Essays on Optimal Taxation and Policy Implication

by

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Submitted in partial fulfillment of the requirements
for the degree of
Doctor of Philosophy
in
Economics

Graduate School of Economics
Hitotsubashi University

2018

Acknowledgments

This dissertation is complied from research papers that I wrote at the Graduate School of Economics of Hitotsubashi University. I would like to express my sincere gratitude to my advisor, Professor Motohiro Sato for his guidance, support, and generosity throughout my study in M.S. and Ph.D. courses. Without his invaluable help and advice, this work would never have been completed. I appreciate Associate Professor Shigeki Kunieda, Professor Takashi Oshio, Professor Shinji Yamashige, and Assistant Professor Izumi Yokoyama for their thoughtful comments and instructions. I am deeply grateful to Nobuo Akai, Hiroki Arato, Shun-ichiro Bessho, Takeo Hori, Kaname Miyagishima, Yukihiro Nishimura, Hikaru Ogawa, and Yoshihiro Takamatsu for their discussion. Thanks also go to friends of mine, especially Shigeo Morita and Shuichi Tsugawa. Finally, I wish to acknowledge my indebtedness to my family for their support and encouragement.

Contents

Chapter 1

General Introduction

1.1 Introduction

This dissertation consists of three chapters studying the optimal nonlinear taxation under individuals differing in their innate ability to earn income. Mirrlees (1971) is the foundational work of optimal tax that suggests a way to formalize the government's problem dealing explicitly with unobserved heterogeneity. The government can observe their labor income depending on ability and labor effort, but it can observe neither ability nor labor effort. The objective of the government is to maximize social welfare through income redistribution (i.e., equity) without changing individual behaviors (e.g., labor responses) which affect tax revenues (i.e., efficiency). Contrary on the first-best environment, the unobserved heterogeneity implies that the government cannot employ lump-sum tax, which leads to distortions. The unfeasible of lump-sum tax stems from *self-selection constraint* to deter high-ability individuals from mimicking low-ability ones. According to the *revelation principle* which is the classic game theoretic result, any optimal allocation can be achieved through the tax system inducing high-ability taxpayers to reveal their true types. Under the binding *self-selection constraint*, the government must rely on a distortionary nonlinear income tax to prevent high-ability from mimicking low-ability. Hence, lump-sum tax is unavailable. From the fact, since the government cannot implement the redistribution without distorting labor responses, *equity and efficiency tradeoff* occurs in the Mirrlees framework. Here, the following question appears: how should the government design income tax schedules given *equity and efficiency tradeoff*?

Mirrlees (1971) characterizes the optimal marginal income tax rate formula under a continuum of individuals who differ in skill levels. The most famous property of optimal income tax schedules is that the optimal marginal tax rate must be nonnegative and equal zero at the top and the bottom. Many theorists examine the condition under which the properties hold. Seade (1982) derives that the marginal tax rate is nonnegative if leisure is non-inferior and agent monotonicity holds. Sadka (1976) and Seade (1977) show that, when the income distribution is bounded, the marginal tax rate should be zero at the top income earner. Seade (1977) demonstrates that the marginal tax rate at the bottom is zero if there is no bunching at the bottom, that is, labor supply is bounded away from zero. However, Ebert (1992) show that the marginal tax rate at the bottom is positive if there is bunching at the bottom. Boadway and Jacquet (2008) find that the marginal income tax rate at the bottom is positive under a maximin criterion, i.e., Rawlsian. Diamond (1998) and Saez (2001) argue that the upper part of the skill and income distribution are much better approximated by unbounded distribution, especially *un-truncated* Pareto distribution. Under the situation, the top marginal tax rate is asymptotically positive.

As mentioned above, these properties in the Mirrlees model are of little practical relevance for tax policy. An application of sufficient statics approach to optimal tax rates launches. Using a quasi-linear utility function, Diamond (1998) decompose the first-order condition in terms of optimal tax rates into three terms, which is called *ABC*-formula, and clarifies three key parameters that determine optimal tax rates: the taxable income elasticity, the shape of the income distribution, and the social welfare weights. Saez (2001) generalizes this work by assuming utility function with income effect and elucidates the economic effects that characterize the optimal tax rate using a tax perturbation approach. He shows that the optimal income tax formula is numerically carried out using the ability distribution derived from the observed income distribution and the elasticities of labor supply presented by empirical studies. Therefore, the optimal marginal income tax formula is described by real economic factors which can be measured empirically.

Mirrlees (1971) supposed that the only information available to the government is individual's labor income. However, it is possible that the government recognizes much information such as age, gender, height, and physical attribute. Akerlof (1978) explored the effect of income taxes that are contingent on personal characteristics which is called tagging on redistribution. He considered two categories based on an observable characteristic: one category consists of low-ability individuals only and the other consists of low- and high-ability ones. This situation implies that the characteristic or a tag is correlated with earning abilities. In this case, he showed that social welfare increases because tagging allows the government to redistribute between two tagged groups differing in the ability distribution. As a result, the utility of taxpayers in the group consisting only in low-ability is improved relative to the tax system without tagging. However, utility levels of two low-ability individuals are different because taxpayers in the group consisting only of low-ability individuals obtain higher utilities from the use of tagging. The important implication is that tagging reinforces the redistribution but violates the principle of horizontal equity.[1] Boadway and Pestieau (2006)

[1]Akerlof (1978) assume that the government can identify perfectly the person who is eligible to entry

consider a model with two-types of individuals where there are two groups formulated by observable characteristics, one of which has a higher proportion of high-ability individuals than the other. Assuming quasi-linear preferences, they show that tagging, implementing the redistribution from the group with a higher proportion of high-ability individuals to the group with a higher proportion of low-ability ones, enhances the social welfare. Cremer et al. (2010) extends the model with a continuum of individuals who are separated into two groups with different ability distribution. Assuming quasi-linear preferences and maximin criterion, they prove that if one group first-order stochastically dominates the other group, the redistribution from the former group to the latter group is desirable.

In the traditional optimal income taxation model à la Mirrlees (1971), labor supply is a one-dimensional variable expressing hours of work. However, labor supply reflects the other dimensions. For example, the accumulation of human capital through education arguments effective labor supply given hours of work, as Rosen (1980) mentions. Therefore, the distortion of taxes on the other dimensions of labor supply have been ignored. Jacobs (2005) develops the model of optimal linear income taxation with endogenous human capital to capture the effect of the taxation on both working and learning decision. He shows that the trade-off between efficiency and equity worsens. Jacobs (2005) does not consider a case in which the government subsidizes education, however, many OECD countries heavily subsidize higher education. Bovenberg and Jacobs (2005) extend the model by allowing the government to subsidize education and examine the role of education subsidies under linear and nonlinear income taxation. They demonstrate that optimal education subsidies completely eliminate the distortion caused by labor income taxes on learning decision and ensure the efficient level of education.

Here, we introduce two powerful results that have guided intuition about the optimal taxation of goods and services. The first paper is Diamond and Mirrlees (1971) examining the optimal commodity tax problem for a general production technology. They show that, if production exhibits constant returns to scale or decreasing returns to scale and profits are taxed at 100%, the optimal tax system leaves the economy on production possibilities frontier. This means that the use of non-uniform taxes across productive factors, which distorts the allocation of factor inputs, is not desirable. This finding is well known as the *production efficiency theorem*. The implication is that taxation on intermediate goods is not desirable. This result holds under a Mirrleesian model of nonlinear income taxation as well. In fact, Guesnerie and Seade (1982) and Weymark (1987) show that production efficiency is optimal under the setting. However, they suppose that factor prices are fixed exogenously. Naito (1999) extended the model by allowing individuals' wages to be endogenously determined as

social insurance programs. Persons (1996) consider a situation in which eligibility screening is subject to two-sided classification error: the eligible person is not incorrectly tagged (type I errors) and the ineligible person incorrectly receive a tag (type II errors).

in Stiglitz (1982), and show that a deviation from production efficiency is welfare improving. Saez (2004a) criticized that previous studies ignore long-term decisions such as human capital accumulation and introduce the endogenous human capital formation into the model. In contrast with Naito (1999), Saez (2004a) showed that production efficiency theorem is valid. However, Naito (2004) explores the robustness of the result of Saez (2004a) when high-ability individuals have comparative advantage in the sense that the relative return from accumulating skilled human capital to unskilled human capital is higher than that of low-ability ones. In this case, production efficiency theorem fails again.[2]

Before proposing the second paper, we introduce the previous literatures analyzing optimal structure of commodity taxation. The seminal contribution is the paper of Ramsey (1927) that consider a situation in which the government can only imposes taxes on commodities to collect a given amount of tax revenue. Ramsey (1927) show that commodity taxation on a good should be imposed in inverse proportion to the individual's elasticity of demand for the good. This means that commodities with inelastic demand are levied more heavily to minimize efficiency costs. Diamond (1975) and Mirrlees (1975) consider an economy in which there are many individuals with non-identical preferences in the Ramsey formation. The extension introduces equity concerns into the model. Corlett and Hague (1953) analyzes a role of untaxed good in the optimal commodity taxation and find that the good, which is more complementary with the untaxed good such as leisure, should be taxed more heavily. This suggests that the government should employ differential commodity taxes as long as preferences are not separable between consumption and leisure. Atkinson and Stiglitz (1972) and Deaton (1981) provide sufficient conditions for uniform taxation to be optimal. However, these papers restrict tax policy instruments available to the government to commodity taxes. It is obviously problematic to rule out the other tax policies. Using the Mirrlees framework, Atkinson and Stiglitz (1976) examines the optimal mixed taxation when the government can employ both commodity taxation and income taxation, which is the second paper.

Atkinson and Stiglitz (1976) investigates the optimal design of taxes on final goods under nonlinear income taxation. The purpose of the paper is to clarify the question of direct versus indirect taxation which is the ongoing research on optimal taxation. The answer is that, if identical preferences are weakly separable between consumption and leisure, the optimal commodity taxation is uniform under nonlinear income taxation.[3] On the other hand, if the

[2] There are several papers related to production efficiency result. See Guesnerie (2001), Pirttilä and Tuomala (2001), Spector (2001), Blackorby and Brett (2004), and Gaube (2005).

[3] It is well known that commodity taxes play a role as corrective taxes in the presence of goods that generate externalities, which is called *Pigouvian taxes or subsidies.* This is a exception of Atkinson and Stiglitz result. Sandmo (1975) originally studies this problem under linear commodity taxes and then Cremer et al. (1998) reexamine the optimal tax design problem in the presence of externalities when individuals differ in tastes and earning abilities and the government employs both linear or nonlinear commodity and nonlinear income taxation.

condition is not satisfied, final goods should be taxed since commodity taxation alleviates distortion on labor supply caused by labor income taxation through the complementarity between commodity and leisure, in other words, commodity taxes boost labor supply. Christiansen (1984) considers a continuum of individuals and explore how introducing a little bit of commodity tax or subsidy affect the welfare, starting from the tax system with nonlinear income taxes only. Deaton (1979) examines the choice between direct and indirect taxation under linear labor income taxation, and shows that differential commodity taxation is suboptimal if the weakly separability condition in addition to linear Engel curves for goods hold. Jacobs and Boadway (2014) present a full characterization of optimal linear commodity tax rules under nonlinear income taxes implemented by empirically observable parameters. The strong assumption to obtain Atkinson-Stiglitz theorem is that all individuals have identical preferences. This means that those who have the same amount of disposable income would buy the same amount of goods. Saez (2002a) argues that this homogeneity in tastes for goods is unrealistic and explores whether the Atkinson-Stiglitz theorem is robust under heterogeneous preferences.[4] Saez (2002a) demonstrates that separability between consumptions and leisure is not sufficient condition to obtain the Atkinson-Stiglitz theorem under the situation and present the conditions necessary to restore the Atkinson-Stiglitz theorem. Therefore, the Atkinson-Stiglitz theorem is not justified under the realistic assumption. Cremer et al. (2001) propose a special case in which heterogeneous tastes occur before the publication of Saez (2002a). Cremer et al. (2001) considers a situation where individuals differ in not only earning ability but also initial endowment in contrast with the Mirrleesian model. This is an important step from the unidimensional setting to the multidimensional setting, however, note that they assume a discrete of individuals, not a continuum of individuals as in Mirrlees (1971). The second source of heterogeneity causes heterogeneous tastes and then the Atkinson-Stiglitz theorem no longer holds.

According to the earlier contribution of Ordover and Phelps (1979), another implication of the Atkinson-Stiglitz result is that capital income should not be taxed when consumption in each period is weakly separable with leisure.[5] As in the model examining optimal commodity taxes, general equilibrium effects and heterogeneous preferences in consumption are driving forces to break down the Atkinson-Stiglitz result (see Pirttilä and Tuomala (2001) and Saez (2002a)). However, the dynamic setting is used in analyzing the optimal capital taxation compared to the optimal commodity taxation, which is a crucial difference between the two

[4]Boadway et al. (2002) suggest the properties of optimal nonlinear income taxes under heterogeneous preferences for leisure. Choné and Laroque (2010) extends the model with finite types to the model with a continuum of types and characterize the condition in which the optimal marginal tax rate at the bottom is negative. Lockwood and Weinzierl (2015) analyze the policy implication when preferences for leisure are irrelevant from the viewpoint of redistribution.

[5]Judd (1985) and Chamley (1986) consider a dynamic Ramsey problem in which the government employs linear taxes on labor and capital income. It is shown that capital income tax rate is zero in the long run.

models. In the dynamic setting, two main problems appear which is not considered in the static model. The first problem is uncertainty. In the Mirrleesian framework, heterogeneous earning abilities among individuals are fixed, that is, are constant over time. However, earning abilities which are realized in the future is unknown. The "new dynamic public finance" literature deals with labor productivity risk which leads to non-zero capital income taxes. For example, Golosov et al. (2003) allow abilities to evolve stochastically over time and show that capital income should be taxed to provide social insurance against labor productivity shocks. The second problem is time consistency. Using information revealed by taxpayers in the first period, the government may not commit redistributive policy in the second period since it has an incentive to re-optimize its policy. If the taxpayers can recognize such a behavior, they may adjust their behavior to conceal information, which is called the ratchet effect. Therefore, when the government lacks commitment, tax policies must satisfy the requirements of time consistency. Brett and Weymark (2008) show that capital income taxes are needed under the lack of commitment when skill abilities are non-stochastic. Farhi et al. (2012) study nonlinear labor and capital income taxation in a dynamic Mirrleesian model without commitment. The novel finding is that capital income taxation is progressive as well as positive.

Mirrlees (1971) focuses only on the case in which individuals can respond along *intensive margin* by varying the intensity of work on the job given income tax schedules. According to the empirical literature as with Heckman (1993), there is the evidence of labor supply responses along *extensive margin*. In other words, they can decide whether or not to participate in the labor force. In particular, when the government designs the optimal transfer program, it is important to take account of the response along extensive margin. This is because the empirical evidence suggests that the transfer program as in the Earned Income Tax Credit (EITC) in the United States positively affect labor force participation. On the other hand, the Negative Income Tax (NIT) has adverse effects on the labor force. Saez (2002b) examines the optimal transfer scheme when individuals can respond along both intensive and extensive margin. Saez (2002b) numerically show that if the participation elasticity is substantial, the optimal program is the EITC system and the negative marginal tax rate is optimal. This findings are different from the result of Mirrlees (1971) that the optimal marginal income tax rate is nonnegative which means that the NIT system is optimal.[6] Choné and Laroque (2011) and Jacquet et al. (2013) investigate the optimal income taxation in random participation models with multidimensional heterogeneity when labor supply reacts along the extensive margin. Especially, Jacquet et al. (2013) suggest analytical results about optimal tax sched-

[6] Jacquet and de Gaer (2011) examine the optimal income taxation when individuals differ in skill levels and preferences for labor and respond along extensive margin. The social welfare function satisfying the equality of opportunity principles, requiring that income inequalities stemming from skill level (tastes for labor) are (not) compensated, leads to the NIT program.

ules under two margins of labor supply responses. It is shown that both participation tax rates and marginal tax rates are positive at the optimum under Maximin preferences, and the same results hold under Benthamite preferences if the marginal social weight for lowest types is equal to or less than one. Put it differently, the NIT program is desirable in this case. However, using US data, their numerical simulations illustrate that negative participation tax rates at the bottom and nonnegative marginal tax rates everywhere are optimal under Benthamite preferences.[7]

Applying the decision-making at the extensive margin to several situations, there are growing body of literatures on the optimal income taxation in random participation models with multidimensional heterogeneity. For instance, the following literatures explore the impact of the income taxation on the occupational choice margin. Rothchild and Scheuer (2013) consider a two sector Roy (1951) model with endogenous wages and individuals differing in productivity and sector-specific skill have an occupational choice to work in either. They show that if sectoral inputs are complementary, optimal income tax schedules are less progressive than the corresponding schedules in the standard optimal taxation model without an occupational choice. Gomes et al. (2014) extend the model of Rothchild and Scheuer (2013) by allowing the government to differentiate income tax schemes between two sectors, but abstracts from the general equilibrium effect. The main finding is that tagging leads to the invalidity of Diamond-Mirrlees production efficiency theorem. Rothchild and Scheuer (2016) consider a situation where individuals can engage in either traditional activities or rent-seeking activities which generate negative externalities on traditional activities. In the presence of externalities across sectors, the deviation of Pigouvian corrective taxes is optimal unless rent-seeking income could be directly identified. Scheuer (2014) examines the optimal taxation on both profits and labor income when individuals can choose two occupations: workers or entrepreneurs. Under the endogenous firm formation, production efficiency is optimal if the government can observe profits and labor income separately. Also, the following papers focus on the locational choice of individuals. Lehmann et al. (2014) examine how the optimal income tax schemes are characterized when two governments compete under the mobility of labor. Individuals differ in the productivity and mobility cost and choose the location in response to income taxes.[8] They show that the shape of the marginal income tax rate at Nash equilibrium crucially depends on the slope of the semi-elasticity of migration. Using the location choice model, Blumkin et al. (2015) show that, even if skill distributions

[7]The reason why nonnegative marginal tax rates does not imply positive participation tax rates is that the optimal transfer system is not continuous at lowest earning ability. This means that the government provides a distinct transfer to the non-employed and to workers with lowest earning ability.

[8]Simula and Trannoy (2010) examine how individual's labor mobility affect the optimal nonlinear income tax scheme when mobility cost depends on skill ability. Given the tax schedule in the other country, the government intends to decrease the marginal tax rate to prevent high-skilled individuals with low mobility cost from migrating, which may lead to the negative marginal tax rate.

are unbounded, the zero marginal tax rate at the top holds when skill level and migration cost are independently distributed.

Not only levying any taxes but also providing public goods are important policies for the government. Public goods will be under-provided by the market due to the *free rider problem* caused by properties of public goods: *non-excludability* and *non-rivalry*. From the reason, public economists are concerned with how the government should design the optimal provision of public goods. Samuelson (1954) suggests the criterion for providing public goods, which states that the optimal provision level equates the sum of marginal willingness to pay for the public good to the marginal cost of providing the public good, which is called *the Samuelson rule*. However, this criterion is derived under financing public good provision with non-distortionary taxation. When the public good is financing by linear commodity taxation, Atkinson and Stern (1974) shows that the original Samuelson rule is modified, as noted by Pigou (1928). Recently, Boadway and Keen (1993) integrate the theory of public goods provision and the theory of optimal income taxation. In their model, the role of the government is to mitigate income inequality through providing public good and income redistribution by considering heterogeneous individuals in terms of innate ability as in Mirrleesian economy. They show that the original Samuelson rule is valid if both private consumption and public good are weakly separable with leisure. Edwards et al. (1994) and Nava et al. (1996) deal with a more general setting in which the government deploys both nonlinear income and linear commodity taxes to finance a public good. Pirttilä and Tuomala (2001) show that the Samuelson rule is modified under the endogenous wage. Hellwig (2005) focused on the case in which public goods exhibit non-rivalry in consumption and allow for the possibility of use exclusion, and characterize the optimal level of admission fees to access to the public good. He shows that if the objective of the government is Rawlsian, admission fees should be imposed since they play an important role as the part of redistributive policies. On the other hand, the utilitarian government does not have an incentive to levy admission fees because of no interest in redistribution.

Some public goods are financed by a contribution good such as charitable giving as well as direct government expenditures. In particular, religious organizations in the US is financed only by private contributions since direct government expenditures for the organizations are constitutionally banned. To finance public goods which are privately provided, the government encourages economic activities such as charitable giving through a tax break. Previous literatures analyze how should the government subsidize private contributions for public goods. Under linear income taxes, Saez (2004b) characterizes the optimal subsidy rate on the contribution good which is expressed by empirically estimable parameters, and clarify the sensitivity of the subsidy rate with respect to the change in the size of the price elasticity, the size of crowding out effect of public contribution, and the size of the public good effect of the contribution effect. Diamond (2006) examines the optimal tax treatment of private contributions under nonlinear income taxation when individuals are motivated to donate by *purely altruistic* or *impurely altruistic*. According to Andreoni (1990), *purely altruistic* means that the individual cares nothing for the private gift *per se* and *impurely altruistic* means that the motivation of donation is based on not only altruism but also warm-glow. Diamond (2006) finds that the tax treatment inducing high-ability individuals to contribute more than low-ability ones relaxes the incentive constraint.

Each chapter in this dissertation contributes to the literature on optimal taxation as follows.

Chapter 2 studies optimal region-specific income taxes (i.e., *tagging*) in the presence of geographical mobility. We consider that individuals differ in their preferences for a public good and labor productivities and regions differ according the amount of a public amenity. A single government sets region-specific income taxes knowing that residential decision will be affected by differences in tax treatment across regions, inducing individuals to "vote with their feet". Most of the previous literatures on *tagging* investigate the case in which a tagged group is immutable. In this chapter, a tagged group is variable because individuals can move into the other region, which is described by the decision at the *extensive margin* It is shown that the correlation between preferences for a public good and labor productivities and the curvature of the social welfare function is crucial in characterizing the optimal region-specific income tax schedules. If two characteristics are independently distributed and the first derivative of the social welfare function is strictly convex, the marginal tax rate in the region providing higher quality public service is lower. Numerical simulations present how the correlation between two characteristics affect the shape of differential income tax schedules.

Chapter 3 is based on the joint work with Shigeo Morita. We develop an overlapping generation model of optimal nonlinear labor income taxation with individual's charitable giving to explore optimal capital income taxation. We suppose that individuals can be thought of as *purely altruistic*. Although some empirical studies find that a high tax rate on capital gains leads taxpayers to choose charitable giving as strategies to avoid recognizing taxable gains, there are no theoretical studies investigating optimal capital income taxes affecting the decision in terms of charitable giving. The purpose of this chapter is to explore the optimal design of capital income taxation when individuals can contribute to a public good. It is shown that, even if additive and separable preference between consumption and labor supply is assumed and individuals differ only in earning abilities, marginal capital income tax rates are not zero. This indicates that the Atkinson-Stiglitz theorem does not hold. The point is that heterogeneous tastes for private consumptions endogenously occur.

Chapter 4 examines optimal human capital policies under nonlinear labor and capital income taxes in a two-period setting. We consider that investment in human capital results in not only production value but also consumption value. Most of the previous studies investigating human capital policies focus only on production value from education. The aim of this chapter is to shed light on how human capital policies should be designed when human capital investment directly affects individual's utility as well as labor productivity. It is shown that education subsidies should not offset distortions induced by nonlinear taxes on labor and capital income. This means that education policies does not restore efficiency in the household production, that is, Diamond and Mirrlees production efficiency theorem fails. Moreover, capital income taxation is optimal, which means that the Atkinson-Stiglitz theorem breaks down. These findings stem from heterogeneous preferences that occur when individuals are allowed to choose educational types differing in the ratio between consumption value and production value.

Chapter 2

Differential Income Taxation and Tiebout Sorting

2.1 Introduction

Tiebout (1956) argues that individuals can move to jurisdictions that better satisfy their preferences. The response to local fiscal differences across a large number of local governments leads to efficient allocation of local public goods. In contrast, Tiebout sorting has the dark side leading to income segregation across regions. Bayer and McMillan (2012) show that heterogeneity in housing characteristics, including local public goods such as school quality and crime, lead to increases in income stratification. Verdugo (2016) investigates how a policy allowing immigrants with children to live in public housing in France affects their location choices and shows that cities with higher public housing stocks attract more low-skilled immigrants, implying that spatial differences in public housing may cause income stratification. Using a panel dataset of European regions from 17 countries, Kessing and Strozzi (2016) empirically find that the level of public employment is significantly higher in low productivity regions. Viscusi et al. (2008) empirically find that there is a positive income elasticity for the benefit from clean water, which implies that the difference of lake density across states brings the sorting of people differing in income levels. Therefore, the interference of the federal government with regions of different income distributions plays an important role to improve income inequalities across regions.

The objective of this study is to examine how income tax schedules should be differentiated between two regions with different amenities resulting from the quality of local public goods.[1]

[1] Our study does not consider tax competition among governments since we suppose that the responsibility for redistributive taxation is devolved to a supranational government, such as the European Union. Given that the idea of deeper fiscal integration is suggested in the European policy agenda, Bargain et al. (2013) estimate the effect of replacing with an integrated tax and transfer system on redistribution and fiscal stabilization. Kessing et al. (2015) theoretically examine the optimal nonlinear income tax schedules in each

The differentiation of tax schemes is useful as a screening device that sorts individuals based on income levels since the difference of tax burdens across regions induce individuals to vote with their feet. It allows the government to reinforce interregional transfers that mitigate the dark side of Tiebout sorting.

We consider an economy that comprises individuals who differ in their preference for a public good and labor productivity. Individuals make a labor supply decision on the basis of nonlinear income taxes and a binary one regarding which region to live in without incurring mobility costs.[2] The government can differentiate the quality of public good across regions and design differential income tax schemes for the two regions to maximize social welfare while taking account of the labor supply decision on the intensive margin and participation decision on the extensive margin. While the differentiation of public good is costly, it enhances the redistribution since the difference in tax burdens and amenities from local public goods between two regions separates the population into two categories: one that obtains more benefits from a public good and has higher tax burdens and the other that gains lower benefits and has lower tax burdens.

First, we examine the case in which the government is allowed to implement a lump-sum transfer between two regions, although it cannot differentiate marginal income tax rates. We characterize optimal marginal income tax rates and optimal level of lump-sum transfer. The former result is similar to that derived in Mirrlees (1971). The latter result is characterized as the Ramsey inverse elasticity rule and the direction of transfer is determined by the government's redistributive tastes and the correlation between two characteristics. In particular, if there is no correlation between the characteristics, the inter-regional transfer from the region with higher quality public goods to that with lower quality public goods is desirable. Second, we allow the government to introduce the differentiation of marginal tax rates into the tax system and find that the shape of optimal income tax schedules crucially depends on the government's redistributive tastes and the correlation between two characteristics. Using a tax perturbation method, we analytically demonstrate that, if public goods preferences and labor productivities are independently distributed and the first derivative of the social welfare function is strictly convex, the marginal income tax rate for individuals who receive higher amenities from local public goods is lower. This is because the decrease in tax burdens on the region with higher quality public goods leads to a greater welfare gain

member state designed by an integrated government. In contrast, Morelli et al. (2012), Bierbrauer et al. (2013), and Lehmann et al. (2014) analyze the nonlinear income tax competition between two governments.

[2]Our study is part of large body of papers dealing with optimal nonlinear income taxes in random participation models with multidimensional heterogeneity (e.g., Lehmann et al. (2011, 2014), Jacquet et al. (2013), Rothchild and Scheuer (2013, 2016), Scheuer (2014), Blumkin et al. (2015)). These studies assume individuals differ in ability and other characteristics, such as migration cost, work cost, or cost of setting up a firm, and do not allow the government to separate income tax schedules. By contrast, this study investigates tagging assuming individuals differ in ability and public goods preferences.

generated by inducing individuals to access the region (participation effect) than the welfare loss done by the decrease in tax receipts (mechanical effect). Therefore, introducing the differentiation of marginal income tax rates reinforces redistribution. Further, we numerically assess whether our tax perturbation method is reasonable to understand the shape of optimal income schedules and present the implication of introducing the correlation between two characteristics.

This study draws from the growing body of literature examining separated income tax schedules for groups divided by observable characters, or the so-called "tagging" (e.g., Akerlof (1978), Immonen et al. (1998), Viard (2001), Boadway and Pestieau (2006), Cremer et al. (2010), Mankiw and Weinzierl (2010)). Since Mirrlees (1971) seminal work, the optimal income taxation model has considered a situation in which the government designs a redistributive tax system when individuals have private information in terms of their labor productivities. While their labor productivities are unobservable to the government, there are several individual characteristics that the government can observe, such as age, gender, and disability status, which are correlated with their labor productivities. Akerlof (1978) shows that the use of categorical information (also called "tagging") is welfare improving from the viewpoint of utilitarianism, since it allows redistribution not only within each tagged group but also between groups.[3] Therefore, if the government reflects observable characteristics that are correlated with abilities in the tax system, it can reinforce the redistributive tax system. In particular, Boadway and Pestieau (2006) and Cremer et al. (2010) analytically examine optimal income taxation with tagging in an economy comprising two groups, of which one has a higher proportion of high-ability individuals, and conclude that, in this case, the tax system with inter-group transfers will be more redistributive compared to standard optimal taxation model of Mirrlees (1971) and Saez (2001). The crucial difference is that we consider two regions, which are variable categories, as the tag, which means that individuals engage in decision making on the intensive and extensive margin. The labor responses along the extensive margin where regions to work is empirically found by Kleven et al. (2013) who present the sorting effect that low taxes attract high ability individuals. Therefore, the government must pay attention to two types of distortion in individual labor supply when implementing income taxation. By contrast, much of the previous literature supposes that a tagged group is immutable, that is, individuals respond along the intensive margin only.[4] Therefore, we aim to explore how responses along the extensive margin affect the differentiation of income

[3] It is well known that tagging violates the principle of horizontal equity and therefore, is limited in practice. However, Weinzierl (2014) uses the equal sacrifice principle as a comprehensive criterion in that tagging, not the horizontal equity principle, is limited and shows that tagging is justified because the deviation from the equal sacrifice principle is small when observable characteristics are strongly correlated with abilities.

[4] Indeed, previous works have considered demographic characteristics such as age and gender or health conditions including illnesses or disabilities as observable characters. In this case, individuals do not make decisions along the extensive margin since they cannot change groups.

taxes.

This paper is not the first to examine differential income taxation in a variable category (e.g., Kleven et al. (2006, 2009), Gomes et al. (2014), Kessing et al. (2015)). Our study is closely related to Kleven et al. (2006, 2009), who examine how the government should differentiate income tax schedules, considering whether the spouse works as a tag, and numerically investigate the impact of introducing the correlation between ability and work cost on the tax system.[5] They show that the household in which the spouse (does not) works faces lower (higher) marginal tax rates under no correlation between two characteristics and that introducing the correlation is not significant, that is, the theoretical result does not overturn. Our study differs in three ways from their framework. First, the applications of our findings pertain to the design of the optimal inter-regional transfer program related to the difference in the quality of public goods between regions. Second, while they examine tagging under the existing of a tagged group depending on whether the spouse (does not) works, we consider a situation in which a tagged group is endogenously generated by the differentiation of the quality level across regions which is costly. We emphasize that the endogenous policy instrument can promote the redistribution. Third, it attempts to clarify the difference in tagging between immutable and variable categories and shows that the government must take account of the participation effect in addition to the mechanical effect. We numerically demonstrate that the participation effect caused by labor mobility decreases (increases) the marginal tax rate in region A (region B), which implies that it weakens the differentiation of marginal tax rates on the basis of a positive correlation. However, compared to Kleven et al. (2006, 2009), we find that differentiation due to the mechanical effect slightly remains if the correlation between characteristics is strong, that is, the correlation can be an important parameter as in previous studies examining tagging on immutable categories. To the best of our knowledge, there is no theoretical model that elucidates the gap in policy implications for immutable and variable categories in the presence of the correlation. Therefore, it is the main contribution of our paper.

This remainder of this paper is organized as follows. Section 2.2 describes the framework of the basic model. Section 2.3 characterizes differential income tax schedules with non-differentiated marginal tax rates, which is the benchmark result in our study. Section 2.4 shows that it is desirable to introduce differentiated marginal tax rates and section 2.5

[5]Gomes et al. (2014) examine the effect of sector-specific income taxes on production efficiency when individuals with sector-specific abilities choose a sector to work in. They characterize a sufficient condition in which production inefficiency is optimal, although they do not investigate how income tax schemes are differentiated across two sectors. On the other hand, Kessing et al. (2015) investigates differential income taxation on two regions from the viewpoint of the central government, as in the present study. They assume that one is the more productive region, that is, if individuals live in this region, their productivities are enhanced, and numerically find that the shape of optimal differential income taxation dramatically changes in response to migration elasticity. However, they ignore the correlation between two characteristics in numerically analyzing optimal marginal income tax rates.

presents the numerical results. Section 2.6 offers concluding remarks.

2.2 Model

We consider an economy consisting of individuals who are characterized by the preference for a public good and labor productivity denoted by θ and w. The two types of characteristics (θ, w) are distributed according to the cumulative distribution function $F(\theta, w)$ with the strictly positive and continuously differentiable density function $f(\theta, w)$ over $[\underline{\theta}, \overline{\theta}] \times [\underline{w}, \overline{w}]$. We assume that $0 = \underline{\theta} < \overline{\theta} < \infty$ and $0 < \underline{w} < \overline{w} < \infty$, and $\overline{\theta}$ is sufficiently large. The size of population is normalized to 1. We consider two regions indexed by $i = A, B$ and there is a same type of public good in each region, denoted by G_i. The government intends to set higher quality levels in region A, that is, $G_A \equiv \Delta G + G_B \geq G_B$, where $\Delta G \geq 0$ denotes the difference in the quality of public goods between regions. While each public good should be set at the same quality if $\Delta G = 0$, quality levels of public good should be differentiated across regions if $\Delta G > 0$. We assume that G_B is given to focus on examining whether an endogenous policy instrument ΔG is positive or zero. Subsequently, we show that ΔG plays a key role to implement differentiated income tax schedules. In the model, we suppose local public goods with rivalness, which is modeled as the effect on the production (see footnote 6 for details). As used in Diamond (1998), the utility function of individuals in region i is described by

$$U_i = \theta G_i + x_i - v(\ell_i) \tag{2.1}$$

where x_i denotes the private consumption of individuals in region i, and ℓ_i is the labor supply of individuals in region i. On the other hand, $v(\cdot)$ denotes the disutility of labor supply and is strictly increasing, strictly convex, and continuously differentiable.

The government can observe the labor income of individuals in region i, denoted by $z_i \equiv w\ell_i$, and thus, can levy nonlinear income taxes depending on each region, denoted by $T_i(z_i)$. The budget constraint which individuals in region i face is given by $x_i = z_i - T_i(z_i)$.

2.2.1 Intensive margin

Individuals with type vector (θ, w) in region i choose the amount of labor supply by solving the following optimization problem:

$$max_{\ell_i} \; U_i = \theta G_i + w\ell_i - T_i(w\ell_i) - v(\ell_i)$$

The first-order condition yields

$$\frac{v'(\ell_i)}{w} = 1 - T_i'(w\ell_i) \qquad \forall w \tag{2.2}$$

where $v'(\cdot) \equiv \frac{\partial v}{\partial \ell}$ denotes the marginal disutility of labor.

Let us denote the indirect utility function of individuals in region A by $\theta G_A + V_A(w) \equiv \theta G_A + x_A(w) - v(\ell_A(w))$ and those in region B as $\theta G_B + V_B(w) \equiv \theta G_B + x_B(w) - v(\ell_B(w))$, where $x_i(w)$ and $\ell_i(w)$ are the private consumption and labor supply of individuals in region i with labor productivity w.

We define the elasticity of labor supply with respect to the net-of-tax wage rate $1 - T_i'$ as

$$\epsilon_i \equiv \frac{1 - T_i'}{\ell_i} \frac{\partial \ell_i}{\partial 1 - T_i'} = \frac{v'(\ell_i)}{\ell_i v''(\ell_i)} \tag{2.3}$$

From optimized individual behavior (equation (2.2)), we have $\epsilon \equiv \epsilon_A = \epsilon_B$ if $T_A' = T_B'$.

