly in the CA sector than in final sector and more strongly in the final sector than in the CD sector:*

$$\frac{z_{1L}}{z_{1L}^a} > \frac{x_L}{x_L^a} > 1 > \frac{z_{2L}}{z_{2L}^a}.$$

Proof. In Appendix. □

Aggregate Welfare After the opening of trade, the mass of entrants in the final producers do not change but the cutoff for final producers rise. Thus, the mass of varieties of final goods decrease and some final goods become sold with lower quality or higher prices, both of which tend to decrease the consumer welfare. What is the total impact of trade on the aggregate welfare? Appendix shows that the welfare theorem continues to hold under costly trade. Therefore, the opening of costly trade increases the welfare of each country.

Proposition 4. *(1) The costly trade equilibrium maximizes the world welfare. (2) The opening of costly trade increases the welfare of each country.*

5 Conclusion

This paper has analyzed location, selection and matching of global supply chains in a tractable general equilibrium model. The model demonstrates a new gain from trade associated with firm heterogeneity. Trade improves the world welfare by changing matching of firms in the world positive assortative by capability. A companion empirical paper, Sugita, Teshima and Seira (2015), identifies this welfare-improving rematching in an actual trade liberalization episode.

The model employs several simplifying assumptions for tractable analysis. Relaxing some of them will make it possible to study further implications of firm-to-firm matching in international

[17] The larger increase in the domestic capability cutoff in the CA sector compared to the CD sector is similar to Bernard, Redding and Schott (2007), in which the domestic productivity cutoff rises more in the CA sector than in the CD sector.

trade. The first one is to allow countries asymmetric so that wages differ across countries. When factor prices are different across countries, firm's capability depends not only on its technology such as product quality and productivity, but also on factor prices. At the same time, factor prices will depend on matching patterns, that is, with whom local firms form global supply chains. The second possible extension is to allow randomness in meeting. Since Shimer and Smith (2000), the Becker type assortative matching model is extended to include randomness in meeting (see Smith, 2011, for survey). These studies find positive assortative matching does not exactly hold, but does hold on average with some deviations. This will be a more realistic picture of actual matching in data, though I guess many of the current results shown in a model without randomness will continue to hold in a model with randomness. The third is to formulate a micro-foundation of θ. An example is to introduce supplier's learning technology from buyers. This extension allows us to analyze how the extent of learning by exporting depends on the characteristics of importers and exporters. Finally, the current model is silent about boundaries of firms. For instance, the current model implicitly assumes complete contracts in the matching stage. To incorporate the theory of multinational firms by Antras (2003) based on contract incompleteness would be an interesting task for future research.

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Appendix (Not for Publication)

A1. Solving for an Autarky Equilibrium

Production Stage

Utility Maximization

The maximization problem for the representative consumer is:

$$\max_{q(\cdot)} U = \left[\int_{\omega \in \Omega} \theta(\omega)^{\alpha} q(\omega)^{\rho} d\omega \right]^{1/\rho} \tag{A.1}$$
$$\text{subject to} \int_{\omega \in \Omega} p(\omega)\, q(\omega)\, d\omega = I,$$

where $q(\omega)$ is quantity demanded for variety ω of final good, $p(\omega)$ is the price of ω, $\theta(\omega)$ is the team capability of ω and I is the aggregate income. Setting up a Lagrangian

$$\mathcal{L} = \left[\int_{\omega \in \Omega} \theta(\omega)^{\alpha} q(\omega)^{\rho} d\omega \right]^{1/\rho} + \lambda \left[I - \int_{\omega \in \Omega} p(\omega)\, q(\omega)\, d\omega \right],$$

I obtain the first order conditions as:

$$\frac{\partial \mathcal{L}}{\partial q(\omega)} = \theta(\omega)^{\alpha} q(\omega)^{\rho-1} U^{(1-\rho)/\rho} - \lambda p(\omega) = 0 \text{ for all } \omega \in \Omega. \tag{A.2}$$

For any two varieties ω and ω', we have

$$\begin{aligned}
\left(\frac{\theta(\omega')}{\theta(\omega)} \right)^{\alpha} \left(\frac{q(\omega')}{q(\omega)} \right)^{\rho-1} &= \frac{p(\omega')}{p(\omega)} \\
\frac{q(\omega')}{q(\omega)} &= \left(\frac{p(\omega')}{p(\omega)} \right)^{\frac{1}{\rho-1}} \left(\frac{\theta(\omega')}{\theta(\omega')} \right)^{\frac{\alpha}{1-\rho}} \\
\frac{p(\omega')q(\omega')}{p(\omega)q(\omega)} &= \left(\frac{p(\omega')}{p(\omega)} \right)^{1-\sigma} \left(\frac{\theta(\omega')}{\theta(\omega)} \right)^{\alpha\sigma} \\
p(\omega')q(\omega') &= \left(\frac{p(\omega')}{p(\omega)} \right)^{1-\sigma} \left(\frac{\theta(\omega')}{\theta(\omega)} \right)^{\alpha\sigma} p(\omega)q(\omega).
\end{aligned}$$

Integrating both sides with respect to $\omega' \in \Omega$, we obtain

$$\begin{aligned}
\int_{\omega' \in \Omega} p(\omega')q(\omega')d\omega' &= \frac{q(\omega)}{p(\omega)^{-\sigma}\, \theta(\omega)^{\alpha\sigma}} \int_{\omega' \in \Omega} \theta(\omega')^{\alpha\sigma} p(\omega')^{1-\sigma} d\omega' \\
I &= \frac{q(\omega)}{p(\omega)^{-\sigma}\, \theta(\omega)^{\alpha\sigma}} P^{1-\sigma} \\
q(\omega) &= \frac{I\theta(\omega)^{\alpha\sigma}\, p(\omega)^{-\sigma}}{P^{1-\sigma},},
\end{aligned}$$

where $P \equiv \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}\theta\left(\omega\right)^{\alpha\sigma} d\omega\right]^{1/(1-\sigma)}$ is the price index. Therefore,

$$q(p,\theta) = \frac{I\theta^{\alpha\sigma}p^{-\sigma}}{P^{1-\sigma}} \tag{A.3}$$

Profit Maximization

Facing the demand function (A.3), a team with capability θ sets the price to maximize the team profits:

$$\max_p (p - c\theta^\beta)q(p,\theta) - f \text{ subject to } (A.3).$$

The first order condition is

$$\begin{aligned}
(p - c\theta^\beta)\frac{\partial q(p,\theta)}{\partial p} + q(p,\theta) = 0 \\
-\sigma(p - c\theta^\beta)\frac{q(p,\theta)}{p} + q(p,\theta) = 0 \\
-\sigma(p - c\theta^\beta) + p = 0 \\
p = \left(\frac{\sigma}{\sigma-1}\right)c\theta^\beta = \frac{c\theta^\beta}{\rho}
\end{aligned}$$

A team chooses the optimal price $p(\theta) = \frac{c\theta^\beta}{\rho}$.

Let $A \equiv \frac{I}{\sigma}\left(\frac{\rho P}{c}\right)^{\sigma-1}$ and $\gamma \equiv \alpha\sigma - \beta\left(\sigma-1\right)$. Team's output $q\left(\theta\right)$, revenue $R(\theta)$, costs $C(\theta)$, and profits $\Pi(\theta)$ become:

$$\begin{aligned}
q\left(\theta\right) &= IP^{\sigma-1}\left(\frac{\rho}{c}\right)^\sigma \theta^{(\alpha-\beta)\sigma}; \\
R(\theta) &= p(\theta)q\left(\theta\right) \\
&= I\left(\frac{\rho P}{c}\right)^{\sigma-1}\theta^{(\alpha-\beta)\sigma+\beta} \\
&= \sigma A\theta^\gamma; \\
C(\theta) &= c\theta^\beta q\left(\theta\right) + f \\
&= \frac{I}{\rho}\left(\frac{\rho P}{c}\right)^{\sigma-1}\theta^{(\alpha-\beta)\sigma+\beta} + f \\
&= \left(\sigma-1\right)A\theta^\gamma + f; \\
\Pi\left(\theta\right) &= R(\theta) - C(\theta) = A\theta^\gamma - f.
\end{aligned}$$

I normalize $\gamma = 1$.

Matching Stage

In stable matching, matching functions, $m_i(x)$, and profit functions, $\pi_X(x)$ and $\pi_i(z_i)$, satisfy two conditions: (1) (individual rationality) all firms receive non-negative profits; (2) (pair-wise stability) each firm is the optimal partner for the other team members. The (pair-wise) stability condition implies that a final producer maximizes its residual profit:

$$\pi_X(x) = \max_{z_1,z_2} \Pi(xz_1z_2) - \pi_1(z_1) - \pi_2(z_2). \tag{A.4}$$

The first order condition for (A.4) becomes:

$$\pi_i'(m_i(x)) = Axm_j(x) > 0. \tag{A.5}$$

The stability condition also implies that Z_i suppliers maximize residual profits:

$$\pi_i(m_i(x)) = \max_{x',z_j} \Pi(x'm_i(x)z_j) - \pi_X(x') - \pi_j(z_j).$$

The first order condition becomes:

$$\pi_X'(x) = Am_1(x)m_2(x) > 0. \tag{A.6}$$

As will be shown in section A2 below, stable matching is positive assortative, $m_i'(x) > 0$. Therefore, from (A.5) and (A.6), profit functions are increasing: $\pi_X'(x) > 0$ and $\pi_i'(z_i) > 0$. The individual rationality condition implies that capability cutoffs exists, x_L, z_{1L}, and z_{2L}, such that

$$z_{iL} = m_i(x_L),\ \theta_L = x_L z_{1L} z_{2L} \text{ and } \pi_X(x_L) = \pi_i(z_{iL}) = 0. \tag{A.7}$$

Integrating the first order condition (A.6) with an initial condition (A.7), I obtain final producer's profits as:

$$\begin{aligned}
\pi_X(x) &= \pi_X(x_L) + \int_{x_L}^{x} \pi_X'(t)dt \\
&= \int_{x_L}^{x} \pi_X'(t)dt \\
&= A\int_{x_L}^{x} m_1(x)m_2(x)\,dt.
\end{aligned} \tag{A.8}$$

Integrating the first order condition (A.5) with an initial condition (A.7), I obtain Z_i supplier's profits as:

$$\begin{aligned}
\pi_i\left(m_i\left(x\right)\right) &= \pi_i(m_i(x_L)) + \int_{x_L}^{x} \pi_i'(m_i(t))m_i'(t)dt \\
&= \int_{x_L}^{x} \pi_i'(m_i(t))m_i'(t)dt \\
&= A\int_{x_L}^{x} x m_j(t)m_i'(t)dt \\
&= A\int_{x_L}^{x} m_1(t)m_2\left(t\right)\eta_i(t)dt,
\end{aligned} \tag{A.9}$$

where $\eta_i(x) \equiv x m_i'(x)/m_i(x)$.

Stable matching functions satisfy the matching market clearing conditions:

$$M_{Xe}\left[1 - G\left(x\right)\right] = M_{ie}\left[1 - G\left(m_i\left(x\right)\right)\right] \text{ for all } x \geq x_L. \tag{A.10}$$

The left hand side in (A.10) is the mass of final producers with higher capability than x, while the right hand side in (A.10) is the mass of Z_i suppliers with higher capability than $m_i\left(x\right)$. PAM requires equation (A.10) to hold for all final producers with higher capability than the domestic cutoff.