2.2.2 Extensive margin

Individuals choose a region to live in without migration cost, which means that labor is perfectly mobile. Individuals with type vector (θ, w) obtain utility $\theta G_A + V_A(w)$ if they have access to region A and utility $\theta G_B + V_B(w)$ if they access region B. Therefore, they access region A if and only if

$$\theta \geq \frac{V_B(w) - V_A(w)}{\Delta G} \equiv \hat{\theta}(w) \tag{2.4}$$

We interpret $\hat{\theta}(w)$ as the net gain from living in region B. This means if the preference for the public good by individuals with labor productivity w is greater (lower) than the threshold $\hat{\theta}(w)$, they (do not) access region A. The change of ΔG, T_A, and T_B affect $\hat{\theta}(w)$. If ΔG increases, $\hat{\theta}(w)$ for any w decreases since more individuals attract region A. Also, income taxes of individuals with w in region A increase, the individuals move into region B, that is, $\hat{\theta}(w)$ increases.

Here, we denote the entire labor productivity and preference for a public good density by $f(w)$ and $f(\theta)$. If θ and w are independently distributed, the density of joint distribution $f(\theta, w)$ is expressed by $f(w)f(\theta)$. For each labor productivity, the conditional density of the preference for a public good in region A is $f_A^c(w) \equiv \int_{\hat{\theta}(w)}^{\overline{\theta}} f(\theta|w)d\theta$ and that in region B is $f_B^c(w) \equiv \int_{\underline{\theta}}^{\hat{\theta}(w)} f(\theta|w)d\theta$. Therefore, the skill density in region i is $f_i^c(w)f(w)$ denoted by $f_i(w)$ and the corresponding cumulative distribution function is $\int_{\underline{w}}^{w} f_i(x)dx$ denoted by $F_i(w)$. The entire population in region i is $\int_{\underline{w}}^{\overline{w}} f_i(w)dw$ denoted by N_i.

2.2.3 Government

The budget constraint of the government takes the following form:

$$\int_{\underline{w}}^{\overline{w}} T_A(z_A(w))f_A(w)dw + \int_{\underline{w}}^{\overline{w}} T_B(z_B(w))f_B(w)dw = \phi(G_A, N_A) + \phi(G_B, N_B) \tag{2.5}$$

Each term on the left-hand side represents the aggregate revenue from income taxes imposed on individuals in region i. On the other hand, $\phi(\cdot)$ is a cost function of a public good that captures not only provision cost but also congestion cost.[6] For simplicity, we assume that the cost function is the following functional form: $\phi(G_i, N_i) = \phi(G_i)N_i$, where $\phi(G_i)$ is a strictly increasing, strictly convex, and continuously differentiable function with $\phi(0) = 0$, $\phi_G(0) = 0$, and $\lim_{G\to\infty} \phi_G(G) = \infty$. Here, we define $\frac{\partial\phi}{\partial G} \equiv \phi_G$ as the marginal provision cost and $\frac{\partial\phi}{\partial N_i} \equiv \phi_{N_i}$ as the marginal congestion cost.

We focus on the Bergson-Samuelson criterion, which is represented as follows:

$$\mathcal{W} \equiv \int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}(w)}^{\overline{\theta}} W(\theta G_A + V_A(w))f(\theta, w)d\theta + \int_{\underline{\theta}}^{\hat{\theta}(w)} W(\theta G_B + V_B(w))f(\theta, w)d\theta \right] dw \tag{2.6}$$

where W is a strictly increasing and concave function, that is, $W' > 0$ and $W'' < 0$.

In the second best environment, the government cannot observe labor productivity, which is individuals' private information. As per the revelation principle, it suffices to induce individuals to reveal their true types of labor productivity to maximize the objectives of the government. As shown in Mirrlees (1971), the first-order incentive compatibility constraint in region i is given by[7]

$$V_i'(w) = \frac{\ell_i(w)}{w} v'(\ell_i(w)) \qquad \forall w \tag{2.7}$$

This is the necessary condition to meet the incentive constraint. Hereafter, we assume that the sufficient condition is satisfied, that is, monotonicity conditions hold.

[6] To express the congestion effect, the functional form is followed by McGuire (1974) model. The assumption means that the number of residents causes the production effect. This reflects maintenance costs associated with utilization. On the other hand, Buchanan (1965) model assumes that the number of residents directly affects the perceived amount of public good, that is, the congestion effect can be observed in the utility function. This reflects the disutility of crowding, that is, the more people who use the facility, the more crowded it becomes and the less it is available. Our setting can be verified by the following local public goods. According to World Bank estimates in 2013, public education in high-income countries is a suitable example. This is because the pupil-teacher ratio in primary schooling in Sweden was 11.8, but 36.4 in Zimbabwe. Also, people can utilize water and sewer service within a sanitary district without the disutility of crowding. However, costs to maintain facilities providing those services are needed.

[7] In our setting, the government needs to take account of a move-and-mimic which means that individuals have an incentive to mimic those in the other region. However, if the incentive constraint within each region and equation (2.4) are satisfied, the government can prevent the move-and-mimic. This result follows from separability of work preferences from public goods preferences, which is consistent with Blackorby et al. (2007). Therefore, we consider only the incentive constraint within a region.

Before investigating the property of the optimal tax system, it is useful to define the marginal social welfare weight for individuals with labor productivity w in region i denoted by $g_i(w)$.

$$g_A(w) \equiv \frac{\int_{\hat{\theta}(w)}^{\overline{\theta}} W'(\theta G_A + V_A(w)) f(\theta|w) d\theta}{\gamma f_A^c(w)}, \; g_B(w) \equiv \frac{\int_{\underline{\theta}}^{\hat{\theta}(w)} W'(\theta G_B + V_B(w)) f(\theta|w) d\theta}{\gamma f_B^c(w)}$$

where, γ is the multiplier associated with the budget constraint (2.5), and $g_i(w)$ measures the relative value of the government that gives an additional 1\$ to individuals with labor productivity w in region i. Thus, if the government has redistributive tastes, $g_i(w)$ is decreasing in w, which allows income tax schedules to be progressive in region i. Moreover, as shown later, these parameters are crucially related to the optimal redistribution between two regions as well as within each region.

2.3 Non-differentiated marginal tax rates

First, we illustrate the benchmark case in which the government designs differential income tax schedules with the same marginal tax rates, that is, $T' \equiv T'_A = T'_B$. In this case, we allow the government to make the lump-sum transfer E within two regions, where $E \equiv T_A - T_B$ and E is constant in w. Before characterizing the optimal tax policy, we show that it suffices to satisfy the following constraints to solve the optimization problem.

It is sufficient to meet either the incentive constraint in region A or B. Under the same marginal tax rates, the labor supply of individuals in region A is the same as that in region B from equation (2.2), that is, $\ell \equiv \ell_A = \ell_B$ and $z \equiv z_A = z_B$. Therefore, each incentive constraint coincides. Without loss of generality, we take account of the incentive constraint in region B, that is,

$$V'_B(w) = \frac{\ell(w)}{w} v'(\ell(w)) \qquad \forall w \tag{2.8}$$

Second, the threshold $\hat{\theta}(w)$ becomes constant in w. From the definition of $\hat{\theta}(w)$, the first derivative of $\hat{\theta}(w)$ is $\frac{V'_B(w) - V'_A(w)}{\Delta G}$. Since $V'_A(w) = V'_B(w)$ holds from the incentive constraint given that $\ell_A = \ell_B$, $\hat{\theta}(w)$ takes a constant value defined as $\hat{\theta}$. This result allows for a further interpretation of equation (2.4). In this case, equation (2.4) can be rewritten as

$$\theta \Delta G \geq E = \hat{\theta} \Delta G \tag{2.9}$$

That is, individuals prefer to access region A if benefit $\theta \Delta G$ they draw from the additional enjoyment of a public good exceeds additional taxes E.

Finally, using $E = \hat{\theta}\Delta G$, budget constraint (2.5) can be rewritten as follows:

$$\int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta}\Delta G f(\theta) d\theta + \int_{\underline{w}}^{\overline{w}} T_B(z(w)) f(w) dw = \phi(G_A, N_A) + \phi(G_B, N_B) \tag{2.10}$$

In addition, substituting $V_A = -\hat{\theta}\Delta G + V_B$ into social welfare function (2.6), the following denoted by $\hat{\mathcal{W}}$ is obtained:

$$\hat{\mathcal{W}} \equiv \int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}}^{\overline{\theta}} W([\theta - \hat{\theta}]\Delta G + \theta G_B + V_B(w)) f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W(\theta G_B + V_B(w)) f(\theta, w) d\theta \right] dw \tag{2.11}$$

In sum, the government faces with the problem of choosing $V_B(w)$, $\ell(w)$, $\hat{\theta}$, and ΔG to maximize social welfare function (2.11) subject to budget constraint (2.10) and incentive constraint (2.8):

$$\begin{aligned} \max_{V_B(w), \ell_B(w), \hat{\theta}, \Delta G} \quad \hat{\mathcal{W}} \quad s.t. \quad & V_B'(w) = \frac{\ell(w)}{w} v'(\ell(w)) \quad and \\ & \int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta}\Delta G f(\theta) d\theta + \int_{\underline{w}}^{\overline{w}} T_B(z(w)) f(w) dw = \phi(G_A, N_A) + \phi(G_B, N_B) \end{aligned} \tag{2.12}$$

The corresponding Lagrangian is

$$\begin{aligned} \mathcal{L} = \hat{\mathcal{W}} + & \int_{\underline{w}}^{\overline{w}} \lambda(w) \left[\frac{\ell(w)}{w} v'(\ell(w)) - V_B'(w) \right] dw \\ & + \gamma \left[\int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta}\Delta G f(\theta) d\theta + \int_{\underline{w}}^{\overline{w}} T_B(z(w)) f(w) dw - \phi(G_A, N_A) - \phi(G_B, N_B) \right] \end{aligned} \tag{2.13}$$

where γ is the Lagrangian multiplier in the resource constraint and $\lambda(w)$ is the co-state variable in the incentive constraint.

Whether the government implements a region-specific income tax schedules, i.e., $E > 0$, crucially depends on $\Delta G > 0$ and $0 < \hat{\theta} < \overline{\theta}$ since the government needs to divide the population into two groups by differentiating the quality of public good across two regions. The question is whether it is desirable to set at the different quality level across regions because it is possible to set at the same quality level and not to give an incentive to migrate into region A, that is, $\Delta G = 0$ and $\hat{\theta} = \overline{\theta}$. First order conditions in terms of $\hat{\theta}$ and ΔG (equation (2.B.5) and (2.B.6) in Appendix 2.B) are satisfied at $\Delta G = 0$ and $\hat{\theta} = \overline{\theta}$, however, this critical point does not correspond to a maximum of $\hat{\mathcal{W}}$. As shown in Appendix 2.A, the government wants to induce individuals to migrate into region A to differentiate quality levels, i.e., $\Delta G > 0$ and $\hat{\theta} < \overline{\theta}$. Intuitively, since the provision cost of ΔG can be covered with the revenue from the difference of income tax across regions, it is welfare-improving without violating the government's budget constraint. In addition, $\hat{\theta}$ is positive due to a congestion cost, which means that all individuals cannot live in region A. The next proposition summarizes the statement.

Proposition 2.1. *At the optimum, both ΔG and $\hat{\theta}$ are interior solutions.*

Therefore, the difference of quality across regions acts as a tag which is endogenously formulated by a policy instrument, and it allows us to examine optimal region-specific income tax schedules. The first-order conditions are given in Appendix 2.B. Rearranging the first-order conditions, we can obtain the following.

Proposition 2.2. *Under non-differentiated marginal tax rates, the optimal marginal income tax rate and optimal level of lump-sum transfer are characterized by*

$$\frac{T'(z(w))}{1-T'(z(w))} = \left[1+\frac{1}{\epsilon}\right]\frac{1}{wf(w)}\int_{w}^{\overline{w}}[1-\overline{g}(x)]f(x)dx \tag{2.14}$$

$$\frac{E-(\phi_{N_A}-\phi_{N_B})}{E} = \frac{1}{\eta N_A}\left[N_A\int_{\underline{w}}^{\overline{w}} g_B(w)f_B^c(w)f(w)dw - N_B\int_{\underline{w}}^{\overline{w}} g_A(w)f_A^c(w)f(w)dw\right] \tag{2.15}$$

$$\frac{\int_{\underline{w}}^{\overline{w}}\int_{\hat{\theta}}^{\overline{\theta}}(\theta-\hat{\theta})W'(\theta G_A+V_A(w))f(\theta,w)d\theta dw}{\gamma} + \frac{\partial E(1-F(\hat{\theta}))}{\partial \Delta G} = \phi_{G_A} \tag{2.16}$$

where $\overline{g}(x) \equiv f_A^c(x)g_A(x) + f_B^c(x)g_B(x)$ is the average social marginal welfare weight for individuals with labor productivity w and $\eta \equiv -\frac{\partial 1-F(\frac{E}{G})}{\partial E}\frac{E}{1-F(\frac{E}{G})}$ is the migration elasticity with respect to E in region A.

These derivations are also included in Appendix 2.B. Equation (2.14) is the traditional formula for optimal marginal income tax rate obtained by Mirrlees (1971) under no income effect. The heuristic derivation is followed by Saez (2001).

Equation (2.15) is the Ramsey inverse elasticity rule in terms of lump-sum transfers. The amount of lump-sum transfers charged is determined by two main terms. First, the elasticity of demands for additional taxes η in the denominator represents distortions, that is, a decrease in individuals accessing region A, created by imposing additional taxes. If η is highly inelastic, the level of lump-sum transfers tends to increase. Second, the numerator expresses the net welfare gains from the redistribution between regions and the first and second terms in the numerator describe the government's redistributive tastes for each region. If the government prefers to redistribute from region A to B, that is, the first term in the bracket on the right-hand side is greater than the second term, additional taxes are charged above the marginal congestion cost to increase revenues from lump-sum transfers and raise consumption levels. Therefore, whether the level of lump-sum transfer deviates from net marginal congestion cost $\phi_{N_A} - \phi_{N_B}$ crucially depends on the sign of the numerator. Since the Ramsey formula above is very general, making the sign of the numerator ambiguous, we present a special case in which the sign is determined by placing assumptions on the correlation between two characteristics.

Equation (2.16) is the modified Samuelson condition. The first term is the sum of the marginal rate of substitution of individuals in region A and the term in the right hand side is the marginal cost to provide more public goods in region A. Note that even if public goods and consumption are separable with labor as in equation (2.1), the original Samuelson condition is not replicated since the novel term which is the second term in the left hand side of equation (2.16) appears unlike Boadway and Keen (1993). This term describes the marginal benefit which reflects the additional revenue gain arising from the increase of tax burden in region A due to the increase of the quality of public goods in region A. Therefore, over-provision is optimal.

2.3.1 Heuristic derivation and interpretation of the Ramsey inverse elasticity formula

Here, we provide the heuristic derivation for equation (2.15) to help with intuition. We suppose a situation in which the government marginally increases additional taxes E. Let dE be a small tax reform for the lump-sum transfer. First, a small reform, such that E increases, distorts the decision making on the extensive margin. That is, individuals with lower preferences for a public good tend to access region B, which amounts to the size of $f(\hat{\theta})d\hat{\theta}$. Therefore, revenues from additional taxes E decrease. In addition, the decrease in public good users from region A mitigates net marginal congestion cost $\phi_{N_A} - \phi_{N_B}$. As a result, we can express the participation effect denoted by dP as follows:

$$dP = -(E - (\phi_{N_A} - \phi_{N_B}))f(\hat{\theta})d\hat{\theta}$$

Moreover, using $dE = d\hat{\theta} \cdot \Delta G$ obtained from equation (2.9), gives us

$$dP = -\frac{E - (\phi_{N_A} - \phi_{N_B})}{\Delta G} f(\hat{\theta})dE$$

Therefore, the participation effect exhibits a net efficiency loss from imposing additional taxes. Second, a small perturbation that uniformly increases additional taxes E affects tax revenues from income taxes from region A without behavioral responses and the net mechanical effect denoted by dM is measured as follows:

$$dM = \int_{\underline{w}}^{\overline{w}} (1 - g_A(x)) f_A(x) dx \times dE$$

Rearranging this and then substituting equation (2.B.14) in Appendix 2.B yields

$$\begin{aligned} dM &= \frac{1}{\gamma}\left[\gamma(1 - F(\hat{\theta})) - \int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(w)) f(\theta, w) d\theta\right] \times dE \\ &= \left[N_A \int_{\underline{w}}^{\overline{w}} g_B(w) f_B^c(w) f(w) dw - N_B \int_{\underline{w}}^{\overline{w}} g_A(w) f_A^c(w) f(w) dw\right] \times dE \end{aligned}$$

The increase in additional taxes E amounts to revenue $N_A dE$, which increases the level of private consumption by $N_A dE$ units. Therefore, the first term on the right-hand side is the welfare gain from an increase in the private consumptions of individuals in region B. On the other hand, although the tax burdens of individuals in region A decrease $N_A dE$ units, the level of private consumptions decreases $N_B dE$ units since they are levied dE. As a result, the second term on the right-hand side represents the welfare loss from the decrease in the private consumptions of individuals in region A. That is, the term on the right-hand side is interpreted as the net welfare gain from redistribution. In sum, we must have $dP + dM = 0$ at the optimum, which leads to equation (2.15). Put differently, equation (2.15) implies an *equity − efficiency tradeoff*.

2.3.2 Special cases for the Ramsey inverse elasticity formula

The determinants for whether additional taxes should be charged above the marginal congestion cost are the correlation between θ and w and the government's redistributive tastes, as seen in equation (2.15). However, we do not know the direction of the optimal tax policy in general since the formula is complicated.

Here, we assume that θ and w are independently distributed. In this case, equation (2.15) is transformed as follows:

$$\frac{E - (\phi_{N_A} - \phi_{N_B})}{E} = \frac{N_B}{\eta} \int_{\underline{w}}^{\overline{w}} (g_B(w) - g_A(w)) f(w) dw \tag{2.17}$$

If social welfare is a strictly concave function as in equation (2.6), the sign of the equation is positive because $g_B(w) - g_A(w)$ is positive for any labor productivities given the concavity of the social welfare function. In other words, the redistribution from region A to B causes net welfare gains. Therefore, we can summarize the statement as follows:

Corollary 2.1. *If θ and w are independently distributed, the level of lump-sum transfer* *exceeds the net marginal congestion cost.*

It is more interesting to examine how equation (2.15) is characterized in relaxing the assumption in the strictly concavity of W. We define a weighted utilitarian social objective with type-specific weights denoted by $\beta(\theta, w)$ as:

$$\int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}}^{\overline{\theta}} \beta(\theta, w)\{\theta G_A + V_A(w)\} f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} \beta(\theta, w)\{\theta G_B + V_B(w)\} f(\theta, w) d\theta \right] dw \quad (2.18)$$

First of all, we consider a case in which $\beta(\theta, w)$ is a strictly decreasing function with respect to both θ and w. This means that the weighted utilitarian social objective is structurally identical with the Bergson-Samuelson criterion. Thus, the conclusion under the Bergson-Samuelson criterion remains. However, if the type-specific weights depend only on w, that is, $\beta(\theta, w)$ is constant with θ, $g_B(w)$ equals to $g_A(w)$ for any w. This means that if θ and w are independently distributed, equation (2.15) reduces to:

$$\frac{E - (\phi_{N_A} - \phi_{N_B})}{E} = 0 \quad (2.19)$$

This implies that the government's motivation of inter-regional redistribution under no information in terms of correlation between θ and w stems from inequalities due to the difference of public goods quality across regions. Indeed, if the type-specific weights depend only on θ and are strictly decreasing in θ, $g_B(w) > g_A(w)$ holds, and thus, the tax system requires net transfer to region B under independence between θ and w. The social objective with type-specific weights depending only on w reflects the notion of equality of opportunity, i.e., individuals are responsible for public goods preferences but not for innate skills. Therefore, the government attains the first-best rule for the assignment of people across regions. The arguments are summarized as follows:

Corollary 2.2. *Suppose the weighted utilitarian social objective and independence between θ and w. As long as type-specific weights $\beta(\theta, w)$ depend on θ and are strictly decreasing in θ, the tax system calls for net transfer to region B. However, if the type-specific weights depend only on w, the level of lump-sum transfer coincides with the net marginal congestion cost.*

Finally, we investigate a special case where $\beta(\theta, w)$ depends on neither θ nor w. This means that the weighted utilitarian social objective becomes a Benthamite criterion that values efficiency more than equity.[8] In this case, even if θ is correlated with w, equation (2.19) is valid. This is because there is no incentive for the government to reinforce the

[8]Note that the weighted utilitarian social objective corresponds to a Rawlsian social objective if type-specific weights satisfy $\beta(\theta, \underline{w}) > 0$ and $\beta(\theta, w) = 0 \ \forall w > \underline{w}$. Under the Rawlsian criterion, equation (2.15)

redistribution using the additional information for income inequalities across regions. This is given in the following corollary.

Corollary 2.3. *Suppose that type-specific weights are constant with θ and w, i.e., the Benthamite criterion. Even if there is the correlation between θ and w, the level of lump-sum transfer coincides with the net marginal congestion cost.*

2.4 Differentiated marginal tax rates

In this section, we examine the effect of introducing the differentiation of marginal income tax rates between two regions. If the government is able to design differential income tax schedules with differentiated marginal tax rates, it faces with the problem of choosing $V_i(w)$, $\ell_i(w)$ for $i = A, B$, $\hat{\theta}(w)$, and ΔG to maximize social welfare function (2.6) subject to budget constraint (2.5), incentive constraints (2.7), and participation constraint (2.4). Therefore, the optimization problem is formulated as follows:

$$\max_{V_i(w),\ell_i(w),\hat{\theta}(w),\Delta G} \quad \mathcal{W} \quad s.t. \quad V_i'(w) = \frac{\ell_i(w)}{w} v'(\ell_i(w)), \quad \hat{\theta}(w)\Delta G + V_A(w) = V_B(w) \quad and$$
$$\int_{\underline{w}}^{\overline{w}} T_A(z_A(w)) f_A(w) dw + \int_{\underline{w}}^{\overline{w}} T_B(z_B(w)) f_B(w) dw = \phi(G_A, N_A) + \phi(G_B, N_B) \tag{2.20}$$

The corresponding Lagrangian is

$$\begin{aligned}\mathcal{L} = \mathcal{W} + \gamma \left[\int_{\underline{w}}^{\overline{w}} T_A(z_A(w)) f_A(w) dw + \int_{\underline{w}}^{\overline{w}} T_B(z_B(w)) f_B(w) dw - \phi(G_A, N_A) - \phi(G_B, N_B) \right] \\ + \sum_{i=A,B} \int_{\underline{w}}^{\overline{w}} \lambda_i(w) \left[\frac{\ell_i(w)}{w} v'(\ell_i(w)) - V_i'(w) \right] dw + \int_{\underline{w}}^{\overline{w}} \mu(w) \left[\hat{\theta}(w)\Delta G + V_A(w) - V_B(w) \right] dw\end{aligned} \tag{2.21}$$

where γ is the Lagrangian multiplier on the resource constraint, $\lambda_i(w)$ is the co-state variable associated with the incentive constraint in region i, and $\mu(w)$ is the co-state variable associated with the participation constraint. The first-order conditions are given in Appendix 2.C and the optimal marginal income tax rate for each region is derived by rearranging them.

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reduces to:

$$\frac{E - (\phi_{N_A} - \phi_{N_B})}{E} = \frac{1}{\eta N_A} \{N_A g_B(\underline{w}) f_B^c(\underline{w}) - N_B g_A(w) f_A^c(\underline{w})\} f(\underline{w})$$

According to Corollary 2.2, if the type-specific weights depend on θ and are strictly decreasing in θ, the government implements the tax system with net lump-sum transfer to region B. On the other hand, if the type-specific weights is constant with θ, the government is not interested in the equity consideration.

Proposition 2.3. *The optimal marginal income tax rate for each region is characterized by*

$$\frac{T'_A(z_A(w))}{1 - T'_A(z_A(w))} = \left[1 + \frac{1}{\epsilon_A}\right] \cdot \frac{1}{wf_A(w)} \cdot \int_w^{\overline{w}} \left[(1 - g_A(x))f^c_A(x) - \Phi(x)\right] f(x)dx \tag{2.22}$$

$$\frac{T'_B(z_B(w))}{1 - T'_B(z_B(w))} = \left[1 + \frac{1}{\epsilon_B}\right] \cdot \frac{1}{wf_B(w)} \cdot \int_w^{\overline{w}} \left[(1 - g_B(x))f^c_B(x) + \Phi(x)\right] f(x)dx \tag{2.23}$$

where $\Phi(x) \equiv \frac{T_A(z_A(x)) - T_B(z_B(x)) - (\phi_{N_A} - \phi_{N_B})}{\Delta G} f(\hat{\theta}(x)|x)$

These formulas describe the optimal differentiated marginal income tax rate for each region. This result is consistent with those in the existing literature (Kleven et al. (2006, 2009), Kessing et al. (2015)). In contrast with the optimal tax rate on the basis of an immutable tag, the novel effect $\Phi(\cdot)$ appears, which negatively (positively) works for tax rates on individuals in region A (region B) if $\Phi(\cdot)$ is positive. The term consists of two terms: $T_A(z_A(w)) - T_B(z_B(w))$ and $\phi_{N_A} - \phi_{N_B}$. The first term expresses the additional tax revenue obtained by inducing individuals to access region A. Thus, if this term is positive, the government intends to decrease (increase) the marginal tax rates in region A (region B) to attract people to the region. The second term is the net marginal congestion cost $\phi_{N_A} - \phi_{N_B}$ for the government, which differs from the previous literature examining variable categories. If the government decreases the marginal tax rate in region A as an incentive to live in region A, efficiency loss occurs in region A owing to the congestion cost. On the other hand, efficiency gain occurs in region B due to population outflow. This mechanism reflects the net marginal congestion cost $\phi_{N_A} - \phi_{N_B}$. If it is positive, this implies that the government increases (decreases) the marginal tax rates in region A (region B) to mitigate the congestion cost in total. As a result, even if the government can extract additional tax revenue inducing individuals to access region A, it must determine income tax schedules while taking account of the congestion cost.

The social welfare criterion affects the differentiation of the marginal income tax rate. If the government has distributional concerns, government redistributive tastes for region A is estimated to be lower than those for region B because the utility of individuals in region A is higher than that of individuals in region B. That is, from the concavity of the social welfare function, we have $g_B > g_A$. Therefore, the government intends to redistribute income from region A to region B by imposing more income taxes on individuals in region A.

2.4.1 Direct proof of optimal differentiated marginal tax rate

We present an intuitive interpretation of formulas in Proposition 2.3 by characterizing optimal marginal nonlinear income tax rates by means of direct derivation as in Saez (2001).

We consider a situation in which the government marginally increases the marginal income tax rates for individuals in region A whose income levels are distributed over $[z_A, z_A + dz_A]$, denoted by dT'_A. This small tax reform causes the following three effects: mechanical, substitution, and participation effect.

Mechanical effect

The rise in marginal income tax rates increases tax receipts without behavioral responses. Since individuals in region A with labor productivity above w_{z_A} must pay the additional payment $dT'_A \times dz_A$, the added net tax receipts amount to

$$\varrho^A_M \equiv \int_{w_{z_A}}^{\overline{w}} (1 - g_A(x)) f_A(x) dx \times dT_A \tag{2.24}$$

where w_{z_A} is the ability of individuals in region A who earn labor income z_A.

Substitution effect

The change in the marginal income tax rate distorts decision making in terms of labor supply. If the marginal income tax rates increase, the tax base decreases by the reduction of labor supply. Thus, a decrease in tax receipts occurs due to behavioral responses. To measure this effect, we rearrange the change in labor income owing to a small change in the marginal income tax rates, denoted by $\frac{dz_A}{dT'_A}$ as follows:

$$dz_A = -\frac{z_A}{1 - T'_A(z_A)} \times \epsilon_A \times dT'_A \tag{2.25}$$

Substituting equation (2.25) with $dT_A(z_A) = T'_A(z_A) dz_A$ yields

$$dT_A(z_A) = -T'_A(z_A) \frac{z_A}{1 - T'_A(z_A)} \times \epsilon_A \times dT'_A \tag{2.26}$$

Let ϱ^A_B be the total reduction of tax receipts from region A brought about by substitution effect. Thus, ϱ^A_B is equal to $dT_A(z_A) \times f_A(w_{z_A}) d\hat{w}$ because individuals whose skill levels are within the interval $[w_{z_A}, w_{z_A} + d\hat{w}]$ are affected by the change in marginal tax rates. Given that $d\hat{w} = \frac{dz_A}{(1+\epsilon_A)\ell}$ is derived using equation (2.2), we have

$$\varrho^A_B = -\frac{T'_A(z_A)}{1 - T'_A(z_A)} \times \frac{\epsilon_A}{1 + \epsilon_A} \times w_{z_A} f_A(w_{z_A}) \times dT_A \tag{2.27}$$

Participation effect

Unlike in the traditional literature examining differential income taxation on immutable categories, our model considers a variable category as a tag. The increase in marginal tax rates induces individuals in region A with $x \geq w_{z_A}$ such that the number of switchers amounts to $f(\hat{\theta}(x), x)d\hat{\theta}(x)$ to drop out of the access to region A. Because their payments change from $T_A(z_A)$ to $T_B(z_B)$, the government's revenue decreases by $T_A(z_A) - T_B(z_B)$ units. In addition, the decrease in the number of individuals in region A alleviates the congestion cost in region A and augments that in region B, measured by $\phi_{N_A} - \phi_{N_B}$. Therefore, a net effect on tax revenues is equal to $-(T_A(z_A) - T_B(z_B)) + (\phi_{N_A} - \phi_{N_B})$. Using $d\hat{\theta}(x) \cdot \Delta G = dT_A$, the total effect on tax receipts is as follows:

$$\varrho_P^A \equiv -\int_{w_{z_A}}^{\overline{w}} \frac{T_A(z_A) - T_B(z_B) - (\phi_{N_A} - \phi_{N_B})}{\Delta G} f(\hat{\theta}(x), x)dx \times dT_A \tag{2.28}$$

As a whole, the three effects need to be offset at the optimum, and accordingly, we have $\varrho_M^A + \varrho_B^A + \varrho_P^A = 0$. Rearranging this, we can obtain the optimal marginal income tax rate in region A in Proposition 2.3.

Using a similar method, the optimal marginal income tax rate in region B in Proposition 2.3 is characterized, where the participation effect is the opposite since the increase in marginal tax rates in region B induces individuals to access region A.

In the traditional literature, a tagged group as an immutable category depends on the mechanical effect and the substitution effect. However, if a tagged group is a variable category, the change in the tax system due to differential income taxes distorts decision making on the extensive margin. Hence, the government takes account of the participation effect on tax revenues when differentiating income taxes.

2.4.2 Tax perturbation method: welfare gains introducing differentiated marginal tax rates

Beginning from the tax system with non-differentiated marginal tax rates at the optimum, we examine how differentiation of marginal tax rates should be introduced. Similar to Kleven et al. (2006, 2009), we consider a little bit of tax reform at any labor productivity w as depicted in Figure 2.1. The tax reform is decomposed into two components. Above labor productivity w, we decrease income taxes on people accessing region A and increase income taxes on people accessing region B. Let dT_A^a and dT_B^a be the small tax reform for each region above labor productivity w. Here, we assume that θ and w are independently distributed. We numerically examine the implication of correlation between two characteristics in section 2.5. Let the change in income taxes on each segment be inversely proportional to the population

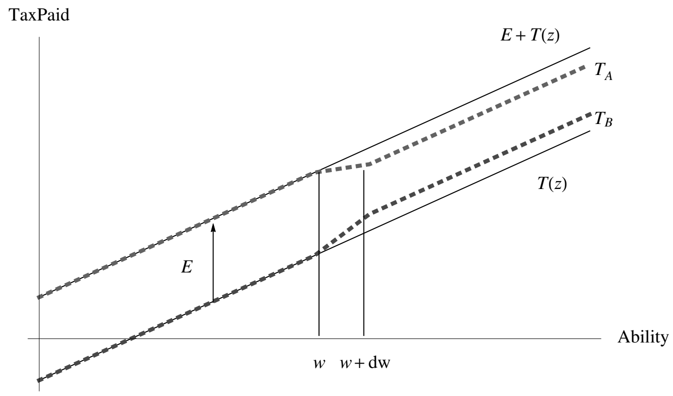

Figure 2.1: Small tax reform perturbation

on the segment; in other words, $dT_A^a = -\frac{dT}{1-F_A(w)}$ and $dT_B^a = \frac{dT}{1-F_B(w)}$. Therefore, the tax reform is revenue neutral.

The tax reform causes three effects. First, an implementation of the tax reform induces individuals with labor productivity above w to access region A. The effect is associated with participation responses. Above w, while individuals accessing region A provide the government with additional revenue $T_A - T_B$, they cause efficiency loss by the amount of the net marginal congestion effect $\phi_{N_A} - \phi_{N_B}$. The number of switchers due to the tax reform amounts to the size of $|f(\hat{\theta}(x), x)d\hat{\theta}(w)|$ at labor productivity $x \geq w$, which is an absolute value. Therefore, a net effect at $x \geq w$ is measured by $(T_A - T_B - (\phi_{N_A} - \phi_{N_B}))|f(\hat{\theta}(x), x)d\hat{\theta}(w)|$. Moreover, given that $d\hat{\theta}(w) \cdot \Delta G = dT_A^a - dT_B^a$, the total effect associated with participation responses denoted by dP is expressed as follows:

$$dP = \int_w^{\overline{w}} \frac{T_A - T_B - (\phi_{N_A} - \phi_{N_B})}{\Delta G} f(\hat{\theta}(x), x)dx \times \left(\frac{1}{1 - F_A(w)} + \frac{1}{1 - F_B(w)}\right) dT \quad (2.29)$$

As we begin with tax systems with non-differentiated marginal tax rates, we have $T'_A = T'_B$ and constant threshold $\hat{\theta}$. Using the assumption of independence between θ and w, equation (2.29) can be rewritten as follows:

$$dP = \frac{E - (\phi_{N_A} - \phi_{N_B})}{E} \frac{\hat{\theta} f(\hat{\theta})}{F(\hat{\theta})(1 - F(\hat{\theta}))} \times dT \quad (2.30)$$

Finally, from the fact that $N_B = F(\hat{\theta})$ and $\eta = \frac{\hat{\theta} f(\hat{\theta})}{1 - F(\hat{\theta})}$, equation (2.30) can be transformed by substituting equation (2.17) as follows:

$$dP = \int_{\underline{w}}^{\overline{w}} (g_B(w) - g_A(w)) f(w) dw \quad (2.31)$$

The sign of dP is positive, implying that the government can collect more tax revenues through the response that individuals participate in region A owing to the tax reform.

Second, a small perturbation that changes the tax burden on each segment directly affects tax revenues without behavioral responses and the net mechanical effect denoted by dM is measured as follows:

$$dM \equiv \int_{w}^{\overline{w}} (1 - g_B(x)) f_B(x) dx \times dT_B^a + \int_{w}^{\overline{w}} (1 - g_A(x)) f_A(x) dx \times dT_A^a \tag{2.32}$$

Since we start from tax systems with non-differentiated marginal tax rates, we have the constant threshold $\hat{\theta}$. By the assumption of independence, dM is transformed as follows:

$$dM = \frac{1}{1 - F(w)} \int_{w}^{\overline{w}} (g_A(x) - g_B(x)) f(x) dx \times dT \tag{2.33}$$

The sign of dM is negative, which means that the mechanical effect caused by the tax reform decreases tax revenues.

Finally, the tax reform associated with the change in marginal income tax rates affects an individual's behaviors with respect to labor supply with labor productivity around w. The decrease in tax rates for individuals in region A increases tax receipts through the promotion of labor responses and the increase in tax rates for individuals in region B reduces tax receipts through the distortion of labor responses. As with the derivation of substitution effects in subsection 2.4.1, we describe the effect on $[w, w + dw]$ in each region, denoted by dB_A^a and dB_B^a.

$$dB_A^a \equiv -\frac{T_A'}{1 - T_A'} \frac{\epsilon_A}{1 + \epsilon_A} w f_A(w) \times dT_A^a \tag{2.34}$$

$$dB_B^a \equiv -\frac{T_B'}{1 - T_B'} \frac{\epsilon_B}{1 + \epsilon_B} w f_B(w) \times dT_B^a \tag{2.35}$$

Since we start from tax systems with non-differentiated marginal tax rates, we have $T_A' = T_B'$, constant threshold $\hat{\theta}$, and $\epsilon_A = \epsilon_B$. Therefore, by the assumption of independence, these substitution effects cancel out.

Here, we denote the total welfare effect by dW, which is the sum of the effects above. As a result, if θ and w are independently distributed, the total welfare effect of introducing differentiated marginal tax rates starting from tax systems with non-differentiated marginal tax rates is as follows:

$$\begin{aligned} dW &= dP + dM \\ &= \left[\underbrace{\int_{\underline{w}}^{\overline{w}} (g_B(w) - g_A(w)) f(w) dw}_{Participation\ Effect} + \underbrace{\frac{1}{1 - F(w)} \int_{w}^{\overline{w}} (g_A(x) - g_B(x)) f(x) dx}_{Net\ Mechanical\ Effect} \right] \times dT \end{aligned} \tag{2.36}$$

The direct welfare effect represents the trade-off between the positive effect due to participation responses and the negative effect associated with the mechanical effect. The first term expresses welfare gain (the tax reform enables the government to reinforce the redistributive tax system, inducing individuals to access region A from the decrease in tax burdens) and the second term reflects welfare loss (the tax reform weakens redistribution by decreasing total tax receipts). As shown in Appendix 2.D, dW is positive if the first derivative of the social welfare function is strictly convex such as the constant rate of risk aversion (CRRA) form $W = V^{1-\pi}/(1-\pi)$, where π measures the government's taste for redistribution. This means that the welfare gain owing to the participation effect exceeds the welfare loss caused by the mechanical effect, and thus, $dW > 0$. In sum, the government can enhance social welfare by implementing the tax reform (Figure 2.1) and the following statement holds.

Proposition 2.4. *If θ and w are independently distributed and the first derivative of the social welfare function is strictly convex, starting from the tax system with non-differentiated marginal tax rates, the social welfare increases by introducing differentiated marginal tax rates, such that the marginal tax rate on individuals who access region A decreases and the marginal tax rate on individuals who access region B increases for any labor productivity w.*

The result presents a novel insight in terms of the characterization of the tax system with tagging. To clarify how the structure of optimal differentiated marginal tax rates are different between immutable and variable categories, we examine the case where individuals cannot move across regions. Under no migration, the optimization problem is to maximize the social welfare function (2.6) subject to budget constraint (2.5) and incentive constraint (2.7), given $\hat{\theta}(w)$. Under no-differentiated marginal tax rates, it is not necessarily that $\hat{\theta}(w)G_A + V_A(w) = \hat{\theta}(w)G_B + V_B(w)$ and $\hat{\theta}(w)$ is constant with w due to the absent of participation constraint (2.4), which means that the sign of $g_B(w) - g_A(w)$ is not determined. For comparison with the results under migration, we consider that $\int_w^{\overline{w}}(g_B(x) - g_A(x))f(x)dx > 0$ for any $w \in (\underline{w}, \overline{w})$.[9] This implies that the government intends to redistribute income from people in region A to people in region B to improve income inequalities. When individuals cannot move across regions, the mechanical effect plays an important role to determine the differentiation of marginal tax rates in both regions since the participation effect vanishes. Consider that the marginal income tax rate in region A increases and that in region B decreases at $w \in (\underline{w}, \overline{w})$ so that $dT_A^a = \frac{dT}{1-F_A(w)}$ and $dT_B^a = -\frac{dT}{1-F_B(w)}$. From equation (2.32), the mechanical effect

[9]Note that the reason why the condition does not hold at $w = \underline{w}$ under no migration is that we have $\int_{\underline{w}}^{\overline{w}} g_A(w)dw = \int_{\underline{w}}^{\overline{w}} g_B(w)f(w)dw$ at the optimum when the government implements the tax system with non-differentiated marginal tax rates. In contrast, the condition is valid at not only any $w \in (\underline{w}, \overline{w})$ but also $w = \underline{w}$ under migration.

under the independence is described as

$$dM = \frac{1}{1-F(w)} \int_{w}^{\overline{w}} (g_B(x) - g_A(x)) f(x) dx \times dT \tag{2.37}$$

which is positive. Since the substitution effect vanishes from equation (2.34) and (2.35), it is desirable that the marginal income tax rate in region A is higher at w while that in region B is lower at w. Thus, the optimal differentiated marginal tax rates in both regions dramatically change depending on whether individuals can move or not, even though the government has the same redistributive tastes that it wants to redistribute income from people in region A to people in region B. Proposition 2.4 suggests that the optimal structure of the tax system under migration is quite reformed compared to that under no migration.

However, we cannot assess whether this result holds even if it allows for the correlation between θ and w. To confirm how income tax schemes at the optimum are differentiated in various situations, we exercise numerical simulations in section 2.5.

2.5 Numerical examples

To illustrate our results at the optimum, we now exercise a simple simulation. The objective is to (i) confirm that, if preferences for public goods and labor productivity are independently distributed, the tax perturbation analysis in section 2.4.2 is consistent with tax reforms implemented at the optimum while checking the robustness with respect to alternative parameters (ii) examine the impact of the correlation between two characteristics on the differentiation of marginal tax rates at the optimum, and (iii) contrast the marginal tax rate of an immutable tag with that of a variable tag under the correlation between two characteristics.

In the simulation, we set the following assumptions. First, we assume that the Bergson-Samuelson criterion is CRRA form. Second, we assume that the disutility of labor $v(\cdot)$ takes the following functional form: $v(\ell_i) = \ell_i^{1+1/e}/(1+1/e)$, where $e > 0$. In this case, $e = \epsilon_A = \epsilon_B$, and thus, the elasticity of labor supply with respect to the net-of-tax wage rate ϵ_i is constant. Third, following by Kleven et al. (2006, 2009), public goods preferences θ are distributed as the power function $F(\theta) = (\theta/\overline{\theta})^\sigma$ with the density function $f(\theta) = \sigma \cdot \theta^{\sigma-1}/\overline{\theta}^\sigma$ on the interval $[\underline{\theta} = 0, \overline{\theta} = 2.5]$, where σ indicates the constant migration elasticity $\hat{\theta}(w) f(\hat{\theta}(w))/F(\hat{\theta}(w))$ in region B. Fourth, we assume that a cumulative distribution function of labor productivity w is a truncated Pareto distribution with parameter $a = 2$ over $[\underline{w} = 1, \overline{w} = 2]$, expressed by $F(w) = [1 - (\underline{w}/w)^a]/[1 - (\underline{w}/\overline{w})^a]$.[10] Fifth, we consider $G_B = 0.05$ and assume that the cost function for the public good takes the following functional form: $\phi(G_i, N_i) = G_i^2 N_i$. In the

[10]Notice that we do not replicate actual income distribution for which country at which period of time in numerical simulations. Our aim is to present that the correlation between two characteristics is an important factor in differentiating marginal tax rates even under an endogenous tagging.

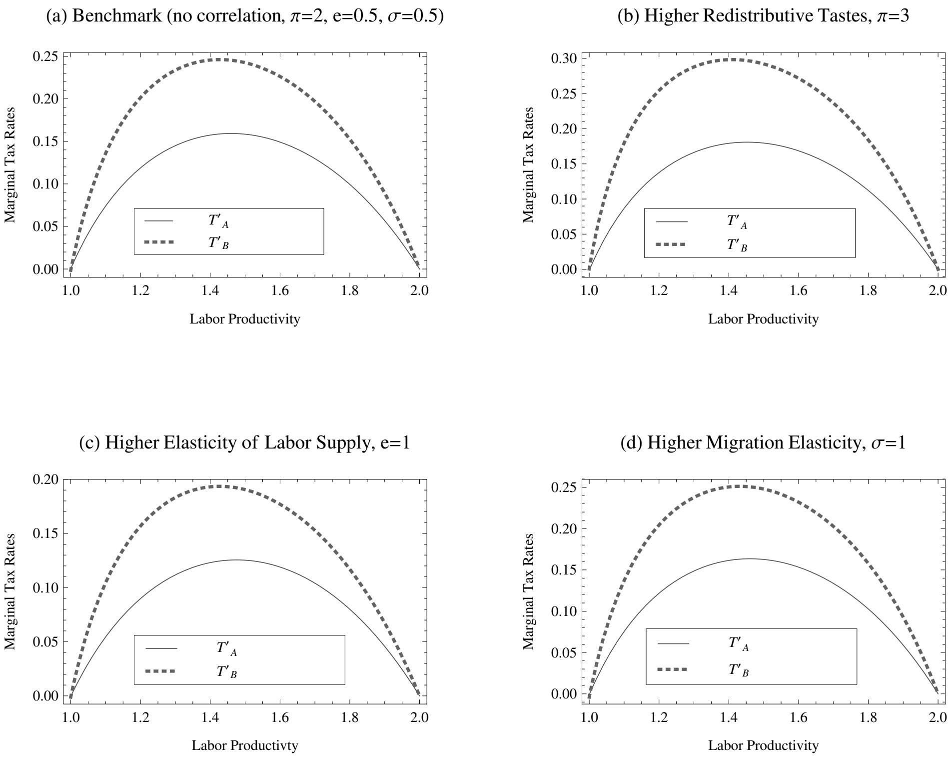

Figure 2.2: Benchmark simulations and sensitivity with alternative parameters

benchmark simulation, we consider $\pi = 2$, $e = 0.5$, and $\sigma = 0.5$ and that the preference for a public good and labor productivity are independently distributed.

We plot optimal marginal tax rates T'_A and T'_B in each figure. Figure 2.2(a) is our benchmark simulation and describes their results. The marginal income tax rate on individuals enjoying a higher quality public good is lower than that on individuals enjoying a lower quality public good, which is in line with Proposition 2.4. Moreover, we present the sensitivity of optimal policies with respect to changes in the parameter values around the benchmark simulation. First, we increase the redistributive taste π from 2 to 3, the result of which is depicted in Figure 2.2(b). We find that all marginal tax rates increase to reinforce the redistribution. Second, we increase the elasticity of labor supply ϵ from 0.5 to 1, whose effect is described in Figure 2.2(c). As expected, all marginal tax rates decrease. Third, we increase migration elasticity σ from 0.5 to 1, the outcome of which is shown in Figure 2.2(d), and find that the increase in migration elasticity has a limited impact on the marginal tax rate. Nevertheless, the marginal tax rate in region A remains lower. Thus, as long as preferences for public goods and labor productivity are independently distributed, the tax reform in our tax perturbation method is implemented at the optimum, regardless of the sensitivity of alternative parameter values.

Here, we examine the implication of introducing a positive and negative correlation be-

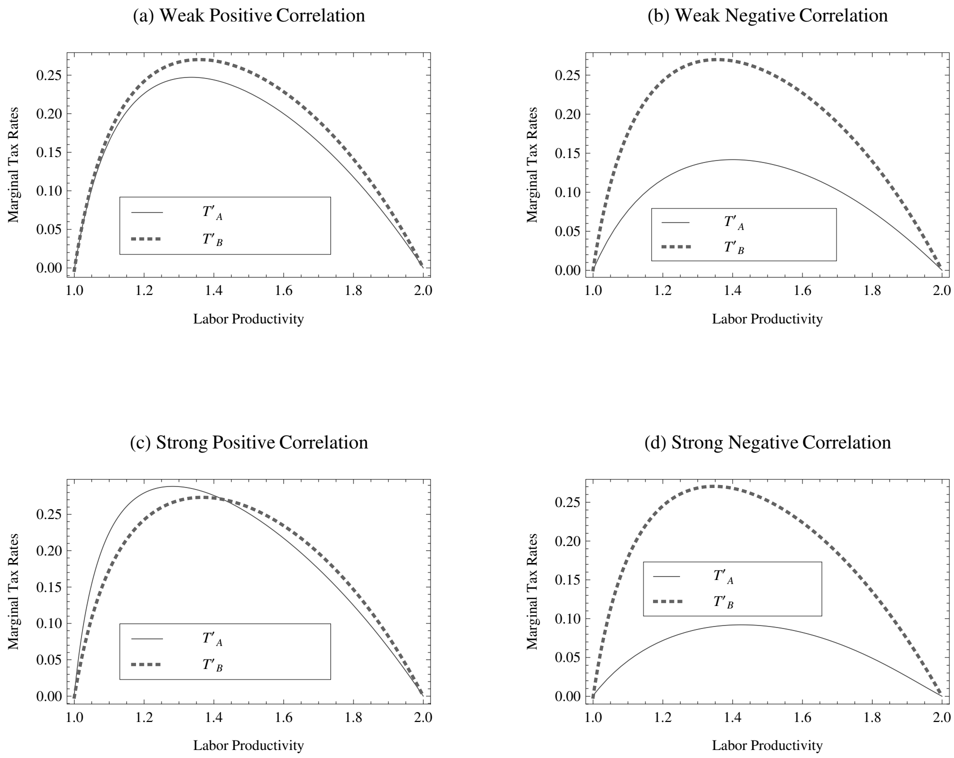

Figure 2.3: Simulations with positive or negative correlation

tween the preference for a public good and labor productivity. We introduce a positive correlation by considering $\overline{\theta}$ as a increasing function of w and a negative correlation $\overline{\theta}$ as a decreasing function of w, as in Kleven et al. (2006, 2009). First, we consider the case in which the preference for a public good is weakly correlated with labor productivity ($\overline{\theta} = 1 + 0.5w$). As shown in Figure 2.3(a), the level of marginal tax rates in the weak positive correlation case is higher than the independent case to reinforce the income redistribution because the inequalities between categories are more serious. In contrast, as depicted in Figure 2.3(b), the level of marginal tax rates in the weak negative correlation case ($\overline{\theta} = 3.5 - 0.5w$) is lower than the independent case because the inequalities are mitigated. The fact that the marginal tax rate in region A is lower remains even though weak correlation is allowed. This is consistent with the findings of Kleven et al. (2006, 2009), who numerically demonstrate that introducing a positive or negative correlation between two characteristics does not overturn the tax perturbation results. Next, we present a situation in which the preference for a public good is strongly correlated with labor productivity. Figure 2.3(c) shows that the marginal tax rates in region A in a strong positive correlation case ($\overline{\theta} = 1.5w$) can be higher than those in region B. Therefore, it is not necessary that the effect of a positive correlation does not overturn the theoretical results obtained from the tax perturbation analysis. Undoubtedly, a strong negative correlation ($\overline{\theta} = 4.5 - 1.5w$) does not affect the relationship between T'_A and

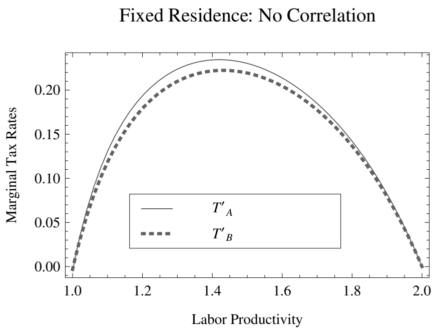

Figure 2.4: Simulations with labor immobility under no correlation

T'_B, as described in Figure 2.3(d).

Finally, we investigate how the optimal tax structure changes depending on whether the category is immutable or variable. Under the independence between two characteristics, we apply an income distribution that is endogenously generated from the result in Figure 2.2(a) when calibrating the marginal tax rate without labor mobility. Figure 2.4 depicts the optimal differentiated marginal tax rate in the independence case with immobile labor. The marginal tax rate in region A is higher than that in region B. From the optimal tax formula under no labor mobility, this means that the government's redistributive tastes are to redistribute income from people in regions A to people in region B.[11] In contrast with Figure 2.2(a), the government increases the tax burden in region A and decreases it in region B to reinforce the redistribution from region A to region B. In the same manner, we conduct numerical simulations under the correlation between two characteristics by applying an income distribution generated from the result in Figures 2.3(c) and 2.3(d). Figure 2.5(a) (Figure 2.5(b)) depicts the optimal differentiated marginal tax rate in the positive correlation case (negative correlation case) with immobile labor. These outcomes imply that the marginal tax rate on the region comprising a higher proportion of individuals with high ability is greater. Note that labor mobility decreases (increases) the marginal tax rate in region A (region B) compared to that under labor immobility, regardless the correlation. This is

[11] Under the independence between θ and w and the constant elasticity of labor supply with respect to the net-of-tax wage rate, the optimal tax formulas are given by

$$\frac{T'_A(z_A(w))}{1-T'_A(z_A(w))} = \left[1+\frac{1}{e}\right]\cdot\frac{1}{wf(w)}\cdot\int_w^{\overline{w}}(1-g_A(x))f(x)dx$$

$$\frac{T'_B(z_B(w))}{1-T'_B(z_B(w))} = \left[1+\frac{1}{e}\right]\cdot\frac{1}{wf(w)}\cdot\int_w^{\overline{w}}(1-g_B(x))f(x)dx$$

From the fact that the marginal tax rate in region A is higher, $\frac{T'_A(z_A(w))}{1-T'_A(z_A(w))} - \frac{T'_B(z_B(w))}{1-T'_B(z_B(w))} = (1+\frac{1}{e})\cdot\frac{1}{wf(w)}\cdot\int_w^{\overline{w}}(g_B(x)-g_A(x))f(x)dx > 0$ for any $w \in (\underline{w}, \overline{w})$. Therefore, the government increases the tax burden in region A to improve income inequalities in region B.

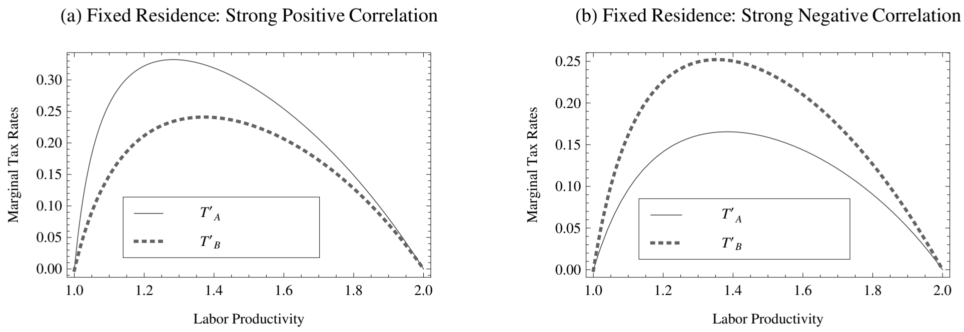

Figure 2.5: Simulations with labor immobility under positive or negative correlation

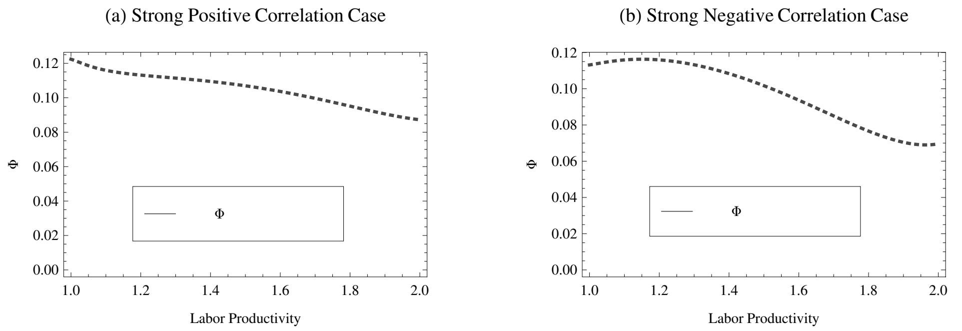

Figure 2.6: Simulations with respect to $\Phi(\cdot)$

because, as shown in Figures 2.6(a) and 2.6(b), $\Phi(\cdot)$, which appears when the labor is mobile, is always positive; this acts the marginal tax rate in region A (region B) as downward (upward) pressure, which in line with Proposition 2.3.[12]

2.6 Concluding Remarks

This study analyzes optimal nonlinear income taxes under spatial differences in terms of the quality of public goods when individuals have two types and determine labor supply along both intensive and extensive margins. The government can design differential income taxes on two regions, of which one has a higher quality public good. We show that the government's redistributive tastes and correlation between preferences for a public good and labor productivity are especially crucial in determining the shape of income tax schedules. In particular, if the preference for a public good and labor productivity are independently distributed and the first derivative of the social welfare function is strictly convex, the marginal income tax rate on individuals enjoying a higher quality public good is lower. This is because the de-

[12]These results cannot be directly compared in general because the redistributive taste g_i can differ depending on whether labor is mobile. However, the simulation result suggests that the the effect of the change in $\Phi(\cdot)$ is crucial to decrease (increase) the marginal tax rate in region A (region B), regardless of the change in g_i.

crease in tax burdens on the region in the presence of higher quality public goods leads to greater welfare gain from individuals being induced to access the region (participation effect) than the welfare loss caused by a decrease in tax receipts (mechanical effect). Moreover, we numerically find that the theoretical results are supported when the correlation between two characteristics is weak. However, the marginal tax rate on individuals enjoying a higher quality public good can be higher when the positive correlation between two characteristics is strong, although labor mobility weakens the differentiation of marginal tax rates on the basis of the positive correlation. Therefore, the present study offers implications for the optimal design of differential income tax schedules in the presence of a correlation between two characteristics, which is in contrast to Kleven et al. (2006, 2009).

Our findings have key implications for applied tax policies. If the aim of the government is to mitigate inequalities between regions in which income distributions slightly differ, the marginal income tax rate on individuals who enjoy higher quality public good services should be lower. However, if income stratification across regions is serious, the differentiation of the marginal tax rates on the basis of the correlation is recommended, as shown in previous studies examining tagging on immutable categories corresponding to the present study, in which individuals do not vote with their feet. Therefore, our novel findings are that the government should put emphasis on the information in terms of serious income stratification across regions in designing income tax schedules.

Our findings can be further applied to the optimal tax and transfer program designed by the central government. Indeed, Boadway and Pestieau (2006) present the federal government with regions of different income distributions as an example. In particular, the results of this study will be useful when a supranational government such as the European Union is transfered the responsibility of redistributive taxation from national governments.

2.7 Appendix

Appendix 2.A

Consider each optimal allocation as $\hat{\theta}^*, \Delta G^*, and\ V_B^*$. If ΔG^* is positive, $\hat{\theta}^* < \overline{\theta}$ is optimal because ΔG^* is zero from equation (2.B.6) evaluated at $\hat{\theta} = \overline{\theta}$. Therefore, it is sufficient to show that ΔG^* is positive. To show it, we assume that ΔG^* is zero, resulting in $\hat{\theta}^* = \overline{\theta}$. From the fact that $\theta(1 - F(\theta)) - \phi_{G_A}(G_B, N_A) = (\theta - \phi_{G_A}(G_B))(1 - F(\theta))$ evaluated at $\Delta G = 0$ is positive under a sufficiently large θ, there exists a pair $\{\Delta G^0, \hat{\theta}^0\}$ such that $\hat{\theta}^0 \Delta G^0 (1 - F(\hat{\theta}^0)) - \phi(\Delta G^0 + G_B, N_A) > 0$. This means that the additional tax revenue per capita is greater than the provision cost per capita. Note that the effect of the change in $\hat{\theta}$ from $\hat{\theta}^* = \overline{\theta}$ to $\hat{\theta}^0 < \overline{\theta}$ on $\phi(G_B, N_B)$ relaxes the budget constraint since $\phi(G_B, N_B)$ increases with respect to $\hat{\theta}$. Therefore, using additional revenues $\int_{\hat{\theta}^0}^{\overline{\theta}} \hat{\theta}^0 \Delta G^0 f(\theta) d\theta$, the government can increase all individuals' utilities by upwardly parallel shifting V_B^* without violating the incentive constraint, denoted by $V_B^0(w)$ ($> V_B^*(w)\ \forall w$). Here, we define the social welfare function achieved by a pair $\{\Delta G^i, \hat{\theta}^i, V_B^i(w)\}$, $i = *, 0$, as

$$\hat{\mathcal{W}}^i \equiv \int_{\underline{w}}^{\overline{w}} \left[\int_{\hat{\theta}^i}^{\overline{\theta}} W([\theta - \hat{\theta}^i]\Delta G^i + \theta G_B + V_B^i(w)) f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}^i} W(\theta G_B + V_B^i(w)) f(\theta, w) d\theta \right] dw$$

Obviously, we have $\hat{\mathcal{W}}^* \geq \hat{\mathcal{W}}^0$. Also, we have the following inequality for all G, $\hat{\theta}$, and $V_B(w)$ from the fact that $\hat{\mathcal{W}}$ is a decreasing function of $\hat{\theta}$:

$$\int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} W(\theta G_B + V_B(w)) f(\theta, w) d\theta dw \leq \hat{\mathcal{W}} \leq \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} W(\theta \Delta G + \theta G_B + V_B(w)) f(\theta, w) d\theta dw$$

Combining inequalities, we have

$$\int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} W(\theta G_B + V_B^0(w)) f(\theta, w) d\theta dw \leq \int_{\underline{w}}^{\overline{w}} \int_{\underline{\theta}}^{\overline{\theta}} W(\theta \Delta G^* + \theta G_B + V_B^*(w)) f(\theta, w) d\theta dw$$

Because ΔG^* is zero and $V_B^0(w)$ is greater than $V_B^*(w)$ for all w, it contradicts with the inequality. Thus, ΔG^* is positive.

Appendix 2.B

Using integration by parts, $\int_{\underline{w}}^{\overline{w}} \lambda(w) V_B'(w)$ is transformed into $\lambda(\overline{w})V_B(\overline{w}) - \lambda(\underline{w})V_B(\underline{w}) - \int_{\underline{w}}^{\overline{w}} \lambda'(w) V_B(w)$. Applying this to the optimization problem with non-differentiated marginal tax rates, the corresponding Lagrangian is rewritten as follows:

$$\begin{aligned} \mathcal{L} = \hat{\mathcal{W}} + \gamma &\left[\int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta} \Delta G f(\theta) d\theta + \int_{\underline{w}}^{\overline{w}} T_B(z(w)) f(w) dw - \phi(G_A, N_A) - \phi(G_B, N_B) \right] \\ &+ \int_{\underline{w}}^{\overline{w}} \lambda(w) \frac{\ell(w)}{w} v'(\ell(w)) dw + \int_{\underline{w}}^{\overline{w}} \lambda'(w) V_B(w) dw - \lambda(\overline{w}) V_B(\overline{w}) + \lambda(\underline{w}) V_B(\underline{w}) \end{aligned} \tag{2.B.1}$$

By the definition of indirect utilities, income taxes are expressed by $T_B(z(w)) = w\ell(w) - V_B(w) - v(\ell(w))$ and substitute this for (2.B.1) yields:

$$\begin{aligned} \mathcal{L} = \hat{\mathcal{W}} + \gamma &\left[\int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta} \Delta G f(\theta) d\theta + \int_{\underline{w}}^{\overline{w}} \Big(w\ell(w) - V_B(w) - v(\ell(w)) \Big) f(w) dw - \Big(\sum_{i=A,B} \phi(G_i, N_i) \Big) \right] \\ &+ \int_{\underline{w}}^{\overline{w}} \lambda(w) \frac{\ell(w)}{w} v'(\ell(w)) dw + \int_{\underline{w}}^{\overline{w}} \lambda'(w) V_B(w) dw - \lambda(\overline{w}) V_B(\overline{w}) + \lambda(\underline{w}) V_B(\underline{w}) \end{aligned} \tag{2.B.2}$$

The first-order conditions associated with $V_B(w)$, $\ell(w)$, $\hat{\theta}$, and ΔG are as follows:

$$\frac{\partial \mathcal{L}}{\partial V_B(w)} = \int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(w)) f(\theta, w) d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(\theta G_B + V_B(w)) f(\theta, w) d\theta + \lambda'(w) - \gamma f(w) = 0 \tag{2.B.3}$$

$$\frac{\partial \mathcal{L}}{\partial \ell(w)} = \lambda(w) \left[\frac{v'(\ell(w))}{w} + \frac{\ell(w)}{w} v''(\ell(w)) \right] + \gamma \left[w - v'(\ell(w)) \right] f(w) = 0 \tag{2.B.4}$$

$$\frac{\partial \mathcal{L}}{\partial \hat{\theta}} = -\gamma \left[\hat{\theta} f(\hat{\theta}) - (1 - F(\hat{\theta})) \right] \Delta G - \int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} \Delta G \cdot W'(\theta G_A + V_A) f(\theta, w) d\theta dw + \gamma (\phi_{N_A} - \phi_{N_B}) f(\hat{\theta}) = 0 \tag{2.B.5}$$

$$\frac{\partial \mathcal{L}}{\partial \Delta G} = \int_{\underline{w}}^{\overline{w}} \int_{\hat{\theta}}^{\overline{\theta}} (\theta - \hat{\theta}) W'(\theta G_A + V_A(w)) f(\theta, w) d\theta dw + \gamma \int_{\hat{\theta}}^{\overline{\theta}} \hat{\theta} f(\theta) d\theta - \gamma \phi_{G_A} = 0 \tag{2.B.6}$$

$$\frac{\partial \mathcal{L}}{\partial V_B(\overline{w})} = -\lambda(\overline{w}) = 0, \ \frac{\partial \mathcal{L}}{\partial V_B(\underline{w})} = \lambda(\underline{w}) = 0 \tag{2.B.7}$$

Integrating (2.B.3) between w and $\overline{w}$ and using the transversality condition (2.B.7) yields:

$$-\frac{\lambda(w)}{\gamma} = \int_{w}^{\overline{w}} \left[1 - \frac{\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(x)) f(\theta|x) d\theta}{\gamma} - \frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(\theta G_B + V_B(x)) f(\theta|x) d\theta}{\gamma} \right] f(x) dx \tag{2.B.8}$$

Rearranging equation (2.B.8) yields:

$$-\frac{\lambda(w)}{\gamma} = \int_{w}^{\overline{w}} \left[1 - \frac{\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(x)) f(\theta|x) d\theta}{\gamma f_A^c(x)} f_A^c(x) - \frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(\theta G_B + V_B(x)) f(\theta|x) d\theta}{\gamma f_B^c(x)} f_B^c(x) \right] f(x) dx \tag{2.B.9}$$

Therefore, by the definition of g_i and $\overline{g}$,

$$-\frac{\lambda(w)}{\gamma} = \int_{w}^{\overline{w}} \left[1 - g_A(x) f_A^c(x) - g_B(x) f_B^c(x) \right] f(x) dx = \int_{w}^{\overline{w}} (1 - \overline{g}(x)) f(x) dx \tag{2.B.10}$$

On the other hand, equation (2.B.4) is transformed as follows:

$$\lambda(w) \frac{v'(\ell(w))}{w} \left[1 + \frac{\ell(w)}{v'(\ell(w))} v''(\ell(w)) \right] + \gamma w \left[1 - \frac{v'(\ell(w))}{w} \right] f(w) = 0 \tag{2.B.11}$$

Substituting equation (2.2) and (2.3) and rearranging, (2.B.11) is rewritten as follows:

$$\frac{T'(z(w))}{1 - T'(z(w))} = -\left[1 + \frac{1}{\epsilon} \right] \frac{\lambda(w)}{\gamma} \frac{1}{w f(w)} \tag{2.B.12}$$

Finally, combining (2.B.10) and (2.B.12) yields (2.14).

We transform (2.B.5) using $E = \hat{\theta}\Delta G$ as follows:

$$\gamma\frac{E-(\phi_{N_A}-\phi_{N_B})}{E}\hat{\theta}f(\hat{\theta}) = \gamma(1-F(\hat{\theta})) - \int_{\underline{w}}^{\overline{w}}\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(w))f(\theta,w)d\theta dw \quad (2.B.13)$$

Furthermore, integrating (2.B.3) between $\underline{w}$ and $\overline{w}$ and using the transversality conditions (2.B.7) yields:

$$\gamma = \int_{\underline{w}}^{\overline{w}}\left[\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(w))f(\theta,w)d\theta + \int_{\underline{\theta}}^{\hat{\theta}} W'(\theta G_B + V_B(w))f(\theta,w)d\theta\right]dw \quad (2.B.14)$$

Substituting (2.B.14) into (2.B.13) and dividing by N_A, the following is obtained:

$$\begin{aligned}\frac{E-(\phi_{N_A}-\phi_{N_B})}{E}\frac{\hat{\theta}f(\hat{\theta})}{1-F(\hat{\theta})} =& \frac{1}{N_A}\left[N_A\int_{\underline{w}}^{\overline{w}}\frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(\theta G_B + V_B(w))f(\theta|w)d\theta}{\gamma}f(w)dw\right.\\ &\left.- N_B\int_{\underline{w}}^{\overline{w}}\frac{\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(w))f(\theta|w)d\theta}{\gamma}f(w)dw\right]\end{aligned} \quad (2.B.15)$$

By the definition of g_i and η, we can rewrite (2.B.15) for (2.15).

Appendix 2.C

Using integration by parts, $\int_{\underline{w}}^{\overline{w}} \lambda_i(w)V_i'(w)$ is transformed into $\lambda_i(\overline{w})V_i(\overline{w}) - \lambda_i(\underline{w})V_i(\underline{w}) - \int_{\underline{w}}^{\overline{w}} \lambda_i'(w)V_i(w)$. Applying this to the optimization problem with differentiated marginal tax rates, the corresponding Lagrangian is rewritten as follows:

$$\begin{aligned}\mathcal{L} = \mathcal{W} + \gamma&\left[\int_{\underline{w}}^{\overline{w}} T_A(z_A(w))f_A(w)dw + \int_{\underline{w}}^{\overline{w}} T_B(z_B(w))f_B(w)dw - \left(\sum_{i=A,B}\phi(G_i,N_i)\right)\right]\\ &+ \sum_{i=A,B}\left[\int_{\underline{w}}^{\overline{w}}\lambda_i(w)\frac{\ell_i(w)}{w}v'(\ell_i(w))dw + \int_{\underline{w}}^{\overline{w}}\lambda_i(w)V_i(w)dw - \lambda_i(\overline{w})V_i(\overline{w}) + \lambda_i(\underline{w})V_i(\underline{w})\right]\\ &+ \int_{\underline{w}}^{\overline{w}}\mu(w)\left[\hat{\theta}(w)\Delta G + V_A(w) - V_B(w)\right]dw\end{aligned} \quad (2.C.1)$$

By the definition of indirect utilities, government's revenues from region i are expressed by $T_i(z_i(w)) = w\ell_i(w) - V_i(w) - v(\ell_i(w))$ and substitute this for (2.C.1) yields:

$$\begin{aligned}\mathcal{L} = \mathcal{W} + \gamma\Bigg[\sum_{i=A,B}\int_{\underline{w}}^{\overline{w}}\Big(w\ell_i(w) - V_i(w) - v(\ell_i(w))\Big)f_i(w)dw - \Big(\sum_{i=A,B}\phi(G_i, N_i)\Big)\Bigg] \\ + \sum_{i=A,B}\Bigg[\int_{\underline{w}}^{\overline{w}}\lambda_i(w)\frac{\ell_i(w)}{w}v'(\ell_i(w))dw + \int_{\underline{w}}^{\overline{w}}\lambda_i(w)V_i(w)dw - \lambda_i(\overline{w})V_i(\overline{w}) + \lambda_i(\underline{w})V_i(\underline{w})\Bigg] \\ + \int_{\underline{w}}^{\overline{w}}\mu(w)\Big[\hat{\theta}(w)\Delta G + V_A(w) - V_B(w)\Big]dw\end{aligned} \tag{2.C.2}$$

The first-order conditions associated with $V_i(w)$, $\ell_i(w)$, and $\hat{\theta}(w)$ are as follows:

$$\frac{\partial\mathcal{L}}{\partial V_A(w)} = \int_{\hat{\theta}(w)}^{\overline{\theta}} W'(\theta G_A + V_A(w))f(\theta, w)d\theta + \lambda'_A(w) - \gamma f_A(w) + \mu(w) = 0 \tag{2.C.3}$$

$$\frac{\partial\mathcal{L}}{\partial V_B(w)} = \int_{\underline{\theta}}^{\hat{\theta}(w)} W'(\theta G_B + V_B(w))f(\theta, w)d\theta + \lambda'_B(w) - \gamma f_B(w) - \mu(w) = 0 \tag{2.C.4}$$

$$\frac{\partial\mathcal{L}}{\partial \ell_A(w)} = \lambda_A(w)\Bigg[\frac{v'(\ell_A(w))}{w} + \frac{\ell_A(w)}{w}v''(\ell_A(w))\Bigg] + \gamma\Big[w - v'(\ell_A(w))\Big]f_A(w) = 0 \tag{2.C.5}$$

$$\frac{\partial\mathcal{L}}{\partial \ell_B(w)} = \lambda_B(w)\Bigg[\frac{v'(\ell_B(w))}{w} + \frac{\ell_B(w)}{w}v''(\ell_B(w))\Bigg] + \gamma\Big[w - v'(\ell_B(w))\Big]f_B(w) = 0 \tag{2.C.6}$$

$$\frac{\partial\mathcal{L}}{\partial \hat{\theta}(w)} = -\gamma\Big[T_A(z_A(w)) - T_B(z_B(w)) - (\phi_{N_A} - \phi_{N_B})\Big]f(\hat{\theta}(w), w) + \mu(w)\Delta G = 0 \tag{2.C.7}$$

$$\frac{\partial\mathcal{L}}{\partial V_i(\overline{w})} = -\lambda_i(\overline{w}) = 0,\ \frac{\partial\mathcal{L}}{\partial V_i(\underline{w})} = \lambda_i(\underline{w}) = 0\ i = A, B \tag{2.C.8}$$

Substituting (2.C.7) for (2.C.3) to delete $\mu(w)$ and dividing by γ yields:

$$\begin{aligned}\frac{\lambda'_A(w)}{\gamma} = f_A(w) - \frac{\int_{\hat{\theta}(w)}^{\overline{\theta}} W'(\theta G_A + V_A(w))f(\theta, w)d\theta}{\gamma} \\ - \frac{T_A(z_A(w)) - T_B(z_B(w)) - (\phi_{N_A} - \phi_{N_B})}{\Delta G}f(\hat{\theta}(w), w)\end{aligned} \tag{2.C.9}$$

By the definition of g_i,

$$\frac{\lambda'_A(w)}{\gamma} = \Bigg[(1 - g_A(w))f^c_A(w) - \frac{T_A(z_A(w)) - T_B(z_B(w)) - (\phi_{N_A} - \phi_{N_B})}{\Delta G}f(\hat{\theta}(w)|w)\Bigg]f(w) \tag{2.C.10}$$

Integrating (2.C.10) between w and $\overline{w}$ and using the transversality condition (2.C.8) yields:

$$-\frac{\lambda_A(w)}{\gamma} = \int_w^{\overline{w}} \left[(1-g_A(x))f_A^c(x) - \frac{T_A(z_A(x)) - T_B(z_B(x)) - (\phi_{N_A} - \phi_{N_B})}{\Delta G} f(\hat{\theta}(x)|x) \right] f(x)dx \tag{2.C.11}$$

On the other hand, (2.C.5) is transformed as follows:

$$\lambda_A(w)\frac{v'(\ell_A(w))}{w}\left[1 + \frac{\ell_A(w)}{v'(\ell_A(w))}v''(\ell_A(w))\right] + \gamma w\left[1 - \frac{v'(\ell_A(w))}{w}\right] f_A(w) = 0 \tag{2.C.12}$$

Substituting equation (2.2) and (2.3) and rearranging,

$$\frac{T_A'(z_A(w))}{1 - T_A'(z_A(w))} = -\left[1 + \frac{1}{\epsilon_A}\right]\frac{\lambda_A(w)}{\gamma}\frac{1}{wf_A(w)} \tag{2.C.13}$$

Finally, combining (2.C.11) and (2.C.13), we can obtain equation (2.22). In the similar way, (2.23) is obtained.