From the Pareto distribution, the matching market clearing condition is written as: for all $x \geq x_L$,

$$\begin{aligned}
&M_{Xe}\left[1 - G\left(x\right)\right] = M_{ie}\left[1 - G\left(m_i\left(x\right)\right)\right] \\
&\quad \Leftrightarrow \frac{M_{Xe}}{x^k} = \frac{M_{ie}}{m_i(x)^k} \\
&\quad \Leftrightarrow m_i(x) = \left(\frac{M_{ie}}{M_{Xe}}\right)^{1/k} x.
\end{aligned} \tag{A.11}$$

Substituting x_L into (A.11), I obtain

$$z_{iL} = m_i(x_L) = \left(\frac{M_{ie}}{M_{Xe}}\right)^{1/k} x_L \text{ and } \frac{m_i(x)}{z_{iL}} = \frac{x}{x_L}. \tag{A.12}$$

From (A.12), I obtain

$$\frac{\theta_X\left(x\right)}{\theta_L} = \frac{x m_1(x) m_2(x)}{x_L z_{IL} z_{2L}} = \left(\frac{x}{x_L}\right)^3. \tag{A.13}$$

The elasticity of matching function is $\eta_i(x) = 1$. Then, (A.8) and (A.9) imply $\pi_X(x) = \pi_i(m_i(x)) =$

$\Pi(\theta_X(x))/3$, which is further simplified from (13) as:

$$\begin{aligned}\pi_X(x) = \pi_i(m_i(x)) &= \frac{1}{3}\Pi(\theta_X(x)) \\ &= \frac{1}{3}\left[A\theta_X(x) - f\right] \\ &= \frac{f}{3}\left[\frac{\theta_X(x)}{\theta_L} - 1\right] \\ &= \frac{f}{3}\left[\left(\frac{x}{x_L}\right)^3 - 1\right]\end{aligned}$$

Entry Stage

Since firms are risk neutral, the expect profit from entry is equal to entry costs in equilibrium:

$$[1 - G(x_L)]\,\bar{\pi}_X = f_{Xe} \text{ and } [1 - G(z_{iL})]\,\bar{\pi}_i = f_{ie}, \tag{A.14}$$

where $\bar{\pi}_X = [1 - G(x_L)]^{-1} \int_{x_L}^{\infty} \pi_X(t)\, dG(t)$ and $\bar{\pi}_i = [1 - G(z_{iL})]^{-1} \int_{z_{iL}}^{\infty} \pi_i(t)\, dG(t)$ are the average profits of active firms.

From integration by parts and $\frac{m_i(x)}{z_{iL}} = \frac{x}{x_L}$, the average profits is simplified as:

$$\begin{aligned}\bar{\pi}_X &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \pi_X(t)\, g(t) dt \\ &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \pi_X'(t)\,[1 - G(t)]\, dt - \frac{1}{1 - G(x_L)}\left[\pi_X(t)\,[1 - G(t)]|_{x_L}^{\infty}\right] \\ &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \pi_X'(t)\,[1 - G(t)]\, dt \\ &= \frac{1}{1 - G(x_L)} A \int_{x_L}^{\infty} m_1(t) m_2(t)\,[1 - G(t)]\, dt \\ &= \frac{f}{1 - G(x_L)} \int_{x_L}^{\infty} \frac{t m_1(t) m_2(t)}{x_L z_{1L} z_{2L}}\left[\frac{1 - G(t)}{t}\right] dt \\ &= \frac{f}{1 - G(x_L)} \int_{x_L}^{\infty} \left(\frac{t}{x_L}\right)^3 \left[\frac{1 - G(t)}{t}\right] dt \\ &= x_L^{k-3} f \int_{x_L}^{\infty} t^{-k+2} dt \\ &= x_L^{k-3} f \left(\frac{x_L^{3-k}}{k-3}\right) \\ &= \frac{f}{k-3}.\end{aligned}$$

Since

$$\begin{aligned}
\left(\frac{g(m_i(x))}{1-G(z_{iL})}\right)m_i'(x) &= \left(\frac{kz_{iL}^k}{m_i(x)^{k+1}}\right)m_i'(x) \\
&= \left(\frac{k}{x}\right)\left(\frac{z_{iL}}{m_i(x)}\right)^k\left(\frac{m_i'(x)x}{m_i(x)}\right) \\
&= \left(\frac{k}{x}\right)\left(\frac{x_L}{x}\right)^k \text{ from (A.12) and } \eta_i(x) = xm_i'(x)/m_i(x) = 1 \\
&= \frac{kx_L^k}{x^{k+1}} \\
&= \frac{g(x)}{1-G(x_L)}
\end{aligned}$$

and $\pi_X(x) = \pi_i(m_i(x))$, the average profit of Z_i suppliers is written as:

$$\begin{aligned}
\bar{\pi}_i &= \frac{1}{1-G(z_{iL})}\int_{z_L}^{\infty}\pi_i\left(z_i\right)g(z_i)dz_i \\
&= \frac{1}{1-G(z_{iL})}\int_{x_L}^{\infty}\pi_i\left(m_i(x)\right)g(m_i(x))m_i'(x)dx \text{ (from } z_i = m_i\left(x\right)) \\
&= \int_{x_L}^{\infty}\pi_i\left(m_i(x)\right)\left(\frac{g(m_i(x))m_i'(x)}{1-G(z_{iL})}\right)dx. \\
&= \int_{x_L}^{\infty}\pi_X\left(x\right)\left(\frac{g(x)}{1-G(x_L)}\right)dx \\
&= \bar{\pi}_X \\
&= \frac{f}{k-3}
\end{aligned}$$

Free entry also implies $M\bar{r} = L$, where M is the mass of active teams and $\bar{r}$ is the average team revenue. Since the average revenue is:

$$\begin{aligned}
\bar{r} &= \frac{1}{1-G(x_L)}\int_{x_L}^{\infty}r(\theta_X\left(x\right))g(x)dx \\
&= \frac{1}{1-G(x_L)}\int_{x_L}^{\infty}\sigma\left[\Pi\left(\theta_X\left(x\right)\right)+f\right]g(x)dx \\
&= \sigma\left(\bar{\pi}_X+\bar{\pi}_1+\bar{\pi}_2+f\right) \\
&= \frac{\sigma kf}{k-3}, \qquad \text{(A.15)}
\end{aligned}$$

The mass of varieties is

$$M = \frac{(k-3)\,L}{k\sigma f}.$$

The cutoff capabilities are obtained from the free entry as

$$\begin{aligned}
x_L &= \left(\frac{1}{1-G(x_L)}\right)^{1/k} = \left(\frac{\bar{\pi}_X}{f_{Xe}}\right)^{1/k} = \left[\frac{f}{f_{Xe}\,(k-3)}\right]^{1/k} \\
z_{iL} &= \left(\frac{1}{1-G(z_{iL})}\right)^{1/k} = \left(\frac{\bar{\pi}_i}{f_{ie}}\right)^{1/k} = \left[\frac{f}{f_{ie}\,(k-3)}\right]^{1/k}.
\end{aligned}$$

From $M = M_{Xe}[1-G(x_L)] = M_{ie}\,[1-G(z_{iL})]$, the mass of entrants are obtained as:

$$\begin{aligned}
M_{Xe} &= \frac{M}{1-G(x_L)} = \frac{L}{f_{Xe}\,(k-3)} \text{ and} \\
M_{ie} &= \frac{M}{1-G(z_{iL})} = \frac{L}{f_{ie}\,(k-3)}.
\end{aligned}$$

From these, matching functions become:

$$m_i(x) = \left(\frac{M_{ie}}{M_{Xe}}\right)^{1/k} x = \left(\frac{f_{Xe}}{f_{ie}}\right)^{1/k} x. \tag{A.16}$$

A2. Welfare Theorem

I prove the welfare theorem for a costly trade equilibrium and consider the cases of autarky and free trade as its special cases when $f_T = \infty$ and $f_T = 0$. Suppose the social planner maximizes a symmetric increasing and concave function of U and U^*. Because Home and Foreign are symmetric, the optimal solution always requires $U = U^*$. Therefore, I consider a problem maximizing U by choosing allocation in the Home market.

In the long run problem, the planner maximizes U chooses the following variables: (V1) Production of teams with capability θ, $q(\theta)$; (V2) the distribution function $H(\theta)$ of θ that is generated by feasible matching; (V3) Mass of international teams M_T, capability cut-offs, x_L, z_{1L} and z_{2L}, and Mass of entrants M_{Xe}, M_{1e}, and M_{2e}. In the short run problem, the planner chooses (V3') Mass of international teams M_T and capability cut-offs, x_L, z_{1L} and z_{2L} for given M_{Xe}, M_{1e}, and M_{2e} instead of (V3). Notice that because of the mirror image structure, finding these variables for Home is sufficient. Following Dhingra and Morrow (2014), I solve the problem in step by step. Step 1 finds an optimal (V1) for given (V2)-(V3) and substitute it to the objective function. Step 2 finds an optimal (V2) maximizing the new objective function for given (V3) and substitute it to the objective function. Step 3 find an optimal (V3). Then, I compare the solution to a decentralized market equilibrium.

In Step1, the planner faces the resource constraint.

$$L = L_e + M \int_{\theta_L}^{\infty} \left[q(\theta)\theta^{\beta} + f\right] dH(\theta) + M_T f_T.$$

where $L_e \equiv M_{Xe} f_{Xe} + \sum_{i=1,2} M_{ie} f_{ie}$ is labor for entry costs, the second term is labor for production, and the third term is fixed trade costs. The team capability cutoff θ_L and the mass of Home international teams M_T are chosen in Step 2. The mass of varieties $M = M_{Xe}[1 - G(x_L)]$ is chosen in Step 3. Entry costs L_e is chosen in Step 4. Therefore, the social planner solves

$$\begin{aligned} \max W \equiv U^\rho =& M \int_{x_L}^{\infty} \theta^\alpha q(\theta)^\rho dH(x) \\ \text{subject to } & M \int_{\theta_L}^{\infty} q(\theta)\theta^\beta dH(\theta) \leq L - L_e - Mf - M_T f_T. \end{aligned} \tag{A.17}$$

The first order condition is

$$\rho\theta^\alpha q(\theta)^{\rho-1} - \lambda\theta^\beta = 0 \text{ for all } \theta \geq \theta_L,$$

where $\lambda > 0$ is the Lagrange multiplier. From the first order condition, I obtain

$$q(\theta) = \left(\frac{\rho}{\lambda}\right)^\sigma \theta^{(\alpha-\beta)\sigma}. \tag{A.18}$$

Substituting this into the resource constraint (A.17) and $\gamma = (\alpha - \beta)\,\sigma + \beta = 1$ lead to

$$\begin{aligned} \left(\frac{\rho}{\lambda}\right)^\sigma M \int_{\theta_L}^{\infty} \theta^{(\alpha-\beta)\sigma+\beta} dH(\theta) = L - L_e - Mf - M_T f_T \\ \left(\frac{\rho}{\lambda}\right)^\sigma = \frac{L - L_e - Mf - M_T f_T}{M \int_{\theta_L}^{\infty} \theta dH(\theta)}. \end{aligned}$$

Substituting (A.18) into W and using $\gamma = (\alpha - \beta)\,\sigma + \beta = 1$, I obtain

$$\begin{aligned} W &= \left(\frac{\rho}{\lambda}\right)^{\sigma-1} M \int_{\theta_L}^{\infty} \theta dH(x) \\ &= \left(\frac{L - L_e - Mf - M_T f_T}{M \int_{\theta_L}^{\infty} \theta dH(\theta)}\right)^{\frac{\sigma-1}{\sigma}} M \int_{\theta_L}^{\infty} \theta dH(x). \\ &= \left(M \int_{\theta_L}^{\infty} \theta dH(x)\right)^{\frac{1}{\sigma}} (L - L_e - Mf - M_T f_T)^{\frac{\sigma-1}{\sigma}}. \end{aligned}$$

In Step 2, when $M = M_{Xe}[1 - G(x_L)]$, M_T, and mass of firms are given, the planner chooses $H(\theta)$ and θ_L to maximize $M \int_{\theta_L}^{\infty} \theta dH(x)$. Since Koopmans and Beckmann (1957) and Shapley and Shubik (1972), a stable matching, that is, PAM is known to maximize the aggregate payoffs $M \int_{\theta_L}^{\infty} \theta dH(x) - Mf - M_T f_T$. Gretsky, Ostroy and Zame (1992) prove this result when agents are continuums as in the current model. Since M and M_T are given in this step, PAM actually maximizes the aggregate team capability $M \int_{\theta_L}^{\infty} \theta dH(x)$. Therefore, the social planner chooses PAM.