Appendix 2.D

Let us start from the separable taxation with non-differentiated marginal income tax rates. Using the assumption of independence between θ and w, $g_A(w)$ and $g_B(w)$ can be rewritten as follows:

$$g_A(w) = \frac{\int_{\hat{\theta}}^{\overline{\theta}} W'(\theta G_A + V_A(w))f(\theta)d\theta}{\gamma(1 - F(\hat{\theta}))},\ g_B(w) = \frac{\int_{\underline{\theta}}^{\hat{\theta}} W'(\theta G_B + V_B(w))f(\theta)d\theta}{\gamma F(\hat{\theta})}$$

By differentiating $g_B(w) - g_A(w)$ with respect to w, we can get the following:

$$g_B'(w) - g_A'(w) = \left[\frac{\int_{\underline{\theta}}^{\hat{\theta}} W''(\theta G_B + V_B(w))f(\theta)d\theta}{\gamma F(\hat{\theta})} - \frac{\int_{\hat{\theta}}^{\overline{\theta}} W''((\theta - \hat{\theta})\Delta G + \theta G_B + V_B(w))f(\theta)d\theta}{\gamma(1 - F(\hat{\theta}))}\right] \cdot V_B'(w) \tag{2.D.1}$$

By the assumption, W'' is strictly increasing. This means that $W''(\hat{\theta}G_B + V_B(w)) > W''(\theta G_B + V_B(w))$ for any $\theta < \hat{\theta}$ and $W''((\theta - \hat{\theta})\Delta G + \theta G_B + V_B(w)) > W''(\hat{\theta}G_B + V_B(w))$ for any $\theta > \hat{\theta}$. Using these relationships, we can obtain $\int_{\underline{\theta}}^{\hat{\theta}} W''(\hat{\theta}G_B + V_B(w))f(\theta)d\theta > \int_{\underline{\theta}}^{\hat{\theta}} W''(\theta G_B + V_B(w))f(\theta)d\theta$ and $\int_{\hat{\theta}}^{\overline{\theta}} W''((\theta - \hat{\theta})\Delta G + \theta G_B + V_B(w))f(\theta)d\theta > \int_{\hat{\theta}}^{\overline{\theta}} W''(\hat{\theta}G_B + V_B(w))f(\theta)d\theta$.

Therefore, the following inequality holds:

$$g_B'(w) - g_A'(w) < \left[\frac{\int_{\underline{\theta}}^{\hat{\theta}} W''(\hat{\theta}G_B + V_B(w))f(\theta)d\theta}{\gamma F(\hat{\theta})} - \frac{\int_{\hat{\theta}}^{\overline{\theta}} W''(\hat{\theta}G_B + V_B(w))f(\theta)d\theta}{\gamma(1 - F(\hat{\theta}))}\right] \cdot V_B'(w) = 0 \tag{2.D.2}$$

Hence, $g_B' - g_A'$ is negative for any w. By using this fact, we can derive the following inequality for any w:

$$\frac{1}{F(w)} \int_{\underline{w}}^{w} (g_B(x) - g_A(x))f(w)dw > g_B(w) - g_A(w) > \frac{1}{1 - F(w)} \int_{w}^{\overline{w}} (g_B(x) - g_A(x))f(w)dw \tag{2.D.3}$$

Here, we rearrange the equation (2.36) as follows:

$$dW = \frac{1}{1 - F(w)}\left[(1-F(w)) \int_{\underline{w}}^{w} (g_B(x)-g_A(x))f(x)dx - F(w) \int_{w}^{\overline{w}} (g_B(w)-g_A(w))f(w)dw\right] \times dT \tag{2.D.4}$$

Using (2.D.3), we can conclude that dW is positive.

Chapter 3

Optimal Capital Income Taxation and Tax Expenditures under Nonlinear Income Taxation

3.1 Introduction

The optimal tax theory plays an important role in designing the income redistribution policy and implementing public projects. Various aspects of this field have been researched for several years; in particular, many economists are concerned with the question: Should capital income be taxed? This question arises from the fact that the government can reinforce redistributive policy by levying taxes on savings; however, it is a form of double taxation. Although it has been discussed whether taxation of capital income is justified, the taxation on capital income is an ongoing research issue.

The objective of this paper is to investigate the desirability of capital income taxes from the viewpoint of economic behaviors that individuals can contribute to public goods. This motivation stems from policy discussion and empirical evidence. In the U.S., one of the important policy discussions is whether the government should tax high income earners more. This is placed on the agenda in a presidential election in 2016. A Sanders's plan is increasing taxes for the very rich, in particular, on their capital gains and charitable giving. This is because, normally, a high tax rate on capital gains leads taxpayers to choose charitable giving as strategies to avoid recognizing taxable gains.[1] Furthermore, Hood et al. (1977) find that,

[0]This chapter is based on a joint work with Shigeo Morita (Faculty of Economics, Fukuoka University) entitled "Optimal Capital Income Taxation in the Case of Private Donations to Public Goods". See Morita and Obara (2016).

[1]According to some empirical studies, Feldstein and Taylor (1976), Clotfelter (1985), and Auten et al. (1992) empirically find that charitable contributions are sensitive to tax rates on capital gains due to avoidance strategies. From the fact, Chetty (2009) estimates the taxable income elasticity (the effect of taxation on

in Canada, the 1971 Tax Reform whose feature was the introduction of a 50 % capital gain tax brought about a decrease in individual charitable donations. More recently, Auten et al. (2002) estimate the price elasticity of donation, which the price is a weighted average of the price of giving cash and appreciated properties. These findings imply that individuals take account of capital income tax when they donate to the public good. The policy debate and empirical evidence suggest that individuals' charitable giving are inseparable when discussing capital income taxation. Therefore, our analysis takes a step towards theoretically clarifying how capital income tax schemes should be designed when individuals can contribute to a public good.

Our analysis comprises a dynamic setting in which individuals live for two periods. We assume that in the first period, individuals can spend a part of their savings on donations to a charity. Conceptually, we regard the private donation to the public good as the charitable giving and utilize the framework of the optimal taxation in the presence of a public good. This setup is in line with Andreoni (1988), Saez (2004b), and Diamond (2006). For simplicity, there are two types of individuals: high- and low-skilled individuals. The government designs three types of tax schedules: nonlinear taxes on labor and capital income and nonlinear subsidies for contributions to a public good. We demonstrate that although a utility function is represented by the preference that private goods are additively separable from leisure, the marginal tax on capital is zero for the high skilled but not low skilled when private contributions are made to public goods. The amount of donation to a public good differs between high- and low-skilled individuals, which affects the marginal rate of substitution between consumption in the first and second period. If a high-skilled individual's valuation of future consumption is higher than a low-skilled individual's one, the distortion on savings behavior for the latter relaxes the self-selection constraint for the former. Thus, the relationship between private consumption and donation to a public good plays an important role in characterizing the optimal tax rates for marginal capital income.

We cite Ordover and Phelps (1979), whose study is an important contribution to the literature on the analysis of capital income taxation. They examine optimal nonlinear taxation on income and savings in an overlapping generation economy in the case that earnings ability is unobservable. Their main conclusion states that if preferences are weakly separable between private goods and leisure, the taxes on savings are redundant. This is consistent with the Atkinson and Stiglitz (1976) theorem. On the other hand, Saez (2002a) investigated the conditions necessary to obtain the Atkinson and Stiglitz (1976) theorem, and show that if individuals have heterogeneous tastes in private consumptions, the results are violated, even though the utility function is weakly separable between private goods and leisure.

This paper is closely related to the model explaining the desirability of capital income

reported taxable income) to measure deadweight loss in the presence of avoidance.

taxes from the heterogeneity of tastes for goods between high-income and low-income earners, which stems from Saez (2002a).[2] However, the taste differentiation which previous literatures refer to is an assumption, e.g. the difference of initial endowments or discount rates (Boadway et al. (2000), Cremer et al. (2001), Diamond and Spinnewijn (2011)). The main difference is that we point out that tastes differentiation can result from individuals' behavior without explicitly assuming additional characteristics. Consequently, we present the desirability of capital income taxes establishing the theoretical foundation that taste differentiation occurs. To the best of our knowledge, this result provides new evidence justifying capital taxation since private donations have not been considered as individual's behaviors in the theory of capital income taxation.

Our models allow the government to give subsidies for contributions to public goods. Prior studies have investigated optimal tax policy assuming that charitable giving exists (Andreoni (1988), Saez (2004b)). The most closely related study in terms of tax treatment of private donations is that of Diamond (2006). Diamond (2006) shows that the welfare-improving effect is achieved by introducing a subsidy on private donations toward a public good under nonlinear income taxes on labor. However, Diamond (2006) does not allow the government to impose income taxes on capital because of a static model. Our interest is the implication of the property of private provision on the capital income tax. We extend the Diamond model as a two-period model to investigate the desirability of capital income taxes. Moreover, Diamond (2006) does not attempt to derive the optimal tax treatment formula. This study rigorously characterizes the tax treatment formula at the optimum level, which depends on the Pigouvian effect, and the effect on the incentive compatibility constraint under a more general utility function.

The optimal tax rate formulas in terms of capital income taxes and subsidies for private donations are conceptually related to the result of Cremer et al. (1998) who examine both optimal linear and nonlinear taxation on commodities and income in the presence of goods that cause externalities. They show that if tastes for private consumption are not identical, goods without externalities are taxed. The crucial difference is that we consider a finite number of individuals as in Diamond (2006) compared to the model of Cremer et al. (1998) who consider an infinite population. In the setting, the change of the contribution to a public good due to a mimicker affects the aggregate level of a public good. If consumption

[2] There are several related papers that attempt the justification of capital income taxes from a variety of aspects. For example, from the commitment issue for the government, Brett and Weymark (2008) show that if the government does not commit to its policy in the next period, capital income taxes should be levied. Christiansen and Tumala (2008) concludes that the government ought to implement capital income taxation in the presence of income shifting, that is, if individuals can shift labor income to capital income since the government cannot observe labor and capital income. Pirttilä and Tuomala (2001) show that capital income taxes are desirable when the wage is endogenously determined, which this approach is followed by Naito (1999).

is not weakly separable with a public good, the mimicker's behavior makes intertemporal substitution between the mimicker and the person being mimicked differ, which this allows for taxation on capital income without exogenously assuming taste differentiation.

Finally, we utilize dynamic setting to verify the desirability of capital income taxes. Recently, there are the growing body of literatures using the new dynamic public finance (NDPF) approach (see Golosov et al. (2007)). The feature of the model is that individual's types can stochastically change over time. Under such an environment, these papers have examined the implementability of nonlinear taxes on labor and capital income over multi-periods. For simplicity, the present paper assumes that individual's types are non-stochastic over time.

This paper is organized as follows. In section 3.2, we describe the framework of the basic model. In section 3.3, we characterize some optimal nonlinear tax formulas. In section 3.4 and 3.5, we explore the robustness of our results. Finally, we present the conclusion in section 3.6.

3.2 The Model

3.2.1 Environment

We consider an economy in which individuals live for two periods: they work in the only first period. There are two types of individuals: high-type and low-type, indexed by $i = H, L$. Type i individuals' wage rate is w^i and we suppose that $w^H > w^L$. Their before-tax income is $y_i \equiv w^i \ell^i$, where ℓ^i denotes individuals' labor supply. The number of type i individuals is defined by π^i, which is a natural number more than 2 and for now, is invariant. [3] The utility function of type i individuals is

$$U(c^i, x^i, G, l^i) = u(c^i, x^i, G) - v(\ell^i) \tag{3.1}$$

where c^i denotes consumption of a private good in the first period, x^i consumption in the second period, and G the amount of public good. Individuals can contribute to the public good, and then the aggregate amount of the public good is

$$G = \sum_i \pi^i g^i + g^G \tag{3.2}$$

[3] Piketty (1993) and Hamilton and Slutsky (2007) show that, with a finite number of individuals, it can achieve the first-best allocation, if an individual's tax schedule depends on the behavior of other individuals. This paper restricts an individual's tax schedule to a function of only the value his/her own labor income, capital income, and private donation to a public good, following by traditional optimal taxation literatures.

where g^i denotes the amount of type i's private donations to a public good and g^G the amount of the public good provision by the government.[4] The sub-utility function $u(\cdot)$ is strictly increasing, concave and twice differentiable, and it also satisfies the Inada condition, and $v(\cdot)$ is strictly increasing, convex and twice differentiable.

3.2.2 Individual's problem

Let s^i denotes savings of type i individuals and r the interest rate. The budget constraints which type i individuals face can be written as follows:

$$c^i + s^i + g^i - \tau(g^i) = y^i - T(y^i) \tag{3.3}$$

$$s^i(1+r) - \Phi(s^i r) = x^i \tag{3.4}$$

where let $\tau(g^i)$ denotes the subsidy for private donation of type i individuals to the public good, $T(y^i)$ the income tax payment, and $\Phi(rs^i)$ the capital income tax payment, which are nonlinear functions of g^i, y^i, and rs^i. Individuals choose c^i, x^i, s^i, g^i, and ℓ^i to maximize the utility function (equation (3.1)) subject to their budget constraints (equations (3.3) and (3.4)). Combining with the first order conditions yields

$$MRS^i_{cx} \equiv \frac{u_c(c^i, x^i, G)}{u_x(c^i, x^i, G)} = (1+r) - r\Phi'(rs^i) \tag{3.5}$$

where $u_c(c^i, x^i, G) \equiv \frac{\partial u}{\partial c^i}$ denotes the marginal utility of consumption in the first period, $u_x(c^i, x^i, G) \equiv \frac{\partial u}{\partial x^i}$ the marginal utility of consumption in the second period, and $\Phi'(rs^i) \equiv \frac{d\Phi}{drs^i}$ the marginal capital income tax rate function corresponding to returns of savings rs^i. The first order condition of donation g^i yields [5]

$$MRS^i_{Gc} \equiv \frac{u_G(c^i, x^i, G)}{u_c(c^i, x^i, G)} = 1 - \tau'(g^i) \tag{3.6}$$

[4] We consider public goods that are financed by not only individuals but also the government such as health, education, and social services. According to Charitable Giving Statistics by National Philanthropic Trust, in the United States, charitable giving from individuals accounts for 71% of total giving, and the majority of giving goes to religious and educational organizations, health, and so on. In particular, a donation to religious organizations is a suitable example for outcomes where Lemma 3.1 implies, because the government in United States cannot contribute to them.

[5] To derive the optimal condition of private contribution to a public good, we introduce the notation $G_{\sim i}$, which is the total amount of a public good contributed by the government and other individuals including others of the same type. The sub-utility function $u(\cdot)$ can be seen as $u(c^i, x^i, G_{\sim i} + g_i)$. As the marginal utility of total amount of the public good $u_G(c^i, x^i, G) \equiv \frac{\partial u}{\partial G}$ is equal to that of private donation to a public good, the first order condition with respect to g^i is given as equation (3.6).

where $\tau'(g^i) \equiv \frac{d\tau}{dg^i}$ is the marginal subsidy rate function of the private donation to a public good.

The optimal labor supply will obey the following condition:

$$MRS^i_{yc} \equiv \frac{v_\ell(\ell^i)}{w^i u_c(c^i, x^i, G)} = 1 - T'(y^i) \tag{3.7}$$

where $v_\ell(\ell^i) \equiv \frac{\partial v}{\partial \ell^i}$ is the marginal disutility of labor supply, and $T'(y^i) \equiv \frac{dT}{dy^i}$ is the marginal labor income tax rate function corresponding to income level y^i. Equation (3.5), (3.6), (3.7) indicate that the corresponding marginal rate of substitution is equal to the price minus the marginal tax rates.

For simplicity, we consider that both wage rate and interest rate are exogenous.[6]

3.2.3 The planning problem

The government designs the optimal tax system to maximize the utilitarian social welfare function which is given by:

$$W = \sum_i \pi^i U(c^i, x^i, G, \frac{y^i}{w^i}) \tag{3.8}$$

The government distributes the total tax revenues from three sorts of tax schedules into tax transfers and the public provision of a public good. Therefore, the budget constraint for the government is [7]

$$\sum_i \pi^i T(y^i) - \sum_i \pi^i \tau(g^i) - s^G - g^G \geq 0 \tag{3.9}$$

$$\sum_i \pi^i \Phi(rs^i) + (1 + r)s^G \geq 0 \tag{3.10}$$

[6] Pirttilä and Tuomala (2001) show that capital income taxation is justified when wages are endogenously determined and the relative wage rate is affected by the amount of savings. By contrast, we assume no general-equilibrium effects of wage rates. This is because we never obtain the novel effect even if we endogenize input prices, that is, the optimal tax formula for capital income just involves the endogenous wage term proposed by Pirttilä and Tuomala (2001). Thus, our model can be seen as the two-period, partial equilibrium version of Pirttilä and Tuomala (2001) model. At the optimum, where only the government contributes to a public good, our model's outcome is consistent with that of their model.

[7] Following Diamond (2006), we assume that there is no response of the government budget constraints to a deviation from individuals' anticipated revealing strategies.

Using the budget constraints that individuals face, these can be equivalently written as

$$\sum_i y^i\pi^i - \sum_i (c^i + s^i + g^i)\pi^i - s^G - g^G \geq 0 \tag{3.11}$$

$$(1+r)\left(\sum_i s^i\pi^i + s^G\right) - \sum_i x^i\pi^i \geq 0 \tag{3.12}$$

The informational assumptions are conventional: the government can observe individuals' donation, labor income, and capital income, while their productivity is never observable. We focus on the case where the government attempts to redistribute from high-type individuals to low-type individuals. This means that the following incentive compatibility constraint preventing high-type individuals from mimicking low-type ones is only binding at the social optimum:

$$U(c^H, x^H, G, \frac{y^H}{w^H}) \geq U(c^L, x^L, \hat{G}, \frac{y^L}{w^H}) \tag{3.13}$$

where $\hat{G} \equiv G - g^H + g^L$ denotes the aggregate level of a public good which is achieved when high-type individuals mimic.

The social planning problem is to maximize the social welfare function (equation (3.8)), subject to the equation for the public good (equation (3.2)), the resource constraints (equations (3.11) and (3.12)), and the incentive compatibility constraints (equation (3.13)), respectively. The Lagrangean corresponding to this planning problem can be formulated as follows:

$$\mathcal{L} = W + \mu\left[\sum_i g^i\pi^i + g^G - G\right] + \gamma_1\left[\sum_i y^i\pi^i - \sum_i (c^i + s^i + g^i)\pi^i - s^G - g^G\right] \tag{3.14}$$
$$+\gamma_2\left[(1+r)\left(\sum_i s^i\pi^i + s^G\right) - \sum_i x^i\pi^i\right] + \lambda\left[U(c^H, x^H, G, \frac{y^H}{w^H}) - U(c^L, x^L, \hat{G}, \frac{y^L}{w^H})\right]$$

Let μ be the Lagrange multiplier of the formation of the aggregate amount of a public good, γ_1 the Lagrange multiplier of the resource constraint in the first period, γ_2 the Lagrange multiplier of the resource constraint in the second period, and λ the Lagrange multiplier of incentive constraint.

3.3 Characterizing the optimal nonlinear tax policies

Here, we present the key features of our model's outcomes. The results imply that the government should design taxes on capital income to be supplement its tax treatment of private donations to a public good.

3.3.1 Optimal nonlinear capital income taxation

Combining the optimality condition regarding c^i and x^i yields the optimal capital income tax rate for type i individuals:

$$\Phi'(rs^L) = \frac{\lambda u_x(c^L, x^L, \hat{G})}{r\pi^L\gamma_2}\left[MRS^L_{cx} - M\hat{R}S_{cx}\right] \tag{3.15}$$

$$\Phi'(rs^H) = 0 \tag{3.16}$$

where $M\hat{R}S_{cx} \equiv \frac{u_c(c^L,x^L,\hat{G})}{u_x(c^L,x^L,\hat{G})}$ denotes the corresponding marginal rate of substitution that the mimicker faces. The derivation is included in Appendix 3.A. Equation (3.15) implies that the deviation of the optimal tax rate on capital income from the Atkinson-Stiglitz theorem depends on the term in the brackets of the right hand side. These equations give the following proposition:

Proposition 3.1.

1. *When a public good has a more complementary (substitutionary) relationship with the consumption good in the first period than in the second, even if individual preferences can be separated between labor and consumption, the marginal capital income tax rate is positive (negative) for type-1 individuals and zero for the type-2 individuals.*

2. *When a public good has no relationship with both the consumption good in the first period and in the second period, the marginal capital income tax rate is zero for both types of individuals.*

The result of Proposition 3.1 is crucially related to the difference between G and $\hat{G}$. At the optimum, the level of a public good is higher when a type-2 individual chooses a truth-telling strategy than a mimicking-one, that is, $G > \hat{G}$. As shown in the Appendix 3.B, it is optimal that only type-2 individuals contribute to the public good, $g^1 = 0$, $g^2 > 0$, and $g^G = 0$.[8]

[8]The intuition is as follows: type-1 individuals do not have their incentive compatibility constraint tightened by private donation to a public good by type-2 individuals and thus inducing type-2 individuals to donate to a public good allows the government to reduce mimicker's utility due to $G > \hat{G}$, that is, it relaxes the binding incentive constraint.

This suggests that inducing type-2 individuals to contribute improves their level of social welfare from the allocation, where no one makes a private donation to public goods. This is consistent with Diamond (2006). At the optimum, the level of public good is higher when a type-2 individual chooses a truth-telling strategy than a mimicking one, that is, $G > \hat{G}$. Assuming that the public good has a stronger complementary relationship with the private good in the first period than in the second, the intertemporal marginal rate of substitution for the mimicker is lower than the corresponding marginal rate of substitution for the mimicked, that is, $MRS^1_{cx} > M\hat{R}S_{cx}$. In other words, the mimicker values the consumption in the second period more than the mimicked (type-1 individuals). This implies that distorting the capital income of type-1 individuals downward hurts the mimicker more than the mimicked and thus relaxes the incentive compatibility constraint. Therefore, the marginal capital income tax rate should be positive. Consequently, individuals' behavior in terms of private donations to a public good creates an informational advantage for the government. Note that this outcome depends on the assumption of finite number of individuals. This is because when there is an infinite population as in Cremer et al. (1998), the level of public good does not change even if a type-2 individual acts as a mimicker, that is, $G = \hat{G}$. This means that capital income taxation is redundant. On the other hand, equation (3.16) shows that the government should not distort type-2 individuals' saving behavior, making zero marginal capital income tax rate desirable.

3.3.2 Optimal nonlinear subsidy for a public good

A new issue emerges owing to the welfare gain from private donations by high-type individuals, that is, how the optimal subsidy for donations is characterized.

The optimality conditions with respect to g^H and c^H gives:

$$\tau'(g^H) = \underbrace{\left[MRS^L_{Gc}\pi^L + MRS^H_{Gc}(\pi^H - 1)\right]}_{\text{Pigouvian effect}} + \underbrace{\frac{\lambda u_c(c^L, x^L, \hat{G})}{\gamma_1 \pi^H}\left[MRS^L_{Gc}\pi^H - M\hat{R}S_{Gc}(\pi^H - 1)\right]}_{\text{The effect of high-type donation on IC constraint}} \tag{3.17}$$

where $M\hat{R}S_{Gc} \equiv \frac{u_G(c^L,x^L,\hat{G})}{u_c(c^L,x^L,\hat{G})}$ denotes the corresponding marginal rate of substitution that the mimicker face. The derivation are included in Appendix 3.C.

The right-hand side of these equations comprises two terms. The first and second terms can be seen as an externality from a public good correcting effect. The first term has an externality effect on the other type, while the second term has an externality effect on other individuals of the same type. Therefore, the terms for optimal tax conditions act as Pigouvian tax and these signs are positive. The third term reflects the marginal effect of private donation on the incentive compatibility constraint.[9] Because $G > \hat{G}$, low-type individuals' marginal utility from a public good is less than that of a mimicker. When private consumption in the first period and public good are complement, we have $MRS^L_{Gc} < \hat{MRS}_{Gc}$. Then, distorting a private donation by high-type individuals upward makes low-type individuals worse off, but leaves mimickers well off. By contrast, using the definition of G and $\hat{G}$, it is easy to show that

$$\frac{\partial G}{\partial g^H} = \pi^H > \frac{\partial \hat{G}}{\partial g^H} = \pi^H - 1$$

This implies that the marginal effect of high-type individuals' donation on G is larger than the effect on $\hat{G}$, thus inducing high-type individuals to contribute appears as a welfare gain. Therefore, "the effect of high-type individuals' donation on the IC constraint" cannot be signed. To sum up, we can conclude as follows.

Proposition 3.2. *At the optimum where only high-type individuals contribute to the public good, optimal subsidy for high-type individuals' private donation differs from the standard Pigouvian subsidy.*

3.3.3 Optimal nonlinear labor income taxation

Finally, we confirm the marginal labor income tax rate for each type of individual, which is deduced as follows:

$$T'(y^L) = \frac{\lambda u_c(c^L, x^L, \hat{G})}{\pi^L \gamma_1} \left[MRS^L_{yc} - \hat{MRS}_{yc} \right] \tag{3.18}$$

$$T'(y^H) = 0 \tag{3.19}$$

where $\hat{MRS}_{yc} \equiv \frac{v_\ell(\ell^L w^L / w^H)}{w^H u_c(c^L, x^L, \hat{G})}$ denotes the corresponding marginal rate of substitution for mimickers. The derivation is included in Appendix 3.D. The bracket in the right hand side of equation (3.18) determines the sign of marginal labor income tax rates for low-type individuals. In the traditional optimal taxation literature, if the single-crossing property is satisfied, the sign of the bracket is positive. However, the sign is ambiguous since the aggregate amount of public goods that mimickers face distort their corresponding marginal rate of substitution. If the utility function is separable between the private consumption in the first period and the total amount of a public good, the bracket is positive. In this case,

[9] The Samuelson rule derived in our model is modified and the social marginal benefit should be equal to the marginal cost of a public good provision and "the effect of high-type donation on the IC constraint." This is consistent with the corresponding rule derived in Diamond (2006). See equation (22) in Diamond (2006).

the pattern of marginal labor income tax rate is consistent with the Stiglitz (1982) model, that is, low-type individuals face a positive marginal tax rate on labor income and high-type individuals face a zero marginal tax rate.

3.4 Three types of agent economy

So far, we suppose two types of individuals following by the Stiglitz (1982) model. The main result under two types of individuals is that marginal capital income tax rates are not zero for low-type individuals and zero for high-type ones. The crucial condition is the difference of private donations to a public good between high-type and low-type. In this section, we explore the robustness of our results analyzing the optimal tax system in the economy consisting of three types of individuals. Now, we introduce middle-type individuals into the previous model, indexed by $i = M$, and suppose that $w^H > w^M > w^L$.

3.4.1 Optimization problem under three types of individuals

We proceed using the previous setting in terms of individuals, firms, and the government, except for the incentive constraint. Under three types of individuals, the number of the restriction requiring that any type has no incentive to mimic the other types increases. Focusing on the downward incentive constraint, the government must prevent not only high-type individuals from mimicking middle-type ones and low-type ones but also middle-type individuals from mimicking low-type ones. This is mathematically formulated as

$$U(c^H, x^H, G, \frac{y^H}{w^H}) \geq U(c^M, x^M, \hat{G}_{HM}, \frac{y^M}{w^H}) \tag{3.20}$$

$$U(c^H, x^H, G, \frac{y^H}{w^H}) \geq U(c^L, x^L, \hat{G}_{HL}, \frac{y^L}{w^H}) \tag{3.21}$$

$$U(c^M, x^M, G, \frac{y^M}{w^M}) \geq U(c^L, x^L, \hat{G}_{ML}, \frac{y^L}{w^M}) \tag{3.22}$$

where $\hat{G}_{HM} \equiv G - g^H + g^M$, $\hat{G}_{HL} \equiv G - g^H + g^L$, and $\hat{G}_{ML} \equiv G - g^M + g^L$.

The social planning problem is to maximize the social welfare function (equation (3.8)), subject to the equation for the public good (equation (3.2)), the resource constraints (equations (3.11) and (3.12)), and the incentive compatibility constraints (equation (3.20), (3.21), and (3.22)), respectively. The Lagrangean corresponding to this planning problem can be formulated as follows:

$$\begin{aligned}
\mathcal{L} = W &+ \mu\Big[\sum_i g^i\pi^i + g^G - G\Big] + \gamma_1\Big[\sum_i y^i\pi^i - \sum_i (c^i + s^i + g^i)\pi^i - s^G - g^G\Big] \\
&+ \gamma_2\Big[(1+r)\Big(\sum_i s^i\pi^i + s^G\Big) - \sum_i x^i\pi^i\Big] \\
&+ \lambda_1\Big[U(c^H, x^H, G, \frac{y^H}{w^H}) - U(c^M, x^M, \hat{G}_{HM}, \frac{y^M}{w^H})\Big] \\
&+ \lambda_2\Big[U(c^H, x^H, G, \frac{y^H}{w^H}) - U(c^L, x^L, \hat{G}_{HL}, \frac{y^L}{w^H})\Big] \\
&+ \lambda_3\Big[U(c^M, x^M, G, \frac{y^M}{w^M}) - U(c^L, x^L, \hat{G}_{ML}, \frac{y^L}{w^M})\Big]
\end{aligned} \tag{3.23}$$

Let μ be the Lagrange multiplier of the formation of the aggregate amount of a public good, γ_1 the Lagrange multiplier of the resource constraint in the first period, γ_2 the Lagrange multiplier of the resource constraint in the second period, λ_1 the Lagrange multiplier of incentive constraint preventing high-type from mimicking middle-type, λ_2 the Lagrange multiplier of incentive constraint preventing high-type from mimicking low-type, and λ_3 the Lagrange multiplier of incentive constraint preventing middle-type from mimicking low-type.

3.4.2 Optimal nonlinear capital income taxation

Under three types of individuals, the optimal capital income tax rate for type i individuals are characterized by:

$$\begin{aligned}
\Phi'(rs^L) = &\frac{\lambda_2 u_x(c^L, x^L, \hat{G}_{HL})}{\gamma_2\pi^L}\Big[MRS^L_{cx} - M\hat{R}S^{HL}_{cx}\Big] \\
&+ \frac{\lambda_3 u_x(c^L, x^L, \hat{G}_{ML})}{\gamma_2\pi^L}\Big[MRS^L_{cx} - M\hat{R}S^{ML}_{cx}\Big]
\end{aligned} \tag{3.24}$$

$$\Phi'(rs^M) = \frac{\lambda_1 u_x(c^M, x^M, \hat{G}_{HM})}{\gamma_2\pi^M}\Big[MRS^M_{cx} - M\hat{R}S^{HM}_{cx}\Big] \tag{3.25}$$

$$\Phi'(rs^H) = 0 \tag{3.26}$$

where $M\hat{R}S^{ij}_{cx} \equiv \frac{u_c(c^j, x^j, \hat{G}_{ij})}{u_x(c^j, x^j, \hat{G}_{ij})}$ denotes the corresponding marginal rate of substitution between private consumption in the first period and the second period for the type i individuals mimicking the type j ones. The derivation is included in Appendix 3.E. As with the results under two types of individuals, the marginal capital tax rate at the top is zero, and the sign of marginal capital tax rates for the person being mimicked depends on the difference of the marginal rate of substitution between intertemporal choice for the mimicker and the person being mimicked. In the subsection, we suppose that all downward incentive constraints are binding, and suggest the following lemma, shown in Appendix 3.F.

Lemma 3.1. *It is optimal that either or both of the high-type or middle-type individuals contribute to a public good and low-type ones and the government do not contribute to a public good.*

The lemma is the extended version of Diamond (2006). The slight difference is that the government has the incentive to design the mechanism so that middle-type individuals donate to a public good to relax the incentive constraint since they are also mimickers. As shown later, there exists the case in which inducing only middle-type individuals to contribute is welfare-improving. Now, using lemma 3.1, we present the robustness of our results under two types of individuals.

Proposition 3.3. *When the public good is more complementary with the consumption good in the first period than in the second period, the statement of Proposition 3.1 remains under three types of individuals.*

Either of the brackets in the right hand side (equation (3.24)) is positive since either or both of G^{HL} and G^{ML} differ from G, as long as all downward incentive constraints are binging. On the other hand, the sign of the marginal capital tax rate for middle-type individuals depends on the difference between the amount of private donations for high-type and middle-type ones. If all downward incentive constraints for mimickers are binding, it is ambiguous.

In the next section, we clarify the sign of all marginal capital tax rates by showing that the crucial condition to determine whether high-type or middle-type contribute is the impact on the incentive constraint.

3.4.3 Special cases for marginal capital tax rates

In the section, we give two special cases to determine the sign of all marginal capital tax rates. So far, we assume that all downward incentive constraints are binding. Here, we loosen constraints as follows: First, we consider that middle-type individuals do not have the incentive to mimic low-type ones, that is, $\lambda_3 = 0$. Second, we consider that high-type individuals do not have the incentive to mimic middle-type and low-type ones, that is, $\lambda_1 = \lambda_2 = 0$.

In the first case, high-type individuals are the only mimickers. This is the same situation as one under two types of individuals. Shown in Appendix 3.F, only contributors are high-type individuals.

Lemma 3.2. *Under $\lambda_3 = 0$, it is optimal that only high-type individuals contribute to a public good, that is, $g^H > 0$, $g^M = 0$, $g^L = 0$, and $g^G = 0$.*

When high-type individuals are only mimickers, there are three possible regimes.

- $\lambda_1 = 0$ and $\lambda_2 > 0$
- $\lambda_1 > 0$ and $\lambda_2 = 0$
- $\lambda_1 > 0$ and $\lambda_2 > 0$

In the first case, high-type individuals will not mimic middle-type ones. The marginal capital tax rate for middle-type individuals is zero from equation (3.2), that is, imposing capital income taxes on middle-type ones based on preferences for intertemporal choices is superfluous without the problem preventing high-type ones from mimicking middle-type ones. As a result, only marginal capital income tax rate for low-type ones is positive. In the second one, high-type individuals will not mimic low-type ones. Thus, the marginal capital tax rate for low-type individuals is zero from equation (3.24). On the other hand, the marginal capital income tax rate for middle-type is positive. Notice that the marginal capital tax rate at the middle is positive, even though it is zero at the bottom and the top. In the third one, both of marginal capital tax rate for middle-type and low-type are positive.

In the second case, middle-type individuals are only mimickers. The marginal capital tax rate for middle-type individuals is obviously zero from equation (3.25), as explained above. Moreover, in this case, the following lemma shown in Appendix 3.F holds.

Lemma 3.3. *Under $\lambda_1 = \lambda_2 = 0$, it is optimal that only middle-type individuals contribute to a public good, that is, $g^H = 0$, $g^M > 0$, $g^L = 0$, and $g^G = 0$.*

That is, inducing only middle-type individuals to contribute enhances the social welfare in the situation that middle-type ones are only mimickers. This result implies that whether individuals contribute to a public good are crucially determined by their mimicking strategy, not their productivity. From lemma 3.3, the second bracket in the right hand side (equation (3.24)) is positive, in other words, the marginal capital tax rate for low-type individuals is positive. This is because the aggregate amount of a public good for the mimicker is greater than one for the person being mimicked, i.e. $G > \hat{G}^{ML}$. Therefore, mimickers prefer private consumptions in the second period more than ones in the second period, which leads to positive marginal tax rates. To sum up, we have the following corollary.[10]

Corollary 3.1. *Consider that a public good is more complementary with the consumption good in the first period than in the second period.*

[10] The others are the following two cases: (i) $\lambda_1 > 0$, $\lambda_2 = 0$, and $\lambda_3 > 0$ (ii) $\lambda_1 = 0$, $\lambda_2 > 0$, and $\lambda_3 > 0$. In these cases, both of high-type and middle-type individuals have the incentive to mimic low-type ones. Therefore, since either or both of them can contribute to a public good, the sign of the bracket in the right hand side (equation (3.25)) is ambiguous. However, the marginal capital income tax rate for the middle type is zero in the latter case.

(i) If high-type individuals are only mimickers, that is, $\lambda_3 = 0$, there are three possible cases: (a) if $\lambda_1 = 0$ and $\lambda_2 > 0$, the marginal capital tax rate is positive for low-type individuals, and zero for middle-type and high-type ones. (b) if $\lambda_1 > 0$ and $\lambda_2 = 0$, the marginal capital tax rate is positive for middle-type individuals, and zero for low-type and high-type ones. (c) if $\lambda_1 > 0$ and $\lambda_2 > 0$, the marginal capital tax rate is positive for low-type and middle-type individuals, and zero for high-type ones.
(ii) If middle-type individuals are only mimickers, that is, $\lambda_1 = \lambda_2 = 0$, the marginal capital tax rate is positive for low-type individuals, and zero for high-type and middle-type ones.

3.4.4 Optimal nonlinear subsidy for a public good

Assuming that all downward incentive constraints are binding, optimal nonlinear marginal subsidy rates for high-type and middle-type individuals are characterized as follows:

$$
\begin{aligned}
\tau'(g^H) = & \underbrace{\left[MRS^H_{G,c}(\pi^H - 1) + MRS^M_{Gc}\pi^M + MRS^L_{Gc}\pi^L \right]}_{\text{Pigouvian effect}} \\
+ & \underbrace{\frac{\lambda_1 u_c(c^M, x^M, \hat{G}_{HM})}{\gamma_1 \pi^H} \left[MRS^M_{Gc}\pi^H - M\hat{R}S^{HM}_{Gc}(\pi^H - 1) \right]}_{\text{The effect of high-type's donation on IC constraint for high-type mimicking middle-type}} \\
+ & \underbrace{\frac{\lambda_2 u_c(c^L, x^L, \hat{G}_{HL})}{\gamma_1 \pi^H} \left[MRS^L_{Gc}\pi^H - M\hat{R}S^{HL}_{Gc}(\pi^H - 1) \right]}_{\text{The effect of high-type's donation on IC constraint for high-type mimicking low-type}} \\
+ & \underbrace{\frac{\lambda_3 u_c(c^L, x^L, \hat{G}_{ML})}{\gamma_1 \pi^H} \left[MRS^L_{Gc}\pi^H - M\hat{R}S^{ML}_{Gc}\pi^H \right]}_{\text{The effect of high-type's donation on IC constraint for middle-type mimicking low-type}}
\end{aligned}
\tag{3.27}
$$

$$
\begin{aligned}
\tau'(g^M) = & \underbrace{\left[MRS^H_{G,c}\pi^H + MRS^M_{Gc}(\pi^M - 1) + MRS^L_{Gc}\pi^L\right]}_{\text{Pigouvian effect}} \\
& + \underbrace{\frac{\lambda_1 u_c(c^M, x^M, \hat{G}_{HM})}{\gamma_1 \pi^M}\left[MRS^M_{Gc}\pi^M - M\hat{R}S^{HM}_{Gc}(\pi^M + 1)\right]}_{\text{The effect of middle-type's donation on IC constraint for high-type mimicking middle-type}} \\
& + \underbrace{\frac{\lambda_2 u_c(c^L, x^L, \hat{G}_{HL})}{\gamma_1 \pi^M}\left[MRS^L_{Gc}\pi^M - M\hat{R}S^{HL}_{Gc}\pi^M\right]}_{\text{The effect of middle-type's donation on IC constraint for high-type mimicking low-type}} \\
& + \underbrace{\frac{\lambda_3 u_c(c^L, x^L, \hat{G}_{ML})}{\gamma_1 \pi^M}\left[MRS^L_{Gc}\pi^M - M\hat{R}S^{ML}_{Gc}(\pi^M - 1)\right]}_{\text{The effect of middle-type's donation on IC constraint for middle-type mimicking low-type}}
\end{aligned} \tag{3.28}
$$

where $M\hat{R}S^{ij}_{gc} \equiv \frac{u_G(c^j, x^j, \hat{G}_{ij})}{u_c(c^j, x^j, \hat{G}_{ij})}$ denotes the corresponding marginal rate of substitution for the type i individuals mimicking the type j ones. The derivation are included in Appendix 3.G. As with the optimal marginal subsidy rate formula under two types of individuals, it consists of Pigouvian term and terms reflecting the effect on the incentive constraint. Under three types of individuals, a new term appears. The fourth term in the right hand side in equation (3.27) expresses the effect on the incentive constraint preventing middle-type individuals from low-type ones, that is, the term is associated with the relaxation of the incentive constraint on the other mimicker owing to high-type's donations. Moreover, this term can be signed because the effect of high-type's donations on G and $\hat{G}^{ML}$ is equivalent. The similar term appears as the third term in the right hand side in equation (3.28).

3.4.5 Optimal nonlinear labor income taxation

We characterize the marginal labor income tax rate for each type under three types of individuals as follows:

$$
\begin{aligned}
T'(y^L) = & \frac{\lambda_2 u_c(c^L, x^L, \hat{G}^{HL})}{\gamma_1 \pi^L}\left[MRS^L_{yc} - M\hat{R}S^{HL}_{yc}\right] \\
& + \frac{\lambda_3 u_c(c^L, x^L, \hat{G}^{ML})}{\gamma_1 \pi^L}\left[MRS^L_{yc} - M\hat{R}S^{ML}_{yc}\right]
\end{aligned} \tag{3.29}
$$

$$
T'(y^M) = \frac{\lambda_1 u_c(c^M, x^M, \hat{G}^{HM})}{\gamma_1 \pi^M}\left[MRS^M_{yc} - M\hat{R}S^{HM}_{yc}\right] \tag{3.30}
$$

$$
T'(y^H) = 0 \tag{3.31}
$$

where $\hat{MRS}_{yc}^{ij} \equiv \frac{v_\ell(\ell^j w^j/w^i)}{w^i u_c(c^j,x^j,\hat{G}^{ij})}$ denotes the corresponding marginal rate of substitution for the type i individuals mimicking the type j ones. The derivation are included in Appendix 3.H. These are extended results with respect to labor income taxation under two types of individuals. As mentioned in section 3.3.3, if single-crossing property is satisfied and the utility function is separable between the private consumption in the first period and the total amount of a public good, the persons being mimicked (middle-type and low-type ones) face a positive marginal tax rate on labor income. Also, no-distortion at the top result remains.

3.5 Linear tax policy

So far, we have assumed that the amount of savings are observable. In this section, we explore the robustness of our results by assuming that the amount of savings is unobservable, that is, the government is not allowed to employ nonlinear capital income taxes, and show that our main conclusion is robust as long as private donations to a public good are observable, otherwise it is ambiguous.

3.5.1 Linear capital income taxation and nonlinear subsidies for private donations

First, we examine a case in which the government observes both private donations to a public good and labor income for each type, but is unable to observe capital income for each type. Therefore, the government can only levy linear tax on savings at rate t_s.

Following the traditional literatures in optimal income taxation, we decompose the individual's problem into two stages. First, each individual chooses the amount of labor supply and private donations to a public good, which determines disposable income $R^i \equiv y^i - T(y^i) - g^i + \tau(g^i)$, given nonlinear labor income taxes and subsidies for private donations. Second, disposable income is allocated into private consumption in the first and the second period. We suppose that individuals anticipate the outcome of the second stage at the first stage. In the second stage, type i individuals choose c^i and x^i to maximize the sub-utility function $u(c^i, x^i, G)$ subject to individual's budget constraint which is given by

$$c^i + s^i = R^i \tag{3.32}$$

$$s^i = q_s x^i \tag{3.33}$$

where $q_s \equiv 1/(1 + r(1 - t_s))$. The first-order condition is given by

$$\frac{u_c(c^i, x^i, G)}{u_x(c^i, x^i, G)} = \frac{1}{q_s} \tag{3.34}$$

On this occasion, we define the sub-indirect utility function for type i individuals as $V^i \equiv V(q_s, R^i, G) \equiv u(c_i^*, x_i^*, G)$, where $c_i^* \equiv c(q_s, R^i, G)$ denotes the optimal solution with respect to the consumption in the first period and $x_i^* \equiv x(q_s, R^i, G)$ the optimal solution with respect to the consumption in the second period.

The government chooses q_s, G, g^1, g^2, g^G, R^1, R^2, y^1, and y^2 to maximize the social welfare subject to the government's budget constraint, and the incentive compatibility constraint, which is expressed by

$$V(q_s, R^2, G) - v(\frac{y^2}{w^2}) \geq V(q_s, R^1, \hat{G}) - v(\frac{y^1}{w^2}) \tag{3.35}$$

where $\hat{V} \equiv V(q_s, R^1, \hat{G})$ indicates the mimicker's sub-indirect utility. Here, we define the optimal solution for the mimicker with respect to the consumption in the first and the second period as $\hat{c}^* \equiv c(q_s, R^1, \hat{G})$ and $\hat{x}^* \equiv x(q_s, R^1, \hat{G})$, respectively.[11]

The first-order conditions are shown in Appendix 3.I. By rearranging these results, we can derive the optimal linear capital income tax rate as follows:

$$\frac{rt_s q_s}{1+r} = \frac{\tilde{\lambda}\frac{\partial \hat{V}}{\partial R^1}(\hat{x}^* - x_1^*)}{-\tilde{\gamma}\sum_i \pi^i \frac{\partial \tilde{x}_i^*}{\partial q_s}} \tag{3.36}$$

where $\tilde{\gamma}$, $\tilde{\lambda}$, and $\tilde{\mu}$ are the Lagrange multipliers. This is shown in Appendix 3.I. Equation (3.39) is consistent with the formula for optimal linear tax rate proposed by Edwards et al. (1994) and Nava et al. (1996). The deviation form the Atkinson-Stiglitz theorem relies on the numerator, that is, the difference of the demand for the consumption in the second period between the mimicker and the person being mimicked. Because only type-2 individuals contribute to a public good as with nonlinear capital tax instruments (see, Appendix 3.J), that is, we have $G > \hat{G}$. Thus, the Atkinson-Stiglitz theorem is not valid. Therefore, even though the government cannot observe savings for each type, weak-separability of preferences between consumption and labor supply is not sufficient condition to make capital income taxation redundant. Note that when there is an infinite population, we obtain $G = \hat{G}$, and then $t_s = 0$.

[11] We omit the first-order conditions with respect to G, y^1, and y^2 since we focus on the characterization of optimal linear capital income tax rates.

3.5.2 Linear capital income taxation and linear subsidies for private donations

Next, this sub-section extends the model in which the government cannot observe both the amount of private donations to a public good and savings for each type. Therefore, the only nonlinear tax instrument is labor income taxation, and the government can only levy linear subsidies on private donations at rate t_g and linear tax on savings at rate t_s.

We now turn to the analysis of individual's behavior. In the second stage, type i individuals choose c^i, x^i, and g^i to maximize the sub-utility function $u(c^i, x^i, G)$ subject to individual's budget constraint which is given by

$$c^i + s^i + q_g g^i = y^i - T(y^i) \equiv R^i \tag{3.37}$$

$$s^i = q_s x^i \tag{3.38}$$

where $q_g \equiv 1 - t_g$ and $q_s \equiv 1/(1 + r(1 - t_s))$. The first-order conditions are given by

$$\frac{u_c(c^i, x^i, g^i + G_{-i})}{u_x(c^i, x^i, g^i + G_{-i})} = \frac{1}{q_s} \tag{3.39}$$

$$\frac{u_G(c^i, x^i, g^i + G_{-i})}{u_c(c^i, x^i, g^i + G_{-i})} = q_g \tag{3.40}$$

where $G_{-i} \equiv (\pi^i - 1)g^i + \pi^j g^j + g^G$, $i \neq j = 1, 2$. Here, let $c^i = c(q_s, q_g, R^i, G_{-i})$, $i = 1, 2$, be the best response function of the consumption in the first period, $x^i = x(q_s, q_g, R^i, G_{-i})$, $i = 1, 2$, the best response function of the consumption in the second period, and $g^i = g(q_s, q_g, R^i, G_{-i})$, $i = 1, 2$, the best response function of private donations to a public good. We can define the sub-indirect utility function for type i as $V^i \equiv V^i(q_g, q_s, R^1, R^2, g^G) \equiv u(c_i^*, x_i^*, g_i^* + G_{-i}^*)$ where the superscript (*) refers to the Nash equilibrium outcome. Under linear tax policy, the incentive constraint preventing high-type individuals from mimicking low-type ones is expressed by:

$$V^2(q_g, q_s, R^1, R^2, g^G) - v(\frac{y^2}{w^2}) \geq \hat{V}(q_g, q_s, R^1, R^2, g^G) - v(\frac{y^1}{w^2}) \tag{3.41}$$

where $\hat{V}(q_g, q_s, R^1, R^2, g^G)$ indicates the mimicker's sub-indirect utility. Here, we define the best response function for the mimicker with respect to the consumption in the first period as $\hat{c} \equiv c(q_g, q_s, R^1, \tilde{G}_{-2})$, the best response function for the mimicker with respect to the consumption in the second period as $\hat{x} \equiv x(q_g, q_s, R^1, \tilde{G}_{-2})$, and the best response function for mimickers with respect to the private donation to a public good as $\hat{g} \equiv g(q_g, q_s, R^1, \tilde{G}_{-2})$, where $\tilde{G}_{-2} = \pi^1 \tilde{g}^1 + (\pi^2 - 1)\tilde{g}^2 + g^G$ and $\tilde{g}^i$ is type i's private donations in the presence of the mimicker. Note that it is not necessarily that $g_1^* = \tilde{g}^1$ or $g_2^* = \tilde{g}^2$ holds since these realize as a Nash equilibrium in contrast with the case in which the government can observe private donations to a public good and thus design the allocation for each type.

To sum up, the government chooses q_s, q_g, R^1, R^2, g^G, y^1, and y^2 to maximize the social welfare subject to the government's budget constraint, and the incentive compatibility constraint.[12] Solving the planning problem yields:

$$\begin{pmatrix} -t_g \\ \frac{rt_s q_s}{1+r} \end{pmatrix} = -\frac{1}{\bar{\gamma}}\Delta^{-1} \begin{pmatrix} \sum_{i\neq j=1,2} \pi^i u_G^i \left(\frac{\partial G_{-i}^*}{\partial q_g} + \frac{\partial G_{-i}^*}{\partial R^i} g_i^* + \frac{\partial G_{-i}^*}{\partial R^j} g_j^* \right) \\ \sum_{i\neq j=1,2} \pi^i u_G^i \left(\frac{\partial G_{-i}^*}{\partial q_s} + \frac{\partial G_{-i}^*}{\partial R^i} g_i^* + \frac{\partial G_{-i}^*}{\partial R^j} g_j^* \right) \end{pmatrix} - \frac{\bar{\lambda}\hat{u}_c}{\bar{\gamma}}\Delta^{-1} \begin{pmatrix} \hat{g} - g_1^* \\ \hat{x} - x_1^* \end{pmatrix}$$
$$- \frac{\bar{\lambda}}{\bar{\gamma}}\Delta^{-1} \begin{pmatrix} u_G^2 \left(\frac{\partial G_{-2}^*}{\partial q_g} + \frac{\partial G_{-2}^*}{\partial R^1} g_1^* + \frac{\partial G_{-2}^*}{\partial R^2} g_2^* \right) - \hat{u}_G \left(\frac{\partial \tilde{G}_{-2}}{\partial q_g} + \frac{\partial \tilde{G}_{-2}}{\partial R^1} g_1^* + \frac{\partial \tilde{G}_{-2}}{\partial R^2} g_2^* \right) \\ u_G^2 \left(\frac{\partial G_{-2}^*}{\partial q_s} + \frac{\partial G_{-2}^*}{\partial R^1} x_1^* + \frac{\partial G_{-2}^*}{\partial R^2} x_2^* \right) - \hat{u}_G \left(\frac{\partial \tilde{G}_{-2}}{\partial q_s} + \frac{\partial \tilde{G}_{-2}}{\partial R^1} x_1^* + \frac{\partial \tilde{G}_{-2}}{\partial R^2} x_2^* \right) \end{pmatrix} \tag{3.42}$$

where Δ^{-1} is the inverse matrix of Δ which denotes the 2×2 matrix as follows:

$$\Delta \equiv \begin{pmatrix} \sum_i \pi^i \frac{\partial g_i^*}{\partial q_g} + \sum_i \pi^i \frac{\partial g_i^*}{\partial R^i} g_i^* + \sum_{i\neq j=1,2} \pi^i \frac{\partial g_i^*}{\partial R^j} g_j^* & \sum_i \pi^i \frac{\partial x_i^*}{\partial q_g} + \sum_i \pi^i \frac{\partial x_i^*}{\partial R^i} g_i^* + \sum_{i\neq j=1,2} \pi^i \frac{\partial x_i^*}{\partial R^j} g_j^* \\ \sum_i \pi^i \frac{\partial g_i^*}{\partial q_s} + \sum_i \pi^i \frac{\partial g_i^*}{\partial R^i} x_i^* + \sum_{i\neq j=1,2} \pi^i \frac{\partial g_i^*}{\partial R^j} x_j^* & \sum_i \pi^i \frac{\partial x_i^*}{\partial q_s} + \sum_i \pi^i \frac{\partial x_i^*}{\partial R^i} x_i^* + \sum_{i\neq j=1,2} \pi^i \frac{\partial x_i^*}{\partial R^j} x_j^* \end{pmatrix}$$

When the government cannot observe private donations to a public good for each types, the optimal tax formula consists of three terms. The first and second terms reflect well known effects respectively: the Pigouvian and non-Pigouvian elements discussed by Cremer et al. (1998).[13] The third term is the novel term, which comes from the different impact of the response of the other individuals to the perturbation in parameters to the mimicker and the high-skilled agent who does not behave as a mimicker. This is an additional information for

[12] Following the argument in footnote 11, we omit the first-order conditions with respect to g^G, y^1, and y^2.

[13] If the public good is additively separable in the utility function, the marginal utility of the public good coincides between a low-skilled and a high-skilled individual, that is, $u_G \equiv u_G^1 = u_G^2$. In this case, the Pigouvian term is simplified as follows.

$$\begin{pmatrix} -t_g \\ \frac{rt_s q_s}{1+r} \end{pmatrix} = -\frac{u_G}{\bar{\gamma}} \begin{pmatrix} \pi^1 + \pi^2 - 1 \\ 0 \end{pmatrix} - \frac{\bar{\lambda}\hat{u}_c}{\bar{\gamma}} \begin{pmatrix} \hat{g} - g_1^* \\ \hat{x} - x_1^* \end{pmatrix}$$
$$- \frac{\bar{\lambda}}{\bar{\gamma}} \begin{pmatrix} u_G \left(\frac{\partial G_{-2}^*}{\partial q_g} + \frac{\partial G_{-2}^*}{\partial R^1} g_1^* + \frac{\partial G_{-2}^*}{\partial R^2} g_2^* \right) - \hat{u}_G \left(\frac{\partial \tilde{G}_{-2}}{\partial q_g} + \frac{\partial \tilde{G}_{-2}}{\partial R^1} g_1^* + \frac{\partial \tilde{G}_{-2}}{\partial R^2} g_2^* \right) \\ u_G \left(\frac{\partial G_{-2}^*}{\partial q_s} + \frac{\partial G_{-2}^*}{\partial R^1} x_1^* + \frac{\partial G_{-2}^*}{\partial R^2} x_2^* \right) - \hat{u}_G \left(\frac{\partial \tilde{G}_{-2}}{\partial q_s} + \frac{\partial \tilde{G}_{-2}}{\partial R^1} x_1^* + \frac{\partial \tilde{G}_{-2}}{\partial R^2} x_2^* \right) \end{pmatrix} \tag{3.43}$$

the government to relax the binding incentive constraint.

The condition to restore Atkinson–Stiglitz theorem crucially depends on private donations to a public good among individuals. If $g_1^* = \tilde{g}_1^* = g_2^* = \tilde{g}_2^*$, we have $G_{-1}^* = \tilde{G}_{-2}$, and then $g_1^* = \hat{g}^*$ and $x_1^* = \hat{x}$. In addition, since it causes $G = \hat{G}$, the third term in the right hand side vanishes. Therefore, Atkinson–Stiglitz theorem remains. However, in contrast with the observability of private donations to a public good, we cannot analytically compare the amount of private donations among individuals since the government cannot directly control their private donations. As the same with the previous sub-section, if there is an infinite population, no person donates to a public good since the private donation does not affect the aggregate amount of public good. Thus, Atkinson–Stiglitz theorem is valid.

3.6 Concluding Remarks

This study is largely relevant to debates on the desirability of capital income taxes. Since Ordover and Phelps (1979) seminal work, a large body of literature has accumulated on whether capital income taxes are required from the viewpoint of heterogeneous tastes in private consumptions, even though the utility function is weakly separable between private goods and leisure. For instance, Boadway et al. (2000), Cremer et al. (2001), and Diamond and Spinnewijn (2011) consider a multidimensional heterogeneity setting in which individuals differ in not only earning abilities but also other characteristics such as initial endowments (bequest or inheritance) and discount rates, which are assumptions. By contrast, this study provides additional economic rationale for capital income taxes from the viewpoint of economic behavior that is, in reality, individuals deduct charitable contributions. Under the standard optimal tax approach, we show that the government should design positive (negative) tax rates on capital income to supplement its redistribution policy when charitable contributions to a public good has a more complementary (substitutionary) relationship with consumption good in the first period than in the second period. This persists even if the additive and separable preference between consumption and labor supply is satisfied and individuals differ in only earning abilities.

The theoretical contribution of this paper is as follows. Although we show that Atkinson–Stiglitz theorem breaks down as a result of heterogeneous preferences, as in the case of Saez (2002a), we justify capital income taxes by clarifying the source of heterogeneity on the basis of individual behavior and not assumptions. It is worth noting that our justification is based on the assumption that there is a finite population. Pirttilä and Tuomala (1997) examine the commodity taxation on an externality-generating good and nonlinear taxation on labor income under the condition of an infinite population. They show that the optimal tax formula reflects the two types of terms, that is, the externality internalizing effect and the influence through the incentive compatibility constraint. When the preference is assumed to be additive and separable between consumption and leisure, policy outcomes in their paper are consistent with standard Pigouvian taxes. However, under the setting of a finite population as our study, the corresponding tax formula includes the novel term, the interaction between the Pigouvian term and the self-selection term, which is the right hand side of equation (3.15). This is true even if the additive and separable preference is assumed.

Our paper derives a condition according to which capital income of low-income earners should be taxed or not. This policy implication depends on the shape of utility function, in particular, the signs of $\frac{\partial^2 u}{\partial c^i \partial G}$ and $\frac{\partial^2 u}{\partial x^i \partial G}$. This is an important issue of empirical study.

3.7 Appendix

Appendix 3.A

The first order conditions associated with c^L, x^L, c^H, x^H, s^L, s^H, and s^G are

$$\frac{\partial \mathcal{L}}{\partial c^L} = \pi^L u_c(c^L, x^L, G) - \gamma_1 \pi^L - \lambda u_c(c^L, x^L, \hat{G}) = 0 \tag{3.A.1}$$

$$\frac{\partial \mathcal{L}}{\partial x^L} = \pi^L u_x(c^L, x^L, G) - \gamma_2 \pi^L - \lambda u_x(c^L, x^L, \hat{G}) = 0 \tag{3.A.2}$$

$$\frac{\partial \mathcal{L}}{\partial c^H} = \pi^H u_c(c^H, x^H, G) - \gamma_1 \pi^H + \lambda u_c(c^H, x^H, G) = 0 \tag{3.A.3}$$

$$\frac{\partial \mathcal{L}}{\partial x^H} = \pi^H u_x(c^H, x^H, G) - \gamma_2 \pi^H + \lambda u_x(c^H, x^H, G) = 0 \tag{3.A.4}$$

$$\frac{\partial \mathcal{L}}{\partial s^i} = -\gamma_1 + \gamma_2(1+r) = 0 \quad i = H, L, G \tag{3.A.5}$$

Substituting equation (3.A.1) and (3.A.2) into equation (3.A.5) yields:

$$\begin{aligned} &\pi^L \Big\{ u_c(c^L, x^L, G) - (1+r) u_x(c^L, x^L, G) \Big\} \\ &= \lambda \Big\{ u_c(c^L, x^L, \hat{G}) - (1+r) u_x(c^L, x^L, \hat{G}) \Big\} \end{aligned} \tag{3.A.6}$$

Combining equation (3.5) with equation (3.A.6) yields:

$$\begin{aligned}&\Big[\pi^L u_x(c^L,x^L,G) - \lambda u_x(c^L,x^L,\hat{G})\Big] r\Phi'(rs^L)\\ &= \lambda u_x(c^L,x^L,\hat{G})\left[\frac{u_c(c^L,x^L,G)}{u_x(c^L,x^L,G)} - \frac{u_c(c^L,x^L,\hat{G})}{u_x(c^L,x^L,\hat{G})}\right]\end{aligned} \tag{3.A.7}$$

Substituting equation (3.A.2) into the term in the brackets of the left hand side, we obtain equation (3.15). Similarly, substituting equation (3.A.3) and (3.A.4) into equation (3.A.5) yields:

$$\begin{aligned}&-\pi^H\Big\{u_c(c^H,x^H,G) - (1+r)u_x(c^H,x^H,G)\Big\}\\ &= \lambda\Big\{u_c(c^H,x^H,G) - (1+r)u_x(c^H,x^H,G)\Big\}\end{aligned} \tag{3.A.8}$$

This can be rewritten as follows:

$$(\pi^H + \lambda)u_x(c^H,x^H,G)r\Phi'(rs^H) = 0 \tag{3.A.9}$$

Equation (3.A.3) implies that $\pi^H + \lambda$ is positive. Then, equation (3.A.9) implies that $\Phi'(rs^H)$ is zero. $\square$

Appendix 3.B

Differentiating $\mathcal{L}$ with respect to g^G, g^L, and g^H implies

$$\frac{\partial \mathcal{L}}{\partial g^G} = -\gamma_1 + \mu \tag{3.B.1}$$

$$\frac{\partial \mathcal{L}}{\partial g^L} = -\gamma_1\pi^L - \lambda u_G(c^L,x^L,\hat{G}) + \mu\pi^L \tag{3.B.2}$$

$$\frac{\partial \mathcal{L}}{\partial g^H} = -\gamma_1\pi^H + \lambda u_G(c^L,x^L,\hat{G}) + \mu\pi^H \tag{3.B.3}$$

If equation (3.B.1) is equal to zero, equation (3.B.3) is as follows.

$$\frac{\partial \mathcal{L}}{\partial g^H} = \lambda u_G(c^L,x^L,\hat{G}) > 0 \tag{3.B.4}$$

In this case, the optimal solution does not exist because of diverging. Therefore, at the optimum, we must have $\frac{\partial \mathcal{L}}{\partial g^G} < 0$ and $g^G = 0$ to satisfy Kuhn-Tucker conditions.

Given this condition, from equation (3.B.2), no contribution to a public good of low-type individuals is optimal, that is, $g^L = 0$. On the other hand, the private donation to a public good of high-type individuals is not zero because the second term in equation (3.B.3) is sufficiently larger than the sum of the first and third term by the Inada condition when g^H is close to zero given $g^L = g^G = 0$. Therefore, g^H is positive. In addition, g^H is an interior solution. As g^H is close to infinity, $\frac{\partial \mathcal{L}}{\partial g^H}$ converges to $-\gamma_1 \pi^H + \mu \pi^H$ which is negative. This implies that g^H must not be corner solution at the optimum.

$\square$

Appendix 3.C

The first order condition associated with G is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial G} &= \pi^L u_G(c^L, x^L, G) + \pi^H u_G(c^H, x^H, G) + \lambda u_G(c^H, x^H, G) \\ &\quad - \lambda u_G(c^L, x^L, \hat{G}) - \mu = 0\end{aligned} \tag{3.C.1}$$

Taking the product of equation (3.C.1) and π^H yields:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial G}\pi^H &= \left\{\pi^L u_G(c^L, x^L, G) + \pi^H u_G(c^H, x^H, G)\right\}\pi^H - \mu\pi^H \\ &\quad + \lambda\left\{u_G(c^H, x^H, G)\pi^H - u_G(c^L, x^L, \hat{G})\pi^H\right\} = 0\end{aligned} \tag{3.C.2}$$

Substituting the first order condition associated with g^H and c^H into (3.C.2) yields:

$$\begin{aligned}&\left\{\pi^L u_G(c^L, x^L, G) + \pi^H u_G(c^H, x^H, G)\right\}\pi^H - \pi^H u_c(c^H, x^H, G) - \lambda u_c(c^H, x^H, G) \\ &\quad + \lambda\left\{u_G(c^H, x^H, G)\pi^H - u_G(c^L, x^L, \hat{G})(\pi^H - 1)\right\} = 0\end{aligned} \tag{3.C.3}$$

Dividing equation (3.C.3) by $\gamma_1 \pi^H$ yields:

$$\begin{aligned}&\frac{u_c(c^H, x^H, G)}{\gamma_1}\left\{\pi^H \frac{u_G(c^H, x^H, G)}{u_c(c^H, x^H, G)} - 1\right\} + \pi^L \frac{u_G(c^L, x^L, G)}{\gamma_1} - \frac{\lambda u_c(c^H, x^H, G)}{\pi^H \gamma_1} \\ &\quad + \frac{\lambda}{\pi^H \gamma_1}\left\{u_G(c^H, x^H, G)\pi^H - u_G(c^L, x^L, \hat{G})(\pi^H - 1)\right\} = 0\end{aligned} \tag{3.C.4}$$

Rearranging equation (3.C.4) yields:

$$\left\{\pi^H \frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)} - 1\right\}\left\{\frac{u_c(c^H,x^H,G)}{\gamma_1} + \frac{\lambda u_c(c^H,x^H,G)}{\pi^H\gamma_1}\right\} + \pi^L \frac{u_G(c^L,x^L,G)}{\gamma_1} - \frac{\lambda u_c(c^L,x^L,\hat{G})}{\pi^H\gamma_1}\frac{u_G(c^L,x^L,\hat{G})}{u_c(c^L,x^L,\hat{G})}(\pi^H-1) = 0 \tag{3.C.5}$$

Substituting equation (3.6) and (3.A.1) into the first term of equation (3.C.5) yields:

$$\left\{\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}\pi^H - \tau'(g^H) - \frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}\right\} + \pi^L \frac{u_G(c^L,x^L,G)}{\gamma_1} - \frac{\lambda u_c(c^L,x^L,\hat{G})}{\pi^H\gamma_1}\frac{u_G(c^L,x^L,\hat{G})}{u_c(c^L,x^L,\hat{G})}(\pi^H-1) = 0 \tag{3.C.6}$$

Substituting equation (3.A.1) into the fourth term of equation (3.C.6) yields:

$$\tau'(g^H) = \pi^L \frac{u_G(c^L,x^L,G)}{u_c(c^L,x^L,G)} + (\pi^H-1)\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)} + \frac{\lambda u_c(c^L,x^L,\hat{G})}{\gamma_1\pi^H}\left\{\frac{u_G^L(c^L,x^L,G)}{u_c(c^L,x^L,G)}\pi^H - \frac{u_G(c^L,x^L,\hat{G})}{u_c(c^L,x^L,\hat{G})}(\pi^H-1)\right\} \tag{3.C.7}$$

Using the notation MRS^i_{Gc}, we can rewrite equation (3.C.7) for (3.17). □

Appendix 3.D

The first order condition associated with y^1 and y^2 are as follows:

$$\frac{\partial \mathcal{L}}{\partial y^L} = -\pi^L v_\ell(\ell^L)\frac{1}{w^L} + \gamma\pi^L + \lambda v_\ell(\ell^L \frac{w^L}{w^H})\frac{1}{w^H} = 0 \tag{3.D.1}$$

$$\frac{\partial \mathcal{L}}{\partial y^H} = -\pi^H v_\ell(\ell^H)\frac{1}{w^H} + \gamma\pi^H - \lambda v_\ell(\ell^H)\frac{1}{w^H} = 0 \tag{3.D.2}$$

Substituting equation (3.A.1) into equation (3.D.1) and rearranging yields:

$$\pi^L u_c(c^L,x^L,G)\left[1 - \frac{v_\ell(\ell^L)}{w^L u_c(c^L,x^L,G)}\right] = \lambda u_c(c^L,x^L,\hat{G})\left[1 - \frac{v_\ell(\ell^L \frac{w^L}{w^H})}{w^H u_c(c^L,x^L,\hat{G})}\right] \tag{3.D.3}$$

By using equation (3.7), equation (3.D.3) can be rewritten:

$$\begin{aligned}&\left[\pi^L u_c(c^L, x^L, G) - \lambda u_c(c^L, x^L, \hat{G})\right] T'(y^L) \\ &= \lambda u_c(c^L, x^L, \hat{G})\left[\frac{v_\ell(\ell^L)}{w^L u_c(c^L, x^L, G)} - \frac{v_\ell(\ell^L \frac{w^L}{w^H})}{w^H u_c(c^L, x^L, \hat{G})}\right]\end{aligned} \tag{3.D.4}$$

From (3.A.1), equation (3.D.4) gives equation (3.18). Similarly, substituting equation (3.A.3) into (3.D.2) and rearranging yields:

$$\begin{aligned}&\pi^H u_c(c^H, x^H, G)\left[1 - \frac{v_\ell(\ell^H)}{w^H u_c(c^H, x^H, G)}\right] \\ &= -\lambda u_c(c^H, x^H, G)\left[1 - \frac{v_\ell(\ell^H)}{w^H u_c(c^H, x^H, G)}\right]\end{aligned} \tag{3.D.5}$$

Using equation (3.7), this can be rewritten as follows.

$$\left[\pi^H u_c(c^H, x^H, G) + \lambda u_c(c^H, x^H, G)\right] T'(y^H) = 0 \tag{3.D.6}$$