When the mass of international teams is given at M_T, PAM is expressed as matching functions $m_i(x)$ satisfying market clearing conditions (A.36) and (A.37) in the costly trade equilibrium. I write these conditions again:

$$\begin{aligned}
2M_{Xe}[1-G(x)] &= (M_{ie}+M_{ie}^{*})\,[1-G(m_i(x))] \text{ for } x \geq x_T \\
M_{Xe}[1-G(x)] &= M_{1e}[1-G(m_1(x))] - M_T = M_{2e}[1-G(m_2(x))] + M_T \text{ for } x \in [x_L, x_T],
\end{aligned} \tag{A.19}$$

where x_T is given by $M_T = s_X M_{Xe}[1-G(x_T)]$.

Since the aggregate team capability becomes $M\int_{\theta_L}^{\infty}\theta dH(x) = M_{Xe}\int_{x_L}^{\infty}\theta_X(x)\,g(x)dx$, the objective function for Step 3 becomes

$$W = \left(M_{Xe}\int_{x_L}^{\infty}\theta_X(x)\,g(x)dx\right)^{\frac{1}{\sigma}}(L - L_e - M_X[1-G(x_L)]f - M_T f_T)^{\frac{\sigma-1}{\sigma}}.$$

Long run problem I first consider Step 3 for the long run problem. Define $v_i \equiv M_{ie}/M_{Xe}$ and $v_T \equiv M_T/M_{Xe}$. Then, conditions (A.19) are expressed as

$$\begin{aligned}
1-G(x) &= \left(\frac{v_1+v_2}{2}\right)[1-G(m_i(x))] \text{ for } x \geq x_T \\
1-G(x) &= v_1[1-G(m_1(x))] - v_T = v_2[1-G(m_2(x))] + v_T \text{ for } x \in [x_L, x_T].
\end{aligned}$$

These conditions determine matching functions as $m_1(x, v_1, v_2, v_T)$ and $m_2(x, v_1, v_2, v_T)$, which is differentiable at $x \in (x_L, x_T) \cup (x_T, \infty)$.

For $x > x_T$, total differentiation leads to the partial derivatives of m_i

$$\begin{aligned}
\frac{\partial m_i}{\partial x} &= \frac{2g(x)}{(v_1+v_2)\,g(m_i(x))} > 0 \\
\frac{\partial m_i}{\partial v_1} &= \frac{\partial m_i}{\partial v_2} = \frac{[1-G(m_i(x))]}{(v_1+v_2)\,g(m_i(x))} = \frac{\partial m_i}{\partial x}\frac{[1-G(m_i(x))]}{2g(x)} > 0 \\
\frac{\partial m_i}{\partial v_T} &= 0.
\end{aligned}$$

For $x < x_T$, total differentiation leads to the partial derivatives of m_i

$$\begin{aligned}
\frac{\partial m_i}{\partial x} &= \frac{g(x)}{v_i g(m_i(x))} > 0 \\
\frac{\partial m_i}{\partial v_i} &= \frac{1-G(m_i(x))}{v_i g(m_i(x))} = \frac{\partial m_i}{\partial x}\frac{[1-G(m_i(x))]}{g(x)}. \\
\frac{\partial m_1}{\partial v_T} &= -\frac{1}{v_1 g(m_1(x))} = -\frac{1}{g(x)}\frac{\partial m_1}{\partial x} \\
\frac{\partial m_2}{\partial v_T} &= \frac{1}{v_2 g(m_2(x))} = \frac{1}{g(x)}\frac{\partial m_2}{\partial x}.
\end{aligned}$$

Define

$$\Theta\left(x_L, v_1, v_2, v_T\right) \equiv \int_{x_L}^{\infty} \theta\left(x, m_1(x, v_1, v_2, v_T), m_2(x, v_1, v_2, v_T)\right) g(x) dx.$$

Then, the partial derivatives of Θ are obtained as follows:

$$\begin{aligned}
\frac{\partial \Theta}{\partial x_L} &= -\hat{\theta}\left(x_L, v_1, v_2, v_T\right) g(x_L) = -\theta_L g(x_L) \\
\frac{\partial \Theta}{\partial v_1} &= \int_{x_L}^{\infty} \frac{\partial \hat{\theta}\left(t, v_1, v_2, v_T\right)}{\partial v_1} g(t) dt \\
&= \int_{x_L}^{x_T} \frac{\partial \theta}{\partial z_1} \frac{\partial m_1}{\partial x} \left[1 - G(m_i\left(t\right))\right] dt + \frac{1}{2} \int_{x_T}^{\infty} \left(\frac{\partial \theta}{\partial z_1} + \frac{\partial \theta}{\partial z_2}\right) \frac{\partial m_1}{\partial x} \left[1 - G(m_1\left(t\right))\right] dt \\
&= \int_{x_L}^{\infty} \frac{\partial \theta}{\partial z_1} \frac{\partial m_1}{\partial x} \left[1 - G(m_i\left(t\right))\right] dt \text{ since } \frac{\partial \theta}{\partial z_1} = \frac{\partial \theta}{\partial z_2} \text{ for } x \geq x_T \\
\frac{\partial \Theta}{\partial v_2} &= \int_{x_L}^{\infty} \frac{\partial \theta}{\partial z_2} \frac{\partial m_2}{\partial x} \left[1 - G(m_2\left(t\right))\right] dt \\
\frac{\partial \Theta}{\partial v_T} &= \int_{x_L}^{x_T} \left[\frac{\partial \theta}{\partial z_2} \frac{\partial m_2}{\partial x} - \frac{\partial \theta}{\partial z_2} \frac{\partial m_2}{\partial x}\right] dt. \qquad \text{(A.20)}
\end{aligned}$$

Using v_i, v_T and Θ, the objective function becomes

$$\begin{aligned}
\ln W &= \frac{1}{\sigma}\left[\ln M_{Xe} + \ln \Theta\left(x_L, v_1, v_2, v_T\right)\right] \\
&\quad + \frac{\sigma - 1}{\sigma} \ln \left[L - M_{Xe}\left(f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T\right)\right].
\end{aligned}$$

I first find an optimal M_{Xe} maximizing this function. The first order condition is

$$-\left(\frac{\sigma - 1}{\sigma}\right) \frac{f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T}{L - M_{Xe}\left[f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T\right]} + \frac{1}{\sigma M_{Xe}} = 0.$$

The mass of entrants of final producers is obtained as

$$M_{Xe} = \frac{L}{\sigma\left[f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T\right]}. \qquad \text{(A.21)}$$

Since

$$L - M_{Xe}\left[f_{Xe} + \sum_{i=1,2} v_i f_{ie} + v_T f_T + \left[1 - G(x_L)\right] f\right] = L - \frac{L}{\sigma} = \left(\frac{\sigma - 1}{\sigma}\right) L,$$

the objective function becomes

$$\sigma \ln W = \ln \Theta \left(x_L, v_1, v_2, v_T\right) - \ln \left[f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T \right] + constant. \tag{A.22}$$

Finally, I find x_T, x_L, v_1, and v_2 that maximize (A.22). The first order conditions are

$$\begin{aligned}
\frac{\partial \sigma \ln W}{\partial x_L} &= \frac{1}{\Theta}\frac{\partial \Theta}{\partial x_L} + \frac{g(x_L) f}{f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_1} &= \frac{1}{\Theta}\frac{\partial \Theta}{\partial v_1} - \frac{f_{1e}}{f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_2} &= \frac{1}{\Theta}\frac{\partial \Theta}{\partial v_2} - \frac{f_{2e}}{f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_T} &= \frac{1}{\Theta}\frac{\partial \Theta}{\partial v_T} - \frac{f_T}{f_{Xe} + \sum_{i=1,2} v_i f_{ie} + [1 - G(x_L)] f + v_T f_T} = 0.
\end{aligned}$$

Using (A.20) and (A.21), these conditions become

$$\begin{aligned}
\frac{\partial \sigma \ln W}{\partial x_L} &= -\frac{\theta_L g(x_L)}{\Theta} + g(x_L) f \frac{\sigma M_{Xe}}{L} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_1} &= \frac{1}{\Theta} \int_{x_L}^{\infty} \frac{\partial \theta}{\partial z_1}\frac{\partial m_1}{\partial x} \left[1 - G(m_i(x))\right] dx - f_{1e}\frac{\sigma M_{Xe}}{L} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_2} &= \frac{1}{\Theta} \int_{x_L}^{\infty} \frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x} \left[1 - G(m_2(x))\right] dx - f_{2e}\frac{\sigma M_{Xe}}{L} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_T} &= \frac{1}{\Theta} \int_{x_L}^{x_T} \left[\frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x} - \frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x}\right] dx - f_T\frac{\sigma M_{Xe}}{L} = 0.
\end{aligned}$$

These conditions are further simplified a

$$\begin{aligned}
\frac{L}{\sigma M_{Xe}\Theta}\theta_L &= f \\
\frac{L}{\sigma M_{Xe}\Theta} \int_{x_L}^{\infty} \frac{\partial \theta}{\partial z_1}\frac{\partial m_1}{\partial x} \left[1 - G(m_i(x))\right] dx &= f_{1e} \\
\frac{L}{\sigma M_{Xe}\Theta} \int_{x_L}^{\infty} \frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x} \left[1 - G(m_2(x))\right] dx &= f_{2e} \\
\frac{L}{\sigma M_{Xe}\Theta} \int_{x_L}^{x_T} \left[\frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x} - \frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x}\right] dx &= f_T.
\end{aligned}$$

Decentralized equilibrium Finally, I compare these first order conditions with conditions characterizing a decentralized equilibrium. First, the price index becomes

$$\begin{aligned}P &= \left[\int_{\omega\in\Omega} p(\omega)^{1-\sigma}\theta\left(\omega\right)^{\alpha\sigma} d\omega\right]^{1/(1-\sigma)} \\ &= \frac{1}{\rho}\left[M_{Xe}\int_{x_L}^{\infty}\theta_X\left(x\right)g(x)dx\right]^{1/(1-\sigma)} \\ &= \frac{1}{\rho}\left[M_{Xe}\Theta\right]^{1/(1-\sigma)}.\end{aligned}$$

The market condition A is written as:

$$\begin{aligned}A &= \frac{I}{\sigma}\left(\rho P\right)^{\sigma-1} \\ &= \frac{L}{\sigma M_{Xe}\Theta}.\end{aligned} \tag{A.23}$$