From (3.A.3), we find that $T'(y^H)$ is zero. □

Appendix 3.E

The first order conditions associated with c^L, x^L, c^M, x^M, c^H, x^H, s^L, s^M, s^H, and s^G are

$$\frac{\partial \mathcal{L}}{\partial c^L} = \pi^L u_c(c^L, x^L, G) - \gamma_1 \pi^L - \lambda_2 u_c(c^L, x^L, \hat{G}_{HL}) - \lambda_3 u_c(c^L, x^L, \hat{G}_{ML}) = 0 \tag{3.E.1}$$

$$\frac{\partial \mathcal{L}}{\partial x^L} = \pi^L u_x(c^L, x^L, G) - \gamma_2 \pi^L - \lambda_2 u_x(c^L, x^L, \hat{G}_{HL}) - \lambda_3 u_x(c^L, x^L, \hat{G}_{ML}) = 0 \tag{3.E.2}$$

$$\frac{\partial \mathcal{L}}{\partial c^M} = \pi^M u_c(c^M, x^M, G) - \gamma_1 \pi^M - \lambda_1 u_c(c^M, x^M, \hat{G}_{HM}) + \lambda_3 u_c(c^M, x^M, G) = 0 \tag{3.E.3}$$

$$\frac{\partial \mathcal{L}}{\partial x^M} = \pi^M u_x(c^M, x^M, G) - \gamma_2 \pi^M - \lambda_1 u_x(c^M, x^M, \hat{G}_{HM}) + \lambda_3 u_x(c^M, x^M, G) = 0 \tag{3.E.4}$$

$$\frac{\partial \mathcal{L}}{\partial c^H} = \pi^H u_c(c^H, x^H, G) - \gamma_1 \pi^H + \lambda_1 u_c(c^H, x^H, G) + \lambda_2 u_c(c^H, x^H, G) = 0 \tag{3.E.5}$$

$$\frac{\partial \mathcal{L}}{\partial x^H} = \pi^H u_x(c^H, x^H, G) - \gamma_2 \pi^H + \lambda_1 u_x(c^H, x^H, G) + \lambda_2 u_x(c^H, x^H, G) = 0 \tag{3.E.6}$$

$$\frac{\partial \mathcal{L}}{\partial s^i} = -\gamma_1 + \gamma_2(1+r) = 0 \quad i = H, M, L, G \tag{3.E.7}$$

Substituting equation (3.E.1) and (3.E.2) into equation (3.E.7) yields:

$$\begin{aligned}
&\pi^L \Big\{ u_c(c^L, x^L, G) - (1+r) u_x(c^L, x^L, G) \Big\} \\
&\quad = \lambda_2 \Big\{ u_c(c^L, x^L, \hat{G}_{HL}) - (1+r) u_x(c^L, x^L, \hat{G}_{HL}) \Big\} \\
&\quad + \lambda_3 \Big\{ u_c(c^L, x^L, \hat{G}_{ML}) - (1+r) u_x(c^L, x^L, \hat{G}_{ML}) \Big\}
\end{aligned} \tag{3.E.8}$$

Combining equation (3.5) with equation (3.E.8) yields:

$$\begin{aligned}
&\Big[\pi^L u_x(c^L, x^L, G) - \lambda_2 u_x(c^L, x^L, \hat{G}_{HL}) - \lambda_3 u_x(c^L, x^L, \hat{G}_{ML}) \Big] r \Phi'(r s^L) \\
&\quad = \lambda_2 u_x(c^L, x^L, \hat{G}_{HL}) \left[\frac{u_c(c^L, x^L, G)}{u_x(c^L, x^L, G)} - \frac{u_c(c^L, x^L, \hat{G}_{HL})}{u_x(c^L, x^L, \hat{G}_{HL})} \right] \\
&\quad + \lambda_3 u_x(c^L, x^L, \hat{G}_{ML}) \left[\frac{u_c(c^L, x^L, G)}{u_x(c^L, x^L, G)} - \frac{u_c(c^L, x^L, \hat{G}_{ML})}{u_x(c^L, x^L, \hat{G}_{ML})} \right]
\end{aligned} \tag{3.E.9}$$

Substituting equation (3.E.2) into the term in the brackets of the left hand side, we obtain equation (3.24). Similarly, substituting equation (3.E.3) and (3.E.4) into equation (3.E.7) yields:

$$\begin{aligned}
&\pi^M \Big\{ u_c(c^M, x^M, G) - (1+r) u_x(c^M, x^M, G) \Big\} \\
&\quad = \lambda_1 \Big\{ u_c(c^M, x^M, \hat{G}_{HM}) - (1+r) u_x(c^M, x^M, \hat{G}_{HM}) \Big\} \\
&\quad - \lambda_3 \Big\{ u_c(c^M, x^M, G) - (1+r) u_x(c^M, x^M, G) \Big\}
\end{aligned} \tag{3.E.10}$$

Combining equation (3.5) with equation (3.E.10) yields:

$$\begin{aligned}
&\Big[\pi^M u_x(c^M, x^M, G) - \lambda_1 u_x(c^M, x^M, \hat{G}_{HM}) + \lambda_3 u_x(c^M, x^M, G)) r \Phi'(r s^M) \Big] \\
&\quad = \frac{\lambda_1 u_x(c^M, x^M, \hat{G}_{HM})}{\gamma_2 \pi^M} \left[\frac{u_c(c^M, x^M, G)}{u_x(c^M, x^M, G)} - \frac{u_c(c^M, x^M, \hat{G}_{HM})}{u_x(c^M, x^M, \hat{G}_{HM})} \right]
\end{aligned} \tag{3.E.11}$$

Substituting equation (3.E.11) into the term in the brackets of the left hand side, we obtain equation (3.25). Finally, substituting (3.E.5) with (3.E.6) into (3.E.7) and rearranging yields that $\Phi'(rs^H)$ is zero. □

Appendix 3.F

Differentiating $\mathcal{L}$ with respect to g^G, g^H, g^M, and g^L implies

$$\frac{\partial \mathcal{L}}{\partial g^G} = -\gamma_1 + \mu \tag{3.F.1}$$

$$\frac{\partial \mathcal{L}}{\partial g^H} = -\gamma_1 \pi^H + \lambda_1 u_G(c^M, x^M, \hat{G}_{HM}) + \lambda_2 u_G(c^L, x^L, \hat{G}_{HL}) + \mu \pi^H \tag{3.F.2}$$

$$\frac{\partial \mathcal{L}}{\partial g^M} = -\gamma_1 \pi^M - \lambda_1 u_G(c^M, x^M, \hat{G}_{HM}) + \lambda_3 u_G(c^L, x^L, \hat{G}_{ML}) + \mu \pi^M \tag{3.F.3}$$

$$\frac{\partial \mathcal{L}}{\partial g^L} = -\gamma_1 \pi^L - \lambda_2 u_G(c^L, x^L, \hat{G}_{HL}) - \lambda_3 u_G(c^L, x^L, \hat{G}_{ML}) + \mu \pi^L \tag{3.F.4}$$

Proof of lemma 3.1

If equation (3.F.1) is equal to zero, equation (3.F.2) is as follows.

$$\frac{\partial \mathcal{L}}{\partial g^H} = \lambda_1 u_G(c^M, x^M, \hat{G}_{HM}) + \lambda_2 u_G(c^L, x^L, \hat{G}_{HL}) > 0 \tag{3.F.5}$$

In this case, the optimal solution with respect to g^H does not exist because of diverging. Therefore, at the optimum, we must have $\frac{\partial \mathcal{L}}{\partial g^G} < 0$ and $g^G = 0$ to satisfy Kuhn-Tucker conditions.

Given this condition, from equation (3.F.4), no contribution to a public good of low-type individuals is optimal, that is, $g^L = 0$. Here, we assume that high-type and middle-type individuals will not donate to a public good, that is, $g^H = g^M = 0$. Under the condition, the sign of $\frac{\partial \mathcal{L}}{\partial g^H}$ is positive because the sum of the second term and the third term in equation (3.F.2) is sufficiently larger than the sum of the first and fourth term by the Inada condition. Therefore, the small increase of g^H is welfare-improving, that is, it is not optimal that both high-type and middle-type will not contribute to a public good. □

Proof of lemma 3.2

Consider $\lambda_3 = 0$. From equation (3.F.3), $\frac{\partial \mathcal{L}}{\partial g^M}$ is negative because $-\gamma_1 + \mu$ is negative as shown above. Therefore, g^M is zero at the optimum. On the other hand, since the second and third term in equation (3.F.2) is sufficiently larger than the sum of the first and fourth term by the Inada condition when g^H is close to zero given $g^L = g^M = g^G = 0$, the small increase of g^H is welfare-improving, that is, g^H is not zero. In addition, g^H is an interior solution. As g^H is close to infinity, $\frac{\partial \mathcal{L}}{\partial g^H}$ converges to $-\gamma_1 \pi^H + \mu \pi^H$ which is negative. This implies that g^H must not be corner solution at the optimum.

$\square$

Proof of lemma 3.3

Consider $\lambda_1 = \lambda_2 = 0$. From equation (3.F.2), $\frac{\partial \mathcal{L}}{\partial g^H}$ is negative because $-\gamma_1 + \mu$ is negative as shown above. Therefore, g^H is zero at the optimum. On the other hand, since the third term in equation (3.F.3) is sufficiently larger than the sum of the first and fourth term by the Inada condition when g^M is close to zero given $g^L = g^H = g^G = 0$, the small increase of g^M is welfare-improving, that is, g^M is not zero. In addition, g^M is an interior solution. As g^M is close to infinity, $\frac{\partial \mathcal{L}}{\partial g^M}$ converges to $-\gamma_1 \pi^M + \mu \pi^M$ which is negative. This implies that g^M must not be corner solution at the optimum. $\square$

Appendix 3.G

The first order condition associated with G is

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial G} &= \pi^H u_G(c^H, x^H, G) + \pi^M u_G(c^M, x^M, G) + \pi^L u_G(c^L, x^L, G) \\ &+ \lambda_1 u_G(c^H, x^H, G) - \lambda_1 u_G(c^M, x^M, \hat{G}^{HM}) + \lambda_2 u_G(c^H, x^H, G) \\ &- \lambda_2 u_G(c^L, x^L, \hat{G}^{HL}) + \lambda_3 u_G(c^M, x^M, G) - \lambda_3 u_G(c^L, x^L, \hat{G}^{ML}) - \mu = 0\end{aligned} \tag{3.G.1}$$

Substituting equation (3.F.2) into (3.G.1) and multiplying π^H yields:

$$\begin{aligned}&\pi^H \Big\{ \pi^H u_G(c^H, x^H, G) + \pi^M u_G(c^M, x^M, G) + \pi^L u_G(c^L, x^L, G) \Big\} \\ &- \gamma_1 \pi^H + \lambda_1 u_G(c^M, x^M, \hat{G}^{HM}) + \lambda_2 \pi^H \Big\{ u_G(c^H, x^H, G) - u_G(c^L, x^L, \hat{G}^{HL}) \Big\} \\ &+ \lambda_2 u_G(c^L, x^L, \hat{G}^{HL}) + \lambda_1 \pi^H \Big\{ u_G(c^H, x^H, G) - u_G(c^M, x^M, \hat{G}^{HM}) \Big\} \\ &+ \lambda_3 \pi^H \Big\{ u_G(c^M, x^M, G) - u_G(c^L, x^L, \hat{G}^{ML}) \Big\} = 0\end{aligned} \tag{3.G.2}$$

Substituting (3.E.5) into (3.G.2) and dividing $\gamma_1\pi^H$ yields:

$$\begin{aligned}
&\frac{1}{\gamma_1}\Big\{\pi^H u_G(c^H,x^H,G)+\pi^M u_G(c^M,x^M,G)+\pi^L u_G(c^L,x^L,G)\Big\}\\
&\quad-\frac{\lambda_1 u_c(c^H,x^H,G)}{\pi^H\gamma_1}-\frac{\lambda_2 u_c(c^H,x^H,G)}{\pi^H\gamma_1}-\frac{u_c(c^H,x^H,G)}{\gamma_1}\\
&\quad+\frac{\lambda_1}{\pi^H\gamma_1}\Big\{\pi^H u_G(c^H,x^H,G)-(\pi^H-1)u_G(c^M,x^M,\hat{G}^{HM})\Big\}\\
&\quad+\frac{\lambda_2}{\pi^H\gamma_1}\Big\{\pi^H u_G(c^H,x^H,G)-(\pi^H-1)u_G(c^L,x^L,\hat{G}^{HL})\Big\}\\
&\quad+\frac{\lambda_3}{\gamma_1}\Big\{u_G(c^M,x^M,G)-u_G(c^L,x^L,\hat{G}^{ML})\Big\}=0
\end{aligned}\tag{3.G.3}$$

Rearranging equation (3.G.3) yields:

$$\begin{aligned}
&\frac{u_c(c^H,x^H,G)}{\gamma_1}\Big\{\pi^H\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}-1\Big\}+\frac{1}{\gamma_1}\Big\{\pi^M u_G(c^M,x^M,G)+\pi^L u_G(c^L,x^L,G)\Big\}\\
&\quad+\Big\{\pi^H\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}-1\Big\}\frac{\lambda_1 u_c(c^H,x^H,G)}{\pi^H\gamma_1}-\frac{\lambda_2}{\gamma_1\pi^H}u_G(c^L,x^L,\hat{G}^{HL})(\pi^H-1)\\
&\quad-\frac{\lambda_1}{\gamma_1\pi^H}u_G(c^M,x^M,\hat{G}^{HM})(\pi^H-1)+\Big\{\pi^H\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}-1\Big\}\frac{\lambda_2 u_c(c^H,x^H,G)}{\pi^H\gamma_1}\\
&\quad+\frac{\lambda_3}{\gamma_1}\Big\{u_G(c^M,x^M,G)-u_G(c^L,x^L,\hat{G}^{ML})\Big\}=0
\end{aligned}\tag{3.G.4}$$

Substituting equation (3.6) and (3.E.5) into (3.G.4) yields:

$$\begin{aligned}
&\Big\{\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}\pi^H-\tau'(g^H)-\frac{u_G(c^H,x^H,G)}{u_c(c^H,x^H,G)}\Big\}-\frac{\lambda_2}{\gamma_1\pi^H}u_G(c^L,x^L,\hat{G}^{HL})(\pi^H-1)\\
&\quad-\frac{\lambda_1}{\gamma_1\pi^H}u_G(c^M,x^M,\hat{G}^{HM})(\pi^H-1)+\frac{\pi^M}{\gamma_1}u_G(c^M,x^M,G)+\frac{\pi^L}{\gamma_1}u_G(c^L,x^L,G)\\
&\quad+\frac{\lambda_3}{\gamma_1}\Big\{u_G(c^M,x^M,G)-u_G(c^L,x^L,\hat{G}^{ML})\Big\}=0
\end{aligned}\tag{3.G.5}$$

Substituting (3.E.1) and (3.E.3) into the second bracket yields:

$$
\begin{aligned}
\tau'(g^H) &= (\pi^H - 1)\frac{u_G(c^H, x^H, G)}{u_c(c^H, x^H, G)} - \frac{\lambda_1}{\gamma_1 \pi^H} u_G(c^M, x^M, \hat{G}^{HM})(\pi^H - 1) \\
&+ u_G(c^L, x^L, G)\left\{\frac{\pi^L}{u_c(c^L, x^L, G)} + \frac{\lambda_2 u_c(c^L, x^L, \hat{G}^{HL})}{\gamma_1 u_c(c^L, x^L, G)} + \frac{\lambda_3 u_c(c^L, x^L, \hat{G}^{ML})}{\gamma_1 u_c(c^L, x^L, G)}\right\} \\
&+ u_G(c^M, x^M, G)\left\{\frac{\pi^M}{u_c(c^M, x^M, G)} + \frac{\lambda_1 u_c(c^M, x^M, \hat{G}^{HM})}{\gamma_1 u_c(c^M, x^M, G)} - \frac{\lambda_3}{\gamma_1}\right\} \\
&- \frac{\lambda_2}{\gamma_1 \pi^H} u_G(c^L, x^L, \hat{G}^{HL})(\pi^H - 1) + \frac{\lambda_3}{\gamma_1}\left\{u_G(c^M, x^M, G) - u_G(c^L, x^L, \hat{G}^{ML})\right\}
\end{aligned}
\tag{3.G.6}
$$

Rearranging (3.G.6) and using the notation MRS^i_{Gc}, we can obtain equation (3.27).

Similarly, substituting equation (3.F.3) into (3.G.1) and multiplying π^M yields:

$$
\begin{aligned}
&\pi^M\left\{\pi^H u_G(c^H, x^H, G) + \pi^M u_G(c^M, x^M, G) + \pi^L u_G(c^L, x^L, G)\right\} \\
&- \gamma_1 \pi^M - \lambda_1 u_G(c^M, x^M, \hat{G}^{HM}) + \lambda_2 \pi^M\left\{u_G(c^H, x^H, G) - u_G(c^L, x^L, \hat{G}^{HL})\right\} \\
&+ \lambda_1 \pi^M\left\{u_G(c^H, x^H, G) - u_G(c^M, x^M, \hat{G}^{HM})\right\} + \lambda_3 u_G(c^L, x^L, \hat{G}^{ML}) \\
&+ \lambda_3 \pi^M\left\{u_G(c^M, x^M, G) - u_G(c^L, x^L, \hat{G}^{ML})\right\} = 0
\end{aligned}
\tag{3.G.7}
$$

Substituting (3.E.3) into (3.G.7) and dividing $\gamma_1 \pi^M$ yields:

$$
\begin{aligned}
&\frac{1}{\gamma_1}\left\{\pi^H u_G(c^H, x^H, G) + \pi^M u_G(c^M, x^M, G) + \pi^L u_G(c^L, x^L, G)\right\} \\
&\quad + \frac{\lambda_1 u_c(c^M, x^M, \hat{G}^{HM})}{\pi^M \gamma_1} - \frac{\lambda_3 u_c(c^M, x^M, G)}{\pi^M \gamma_1} - \frac{u_c(c^M, x^M, G)}{\gamma_1} \\
&\quad + \frac{\lambda_1}{\pi^M \gamma_1}\left\{\pi^M u_G(c^H, x^H, G) - (\pi^M + 1) u_G(c^M, x^M, \hat{G}^{HM})\right\} \\
&\quad + \frac{\lambda_2}{\gamma_1}\left\{u_G(c^H, x^H, G) - u_G(c^L, x^L, \hat{G}^{HL})\right\} \\
&\quad + \frac{\lambda_3}{\pi^H \gamma_1}\left\{\pi^M u_G(c^M, x^M, G) - (\pi^M - 1) u_G(c^L, x^L, \hat{G}^{ML})\right\} = 0
\end{aligned}
\tag{3.G.8}
$$

Rearranging equation (3.G.8) yields:

$$
\begin{aligned}
&\frac{u_c(c^M,x^M,G)}{\gamma_1}\left\{\pi^M\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}-1\right\}+\frac{1}{\gamma_1}\left\{\pi^H u_G(c^H,x^H,G)+\pi^L u_G(c^L,x^L,G)\right\}\\
&-\left\{\pi^M\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}-1\right\}\frac{\lambda_1 u_c(c^M,x^M,G)}{\pi^M\gamma_1}+\frac{\lambda_1 u_G(c^H,x^H,G)}{\gamma_1}\\
&-\frac{\lambda_1}{\gamma_1\pi^M}u_G(c^M,x^M,\hat{G}^{HM})(\pi^M+1)+\frac{\lambda_2}{\gamma_1}(u_G(c^H,x^H,G)-u_G(c^L,x^L,\hat{G}^{HL}))\\
&-\frac{\lambda_3}{\gamma_1\pi^H}u_G(c^L,x^L,\hat{G}^{ML})(\pi^M-1)+\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}\frac{u_c(c^M,x^M,\hat{G}^{HM})\lambda_1}{\gamma_1}\\
&+\left\{\pi^M\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}-1\right\}\frac{\lambda_3 u_c(c^M,x^M,G)}{\pi^M\gamma_1}=0
\end{aligned}
\tag{3.G.9}
$$

Substituting equation (3.6) and (3.E.3) into (3.G.9) yields:

$$
\begin{aligned}
&\left\{\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}\pi^M-\tau'(g^M)-\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}\right\}+\frac{\lambda_1 u_G(c^H,x^H,G)}{\gamma_1}\\
&-\frac{\lambda_1}{\gamma_1\pi^M}u_G(c^M,x^M,\hat{G}^{HM})(\pi^M+1)+\frac{\lambda_2}{\gamma_1}(u_G(c^H,x^H,G)-u_G(c^L,x^L,\hat{G}^{HL}))\\
&-\frac{\lambda_3}{\gamma_1\pi^H}u_G(c^L,x^L,\hat{G}^{ML})(\pi^M-1)+\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}\frac{u_c(c^M,x^M,\hat{G}^{HM})\lambda_1}{\gamma_1}\\
&+\left\{\frac{\pi^H}{\gamma_1}u_G(c^H,x^H,G)+\frac{\pi^L}{\gamma_1}u_G(c^L,x^L,G)\right\}=0
\end{aligned}
\tag{3.G.10}
$$

Substituting (3.E.1) and (3.E.5) into the second bracket yields:

$$
\begin{aligned}
\tau'(g^M)=&(\pi^M-1)\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}+u_G(c^H,x^H,G)\left\{\frac{\pi^H}{u_c(c^H,x^H,G)}-\frac{\lambda_1}{\gamma_1}-\frac{\lambda_2}{\gamma_1}\right\}\\
&+u_G(c^L,x^L,G)\left\{\frac{\pi^L}{u_c(c^L,x^L,G)}+\frac{\lambda_2 u_c(c^L,x^L,\hat{G}^{HL})}{\gamma_1 u_c(c^L,x^L,G)}+\frac{\lambda_3 u_c(c^L,x^L,\hat{G}^{ML})}{\gamma_1 u_c(c^L,x^L,G)}\right\}\\
&-\frac{\lambda_1}{\gamma_1\pi^M}u_G(c^M,x^M,\hat{G}^{HM})(\pi^M+1)+\frac{\lambda_2}{\gamma_1}(u_G(c^H,x^H,G)-u_G(c^L,x^L,\hat{G}^{HL}))\\
&-\frac{\lambda_3}{\gamma_1\pi^H}u_G(c^L,x^L,\hat{G}^{ML})(\pi^M-1)+\frac{u_G(c^M,x^M,G)}{u_c(c^M,x^M,G)}\frac{u_c(c^M,x^M,\hat{G}^{HM})\lambda_1}{\gamma_1}+\frac{\lambda_1 u_G(c^H,x^H,G)}{\gamma_1}
\end{aligned}
\tag{3.G.11}
$$

Rearranging (3.G.11) and using the notation MRS^i_{Gc}, we can obtain equation (3.28). □

Appendix 3.H

The first order condition associated with y^H, y^M, and y^L are as follows:

$$\frac{\partial \mathcal{L}}{\partial y^L} = -\pi^L v_\ell(\ell^L)\frac{1}{w^L} + \gamma_1 \pi^L + \lambda_2 v_\ell(\ell_L \frac{w^L}{w^H})\frac{1}{w^H} + \lambda_3 v_\ell(\ell_L \frac{w^L}{w^M})\frac{1}{w^M} = 0 \tag{3.H.1}$$

$$\frac{\partial \mathcal{L}}{\partial y^M} = -\pi^M v_\ell(\ell^M)\frac{1}{w^M} + \gamma_1 \pi^M + \lambda_1 v_\ell(\ell^M \frac{w^M}{w^H})\frac{1}{w^H} - \lambda_3 v_\ell(\ell^M)\frac{1}{w^M} = 0 \tag{3.H.2}$$

$$\frac{\partial \mathcal{L}}{\partial y^H} = -\pi^H v_\ell(\ell^H)\frac{1}{w^H} + \gamma_1 \pi^H - \lambda_1 v_\ell(\ell^H)\frac{1}{w^H} - \lambda_2 v_\ell(\ell^H)\frac{1}{w^H} = 0 \tag{3.H.3}$$

Substituting equation (3.E.1) into equation (3.H.1) yields:

$$\begin{aligned} \frac{v_\ell(\ell^L)}{w^L}\pi^L &= u_c(c^L, x^L, G)\pi^L - \lambda_2 u_c(c^L, x^L, \hat{G}^{HL}) \\ &+ \lambda_2 v_\ell(\ell_L \frac{w^L}{w^H})\frac{1}{w^H} - \lambda_3 u_c(c^L, x^L, \hat{G}^{ML}) + \lambda_3 v_\ell(\ell_L \frac{w^L}{w^M})\frac{1}{w^M} \end{aligned} \tag{3.H.4}$$

By using equation (3.7), equation (3.H.4) can be rewritten:

$$\begin{aligned} &\Big[\pi^L u_c(c^L, x^L, G) - \lambda_2 u_c(c^L, x^L, \hat{G}_{HL}) - \lambda_3 u_c(c^L, x^L, \hat{G}_{ML})\Big] T'(y^L) \\ &= \lambda_2 u_c(c^L, x^L, \hat{G}^{HL})\Big[\frac{v_\ell(\ell^L)}{w^L u_c(c^L, x^L, G)} - \frac{v_\ell(\ell_L \frac{w^L}{w^H})}{w^H u_c(c^L, x^L, \hat{G}^{HL})}\Big] \\ &+ \lambda_3 u_c(c^L, x^L, \hat{G}^{ML})\Big[\frac{v_\ell(\ell^L)}{w^L u_c(c^L, x^L, G)} - \frac{v_\ell(\ell_L \frac{w^L}{w^M})}{w^M u_c(c^L, x^L, \hat{G}_{ML})}\Big] \end{aligned} \tag{3.H.5}$$

Rearranging equation (3.H.5) gives equation (3.29). Similarly, substituting equation (3.E.3) into equation (3.H.2) yields:

$$\begin{aligned} \frac{v_\ell(\ell^M)}{w^M}\pi^M &= u_c(c^M, x^M, G)\pi^M - \lambda_1 u_c(c^M, x^M, \hat{G}^{HM}) \\ &+ \lambda_1 v_\ell(\ell_M \frac{w^M}{w^H})\frac{1}{w^H} + \lambda_3 u_c(c^M, x^M, G) - \lambda_3 v_\ell(\ell^M)\frac{1}{w^M} \end{aligned} \tag{3.H.6}$$

By using equation (3.7), equation (3.H.6) can be rewritten:

$$\begin{aligned}&\Big[\pi^M u_c(c^M, x^M, G) - \lambda_1 u_c(c^M, x^M, \hat{G}_{HM}) + \lambda_3 u_c(c^M, x^M, G)\Big] T'(y^M) \\ &\quad = \lambda_1 u_c(c^M, x^M, \hat{G}^{HM}) \left[\frac{v_\ell(\ell^M)}{w^M u_c(c^M, x^M, G)} - \frac{v_\ell(\ell_M \frac{w^M}{w^H})}{w^H u_c(c^M, x^M, \hat{G}^{HM})}\right]\end{aligned} \tag{3.H.7}$$

Rearranging equation (3.H.7) gives equation (3.30). Finally, substituting equation (3.E.5) into equation (3.H.3), and then using equation (3.7) and rearranging yields $T'(y^H) = 0$. □

Appendix 3.I

The corresponding Lagrangian is formulated as follows:

$$\begin{aligned}\tilde{\mathcal{L}} &= \tilde{W} + \tilde{\mu}\Big[\sum_i g^i \pi^i + g^G - G\Big] \\ &+ \tilde{\gamma}\Big[\sum_i \pi^i (y^i - g^i - R^i) + \frac{1}{1+r}\sum_i \pi^i (q_s(1+r) - 1) x(q_s, R^i, G) - g^G\Big] \\ &+ \tilde{\lambda}\Big[V(q_s, R^2, G) - v(\frac{y^2}{w^2}) - V(q_s, R^1, \hat{G}) + v(\frac{y^1}{w^2})\Big]\end{aligned} \tag{3.I.1}$$

where $\tilde{\gamma}$, $\tilde{\lambda}$, and $\tilde{\mu}$ are the Lagrange multipliers. The first order conditions associated with q_s, R^1, and R^2 are

$$\frac{\partial \tilde{\mathcal{L}}}{\partial q_s} = \sum_i \pi^i \frac{\partial V^i}{\partial q_s} + \tilde{\gamma} \sum_i \pi^i \left(x_i^* + \frac{q_s(1+r) - 1}{1+r} \frac{\partial x_i^*}{\partial q_s} \right) + \tilde{\lambda} \left(\frac{\partial V^2}{\partial q_s} - \frac{\partial \hat{V}^2}{\partial q_s} \right) = 0 \tag{3.I.2}$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial R^1} = \pi^1 \frac{\partial V^1}{\partial R^1} - \tilde{\gamma}\pi^1 + \tilde{\gamma}\pi^1 \frac{q_s(1+r) - 1}{1+r} \frac{\partial x_1^*}{\partial R^1} - \tilde{\lambda} \frac{\partial \hat{V}^2}{\partial R^1} = 0 \tag{3.I.3}$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial R^2} = \pi^2 \frac{\partial V^2}{\partial R^2} - \tilde{\gamma}\pi^2 + \tilde{\gamma}\pi^2 \frac{q_s(1+r) - 1}{1+r} \frac{\partial x_2^*}{\partial R^2} + \tilde{\lambda} \frac{\partial V^2}{\partial R^2} = 0 \tag{3.I.4}$$

We now combine these constraints by taking

$$\frac{\partial \tilde{\mathcal{L}}}{\partial q_s} + \sum_i \frac{\partial \tilde{\mathcal{L}}}{\partial R^i} x_i^* \tag{3.I.5}$$

From the Roy's identity and the Slutsky decomposition, we can get the following relationships:

$$\frac{\partial V^i}{\partial q_s} = -\frac{\partial V^i}{\partial R^i} \cdot x_i^* \tag{3.I.6}$$

$$\frac{\partial x_i^*}{\partial q_s} = \frac{\partial \tilde{x}_i^*}{\partial q_s} - \frac{\partial x_i^*}{\partial R^i} \cdot x_i^* \tag{3.I.7}$$

where $\tilde{x}_i^*$ indicates the compensated demand function of type i individuals for the consumption in the second period. Using Roy's identity and Slutsky decomposition, this gives

$$\tilde{\gamma}\frac{q_s(1+r)-1}{1+r}\sum_i \pi^i \frac{\partial \tilde{x}_i^*}{\partial q_s} + \tilde{\lambda}\frac{\partial \hat{V}}{\partial R^1}(\hat{x}^* - x_1^*) = 0 \tag{3.I.8}$$

Thus, we can obtain equation (3.36). □

Appendix 3.J

Differentiating $\tilde{\mathcal{L}}$ with respect to g^G, g^1, and g^2 implies

$$\frac{\partial \tilde{\mathcal{L}}}{\partial g^G} = -\tilde{\gamma} + \tilde{\mu} \tag{3.J.1}$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial g^1} = -\tilde{\gamma}\pi^1 - \tilde{\lambda}u_G(q_s, R^1, \hat{G}) + \tilde{\mu}\pi^1 \tag{3.J.2}$$

$$\frac{\partial \tilde{\mathcal{L}}}{\partial g^2} = -\tilde{\gamma}\pi^2 + \tilde{\lambda}u_G(q_s, R^1, \hat{G}) + \tilde{\mu}\pi^2 \tag{3.J.3}$$

Following the same proof as in Appendix 3.B, we conclude that $g^G = g^1 = 0$ and $g^2 > 0$.
□

Appendix 3.K

Using Theorem 1 of Caputo (1996), we can get the following relationships.

$$\frac{\partial V^i}{\partial q_g} = -\phi g_i^* + u_G\frac{\partial G_{-i}^*}{\partial q_g} \tag{3.K.1}$$

$$\frac{\partial V^i}{\partial q_s} = -\phi x_i^* + u_G\frac{\partial G_{-i}^*}{\partial q_s} \tag{3.K.2}$$

$$\frac{\partial V^i}{\partial R^i} = \phi + u_G\frac{\partial G_{-i}^*}{\partial R^i} \tag{3.K.3}$$

$$\frac{\partial V^i}{\partial R^j} = u_G \frac{\partial G^*_{-i}}{\partial R^j},\ i \neq j \tag{3.K.4}$$

where let ϕ be the Lagrange multiplier with respect to individual's budget constraint.