From integration by parts, the average profits of Z_i suppliers can be written as

$$\begin{aligned}f_{ie} &= \int_{z_{iL}}^{\infty}\pi_i\left(z\right)g(z)dz \\ &= \int_{x_L}^{\infty}\pi_i\left(m_i\left(t\right)\right)g(m_i\left(t\right))m_i'\left(t\right)dt \text{ (substitution of } z=m_i(t)) \\ &= \int_{x_L}^{\infty}\pi_i'\left(m_i\left(t\right)\right)m_i'\left(t\right)\left[1-G(m_i\left(t\right))\right]dt \text{ (integration by parts)} \\ &= A\int_{x_L}^{\infty}\frac{\partial\theta}{\partial z_1}\frac{\partial m_i}{\partial x}\left[1-G(m_i\left(t\right))\right]dt\end{aligned}$$

The no arbitrage condition (28) can be written as

$$\begin{aligned}f_T &= \pi_2(z_T)-\pi_1(z_T) \\ &= \int_{x_L}^{x_T}\pi_2'(m_2(t))m_2'(t)dt-\int_{x_L}^{x_T}\pi_1'(m_1(t))m_1'(t)dt \\ &= A\int_{x_L}^{x_T}\left(\frac{\partial\theta}{\partial z_2}\frac{\partial m_2}{\partial x}-\frac{\partial\theta}{\partial z_2}\frac{\partial m_2}{\partial x}\right)dt.\end{aligned}$$

Using (A.23), the cutoff condition, the free entry conditions, and the no arbitrage conditions become

$$\begin{aligned}\frac{L}{\sigma M_{Xe}\Theta}\theta_L &= f \\ \frac{L}{\sigma M_{Xe}\Theta}\int_{x_L}^{\infty}\frac{\partial\theta}{\partial z_1}\frac{\partial m_1}{\partial x}\left[1-G(m_i\left(x\right))\right]dx &= f_{1e} \\ \frac{L}{\sigma M_{Xe}\Theta}\int_{x_L}^{\infty}\frac{\partial\theta}{\partial z_2}\frac{\partial m_2}{\partial x}\left[1-G(m_2\left(x\right))\right]dx &= f_{2e} \\ \frac{L}{\sigma M_{Xe}\Theta}\int_{x_L}^{x_T}\left[\frac{\partial\theta}{\partial z_2}\frac{\partial m_2}{\partial x}-\frac{\partial\theta}{\partial z_2}\frac{\partial m_2}{\partial x}\right]dx &= f_T.\end{aligned}$$

These are the first conditions for the social planner's problem. Therefore, a decentralized long run costly trade equilibrium maximizes the world welfare. In autarky, this means that a decentralized equilibrium maximizes the welfare in each country.

Short run problem Finally, I consider Step 3 for the short run problem. When the mass of entrants are fixed, the planner's problem is to choose x_L and v_T maximizing

$$W = \left(M_{Xe}\Theta(x_L, v_1, v_2, v_T)\right)^{\frac{1}{\sigma}} \left(L - L_e - M_{Xe}\left(v_T f_T + [1 - G(x_L)]\, f\right)\right)^{\frac{\sigma-1}{\sigma}}.$$

This is equivalent to maximize

$$\sigma \ln W = \ln \Theta(x_L, v_1, v_2, v_T) + (\sigma - 1)\ln[L - L_e - M_{Xe}\left(v_T f_T + [1 - G(x_L)]\, f\right)]$$

The first order condition is

$$\begin{aligned}
\frac{\partial \sigma \ln W}{\partial x_L} &= -\frac{\theta_L g(x_L)}{\Theta} + (\sigma - 1)\frac{M_{Xe} g(x_L) f}{L - L_e - M_{Xe}\left(v_T f_T + [1 - G(x_L)]\, f\right)} = 0 \\
\frac{\partial \sigma \ln W}{\partial v_T} &= \frac{1}{\Theta}\int_{x_L}^{x_T}\left[\frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x} - \frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x}\right] dx - (\sigma - 1)\frac{M_{Xe} f_T}{L - L_e - M_{Xe}\left(v_T f_T + [1 - G(x_L)]\, f\right)} = 0
\end{aligned} \tag{A.24}$$

The resource constraint implies

$$\begin{aligned}
L - L_e - M_{Xe} v_T f_T &= M_{Xe}\int_{x_L}^{\infty} C(\theta_X(x)) g(x) dx \\
&= (\sigma - 1)\, A M_{Xe}\int_{x_L}^{\infty}\theta_X(x) g(x) dx + M_{Xe}\left[1 - G(x_L)\right] f \\
&= (\sigma - 1)\, A M_{Xe}\Theta + M_{Xe}\left[1 - G(x_L)\right] f.
\end{aligned}$$

Therefore, it holds that

$$L - L_e - M_{Xe}\left[v_T f_T + [1 - G(x_L)]\, f\right] = (\sigma - 1)\, A M_{Xe}\Theta.$$

Substituting the first order conditions (A.24) leads to

$$\begin{aligned}
A\theta_L &= f \\
A\int_{x_L}^{x_T}\left[\frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x} - \frac{\partial \theta}{\partial z_2}\frac{\partial m_2}{\partial x}\right] dx &= f_T.
\end{aligned}$$

These conditions are the cutoff condition and the no arbitrage condition in a short run costly trade equilibrium. Therefore, the solution to the social planner's problem and a decentralized costly trade equilibrium achieve the same allocation.

A3. Proofs for Lemmas and Propositions

Preparation for the proofs for Lemmas 1, 2, and 3

In the following, I assume $M_{1e} > M_{2e}$ and prove it in Lemma 4. I first introduce the concept of exportable Z_i suppliers.

Definition 1. Home Z_i suppliers with capability z_i are called "exportable" if $\pi_i(z_i) + f_T = \pi_i^*(z_i)$ and "non-exportable" if $\pi_i(z_i) + f_T > \pi_i^*(z_i)$. Similarly, Foreign Z_i suppliers with capability z_i are called exportable if $\pi_i^*(z_i) + f_T = \pi_i(z_i)$ and non-exportable if $\pi_i^*(z_i) + f_T > \pi_i(z_i)$.

The exportability is a necessary (but not sufficient) condition for Z_i suppliers to export. When Home Z_i suppliers with capability z_i are exportable, Foreign final producers are indifferent between Home Z_i suppliers with capability z_i and Foreign Z_i suppliers with the same capability. When Home Z_i suppliers with capability z_i are non-exportable, Foreign final producers strictly prefer Foreign Z_i suppliers with capability z_i to Home Z_i suppliers with the same capability. Therefore, non-exportable Z_i suppliers never export.

Proofs for Lemmas1, 2, and 3 use the following Claims 1, 2, 3, 4, and 5.

Claim 1. All Z_i suppliers are classified as either exportable or non-exportable.

Proof. Suppose $\pi_i(z_i) + f_T < \pi_i^*(z_i)$ holds. Then, no final producer would choose Foreign Z_i suppliers with capability z_i. Therefore, $\pi_i(z_i) + f_T < \pi_i^*(z_i)$ never holds and all Home final producers are classified as either exportable or non-exportable. From the symmetry of the two countries, all Foreign final producers are also classified as either exportable or non-exportable. □

Claim 2. A Z_1 supplier matches with a Z_2 supplier with the same capability if either one of them is exportable.

Proof. From the symmetry of Home and Foreign, it is sufficient to consider the following two cases: (i) an exportable Home Z_i supplier matches with a Foreign Z_j supplier; (ii) an exportable Home Z_i supplier matches with a Home Z_j supplier.

Case (i): Suppose an exportable Home Z_i supplier with capability z_i matches with a Foreign Z_j supplier with capability $z_j \neq z_i$ and a final producer with capability x. From the stability condition (6), the profit of the final producer satisfies the following inequality:

$$\pi_X(x) = \pi_X^*(x) = \Pi(xz_iz_j) - \pi_i(z_i) - \pi_j^*(z_j) - f_T \geq \max_{z_i', z_j'} \Pi\left(xz_i'z_j'\right) - \pi_i\left(z_i'\right) - \pi_j^*\left(z_j'\right) - f_T.$$

From $\pi_j^*(z) = \pi_i(z)$, the above inequality holds if and only if :

$$\Pi(xz_iz_j) - \pi_i(z_i) - \pi_i(z_j) \geq \max_{z_i', z_j'} \Pi\left(xz_i'z_j'\right) - \pi_i\left(z_i'\right) - \pi_i\left(z_j'\right). \tag{A.25}$$

Because the second order condition for maximization in the right hand side of (A.25) requires $\pi_i''(z) > 0$, i.e., $\pi_i(z)$ is a convex function. Let $\tilde{z} \equiv (z_i + z_j)/2$. The convexity of π_i and the quasi-concavity of $\Pi(xz_iz_j)$ imply:

$$\Pi(xz_iz_j) - \pi_i(z_i) - \pi_i(z_j) < \Pi(x\tilde{z}\tilde{z}) - 2\pi_i(\tilde{z}).$$

This inequality contradicts with (A.25). Therefore, only $z_i = z_j$ satisfies (A.25).

Case (ii): an exportable Home Z_i supplier with z_i matches with a Home Z_j supplier with z_j. Suppose $z_i \neq z_j$ and they match with a final producer with x. If the final producer is from Home, the profit satisfies:

$$\pi_X(x) = \Pi(xz_iz_j) - \pi_i(z_i) - \pi_j(z_j) \geq \max_{z_i', z_j'} \Pi\left(xz_i'z_j'\right) - \pi_i\left(z_i'\right) - \pi_j\left(z_j'\right).$$

If the final producer is from Foreign, the profit satisfies:

$$\pi_X^*(x) = \Pi(xz_iz_j) - \pi_i(z_i) - \pi_j(z_j) - 2f_T \geq \max_{z_i', z_j'} \Pi\left(xz_i'z_j'\right) - \pi_i\left(z_i'\right) - \pi_j\left(z_j'\right) - 2f_T.$$

From the mirror-image symmetry, Foreign Z_j supplier with z_i is also exportable, i.e. $\pi_j^*(z_i) + f_T = \pi_j(z_i)$. Because $\pi_j^*(z_i) = \pi_i(z_i)$, this implies $\pi_j(z_i) = \pi_i(z_i) - f_T$. Then, the last two inequalities hold if and only if (A.25) holds. From the argument in case (i), $z_i = z_j$ must hold to satisfy (A.25). □

Claim 3. All suppliers in the CD sectors are non-exportable.

Proof. Suppose Home Z_2 suppliers with capability z are exportable. From Claim 2, Home Z_2 suppliers with capability z match with Home Z_1 suppliers with the same capability. The matching market clearing condition between Home Z_1 suppliers and Home Z_2 suppliers is expressed as:

$$M_{1e} \int_z^\infty s_1^D(t)\, g(t)dt = M_{2e} \int_z^\infty s_2^D(t)\, g(t)dt, \tag{A.26}$$

where $s_i^D(z_i)$ is the share of Home Z_i suppliers matching with Home Z_j suppliers among Home Z_i suppliers with capability z_i. From the definition of exportability, $s_i^D(z_i) < 1$ holds only when Home Z_i suppliers with capability z_i are exportable.

I first show that for given z, only one of either $s_1^D(z)$ or $s_2^D(z)$ can be smaller than unity. Suppose that Home Z_i suppliers with capability z are exportable, i.e. $\pi_i^*(z) = \pi_i(z) + f_T$. The mirror-image structure implies that $\pi_j(z) = \pi_j^*(z) + f_T$. This means that Home Z_j suppliers with capability z are non-exportable because $\pi_j(z) + f_T = \pi_j^*(z) + 2f_T > \pi_j^*(z)$. Therefore, if $s_i^D(z) < 1$, then $s_j^D(z) = 1$.

A differentiation of (A.26) with respect to z leads to:

$$M_{1e}s_1^D(z) = M_{2e}s_2^D(z). \tag{A.27}$$

From $M_{1e} > M_{2e}$, only a combination of $s_2^D(z) = 1$ and $s_1^D(z) = M_{2e}/M_{1e} < 1$ satisfies condition (A.27). Therefore, all Home Z_2 suppliers are non-exportable. From the mirror-image structure, all Foreign Z_1 suppliers are also non-exportable. □

Claim 4. An inequality $m_1(x) \geq m_2(x)$ holds for all $x \geq x_L$. A strict inequality $m_1(x) > m_2(x)$ holds if there exist a positive mass of non-exportable Z_1 suppliers with higher capability than $m_1(x)$.

Proof. Because $s_2^D(z) = 1$ for all $z \geq z_{2L}$ from Claim 3, the market clearing condition (A.26) becomes:

$$M_{1e}\int_{m_1(x)}^{\infty} s_1^D(t)\,g(t)dt = M_{2e}[1 - G(m_2(x))] \text{ for all } x \geq x_L. \tag{A.28}$$

Dividing both sides by M_{2e} and adding $G(m_1(x)) - 1$ to both sides, I obtain:

$$\frac{M_{1e}}{M_{2e}}\int_{m_1(x)}^{\infty}\left(s_1^D(t) - \frac{M_{2e}}{M_{2e}}\right)g(t)dt = G(m_1(x)) - G(m_2(x)) \text{ for all } x \geq x_L. \tag{A.29}$$

Because $s_1^D(z) \geq M_{2e}/M_{1e}$ for all $x \geq x_L$ from (A.27) with $s_2^D(z) = 1$ from Claim 3, the left-hand side of (A.29) is non-negative for all $x \geq x_L$ and strictly positive if there exist a positive mass of non-exportable Z_1 suppliers with higher capability than $m_1(x)$. □

Claim 5. An inequality $\pi_2'(z) \geq \pi_1'(z)$ hold for all z. The inequality becomes strict if there exist a positive mass of non-exportable Z_1 suppliers with higher capability than z.

Proof. (Case 1) Suppose $z \geq \max\{z_{1L}, z_{2L}\}$. Consider two teams with bundles of capability parameters (x, z_1, z_2) and (x', z_1', z_2'), respectively. Suppose $z_2 = z_1' (\equiv \hat{z})$. Claim 4 implies that $z_1 \geq z_2 = z_1' \geq z_2'$. PAM implies $x \geq x'$ and $\theta_X(x) \geq \theta_X(x')$. Therefore, from the first order conditions (8), we obtain:

$$\pi_2'(\hat{z} = m_2(x)) = Axm_1(x) = \frac{A\theta_X(x)}{\hat{z}} \geq \pi_1'\left(\hat{z} = m_1(x')\right) = Ax'm_2(x') = \frac{A\theta_X(x')}{\hat{z}}. \tag{A.30}$$

Suppose there exist a positive mass of non-exportable Z_1 suppliers with higher capability than z. From Claim 4, $x > x'$ and $\theta_X(x) > \theta_X(x)$. Therefore, inequality (A.30) becomes strict.

(Case 2) Suppose $z_{1L} > z \geq z_{2L}$. Then, $\pi_1(z) = \pi_1'(z) = 0$ and $\pi_2'(z) \geq 0$ hold from the first order condition (8). □

Now I am ready to prove Lemmas 1, 2, and 3.

Proof for Lemma 1

Proof. There must exist a positive mass of exportable Home Z_1 suppliers in a trade equilibrium. Consider exportable Home Z_1 suppliers with capability $\tilde{z} > z_L$ in a trade equilibrium. Suppose there exists $z > \tilde{z}$ such that $\pi_1^*(z) - \pi_1(z) < f_T$ on the contrary to the Lemma. Because $\pi_1^*(z) = \pi_2(z)$ for all z and $\pi_1^*(\tilde{z}) - \pi_1(\tilde{z}) = f_T$, the difference in the profit schedules satisfies:

$$\begin{aligned}\pi_1^*(z) - \pi_1(z) &= \pi_1^*(\tilde{z}) - \pi_1(\tilde{z}) + \int_{\tilde{z}}^{z} \left[\pi_1^{*\prime}(u) - \pi_1'(u)\right] du \\ &= f_T + \int_{\tilde{z}}^{z} \left[\pi_2'(u) - \pi_1'(u)\right] du.\end{aligned}$$

The second term in the right-hand side must be non-negative from (A.30), which contradicts $\pi_1^*(z) - \pi_1(z) < f_T$. Therefore, if $\pi_1^*(z') - \pi_1(z') = f_T$ holds for some z', then $\pi_1^*(z) - \pi_1(z) \geq f_T$ holds for all $z \geq z'$. From Claim 1, this means that $\pi_1^*(z) - \pi_1(z) = f_T$ for all $z \geq z'$.

Notice that $z_{2L} = z_{1L}^* < z_{1L} = z_{2L}^*$ from $M_{1e} > M_{2e}$. From $\pi_1^*(z) = \pi_2(z)$ for all z, the difference in the profits of Z_1 suppliers between Home and Foreign is:

$$\pi_1^*(z) - \pi_1(z) = \pi_1^*(z_{2L}) - \pi_1(z_{2L}) + \int_{z_{2L}}^{z} \left[\pi_1^{*\prime}(u) - \pi_1'(u)\right] du.$$

From Clam 5, $\pi_1^{*\prime}(z) - \pi_1'(z) \geq 0$ for all z and $\pi_1^*(z_{2L}) - \pi_1(z_{2L}) = 0$. Therefore, there exists a threshold z_T such that $\pi_1^*(z) - \pi_1(z) = \pi_2(z) - \pi_2^*(z) = f_T$ for all $z \geq z_T$ and $\pi_1^*(z) - \pi_1(z) = \pi_2(z) - \pi_2^*(z) < f_T$ for all $z < z_T$. $\square$

Proof for Lemma 2

Proof. From Claim 3 and Lemma 1, Home Z_1 suppliers with $z_1 \geq z_T$ are exportable, but other Home Z_1 suppliers and all Home Z_2 suppliers are non-exportable. From PAM and Claim 2, there exists x_T such that $m_i(x_T) = z_T$ and $m_1(x) = m_2(x)$ for $x \geq x_T$.

The market clearing condition for matching between Home final producers and Z_1 suppliers is:

$$M_{Xe}[1 - G(x)] = M_{1e}[1 - G(m_1(x))] - M_{1e} \int_{m_1(x)}^{\infty} s_1(t)\, g(t) dt \text{ for all } x \geq x_L, \tag{A.31}$$

where $s_1(z)$ is the share of exporters among Home Z_1 suppliers with capability z. The second term in A.31 is the mass of Home Z_1 suppliers matching with Foreign final producers. The same condition for Foreign final producers is

$$M_{Xe}^*[1 - G(x)] = M_{1e}^*[1 - G(m_1^*(x))] + M_{1e} \int_{m_1^*(x)}^{\infty} s_1(t)\, g(t) dt \text{ for all } x \geq x_L. \tag{A.32}$$

The first term in the right hand side is the mass of Foreign Z_1 suppliers and the second term is the mass of Home Z_1 suppliers. Since $m_1(x) = m_1^*(x) (= m_2(x))$ for all $x \geq x_T$ from the mirror-image structure and Claim 2, adding (A.31) and (A.32) up obtains the global matching market clearing conditions (29). From the mirror-image structure, a similar condition holds for Z_2 producers.

Let M_T be the mass of Home Z_1 suppliers matching with Foreign final suppliers, which is given by $M_T = M_{1e} \int_{m_1(x_T)}^{\infty} s_1(t)g(t)dt$. Substituting M_T in (A.31) and (A.32) and using the mirror image symmetry obtain local matching market clearing conditions (30) . □

Proof for Lemma 3

Proof. Let $s_X(x)$ be the share of importers among Home final producers with capability x and $s_i(z)$ be the share of exporters among Home Z_i suppliers with capability with z. The corresponding shares for Foreign $s_X^*(x)$ and $s_i^*(z)$ are similarly defined.

Since only final producers with $x \geq x_T$ and Z_i suppliers in the comparative sectors with $z \geq z_T$ are exportable, $s_X(x) = s_X^*(x) = 0$ for all $x \leq x_T$, $s_1(z) = s_2^*(z) = 0$ for all $z \leq z_T$, and $s_2(z) = s_1^*(z) = 0$ for all z.

A differentiation of (29) with respect to $x > x_{Tj}$ leads to

$$2M_{Xe}g(x) = (M_{1e} + M_{1e}^*)\, g(m_1(x))m_1'(x) \text{ for } x > x_T \tag{A.33}$$

and a differentiations of (A.31) with respect to $x > x_T$ leads to

$$M_{Xe}g(x) = M_{1e}(1 - s_1(m_1(x)))\, g(m_1(x))m_1'(x) \text{ for } x > x_T. \tag{A.34}$$

A comparison of (A.33) and (A.34) proves $s_1(m_1(x)) = (M_{1e} - M_{1e}^*) / (2M_{1e})$ for all $x > x_T$, which means $s_1(z) = (M_{1e} - M_{1e}^*) / (2M_{1e})$ for all $z \geq z_T$.

The market clearing condition for matching between Foreign final producers and Home Z_1 suppliers is

$$M_{Xe}^* \int_x^{\infty} s_X^*(t)\, g(t)dt = M_{1e} \int_{m_1^*(x)}^{\infty} s_1(t)\, g(t)dt \text{ for } x \geq x_T$$

From $M_{Xe} = M_{Xe}^*$, $m_1(x) = m_1^*(x)$ and $s_X(x) = s_X^*(x)$ for all $x \geq x_T$, the above inequality becomes:

$$M_{Xe} \int_x^{\infty} s_X(t)\, g(t)dt = M_{1e} \int_{m_1(x)}^{\infty} s_1(t)\, g(t)dt \text{ for } x \geq x_T$$

A differentiation of both sides with respect to x leads to

$$M_{Xe}s_X(x)\, g(x) = M_{1e}s_1(m_1(x))\, g(m_1(x))m_1'(x) \text{ for } x > x_T. \tag{A.35}$$

A comparison of (A.34) and (A.35) leads to

$$s_X(x) = \frac{s_1(m_1(x))}{1 - s_1(m_1(x))} = \frac{M_{1e} - M_{1e}^*}{M_{1e} + M_{1e}^*} \text{ for all } x \geq x_T.$$

□

Preparation for proofs for Lemmas 4 and 5 I first prove the following Claims 6 to 8.