The corresponding Lagrangian is formulated as follows:

$$\begin{aligned}\bar{\mathcal{L}} = \bar{W} + \bar{\lambda}\Big[V^2(q_g, q_s, R^1, R^2) - v(\frac{y^2}{w^2}) - \hat{V}(q_g, q_s, R^1, R^2) + v(\frac{y^1}{w^2})\Big] \\ + \bar{\gamma}\Big[\sum_i \pi^i(y^i - R^i) + \sum_i \pi^i(q_g - 1)g_i^* + \frac{1}{1+r}\sum_i \pi^i(q_s(1+r) - 1)x_i^*\Big]\end{aligned} \tag{3.K.5}$$

where $\bar{\gamma}$ and $\bar{\lambda}$ are the Lagrange multipliers. The first order conditions associated with q_s, q_g, R^1, and R^2 are as follows:

$$\begin{aligned}\frac{\partial \bar{\mathcal{L}}}{\partial q_s} = \sum_i \pi^i \frac{\partial V^i}{\partial q_s} + \bar{\lambda}\Big[\frac{\partial V^2}{\partial q_s} - \frac{\partial \hat{V}}{\partial q_s}\Big] \\ + \bar{\gamma}\Big[\frac{1}{1+r}\sum_i \pi^i\Big((1+r)x_i^* + (q_s(1+r) - 1)\frac{\partial x_i^*}{\partial q_s}\Big) + \sum_i \pi^i(q_g - 1)\frac{\partial g_i^*}{\partial q_s}\Big] = 0\end{aligned} \tag{3.K.6}$$

$$\begin{aligned}\frac{\partial \bar{\mathcal{L}}}{\partial q_g} = \sum_i \pi^i \frac{\partial V^i}{\partial q_g} + \bar{\lambda}\Big[\frac{\partial V^2}{\partial q_g} - \frac{\partial \hat{V}}{\partial q_g}\Big] \\ + \bar{\gamma}\Big[\sum_i \pi^i\Big(g_i^* + (q_g - 1)\frac{\partial g_i^*}{\partial q_g}\Big) + \frac{1}{1+r}\sum_i \pi^i(q_s(1+r) - 1)\frac{\partial x_i^*}{\partial q_g}\Big] = 0\end{aligned} \tag{3.K.7}$$

$$\begin{aligned}\frac{\partial \bar{\mathcal{L}}}{\partial R^1} = \pi^1 \frac{\partial V^1}{\partial R^1} + \pi^2 \frac{\partial V^2}{\partial R^1} + \bar{\lambda}\Big[\frac{\partial V^2}{\partial R^1} - \frac{\partial \hat{V}}{\partial R^1}\Big] \\ + \bar{\gamma}\Big[-\pi^1 + \pi^1(q_g - 1)\frac{\partial g_1^*}{\partial R^1} + \pi^2(q_g - 1)\frac{\partial g_2^*}{\partial R^1} \\ + \frac{1}{1+r}\pi^1(q_s(1+r) - 1)\frac{\partial x_1^*}{\partial R^1} + \frac{1}{1+r}\pi^2(q_s(1+r) - 1)\frac{\partial x_2^*}{\partial R^1}\Big] = 0\end{aligned} \tag{3.K.8}$$

$$\begin{aligned}\frac{\partial \bar{\mathcal{L}}}{\partial R^2} = \pi^1 \frac{\partial V^1}{\partial R^2} + \pi^2 \frac{\partial V^2}{\partial R^2} + \bar{\lambda}\Big[\frac{\partial V^2}{\partial R^2} - \frac{\partial \hat{V}}{\partial R^2}\Big] \\ + \bar{\gamma}\Big[-\pi^2 + \pi^2(q_g - 1)\frac{\partial g_2^*}{\partial R^2} + \pi^1(q_g - 1)\frac{\partial g_1^*}{\partial R^2} \\ + \frac{1}{1+r}\pi^2(q_s(1+r) - 1)\frac{\partial x_2^*}{\partial R^2} + \frac{1}{1+r}\pi^1(q_s(1+r) - 1)\frac{\partial x_1^*}{\partial R^2}\Big] = 0\end{aligned} \tag{3.K.9}$$

We now combine these constraints by taking

$$\frac{\partial \bar{\mathcal{L}}}{\partial q_g} + \sum_i \frac{\partial \bar{\mathcal{L}}}{\partial R^i} g_i^* \tag{3.K.10}$$

and

$$\frac{\partial \bar{\mathcal{L}}}{\partial q_s} + \sum_i \frac{\partial \bar{\mathcal{L}}}{\partial R^i} x_i^* \tag{3.K.11}$$

These give

$$\begin{aligned}
&\sum_{i \neq j = 1,2} \pi^i \left[\frac{\partial V^i}{\partial q_g} + \frac{\partial V^i}{\partial R^i} g_i^* + \frac{\partial V^i}{\partial R^j} g_j^* \right] + \bar{\gamma}(q_g - 1) \left[\sum_i \pi^i \frac{\partial g_i^*}{\partial q_g} + \sum_i \pi^i \frac{\partial g_i^*}{\partial R^i} g_i^* + \sum_{i \neq j = 1,2} \pi^i \frac{\partial g_i^*}{\partial R^j} g_j^* \right] \\
&\quad + \bar{\gamma} \frac{q_s(1+r) - 1}{1+r} \left[\sum_i \pi^i \frac{\partial x_i^*}{\partial q_g} + \sum_i \pi^i \frac{\partial x_i^*}{\partial R^i} g_i^* + \sum_{i \neq j = 1,2} \pi^i \frac{\partial x_i^*}{\partial R^j} g_j^* \right] \\
&\quad + \bar{\lambda} \left[\frac{\partial V^2}{\partial q_g} + \frac{\partial V^2}{\partial R^1} g_1^* + \frac{\partial V^2}{\partial R^2} g_2^* - \frac{\partial \hat{V}}{\partial q_g} - \frac{\partial \hat{V}}{\partial R^1} g_1^* - \frac{\partial \hat{V}}{\partial R^2} g_2^* \right] = 0
\end{aligned} \tag{3.K.12}$$

and

$$\begin{aligned}
&\sum_{i \neq j = 1,2} \pi^i \left[\frac{\partial V^i}{\partial q_s} + \frac{\partial V^i}{\partial R^i} g_i^* + \frac{\partial V^i}{\partial R^j} g_j^* \right] + \bar{\gamma}(q_g - 1) \left[\sum_i \pi^i \frac{\partial g_i^*}{\partial q_s} + \sum_i \pi^i \frac{\partial g_i^*}{\partial R^i} x_i^* + \sum_{i \neq j = 1,2} \pi^i \frac{\partial g_i^*}{\partial R^j} x_j^* \right] \\
&\quad + \bar{\gamma} \frac{q_s(1+r) - 1}{1+r} \left[\sum_i \pi^i \frac{\partial x_i^*}{\partial q_s} + \sum_i \pi^i \frac{\partial x_i^*}{\partial R^i} x_i^* + \sum_{i \neq j = 1,2} \pi^i \frac{\partial x_i^*}{\partial R^j} x_j^* \right] \\
&\quad + \bar{\lambda} \left[\frac{\partial V^2}{\partial q_s} + \frac{\partial V^2}{\partial R^1} x_1^* + \frac{\partial V^2}{\partial R^2} x_2^* - \frac{\partial \hat{V}}{\partial q_s} - \frac{\partial \hat{V}}{\partial R^1} x_1^* - \frac{\partial \hat{V}}{\partial R^2} x_2^* \right] = 0
\end{aligned} \tag{3.K.13}$$

Using equation from (3.K.1) to (3.K.4), (3.K.12) and (3.K.13) are transformed as

$$\begin{aligned}
&\sum_{i \neq j = 1,2} \pi^i u_G^i \left[\frac{\partial G_{-i}^*}{\partial q_g} + \frac{\partial G_{-i}^*}{\partial R^i} g_i^* + \frac{\partial G_{-i}^*}{\partial R^j} g_j^* \right] \\
&\quad + \bar{\gamma}(q_g - 1) \left[\sum_i \pi^i \frac{\partial g_i^*}{\partial q_g} + \sum_i \pi^i \frac{\partial g_i^*}{\partial R^i} g_i^* + \sum_{i \neq j = 1,2} \pi^i \frac{\partial g_i^*}{\partial R^j} g_j^* \right] \\
&\quad + \bar{\gamma} \frac{q_s(1+r) - 1}{1+r} \left[\sum_i \pi^i \frac{\partial x_i^*}{\partial q_g} + \sum_i \pi^i \frac{\partial x_i^*}{\partial R^i} g_i^* + \sum_{i \neq j = 1,2} \pi^i \frac{\partial x_i^*}{\partial R^j} g_j^* \right] \\
&\quad + \bar{\lambda} \left[u_G \left(\frac{\partial G_{-2}^*}{\partial q_g} + \frac{\partial G_{-2}^*}{\partial R^1} g_1^* + \frac{\partial G_{-2}^*}{\partial R^2} g_2^* \right) - \hat{u}_G \left(\frac{\partial \tilde{G}_{-2}}{\partial q_g} + \frac{\partial \tilde{G}_{-2}}{\partial R^1} g_1^* + \frac{\partial \tilde{G}_{-2}}{\partial R^2} g_2^* \right) + \hat{u}_c(\hat{g} - g_1^*) \right] \\
&\quad = 0
\end{aligned} \tag{3.K.14}$$

and

$$
\begin{aligned}
&\sum_{i\neq j=1,2}\pi^i u_G^i\left[\frac{\partial G_{-i}^*}{\partial q_s}+\frac{\partial G_{-i}^*}{\partial R^i}g_i^*+\frac{\partial G_{-i}^*}{\partial R^j}g_j^*\right]\\
&\quad+\bar{\gamma}(q_g-1)\left[\sum_i \pi^i\frac{\partial g_i^*}{\partial q_s}+\sum_i \pi^i\frac{\partial g_i^*}{\partial R^i}x_i^*+\sum_{i\neq j=1,2}\pi^i\frac{\partial g_i^*}{\partial R^j}x_j^*\right]\\
&\quad+\bar{\gamma}\frac{q_s(1+r)-1}{1+r}\left[\sum_i \pi^i\frac{\partial x_i^*}{\partial q_s}+\sum_i \pi^i\frac{\partial x_i^*}{\partial R^i}x_i^*+\sum_{i\neq j=1,2}\pi^i\frac{\partial x_i^*}{\partial R^j}x_j^*\right]\\
&\quad+\bar{\lambda}\left[u_G\left(\frac{\partial G_{-2}^*}{\partial q_s}+\frac{\partial G_{-2}^*}{\partial R^1}x_1^*+\frac{\partial G_{-2}^*}{\partial R^2}x_2^*\right)-\hat{u}_G\left(\frac{\partial \tilde{G}_{-2}}{\partial q_s}+\frac{\partial \tilde{G}_{-2}}{\partial R^1}x_1^*+\frac{\partial \tilde{G}_{-2}}{\partial R^2}x_2^*\right)+\hat{u}_c(\hat{x}-x_1^*)\right]\\
&\quad=0
\end{aligned}
\tag{3.K.15}
$$

Using matrix notation, (3.K.14) and (3.K.15) can be rewritten as

$$
\begin{aligned}
\Delta\begin{pmatrix}-t_g\\ \frac{rt_sq_s}{1+r}\end{pmatrix}=&-\frac{1}{\bar{\gamma}}\begin{pmatrix}\sum_{i\neq j=1,2}\pi^i u_G^i\left(\frac{\partial G_{-i}^*}{\partial q_g}+\frac{\partial G_{-i}^*}{\partial R^i}g_i^*+\frac{\partial G_{-i}^*}{\partial R^j}g_j^*\right)\\ \sum_{i\neq j=1,2}\pi^i u_G^i\left(\frac{\partial G_{-i}^*}{\partial q_s}+\frac{\partial G_{-i}^*}{\partial R^i}g_i^*+\frac{\partial G_{-i}^*}{\partial R^j}g_j^*\right)\end{pmatrix}-\frac{\bar{\lambda}\hat{u}_c}{\bar{\gamma}}\begin{pmatrix}\hat{g}-g_1^*\\ \hat{x}-x_1^*\end{pmatrix}\\
&-\frac{\bar{\lambda}}{\bar{\gamma}}\begin{pmatrix}u_G\left(\frac{\partial G_{-2}^*}{\partial q_g}+\frac{\partial G_{-2}^*}{\partial R^1}g_1^*+\frac{\partial G_{-2}^*}{\partial R^2}g_2^*\right)-\hat{u}_G\left(\frac{\partial \tilde{G}_{-2}}{\partial q_g}+\frac{\partial \tilde{G}_{-2}}{\partial R^1}g_1^*+\frac{\partial \tilde{G}_{-2}}{\partial R^2}g_2^*\right)\\ u_G\left(\frac{\partial G_{-2}^*}{\partial q_s}+\frac{\partial G_{-2}^*}{\partial R^1}x_1^*+\frac{\partial G_{-2}^*}{\partial R^2}x_2^*\right)-\hat{u}_G\left(\frac{\partial \tilde{G}_{-2}}{\partial q_s}+\frac{\partial \tilde{G}_{-2}}{\partial R^1}x_1^*+\frac{\partial \tilde{G}_{-2}}{\partial R^2}x_2^*\right)\end{pmatrix}
\end{aligned}
\tag{3.K.16}
$$

where δ_1 indicates the first column vector of Δ. Multiplying equation (3.K.16) by Δ^{-1} yields (3.42). If the public good is additively separable in the utility function, the marginal utility of the public good coincides between a low-skilled and a high-skilled individual. In this case, equation (3.K.16) reduces to

$$
\begin{aligned}
\Delta\begin{pmatrix}-t_g\\ \frac{rt_sq_s}{1+r}\end{pmatrix}=&-\frac{u_G}{\bar{\gamma}}(\pi^1+\pi^2-1)\delta_1-\frac{\bar{\lambda}\hat{u}_c}{\bar{\gamma}}\begin{pmatrix}\hat{g}-g_1^*\\ \hat{x}-x_1^*\end{pmatrix}\\
&-\frac{\bar{\lambda}}{\bar{\gamma}}\begin{pmatrix}u_G\left(\frac{\partial G_{-2}^*}{\partial q_g}+\frac{\partial G_{-2}^*}{\partial R^1}g_1^*+\frac{\partial G_{-2}^*}{\partial R^2}g_2^*\right)-\hat{u}_G\left(\frac{\partial \tilde{G}_{-2}}{\partial q_g}+\frac{\partial \tilde{G}_{-2}}{\partial R^1}g_1^*+\frac{\partial \tilde{G}_{-2}}{\partial R^2}g_2^*\right)\\ u_G\left(\frac{\partial G_{-2}^*}{\partial q_s}+\frac{\partial G_{-2}^*}{\partial R^1}x_1^*+\frac{\partial G_{-2}^*}{\partial R^2}x_2^*\right)-\hat{u}_G\left(\frac{\partial \tilde{G}_{-2}}{\partial q_s}+\frac{\partial \tilde{G}_{-2}}{\partial R^1}x_1^*+\frac{\partial \tilde{G}_{-2}}{\partial R^2}x_2^*\right)\end{pmatrix}
\end{aligned}
\tag{3.K.17}
$$

where δ_1 indicates the first column vector of Δ. Multiplying equation (3.K.17) by Δ^{-1} yields (3.43). $\square$

Chapter 4

Optimal human capital policies under the endogenous choice of educational types

4.1 Introduction

Although investment in human capital plays an important role in enriching lives, it is sensitive to tax policy (Schultz (1961)). In particular, income taxation affects investment in human capital. Labor income taxes prevent individuals from investing in human capital by capturing part of the return to human capital, and capital income taxes distort the choice between physical and human capital. To alleviate tax distortions and foster human capital accumulation, OECD countries heavily subsidize higher education. From the efficiency concern that the government's intervention should not distort individual's decision-making, the optimal design of education policies under labor and capital income taxation is a research issue of interest for many economists.

A common assumption in previous literature on optimal education policies is that investment in human capital results in only a production value. Put differently, these studies have considered that the time invested in education contributes only to labor productivity, which leads to higher wages. However, there is growing empirical evidence for the existence of consumption value (Schaafsma (1976), Lazear (1977), Kodde and Ritzen (1984), Gullason (1989), Heckman et al. (1999), Carneiro et al. (2003), Arcidiacono (2004), and Alstadsæter (2011)). For example, education yields joy and satisfaction in learning new things, meeting new people, and participating in lectures and campus activities. Moreover, higher education generates opportunities for obtaining higher social status and finding interesting jobs. Therefore, the motivation underlining the educational choices of individuals stems from not only production value but also consumption value. In addition, the importance of consumption value or production value differs between individuals. Alstadsæter (2011) shows that teachers' colleges in Norway are an educational type with a higher consumption value and a lower production value than business schools. Walker and Zhu (2003) report a negative wage return to an art degree in the UK, while there is a substantial positive wage return to an engineering degree. This implies that art graduates are willing to forgo future wages to enjoy the consumption value in education. These findings suggest that these returns from education vary across educational types and individuals choose an educational type depending on their own preferences. The present study introduces the consumption value of education into the model and allows individuals to choose an educational type differing in the ratio between consumption value and production value. The set of tax instruments for the government consists of non-linear taxes on labor and capital income and non-linear subsidies on education. The objective of this study is to theoretically investigate how these non-linear optimal tax and subsidy policies should be designed when education has two types of return, and moreover, when the choice of educational type is subject to individuals' control.

Our framework consists of a dynamic setting without uncertainty in which there are two types of individuals who differ only in exogenous ability, that is, a modified version of the Stiglitz (1982) optimal taxation model.[1] These individuals live for two periods. In the first period, they consume, invest in education with a consumption value and a production value, and transfer resources through savings. The former value directly affects individuals' utility and the latter value raises the effective labor supply. In the second period, individuals work and then consume by spending their earnings and assets. Their earnings are a function of ability, labor supply, and education with the production value. We assume that the government can observe labor and capital income and education for each type, but cannot distinguish two types of value in education. This measurement problem does not allow the government to subsidize only the contribution to human capital. Therefore, the government can employ three sorts of non-linear tax schemes: non-linear labor and capital income taxes and non-linear education subsidies.

The first contribution of the study is to show that optimal education policies attaining an efficient level with respect to education choice should be modified under endogenous choice of educational type. Therefore, optimal education policies should not be set at a

[1] There is a growing body of literature analyzing optimal tax policies with human capital investment in a dynamic setting with certainty, for example, Nielsen and Sørensen (1997), Jacobs (2005), Bohacek and Kapička (2008), Jacobs and Bovenberg (2010), Schindler (2011), Kapička (2015), Jacobs and Yang (2016). By contrast, Eaton and Rosen (1980), Hamilton (1987), Anderberg and Andersson (2003), da Costa and Maestri (2007), Anderberg (2009), Grochulski and Piskorski (2010), Jacobs et al. (2012), Kapička and Neira (2015), Schindler and Yang (2015), Stantcheva (2015), and Findeisen and Sachs (2016) investigate optimal tax policies in the presence of a stochastic risk factor on endogenous human capital formation in a dynamic setting.

level to achieve efficiency concerns, which means that the production efficiency theorem of Diamond and Mirrlees (1971) breaks down. The second contribution of this study is to show that an individual's behavior reflecting a choice of educational types can justify taxation on capital income even if the utility function is separable between consumption and labor supply. This result presents the case in which the theorem of Atkinson and Stiglitz (1976) fails. These findings crucially depend on preference heterogeneity in educational types, which are endogenously generated. Allowing for choice of educational types, low-type individuals prefer production value to consumption value more than high-type individuals. Following the logic of Saez (2002a), the additional information is useful to relax the binding incentive constraint, and therefore, the standard result is modified. As usual, high-type individuals face zero marginal tax rate on labor income and education, that is, the result with no-distortion at the top remains. The present study highlights the importance of recognizing individuals' choice of educational types when implementing education policies.

Since the seminal contribution of the information-based approach to tax policy emanated from Mirrlees (1971), many economists have analyzed how education policies should be designed under non-linear labor income taxes when individuals have private information. Our study is closely related to Bovenberg and Jacobs (2005), who introduce education choices as one of the individual's behaviors into the framework of Mirrlees (1971) and show that the role of education subsidies is to eliminate the distortion on educational efforts induced by labor income taxes.[2] The findings suggest that education subsidies restore efficiency in education choices, that is, the Diamond–Mirrlees production efficiency theorem is valid. Moreover, the findings demonstrate that the result continues even in the presence of non-pecuniary benefit in education as long as the utility function is separable between work effort and non-pecuniary benefit in education. Our study differs in two ways from the framework of Bovenberg and Jacobs (2005). First, we extend their model as a two-period setting to explore the desirability of capital income taxes. Second, the authors assume that the choice of consumption value in education is exogenous, as in Alstadsæter (2003). The present study assumes that the choice of consumption value in education is endogenous.[3] Under the setting, we address the desirability of capital income taxation in addition to the distortion on learning, which differs

[2]Ulph (1977), Hare and Ulph (1979), and Krause (2006) are previous works exploring optimal tax systems with both income taxes and education expenditure. However, since these studies focus on publicly provided education, individuals do not have decision-making in terms of educational effort. Tuomala (1986) examines how educational choices should be reflected in optimal income taxation by allowing individuals to choose their educational choices, but education subsidies are not introduced in his model. By contrast, the present study analyzes education policies under non-linear income taxes when individuals can decide not only the level of labor supply but also educational effort, in line with Bovenberg and Jacobs (2005).

[3]Malchow-Møller et al. (2011) examine linear progressive taxes on labor income and tuition fees under endogenous choice of educational types when capital income tax is given exogenously. However, we investigate the optimal design of income taxes and education policies in the context of non-linear taxation. Furthermore, we allow the government to optimize capital income tax and show that capital income tax is not superfluous.

in the statement of Bovenberg and Jacobs (2005).

In this study, the government can employ not only non-linear labor income taxes and education subsidies but also non-linear capital income taxes. It is well known that Ordover and Phelps (1979) examine optimal non-linear taxation on income and savings in an overlapping-generations economy in the case of unobservable earnings ability, and state that if preferences are weakly separable between private goods and leisure, then taxes on savings are redundant. This is consistent with the Atkinson and Stiglitz (1976) theorem. Compared to the result, Jacobs and Bovenberg (2010) show that capital income taxation is useful to alleviate the tax distortion caused by labor income taxes instead of education subsidies when part of educational investment is non-verifiable, even under the weak separability condition. However, the authors also conclude that capital income taxes drop to zero as soon as all educational investments are verifiable. The present study demonstrates that even if all educational investments were verifiable, capital income taxation would not become redundant, because of heterogeneous preferences in educational types. The findings are closely related to the model explaining the desirability of capital income taxes based on heterogeneous tastes for goods between high- and low-income earners, which stems from Saez (2002a). However, the extant literature treats differentiation in taste based on initial endowments and discount rates as an assumption (Boadway et al. (2000), Cremer et al. (2001), Diamond and Spinnewijn (2011)). Thus, we present the desirability of capital income taxes by establishing the theoretical foundation that taste differentiation occurs and results from individuals' behavior, without explicitly assuming additional characteristics.

The remainder of this paper is organized as follows. Section 4.2 describes the basic framework of the model. Section 4.3 characterizes and investigates optimal tax policies. Section 4.4 concludes.

4.2 The model

We consider a partial-equilibrium two-period model without uncertainty. The economy consists of two types of individuals who live for two periods, $t = 1, 2$, high-ability and low-ability, indexed by $i = H, L$. The population size is normalized to one. The proportion of high-ability individuals is π_H and the proportion of low-ability individuals is π_L. All individuals are supposed to invest in education. The amount of educational investment for type-i is denoted by q_i, whose price is normalized to one. For example, q_i can be interpreted as years spent in formal education. Let educational investments q_i consist of consumption value and production value. h_i is the share of q_i with consumption value. Correspondingly, $1 - h_i$ is the share of q_i with production value. We assume that h_i is an endogenous variable over $[0, 1]$, that is, individuals can choose any combination of consumption value and production value.

Therefore, $x_i \equiv h_i q_i$ is consumption value, which directly affects utility, and $e_i \equiv (1-h_i)q_i$ is production value, which augments effective labor supply. An individual's preference for type i is defined over consumption in the first period c_i^1, consumption in the second period c_i^2, consumption value in education x_i, and work effort ℓ_i. We assume separability between consumption in the first and second periods, c_i^1 and c_i^2, and work effort ℓ_i, following the type of preferences in Diamond (1998) without income effects, and between work effort ℓ_i and consumption value x_i. Then, type i's preference is expressed by

$$U(c_i^1, c_i^2, x_i, \ell_i) = u(c_i^1, c_i^2, x_i) - v(\ell_i) \tag{4.1}$$

Following conventional assumptions, we assume that $u(\cdot)$ is twice differentiable, strictly concave, and strictly increasing while $v(\cdot)$ is twice differentiable, strictly convex, and strictly increasing.

The accumulation of human capital is given by $g_i = a_i \phi(e_i)$, where a_i is the exogenous ability to benefit from educational investment and $\phi(\cdot)$ is the production function for human capital, where $\phi(\cdot)$ is twice differentiable, strictly increasing, and strictly concave, that is, $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$. We suppose $a_H > a_L$, that is, high-ability individuals can learn more effectively from the same amount of educational investment. The elasticity of the production function for type i is defined as $\eta_i \equiv \frac{e_i}{g_i}\frac{\partial g_i}{\partial e_i}$. In the setting, we obtain $\eta_i = \frac{e_i \phi'(e_i)}{\phi(e_i)}$, which is constant with respect to ability and labor supply.[4]

We denote labor income of type i by $Y_i \equiv g_i \ell_i = a_i \phi(e_i) \ell_i$.[5] The government can observe labor income, capital income, and educational investment for each type, and thus, it can levy non-linear labor income taxes $T(Y_i)$, capital income taxes $\Phi(rs_i)$, and education subsidies $S(q_i)$ for type-i individuals, where r is the interest rate and s_i is the savings of type-i individuals.[6] However, the government cannot distinguish consumption value and production value.

In the first period, individuals with a common level of initial assets s_0 consume and invest

[4] Maldonado (2008) examines education policies under the assumption of complementarity between ability and educational investment. In this setting, the elasticity of the production function can vary with ability. Jacobs and Bovenberg (2011) generalize the model of Maldonado (2008) by allowing for the elasticity of the production function to depend on not only ability but also labor supply.

[5] The interpretation is that Y_i is the product of the wage rate, which is normalized to one, and effective labor supply $g_i \ell_i$. Alternatively, if we consider $\phi(e_i)\ell_i$ as the effective labor supply, we can interpret ability a_i as the wage rate per effective labor supply.

[6] Pirttilä and Tuomala (2001) show that the production efficiency theorem breaks down and capital income taxes are required under endogenous factor prices determined in general equilibrium. In addition, Jacobs (2013) presents the implication of optimal education policies in the presence of the general equilibrium effect, and shows that non-linear education policies play a redistributive role, which leads to the production efficiency theorem breaking down. In the model, we assume no general-equilibrium effects of input prices to clarify our contribution, and therefore, wage rates and interest rates are exogenous.

in education. The first-period budget constraint is given by

$$c_i^1 + s_i + q_i - S(q_i) = s_0 \tag{4.2}$$

In the second period, the individuals consume, work, and consume their assets or repay their debts. The second-period budget constraint is given by

$$c_i^2 = Y_i - T(Y_i) + (1+r)s_i - \Phi(rs_i) \tag{4.3}$$

4.2.1 Individual's behavior

Consider the following individual optimization problem with two stages. First, type-i individuals choose consumption in the first and second periods, c_i^1 and c_i^2, savings s_i, educational investment q_i, and labor income Y_i, given three sorts of tax instruments. Second, q_i is allocated into each return from education, x_i and e_i, by choosing h_i. In the first stage, each type of individual anticipates the outcome of the second stage.

First, we consider the individual's problem in the second stage. Given c_i^1, c_i^2, s_i, q_i, and Y_i, individuals with type i choose h_i to maximize their utility. Formally, the optimization problem is as follows.

$$\max_{h_i} \quad u(c_i^1, c_i^2, x_i) - v(\frac{Y_i}{a_i \phi(e_i)}) \tag{4.4}$$

The first-order condition for type i is

$$v_\ell(\ell_i) \frac{\ell_i \phi'(e_i)}{\phi(e_i)} = u_x(c_i^1, c_i^2, x_i) \tag{4.5}$$

where $u_x(c_i^1, c_i^2, x_i) \equiv \frac{\partial u(c_i^1, c_i^2, x_i)}{\partial x_i}$ denotes the marginal utility of consumption value in education for type i and $v_\ell(\ell_i) \equiv \frac{\partial v(\ell_i)}{\partial \ell_i}$ denotes the marginal disutility of labor for type i. This condition indicates that the marginal utility of consumption value in education should equal that of production value in education. Equation (4.5) yields optimal choice in terms of each value $h_i^* \equiv h^i(c_i^1, c_i^2, q_i, Y_i)$. Note that h_i^* depends on exogenous ability a_i. Let $x_i^* \equiv h_i^* q_i$ be the optimal solution with respect to consumption value in education for type i and $e_i^* \equiv (1-h_i^*)q^i$ be the optimal solution with respect to production value in education for type i. Here, we define $V^i \equiv V^i(c_i^1, c_i^2, q_i, Y_i) \equiv u(c_i^1, c_i^2, x_i^*) - v(\frac{Y_i}{a_i \phi(e_i^*)})$ as the indirect utility for type i.

In the first stage, type-i individuals choose c_i^1, c_i^2, s_i, q_i, and Y_i to maximize their indirect utility in the second stage subject to the individual's budget constraint (equations (4.2) and

(4.3)). This is formally defined as

$$\begin{aligned}
&\max_{c_i^1,c_i^2,s_i,q_i,Y_i} \quad V^i(c_i^1,c_i^2,q_i,Y_i) \\
&s.t. \quad c_i^1 + s_i + q_i - S(q_i) = s_0 \\
&\qquad\quad c_i^2 = Y_i - T(Y_i) + (1+r)s_i - \Phi(rs_i)
\end{aligned} \tag{4.6}$$

This problem yields the first-order conditions:

$$MRS^i_{c^1q} \equiv \frac{u_x(c_i^1,c_i^2,x_i^*)}{u_c^1(c_i^1,c_i^2,x_i^*)} = \frac{v_\ell(\ell_i)}{u_c^1(c_i^1,c_i^2,x_i^*)}\frac{\ell_i\phi'(e_i^*)}{\phi(e_i^*)} = 1 - S'(q_i) \tag{4.7}$$

$$MRS^i_{c^2\ell} \equiv \frac{v_\ell(\ell_i)}{a_i\phi(e_i^*)u_c^2(c_i^1,c_i^2,x_i^*)} = 1 - T'(Y_i) \tag{4.8}$$

$$MRS^i_{c^1c^2} \equiv \frac{u_c^1(c_i^1,c_i^2,x_i^*)}{u_c^2(c_i^1,c_i^2,x_i^*)} = 1 + r - r\Phi'(rs_i) \tag{4.9}$$

where $u_c^1(c_i^1,c_i^2,x_i) \equiv \frac{\partial u(c_i^1,c_i^2,x_i)}{\partial c_i^1}$ denotes marginal utility of consumption in the first period, $u_c^2(c_i^1,c^2,x_i) \equiv \frac{\partial u(c_i^1,c^2,x_i)}{\partial c_i^2}$ that in the second period, $S'(q_i) \equiv \frac{\partial S(q_i)}{\partial q_i}$ the marginal subsidy rate for education, $T'(Y_i) \equiv \frac{\partial T(Y_i)}{\partial Y_i}$ the marginal labor income tax rate, and $\Phi'(rs_i) \equiv \frac{\partial \Phi'(rs_i)}{\partial rs_i}$ the marginal capital income tax rate. Combining equations (4.7), (4.8), and (4.9) yields

$$MRT^i_{q\ell} \equiv \frac{Y_i\phi'(e_i^*)}{\phi(e_i^*)} = \frac{1-S'(q_i)}{1-T'(Y_i)}(1 + r - r\Phi'(rs_i)) \tag{4.10}$$

To measure the extent to which the tax (subsidy) instruments decrease (increase) the marginal returns to learning, we denote the total net tax wedge on learning for type i by

$$\begin{aligned}
\Delta_i &\equiv T'(\cdot)\frac{Y_i\phi'(e_i^*)}{\phi(e_i^*)} - r\Phi'(\cdot) - S'(\cdot)(1 + r - r\Phi'(\cdot)) \\
&= \frac{T'(\cdot)}{1-T'(\cdot)}R(1-S'(\cdot)) - r\Phi'(\cdot) - S'(\cdot)R
\end{aligned} \tag{4.11}$$

where $R \equiv 1 + r - r\Phi'(\cdot)$ is the discount factor.[7] The equality is derived using equation (4.10). From equation (4.11), while labor income taxes distort decision-making in terms of

[7] Compared to the model of Bovenberg and Jacobs (2005) in the presence of non-pecuniary benefit, Δ_i includes the distortion on consumption benefit from education caused by labor income taxes. In their model, optimal education subsidies fall compared to labor income tax in order to restore production efficiency, because non-pecuniary benefit can escape the distortion caused by labor income tax. On the other hand, if individuals can choose any combination of non-pecuniary benefit and pecuniary benefit, non-pecuniary benefit cannot escape from the distortion, since the distortion on the choice for production value in education caused by labor income tax indirectly affects the choice for consumption value in education, as presented in equation (4.5).

education, capital income taxes and education subsidies alleviate the distortion caused by labor income taxes. In particular, capital income taxes act as a subsidy for education by raising the present value of the marginal benefit of education. If $\Delta_i = 0$, the intertemporal marginal rate of transformation between education and labor supply for type i $MRT^i_{q\ell}$ equals $1+r$, that is, equation (4.10) coincides with the first-order condition without any tax policy. As in the previous literature, if we consider education and labor supply as two inputs in the household production problem, in the situation of $\Delta_i = 0$, the result of Diamond and Mirrlees (1971) applies, that is, the government should ensure efficiency in the production side of the economy. Jacobs and Bovenberg (2010) show that if all educational investments are verifiable, that is, can be subsidized, then education subsidies eliminate the entire distortion on education due to labor income taxes without levying capital income taxes, which implies that the production efficiency theorem of Diamond and Mirrlees (1971) in addition to the theorem of Atkinson and Stiglitz (1976) are desirable. Our concern is whether both these theorems are robust, even if individuals possess endogenous choice of educational type.

4.2.2 The government

The objective of the government is to maximize the sum of indirect utility for type i, which is expressed by

$$W = \pi_H V^H(c^1_H, c^2_H, q_H, Y_H) + \pi_L V^L(c^1_L, c^2_L, q_L, Y_L) \tag{4.12}$$

The government levies a non-linear tax on labor income and capital income to subsidize human capital investment. The budget constraint of the government takes the following form:

$$\sum_{i=H,L} \pi_i \left[-S(q_i) + \frac{1}{1+r}(T(Y_i) + \Phi(rs_i)) \right] = 0 \tag{4.13}$$

Using the budget constraint that individuals face, equation (4.13) can be rewritten as

$$\sum_{i=H,L} \pi_i \left[s_0 - c^1_i - q_i + \frac{1}{1+r}(Y_i - c^2_i) \right] = 0 \tag{4.14}$$

The informational assumptions are in line with the optimal taxation literature analyzing the second-best allocation: the government cannot directly observe labor supply and ability. By the revelation principle, the government must design the allocation to induce individuals to reveal their true types. We focus on the case in which the incentive constraint preventing high-ability individuals from mimicking low-ability individuals is binding. Therefore, the incentive constraint is

$$V^H(c^1_H, c^2_H, q_H, Y_H) \geq V^H(c^1_L, c^2_L, q_L, Y_L) \tag{4.15}$$

Note that $\hat{V} \equiv V^H(c_L^1, c_L^2, q_L, Y_L)$ denotes the indirect utility that high-ability individuals (mimickers) obtain when choosing the allocation of low-ability individuals (the person being mimicked). Given c_L^1, c_L^2, q_L, and Y_L, this is formally defined as

$$V^H(c_L^1, c_L^2, q_L, Y_L) \equiv \max_{\hat{h}} u(c_L^1, c_L^2, \hat{x}) - v(\frac{Y_L}{a_H \phi(\hat{e})}) \tag{4.16}$$

where, $\hat{x} \equiv \hat{h} q_L$ denotes the consumption value for the mimicker and $\hat{e} \equiv (1 - \hat{h}) q_L$ the production value for the mimicker. The first-order condition is as follows.

$$v_\ell(\hat{\ell}) \frac{\hat{\ell}\phi'(\hat{e})}{\phi(\hat{e})} = u_x(c_L^1, c_L^2, \hat{x}) \tag{4.17}$$

Let $\hat{\ell} \equiv \frac{Y_L}{w_H a_H \phi(\hat{e})}$ be the labor supply of the mimicker. Equation (4.17) yields the optimal choice in terms of each value for mimickers, denoted by $\hat{h}^* \equiv h^H(c_L^1, c_L^2, q_L, Y_L)$. We define $\hat{x}^* \equiv \hat{h}^* q_L$ as optimal solution with respect to consumption value in education for mimickers and $\hat{e}^* \equiv (1 - \hat{h}^*) q_L$ as the optimal solution with respect to production value in education for mimickers.

In summary, the government maximizes the social welfare function (4.12) subject to the government's budget constraint (4.14) and the incentive constraint (4.15) by choosing the allocation with respect to consumption in the first and second periods, educational investment, and labor income for each type. The corresponding Lagrangian is

$$\begin{aligned} \max_{\{c_i^1, c_i^2, q_i, Y_i\}_i} \mathcal{L} = {} & \pi_H V^H(c_H^1, c_H^2, q_H, Y_H) + \pi_L V^L(c_L^1, c_L^2, q_L, Y_L) \\ & + \lambda \Big[V^H(c_H^1, c_H^2, q_H, Y_H) - V^H(c_L^1, c_L^2, q_L, Y_L) \Big] \\ & + \gamma \Big[\sum_{i=H,L} \pi_i \{ s_0 - c_i^1 - q_i + \frac{1}{1+r}(Y_i - c_i^2) \} \Big] \end{aligned} \tag{4.18}$$

Let γ be the Lagrange multiplier of the government's budget constraint and λ the Lagrange multiplier of the incentive constraint.

4.3 Optimal tax policy

From the first-order conditions with respect to equation (4.18), we characterize the optimal marginal subsidy rate on education and the optimal marginal labor income and capital income tax rate for each type (Appendix 4.A):

$$S'(q_H) = 0 \tag{4.19}$$

$$\begin{aligned} S'(q_L) &= \frac{\lambda u_c^1(c_L^1, c_L^2, \hat{x}^*)}{\gamma \pi_L}\left[\frac{u_x(c_L^1, c_L^2, x_L^*)}{u_c^1(c_L^1, c_L^2, x_L^*)} - \frac{u_x(c_L^1, c_L^2, \hat{x}^*)}{u_c^1(c_L^1, c_L^2, \hat{x}^*)}\right] \\ &\equiv \frac{\lambda u_c(c_L, \hat{x}^*)}{\gamma \pi_L}\left[MRS_{c^1q}^L - M\hat{R}S_{c^1q}\right] \end{aligned} \tag{4.20}$$

$$T'(Y_H) = 0 \tag{4.21}$$

$$\begin{aligned} \frac{T'(Y_L)}{1+r} &= \frac{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}{\gamma \pi_L}\left[\frac{v_\ell(\ell_L)}{u_c^2(c_L^1, c_L^2, x_L^*)}\frac{1}{a_L \phi(e_L^*)} - \frac{v_\ell(\hat{\ell})}{u_c^2(c_L^1, c_L^2, \hat{x}^*)}\frac{1}{a_H \phi(\hat{e}^*)}\right] \\ &\equiv \frac{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}{\gamma \pi_L}\left[MRS_{c^2\ell}^L - M\hat{R}S_{c^2\ell}\right] \end{aligned} \tag{4.22}$$

$$\Phi'(rs_H) = 0 \tag{4.23}$$

$$\begin{aligned} \frac{\Phi'(rs_L)}{1+r} &= \frac{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}{r\gamma \pi_L}\left[\frac{u_c^1(c_L^1, c_L^2, x_L^*)}{u_c^2(c_L^1, c_L^2, x_L^*)} - \frac{u_c^1(c_L^1, c_L^2, \hat{x}^*)}{u_c^2(c_L^1, c_L^2, \hat{x}^*)}\right] \\ &\equiv \frac{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}{r\gamma \pi_L}\left[MRS_{c^1c^2}^L - M\hat{R}S_{c^1c^2}\right] \end{aligned} \tag{4.24}$$

From these results, we clarify our concerns about (i) the sign of $\Phi'(\cdot)$, that is, the justification of capital income taxes, and (ii) the sign of Δ_i, that is, the direction of the overall distortion on education induced by the three sorts of tax instruments. Before investigating these concerns, we present the following lemma.