Claim 6. For any $x > 1$ and any $n < k$,

$$\frac{1}{1 - G(x)} \int_x^\infty \left(\frac{t}{x}\right)^n g(t)dt = \frac{k}{k-n}.$$

Proof. It holds that

$$\begin{aligned} \frac{1}{1 - G(x)} \int_x^\infty \left(\frac{t}{x}\right)^n g(t)dt &= kx^{k-n} \int_x^\infty t^{-(k-n)-1} dt \\ &= kx^{k-n} \left(\frac{x^{-(k-n)}}{k-n}\right) \\ &= \frac{k}{k-n}. \end{aligned}$$

□

Claim 7. Let $\theta_X(x) \equiv x m_1(x) m_2(x)$ and $\theta_i(z_i) \equiv z_i m_i^{-1}(z_i) m_j\left(m_i^{-1}(z_i)\right)$ be the team capability for final producers with capability x and Z_i suppliers with capability z_i, respectively.

$$\frac{\theta_X(x)}{\theta_L} > \left(\frac{x}{x_L}\right)^3, \; \frac{\theta_1(z_1)}{\theta_L} > \left(\frac{z_1}{z_{1L}}\right)^3, \text{ and } \frac{\theta_2(z_2)}{\theta_L} < \left(\frac{z_2}{z_{2L}}\right)^3.$$

Proof. For final producers with capability $x \geq x_T$, from Lemma 3, matching market clearing conditions (29) can rewritten as

$$\begin{aligned} M_{1e}\left[1 - G(m_1(x))\right] &= M_{Xe}\left[1 - G(x)\right] + s_X M_{Xe}[1 - G(x)] \\ M_{2e}\left[1 - G(m_2(x))\right] &= M_{Xe}\left[1 - G(x)\right] - s_X M_{Xe}[1 - G(x)]. \end{aligned} \tag{A.36}$$

The second term of the right hand side in the first equation represents the mass of Foreign final producers matching with Home Z_1 suppliers with higher capability than $m_1(x)$. The second term in the right hand side in the second equation represents the mass of Home final producers matching with Foreign Z_2 suppliers with higher capability than $m_2(x)$. For final producers with capability $x \leq x_T$, from $M_T = s_X M_{Xe}[1 - G(x_T)]$, matching market clearing conditions (30) can rewritten as

$$\begin{aligned} M_{1e}\left[1 - G\left(m_1(x)\right)\right] &= M_{Xe}\left[1 - G\left(x\right)\right] + s_X M_{Xe}[1 - G(x_T)] \\ M_{2e}\left[1 - G\left(m_2(x)\right)\right] &= M_{Xe}\left[1 - G\left(x\right)\right] - s_X M_{Xe}[1 - G(x_T)]. \end{aligned} \tag{A.37}$$

Define

$$\delta\left(x, x_T\right) \equiv \max\left\{1, \frac{1 - G(x_T)}{1 - G(x)}\right\} = \max\left\{1, \left(\frac{x}{x_T}\right)^k\right\}.$$

Then, it is possible to summarize (A.36) and (A.37) as

$$\begin{aligned} M_{1e}\left[1 - G\left(m_1(x)\right)\right] &= M_{Xe}\left[1 - G\left(x\right)\right]\left[1 + s_X \delta\left(x, x_T\right)\right] \\ M_{2e}\left[1 - G\left(m_2(x)\right)\right] &= M_{Xe}\left[1 - G\left(x\right)\right]\left[1 + s_X \delta\left(x, x_T\right)\right]. \end{aligned} \tag{A.38}$$

From these, matching functions are obtained as

$$m_1(x) = \left(\frac{M_{1e}}{M_{Xe}\left(1 + s_X \delta\left(x, x_T\right)\right)}\right)^{1/k} x \text{ and } m_2(x) = \left(\frac{M_{2e}}{M_{Xe}\left(1 - s_X \delta\left(x, x_T\right)\right)}\right)^{1/k} x,$$

and a capability function,

$$\theta_X\left(x\right) = \left(\frac{M_{1e} M_{2e}}{M_{Xe}^2\left(1 - s_X^2 \delta\left(x, x_T\right)^2\right)}\right)^{1/k} x^3. \tag{A.39}$$

Then, it holds that

$$\frac{\theta_X\left(x\right)}{\theta_L} = \left(\frac{x}{x_L}\right)^3 \left[\frac{1 - \left(s_X \delta\left(x_L, x_T\right)\right)^2}{1 - \left(s_X \delta\left(x, x_T\right)\right)^2}\right]^{1/k} > \left(\frac{x}{x_L}\right)^3.$$

The market clearing conditions (A.38) can also be written as

$$\begin{aligned} M_{ie}\left[1 - G(z_i)\right]\left(1 - s_i \delta\left(z_i, z_T\right)\right) &= M_{Xe}[1 - G(m_i^{-1}(z_i))] \\ M_{ie}\left[1 - G(z_i)\right]\left(1 - 2 s_i \delta\left(z_i, z_T\right)\right) &= M_{je}\left[1 - G\left(m_j\left(m_i^{-1}(z_i)\right)\right)\right], \end{aligned}$$

where $s_i \equiv \left(M_{ie} - M_{je}\right)/2M_{ie}$. From these, I obtain matching functions as

$$m_i^{-1}\left(z_i\right) = \left[\frac{M_{Xe}}{M_{ie}\left(1 - s_i \delta\left(z_i, z_T\right)\right)}\right]^{1/k} z_i \text{ and } m_j\left(m_i^{-1}\left(z_i\right)\right) = \left[\frac{M_{je}}{M_{ie}\left(1 - 2 s_i \delta\left(z_i, z_T\right)\right)}\right]^{1/k} z_i.$$

and a capability function as

$$\theta_i(z_i) = \left[\frac{M_{Xe}M_{je}}{M_{ie}^2(1-s_i\delta(z_i,z_T))(1-2s_i\delta(z_i,z_T))}\right]^{1/k} z_i^3.$$

Then, it holds that

$$\frac{\theta_i(z_i)}{\theta_L} = \left[\frac{(1-s_i\delta(z_{iL},z_T))(1-2s_i\delta(z_{iL},z_T))}{(1-s_i\delta(z_i,z_T))(1-2s_i\delta(z_i,z_T))}\right]^{1/k}\left(\frac{z_i}{z_{iL}}\right)^3.$$

From $s_1 > 0$ and $s_2 < 0$, I obtain that $\theta_1(z_1)/\theta_L > (z_1/z_{1L})^3$ for $z_1 > z_{1L}$ and $\theta_2(z_2)/\theta_L < (z_2/z_{2L})^3$ for $z_2 > z_{2L}$. □

Claim 8. In the long run trade equilibrium, it holds that $\bar{\pi}_X > \bar{\pi}_X^a$, $\bar{\pi}_1 > \bar{\pi}_1^a$ and $\bar{\pi}_2 < \bar{\pi}_2^a$.

Proof. From (14) and $[1-G(x)] = xg(x)/k$,

$$\begin{aligned}
\bar{\pi}_X &= \frac{1}{1-G(x_L)}\int_{x_L}^{\infty}\pi_X'(t)[1-G(t)]dt \\
&= \frac{1}{1-G(x_L)}\int_{x_L}^{\infty}\pi_X'(t)t\left(\frac{g(t)}{k}\right)dt \\
&= \frac{1}{k}\left[\frac{1}{1-G(x_L)}\int_{x_L}^{\infty}Atm_1(t)m_2(t)g(t)dt\right]. \\
&= \frac{f}{k}\left[\frac{1}{1-G(x_L)}\int_{x_L}^{\infty}\frac{\theta_X(t)}{\theta_L}g(t)dt\right].
\end{aligned} \tag{A.40}$$

From Claim 6, the average profits of final producers in autarky become

$$\begin{aligned}
\bar{\pi}_X^a &= \frac{f}{k}\left[\frac{1}{1-G(x_L^a)}\int_{x_L^a}^{\infty}\frac{\theta_X^a(t)}{\theta_L^a}g(t)dt\right] \\
&= \frac{f}{k}\left[\frac{1}{1-G(x_L^a)}\int_{x_L^a}^{\infty}\left(\frac{t}{x_L}\right)^3 g(t)dt\right] \\
&= \frac{f}{k}\left[\frac{k}{k-3}\right] \\
&= \frac{f}{k}\left[\frac{1}{1-G(x_L)}\int_{x_L}^{\infty}\left(\frac{t}{x_L}\right)^3 g(t)dt\right]
\end{aligned} \tag{A.41}$$

From Claim 7, (A.40) and (A.41), $\bar{\pi}_X > \bar{\pi}_X^a$.

The first order condition (8) is written as

$$\pi_i'(z_i) = A\frac{\theta_i(z_i)}{z_i} = \frac{f}{z_i}\frac{\theta_i(z_i)}{\theta_L}.$$

From this, the average profit for Z_i suppliers becomes

$$\bar{\pi}_i = \frac{1}{1-G(z_{iL})}\int_{z_{iL}}^{\infty}\pi_i'(t)[1-G(t)]dt$$
$$= \frac{1}{1-G(z_{iL})}\int_{z_{iL}}^{\infty}\frac{f}{t}\frac{\theta_i(t)}{\theta_L}\left(\frac{tg(t)}{k}\right)dt$$
$$= \left(\frac{f}{k}\right)\frac{1}{1-G(z_{iL})}\int_{z_{iL}}^{\infty}\left(\frac{\theta_i(t)}{\theta_L}\right)g(t)dt. \tag{A.42}$$

From Claim 6, the average profit of Z_i suppliers in autarky is

$$\bar{\pi}_i^a = \left(\frac{f}{k}\right)\frac{1}{1-G(z_{iL}^a)}\int_{z_{iL}^a}^{\infty}\left(\frac{\theta_i^a(t)}{\theta_L^a}\right)g(t)dt \tag{A.43}$$
$$= \left(\frac{f}{k}\right)\frac{1}{1-G(z_{iL}^a)}\int_{z_{iL}^a}^{\infty}\left(\frac{t}{z_{iL}^a}\right)^3 g(t)dt$$
$$= \left(\frac{f}{k}\right)\left(\frac{3}{k-3}\right)$$
$$= \left(\frac{f}{k}\right)\frac{1}{1-G(z_{iL})}\int_{z_{iL}}^{\infty}\left(\frac{t}{z_{iL}}\right)^3 g(t)d$$

From Claim 7, (A.42), and (A.43), it holds that $\bar{\pi}_1 > \bar{\pi}_1^a$ and $\bar{\pi}_2 < \bar{\pi}_2^a$. □

Claim 9. The free entry conditions are written as

$$f_{Xe} = \int_{x_L}^{\infty}\pi_X'(t)[1-G(t)]dt \text{ and } f_{ie} = \int_{x_L}^{\infty}\pi_X'(t)\eta_i(t)[1-G(t)]dt. \tag{A.44}$$

Proof. From integration by parts, the free entry condition for final producers becomes:

$$f_{Xe} = \int_{x_L}^{\infty}\pi_X(t)\,g(t)dt$$
$$= \int_{x_L}^{\infty}\pi_X'(t)\,[1-G(t)]\,dt + \lim_{x\to\infty}\pi_X(x)\,[1-G(x)] - \pi_X(x_L)\,[1-G(x)_L]$$
$$= \int_{x_L}^{\infty}\pi_X'(t)\,[1-G(t)]\,dt.$$

Since from f.o.c. (8)

$$\pi_i'(m_i(x))m'(x) = Am_i(x)m_j(x)\left(\frac{xm_i'(x)}{m_i(x)}\right)$$
$$= \pi_X'(x)\eta_i(x),$$

the free entry condition for Z_i suppliers becomes:

$$\begin{aligned} f_{ie} &= \int_{z_{iL}}^{\infty} \pi_i(z)\, g(z) dz \\ &= \int_{x_L}^{\infty} \pi_i(m_i(t))\, g(m_i(t)) m_i'(t)\, dt \text{ (substitution of } z = m_i(t)) \\ &= \int_{x_L}^{\infty} \pi_i'(m_i(t))\, m_i'(t)\, [1 - G(m_i(t))]\, dt \text{ (integration by parts)} \\ &= \int_{x_L}^{\infty} \pi_X'(t)\, \eta_i(t)\, [1 - G(m_i(t))]\, dt. \end{aligned}$$

□

Proof for Lemma 4

(i) From (A.44) and $[1 - G(x)] = xg(x)/k$, the free entry condition for final producers becomes:

$$\begin{aligned} \bar{\pi}_X &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \pi_X'(t) [1 - G(t)] dt \\ &= \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} \frac{t\pi_X'(t)}{k} g(t) dt. \\ &= \left(\frac{1}{k}\right) \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} A\theta_X(t) g(t) dt \\ &= \left(\frac{1}{k\sigma}\right) \frac{1}{1 - G(x_L)} \int_{x_L}^{\infty} r(\theta_X(t))\, g(t) dt \\ &= \left(\frac{1}{k\sigma}\right) \bar{r} \\ &= \left(\frac{1}{k\sigma}\right) \frac{L}{M}. \end{aligned}$$

From (18), the mass of entrants becomes:

$$M_{Xe} = \frac{M}{1 - G(x_L)} = \frac{\bar{\pi}_X}{f_{Xe}} M = \frac{L}{f_{Xe} k\sigma} = M_{Xe}^a.$$

(ii)(iii) I first prove (ii) and (iii) using Claims 10 to 12.

Claim 10. Let $\eta_i(x) \equiv x m_i'(x) / m_i(x)$. (i) In autarky, $\eta_i(x) = 1$ for all $x \geq x_L$. (ii) Under trade,

$$\eta_i(x) = 1 \text{ if } x > x_T \text{ and } \eta_i(x) = \frac{M_{Xe}[1 - G(x)]}{M_{ie}[1 - G(m_i(x))]} \text{if } x \in [x_L, x_T).$$

Proof. (i) Because autarky matching functions (11) are linear in x, $\eta_i(x) = 1$ for all $x \geq x_L$. (ii) From (32), matching functions under trade are also linear and $\eta_i(x) = 1$ for $x > x_T$. From (30), $m_i(x)$ satisfies:

$$\left(\frac{1}{m_i(x)}\right)^k = \frac{M_{Xe}}{M_{ie}} \left(\frac{1}{x}\right)^k + \left(\frac{1}{z_T}\right)^k - \frac{M_{Xe}}{M_{ie}} \left(\frac{1}{x_T}\right)^k \text{ for } x \in [x_L, x_T].$$

From the implicit function theorem, the derivatives of both sides by x is

$$-k\left(\frac{1}{m_i(x)}\right)^{k+1} m_i'(x) = -k\frac{M_{Xe}}{M_{ie}}\left(\frac{1}{x}\right)^{k+1} \text{ for } x \in [x_L, x_T].$$

From this,

$$\eta_i(x) = \frac{m_i'(x)x}{m_i(x)} = \frac{M_{Xe}}{M_{ie}}\left(\frac{m_i(x)}{x}\right)^k = \frac{M_{Xe}[1-G(x)]}{M_{ie}[1-G(m_i(x))]} \text{ for } x \in [x_L, x_T). \tag{A.45}$$

□

Claim 11. The mass of entrants satisfies:

$$M_{1e}f_{1e} + M_{2e}f_{2e} = 2M_{Xe}f_{Xe}. \tag{A.46}$$

Proof. From Claim 9 and Claim 10, the free entry condition for Z_i suppliers (A.44) becomes

$$\begin{aligned} f_{ie} &= \int_{x_L}^{\infty} \pi_X'(t)\,\eta_i(t)\,[1-G(m_i(t))]\,dt \\ &= \int_{x_T}^{\infty} \pi_X'(t)\,[1-G(m_i(t))]\,dt + \frac{M_{Xe}}{M_{ie}}\int_{x_L}^{x_T} \pi_X'(t)\,[1-G(t)]\,dt. \end{aligned} \tag{A.47}$$

Because $2M_{Xe}[1-G(x)] = \sum_{i=1,2} M_{ie}[1-G(m_i(x))]$ for all $x \geq x_T$, it follows that:

$$\begin{aligned} M_{1e}f_{1e} + M_{2e}f_{2e} &= 2M_{Xe}\int_{x_L}^{\infty} \pi_X'(t)\,[1-G(t)]\,dt \text{ (from (A.47))} \\ &= 2M_{Xe}f_{Xe} \text{ (from (A.44)).} \end{aligned}$$

□

Claim 12.

$$\frac{M_{1e}}{M_{Xe}} > \frac{M_{1e}^a}{M_{Xe}^a} > \frac{M_{2e}^a}{M_{Xe}^a} > \frac{M_{2e}}{M_{Xe}} \text{ and } \frac{M_{1e}+M_{2e}}{2M_{Xe}} > \frac{M_{1e}^a + M_{2e}^a}{2M_{Xe}^a}.$$

Proof. Notice that

$$\begin{aligned} f_{1e} &= f_{Xe}\frac{M_{Xe}^a}{M_{1e}^a} \text{ from (18)} \\ &= \frac{M_{Xe}^a}{M_{1e}^a}\int_{x_L}^{\infty} \pi_X'(t)[1-G(t)]dt \text{ from Claim 9} \\ &= \int_{x_T}^{\infty} \pi_X'(t)\left(\frac{M_{Xe}^a}{M_{1e}^a}\right)[1-G(t)]\,dt + \frac{M_{Xe}^a}{M_{1e}^a}\int_{x_L}^{x_T} \pi_X'(t)\,[1-G(t)]\,dt \\ &= \int_{x_T}^{\infty} \pi_X'(t)\,[1-G(m_1^a(t))]\,dt + \frac{M_{Xe}^a}{M_{1e}^a}\int_{x_L}^{x_T} \pi_X'(t)\,[1-G(t)]\,dt \text{ from (10).} \end{aligned} \tag{A.48}$$

Because $m_1^a(x) > m_1(x)$ for $x \geq x_T$, a comparison of (A.47) and (A.48) proves that $M_{1e}/M_{Xe} > M_{1e}^a/M_{Xe}^a$. From $M_{Xe} = M_{Xe}^a$, $M_{1e} > M_{1e}^a$. Since $M_{1e}^a f_{1e} + M_{2e}^a f_{2e} = M_{Xe}^a f_{Xe}$ also holds from

(18), it holds

$$M_{2e}f_{2e} = 2M_{Xe}f_{Xe} - M_{Xe}f_{1e} > 2M^a_{Xe} - M_{1e}f_{1e} = M^a_{2e}f_{2e}.$$

Thus, $M_{2e} > M^a_{2e}$. □

Preparation for Proofs for Lemma 5 and Lemma 6

Claim 13. In a long run equilibrium, $\bar{\pi}_1 > \bar{\pi}_X > \bar{\pi}_2$.

Proof. From (29), (A.47) and Claim 10, the free entry condition for Z_i suppliers becomes

$$\begin{aligned}
f_{ie} &= \int_{x_T}^{\infty} \pi'_X(t)\left[1 - G\left(m_i(t)\right)\right] dt + \left(\frac{M_{Xe}}{M_{ie}}\right) \int_{x_L}^{x_T} \pi'_X(t)\left[1 - G\left(t\right)\right] dt \\
&= \left(\frac{2M_{Xe}}{M_{1e} + M_{2e}}\right) \int_{x_T}^{\infty} \pi'_X(t)\left[1 - G\left(t\right)\right] dt + \left(\frac{M_{Xe}}{M_{ie}}\right) \int_{x_L}^{x_T} \pi'_X(t)\left[1 - G\left(t\right)\right] dt \text{ from (29)} \\
&= \left(\frac{M_{Xe}}{M_{ie}}\right) \int_{x_L}^{\infty} \pi'_X(t)\left[1 - G\left(t\right)\right] dt + \left(\frac{2M_{Xe}}{M_{1e} + M_{2e}} - \frac{M_{Xe}}{M_{ie}}\right) \int_{x_T}^{\infty} \pi'_X(t)\left[1 - G\left(t\right)\right] dt \\
&= \left(\frac{M_{Xe}}{M_{ie}}\right) \left[\int_{x_L}^{\infty} \pi'_X(t)\left[1 - G\left(t\right)\right] dt + \left(\frac{M_{ie} - M_{je}}{M_{1e} + M_{2e}}\right) \int_{x_T}^{\infty} \pi'_X(t)\left[1 - G\left(t\right)\right] dt\right]
\end{aligned}$$

Using $M_T = s_X M_{Xe}[1 - G\left(x_T\right)]$, $s_X = (M_{1e} - M_{2e})/(M_{1e} + M_{2e})$, and $M = M_{Xe}[1 - G(x_L)]$, the above equation becomes

$$\begin{aligned}
f_{1e} &= \frac{M}{M_{1e}} \left[\int_{x_L}^{\infty} \pi'_X(t)\left(\frac{1 - G\left(t\right)}{1 - G(x_L)}\right) dt + \frac{s_X\left(1 - G(x_T)\right)}{1 - G(x_L)} \int_{x_T}^{\infty} \pi'_X(t)\left(\frac{1 - G\left(t\right)}{1 - G(x_T)}\right) dt\right] \\
&= \frac{M}{M_{1e}} \left[\bar{\pi}_X + \frac{M_T}{M}\bar{\pi}_T\right] \qquad \text{(A.49)}
\end{aligned}$$

and

$$\begin{aligned}
f_{2e} &= \frac{M}{M_{2e}} \left[\int_{x_L}^{\infty} \pi'_X(t)\left(\frac{1 - G\left(t\right)}{1 - G(x_L)}\right) dt - \frac{s_X\left(1 - G(x_T)\right)}{1 - G(x_L)} \int_{x_T}^{\infty} \pi'_X(t)\left(\frac{1 - G\left(t\right)}{1 - G(x_T)}\right) dt\right] \\
&= \frac{M}{M_{2e}} \left[\bar{\pi}_X - \frac{M_T}{M}\bar{\pi}_T\right], \qquad \text{(A.50)}
\end{aligned}$$

where $\bar{\pi}_T \equiv [1 - G(x_T)]^{-1} \int_{x_T}^{\infty} \pi'_X(t)[1 - G(t)]dt$. From the free entry conditions, equation $(A.49)$

is written as

$$
\begin{aligned}
\bar{\pi}_X + \frac{M_T}{M}\bar{\pi}_T &= \frac{M_{1e}f_{1e}}{M} \\
\frac{f_{Xe}}{1-G(x_L)} + \frac{M_T}{M}\bar{\pi}_T &= \left(\frac{M_{1e}f_{1e}}{M_{Xe}f_{Xe}}\right)\frac{f_{Xe}}{1-G(x_L)} \\
\frac{M_T}{M}\bar{\pi}_T &= \left(\frac{M_{1e}f_{1e}}{M_{Xe}f_{Xe}} - 1\right)\bar{\pi}_X \\
\frac{M_T}{M}\bar{\pi}_T &= \left(\frac{M_{1e}f_{1e} - M_{2e}f_{2e}}{2M_T f_{Xe}}\right)\left(\frac{M_T}{M_{Xe}}\right)\bar{\pi}_X \text{ from (A.46)} \\
\frac{M_T}{M}\bar{\pi}_T &= \frac{M_T}{M_{Xe}\left[1-G(x')\right]}\bar{\pi}_X \text{ from (A.56)} \\
\frac{M_T}{M}\bar{\pi}_T &= \frac{M_T}{M'}\bar{\pi}_X,
\end{aligned}
\tag{A.51}
$$

where $M' \equiv M_{Xe}\left[1-G(x')\right]$.