Lemma 4.1. *The production value of education for low-type individuals is greater than that of mimicker, that is, $e_L^* > \hat{e}^*$. On the other hand, the consumption value of education for low-type individuals is lower than mimicker's one, that is $x_L^* < \hat{x}^*$.*

This proof is shown in Appendix 4.B. This result stems from the fact that mimickers with higher productivity can earn labor income for low-type individuals with less production value in education. Therefore, mimickers allocate educational investment into consumption value in education more than low-type individuals.

4.3.1 Optimal capital income taxation

First, we investigate the desirability of capital income taxes. Whereas the marginal capital income tax rate is zero for high-type individuals from equation (4.23), the situation is different for low-type individuals. Note that deviating from the Atkinson–Stiglitz theorem crucially relies on the sign of the bracket on the right-hand side of equation (4.24), which is determined by the complementarity of consumption in each period with consumption value in education. For example, education can affect the level of consumption such as books, computers, and tobacco. Let us consider the situation in which consumption in the second period is more complementary to the consumption value than consumption in the first period. In this case, the bracket is positive, which means that $\Phi'(\cdot)$ is positive. Therefore, the Atkinson–Stiglitz theorem breaks down, as capital income taxation is not redundant. The intuition is as follows: since the mimicker values consumption in the second period more than the mimicked because of $MRS^L_{c^1c^2} > M\hat{R}S_{c^1c^2}$, imposing capital income taxes hurts the mimicker more than the mimicked, and thus, the government relaxes the incentive constraint for high-type individuals.

According to lemma 4.1, the justification of capital income taxes stems from heterogeneous preferences for educational types between high- and low-income earners, which are endogenously generated. The heterogeneous preferences allow the intertemporal marginal rate of substitution to vary between the mimicker and the mimicked, and thus, gives additional information to relax the incentive constraint to the government. In contrast to Saez (2002a), the findings of our study show the desirability of capital income taxes under a case in which individuals differ in a single dimension. The following proposition summarizes the main results of this section.

Proposition 4.1. *When the consumption value in education is more (less) complementary to consumption in the second period than in the first, the marginal capital income tax rate is positive (negative) for low-type individuals and zero for high-type individuals.*

However, the Atkinson–Stiglitz theorem can be restored if we assume that preferences are weakly separable in the sense of the following functional form: $u(c^1_L, c^2_L, x) = u(f(c^1_L, c^2_L), x)$. In that case, the sign of the bracket is zero, owing to no impact of heterogeneous tastes on the intertemporal marginal rate of substitution. Therefore, capital income taxes are no longer required.

4.3.2 Production inefficiency

The next item of interest is examination of the sign of Δ_i, that is, the direction of the overall distortion on education caused by the three kinds of tax instruments used in this study. To observe this, we combine equations (4.20), (4.22), and (4.24), which yields (Appendix 4.C):

$$\begin{aligned}\frac{\Delta_L}{1+r} &= \frac{\lambda u_x(c^1_L, c^2_L, \hat{x}^*)}{\gamma \pi_L}\left[\frac{Y_L\phi'(\hat{e}^*)}{\phi(\hat{e}^*)} - \frac{Y_L\phi'(e^*_L)}{\phi(e^*_L)}\right] \\ &\equiv \frac{\lambda u_x(c^1_L, c^2_L, \hat{x}^*)}{\gamma \pi_L}\left[M\hat{R}T_{q\ell} - MRT^L_{q\ell}\right]\end{aligned} \tag{4.25}$$

In contrast to Bovenberg and Jacobs (2005), when individuals have education choice between consumption value and production value, the novel term appears even if the utility function is separable between work effort and consumption value.[8][9] The deviation from household production efficiency crucially depends on the sign of the bracket on the right-hand side of equation (4.25), which creates an informational advantage for the government. From lemma 4.1, the intertemporal marginal rate of transformation between education and labor supply for the mimicker is greater than that for low-type individuals, that is, we obtain $M\hat{R}T_{q\ell} > MRT^L_{q\ell}$.[10] Therefore, $\Delta_L > 0$ is optimal, which means that education subsidies for low-type individuals should not completely offset the distortions of labor income taxes that are alleviated (augmented) by positive (negative) capital income taxes. The intuition is that distortions in learning for low-type individuals damages the mimicker more than the mimicked, and thereby relaxes the binding incentive constraint, since the mimicker prefers education to labor supply measured by the present value relative to low-type individuals. It then follows that the Diamond–Mirrlees production efficiency theorem breaks down. From equation (4.19), for high-type individuals, education subsidies are redundant, since learning is not distorted by labor and capital income taxes from equations (4.21) and (4.23), which implies $\Delta_H = 0$. In summary, we propose as follows.

Proposition 4.2. *Low-ability types face a downward distortion on learning, that is, Δ_L is positive; high-ability types face no distortion on learning, that is, $\Delta_H = 0$.*

The main result also stems from heterogeneous tastes in educational types from lemma 4.1. As the bracket of equation (4.25) shows, the sign of the bracket is positive as long as heterogeneity occurs. Therefore, if individuals differ in exogenous skill ability and the incentive constraint is binding, then production inefficiency is always desirable in our model.

4.3.3 Optimal labor income taxation and education subsidies

Finally, we check the sign of $S'(q_i)$ and $T'(Y_i)$. For high-type individuals, both marginal subsidy and tax rate are zero from equations (4.19) and (4.21). In other words, no distortion for

[8] As mentioned in footnote 7, labor income taxes distort decision-making in terms of educational choice between consumption and production value. Consequently, consumption benefit in education cannot escape from the distortion induced by labor income taxes. Equation (4.25) implies that education subsidies should offset the distortion on the benefit. Thus, the first term in the bracket of equation (4.43) in the model of Bovenberg and Jacobs (2005) disappears.

[9] Under non-separability between work effort and consumption value, education subsidies increase or decrease to mitigate the distortion stemming from redistributive taxes, depending on the complementarity between them (see Bovenberg and Jacobs (2005)).

[10] The first derivative of $\frac{\phi'(e)}{\phi(e)}$ with respect to e is

$$\frac{\partial \frac{\phi'(e)}{\phi(e)}}{\partial e} = \frac{\phi''(e)}{\phi(e)} - \left(\frac{\phi'(e)}{\phi(e)}\right)^2$$

From the concavity of $\phi(\cdot)$, it is negative. Thus, we can obtain $M\hat{R}T_{q\ell} > MRT^L_{q\ell}$ from lemma 4.1.

the top result holds. On the other hand, the sign of $S'(q_L)$ and $T'(Y_L)$ for low-type individuals depends on the complementarity or substitution between consumption and consumption value. First, we take the marginal subsidy rate on education for low-type individuals in equation (4.20). Using lemma 4.1, if consumption value is complementary to or has no relationship with consumption in the first period, that is, $u_{c^1x} \geq 0$, then the sign of $S(q_L)$ is positive which means that education is subsidized.[11] This is because the single-crossing property holds, in other words, the marginal rate of substitution between consumption and consumption value in education for the mimicker is lower than that for low-type individuals. The intuition of positive $S'(q_L)$ is that the mimicker prefers consumption value over education, and therefore, distorting educational efforts downward can relax the incentive constraint. On the other hand, if consumption value in education is substitute for consumption, that is, $u_{c^1x} < 0$, then $S'(q_L)$ cannot be signed, since the single-crossing property does not hold. Second, we take the marginal tax rate on labor income for low-type individuals in equation (4.22). Following the standard result in a two-class economy suggested by Stiglitz (1982), the marginal labor income tax rate is positive for low-type individuals if a single-crossing condition holds. However, as for the discussion on marginal subsidy rates, it is ambiguous whether the single-crossing condition holds in our model. To observe this, we rewrite the marginal income tax rate for low-type individuals by substituting equations (4.5) and (4.17) into (4.22), as follows:

$$\begin{aligned}\frac{T'(Y_L)}{1+r} &= \frac{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}{\gamma \pi_L}\left[\frac{u_x(c_L^1, c_L^2, x_L^*)}{u_c^2(c_L^1, c_L^2, x_L^*)}\frac{\phi(e_L^*)}{Y_L \phi'(e_L^*)} - \frac{u_x(c_L^1, c_L^2, \hat{x}^*)}{u_c^2(c_L^1, c_L^2, \hat{x}^*)}\frac{\phi(\hat{e}^*)}{Y_L \phi'(\hat{e}^*)}\right] \\ &= \frac{\lambda u_c(c_L^1, c_L^2, \hat{x}^*)}{\gamma \pi_L}\left[\frac{MRS_{cq}^L}{MRT_{q\ell}} - \frac{M\hat{R}S_{cq}}{M\hat{R}T_{q\ell}}\right]\end{aligned} \tag{4.26}$$

From the fact $M\hat{R}T_{q\ell} > MRT_{q\ell}$, the sign of $T'(Y_L)$ crucially depends on the sign of u_{c^2x}. If consumption value in education is complementary to or has no relationship with consumption in the second period, that is, $u_{c^2x} \geq 0$, $M\hat{R}S_{cq}$ is less than MRS_{cq} which leads to the conclusion that the single-crossing property holds. This case means that the marginal labor income tax rate is positive at the bottom. However, if consumption value in education is a substitute for consumption, that is, $u_{c^2x} < 0$, then $T'(Y_L)$ cannot be signed, since the single-crossing condition does not hold. As a result, the marginal income tax rate is not necessarily positive for low-type individuals. Thus, we summarize the sign of the marginal subsidy rate on education and the marginal tax rate on labor income as follows.

[11] The first derivative of the marginal rate of substitution between consumption in the first period and consumption value is

$$\frac{\partial \frac{u_x(c^1,c^2,x)}{u_c^1(c^1,c^2,x)}}{\partial x} = \frac{u_{xx}}{u_c^1} - \frac{u_x}{u_c^1}u_{c^1x}$$

If $u_{c^1x} \geq 0$, it is a decreasing function with respect to consumption value. Thus, the sign of the bracket on the right-hand side in equation (4.20) is positive, which means $S'(q_L) > 0$.

Proposition 4.3. *For low-type individuals, when consumption value in education is complementary to or has no relationship with consumption in the first and second periods, both the marginal subsidy rate on education and the marginal tax rate on labor income are positive, and otherwise, the sign of either or both of them is ambiguous. For high-type individuals, both marginal education subsidy and labor income tax rate are zero.*

As a result, a sufficient condition to hold each single-crossing property satisfies a complementary relationship between consumption value in education and consumption in the first and second periods, that is, $u_{c^1x} \geq 0$ and $u_{c^2x} \geq 0$. For example, the sub-utility function is the constant elasticity of substitution (CES) form: $u(c_i^1, c_i^2, x_i) = (\alpha(c_i^1)^{-\rho} + \beta(c_i^2)^{-\rho} + \delta x_i^{-\rho})^{-\frac{1}{\rho}}$ with $\alpha + \beta + \delta = 1$ and $\rho \geq -1$. In this case, the marginal subsidy rate on education and labor income tax rate for low-type individuals are always positive, because $u_{c^1x} \geq 0$ and $u_{c^2x} \geq 0$ regardless of the level of ρ.[12]

4.4 Concluding Remarks

This study examines optimal human capital policies under non-linear labor and capital income taxes when education has consumption value and production value, and individuals can choose an educational type. The former value generates a direct utility gain and the latter value promotes effective labor supply. Since the government can observe labor income, capital income, and educational investment, but is unable to distinguish the two types of returns from education, it can implement non-linear labor and capital income taxes and subsidies for education. To the best of our knowledge, no previous study characterizes optimal education policies under non-linear income tax instruments when individuals have endogenous choice of educational types.

Under endogenous choice of educational types, the optimal tax policies are modified in this study. First, we show that capital income taxation can be necessary for low-type individuals, even when individuals differ in a single dimension; this result is in contrast to several studies that have highlighted how the theorem of Atkinson and Stiglitz (1976) breaks down when individuals differ along more than one dimension. Second, the direction of the overall distortion on learning induced by the three sorts of tax instruments shifts down, which means that the production efficiency theorem of Diamond and Mirrlees (1971) fails. The two novel findings stem from the preference heterogeneity in education between the mimicker and the mimicked, which is endogenously generated. The additional information is

[12] The cross-derivative of the sub-utility function is

$$u_{c^1x} = \alpha\delta(1+\rho)(\alpha(c_i^1)^{-\rho} + \beta(c_i^2)^{-\rho} + \delta x_i^{-\rho})^{-\frac{1}{\rho}-2}(c_i^1 x_i)^{-\rho-1}$$

Therefore, it is non-negative for any $\rho \geq -1$. Using a similar method, we obtain $u_{c^2x} \geq 0$.

useful to relax the incentive constraint and thus, the deviation from the tax policy attaining efficiency is optimal. On the other hand, the result of no distortion at the top remains. The present study highlights the importance of recognizing individuals' choice of educational types when implementing education policies. However, note that traditional results remain if the government can observe the share of educational investment with consumption value (see Appendix 4.D).

4.5 Appendix

Appendix 4.A

Using the envelope theorem, we obtain the following results:

$$\frac{\partial V^i}{\partial c_i} = u_c(c_i^1, c_i^2, x_i^*) \tag{4.A.1}$$

$$\frac{\partial \hat{V}}{\partial c_L} = u_c(c_L^1, c_L^2, \hat{x}^*) \tag{4.A.2}$$

$$\frac{\partial V^i}{\partial q_i} = u_x(c_i^1, c_i^2, x_i^*) \tag{4.A.3}$$

$$\frac{\partial \hat{V}}{\partial q_L} = u_x(c_L^1, c_L^2, \hat{x}^*) \tag{4.A.4}$$

$$\frac{\partial V^i}{\partial Y_i} = \frac{v_\ell(\ell_i)}{a_i \phi(e_i^*)} \tag{4.A.5}$$

$$\frac{\partial \hat{V}}{\partial Y_L} = \frac{v_\ell(\hat{\ell})}{a_H \phi(\hat{e}^*)} \tag{4.A.6}$$

Using (4.A.1)–(4.A.6), the first-order conditions associated with c_i^1, c_i^2, q_i, and Y_i, $i = H, L$, are as follows:

$$\frac{\partial \mathcal{L}}{\partial c_L^1} = \pi_L u_c^1(c_L^1, c_L^2, x_L^*) - \lambda u_c^1(c_L^1, c_L^2, \hat{x}^*) - \gamma \pi_L = 0 \tag{4.A.7}$$

$$\frac{\partial \mathcal{L}}{\partial c_H^1} = \pi_H u_c^1(c_H^1, c_H^2, x_H^*) + \lambda u_c^1(c_H^1, c_H^2, x_H^*) - \gamma \pi_H = 0 \tag{4.A.8}$$

$$\frac{\partial \mathcal{L}}{\partial c_L^2} = \pi_L u_c^2(c_L^1, c_L^2, x_L^*) - \lambda u_c^2(c_L^1, c_L^2, \hat{x}^*) - \frac{1}{1+r}\gamma \pi_L = 0 \tag{4.A.9}$$

$$\frac{\partial \mathcal{L}}{\partial c_H^2} = \pi_H u_c^2(c_H^1, c_H^2, x_H^*) + \lambda u_c^2(c_H^1, c_H^2, x_H^*) - \frac{1}{1+r}\gamma \pi_H = 0 \tag{4.A.10}$$

$$\frac{\partial \mathcal{L}}{\partial q_L} = \pi_L u_x(c_L^1, c_L^2, x_L^*) - \lambda u_x(c_L^1, c_L^2, \hat{x}^*) - \gamma \pi_L = 0 \tag{4.A.11}$$

$$\frac{\partial \mathcal{L}}{\partial q_H} = \pi_H u_x(c_H^1, c_H^2, x_H^*) + \lambda u_x(c_H^1, c_H^2, x_H^*) - \gamma\pi_H = 0 \tag{4.A.12}$$

$$\frac{\partial \mathcal{L}}{\partial Y_L} = -\pi_L v_\ell(\ell_L)\frac{1}{a_L\phi(e_L^*)} + \lambda v_\ell(\hat{\ell})\frac{1}{a_H\phi(\hat{e}^*)} + \frac{1}{1+r}\gamma\pi_L = 0 \tag{4.A.13}$$

$$\frac{\partial \mathcal{L}}{\partial Y_H} = -\pi_H v_\ell(\ell_H)\frac{1}{a_H\phi(e_H^*)} - \lambda v_\ell(\ell_H)\frac{1}{a_H\phi(e_H^*)} + \frac{1}{1+r}\gamma\pi_H = 0 \tag{4.A.14}$$

First, we derive the marginal subsidy rate on education at the optimum. Combining (4.A.7) with (4.A.11) yields

$$\pi_L\{u_x(c_L^1, c_L^2, x_L^*) - u_c^1(c_L^1, c_L^2, x_L^*)\} = \lambda\{u_x(c_L^1, c_L^2, \hat{x}^*) - u_c^1(c_L^1, c_L^2, \hat{x}^*)\} \tag{4.A.15}$$

Rearranging (4.A.15) and substituting equation (4.7) yields

$$\{\pi_L u_c^1(c_L^1, c_L^2, x_L^*) - \lambda u_c^1(c_L^1, c_L^2, \hat{x}^*)\}S'(q_L) = \lambda u_c^1(c_L^1, c_L^2, \hat{x}^*)\left[\frac{u_x(c_L^1, c_L^2, x_L^*)}{u_c(c_L^1, c_L^2, x_L^*)} - \frac{u_x(c_L^1, c_L^2, \hat{x}^*)}{u_c(c_L^1, c_L^2, \hat{x}^*)}\right] \tag{4.A.16}$$

Substituting (4.A.7) into the term in the brackets of the left-hand side, we obtain equation (4.20). Similarly, combining (4.A.8) with (4.A.12) yields

$$\pi_H\{u_x(c_H^1, c_H^2, x_H^*) - u_c^1(c_H^1, c_H^2, x_H^*)\} = -\lambda\{u_x(c_H^1, c_H^2, x_H^*) - u_c^1(c_H^1, c_H^2, x_H^*)\} \tag{4.A.17}$$

Using equation (4.7), this can be rewritten as follows:

$$(\pi_H + \lambda)u_c^1(c_H^1, c_H^2, x_H^*)S'(q_H) = 0 \tag{4.A.18}$$

From (4.A.8), $\pi_H + \lambda$ is positive, which implies that $S'(q_H)$ is zero.

Second, we turn to the derivation of the optimal marginal labor income tax rate. Combining (4.A.9) with (4.A.13) yields

$$\pi_L\left[u_c^2(c_L^1, c_L^2, x_L^*) - v_\ell(\ell_L)\frac{1}{a_L\phi(e_L^*)}\right] = \lambda\left[u_c(c_L^1, c_L^2, \hat{x}^*) - v_\ell(\hat{\ell})\frac{1}{a_H\phi(\hat{e}^*)}\right] \tag{4.A.19}$$

Rearranging (4.A.19) and then substituting equation (4.8) yields

$$\begin{aligned}&\{\pi_L u_c^2(c_L^1, c_L^2, x_L^*) - \lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)\}T'(Y_L)\\ &\quad = \lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)\left[\frac{v_\ell(\ell_L)}{u_c^2(c_L^1, c_L^2, x_L^*)}\frac{1}{a_L\phi(e_L^*)} - \frac{v_\ell(\hat{\ell})}{u_c^2(c_L^1, c_L^2, \hat{x}^*)}\frac{1}{a_H\phi(\hat{e}^*)}\right]\end{aligned} \tag{4.A.20}$$

Substituting (4.A.9) into the term in the brackets of the left-hand side, we obtain equation

(4.22). Similarly, combining (4.A.10) with (4.A.14) yields

$$\pi_H\left[u_c^2(c_H^1, c_H^2, x_H^*) - v_\ell(\ell_H)\frac{1}{a_H\phi(e_H^*)}\right] = -\lambda\left[u_c^2(c_H^1, c_H^2, x_H^*) - v_\ell(\ell_H)\frac{1}{a_H\phi(e_H^*)}\right] \quad (4.A.21)$$

Using equation (4.8), (4.A.21) can be rewritten as follows:

$$(\pi_H + \lambda)u_c^2(c_H^1, c_H^2, x_H^*)T'(Y_H) = 0 \quad (4.A.22)$$

From (4.A.10), $\pi_H + \lambda$ is positive, which implies that $T'(Y_H)$ is zero.

Finally, we derive the optimal marginal capital income tax rate. Combining (4.A.7) with (4.A.9) yields

$$\pi_L\{u_c^1(c_L^1, c_L^2, x_L^*) - (1+r)u_c^2(c_L^1, c_L^2, x_L^*)\} = \lambda\{u_c^1(c_L^1, c_L^2, \hat{x}^*) - (1+r)u_c^2(c_L^1, c_L^2, \hat{x}^*)\} \quad (4.A.23)$$

Rearranging (4.A.23) and substituting equation (4.9) yields

$$\{\pi_L u_c^2(c_L^1, c_L^2, x_L^*) - \lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)\}r\Phi'(rs_L) = \lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)\left[\frac{u_c^1(c_L^1, c_L^2, x_L^*)}{u_c^2(c_L^1, c_L^2, x_L^*)} - \frac{u_c^1(c_L^1, c_L^2, \hat{x}^*)}{u_c^2(c_L^1, c_L^2, \hat{x}^*)}\right] \quad (4.A.24)$$

Substituting (4.A.9) into the term in the brackets of the left-hand side, we obtain equation (4.24). Similarly, combining (4.A.8) with (4.A.10) yields

$$\pi_H\{u_c^1(c_H^1, c_H^2, x_H^*) - (1+r)u_c^2(c_H^1, c_H^2, x_H^*)\} = -\lambda\{u_c^1(c_H^1, c_H^2, x_H^*) - (1+r)u_c^2(c_H^1, c_H^2, x_H^*)\} \quad (4.A.25)$$

Using equation (4.9), this can be rewritten as follows:

$$(\pi_H + \lambda)u_c^2(c_H^1, c_H^2, x_H^*)r\Phi'(rs_H) = 0 \quad (4.A.26)$$

From (4.A.8), $\pi_H + \lambda$ is positive, which implies that $\Phi'(rs_H)$ is zero.

$\square$

Appendix 4.B

Consider low-type individual's optimization problem in the second stage. Given Y_L, c_L^1, c_L^2, s_L, and q_L, the first-order condition with respect to h_L is given by

$$\frac{\partial \mathcal{L}}{\partial h_L} = u_x(c_L^1, c_L^2, x_L)q_L - v_\ell(\frac{Y_L}{a_L\phi(e_L)})\frac{Y_L}{a_L\phi(e_L)}\frac{\phi'(e_L)}{\phi(e_L)}q_L = 0 \quad (4.B.1)$$

Moreover, the second-order condition with respect to h_L is as follows:

$$\begin{aligned}\frac{1}{q_L^2}\frac{\partial^2\mathcal{L}}{\partial h_L^2} &= u_{xx}(c_L^1, c_L^2, x_L) - v_{\ell\ell}(\frac{Y_L}{a_L\phi(e_L)})\left(\frac{Y_L}{a_L\phi(e_L)}\frac{\phi'(e_L)}{\phi(e_L)}\right)^2 \\ &- 2v_\ell(\frac{Y_L}{a_L\phi(e_L)})\frac{Y_L}{a_L\phi(e_L)}\left(\frac{\phi'(e_L)}{\phi(e_L)}\right)^2 + v_\ell(\frac{Y_L}{a_L\phi(e_L)})\frac{Y_L}{a_L\phi(e_L)}\frac{\phi''(e_L)}{\phi(e_L)} < 0\end{aligned} \tag{4.B.2}$$

Therefore, h_L^* is a locally maximized solution, because the second-order condition is satisfied from the assumption of $u(\cdot)$, $v(\cdot)$, and $\phi(\cdot)$ on the curvature. Now, we present the comparative statics of an individual's behavior due to the change of a_L. From equation (4.B.1), we derive it as follows:

$$\begin{aligned}\frac{\partial h^L}{\partial a_L}\frac{\partial^2\mathcal{L}}{\partial h_L^2} &= -v_{\ell\ell}(\frac{Y_L}{a_L\phi(e_L)})\left(\frac{Y_L}{a_L\phi(e_L)}\right)^2\frac{\phi'(e_L)}{\phi(e_L)}\frac{1}{a_L}q_L \\ &- v_\ell(\frac{Y_L}{a_L\phi(e_L)})\frac{Y_L}{(a_L)^2\phi(e_L)}\frac{\phi'(e_L)}{\phi(e_L)}q_L\end{aligned} \tag{4.B.3}$$

Since the sign of the sum of the two terms on the right-hand side is negative, the sign of $\frac{\partial h^L}{\partial a_L}$ is positive. Here, note that the mimicker faces the same allocations in the second stage. Thus, we can conclude that $x_L^* < \hat{x}^*$ because of $a_H > a_L$. In addition, we obtain $e_L^* > \hat{e}^*$. $\square$

Appendix 4.C

Using equations (4.5) and (4.17), equation (4.20) can be rewritten as follows:

$$S'(q_L) = \frac{\lambda u_c^1(c_L^1, c_L^2, \hat{x}^*)}{\gamma\pi_L}\left[\frac{v_\ell(\ell_L)}{u_c^1(c_L^1, c_L^2, x_L^*)}\frac{\ell_L\phi'(e_L^*)}{\phi(e_L^*)} - \frac{v_\ell(\hat{\ell})}{u_c^1(c_L^1, c_L^2, \hat{x}^*)}\frac{\hat{\ell}\phi'(\hat{e}^*)}{\phi(\hat{e}^*)}\right] \tag{4.C.1}$$

By the definition of ℓ and $\hat{\ell}$,

$$S'(q_L) = \frac{\lambda u_c^1(c_L^1, c_L^2, \hat{x}^*)}{\gamma\pi_L}\frac{v_\ell(\ell_L)}{u_c^1(c_L^1, c_L^2, x_L^*)}\frac{Y_L}{a_L\phi(e_L^*)}\frac{\phi'(e_L^*)}{\phi(e_L^*)} - \frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\frac{Y_L\phi'(\hat{e}^*)}{\phi(\hat{e}^*)} \tag{4.C.2}$$

Here, we rearrange equation (4.22) as follows:

$$\frac{\lambda u_c^1(c_L^1, c_L^2, \hat{x}^*)}{\gamma\pi_L}\frac{v_\ell(\ell_L)}{u_c^1(c_L^1, c_L^2, x_L^*)}\frac{1}{a_L\phi(e_L^*)} = \Gamma\left[\frac{T'(Y_L)}{1+r} + \frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\right] \tag{4.C.3}$$

where, $\Gamma \equiv \frac{u_c^2(c_L^1, c_L^2, x_L^*)}{u_c^1(c_L^1, c_L^2, x_L^*)}\frac{u_c^1(c_L^1, c_L^2, \hat{x}^*)}{u_c^2(c_L^1, c_L^2, \hat{x}^*)}$

Substituting (4.C.3) into the first term of (4.C.2),

$$S'(q_L) = -\frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\frac{Y_L\phi'(\hat{e}^*)}{\phi(\hat{e}^*)} + \frac{Y_L\phi'(e_L^*)}{\phi(e_L^*)}\Gamma\left[\frac{T'(Y_L)}{1+r} + \frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\right] \tag{4.C.4}$$

On the other hand, we rearrange equation (4.24) as follows:

$$\Gamma = 1 - \frac{r\gamma\pi_L}{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}\frac{u_c^2(c_L^1, c_L^2, x_L^*)}{u_c^1(c_L^1, c_L^2, x_L^*)}\frac{\Phi'(rs_L)}{1+r} \tag{4.C.5}$$

Substituting (4.C.5) into (4.C.4) yields

$$\begin{aligned} S'(q_L) = &-\frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\left[\frac{Y_L\phi'(\hat{e}^*)}{\phi(\hat{e}^*)} - \frac{Y_L\phi'(e_L^*)}{\phi(e_L^*)}\right] + \frac{Y_L\phi'(e_L^*)}{\phi(e_L^*)}\frac{T'(Y_L)}{1+r} \\ &- \frac{Y_L\phi'(e_L^*)}{\phi(e_L^*)}\left[\frac{T'(Y_L)}{1+r} + \frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\right]\frac{\gamma\pi_L}{\lambda u_c^2(c_L^1, c_L^2, \hat{x}^*)}\frac{u_c^2(c_L^1, c_L^2, x_L^*)}{u_c^1(c_L^1, c_L^2, x_L^*)}\frac{r\Phi'(rs_L)}{1+r} \end{aligned} \tag{4.C.6}$$

Using equations (4.5), (4.7), and (4.22), the last term of equation (4.C.6) reduces to $(1 - S'(q_L))\frac{r\Phi'(\cdot)}{1+r}$. Therefore, (4.C.6) can be rewritten as follows:

$$-S'(q_L)+\frac{Y_L\phi'(e_L^*)}{\phi(e_L^*)}\frac{T'(Y_L)}{1+r}-(1-S'(q_L))\frac{r\Phi'(\cdot)}{1+r} = \frac{\lambda}{\gamma\pi_L}\frac{v_\ell(\hat{\ell})}{a_H\phi(\hat{e}^*)}\left[\frac{Y_L\phi'(\hat{e}^*)}{\phi(\hat{e}^*)}-\frac{Y_L\phi'(e_L^*)}{\phi(e_L^*)}\right] \tag{4.C.7}$$

Using equation (4.10) and the definition of Δ, the left-hand side equals $\frac{\Delta}{1+r}$. Finally, applying equation (4.17) to the right-hand side, we obtain equation (4.25). □

Appendix 4.D

In this Appendix, we now show that both the Atkinson and Stiglitz (1976) theorem and the Diamond and Mirrlees (1971) production efficiency theorem are valid when the government can observe the share of educational investment with consumption value, h_i. In other words, the government can control allocations of consumption value x_i and production value e_i for type i. In the setting, education subsidies are redefined as $S(x_i, e_i)$ depending on each component separately.

When h_i is observable, the individual optimization problem is formulated by

$$\begin{aligned} \max_{c_i^1, c_i^2, s_i, x_i, e_i, Y_i} \quad & u(c_i^1, c_i^2, x_i) - v(\frac{Y_i}{a_i\phi(e_i)}) \\ s.t. \quad & c_i^1 + s_i + x_i + e_i - S(x_i, e_i) = s_0 \\ & c_i^2 = Y_i - T(Y_i) + (1+r)s_i - \Phi(rs_i) \end{aligned} \tag{4.D.1}$$

This problem yields the first-order conditions:

$$MRS^i_{c^1x_i} \equiv \frac{u_x(c_i^1, c_i^2, x_i)}{u_c^1(c_i^1, c_i^2, x_i)} = 1 - \frac{\partial S(x_i, e_i)}{\partial x_i} \equiv 1 - S_{x_i}(x_i, e_i) \tag{4.D.2}$$

$$MRS^i_{c^1e_i} \equiv \frac{v_\ell(\ell_i)}{u_c^1(c_i^1, c_i^2, x_i)} \frac{\ell_i \phi'(e_i)}{\phi(e_i)} = 1 - \frac{\partial S(x_i, e_i)}{\partial e_i} \equiv 1 - S_{e_i}(x_i, e_i) \tag{4.D.3}$$

$$MRS^i_{c^2\ell} \equiv \frac{v_\ell(\ell_i)}{a_i \phi(e_i^*) u_c^2(c_i^1, c_i^2, x_i^*)} = 1 - T'(Y_i) \tag{4.D.4}$$

$$MRS^i_{c^1c^2} \equiv \frac{u_c^1(c_i^1, c_i^2, x_i)}{u_c^2(c_i^1, c_i^2, x_i)} = 1 + r - r\Phi'(rs_i) \tag{4.D.5}$$

Combining (4.D.3), (4.D.4), and (4.D.5) yields

$$MRT^i_{e_i\ell} \equiv \frac{Y_i \phi'(e_i)}{\phi(e_i)} = \frac{1 - S_{e_i}(x_i, e_i)}{1 - T'(Y_i)}(1 + r - r\Phi'(rs_i)) \tag{4.D.6}$$

From the same manner of equation (4.11), the total net tax wedge on learning for type i is

$$\Delta_i = \frac{T'(\cdot)}{1 - T'(\cdot)} R(1 - S_{e_i}(\cdot)) - r\Phi'(\cdot) - S_{e_i}(\cdot)R \tag{4.D.7}$$

We now turn to the analysis of the government optimization problem. The objective of the government, the public budget constraint, and the incentive constraint are given by

$$W = \sum_i \pi^i \{u(c_i^1, c_i^2, x_i) - v(\frac{Y_i}{a_i \phi(e_i)})\} \tag{4.D.8}$$

$$\sum_{i=H,L} \pi_i \left[s_0 - c_i^1 - x_i - e_i + \frac{1}{1+r}(Y_i - c_i^2) \right] = 0 \tag{4.D.9}$$

$$u(c_H^1, c_H^2, x_H) - v(\frac{Y_H}{a_H \phi(e_H)}) \geq u(c_L^1, c_L^2, x_L) - v(\frac{Y_L}{a_H \phi(e_L)}) \tag{4.D.10}$$

Therefore, the government maximizes the social welfare function (4.D.8) subject to the government's budget constraint (4.D.9) and the incentive constraint (4.D.10) by selecting the allocations with respect to c_i^1, c_i^2, x_i, e_i, and Y_i for type i. The corresponding Lagrangian is

$$\begin{aligned} \max_{\{c_i^1, c_i^2, x_i, e_i, Y_i\}_i} \mathcal{L} = & \sum_i \pi^i \{u(c_i^1, c_i^2, x_i) - v(\frac{Y_i}{a_i \phi(e_i)})\} \\ & + \gamma \Big[\sum_{i=H,L} \pi_i \{s_0 - c_i^1 - x_i - e_i + \frac{1}{1+r}(Y_i - c_i^2)\} \Big] \\ & + \lambda \Big[u(c_H^1, c_H^2, x_H) - v(\frac{Y_H}{a_H \phi(e_H)}) - u(c_L^1, c_L^2, x_L) + v(\frac{Y_L}{a_H \phi(e_L)}) \Big] \end{aligned} \tag{4.D.11}$$

Let γ be the Lagrange multiplier of the government's budget constraint and λ the Lagrange multiplier of the incentive constraint. From the first-order conditions, we characterize optimal conditions for each type with respect to the marginal tax rate on educational investment with consumption value, the marginal subsidy rate for educational investment with production value, and the marginal labor and capital income tax rate as follows.

$$S_{x_H}(x_H, e_H) = S_{x_L}(x_L, e_L) = 0 \tag{4.D.12}$$

$$\Phi'(rs_H) = \Phi'(rs_L) = 0 \tag{4.D.13}$$

$$T'(Y_H) = 0 \tag{4.D.14}$$

$$\frac{T'(Y_L)}{1+r} = \frac{\lambda u_c^2(c_L^1, c_L^2, x_L)}{\gamma \pi_L} \left[\frac{v_\ell(\ell_L)}{u_c^2(c_L^1, c_L^2, x_L)} \frac{1}{a_L \phi(e_L)} - \frac{v_\ell(\hat{\ell})}{u_c^2(c_L^1, c_L^2, x_L)} \frac{1}{a_H \phi(e_L)} \right] > 0 \tag{4.D.15}$$

$$S_{e_H}(x_H, e_H) = 0 \tag{4.D.16}$$

$$S_{e_L}(x_L, e_L) = T'(Y_L) \tag{4.D.17}$$

Equation (4.D.12) and (4.D.13) mean that the differential tax on educational investment with consumption value and capital income is superfluous and correspond to the canonical result of Atkinson and Stiglitz (1976), which states that nonlinear income taxes are only needed if the utility function is weakly separable between consumption and labor supply. Equation (4.D14) and (4.D15) are consistent with the result of Stiglitz (1982) analyzing optimal income taxation in the economy consisting of two types of individuals. The marginal income tax rate is zero at the top and positive at the bottom. Equation (4.D16) and (4.D17) suggest that the marginal subsidy rate for educational investment with production value should be equal to the marginal labor income tax rate. This result implies that the government ought to improve the distortion of individual's behavior with respect to human capital investment due to labor income taxation by subsidizing educational investment. From the above results, we have $\Delta_H = \Delta_L = 0$, which means that the Diamond-Mirrlees production efficiency theorem holds. This is consistent with Bovenberg and Jacobs (2005).

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