From (A.49), (A.51), and $M_{1e}[1-G(z_{1L})] = M + M_T$, the free entry condition for Z_1 suppliers becomes:

$$
\begin{aligned}
\bar{\pi}_1 &= \frac{f_{1e}}{1-G(z_{1L})} \\
&= \frac{M}{M_{1e}[1-G(z_{1L})]}\left[\bar{\pi}_X + \frac{M_T}{M}\bar{\pi}_T\right] \\
&= \left(\frac{M}{M+M_T}\right)\left(\frac{M'+M_T}{M'}\right)\bar{\pi}_X. \\
&= \left(\frac{M}{M'}\right)\left(\frac{M'+M_T}{M+M_T}\right)\bar{\pi}_X.
\end{aligned}
$$

From (A.50), (A.51), and $M_{2e}[1-G(z_{2L})] = M - M_T$, the free entry condition for Z_2 suppliers becomes

$$
\begin{aligned}
\bar{\pi}_2 &= \frac{f_{2e}}{1-G(z_{2L})} \\
&= \frac{M}{M_{2e}[1-G(z_{2L})]}\left[\bar{\pi}_X + \frac{M_T}{M}\bar{\pi}_T\right] \\
&= \left(\frac{M}{M'}\right)\left(\frac{M'-M_T}{M-M_T}\right)\bar{\pi}_X.
\end{aligned}
$$

Since

$$
\frac{M'+M_T}{M+M_T} - \frac{M'-M_T}{M-M_T} = \frac{2M_T(M-M')}{(M+M_T)(M-M_T)} > 0,
$$

$\bar{\pi}_1 > \bar{\pi}_X > \bar{\pi}_2$ holds. □

Proof for Lemma 5

Proof. Defining x'_1, x'_2, z'_1, and z'_2 such that $z'_1 = m_1(x'_1) = m^a_1(x'_1)$ and $z'_2 = m_2(x'_2) = m^a_2(x'_2)$, I will show $x'_1 = x'_2 \equiv x'$ and $x' > x_L$. Notice that from (32), x'_1 and x'_2 are smaller than x_T. From definition, these variables satisfy:

$$M_{Xe}\left[1-G\left(x'_1\right)\right] = M_{1e}\left[1-G\left(z'_1\right)\right] - M_T, \tag{A.52}$$

$$M^a_{Xe}\left[1-G\left(x'_1\right)\right] = M^a_{1e}\left[1-G\left(z'_1\right)\right], \tag{A.53}$$

$$M_{Xe}\left[1-G\left(x'_2\right)\right] = M_{2e}\left[1-G\left(z'_2\right)\right] + M_T, \tag{A.54}$$

$$M^a_{Xe}\left[1-G\left(x'_2\right)\right] = M^a_{2e}\left[1-G\left(z'_2\right)\right]. \tag{A.55}$$

From straightforward manipulations, we obtain:

$$\begin{aligned} M_T &= M^a_{1e}\left[1-G\left(z'_1\right)\right]\left(\frac{M_{1e}}{M^a_{1e}}\right) - M_{Xe}\left[1-G\left(x'_1\right)\right] \text{ from (A.52)} \\ &= M^a_{Xe}\left[1-G\left(x'_1\right)\right]\left(\frac{M_{1e}}{M^a_{1e}}\right) - M_{Xe}\left[1-G\left(x'_1\right)\right] \text{ from (A.53)} \\ &= M^a_{Xe}\left[1-G\left(x'_1\right)\right]\left[\frac{M_{1e}}{M^a_{1e}} - \frac{M_{Xe}}{M^a_{Xe}}\right] \end{aligned}$$

and

$$\begin{aligned} M_T &= M_{Xe}\left[1-G\left(x'_2\right)\right] - M^a_{2e}\left[1-G\left(z'_2\right)\right]\left(\frac{M_{2e}}{M^a_{2e}}\right) \text{ from (A.54)} \\ &= M_{Xe}\left[1-G\left(x'_2\right)\right] - M^a_{Xe}\left[1-G\left(x'_2\right)\right]\left(\frac{M_{2e}}{M^a_{2e}}\right) \text{ from (A.55)} \\ &= M^a_{Xe}\left[1-G\left(x'_2\right)\right]\left[\frac{M_{Xe}}{M^a_{Xe}} - \frac{M_{2e}}{M^a_{2e}}\right]. \end{aligned}$$

From (18) and (A.46), we have:

$$\begin{aligned} \left(\frac{M_{1e}}{M^a_{1e}} - \frac{M_{Xe}}{M^a_{Xe}}\right) - \left(\frac{M_{Xe}}{M^a_{Xe}} - \frac{M_{2e}}{M^a_{2e}}\right) &= \frac{M_{Xe}}{M^a_{Xe}}\left[\frac{M_{1e}}{M_{Xe}}\frac{M^a_{Xe}}{M^a_{1e}} + \frac{M_{2e}}{M_{Xe}}\frac{M^a_{Xe}}{M^a_{2e}} - 2\right] \\ &= \frac{M_{Xe}}{M^a_{Xe}}\left[\frac{M_{1e}}{M_{Xe}}\frac{f_{1e}}{f_{Xe}} + \frac{M_{2e}}{M_{Xe}}\frac{f_{2e}}{f_{Xe}} - 2\right] \\ &= 0. \end{aligned}$$

Therefore, $x'_1 = x'_2 \equiv x'$.

By definition of x', $\theta^a_X(x') = \theta_X(x') = x'z'_1z'_2$. From autarky matching functions,

$$\theta^a_X(x) = x^3\left(\frac{M_{1e}M_{2e}}{M^2_{Xe}}\right)^{1/k} = \theta^a_X(x')\left(\frac{x}{x'}\right)^3.$$

In a costly trade equilibrium, it holds from (A.39):

$$
\begin{aligned}
\theta_X(x) &= \left(\frac{M_{1e}M_{2e}}{M_{Xe}^2}\right)^{1/k} x^3 \left[\frac{1}{1-\left(s_X\delta\left(x,x_T\right)\right)^2}\right]^{1/k} \\
&= \theta_X(x')\left(\frac{x}{x'}\right)^3 \left[\frac{1-\left(s_X\delta\left(x',x_T\right)\right)^2}{1-\left(s_X\delta\left(x,x_T\right)\right)^2}\right]^{1/k} \\
&= \theta_X^a(x')\left(\frac{x}{x'}\right)^3 \psi\left(x\right), \\
&= \theta_X^a(x)\psi\left(x\right),
\end{aligned}
$$

where $\psi\left(x\right) \equiv \left[1-\left(s_X\delta\left(x',x_T\right)\right)^2\right]^{1/k} / \left[1-\left(s_X\delta\left(x,x_T\right)\right)^2\right]^{1/k}$. Since $\delta(x,x_T) = (x/x_T)^k$ for $x < x_T$,$\psi'\left(x\right) > 0$ for $x < x_T$. From$\psi\left(x'\right) = 1$, $\psi\left(x\right) > 1$ for $x > x'$ and $\psi(x) < 1$ for $x < x'$. Therefore,$\theta_X(x) > \theta_X^a(x)$ for $x > x'$ and $\theta_X(x) < \theta_X^a(x)$ for $x < x'$.

Finally, I prove $x' > x_L$. From (A.52) and (A.53),

$$
\begin{aligned}
M_{Xe}\left[1-G\left(x'\right)\right] &= M_{1e}\left(\frac{M_{Xe}^a}{M_{1e}^a}\right)\left[1-G\left(x'\right)\right] - M_T \\
M_T &= \left[1-G(x')\right]\left[\left(\frac{M_{Xe}^a}{M_{1e}^a}\right)M_{1e} - M_{Xe}\right] \\
\frac{1}{1-G\left(x'\right)} &= \frac{1}{M_T}\left[M_{Xe} - \left(\frac{M_{Xe}^a}{M_{1e}^a}\right)M_{1e}\right] \\
&= \frac{1}{M_T}\left[\frac{M_{1e}f_{1e}}{f_{Xe}} - M_{Xe}\right] \\
&= \frac{1}{M_T}\left[\frac{2M_{1e}f_{1e} - 2M_{Xe}f_{Xe}}{2f_{Xe}}\right] \\
&= \frac{M_{1e}f_{1e} - M_{2e}f_{2e}}{2M_T f_{Xe}}.
\end{aligned}
\tag{A.56}
$$

Let the mass of surviving Home Z_i suppliers be M_i. Since $M_i = M_{ie}\left[1-G\left(z_{iL}\right)\right]$ and $f_{ie} = \left[1-G\left(z_{iL}\right)\right]\bar{\pi}_i$, the numerator of (A.56) becomes

$$
\begin{aligned}
M_{1e}f_{1e} - M_{2e}f_{2e} &= M_1\left(\frac{f_{1e}}{1-G\left(z_{1L}\right)}\right) - M_2\left(\frac{f_{2e}}{1-G\left(z_{2L}\right)}\right) \\
&= M_1\bar{\pi}_1 - M_2\bar{\pi}_2.
\end{aligned}
\tag{A.57}
$$

Let the mass of surviving Home final producers be M. From (A.46), it holds that

$$
\begin{aligned}
2M\bar{\pi}_X &= 2M_{Xe}\left[1-G(x_L)\right]\bar{\pi}_X \\
&= 2M_{Xe}f_{Xe} \\
&= M_{1e}f_{1e} + M_{2e}f_{2e} \\
&= M_1\bar{\pi}_1 + M_2\bar{\pi}_2
\end{aligned}
\tag{A.58}
$$

Since the mass of surviving Z_i suppliers satisfy $M_1 = M + M_T$ and $M_2 = M - M_T$, the share of international teams becomes

$$\frac{M_T}{M} = \frac{M_1 - M_2}{M_1 + M_2}. \tag{A.59}$$

From (A.56) and the free entry conditions, I obtain

$$\begin{aligned}
\frac{1}{1 - G(x')} - \frac{1}{1 - G(x_L)} &= \frac{M_{1e}f_{1e} - M_{2e}f_{2e}}{2M_T f_{Xe}} - \frac{\bar{\pi}_X}{f_{Xe}} \\
&= \frac{M\bar{\pi}_X}{M_T f_{Xe}} \left[\frac{M_{1e}f_{1e} - M_{2e}f_{2e}}{2M\bar{\pi}_X} - \frac{M_T}{M} \right] \\
&= \frac{M\bar{\pi}_X}{M_T f_{Xe}} \left[\frac{M_1\bar{\pi}_1 - M_2\bar{\pi}_2}{M_1\bar{\pi}_1 + M_2\bar{\pi}_2} - \frac{M_1 - M_2}{M_1 + M_2} \right] \text{ from (A.56), (A.58), and (A.59)} \\
&= \frac{M\bar{\pi}_X}{M_T f_{Xe}} \left[\frac{2M_1 M_2 (\bar{\pi}_1 - \bar{\pi}_2)}{(M_1\bar{\pi}_1 + M_2\bar{\pi}_2)(M_1 + M_2)} \right] > 0. \qquad \text{(A.60)}
\end{aligned}$$

From Claim 13, $\bar{\pi}_1 > \bar{\pi}_2$, and the inequality (A.60), I obtain $x' > x_L$. □

Proof for Lemma 6

Proof. The free entry conditions imply

$$\frac{\bar{\pi}_X}{\bar{\pi}_X^a} = \frac{1 - G(x_L^a)}{1 - G(x_L)} = \left(\frac{x_L}{x_L^a}\right)^k \text{ and } \frac{\bar{\pi}_i}{\bar{\pi}_i^a} = \frac{1 - G(z_{iL}^a)}{1 - G(z_{iL})} = \left(\frac{z_{iL}}{z_{iL}^a}\right)^k.$$

From Claim 13 and $\bar{\pi}_1^a = \bar{\pi}_X^a = \bar{\pi}_2^a$, I obtain

$$\frac{\bar{\pi}_1}{\bar{\pi}_1^a} > \frac{\bar{\pi}_X}{\bar{\pi}_X^a} > \frac{\bar{\pi}_2}{\bar{\pi}_2^a} \Rightarrow \frac{z_{1L}}{z_{1L}^a} > \frac{x_L}{x_L^a} > \frac{z_{2L}}{z_{2L}^a}.$$

Since $\bar{\pi}_X > \bar{\pi}_X^a$ and $\bar{\pi}_2 < \bar{\pi}_2^a$ from Claim 8, the free entry conditions imply $\frac{x_L}{x_L^a} > 1 > \frac{z_{2L}}{z_{2L}^a}$. □